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Part I

Pre-School

Chapter 1

Numbers

This is content that is only quick concepts, theorems, ideas, definitions.

1.1 Natural numbers

Definition 1.1 (Natural Numbers). *The set*[a] *of numbers*

$$\{1, 2, 3, \ldots\}$$

is called a natural numbers denoted as $\mathbb{N}$

[a] A set is a well defined collection of objects

If we add the number 0 to the set of natural numbers we get the set of *Whole Numbers*

$$\{0, 1, 2, 3, \ldots\}$$

1.2 Primes and Composite

In the set of naturals we first try and understand what are prime and composite numbers.

1.2.1 Squares and cubes of numbers

Definition 1.2 (a^2). *a^2 read as "a square" means $a \times a$*

Definition 1.3 (a^3). *a^3 read as "a cube" means $a \times a \times a$ i.e. a multiplied to itself 3 times.*

Definition 1.4. *So a^n is read as "a raised to n " and is*

$$\underbrace{a \times a \times a \times \cdots \times a}_{n}$$

Problem 1.2.1. *what is a^{10} ?*

Is a multiplied 10 times.

Problem 1.2.2. *What is 3^4 ? And how it is read?*

$3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$ And it is read as "3 raised to 4" s

The divisors of 26 are $1, 2, 13, 26$ & divisors of 23 are $1, 23$ only.

1.2.2 Square roots, cube roots and general

Problem 1.2.3. *Solve the following problems.*

1. *What is square root of 4 : $2 \times 2 = 4$ hence $\sqrt{4} = 2$*
2. *What is square root of 9 : $3 \times 3 = 9$ hence $\sqrt{9} = 3$*

1.2.3 List of square of naturals

- $1^2 = 1$
- $2^2 = 4$
- $3^2 = 9$
- $4^2 = 16$
- $5^2 = 25$
- $6^2 = 36$
- $7^2 = 49$
- $8^2 = 64$
- $9^2 = 81$
- $10^2 = 100$
- $11^2 = 121$
- $12^2 = 144$
- $13^2 = 169$
- $14^2 = 196$
- $15^2 = 225$
- $16^2 = 256$
- $17^2 = 289$
- $18^2 = 324$
- $19^2 = 361$
- $20^2 = 400$

Definition 1.5 (Primes). *A positive integers that is divisible by itself and 1 is called a prime number.*

Like, $2, 3, 5, 7, 11, 13, 17, 19, 23$, $29, 31, 37, 41, 43, \ldots$ are some of the primes which are divisible by itself and 1

Definition 1.6 (Composite). *A positive integer that has a divisor that is different from itself and 1.*

Like, $4 = 2 \times 2 = 4 \times 1$

Remember : (How to check prime). *Our objective is to check n is a prime or composite ?*

- *We start with primes $2, 3, 5, \ldots$ and see if they divide n or not !*
- *We check all primes till the prime squares are less than n.*

Problem 1.2.4. *Check if 101 is a prime or not ?*

Solution.
- 2 does not divide 101 and $2^2 = 4 < 101$ so we continue to the next prime.
- 3 does not divide[1] 101 and also $3^2 = 9 < 101$
- 5 does not divide 101 (since last digit of 101 is not 0 or 5) and also $5^2 = 25 < 101$
- 7 does not divide 101 and $7^2 = 49 < 101$ and hence we go to the next prime number
- $11^2 > 101$ and hence stop the process.

□

List of primes

$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 57, \ldots$

Challenging Problem 1.2.1. *In Sieve of Erasthothenes, we remove all multiples of primes to get the leftover only primes. To find all primes in first 100 naturals till what primes multiples should we cancel ? How about first 1000 naturals till what primes multiples should we remove?*

Solution. for the first part we will go till prime number 7. And second part prime number 31. □

Problem 1.2.5. *What are the primes in first 100 naturals? Use the above problem discussion.*

Solution. [Soham's solution] Soham observed that primes $5, 7$ are of the form $6k - 1$ and $6k + 1$ and that pattern follows further. Hence he saw that it becomes the shortcut for finding the primes. And this is true. □

1.3 Integers

Definition 1.7 (Integers). *Set of numbers*

$$\{0, +1, +2, +3, \ldots, -1, -2, -3, \ldots\}$$

are called integers and are represented by letter $\mathbb{Z}$.

Moreover we should also known the set of

- Negative integers $= \{-1, -2, -3, \ldots\}$
- Positive integers $= (1, 2, 3, \ldots) = \mathbb{N}$
- Non-negative integers $= \{0, 1, 2, 3, \ldots\}$
- Non-positive integers $= \{0, -1, -2, -3, \ldots\}$

1.3.1 Operations on integers

Addition

Theorem 1.1 (product of sign).
- *product of same sign is $+$ i.e. $(+)(+) = +, (-)(-) = +$*
- *product of opp signs is $-$ i.e. $(+)(-) = (-)(+) = -$*

- $2 + 3 = 5$
- $-2 + 3 = 1$
- $2 - 3 = -1$
- $-2 - 3 = -5$

Multiplication of Integers

While multiplying two integers we need to multiply the numbers and then also multiply their

Remember : . *Division of integers are the rational numbers and hence we study that in the next section of rational numbers.*

1.4 Rational Numbers

Definition 1.8. *Any number[a] that can be written as ratio of two integers, i.e.*

$$\left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$$

[a]Division by zero is not defined

There is another way to represent set of rationals, that is in decimal form.
$\frac{1}{2} = 0.5$ is written in both forms.

Remember : . *All rationals numbers can be either ratio or decimal form. But in decimal form all ratios may not be terminating, like $\frac{1}{3} = 0.33333333333\ldots$ So there is a short way to represent repetition in such numbers and hence*

[1]A number is divisible by 3 iff 3 divides sum of its digits

$1/3 = 0.333333\ldots = 0.\overline{3}$

Problem 1.4.1. *Write the compact form of the following decimal numbers.*

- $0.777777\ldots$
- $0.121212\ldots$
- $0.123123123\ldots$
- $0.123417373737\ldots$

Solution.
- $0.777777\ldots = 0.\overline{7}$
- $0.121212\ldots = 0.\overline{12}$
- $0.123123123\ldots = 0.\overline{123}$
- $0.123417373737\ldots = 0.1234\overline{173}$

□

Remember : . • $-\dfrac{a}{b} = \dfrac{-a}{b} = \dfrac{a}{-b}$

For example, $\dfrac{-2}{3} = -\dfrac{2}{3} = -0.66666\cdots = -0.\overline{6}$

1.4.1 Conversion of ratio form to decimal form of rationals

Problem 1.4.2. *Find the decimal form of the rationals* $1/2, 3/4, 6/7, 13/6$

Solution. Following are the representations

1. $\frac{1}{2} = 0.5 = 0.500000\ldots = 0.5\overline{0}$
2. $\frac{3}{4} = 0.75$
3. $\frac{6}{7} = 0.857142857142857142\cdots = 0.\overline{857142}$

□

Theorem 1.2. *Rational numbers in decimal form always have recurring parts. For example, in* $0.12343434\ldots$ *we have* 34 *is the recurring part.*

Remember : . *Rational Number* $\iff$ *in decimal form has recurring terms. So if a decimal number has no recurring terms then its a Irrational number*[a]

[a] Since this behaviour is so weird they were called as irrational numbers, means without logic

Problem 1.4.3. *Convert the following rationals from decimal form to ratio form for*

1. $0.2222222\ldots$
2. $0.232323\ldots$
3. $0.1234512345\ldots$
4. $12.\overline{3}$

Solution. 1. Let $x = 0.2222222\ldots \implies 10x = 2.2222\ldots$

□

1.4.2 Revision

1. Evaluate the value of
   a) $2 + 3 - 6 + 4 \times 12 \div 3$
   b) $12 \div 3 \div 2 \times 6 + 1$
2. Identify the rational and irrational of the list
   a) $2.343434\ldots$
   b) 23.61
   c) $4/3$
   d) $1.23456794885054394505\ldots$
   e) 37
3. Which of the following numbers are prime, and which are composite
   a) 37
   b) 437
   c) 91
   d) 11111
   e) $37 \div 37$
4. State which of the following definitions are true or false
   a) If a number is ratio of two integers then its called a prime
   b) If a number is ratio of two integers then its called a Irrational.
   c) Number 123.45678 is a rational number since its pattern is recurring
   d) A natural number is composite if its divisible by any number other than itself and 1.
5. Do the following calculations
   a) $\frac{2}{3} + \frac{7}{6}$
   b) $\frac{3}{7} + \frac{2}{14}$
   c) $\frac{2}{3} + \frac{3}{2}$
   d) $\frac{11}{5} + \frac{2}{7}$
   e) $\frac{2}{3} - \frac{1}{7}$
   f) $\frac{2}{3} + \frac{6}{11}$

6. Evaluate the following
   a) $2 + (-3) - 6 + 4 =$
   b) $2 \times (-3) =$
   c) $6 \times (-3) =$
   d) $(-3) \times (-4) =$
   e) $12 \times (-6) =$
   f) $(-2) \times (-3) \times (-4) =$

1.5 Rationals as Fractions

We already know that rationals are ratio of integers. That gives rise to study of fractions.

Equivalent fractions

Fractions when multiplied by the same number in numerator and denominator are called Equivalent fractions, they have the same value.

Problem 1.5.1. *Can we say $\frac{4}{6}$ and $\frac{2}{3}$ are Equivalent fractions*

1.5.1 Proper Fractions

Definition 1.9 (Proper fraction)**.** *Fractions in which numerator value is less than the denominator are called proper fractions. eg.* $2/7, 3/11$ *are proper fractions*

Remember : . *In proper fractions since the numerator is less than denominator we get that* proper fractions are less than 1

1.5.2 Improper fractions

Definition 1.10 (Improper fractions)**.** *Fractions in which numerator is greater than or equal to the denominator are called* improper fractions *e.g.* $2/2, 10/3, 9/7$ *are improper fractions.*

Remember : . *Improper fractions* $\Longrightarrow$ *Mixed fractions.*

1.5.3 Mixed Fractions

Definition 1.11 (Mixed Fractions)**.** *When improper fractions are converted to proper fractions we get mixed fractions, where we have integral part as well as fractions*

For example, $\frac{11}{7}$ is a improper fraction. We can write $\dfrac{11}{7} = 1 + \dfrac{4}{7}$. As convenient way to write mixed fraction $\frac{11}{7}$ is written as $1\frac{4}{7}$

Problem 1.5.2. *Convert the following fractions into mixed fractions. Also state them in convenient form.*

1. 11/7
2. 23/11
3. 111/3
4. 33/44

Solution. 1. $\frac{11}{7} = 1 + \frac{4}{7} = 1\frac{4}{7}$

2. $\frac{23}{11} = 2 + \frac{1}{11} = 2\frac{1}{11}$

3. $\frac{111}{3} = 37 + \frac{0}{3} = 37$

4. $\frac{33}{44} = 3/4$ which is in proper form and hence there is no mixed form. Or this can be written as $0\frac{3}{4}$

□

Remember : . *To convert mixed fraction* $a\frac{b}{c}$ *we get* $\frac{a \times c + b}{c}$ *since* $a\frac{b}{c} = a + \frac{b}{c}$

Problem 1.5.3. *1.* $2\frac{3}{4} =$

2. $3\frac{4}{5} =$

1.5.4 Equivalent fractions

Reducing the fractions to lowest forms is of huge help in solving operations with fractions.

Remember : . *Steps :*

- *First factorise numerator and denominator*
- *Cancel out the common factors from Nr. and Dr.*

$$\begin{aligned}\frac{210}{385} &= \frac{2 \times 3 \times 5 \times 7}{5 \times 7 \times 11} \\ &= \frac{2 \times 3}{11} \\ &= \frac{6}{11}\end{aligned}$$

Problem 1.5.4. *Reduce the following fractions to lowest forms*

1. $\frac{44}{88} =$

2. $\frac{44}{36} =$

3. $\frac{42}{77} =$

4. $\frac{128}{3072} =$

5. $\frac{210}{1001} =$

1.5.5 Operations on Fractions

Fractions like numbers can be added, subtracted, multiplied and divided. So we will study these operations on fractions.

Addition

We will learning this by way of solving problems.

Theorem 1.3. *Two fractions can be added only after making the denominators same.*

TYPE 1 : Denominators are multiples of each other or a specific number

$$\begin{aligned}\frac{2}{3} + \frac{7}{6} &= \frac{2 \times 2}{2 \times 3} + \frac{7}{6} \\ &= \frac{4}{6} + \frac{7}{6} \\ &= \frac{11}{6}\end{aligned}$$

the above two fractions $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions

Problem 1.5.5. *Find the sum of the following fractions*

1. $\frac{2}{11} + \frac{3}{44}$

2. $\frac{2}{35} + \frac{11}{63}$

3. $\frac{5}{42} + \frac{11}{48}$

Solution. Following are the solutions

1. $\frac{2}{11} + \frac{3}{44} = \frac{2 \times 4}{11 \times 4} + \frac{3}{44} = \frac{8}{44} + \frac{3}{44} = \frac{11}{44} = \frac{1}{4}$ (finally the solution was brought into lowest form)

2. $\frac{2}{35} + \frac{11}{63} = \frac{2}{5 \times 7} + \frac{11}{9 \times 7} = \frac{2 \times 9}{5 \times 7 \times 9} + \frac{11 \times 5}{5 \times 7 \times 9} = \frac{18}{315} + \frac{55}{315} = \frac{18+55}{315} = \frac{73}{315}$

   observe in the denominators out of $5, 7, 9$ numbers one is missing in the other.

3. $\frac{5}{42} + \frac{11}{48} = \frac{5}{6 \times 7} + \frac{11}{6 \times 8} = \frac{5 \times 8}{8 \times 6 \times 7} + \frac{11 \times 7}{6 \times 8 \times 7} = \frac{40}{336} + \frac{77}{336} = \frac{117}{336}$

□

Problem 1.5.6. *1.* $\frac{3}{8} + \frac{11}{64}$

2. $\frac{6}{7} + \frac{7}{49}$

3. $\frac{3}{20} + \frac{4}{30}$

4. $\frac{1}{6} + \frac{5}{12} + \frac{3}{20}$

5. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

Solution. 1. $\frac{3}{8} + \frac{11}{64} = \frac{3 \times 8}{8 \times 8} + \frac{11}{64} = \frac{24+11}{64} = \frac{35}{64}$

2. $\frac{6}{7}+\frac{1}{7}=\frac{7}{7}=1$
3. $\frac{3}{20}+\frac{2}{15}+\frac{3\times3}{60}+\frac{2\times4}{60}=\frac{9+8}{60}=\frac{17}{60}$
4. $\frac{10+25+9}{60}=\frac{44}{60}$
5. $\frac{4+2+1}{8}=\frac{7}{8}$

□

Denominators are not multiples

So like $\dfrac{1}{7}+\dfrac{2}{11}$ kind of addition of fractions. There are no multiples to help us to set the denominators same.

So in such scenarios we go to *Cross multiplication method*

$$\frac{1}{7}+\frac{2}{11}=\frac{1\times 11+2\times 7}{7\times 11}=\frac{11+14}{77}=\frac{25}{77}$$

Remember : . *In this method, the fractions can be proper or improper. You can just go with the method to solve the problems.*

Problem 1.5.7. *1.* $\frac{2}{3}+\frac{5}{8}$

2. $\frac{1}{13}+\frac{3}{7}$

3. $\frac{3}{44}+\frac{7}{88}$

Solution. 1. $\frac{2\times8+5\times3}{3\times8}=\frac{16+15}{24}=\frac{31}{24}$

2. $\frac{1\times7+3\times13}{13\times7}=\frac{7+39}{91}=\frac{46}{91}$

3. $\frac{3\times2}{44\times2}+\frac{7}{88}=\frac{6+7}{88}=\frac{13}{88}$ so for this problem since we see the multiples in the denominator we go by the previous method and not use the cross multiplication rule.

□

Subtraction of Fractions

Just like addition of fraction, subtraction too works in a similar way. Use the same ideas discussed above.

We have to recall the rules

Remember : . [a]

- $(+)\times(+)=+$
- $(+)\times(-)=-$
- $(-)\times(-)=+$

[a] Same sign product is + and opposite sign product is −

Problem 1.5.8. *Solve the following*

1. $\frac{2}{3}-\frac{3}{11}$

2. $\frac{3}{44}-\frac{7}{28}$

3. $\frac{11}{58}-\frac{3}{87}$

4. $\frac{2}{7}-\frac{3}{11}$

5. $\frac{1}{9}-\frac{2}{3}$

6. $\frac{1}{3}-\frac{1}{2}$

7. $\frac{1}{2}-\frac{2}{3}-\frac{3}{4}$ *(you can see that the LCM of the denominator is 12)*

Solution. 1. $\frac{2\times11-3\times3}{3\times11}=\frac{22-9}{33}=\frac{13}{33}$ Its the cross multiplication rule used in case of addition.

2. $\frac{3}{4\times11}-\frac{\cancel{7}}{4\times\cancel{7}}=\frac{3}{4\times11}-\frac{1}{4}=\frac{3}{4\times11}-\frac{11}{4\times11}=\frac{3-11}{44}=\frac{-8}{44}$

3. $\frac{11}{2\times29}-\frac{3}{3\times29}=\frac{11}{2\times29}-\frac{1}{29}=\frac{11}{2\times29}-\frac{2}{2\times29}=\frac{11-2}{2\times29}=\frac{9}{58}$

4. LCM of $2,3,4$ is 12, So we get $\frac{6}{12}-\frac{8}{12}-\frac{9}{12}=\frac{6-8-9}{12}=\frac{-11}{12}$

□

Multiplication

We will learn multiplication of fractions using examples.

Remember : . $\frac{A}{B}\times\frac{C}{D}=\frac{AC}{BD}$

Problem 1.5.9. *1.* $\frac{1}{2}\times\frac{3}{4}=$

2. $\frac{2}{3}\times\frac{3}{2}=$

3. $\frac{1}{2}\times\frac{3}{4}\times\frac{5}{6}=$

4. $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{100}{101}=$ *(think using cross cancellation)*

Division of fractions

Remember : . *Reciprocal of a fraction* $\dfrac{a}{b}$ *is* $\dfrac{b}{a}$

Theorem 1.4. *In division,*

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Problem 1.5.10. *1.* $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

2. $\frac{\left(\frac{3}{4}\right)}{\left(\frac{7}{8}\right)} = \frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$

3. $\frac{\left(\frac{7}{8}\right)}{14} = \frac{7}{8} \div 14 = \frac{7}{8} \times \frac{1}{14} = \frac{1}{16}$

Problem 1.5.11. *1.* $\frac{2}{3} \times \frac{3}{16} = \frac{2\times 3}{3\times 16} = \frac{1}{8}$

2. $\frac{9}{11} \div \frac{81}{88} = \frac{9}{11} \times \frac{88}{81} = \frac{9\times 88}{11\times 81} = \frac{8}{9}$

3. $\frac{63}{8} \div \frac{7}{64} = \frac{63}{8} \times \frac{64}{7} = \frac{9\times 8}{1} = 72$

4. $\frac{2}{3} \times \frac{3}{4} \div \frac{4}{5} = \frac{2}{3} \times \frac{3}{4} \times \frac{5}{4} = \frac{5}{8}$

5. $\frac{4}{5} \div \frac{5}{6} \times \frac{6}{7} = \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} = \frac{144}{175}$

1.6 Decimals (Rational)

Definition 1.12 (Decimal)**.** A decimal can be rational or an irrational decimal. *e.g like* 1.232323 . . . *is a decimal which is rational since the pattern is recurring or like* 1.23435343536463434 . . . *is a decimal number which is non-recurring and hence is a irrational number.*

In our study of this section on decimals we will limit ourself to only rational decimals.

Representing decimals as fractions helps in understanding decimals much better.

1.6.1 Decimals as fractions

We can write every decimal as a fraction

Remember : . *While converting decimal into fraction, we will divide the number without decimal with that many powers of 10.*

Problem 1.6.1. *Write the fraction form of the following decimals*

1. 1.22

2. 2.345

3. 10.09

Solution. 1. $1.22 = \frac{122}{100}$

2. $2.345 = \frac{2345}{1000}$

3. $10.09 = \frac{1009}{100}$

□

Remember : (Equivalent Decimals)**.** *Decimals that have same value are called Equivalent decimals. Like* $2.1 = 2.10 = 2.100 = \cdots$

Problem 1.6.2. *Write the following in decimal form.*

1. $\frac{34}{100}$

2. $\frac{123}{25}$

3. $\frac{111}{125}$

4. $\frac{7}{20}$

5. $\frac{23}{1000}$

1.6.2 Conversion of Decimals to fractions

Obviously the conversion is permissible to only decimals that are rationals.

Method I (Write denominator as power of 10)

Remember : (convert to power of 10 in deno.). *Steps are ..*

- *convert the denominator to power of* 10
- *And cancel the multiples of* $2, 5$ *to get fraction in lowest form.*

Method II (Using basic algebra)

Remember : . *This method has to be learnt only after linear equations are studied by a student.*

Remember : . *This method we will exhibit using an example. so the problem is write fraction form of* $12.23232323\ldots$, *See the following steps*

- $x = 1.232323\ldots$ *so the recurring pattern is of two places hence we multiply by* 10^2 *(observe power 2 since there are two digit recurring pattern)*

1.6.3 Operations on Decimals

Addition & Subtraction

Remember : . *Align the decimal point while adding/substracting the Decimals*

Problem 1.6.3.
- $12 + 1.34 = 12.00 + 1.34 = 13.34$ *best done when written vertically*
- $1.234 + 0.03 = 1.234 + 0.030 = 1.264$
- $34.6 + 9.999 = 34.600 + 9.999 = 44.599$
- $1.234 - 88.34 = -87.106$ *(solve as you do in integers like larger from smaller and put the sign of the larger number)*
- $-1.2 - 2.43 = -1.63$
- $0.033 - 1.234 = -1.201$
- $-2.222 + 3.24311 = 1.02111$

Multiplication

Remember : . *While multiplying two decimals,*

- *Add the number of decimal places in both the numbers,*
- *Multiply the numbers without decimals, and*
- *in the final product, put the decimal sign after the (step 1) total sum of decimals calculated.*

Lets see the above steps with an example

$$1.23 \times 2.3$$

- 1.23 *has two decimal places, and* 2.3 *has one decimal place, so total we get 3 decimals places to be placed in the final product.*
- *Multiply* 123×23 *to get* 2829
- *Now we place the 3 decimal sum in the final outcome* 2829 *to get the final answer as* $\boxed{2.829}$

Problem 1.6.4. 1. $11.1 \times 12.23 = 135.753$

2. $2.3 \times 0.03 = 0.069$

Observe the above rule, comes from this simple illustration.

$$\begin{aligned} 11.1 \times 12.23 &= \frac{111}{10} \times \frac{1223}{100} \\ &= \frac{111 \times 1223}{1000} \\ &= \frac{135753}{1000} \\ &= 135.753 \end{aligned}$$

Division of Decimals

Problem 1.6.5. 1. $\frac{1.24}{0.04} = \frac{124}{4}$ *(on multiplying by 100)* $= 31$

2. $\frac{1.24}{0.4} = 1.24 \div 0.4 = \frac{124}{100} \div \frac{4}{10} = \frac{124}{100} \times \frac{10}{4} = \frac{31}{10} = 3.1$

Chapter 2

Algebraic expression & equations

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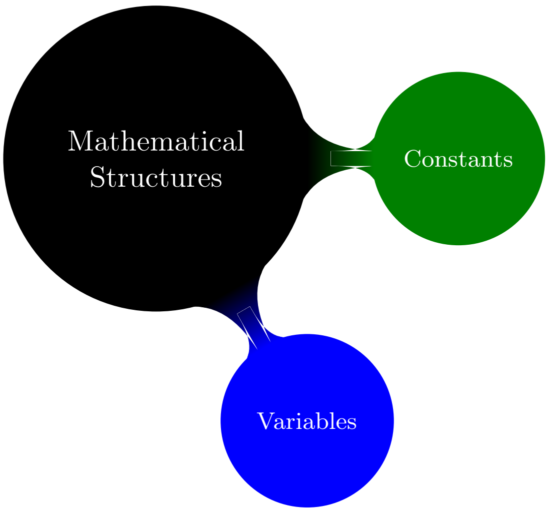
2.1 Constants and Variables

Matheamtics is made up of two aspects,

- Constants
- Variables



2.1.1 Constant

Numbers like naturals, wholes, integers, rationals, irrationals all have a fixed value. Like 7 will remain to be 7 only, its value will not change.

2.1.2 Variable

Mathematically quantity whose value is unknown (for the time-being).

Usually the last few alphabets – x, y, z, w are used to represent variables. So variables word comes from the word *vary* means 'that which changes'.

Next we study what we can create using *constants* & *variables.*

Term

Definition 2.1 (Term)**.** *A term is a quantity made by multiplication of variables and constants.*

For example, $2 \times x, x \times x, 2 \times x \times x \times y$

Remember : . *Multiplication of variable and constant is for the sake of convenience shown without the multiplication sign* $\times$*.*

- *So* $3 \times x$ *is written as* $3x$
- $x \times x$ *is shown as* x^2
- $x \times y$ *is shown as* xy
- $x \times x \times y$ *is shown as* x^2y *(and is read as 'x square y')*
- $10 \times x \times x \times x \times y$ *as* $10x^3y$ *(read as '10 x cube y') or (read as* 10 *into* x *raised to* 3 y *)*

2.1.3 Expression

Definition 2.2. *A variable containing mathematical structure is called an* expression*.*

Remember : . *Expression should have atleast one term of variable.*

For example, x, $x + x$, $x + 3$ are expressions.

Problem 2.1.1. *1. Is 34 an expression ? – is not an expression since its made up of only constants.*

2. How many terms are there in $4x^3 + 3x + 2$ – here there are three terms, namely $4x^3$ is the first term, $3x$ is the second term and 2 is the third term.

3. How many terms are there in $3x + 4$ – there are two terms in this expression.

Remember : .
- *A* term *is created when we multiply a constant and a variable(s) together. like $10x, xy, 100x^2y$ are all terms. also called as* monomials
- *When we add/subtract more than one terms we get an expression. like $10x + 3y, 5x + 3, 3x + 2y - 5$*

Problem 2.1.2. *Find the terms, number of terms in the following expressions*
- $3x + 2y - 1$
- $4x^2 - 2$

Solution.
- The terms are $3x, 2y, -1$ and there are 3 terms.
- The terms are $4x^2, -2$ and there are 2 terms.

□

2.1.4 Like terms

If two terms have the same variable (with power) like $4x$, $7x$ then we call them like terms.

Problem 2.1.3. *Can we say*
- *$3x^2$ and $7x$ are like terms ? they are not like since they difer in power.*
- *$5x^3$ and $11x^3$ are like terms ? yes!*

2.1.5 Operations on expressions

Addition & Subtraction

Observe the following

- $x + 0 = x$
- $x + x = 2x$
- $x + 3x = 4x$
- $x - 3x = -23x$
- $x + x + x = 3x$
- $x + x + y + y + y = 2x + 3y$
- $(x + 2y) + 3y = x + 2y + 3y = x + 5y$
- $(x - 2y) + 3y = x\underline{-2y + 3y} = x + y$
- $(x - 2y) - 3y = x - 2y - 3y = x - 5y$

Problem 2.1.4. *Find the value of $2x - 5x$*

Solution. $2x - 5x = -3x$ □

Problem 2.1.5. *Find the value of $2x - 3x - 5x$*

Solution. $2x - 3x - 5x = -6x$ □

Problem 2.1.6. *Find the value of the following expressions*

1. $x + 3y - 2x - 6y$

2. $x + 3y - 2x + 6y$

3. $6x + z - y + 3y + x - 7z$

Solution. 1. $x + 3y - 2x - 6y = (x - 2x) + (3y - 6y) = -x - 3y$

2. $x + 3y - 2x + 6y = (x - 2x) + (3y + 6y) = -x + 9y$

3. $6x + z - y + 3y + x - 7z = (6x + x) + (-y + 3y) + (z - 7z) = 7x + 2y - 6z$

□

In addition we add the like terms and leave the unlike terms as it is.

Scalar multiplication

Multiplying an expression with a constant is called scalar multiplication.

Remember : . *If a shed has a donkey(x) and a monkey(y), mathematically we will represent total animals in the shed as $x + y$ (since we cannot add these two different animals). Imagine there is a spell which doubles the number of items on which its done, so if we run the spell on the shed, then how will we represent the outcome?*

$x + y$ animals will become double – so there will

be

$$\begin{aligned} 2(x+y) &= (x+y)+(x+y) \\ &= x+y+x+y \\ &= x+x+y+y \\ &= (x+x)+(y+y) \\ &= 2x+2y \end{aligned}$$

This is called distributive law

Observe

- $2 \times x = 2x$
- $2 \times (x+y) = 2(x+y) = 2x+2y$
- $7(x+3y-2) = 7x+21y-14$
- $4(x+3) = 4x+12$

Theorem 2.1 (Distributive law)**.**

$$a(x+y) = ax+ay$$

$$a(x-y) = ax-ay$$

Multiplication distributives about addition or subtraction

Remember : .

- $-(-a) = (-1)(-a) = +a$
- $-(a) = -a$
- $+(-a) = -a$

Problem 2.1.7.

- $-(-2) = (-1)(-2) = +2$

Problem 2.1.8. *Find the value of*

- $-(2x+y) = (-1)(2x+y) = -2x-y$

Problem 2.1.9.

1. $2(x+y) = 2x+2y$
2. $z(x+y) = zx+zy$
3. $3(2x+y) = 6x+3y$
4. $11(x-y) = 11x-11y$
5. $-2(x+y) = -2x-2y$ (remember $(-)(+) = -$)
6. $3(x+y-z) = 3x+3y-3z$
7. $4(x-2y-3z) = 4x-8y-12z$
8. $-(2x-3y) = -(2x)-(-3y) = -2x+3y$
9. $-3(-2x-3y) = (-3)(-2x)+(-3)(-3y) = +6x+9y$
10. $-(2x-3y) =$ (we are using $\alpha(\beta+\gamma) = \alpha\beta+\alpha\gamma$)
11. $-(x-11y) = -x-(-11y) = -x+11y$
12. $-4(-3x+y) = (-4)(-3x)+(-4)(y) = 12x-4y$
13. $2(2x+4y) = 4x+8y$
14. $-(-x-2y) = x+2y$

2.2 Revision

1. What is a constant ? Give an example.
2. What is a variable ? Give an example.
3. How can $3 \times x \times y$ be written in short notation?
4. What is an expression ? Can $3x+y$ be called an expression? If yes then how many terms are there in this expression $3x+y$?
5. How many terms are there in the following expressions
   a) $3x+2$
   b) x^2+x+y
6. What is scalar multiplication?
7. What is distributive law?
8. Can you prove distributive law using area of a rectangle ?
9. Simplify the following expressions
   a) $(x+y)+z$
   b) $-x(-y+x)$
   c) $2x(2+3x)$
   d) $33(2-3x)$
   e) $3(-x-y)$
   f) $-2(-2x+4y)$
   g) $-3(2x+3y)$
   h) $-4(4-3x)$
   i) $x(2+z)$
10. Simplify the following expressions
    a) $2x+4x+5x$
    b) $4x+3y+7x+y$
    c) $(23x + y) -$ $(2+3x-y)$
    d) $(x+y)+3(-2x+3y)$
    e) $-2x(-x+y) =$

2.2.1 Solutions

1. A constant is a mathematical term whose value does not change like integers, rationals, real numbers. example is 2
2. A mathematical quantity whose value changes, and is unknown initially. example $3x$.
3. In short notation we can write it as $3xy$

4. An expression is made up of one or more variable terms. So $3x + y$ is an expression with two terms.

5. $3x + 2$ has two terms, and $x^2 + x + y$ has three terms.

6. When any quantity is multiplied by a *constant* it is called scalar multiplication.

7. Distributive law is the rule by which we calculate how multiplication works over addition or subtraction. So $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$ where α, β, γ are mathematical quantities that can be constant or a variables.

8. -

9. a)

$$2x + 4x + 5x = 11x$$

b)

$$\begin{aligned} 4x + 3y + 7x + y &= 4x + 7x + 3y + y \\ &= 11x + 4y \end{aligned}$$

c)

$$\begin{aligned} (23x + y) - (2 + 3x - y) &= (23x + y) - 2 - 3x + y \\ &= 23x - 3x + y + y - 2 \\ &= 20x + 2y - 2 \end{aligned}$$

d)

$$\begin{aligned} (x + y) + 3(-2x + 3y) &= x + y - 6x + 9y \\ &= -5x + 10y \end{aligned}$$

Review Test

1. Find the following values

   a) $(-11) - 16$
   b) $2 - 111$
   c) $-31 + 87$
   d) $-11 - 23$
   e) $-11 - (-10)$
   f) $3 - (-17)$
   g) $4 + (-3)(2)$
   h) $23 - (6) - 88$
   i) $63 + 71 - 128$
   j) $6 - (-6)$
   k) $(-3)(-6) + (-11)7$

2. Are the following expressions or not?

   - $9x + 7$
   - x
   - $x^2 + 3x + 2$
   - 77

3. How many terms are there in

   - $x^2 + 3$
   - $x^3 + x^2 + x + 1$
   - 33

4. Simplify $3x + 2 + 8x + 17$

5. Using distributive law, simplify

   a) $2(x + 1)$
   b) $3(3x + 11)$
   c) $-3(3 - x)$
   d) $x(x + 3)$

6. Simplify $-(4x - 3)$

7. Simplify $(7x - 2) - (4x - 3)$

8. Simplify the following

   a) $(x + 3) + (3x + 1)$
   b) $-(4x + 3) - (3 - 6x)$
   c) $11x - (3 - 4x)$
   d) $x^2 + x + 1 - \left(3x^2 + 6x - 3\right)$

solutions

1. a) -27
   b) -109
   c) 56
   d) -34
   e) -1
   f) 20
   g) -2
   h) -71
   i) 6
   j) 12
   k) -59
2. - yes
   - yes
   - yes
   - no
3. - 2
   - 4
   - Is not an expression,undefined
4. $11x + 19$
5. a) $2x + 2$
   b) $9x + 33$
   c) $-9 + 3x$
   d) $x^2 + 3x$
6. $-4x + 3$
7. $3x + 1$
8. a) $4x + 4$
   b) $2x$
   c) $15x - 3$
   d) $-2x^2 - 5x + 4$

2.3 Multiplication of expression

Problem 2.3.1. *What is* $(x+2)(x+3)$

Solution.

$$\begin{aligned}(x+2)(x+3) &= x(x+3) + 2(x+3)\\ &= \left(x^2+3x\right) + (2x+6)\\ &= x^2+5x+6\end{aligned}$$

□

Theorem 2.2 (product of expression)**.**

$$(a+b)(x+y) = a(x+y) + b(x+y)$$

Problem 2.3.2. *Solve the following problems*

1. $(x+2)(x-3) =$
2. $\left(x^2+2\right)\left(x^2+1\right) =$
3. $(x+3)\left(x^2+2\right) =$

Problem 2.3.3. *Solve the following products.*

1. $(x+1)(2x+3) =$
2. $(x+1)\left(x^2+x+1\right) =$
3. $2(3x+4) =$
4. $(x-1)\left(x^2+x+1\right)$

Division of expressions makes sense when we study polynomials.

2.4 Opposite (Inverse) operations

Definition 2.3. *Every operation may have its opposite operation*

- *Opposite operation of addition* $+$ *is substraction* $-$ *and vice-versa.*
- *Opposite operation of multiplication* $\times$ *is division* $\div$ *and vice-versa.*

Problem 2.4.1.
- $a+2-2=a$ *hence we say that* $+2$ *and* -2 *are opposite operations of each other.*
- $a\times 10\div 10=a$ *so* $\times 10$ *and* $\div 10$ *are opposite operations as they take the situation back to the original situation.*

Lemma 2.1. *Opposite operations if applied in a sequence then they take back the system to the earlier state.*

2.5 Equations

Definition 2.4. *If two expressions are equated then we say we have an equation.*
For eg. $2x+3=x-3$, $4x-3=0$ *and* $10=3$

An equation as in the definition, we are equating two expressions. Now the next question is when are the two expressions equal? For finding that, we need to use the concept of opposite operations.

Lemma 2.2 (Rules of solving an equation)**.** *Mathematically, on*

- *Adding same number on both sides of an equation or*
- *subtracting same number on both sides of an equation or*
- *multiplying same number on both sides of an equation or*
- *dividing same number on both sides of an equation*

the equation is still valid.

Remember : .
- *A value **satisfies** an equation, means the equality holds. like in the equation* $x+3=10$ *putting* $x=7$ *satisfies the equation, while* $x=2$ *does not **satisfy** the equation.*
- *To solve a equation is to find the variable value , like in the equation* $2x+10=16$, *to solve is to find the value of* x *which satisfies the equation.*

Problem 2.5.1. *Solve* $x-3=4$

Solution. To solve the equation, we need to find the value of x that will satisfy the equation.

Here we need to find the value of x, so we carry the following steps.

$$\begin{aligned} x-3&=4\\ x-3{\color{red}+3}&=4{\color{red}+3}\\ \cancelto{0}{x-3{\color{red}+3}}&=4{\color{red}+3}\\ x&=4+3=7 \end{aligned}$$

□

Problem 2.5.2. *Solve* $4x = 64$

Solution. Since here we need to find x hence we need to get rid of 4 from the LHS. And since 4 is multiplied on the LHS, we divide by 4 on both the sides.

$$\begin{aligned} 4x&=64\\ \frac{\cancel{4}\times x}{\cancel{\color{red}4}}&=\frac{64}{\color{red}4}\\ 1\times x&=16\\ x&=16 \end{aligned}$$

□

Problem 2.5.3. *Solve* $\frac{x}{3} = 12$

Solution. Here 3 is dividing x. So to remove 3 from the LHS we need to multiply 3 on both sides.

$$\begin{aligned} \frac{x}{3}&=12\\ \frac{x}{3}\times 3&=12\times 3\\ \frac{x}{\cancel{3}}\times \cancel{3}&=12\times 3\\ x\times 1&=36\\ x&=36 \end{aligned}$$

□

Problem 2.5.4. *Solve the following equations*

1. $x + 3 = 11$
2. $3x = 15$
3. $\frac{x}{4} = 11$
4. $3 - x = 2$ *(tricky)*

Solution.
1. Since on the LHS we have $+3$ when its taken on the other sides of equality it substracts, becomes opposite -3 and hence we get $x = 11 - 3$ and hence we get $x = 8$
2. Since 3 on the LHS is multiplied when it goes on the other side of equality, it divides 15 to get $x = \frac{15}{3}$.
3. Since 4 is dividing on the LHS and hence on going other side of equality it multiplies and hence we get $x = 4 \times 11 = 44$
4.

□

Remember : . *When problems get complicated we take the following shortcut,*

- *We need to note that at the end of solving, we only need to have x in the LHS and all the constants (numbers) on the RHS.*
- *To achieve this, we need to see what all operations are done on x on the LHS.*
- *The last operation (or number) has to be taken on the RHS.*

Problem 2.5.5. *First state which is the last operation in the following problems, and then Solve the following*

1. $3x - 12 = 12$
2. $\frac{x}{4} - 3 = 5$
3. $\frac{x-1}{2} = 3$

Solution.
1. First x is multiplied by 3 and then 12 is subtracted, hence the last operation is -12
2. Here -3 is the last operation, hence we take that first on the RHS.
3. Here divide by 2 is the last operation and hence we take it on the RHS and multiply to the RHS.

□

Problem 2.5.6. *First state the last operations in the LHS, and then solve.*

1. $2x + 3 = 7$
2. $\frac{x-3}{4} = 5$
3. $\frac{11x+3}{2} = -4$
4. $4x - 3 = 17$
5. $\frac{x}{2} = \frac{7}{2}$
6. $11x + 3 = -8$

7. $(11x+3)+3=6$
8. $12x+3=9$
9. $(x+3)-4x=0$
10. $2(3x+4)-4=12$

Remember : . *You may need the following while solving problems,*

- $\dfrac{a}{a}=1$
- $\dfrac{0}{a}=0$
- $\dfrac{0}{0}=$ *Undefined.*
- $\dfrac{a}{0}=$ *undefined*

Appendix A

Documentation

1. In order to make the appendix point to the page added in footnotes instead of using `\hyperref` we should use autoref. Have shown its use in theorem **??**

2. To write todo notes we use the package todonotes. location of the documentation is http://tug.ctan.org/macros/latex/contrib/todonotes/todonotes.pdf

3. to add hints in questions - like highlighting of this kind we use the key macro hint

4. `\clearpage` takes from two column document page to next page

5. How to add the bibliography to the memoir document

   ```
   // before the preamble.
   \usepackage{natbib}
   // before index where we need the references
   \bibliographystyle{plain}
   \bibliography{bibliography} // this is the .bib file.
   ```

   Also note that we have to run in this order first latex in vscode, then in terminal 'bibtex main', then two more times run latex in vscode. – you should get the reference and citation. just add `\cite{}` to get the citation [1] like this

6. table of contents and sections depth is set by commands `\setcounter{tocdepth}{1}` means set till subsection. 3 means till subsubsection. and for section `\setcounter{secnumdepth}{2}`

7. the `\usepackage{etoc}` and `\localtableofcontents` to get minitoc on

8. settings that need to be changed if pdf has to be made dark. check the following setting in `settings.json`

   `//` this set of codes are for darkmode to be added into the vscode. settings .json.

   ```
   "latex-workshop.view.pdf.color.dark.pageColorsForeground": "#ffffff",
   "latex-workshop.view.pdf.color.dark.backgroundColor": "#ffffff",
   "latex-workshop.view.pdf.invertMode.enabled": "auto",
   "latex-workshop.view.pdf.invert": 1,
   ```