On the Limits of Black-Box Techniques for Minimizing Cryptographic Assumptions

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Outline

Preliminaries

- Reductionism underlying modern cryptography

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- Relating cryptographic primitives

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Contributions

- Overview

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- (Non-)black-box construction of unbiasable verifiable random functions (VRFs)

Some Cryptographic Primitives

Collision-Resistant Hash Function

Verifiable Random Function

Public-Key Encryption

Key Agreement

Oblivious Transfer

One-Way Function

Pseudorandom Generator

Pseudorandom Function

Digital Signatures

Commitment Scheme

Secret-Key Encryption

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Security based on computational hardness

- $\mathsf{P} = \mathsf{NP} \implies$ most cryptographic primitives can be broken efficiently.
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Which assumption is best?

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Hardness of LWE

Reductionism

What is a minimal assumption for a given primitive?

Reductionism

What is a cryptographic reduction?

Reductionism

Cryptographic construction/reduction:

$$\mathsf{A} \longrightarrow \mathsf{B}$$

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$$A \longrightarrow B$$

Construction: $\boxed{\Pi_B} \longleftrightarrow \boxed{\Pi_A}$

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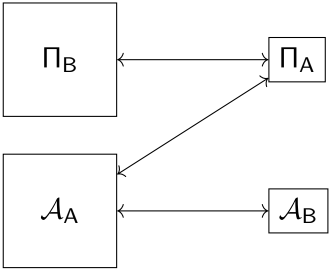

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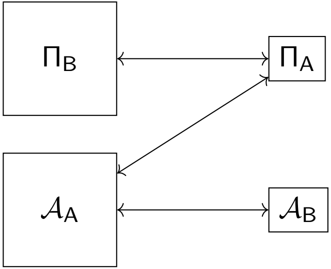

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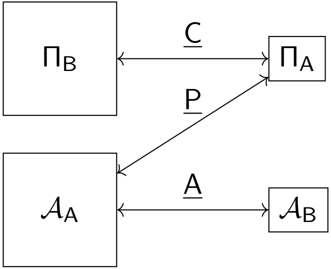

CAP taxonomy [BBF13]:

Construction	Adversary	Primitive
$\in \{B, N\}$	$\in \{B, N\}$	$\in \{B, N\}$

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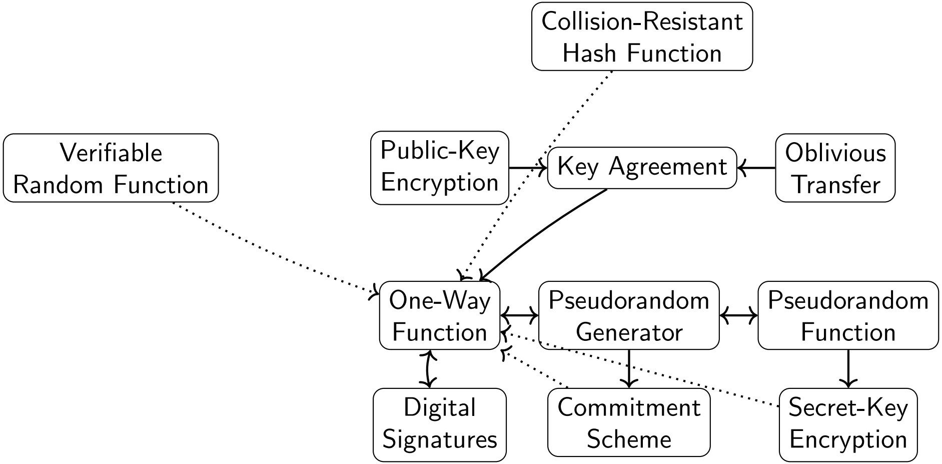

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Oracle separations [IR90; IR89; HR04]

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- Relative to some oracle, primitive OWF exists but KA does not: OWF $\not\leq$ KA.

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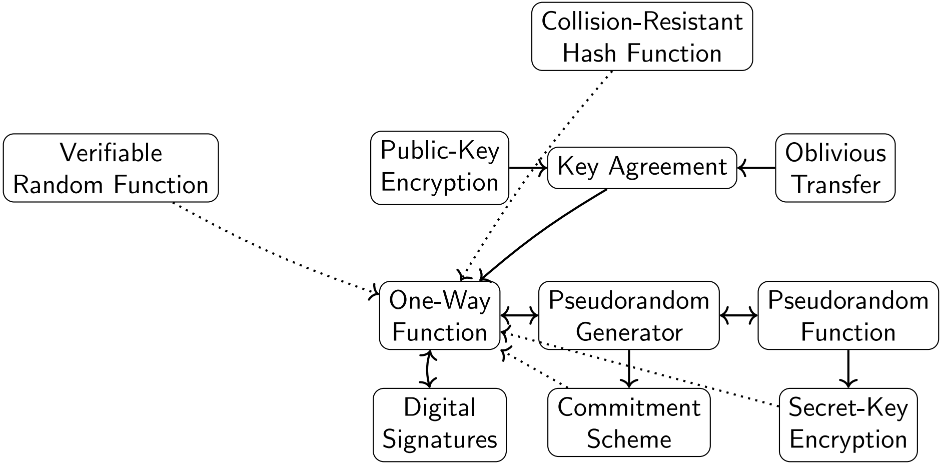

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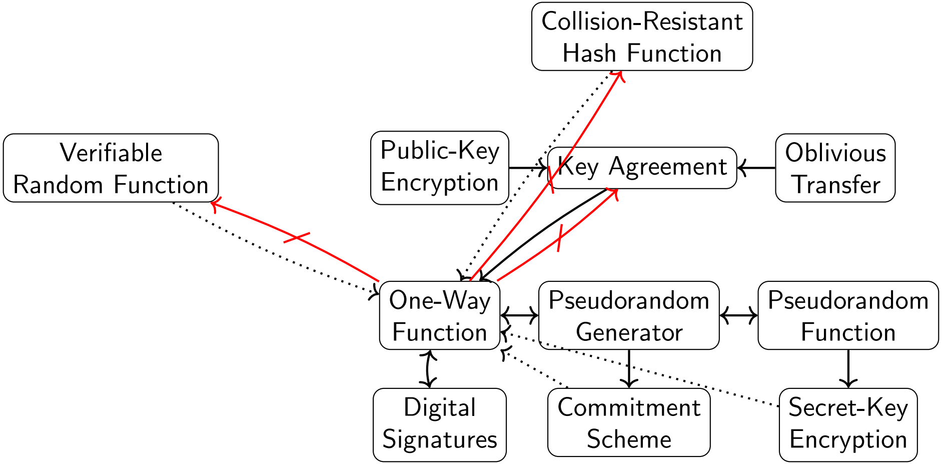

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Meta-reductions [BV98; Cor02; Pas11]

- Rules out NBN-reductions.
- Often requires special properties of the reduction (algebraicity, small loss, non-interactive games, etc.).

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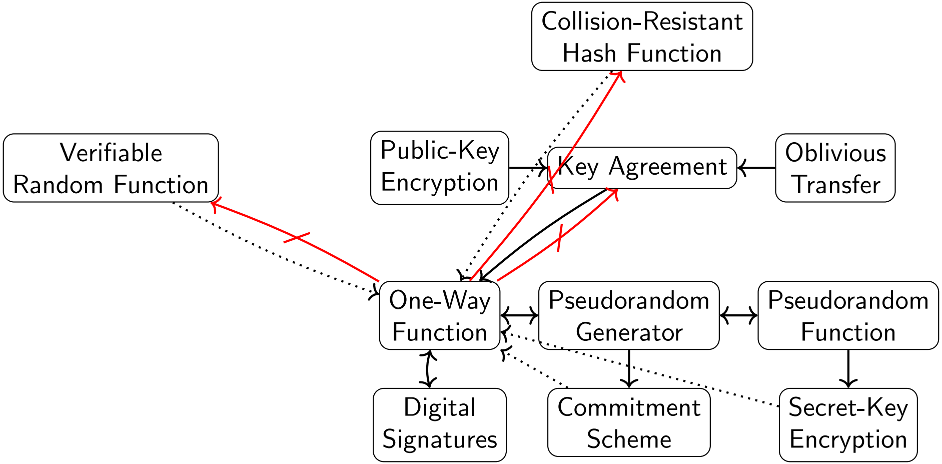

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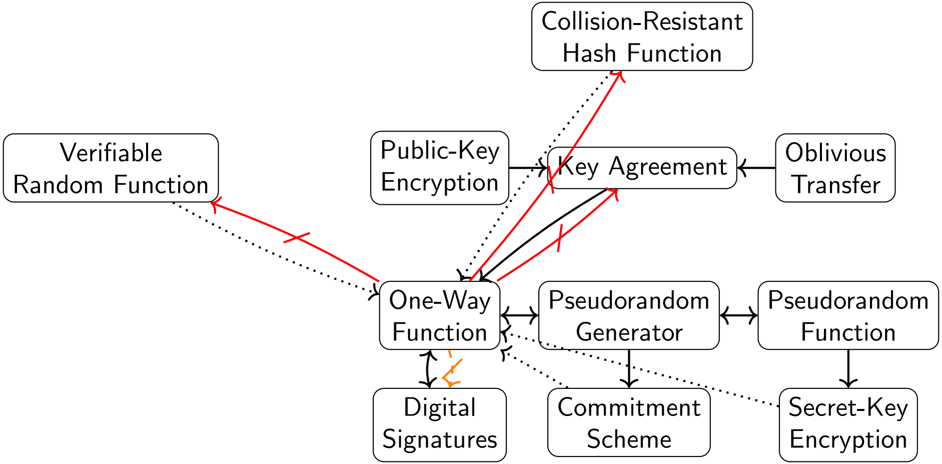

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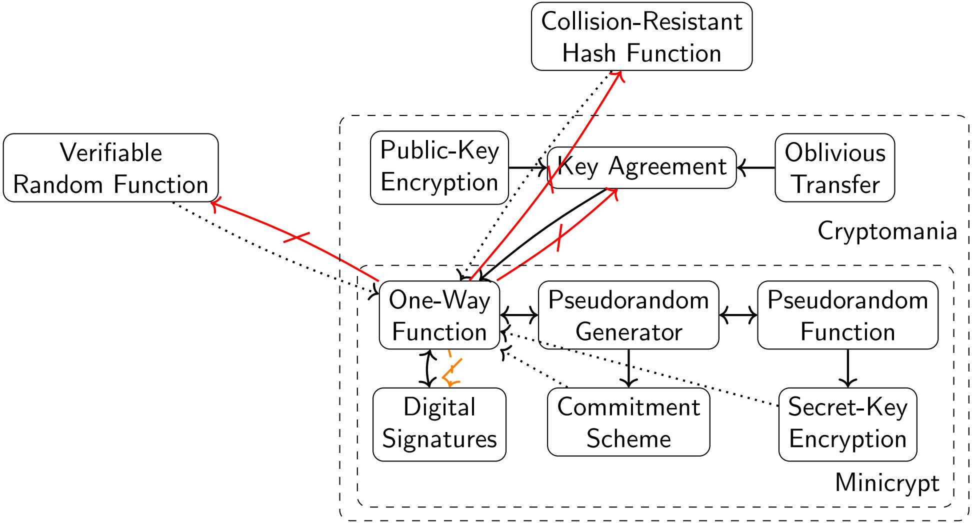

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- [BHK+22]: Meta-reductions for VRFs from standard group assumptions
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- [Bra24]: Non-black-box lower bounds for Levin–Kolmogorov complexity

Other works (not in the thesis)

- [BMM+23]: Tight setup bounds for secure multi-party computation with identifiable abort
- [BHK+24]: Tightly secure blind signatures in pairing-free groups
- [BFM24]: A formal treatment of key transparency
- [BNG+25]: Constrained VRFs from novel pairing-based assumptions

Definition of VRF

Verifiable Random Function [MRV99]

A *verifiable random function* (VRF) consists of three algorithms:

- $\mathsf{Gen}(1^\lambda) \to (\mathsf{vk}, \mathsf{sk})$
- $\mathsf{Eval}(\mathsf{sk}, x) \to (y \in \{0,1\}, \pi)$
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- Rules out algebraic NBN-reductions from various non-interactive assumptions to pairing-based VRFs with short proofs.
- For a set of natural VRFs constructions:
  the shorter the VRF proof, the stronger the assumption needed.

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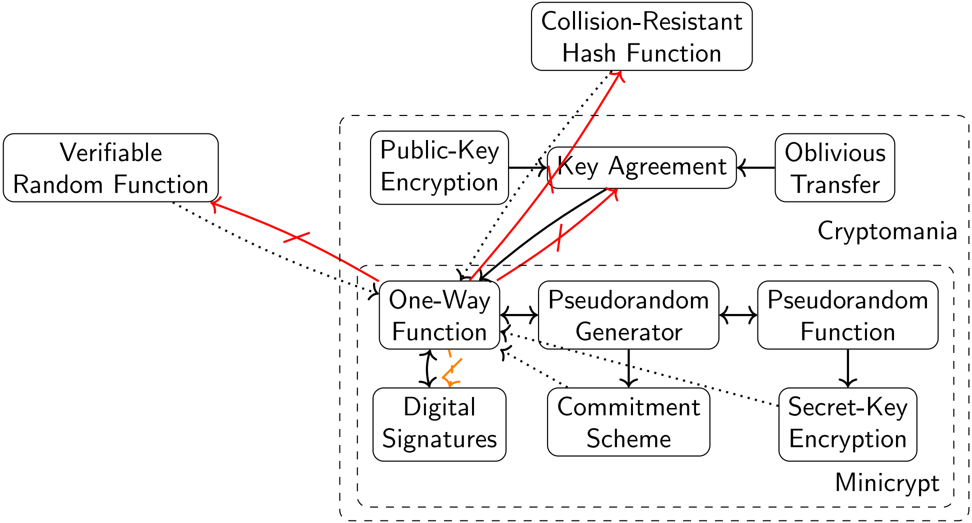

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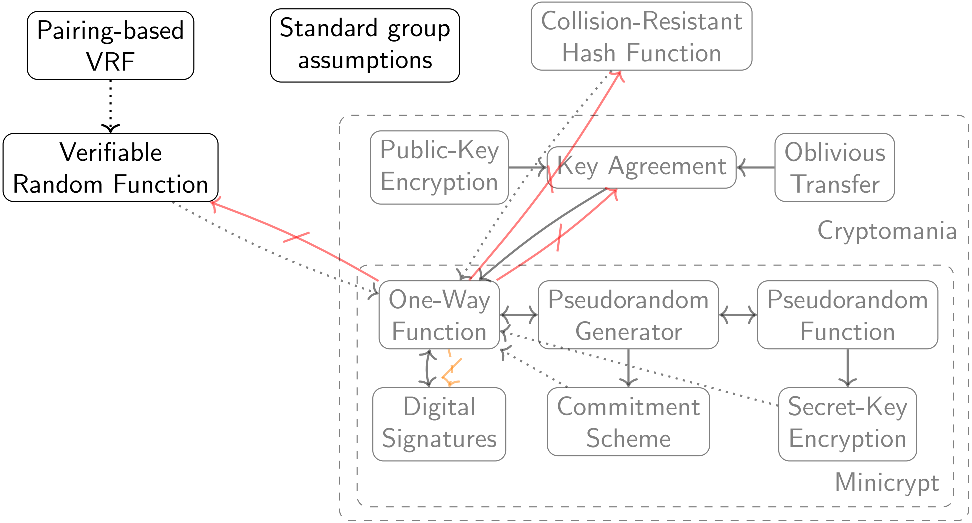

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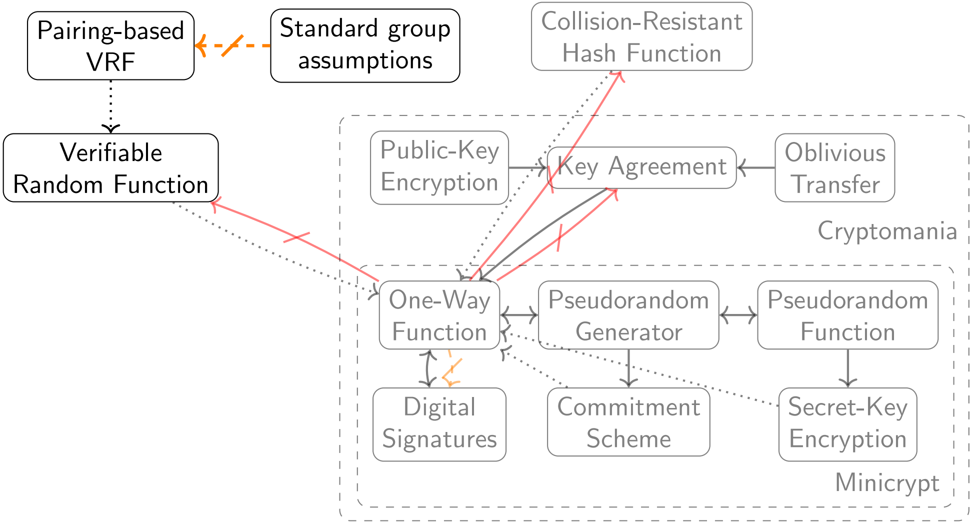

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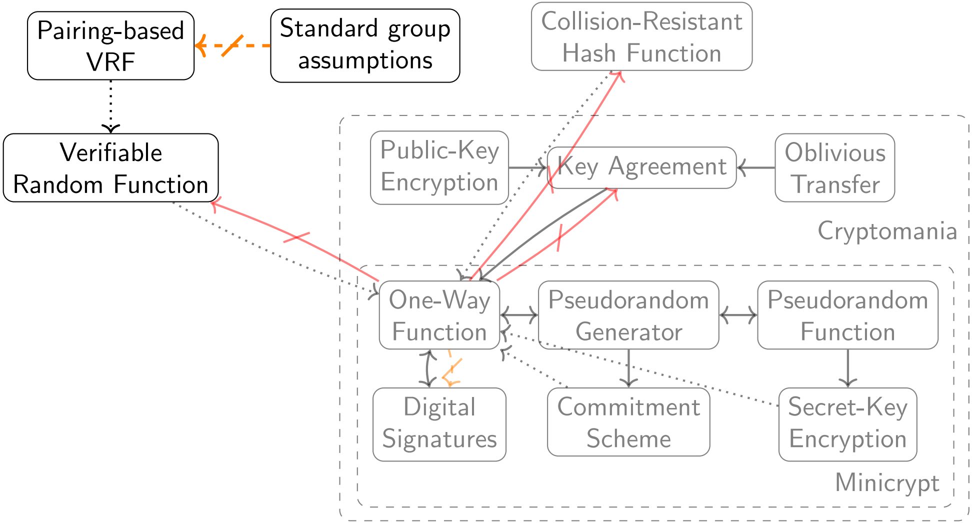

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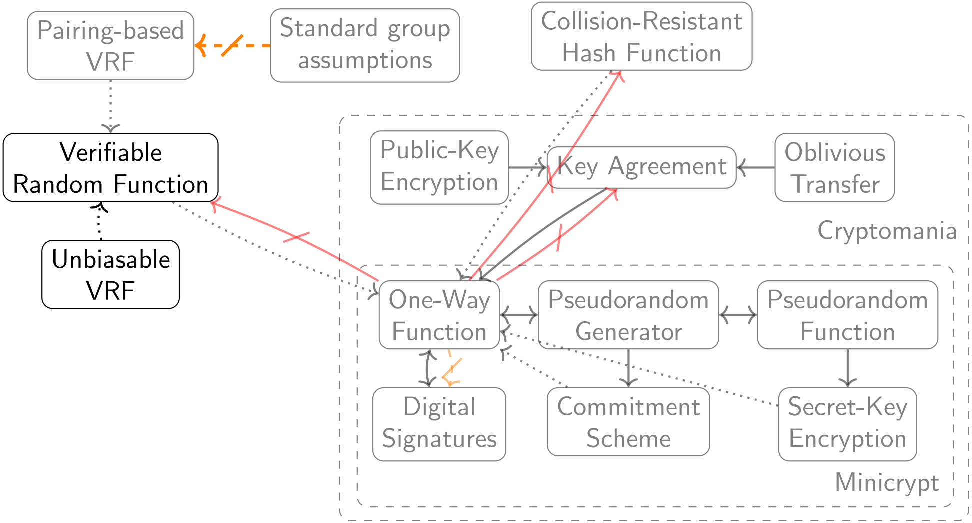

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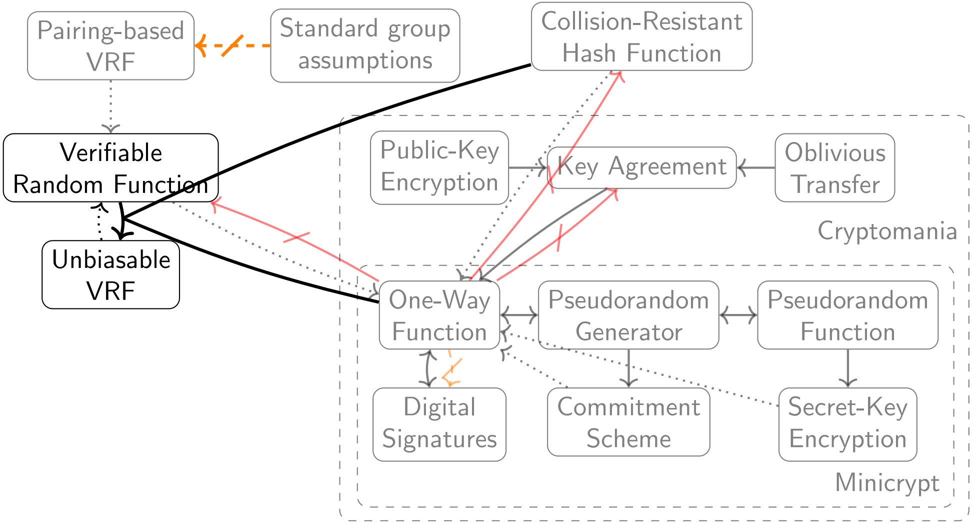

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- First unconditional lower bound MKtP $\notin$ DTIME$[\mathcal{O}(n)]$.
- Conditional lower bounds MKtP $\notin$ DTIME$[\mathfrak{t}(n)] \cap \mathrm{Heur}_{0,o(1/\mathfrak{t}(n))}$DTIME$[\mathcal{O}(n)]$.

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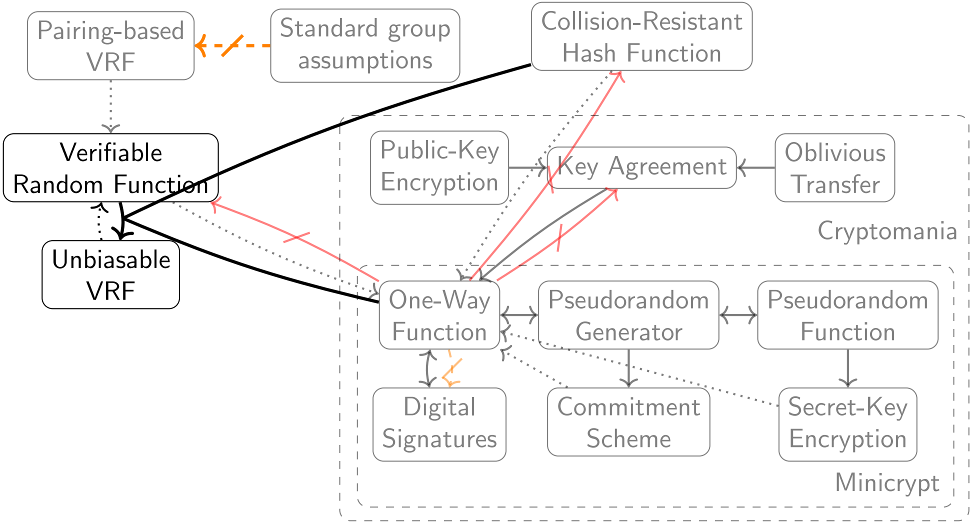

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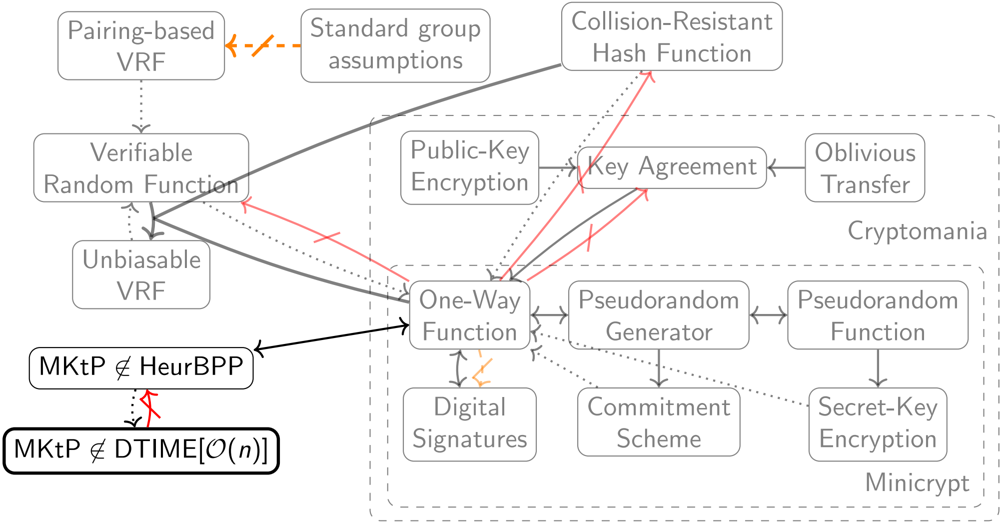

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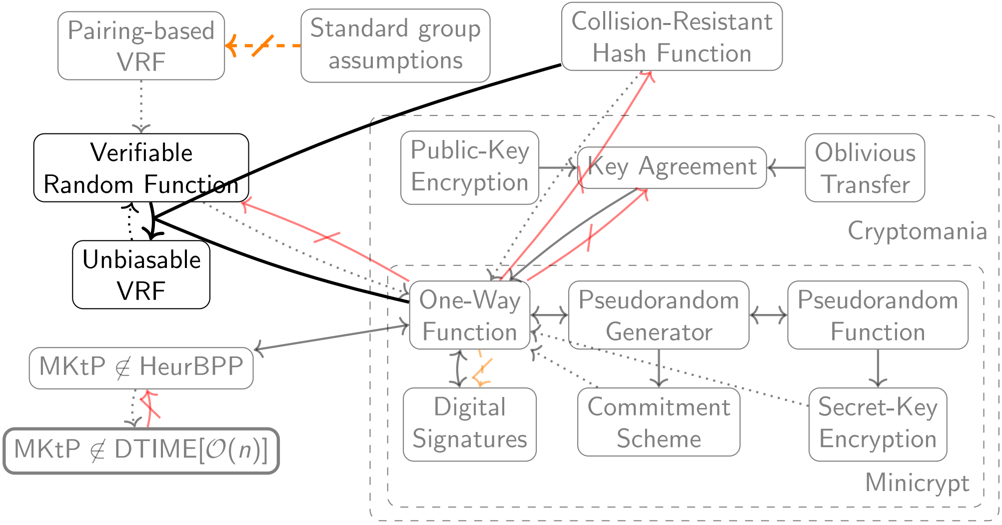

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- Unbiasability [GS24]:
  random preimage $\implies$ random image even for maliciously chosen vk

Simplified Unbiasability Game

$\mathcal{C}(1^\lambda)$ $\mathcal{A}(1^\lambda; r_\mathcal{A})$

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$x \leftarrow \{0,1\}^\lambda$

find y_0, π : $\mathsf{VRF.Verify}(\mathsf{vk}, x, y_0, \pi) = 1$

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$\xrightarrow{y_b}$

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$$\xrightarrow{\quad y_b \quad}$$

$$\xleftarrow{\quad b' \quad}$$

return $b' \stackrel{?}{=} b$

Construction Overview

Construction [Bra24]

$$\begin{pmatrix} \text{VRF} \\ \text{one-way permutation (OWP)} \\ \text{collision-resistant hash function (CRH)} \end{pmatrix} \implies \text{unbiasable VRF}$$

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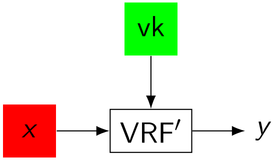
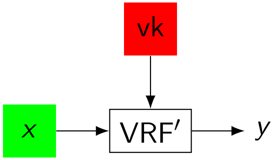
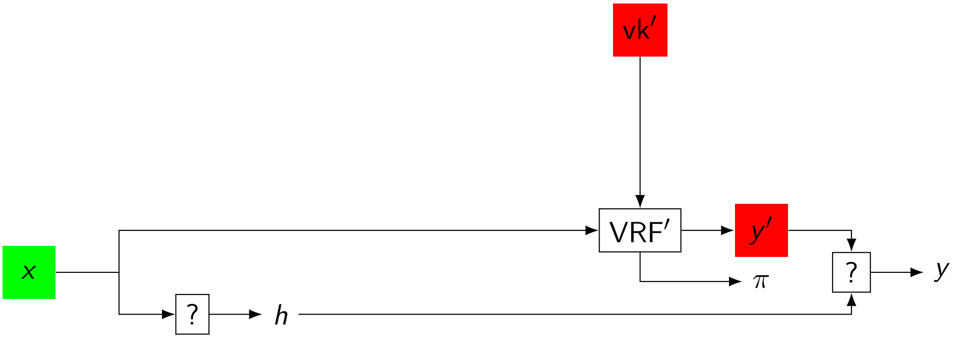
	

Construction

Pseudorandomness ☑ | Unbiasability ☐



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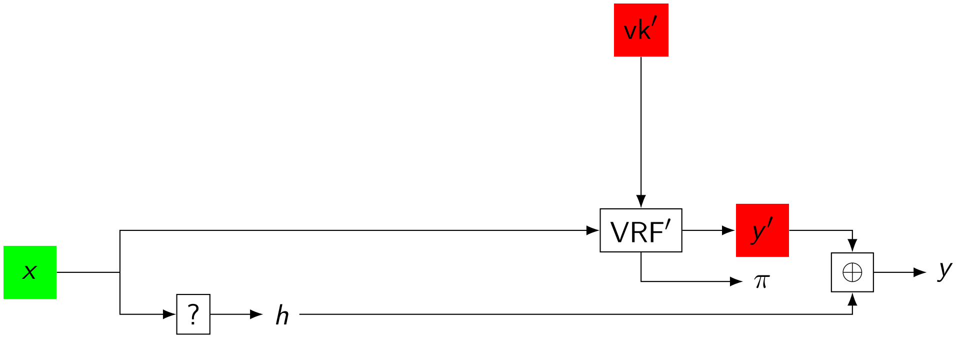

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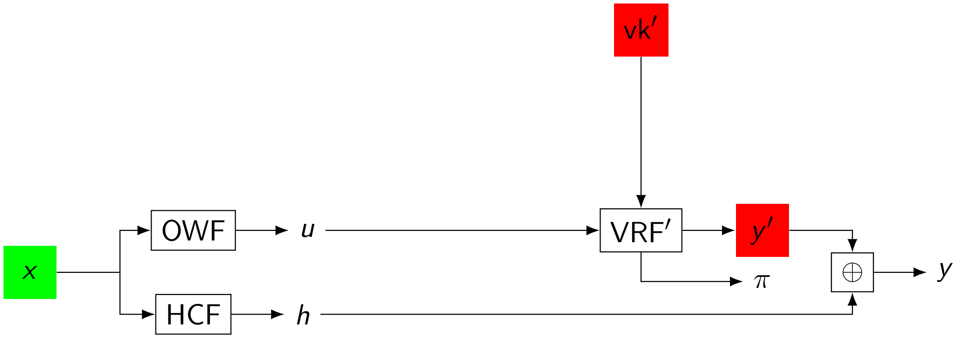

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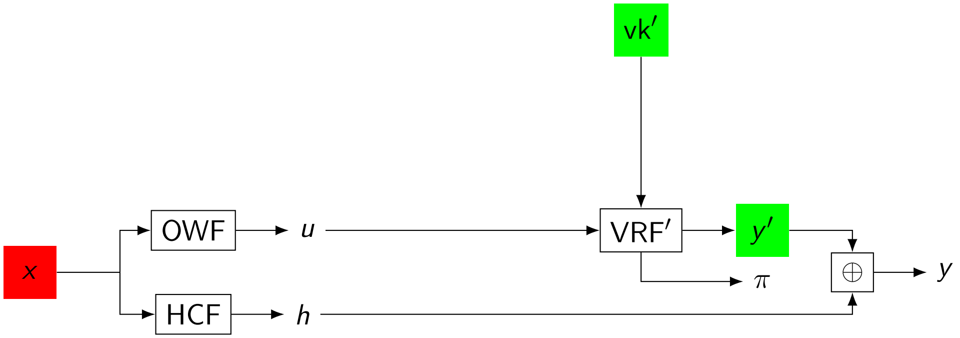

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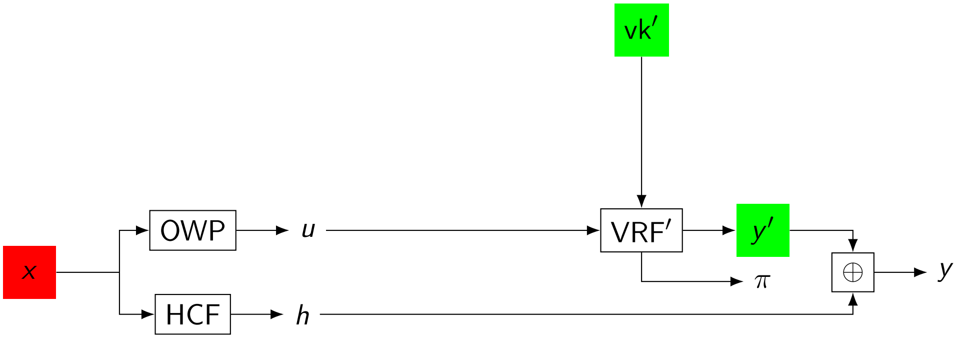

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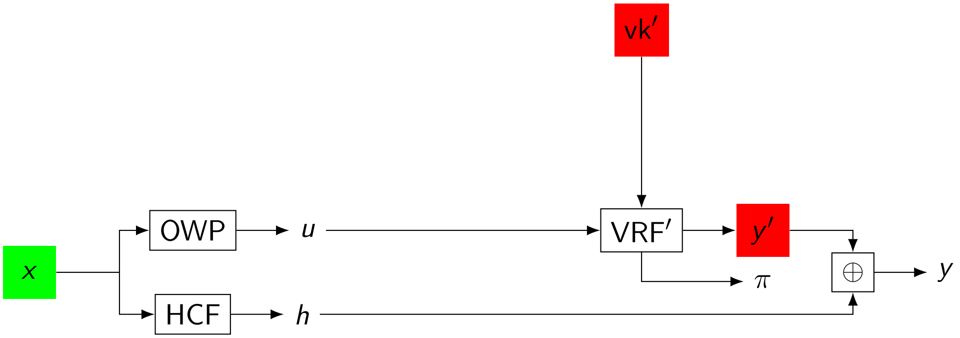

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$x \leftarrow \{0,1\}^\lambda$

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$\xrightarrow{\hspace{2em}y_b\hspace{2em}}$

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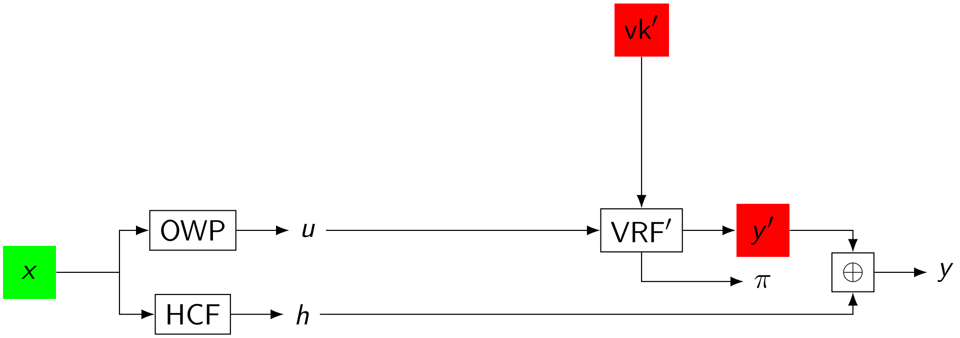

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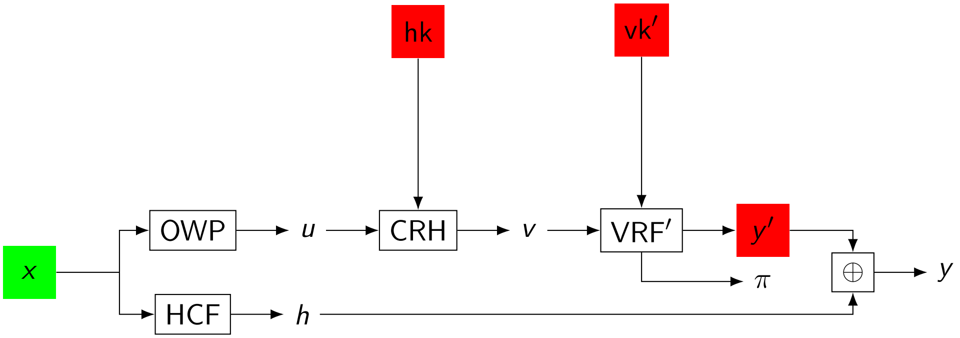

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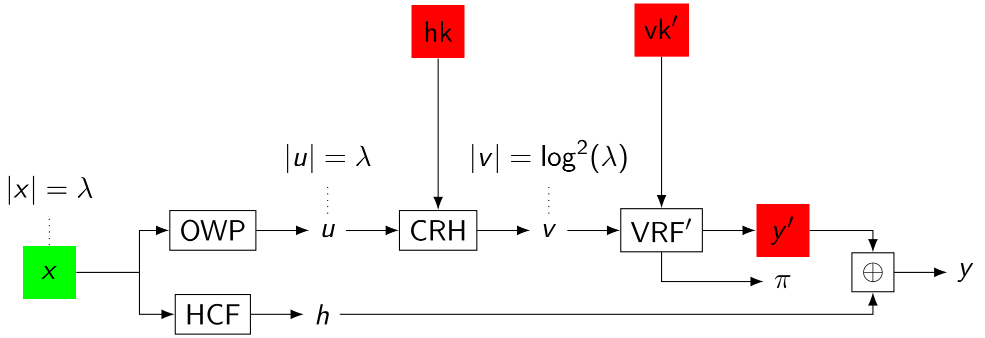

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$\mathcal{C}(1^\lambda)$ $\qquad\qquad$ $\mathcal{A}(1^\lambda; r_\mathcal{A})$

$\xleftarrow{\mathsf{vk}}$

$x \leftarrow \{0,1\}^\lambda$

find $y_0, \pi : \mathsf{VRF.Verify}(\mathsf{vk}, x, y_0, \pi) = 1$

$y_1 \leftarrow \{0,1\}$

$b \leftarrow \{0,1\}$

$\xrightarrow{y_b}$

$\xleftarrow{b'}$

return $b' \stackrel{?}{=} b$

Simplified Unbiasability Reduction

$\mathcal{C}(1^\lambda)$		$\mathcal{R}(1^\lambda)$		$\mathcal{A}(1^\lambda; r_{\mathcal{A}})$
$x \leftarrow \{0,1\}^\lambda$				
$u := \mathsf{OWP}(x)$				
$h := \mathsf{HCF}(u)$				
	$\xrightarrow{\quad u \quad}$		$\xleftarrow{\quad \mathsf{vk} = (\mathsf{hk}, \mathsf{vk}') \quad}$	
		$v := \mathsf{CRH}(\mathsf{hk}, u) \in \{0,1\}^{\log^2(\lambda)}$		
		find $y_0', \pi' : \mathsf{VRF}'.\mathsf{Verify}(\mathsf{vk}', v, y_0', \pi') = 1$		
			$\xrightarrow{\quad y := y_0' \overset{!}{=} y' \oplus h \quad}$	
			$\xleftarrow{\quad b' \quad}$	
		if $b' = 0$ **then** $h' := 0$		
		else $h' := 1$		
	$\xleftarrow{\quad h' \quad}$			
return $h' \overset{?}{=} h$				

Simplified Unbiasability Reduction

$\mathcal{C}(1^\lambda)$ | $\mathcal{R}_{\mathsf{ek}'}(1^\lambda)$ | $\mathcal{A}(1^\lambda; r_\mathcal{A})$

$x \leftarrow \{0,1\}^\lambda$

$u := \mathsf{OWP}(x)$

$h := \mathsf{HCF}(u)$

$\xrightarrow{\quad u \quad}$ $\qquad$ $\xleftarrow{\quad \mathsf{vk} = (\mathsf{hk}, \mathsf{vk}') \quad}$

$v := \mathsf{CRH}(\mathsf{hk}, u) \in \{0,1\}^{\log^2(\lambda)}$

$y'_0 := \mathsf{VRF}'.\mathsf{LatEval}(\mathsf{ek}', v)$

$\xrightarrow{\quad y := y'_0 \stackrel{!}{=} y' \oplus h \quad}$

$\xleftarrow{\quad b' \quad}$

if $b' = 0$ **then** $h' := 0$

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$\xleftarrow{\quad h' \quad}$

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BNN-Reduction with Polynomial Loss [Bra24]

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Latent VRF

A VRF VRF' is (perfectly) *latent* if there exists a deterministic polynomial-time algorithm $\mathsf{LatEval}'$ such that

$$\forall \mathsf{vk}' \; \exists \mathsf{ek}' \; \forall x \left(\begin{array}{lcl} y' := \mathsf{LatEval}'(\mathsf{ek}', x) \neq \bot & \Longrightarrow & y' \text{ is the valid image} \\ y' := \mathsf{LatEval}'(\mathsf{ek}', x) = \bot & \Longrightarrow & \text{no valid image exists} \end{array} \right).$$

Simplified Unbiasability Reduction

$\mathcal{C}(1^\lambda)$ | $\mathcal{R}(1^\lambda)$ | $\mathcal{A}(1^\lambda; r_{\mathcal{A}})$

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$v := \mathsf{CRH}(\mathsf{hk}, u) \in \{0,1\}^{\log^2(\lambda)}$

find y_0', π' : $\mathsf{VRF}'.\mathsf{Verify}(\mathsf{vk}', v, y_0', \pi') = 1$

$\xrightarrow{\quad y := y_0' \stackrel{!}{=} y' \oplus h \quad}$

$\xleftarrow{\quad b' \quad}$

if $b' = 0$ **then** $h' := 0$

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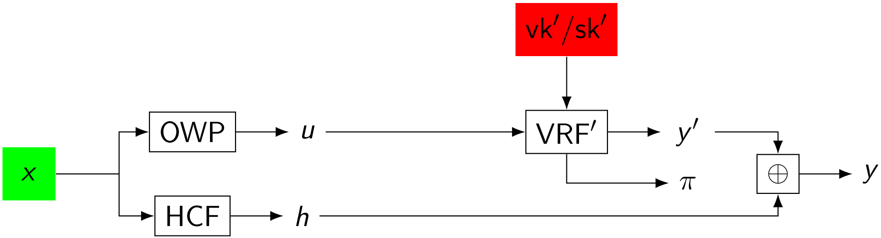

Contributions

Meta-reductions for VRFs [BHK$^+$22]

- Rules out algebraic NBN-reductions from various non-interactive assumptions to pairing-based VRFs with short proofs.
- For a set of natural VRFs constructions:
  the shorter the VRF proof, the stronger the assumption needed.

(Non-)black-box reductions for unbiasable VRFs [Bra25]

- BBB-reduction: injective OWF, CRH, VRF $\implies$ unbiasable VRF (subexp. loss)
- BNN-reduction: injective OWF, VRF $\implies$ unbiasable VRF (poly. loss)

Non-black-box lower bounds for Levin–Kolmogorov complexity [Bra24]

- First unconditional lower bound MKtP $\notin$ DTIME$[\mathcal{O}(n)]$.
- Conditional lower bounds MKtP $\notin$ DTIME$[\mathfrak{t}(n)] \cap \mathrm{Heur}_{0,\mathrm{o}(1/\mathfrak{t}(n))}$DTIME$[\mathcal{O}(n)]$.

Conclusion

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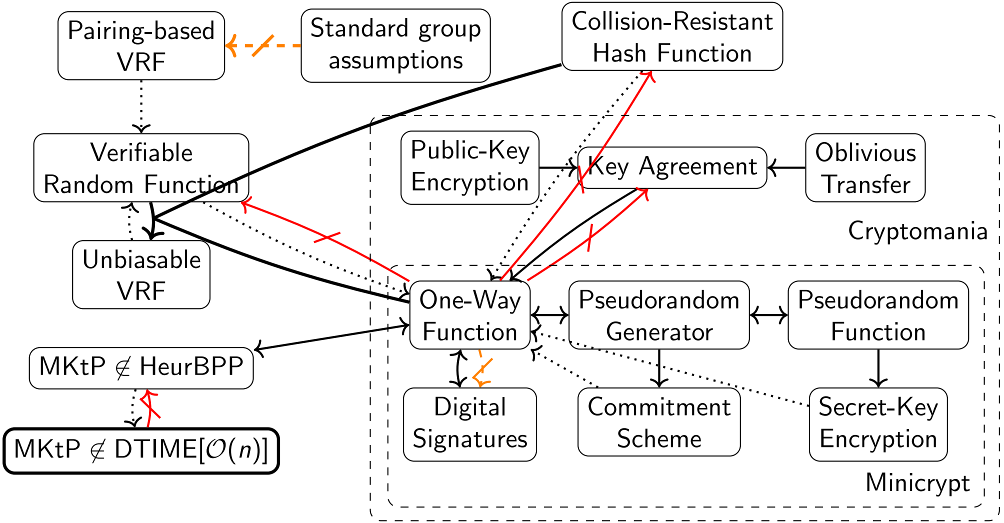
- BBB-constructions are conceptually simple but often lack feasibility or efficiency.
- Separations can formally explain insufficiencies of BBB-constructions.
- Obfuscation-/NIZK/NIWI-based constructions are often NAP.
- CNP-constructions are under-explored (non-black-box use of the adversary).

Construction cookbook

1. Try a BBB-construction.
2. Try a meta-reduction or oracle separation.
3. Try a CNP- or NAP-construction.

Thank you!

Some Cryptographic Primitives



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Lower bounds for Levin–Kolmogorov complexity [Bra24]

Context

- Levin's notion [Lev84] of Kolmogorov complexity:
$$\mathrm{Kt}(x) := \min\{|\Pi| + \lceil \log(t) \rceil \mid \Pi(\varepsilon) = x \text{ in } t \text{ steps}\}$$
- Set of low-complexity strings: $\mathsf{MKtP} := \{x \mid \mathrm{Kt}(x) \leq |x|\} \in \mathsf{E} := \mathsf{DTIME}[2^{\mathcal{O}(n)}]$
- Liu and Pass [LP21]:
$$\exists\mathsf{OWF} \iff \mathsf{MKtP} \notin \mathsf{HeurBPP}$$
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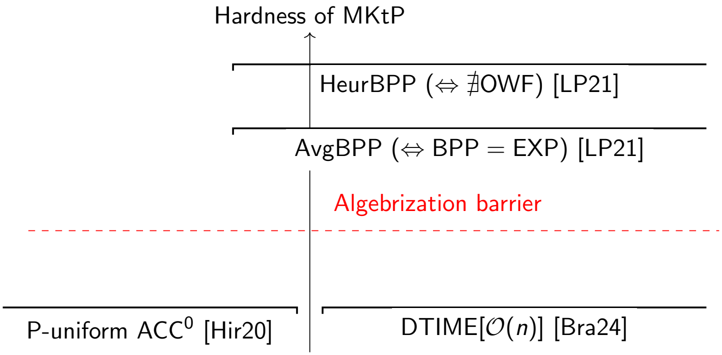

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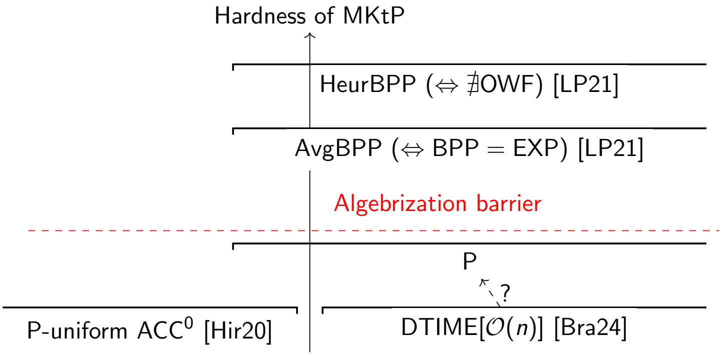

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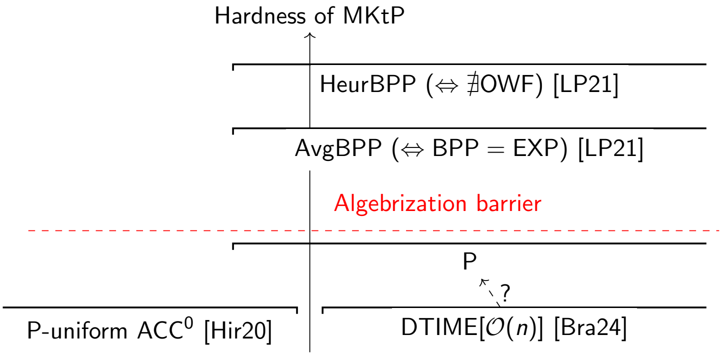

[RS22] oracle: approx. Kt within factor $(1+\epsilon)$ in linear time and BPP $=$ EXP