

On the Limits of Black-Box Techniques for Minimizing Cryptographic Assumptions

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Outline

Preliminaries

- ▶ Reductionism underlying modern cryptography

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- ▶ Relating cryptographic primitives

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Contributions

- ▶ Overview

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Contributions

- ▶ Overview
- ▶ (Non-)black-box construction of unbiased verifiable random functions (VRFs)

Some Cryptographic Primitives

Collision-Resistant
Hash Function

Verifiable
Random Function

Public-Key
Encryption

Key Agreement

Oblivious
Transfer

One-Way
Function

Pseudorandom
Generator

Pseudorandom
Function

Digital
Signatures

Commitment
Scheme

Secret-Key
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Security based on computational hardness

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Which assumption is best?

Shor [Sho94]

~~Hardness
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Hardness
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?

Reductionism

What is a minimal assumption for a given primitive?

Reductionism

What is a cryptographic reduction?

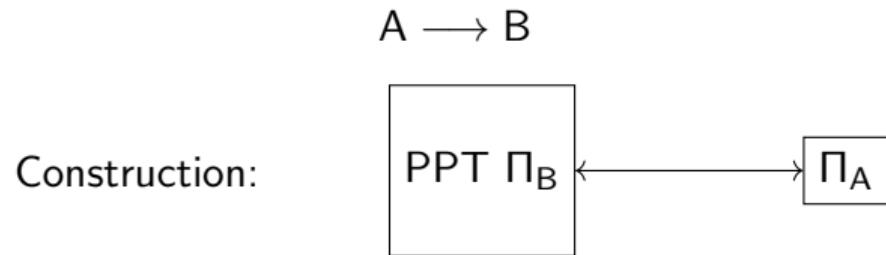
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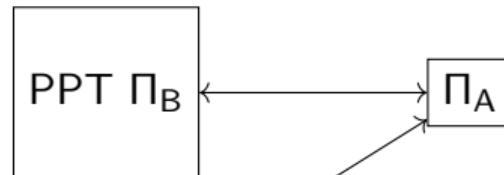


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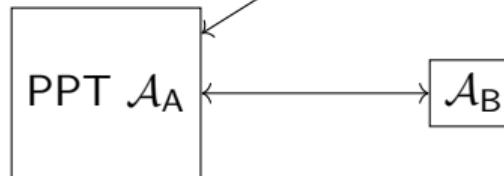
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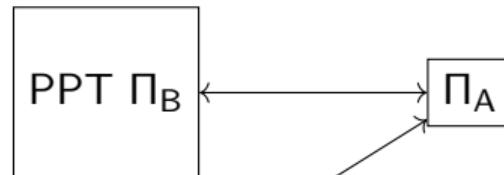


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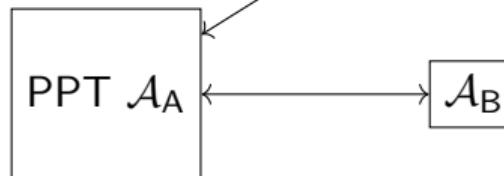
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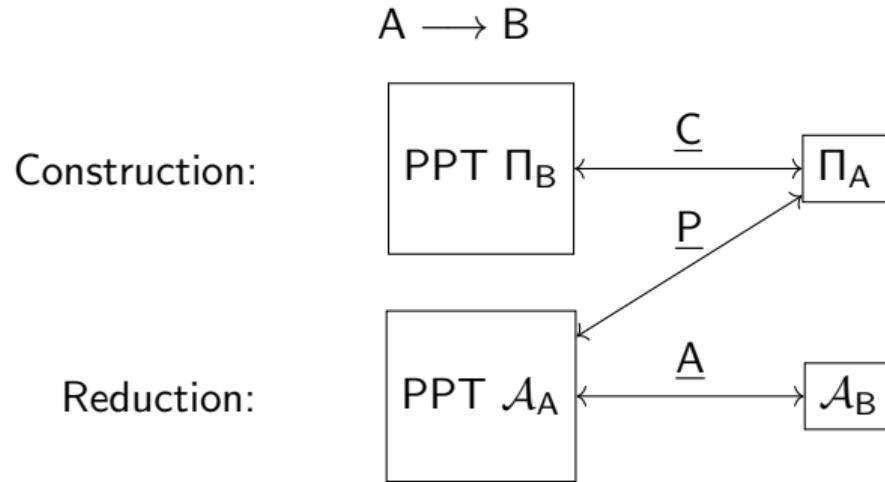


Reduction:



Reductionism

Cryptographic construction/reduction:



CAP taxonomy [BBF13]:

<u>Construction</u>	<u>Adversary</u>	<u>Primitive</u>
$\in \{B, N\}$	$\in \{B, N\}$	$\in \{B, N\}$

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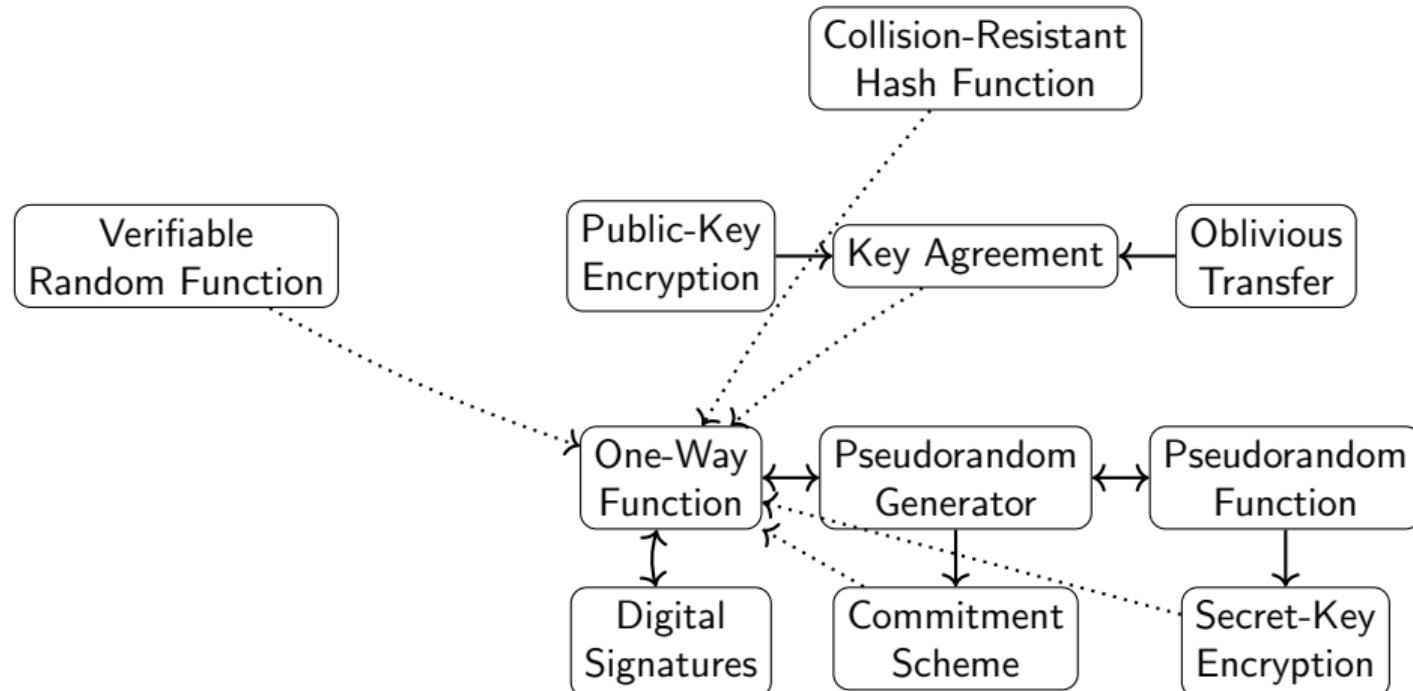
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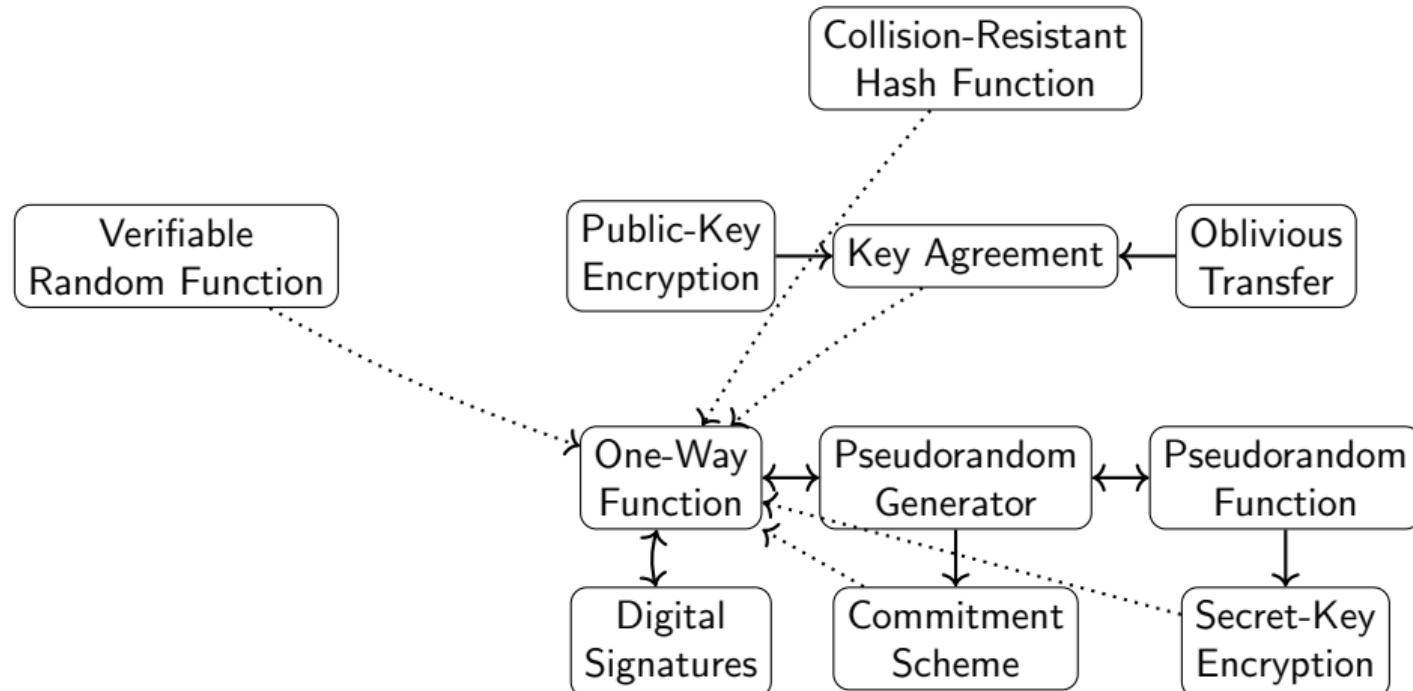
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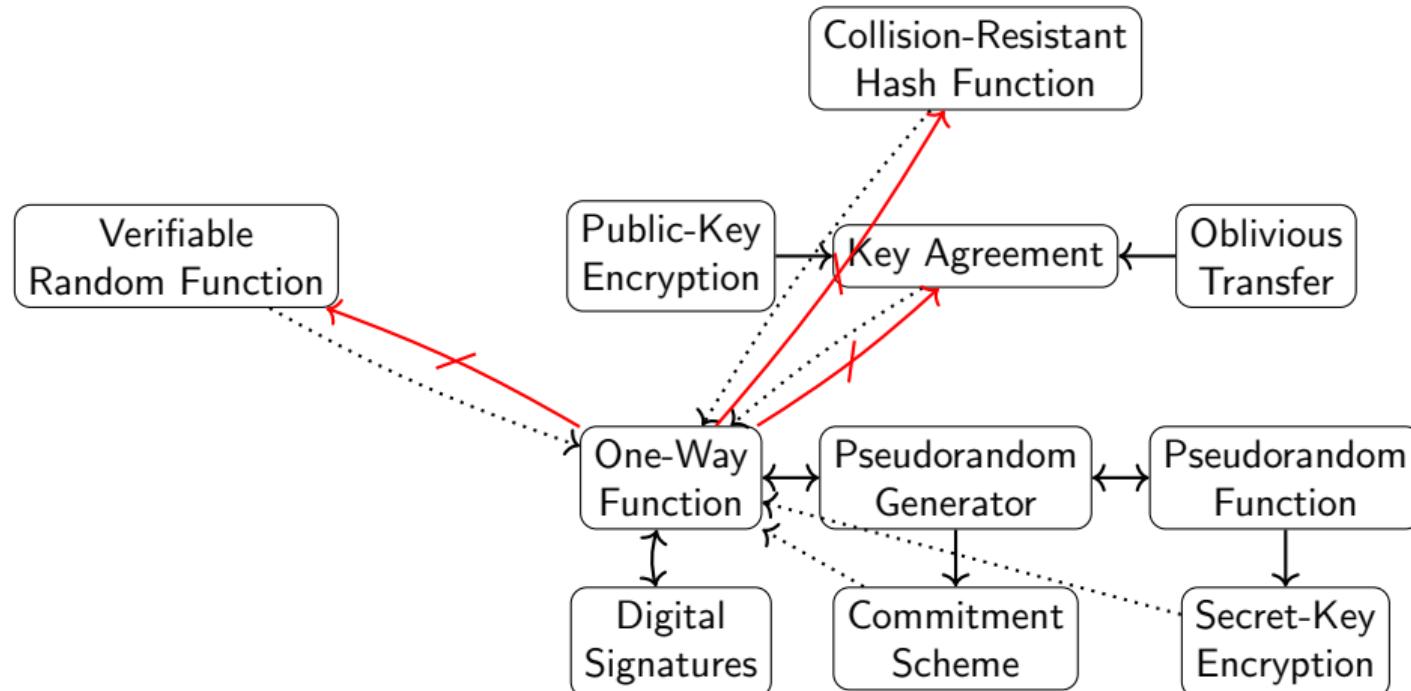
Oracle separations [IR90; IR89; HR04]

- ▶ Rules out (in particular) BBB-reductions.
- ▶ Relative to some oracle, primitive OWF exists but KA does not: $\text{OWF} \not\leq \text{KA}$.

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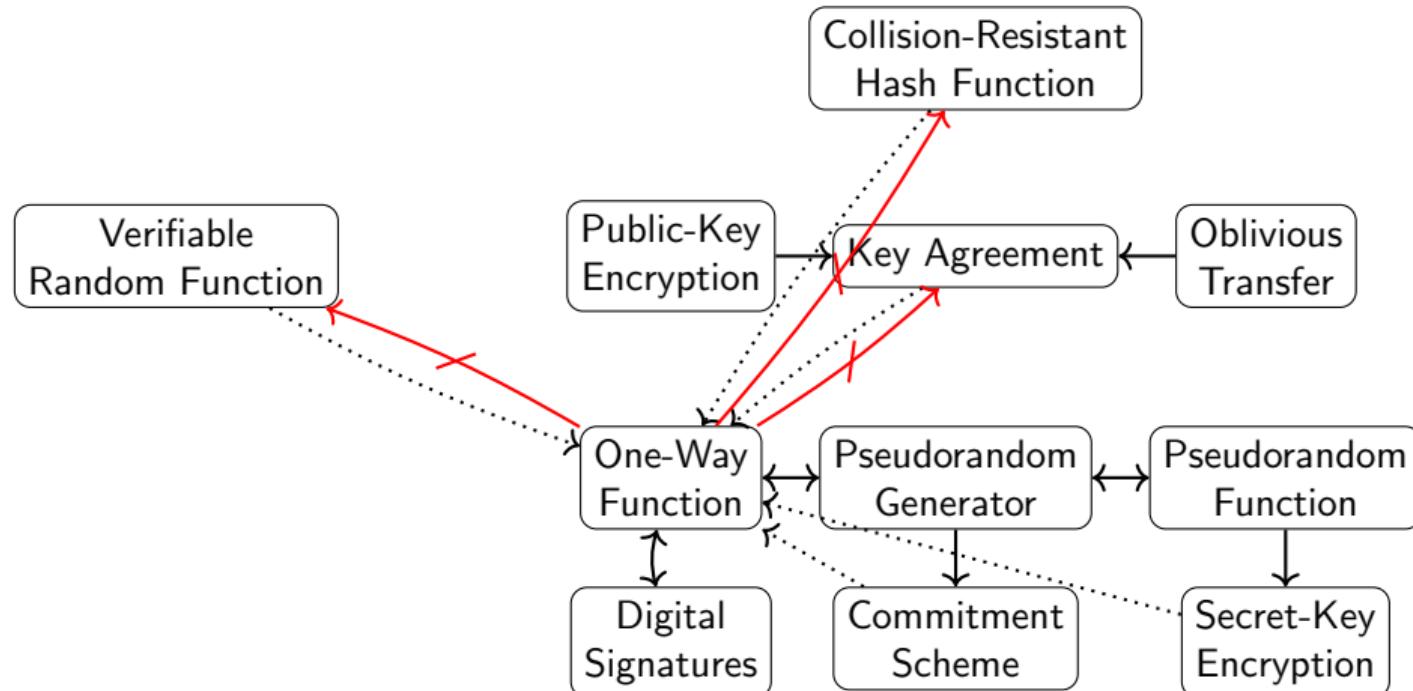
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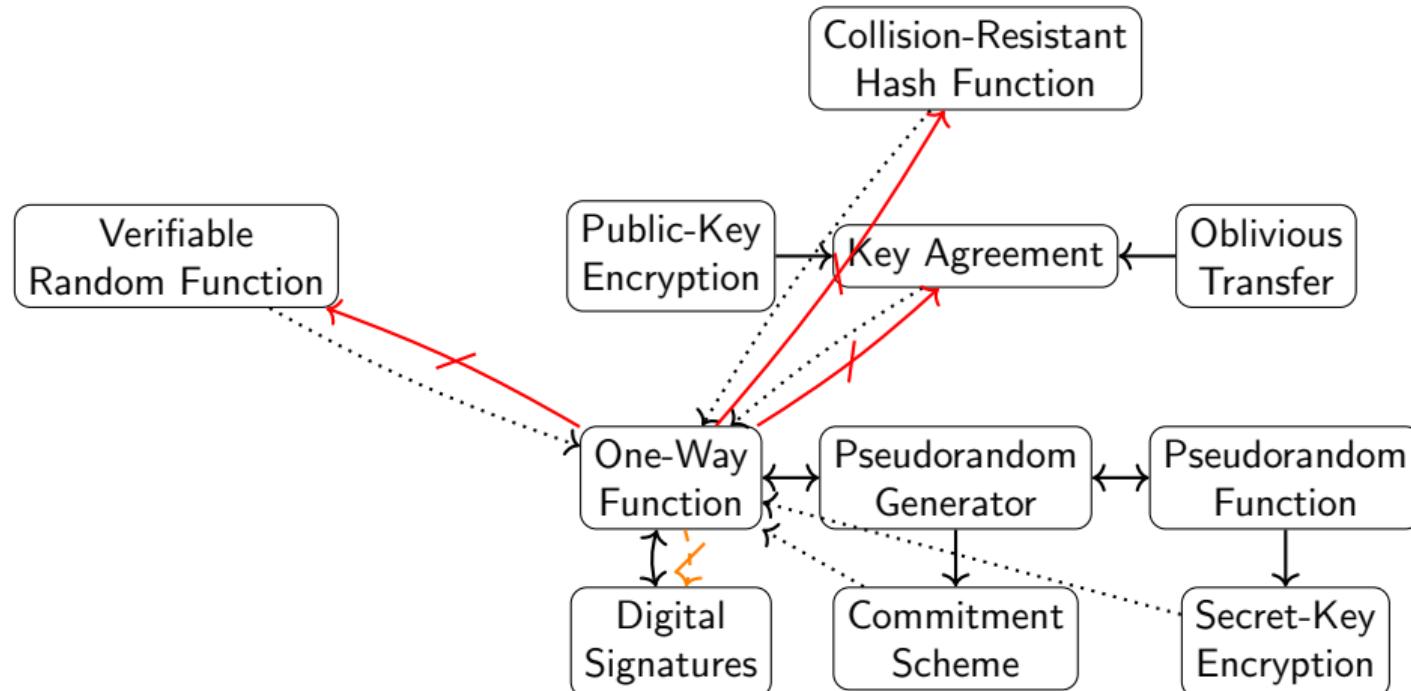
Meta-reductions [BV98; Cor02; Pas11]

- ▶ Rules out NBN-reductions.
- ▶ Often requires special properties of the reduction (algebraicity, small loss, non-interactive games, etc.).

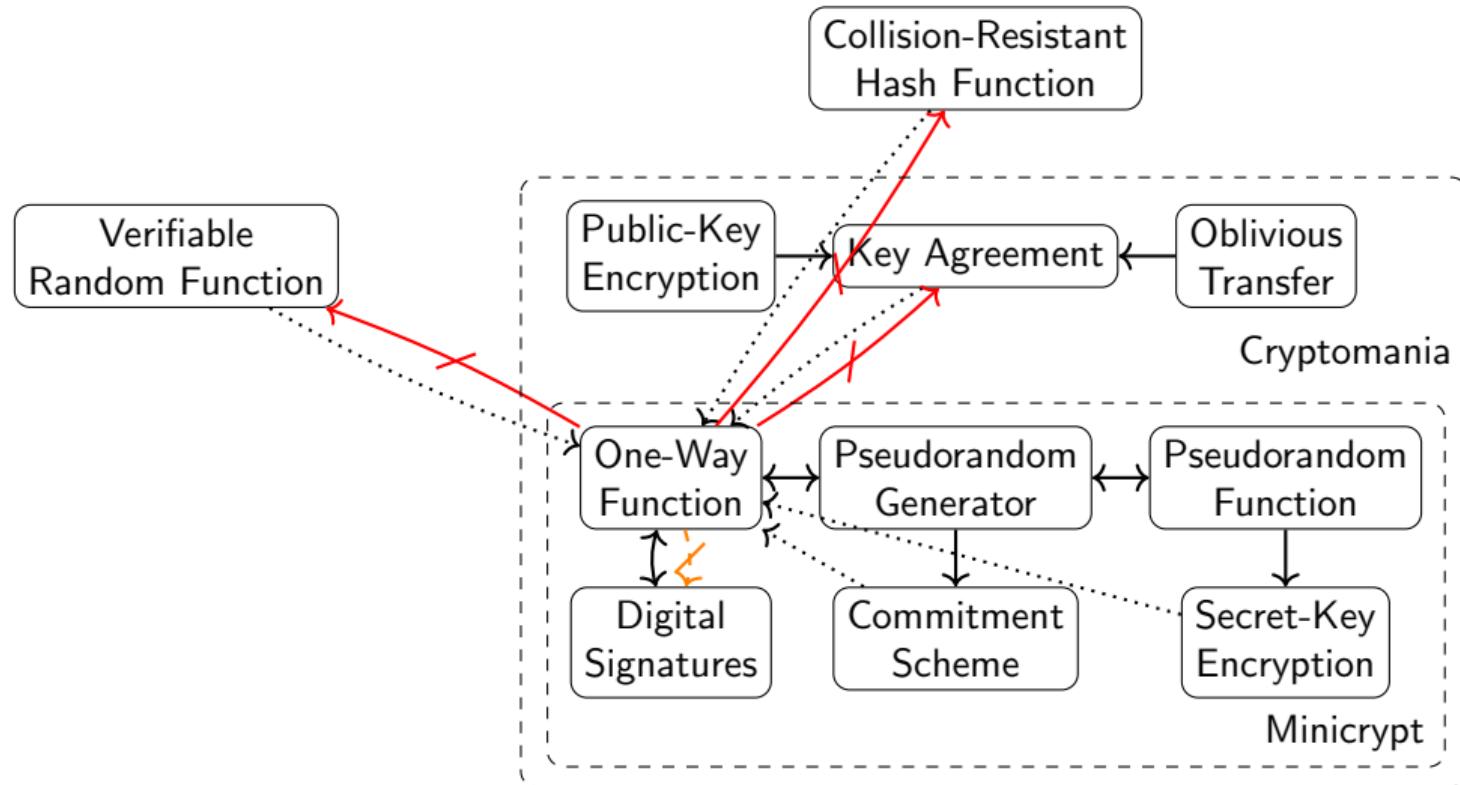
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Some Cryptographic Primitives [Imp95]



Contributions

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Other works (not in the thesis)

- ▶ [BMM⁺23]: Tight setup bounds for secure multi-party computation with identifiable abort
- ▶ [BHK⁺24]: Tightly secure blind signatures in pairing-free groups
- ▶ [BFM24]: A formal treatment of key transparency
- ▶ [BNG⁺25]: Constrained VRFs from novel pairing-based assumptions

Definition of VRF

Verifiable Random Function [MRV99]

A *verifiable random function* (VRF) consists of three algorithms:

- ▶ $\text{Gen}(1^\lambda) \rightarrow (\text{vk}, \text{sk})$
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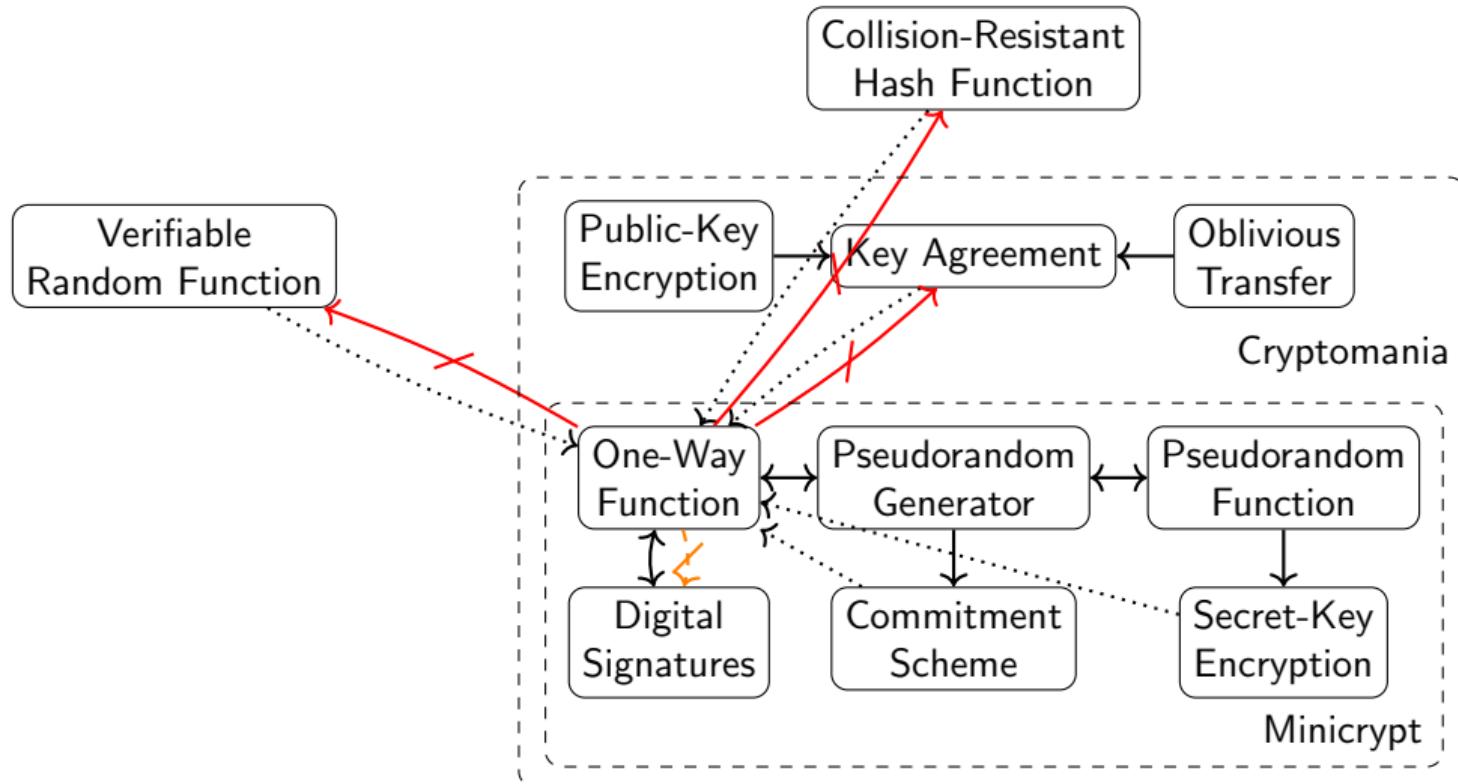
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- ▶ For a set of natural VRFs constructions:
the shorter the VRF proof, the stronger the assumption needed.

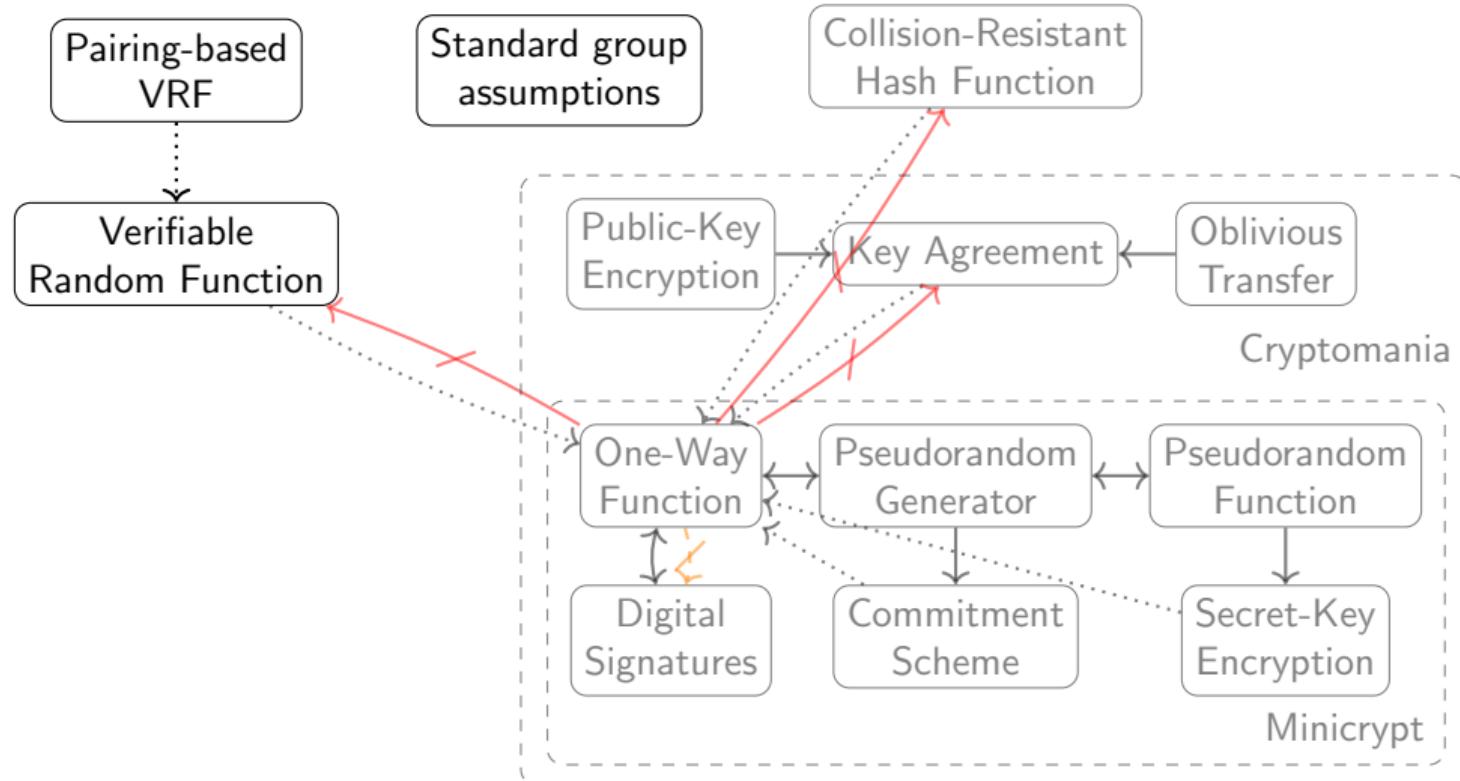
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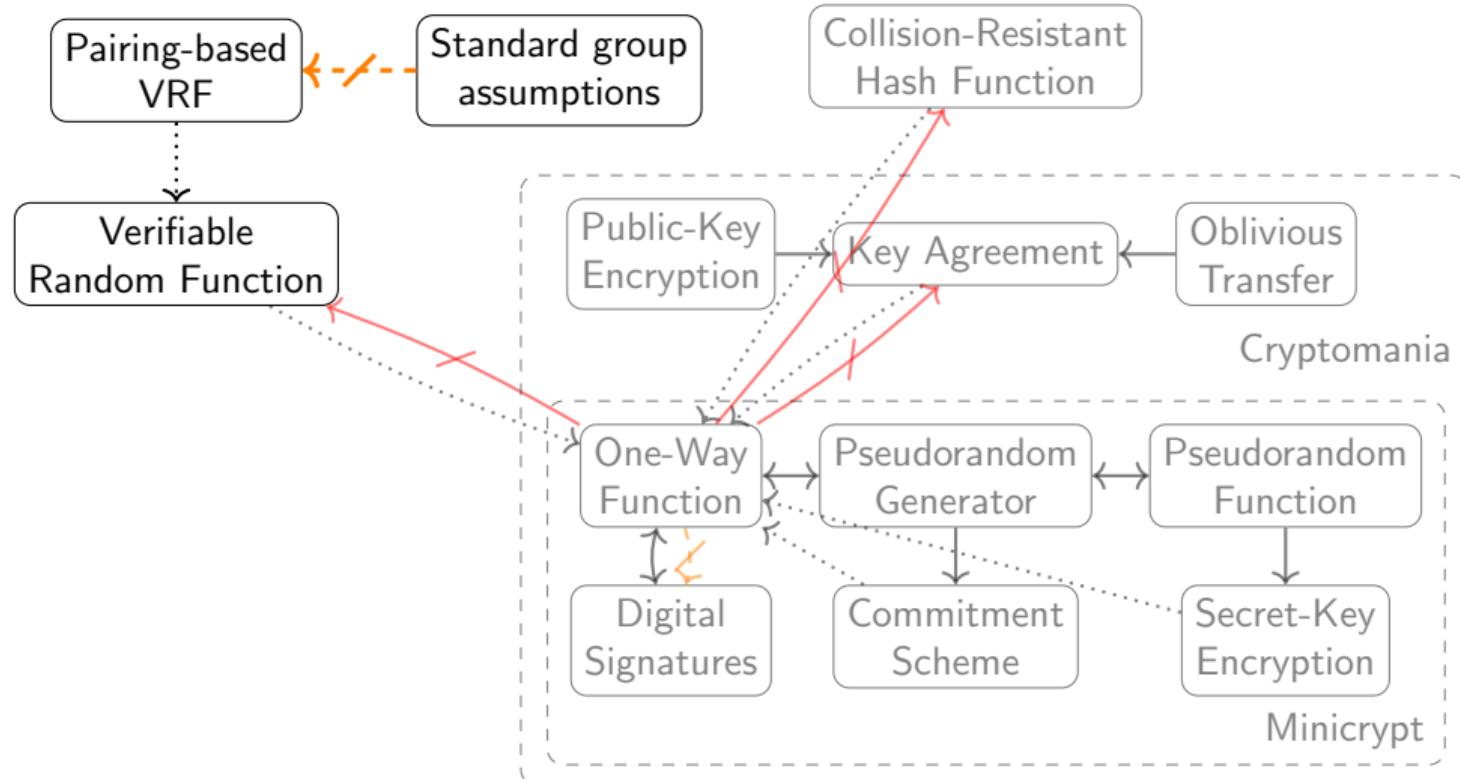
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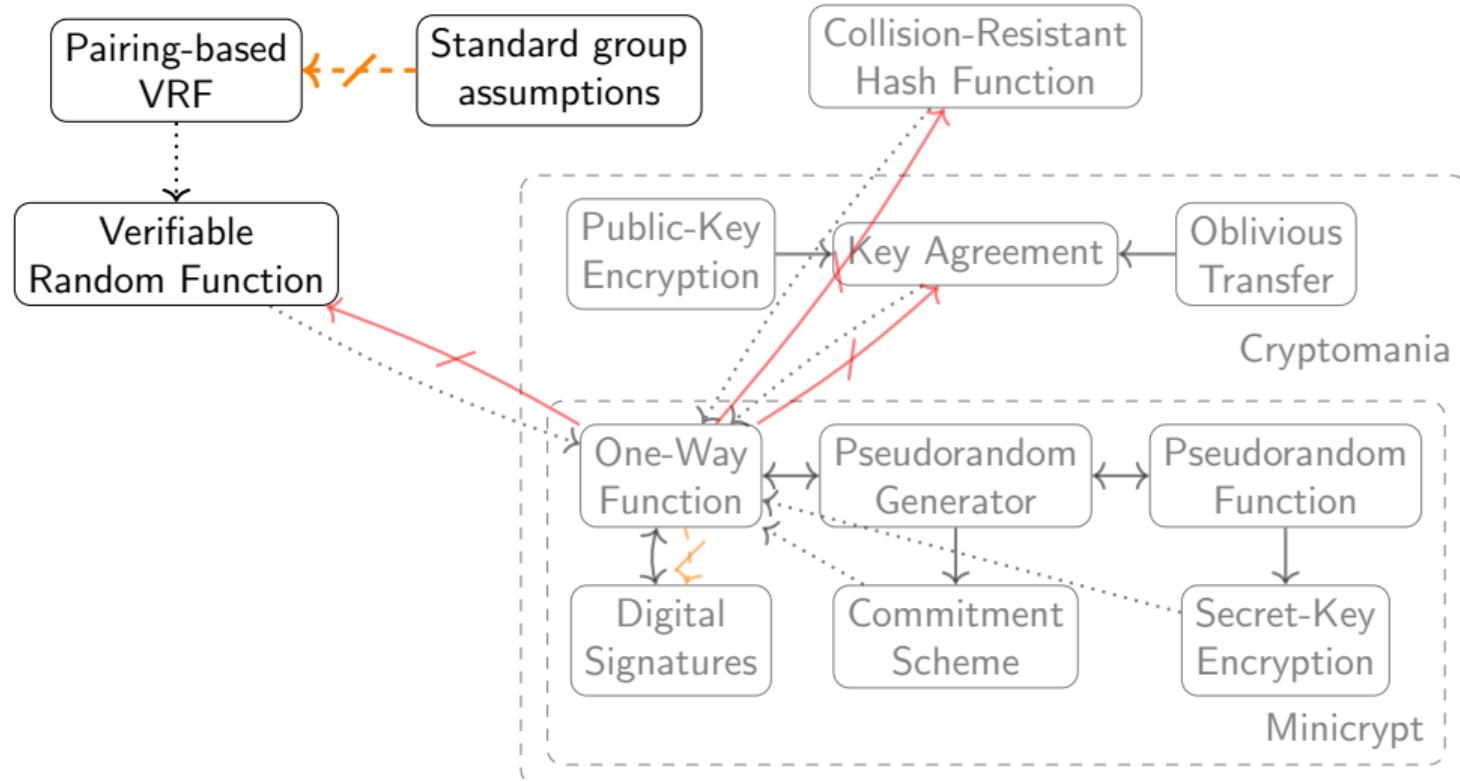
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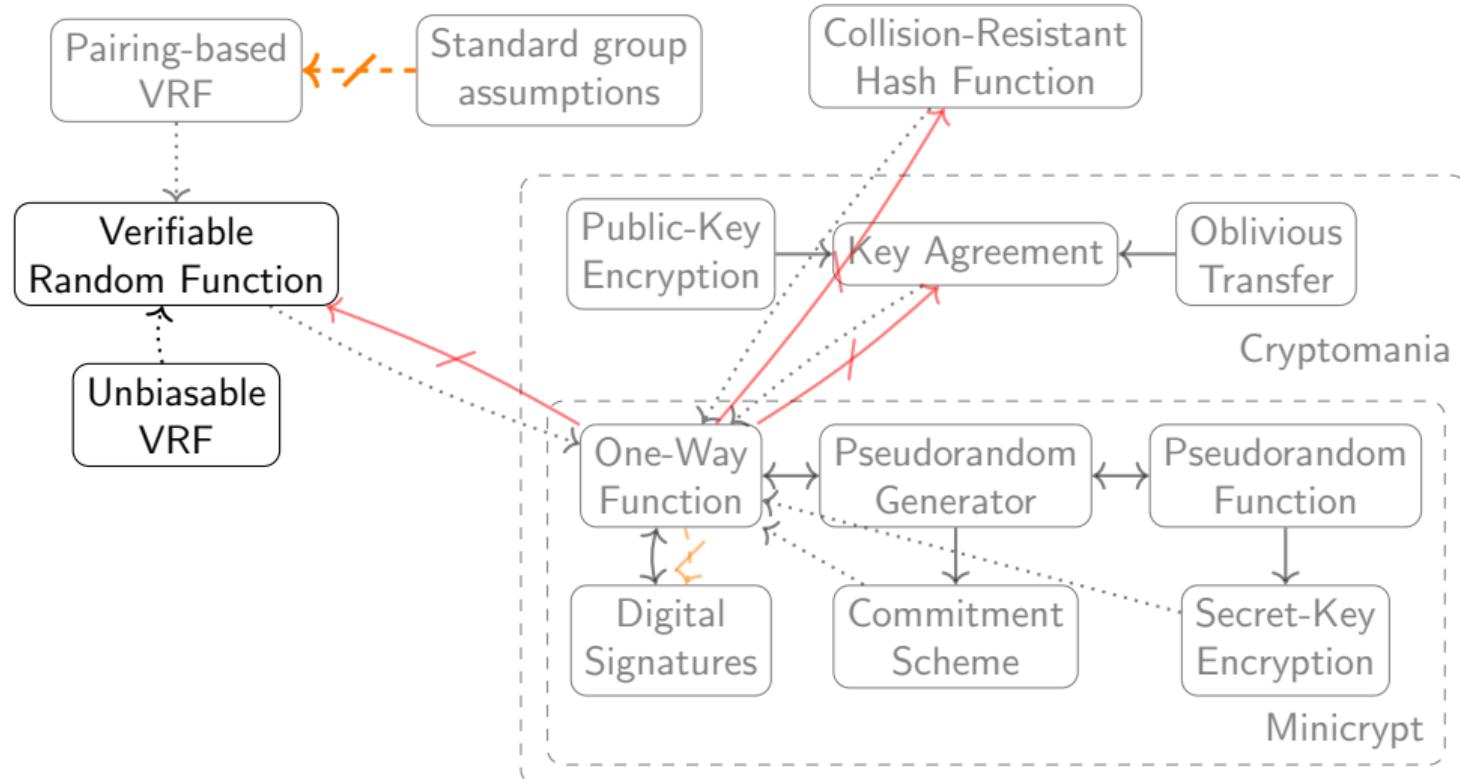
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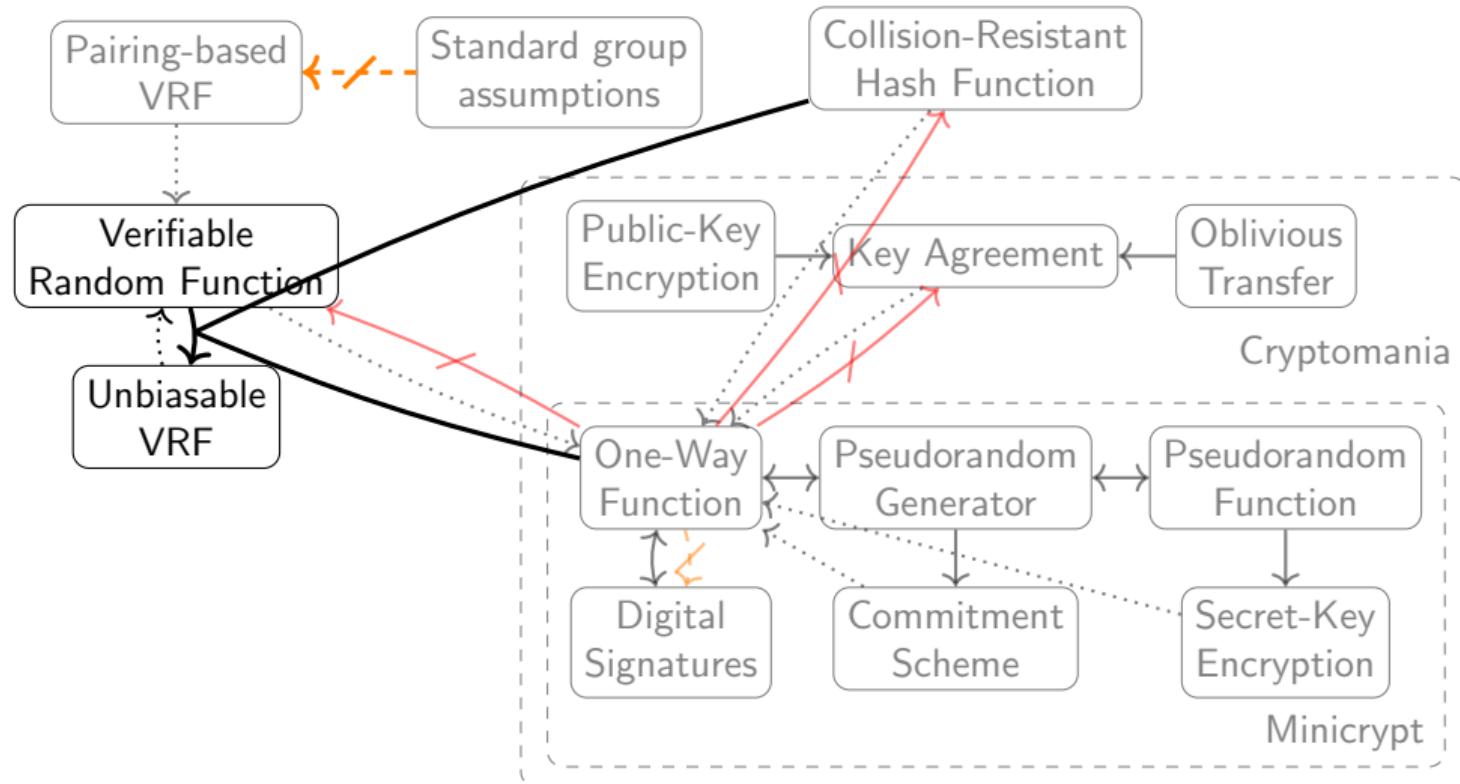
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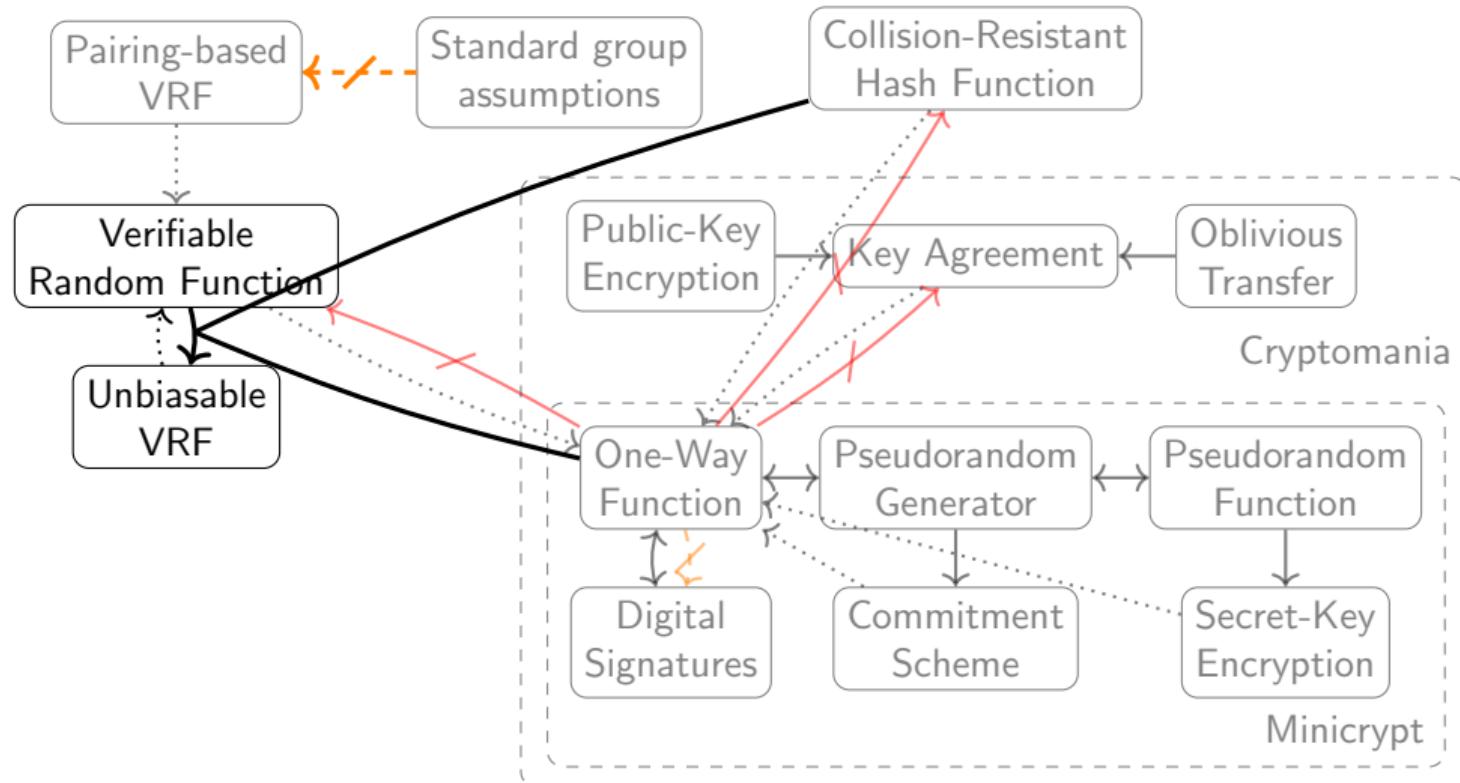
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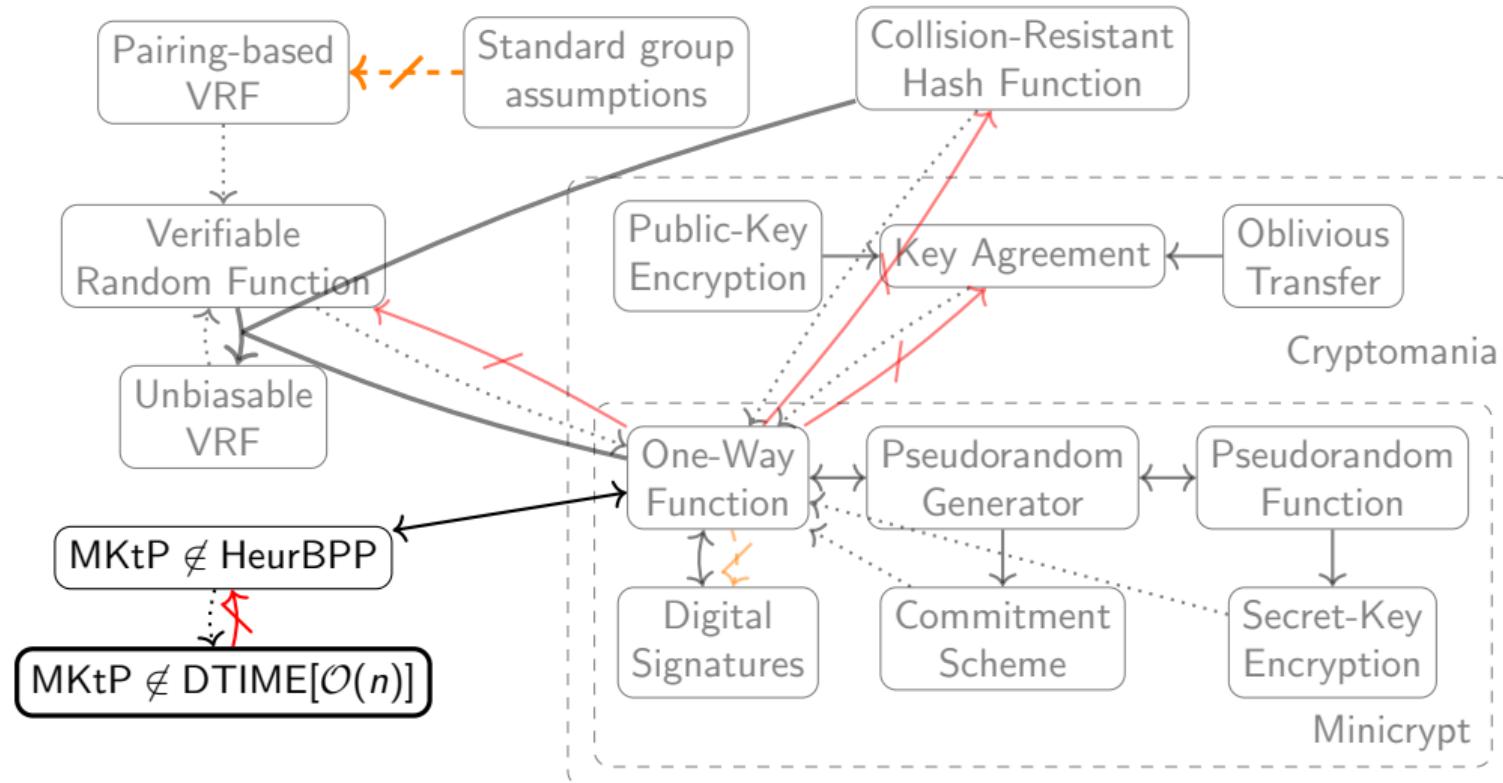
Non-black-box lower bounds for Levin–Kolmogorov complexity [Bra24]

- ▶ First unconditional lower bound MKtP \notin DTIME[$\mathcal{O}(n)$].
- ▶ Conditional lower bounds MKtP \notin DTIME[$t(n)$] \cap Heur_{0,o(1/t(n))}DTIME[$\mathcal{O}(n)$].

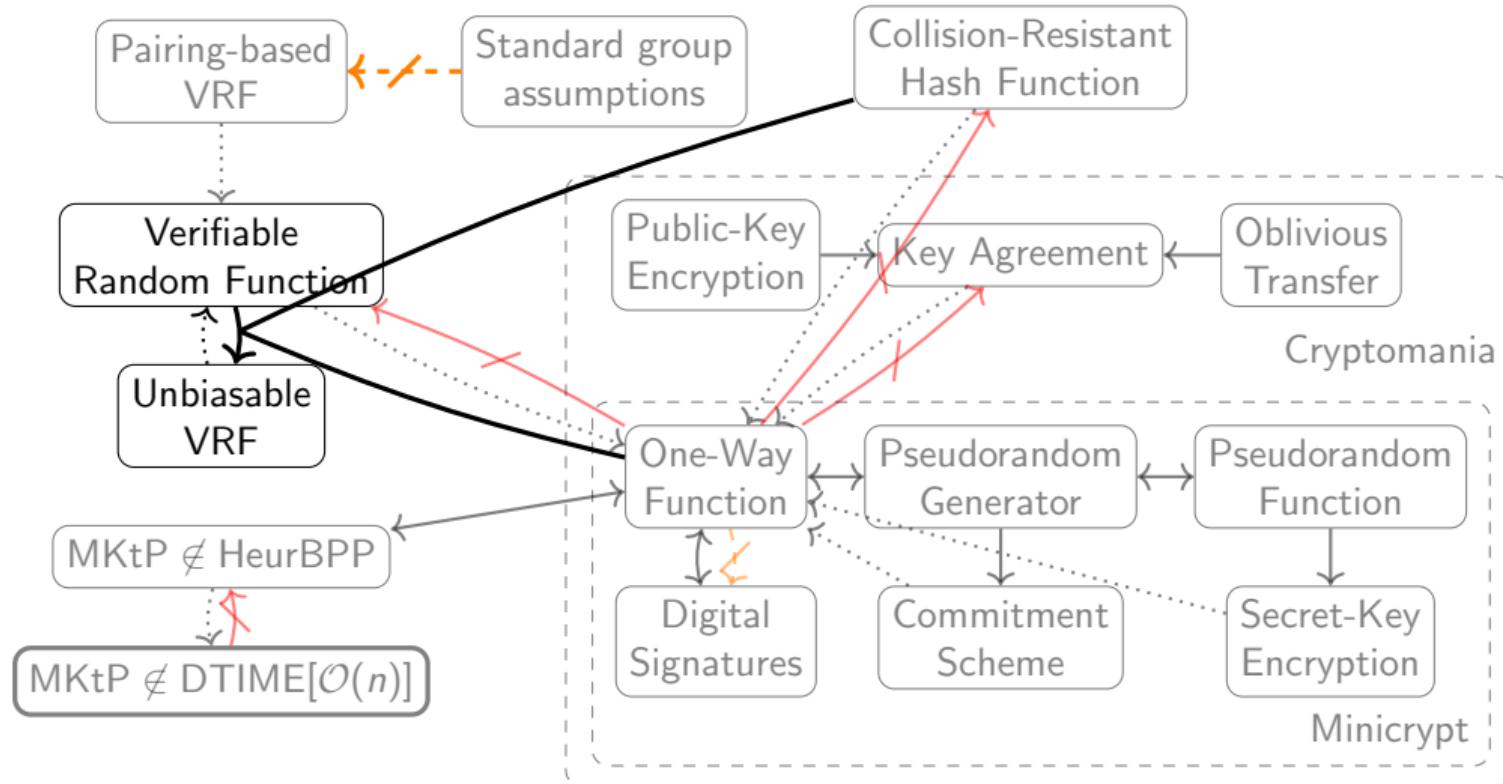
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- ▶ Unbiasability [GS24]:
random preimage \implies random image even for maliciously chosen vk

Simplified Unbiasability Game

$$\mathcal{C}(1^\lambda)$$

$$\mathcal{A}(1^\lambda; r_{\mathcal{A}})$$

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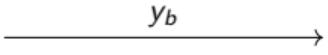
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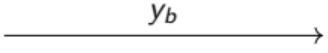
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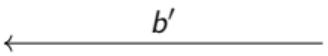
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Construction Overview

Construction [Bra24]

$$\begin{pmatrix} \text{VRF} \\ \text{one-way permutation (OWP)} \\ \text{collision-resistant hash function (CRH)} \end{pmatrix} \implies \text{unbiasable VRF}$$

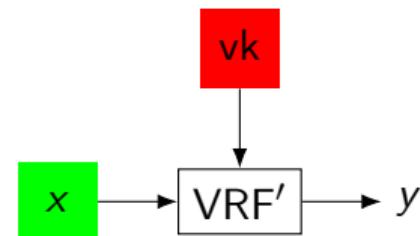
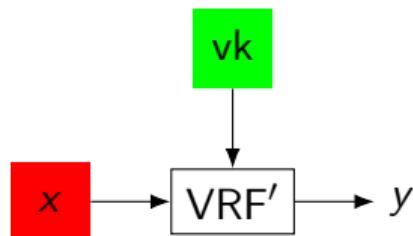
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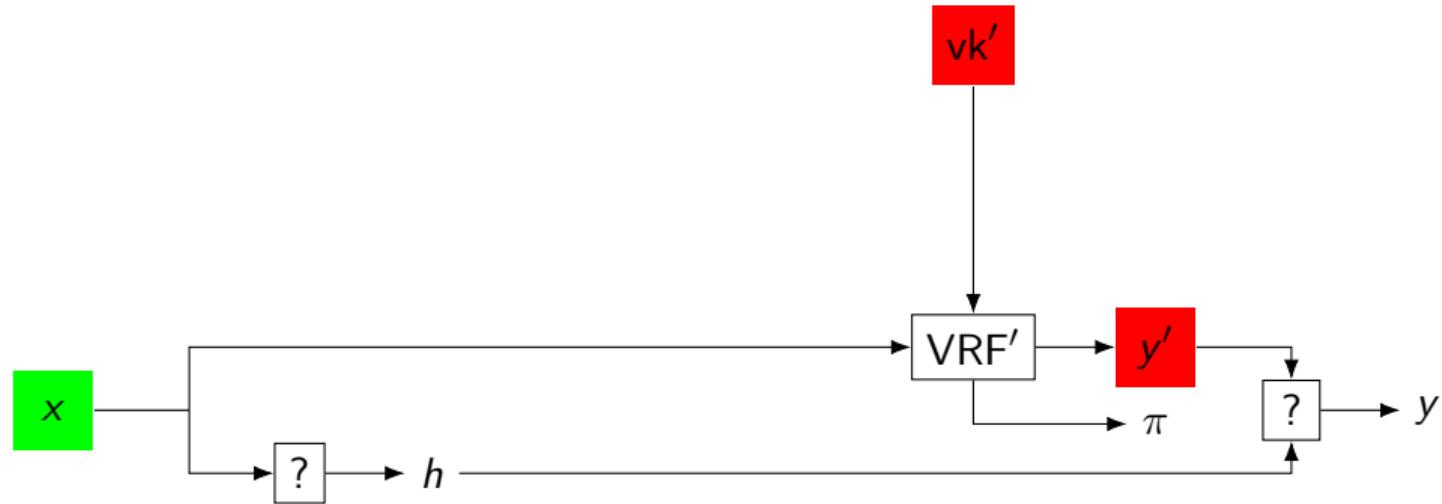
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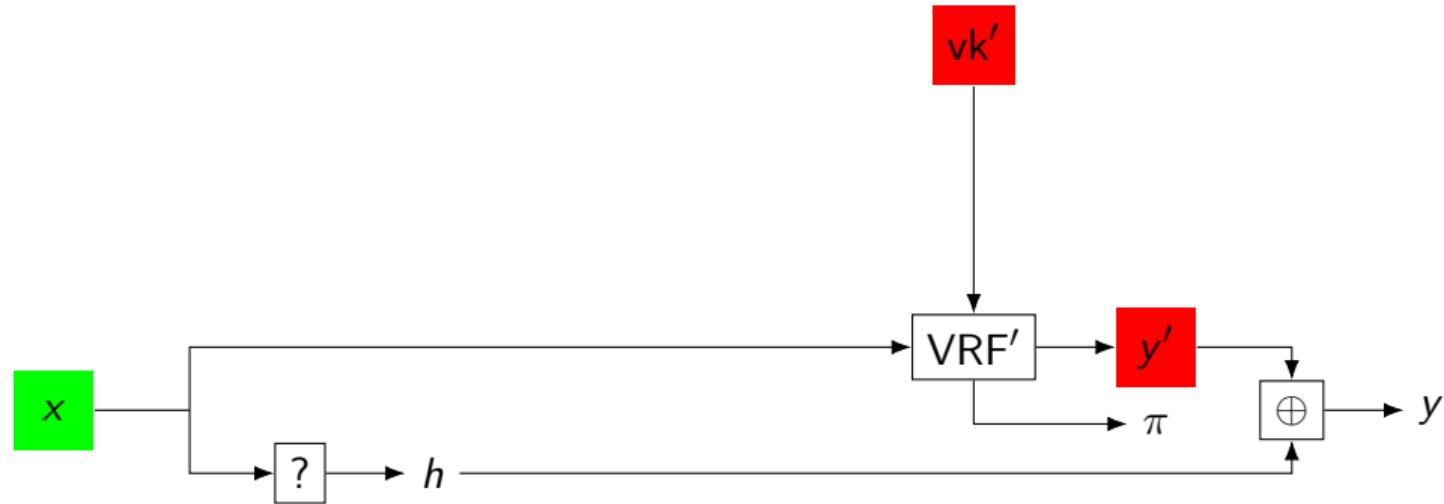
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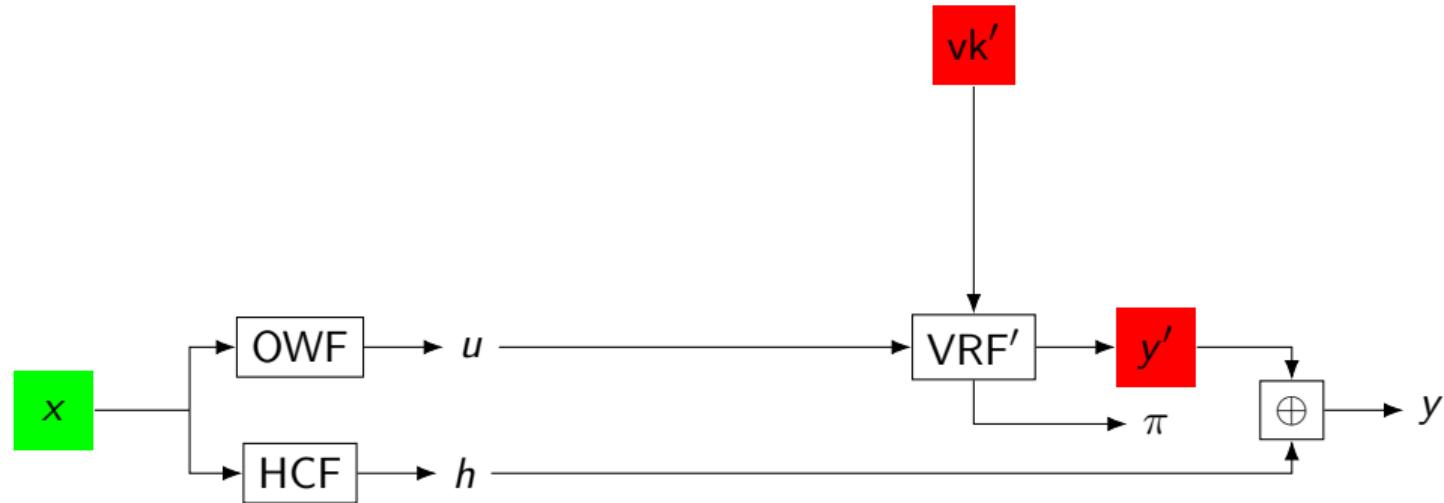
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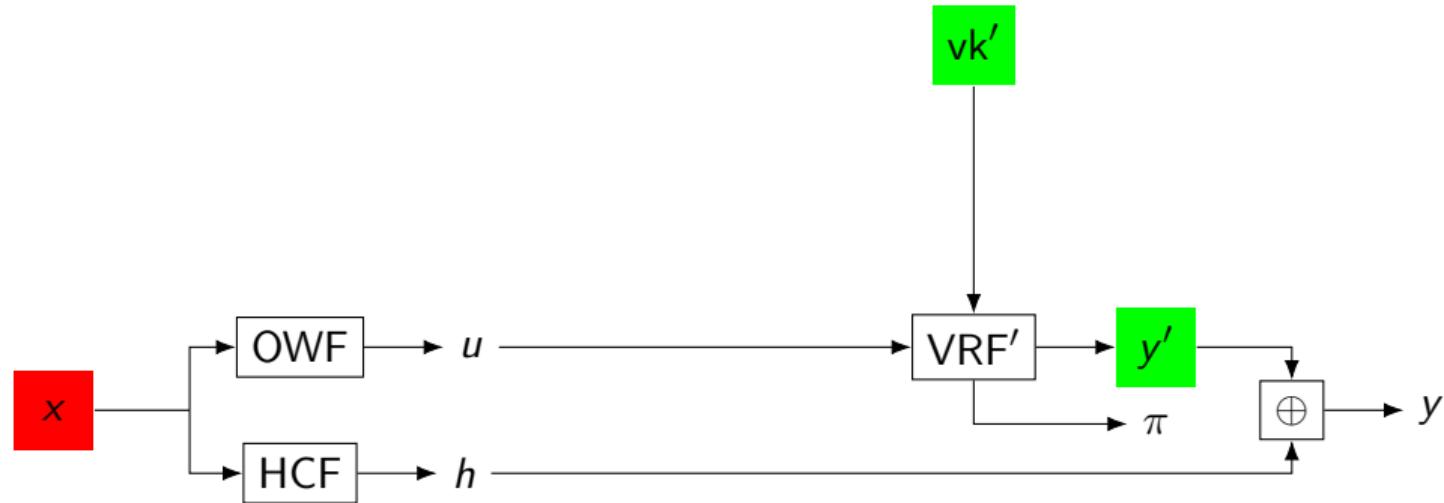
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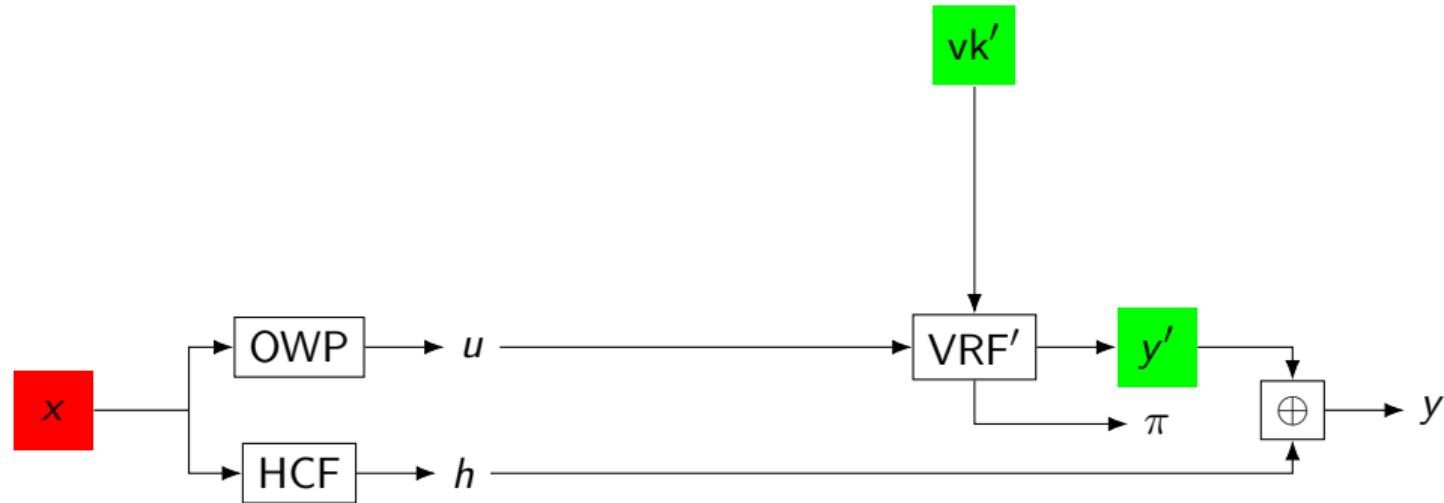
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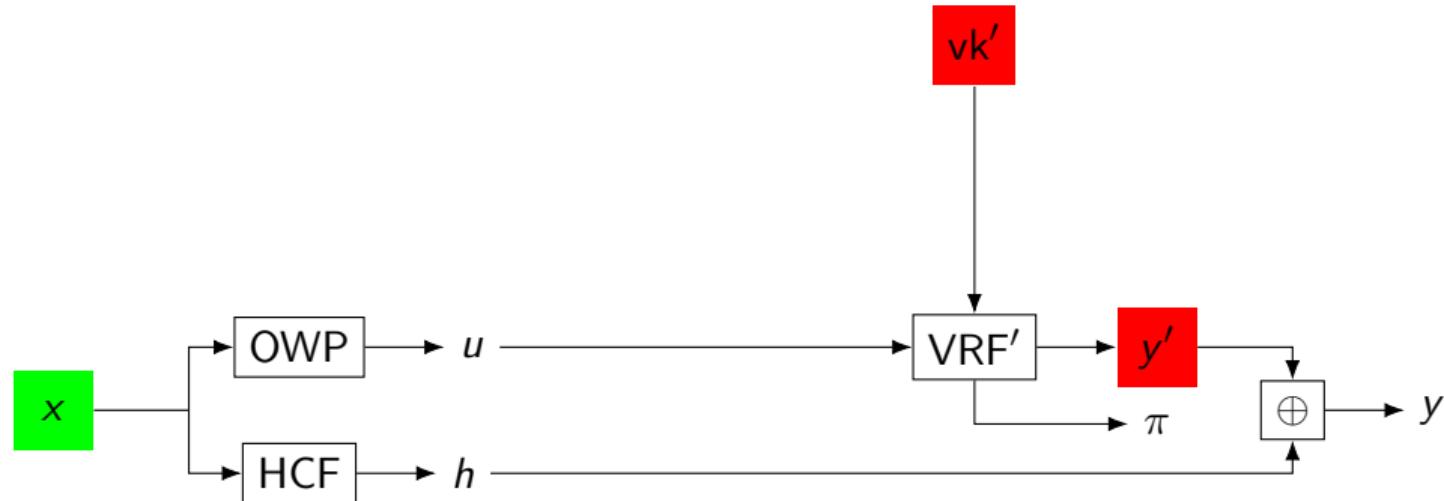
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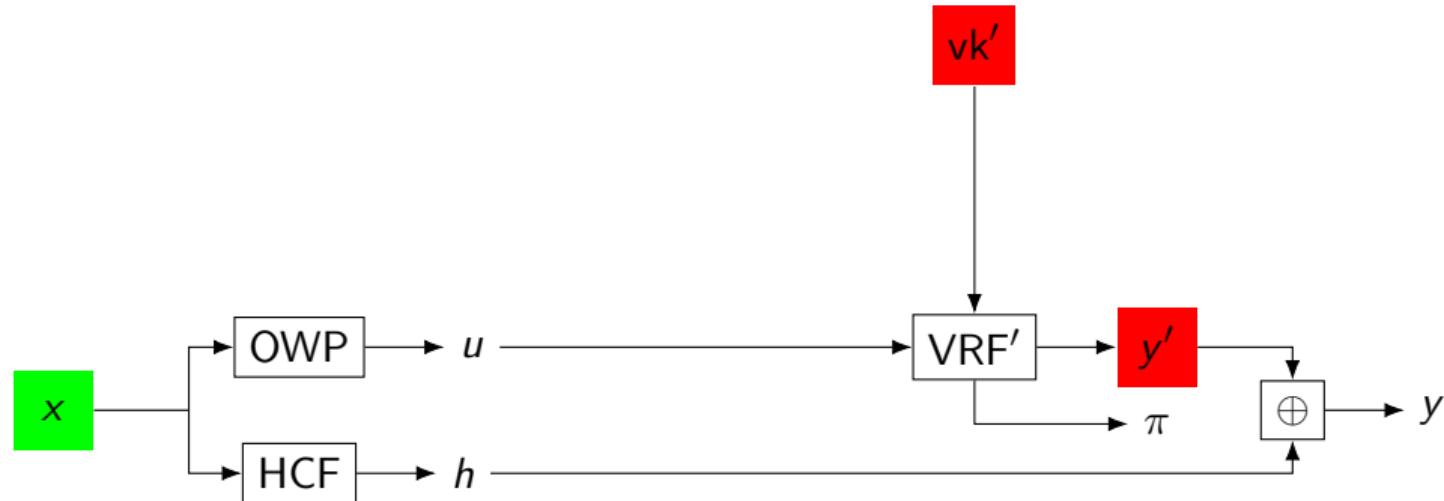
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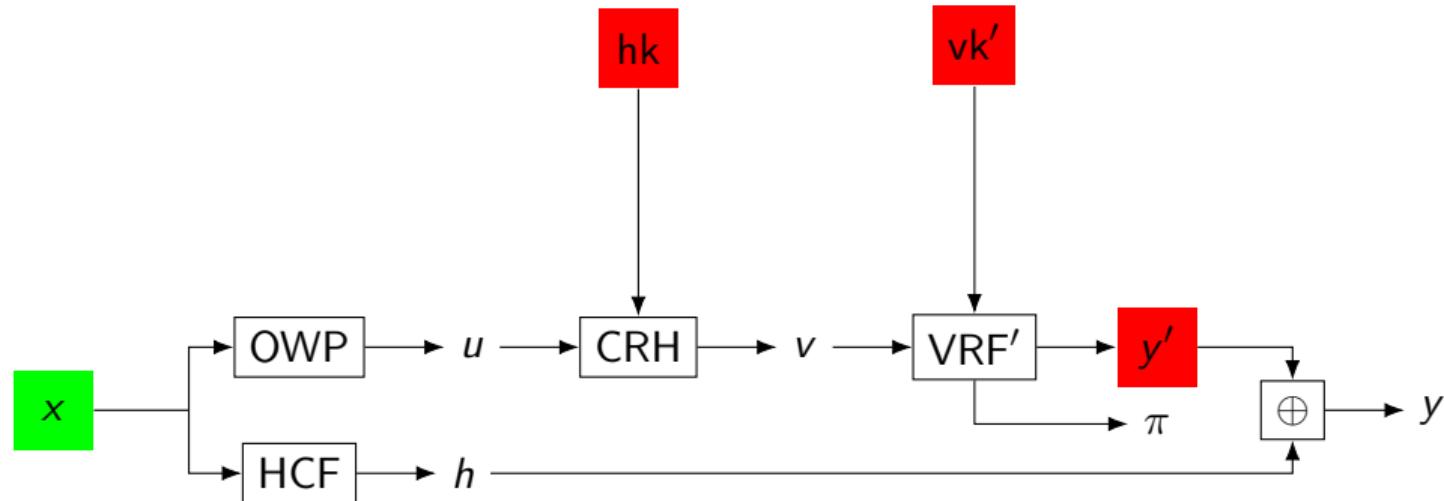
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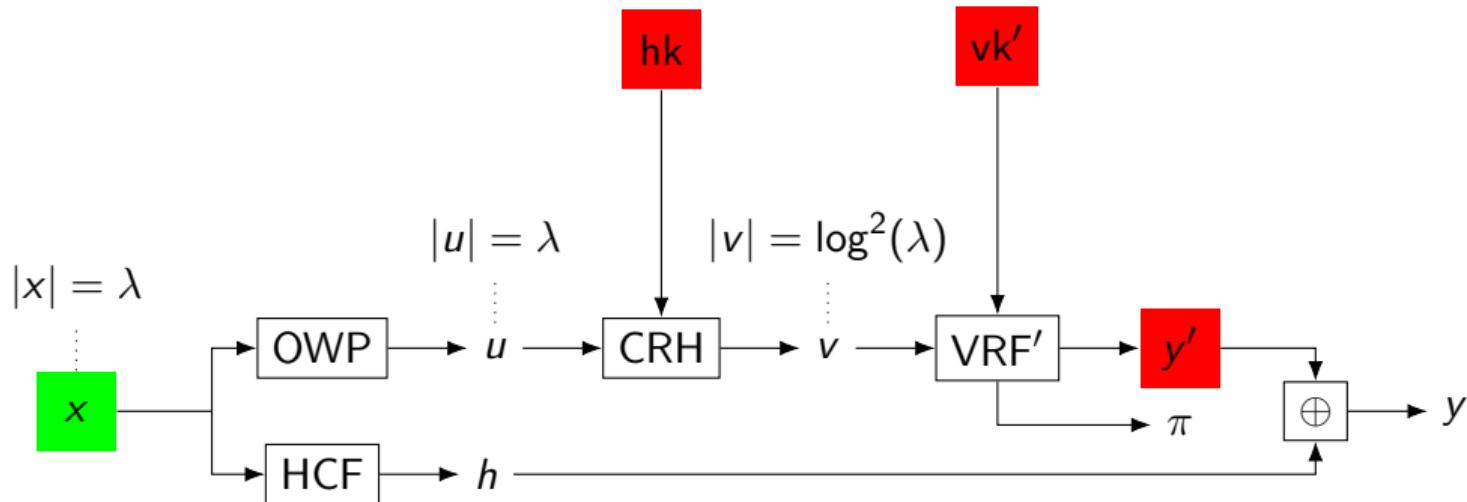
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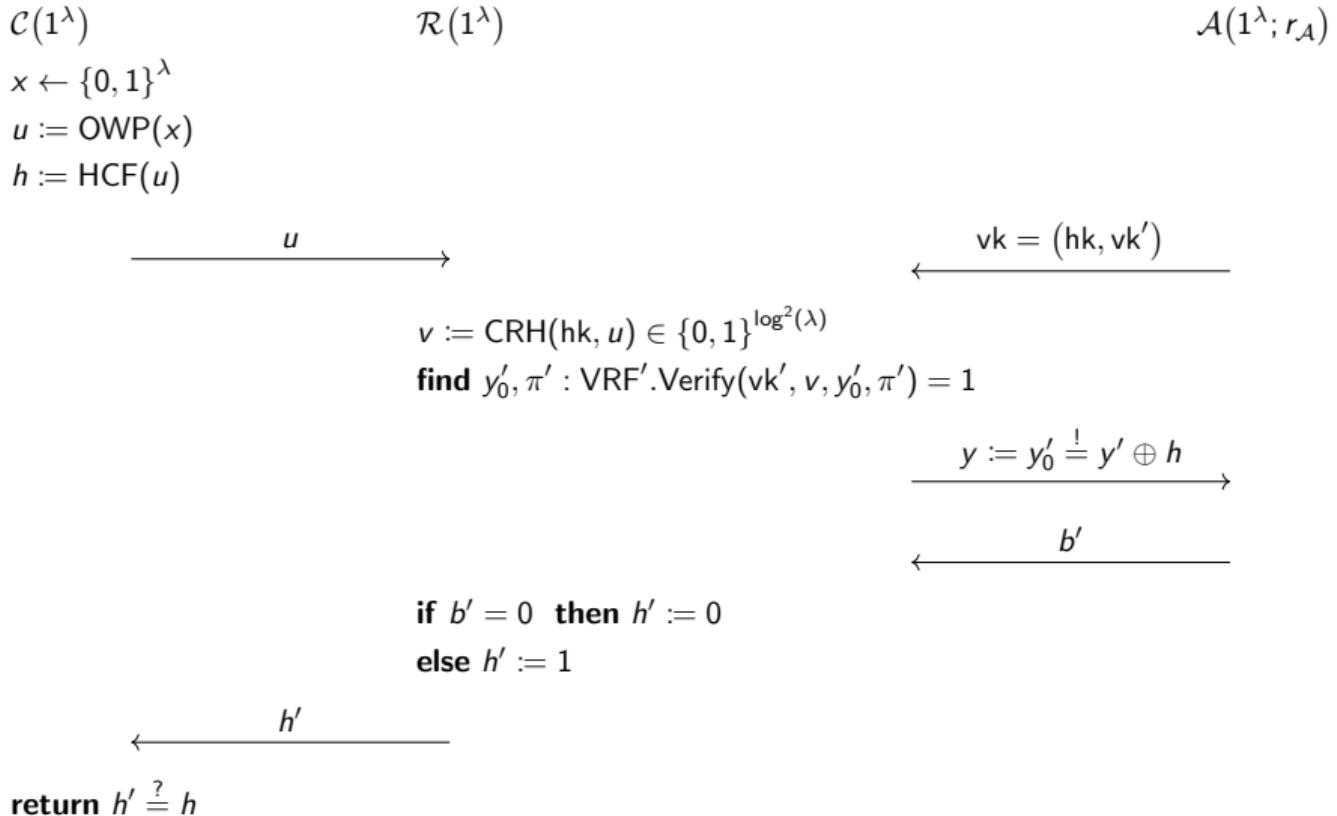
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y_b

b'

Simplified Unbiasability Reduction



Simplified Unbiasability Reduction

$$\mathcal{C}(1^\lambda) \quad \mathcal{R}_{\text{ek}'}(1^\lambda) \quad \mathcal{A}(1^\lambda; r_{\mathcal{A}})$$

$$x \leftarrow \{0, 1\}^\lambda$$

$$u := \text{OWP}(x)$$

$$h := \text{HCF}(u)$$

$$\xrightarrow{u} \qquad \qquad \qquad \xleftarrow{\text{vk} = (\text{hk}, \text{vk}')}$$

$$v := \text{CRH}(\text{hk}, u) \in \{0, 1\}^{\log^2(\lambda)}$$

$$y'_0 := \text{VRF}'\text{-LatEval}(\text{ek}', v)$$

$$\xrightarrow{y := y'_0 \stackrel{!}{=} y' \oplus h}$$

$$\xleftarrow{b'}$$

if $b' = 0$ **then** $h' := 0$
else $h' := 1$

$$\xleftarrow{h'}$$

return $h' \stackrel{?}{=} h$

BNN-Reduction with Polynomial Loss [Bra24]

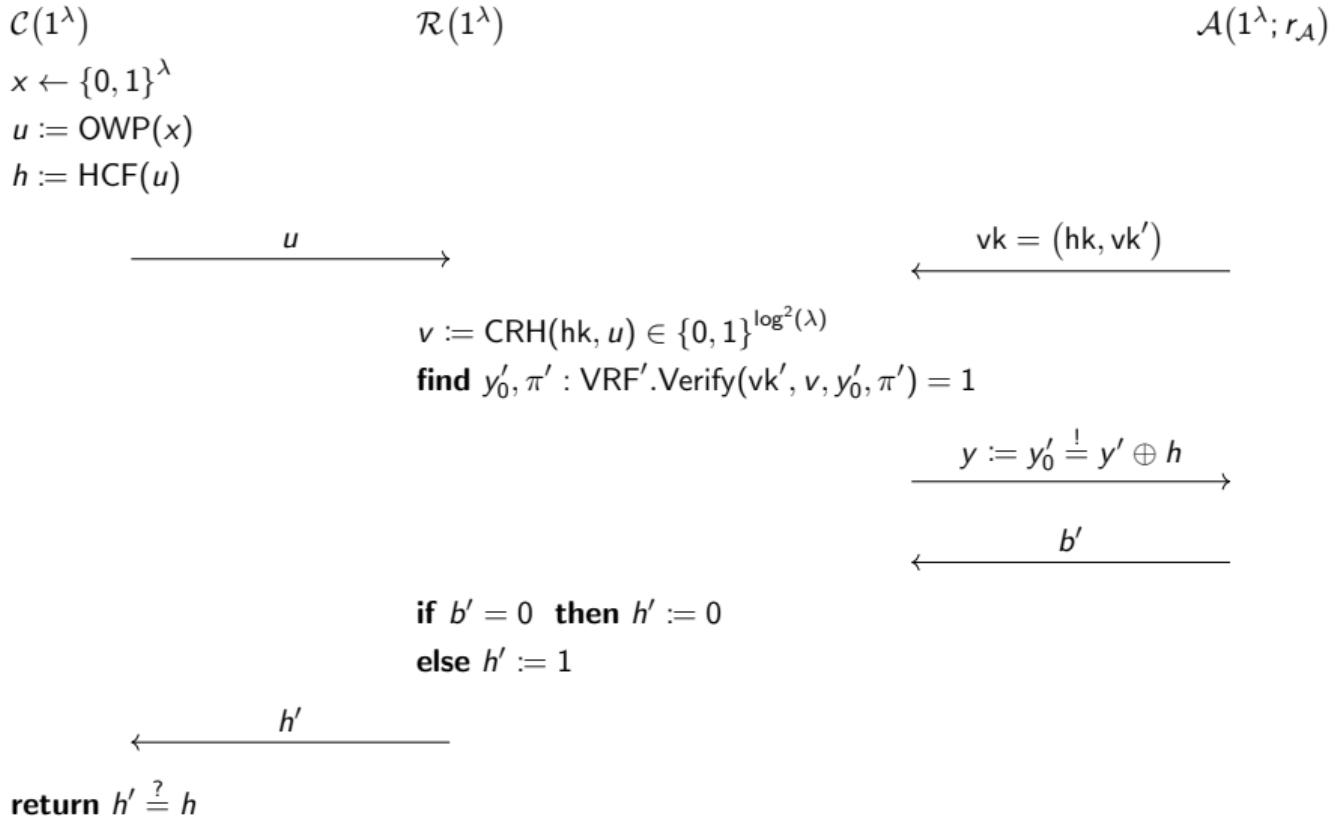
BNN-Reduction with Polynomial Loss [Bra24]

Latent VRF

A VRF VRF' is (perfectly) *latent* if there exists a deterministic polynomial-time algorithm $\text{LatEval}'$ such that

$$\forall v k' \exists e k' \forall x \left(\begin{array}{ll} y' := \text{LatEval}'(e k', x) \neq \perp & \implies y' \text{ is the valid image} \\ y' := \text{LatEval}'(e k', x) = \perp & \implies \text{no valid image exists} \end{array} \right).$$

Simplified Unbiasability Reduction



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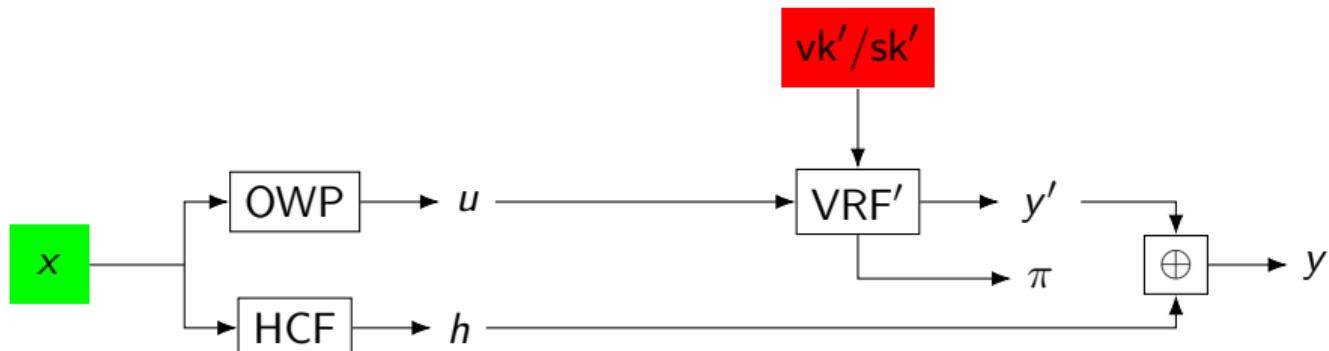
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Contributions

Meta-reductions for VRFs [BHK⁺22]

- ▶ Rules out algebraic NBN-reductions from various non-interactive assumptions to pairing-based VRFs with short proofs.
- ▶ For a set of natural VRFs constructions:
the shorter the VRF proof, the stronger the assumption needed.

(Non-)black-box reductions for unbiased VRFs [Bra25]

- ▶ BBB-reduction: injective OWF, CRH, VRF \implies unbiased VRF (subexp. loss)
- ▶ BNN-reduction: injective OWF, VRF \implies unbiased VRF (poly. loss)

Non-black-box lower bounds for Levin–Kolmogorov complexity [Bra24]

- ▶ First unconditional lower bound MKtP \notin DTIME[$\mathcal{O}(n)$].
- ▶ Conditional lower bounds MKtP \notin DTIME[$t(n)$] \cap Heur_{0,o(1/t(n))}DTIME[$\mathcal{O}(n)$].

Conclusion

Conclusion

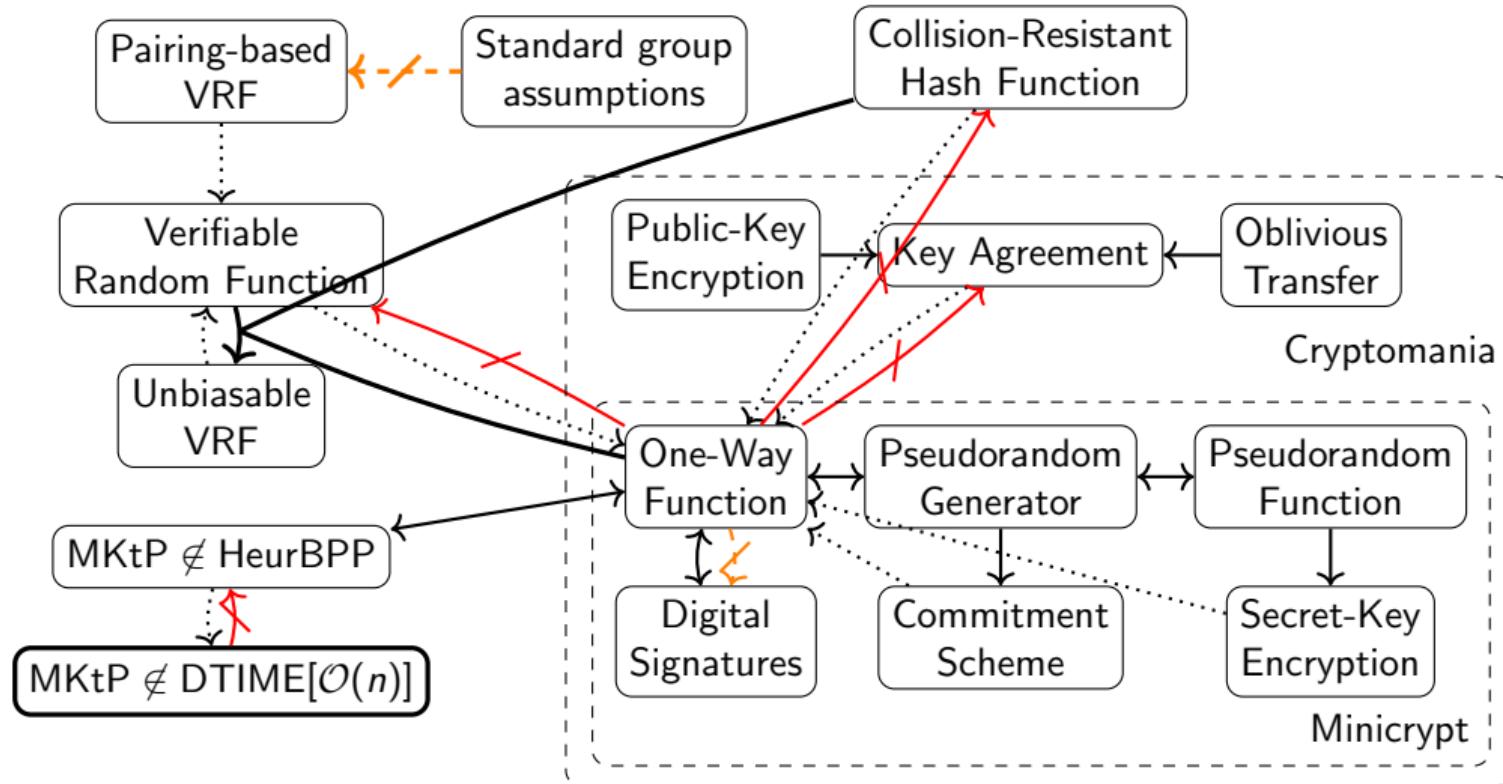
- ▶ BBB-constructions are conceptually simple but often lack feasibility or efficiency.
- ▶ Separations can formally explain insufficiencies of BBB-constructions.
- ▶ Obfuscation-/NIZK/NIWI-based constructions are often NAP.
- ▶ CNP-constructions are under-explored (non-black-box use of the adversary).

Construction cookbook

1. Try a BBB-construction.
2. Try a meta-reduction or oracle separation.
3. Try a CNP- or NAP-construction.

Thank you!

Some Cryptographic Primitives



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Lower bounds for Levin–Kolmogorov complexity [Bra24]

Context

- ▶ Levin's notion [Lev84] of Kolmogorov complexity:

$$Kt(x) := \min\{|\Pi| + \lceil \log(t) \rceil \mid \Pi(\varepsilon) = x \text{ in } t \text{ steps}\}$$

- ▶ Set of low-complexity strings: $MKtP := \{x \mid Kt(x) \leq |x|\} \in E := \text{DTIME}[2^{\mathcal{O}(n)}]$
- ▶ Liu and Pass [LP21]:

$$\exists OWF \iff MKtP \notin \text{HeurBPP}$$

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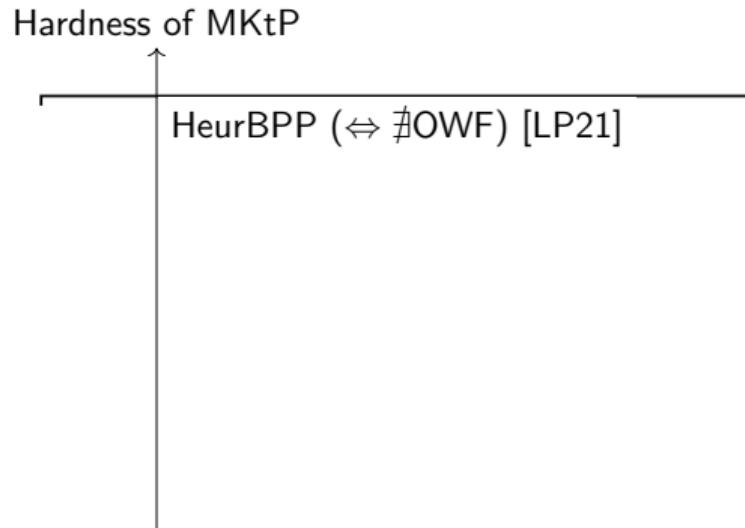
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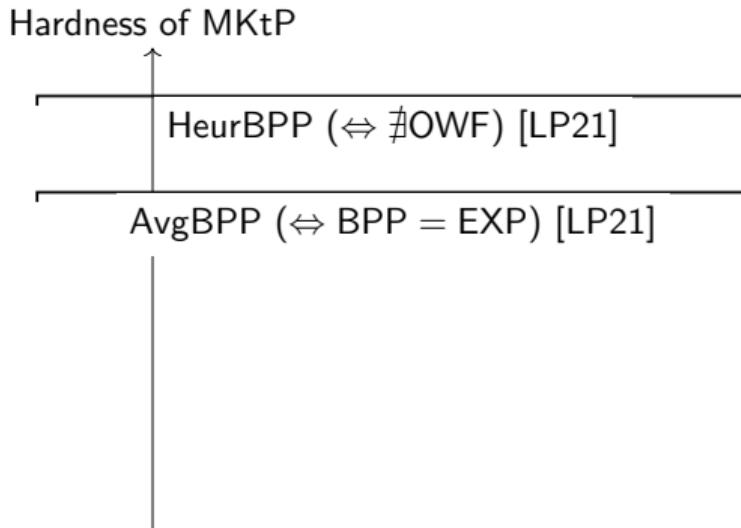
Hardness of MKtP



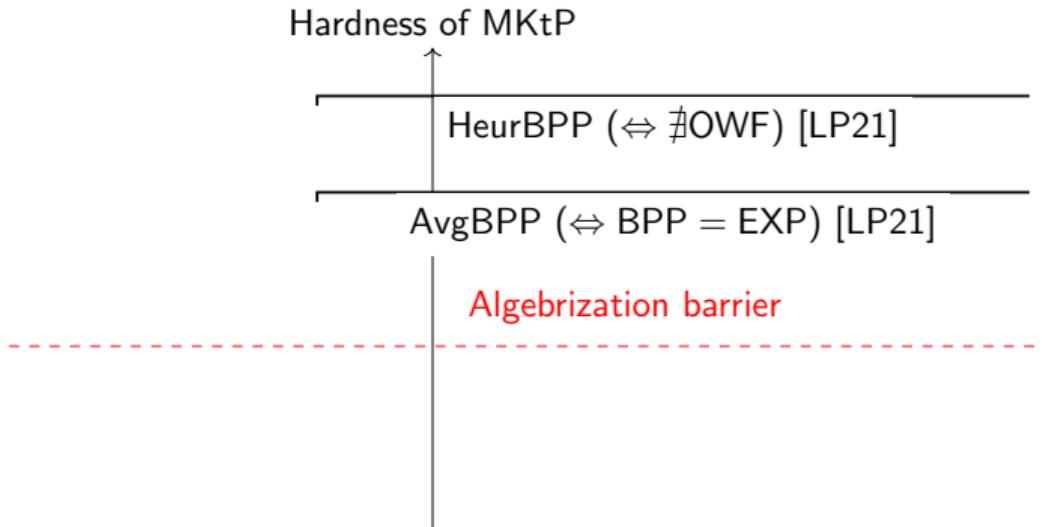
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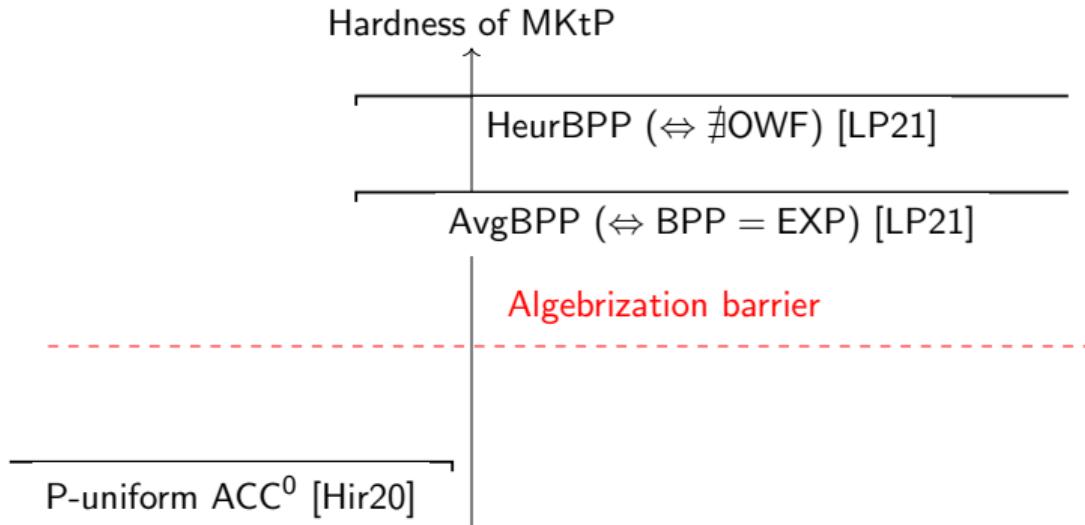
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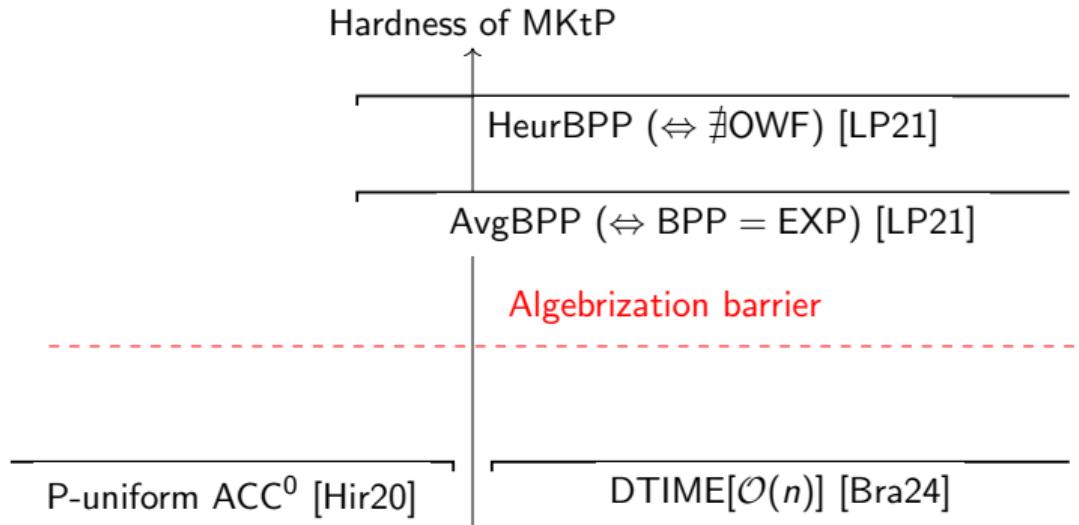
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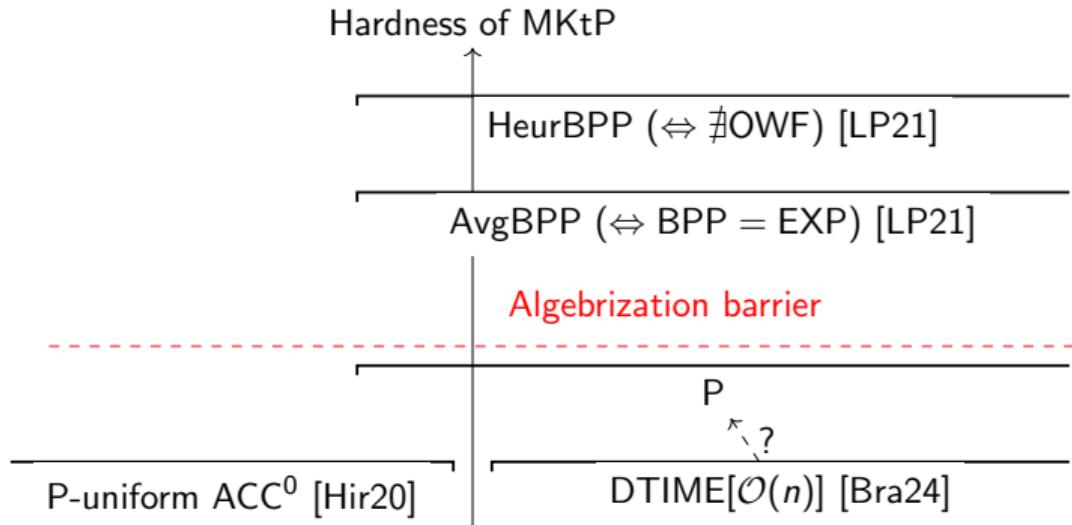
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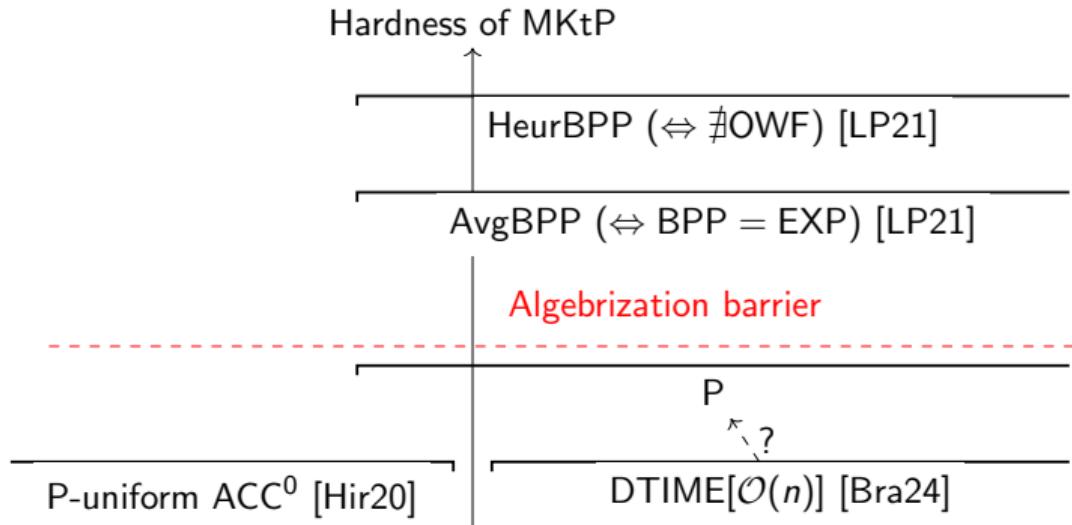
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[RS22] oracle: approx. Kt within factor $(1 + \epsilon)$ in linear time and $BPP = EXP$