Cboe VIX®

White Paper

Cboe Volatility Index®

Contents

Introduction

In 1993, Cboe Global Markets, Incorporated® (Cboe®) introduced the Cboe Volatility Index® (VIX® Index), which was originally designed to measure the market's expectation of 30-day volatility implied by at-the-money S&P 100® Index (OEX® Index) option prices. The VIX Index soon became the premier benchmark for U.S. stock market volatility. It is regularly featured in the Wall Street Journal, Barron's and other leading financial publications, as well as business news shows on CNBC, Bloomberg TV and CNN/Money, where the VIX Index is often referred to as the "fear gauge."

Ten years later in 2003, Cboe together with Goldman Sachs, updated the VIX Index to reflect a new way to measure expected volatility, one that continues to be widely used by financial theorists, risk managers and volatility traders alike. The new VIX Index is based on the S&P 500® Index (SPX℠), the core index for U.S. equities, and estimates expected volatility by aggregating the weighted prices of SPX puts and calls over a wide range of strike prices. By supplying a script for replicating volatility exposure with a portfolio of SPX options, this new methodology transformed the VIX Index from an abstract concept into a practical standard for trading and hedging volatility.

In 2014, Cboe enhanced the VIX Index to include series of SPX Weeklys℠. First introduced by Cboe in 2005, weekly options are now available on hundreds of indexes, equities, ETFs and ETNs and have become a very popular and actively-traded risk management tool. Today, SPX Weeklys account for one-third of all SPX options traded, and average close to 350,000 contracts traded per day[1].

The inclusion of SPX Weeklys allows the VIX Index to be calculated with S&P 500 Index option series that most precisely match the 30-day target timeframe for expected volatility that the VIX Index is intended to represent. Using SPX options with more than 23 days and less than 37 days to expiration ensures that the VIX Index will always reflect an interpolation of two points along the S&P 500 volatility term structure.

Cboe began dissemination of the VIX Index outside of U.S. trading hours in April 2016. The VIX index is now available during "extended trading hours" between 2 a.m. and 8:15 a.m CT, as well as during regular trading hours between 8:30 a.m. and 3:15 p.m. CT. As part of the VIX Index expansion, Cboe implemented a smoothing algorithm for VIX values disseminated during both extended and regular market hours.

Volatility as a tradable asset: VIX Futures & Options

On March 24, 2004, Cboe introduced the first exchange-traded VIX futures contract on its new, all-electronic Cboe Futures Exchange℠ (CFE®). Two years later in February 2006, Cboe launched VIX options, the most successful new product in Cboe history. In 2015, combined trading activity in VIX options and futures grew to nearly 800,000 contracts per day.

The negative correlation of volatility to stock market returns is well documented and suggests a diversification benefit to including volatility in an investment portfolio. VIX futures and options are designed to deliver pure volatility exposure in a single, efficient package. Cboe/CFE provides a continuous, liquid and transparent market for VIX products that are available to all investors from the smallest retail trader to the largest institutional money managers and hedge funds.

[1] *Based on 2015 Volume.*

Beyond the VIX Index

In addition to the VIX Index, Cboe calculates several other broad market volatility indexes including the Cboe Short-Term Volatility Index (VXST℠) - which reflects 9-day expected volatility of the S&P 500 Index, the Cboe S&P 500® 3-Month Volatility Index (VXV℠) and the Cboe S&P 500® 6-Month Volatility Index (VXMT℠). Cboe also calculates the Nasdaq-100® Volatility Index (VXN℠), Cboe DJIA® Volatility Index (VXD℠) and the Cboe Russell 2000® Volatility Index (RVX℠). Currently, RVX futures are listed on CFE and RVX options trade on Cboe.

Historical Prices: The VIX Index and Other Volatility Indexes

Perhaps one of the most valuable features of the VIX Index is the existence of more than 25 years of historical prices. This extensive data set provides investors with a useful perspective of how option prices have behaved in response to a variety of market conditions. Price history for the original Cboe Volatility Index (VXO) based on OEX options is available from 1986 to the present. Cboe has created a similar historical record for the new VIX Index dating back to 1990 so that investors can compare the new VIX Index with VXO, which reflects information about the volatility "skew" or "smile." Historical prices for the VIX Index, VXO and Cboe's other volatility indexes may be found on the Cboe website at http://www.cboe.com/micro/IndexSites.aspx under Cboe Volatility Indexes.

The VIX Index Calculation: Step-by-Step

Stock indexes, such as the S&P 500, are calculated using the prices of their component stocks. Each index employs rules that govern the selection of component securities and a formula to calculate index values.

The VIX Index is a volatility index comprised of options rather than stocks, with the price of each option reflecting the market's expectation of future volatility. Like conventional indexes, the VIX Index calculation employs rules for selecting component options and a formula to calculate index values.

The generalized formula used in the VIX Index calculation[§] is:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \quad \textbf{(1)}$$

Where

σ	$VIX/100 \Rightarrow \textbf{VIX} = \sigma \times 100$
T	Time to expiration
F	Forward index level derived from index option prices
K_0	First strike below the forward index level, F
K_i	Strike price of i^{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$.
ΔK_i	Interval between strike prices – half the difference between the strike on either side of K_i: $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$
R	Risk-free interest rate to expiration
$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike K_i.

[§] *Please see "More than you ever wanted to know about volatility swaps" by Kresimir Demeterfi, Emanuel Derman, Michael Kamal and Joseph Zou, Goldman Sachs Quantitative Strategies Research Notes, March 1999.*

Getting Started

The VIX Index measures 30-day expected volatility of the S&P 500 Index. The components of the VIX Index are near- and next-term put and call options with more than 23 days and less than 37 days to expiration. These include SPX options with "standard" 3rd Friday expiration dates and "weekly" SPX options that expire every Friday, except the 3rd Friday of each month. Once each week, the SPX options used to calculate the VIX Index "roll" to new contract maturities. For example, on the second Tuesday in October, the VIX Index would be calculated using SPX options expiring 24 days later (i.e., "near-term") and 31 days later (i.e., "next-term"). On the following day, the SPX options that expire in 30 calendar days would become the "near-term" options and SPX options that expire in 37 calendar days would be the "next-term" options.

In this hypothetical example, the near-term options are "standard" SPX options with 25 days to expiration, the next-term options are P.M.-settled SPX Weeklys with 32 days to expiration; and the calculation reflects prices observed at 9:46 a.m. Chicago time. For the purpose of calculating time to expiration, "standard" SPX options are deemed to expire at the open of trading on SPX settlement day - the third Friday of the month , and "weekly" SPX options are deemed to expire at the close of trading (i.e., 3:00 p.m. CT).

The VIX Index calculation measures time to expiration, T, in calendar days and divides each day into minutes in order to replicate the precision that is commonly used by professional option and volatility traders. The time to expiration is given by the following expression:

$$T = \{ M_{\text{Current day}} + M_{\text{Settlement day}} + M_{\text{Other days}} \} / \text{Minutes in a year}$$

Where

$M_{\text{Current day}}$	minutes remaining until midnight of the current day
$M_{\text{Settlement day}}$	minutes from midnight until 8:30 a.m. for "standard" SPX expirations; or minutes from midnight until 3:00 p.m. for "weekly" SPX expirations
$M_{\text{Other days}}$	total minutes in the days between current day and expiration day

Using 9:46 a.m. as the time of the calculation, T for the near-term and next-term options, T_1 and T_2, respectively, is:

$$T_1 = \{854 + 510 + 34{,}560\} / 525{,}600 = 0.0683486$$

$$T_2 = \{854 + 900 + 44{,}640\} / 525{,}600 = 0.0882686$$

The risk-free interest rates, R_1 and R_2, are yields based on U.S. Treasury yield curve rates (commonly referred to as "Constant Maturity Treasury" rates or CMTs), to which a cubic spline is applied to derive yields on the expiration dates of relevant SPX options. As such, the VIX Index calculation may use different risk-free interest rates for near- and next-term options. In this example, assume that $R_1 = 0.0305\%$ for the near-term options and that $R_2 = 0.0286\%$ for the next-term options. Note in this example, T_2 uses a value of 900 for $M_{\text{Settlement day}}$, which reflects the 3:00 p.m. expiration time of the next-term SPX Weeklys options. Since many of the interim calculations are repetitive, only representative samples appear below. The complete set of SPX option data and calculations may be found in *Appendix 1*.

Step 1: Select the options to be used in the VIX Index calculation

The selected options are out-of-the-money SPX calls and out-of-the-money SPX puts centered around an at-the-money strike price, K_0. Only SPX options quoted with non-zero bid prices are used in the VIX Index calculation.

One important note: as volatility rises and falls, the strike price range of options with non-zero bids tends to expand and contract. As a result, the number of options used in the VIX Index calculation may vary from month-to-month, day-to-day and possibly, even minute-to-minute.

For each contract month:

- Determine the forward SPX level, F, by identifying the strike price at which the absolute difference between the call and put prices is smallest. The call and put prices in the following table reflect the average of each option's bid / ask quotation. As shown below, the difference between the call and put prices is smallest at the 1965 strike for the near- and the 1960 strike for the next-term options.

Near Term Options			
Strike Price	**Call**	**Put**	**Difference**
1940	38.45	15.25	23.20
1945	34.70	16.55	18.15
1950	31.10	18.25	12.85
1955	27.60	19.75	7.85
1960	24.25	21.30	2.95
1965	**21.05**	**23.15**	**2.10**
1970	18.10	25.05	6.95
1975	15.25	27.30	12.05
1980	12.75	29.75	17.00

Next Term Options			
Strike Price	**Call**	**Put**	**Difference**
1940	41.05	18.80	22.25
1945	37.45	20.20	17.25
1950	34.05	21.60	12.45
1955	30.60	23.20	7.40
1960	**27.30**	**24.90**	**2.40**
1965	24.15	26.90	2.75
1970	21.10	28.95	7.85
1975	18.30	31.05	12.75
1980	15.70	33.50	17.80

Using the **1965** call and put in the near-term, and the **1960** call and put in the next-term contract applied to the formula:

$$F = \text{Strike Price} + e^{RT} \times (\text{Call Price} - \text{Put Price})$$

the forward index prices, F_1 and F_2, for the near- and next-term options, respectively, are:

$$F_1 = 1965 + e^{(0.000305 \times 0.0683486)} \times (21.05 - 23.15) = 1962.89996$$

$$F_2 = 1960 + e^{(0.000286 \times 0.0882686)} \times (27.30 - 24.90) = 1962.40006$$

- Next, determine K_0 - the strike price immediately below the forward index level, F - for the near- and next-term options. In this example, **$K_{0,1}$ = 1960 and $K_{0,2}$ = 1960.**
- Select out-of-the-money put options with strike prices < K_0. Start with the put strike immediately lower than K0 and move to successively lower strike prices. Exclude any put option that has a bid price equal to zero (i.e., no bid). As shown below, once two puts with consecutive strike prices are found to have zero bid prices, no puts with lower strikes are considered for inclusion. (Note that the 1350 and 1355 put options are not included despite having non-zero bid prices.)

Put Strike	Bid	Ask	Include?
1345	0	0.15	Not considered following two zero bids
1350	**0.05**	**0.15**	
1355	**0.05**	**0.35**	
1360	0	0.35	No
1365	0	0.35	No
1370	0.05	0.35	Yes
1375	0.1	0.15	Yes
1380	0.1	0.2	Yes

- Next, select out-of-the-money call options with strike prices > K_0. Start with the call strike immediately higher than K_0 and move to successively higher strike prices, excluding call options that have a bid price of zero. As with the puts, once two consecutive call options are found to have zero bid prices, no calls with higher strikes are considered. (Note that the 2225 call option is not included despite having a non-zero bid price.)

Call Strike	Bid	Ask	Include?
2095	0.05	0.35	Yes
2100	0.05	0.15	Yes
2120	0	0.15	No
2125	0.05	0.15	Yes
2150	0	0.1	No
2175	0	0.05	No
2200	0	0.05	Not considered following two zero bids
2225	**0.05**	**0.1**	
2250	0	0.05	

- Finally, select **both** the put and call with strike price K_0. Notice that two options are selected at K_0, while a single option, either a put or a call, is used for every other strike price.
- The following table contains the options used to calculate the VIX Index in this example. The VIX Index uses the average of quoted bid and ask, or mid-quote, prices for each option selected. The K_0 put and call prices are averaged to produce a single value. The price used for the 1960 strike in the near-term is, therefore, (24.25 + 21.30)/2 = 22.775; and the price used in the next-term is (27.30 + 24.90)/2 = 26.10.

Near term Strike	Option Type	Mid-quote Price
1370	Put	0.2
1375	Put	0.125
1380	Put	0.15
.	.	.
1950	Put	18.25
1955	Put	19.75
1960	**Put/Call Average**	**22.775**
1965	Call	21.05
1970	Call	18.1
.	.	.
2095	Call	0.2
2100	Call	0.1
2125	Call	0.1

Next term Strike	Option Type	Mid-quote Price
1275	Put	0.075
1325	Put	0.15
1350	Put	0.15
.	.	.
1950	Put	21.60
1955	Put	23.20
1960	**Put/Call Average**	**26.10**
1965	Call	24.15
1970	Call	21.10
.	.	.
2125	Call	0.1
2150	Call	0.1
2200	Call	0.08

Step 2: Calculate volatility for both near-term and next-term options

Applying the VIX formula (1) to the near-term and next-term options with time to expiration of T_1 and T_2, respectively, yields:

$$\sigma^2_1 = \frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_1 T_1} Q(K_i) - \frac{1}{T_1}\left[\frac{F_1}{K_0} - 1\right]^2$$

$$\sigma^2_2 = \frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_2 T_2} Q(K_i) - \frac{1}{T_2}\left[\frac{F_2}{K_0} - 1\right]^2$$

The VIX Index is an amalgam of the information reflected in the prices of all of the selected options. The contribution of a single option to the VIX value is proportional to ΔK and the price of that option, and inversely proportional to the square of the option's strike price.

Generally, ΔK_i is half the difference between the strike prices on either side of K_i. For example, the ΔK for the next-term 1325 Put is 37.5: $\Delta K_{1325\ Put} = (1350 - 1275)/2$. At the upper and lower edges of any given strip of options, ΔK_i is simply the difference between K_i and the adjacent strike price. In this example, the 1370 Put is the lowest strike in the strip of near-term options and 1375 is the adjacent strike. Therefore, $\Delta K_{1370\ Put} = 5$ (i.e., 1375 – 1370).

The contribution of the near-term 1370 Put is given by:

$$\frac{\Delta K_{1370\,Put}}{K^2_{1370\,Put}} e^{R_1 T_1} Q(1370\,Put)$$

$$\frac{\Delta K_{1370\,Put}}{K^2_{1370\,Put}} e^{R_1 T_1} Q(1370\,Put) = \frac{5}{1370^2} e^{.000305\ (0.0683486)} (0.20) = 0.0000005328$$

A similar calculation is performed for each option. The resulting values for the near-term options are then summed and multiplied by $2/T_1$. Likewise, the resulting values for the next-term options are summed and multiplied by $2/T_2$. The table below summarizes the results for each strip of options.

Near term Strike	Option Type	Mid-quote Price	Contribution by Strike
1370	Put	0.2	0.0000005328
1375	Put	0.125	0.0000003306
1380	Put	0.15	0.0000003938
.	.	.	.
1950	Put	18.25	0.0000239979
1955	Put	19.75	0.0000258376
1960	**Put/Call Average**	**22.775**	**0.0000296432**
1965	Call	21.05	0.0000272588
1970	Call	18.1	0.0000233198
.	.	.	.
2095	Call	0.2	0.0000002278
2100	Call	0.1	0.0000003401
2125	Call	0.1	0.0000005536
$\frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_1 T_1} Q(K_i)$			**0.018495**

Near term Strike	Option Type	Mid-quote Price	Contribution by Strike
1275	Put	0.075	0.0000023069
1325	Put	0.15	0.0000032041
1350	Put	0.15	0.0000020577
.	.	.	.
1950	Put	21.6	0.0000284031
1955	Put	23.2	0.0000303512
1960	**Put/Call Average**	**26.1**	**0.0000339711**
1965	Call	24.15	0.0000312732
1970	Call	21.1	0.0000271851
.	.	.	.
2125	Call	0.1	0.0000005536
2150	Call	0.1	0.0000008113
2200	Call	0.075	0.0000007748
$\frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_2 T_2} Q(K_i)$			**0.018838**

Next, calculate $\frac{1}{T}\left[\frac{F}{K_0}-1\right]^2$ for the near-term (T_1) and next-term (T_2):

$$\frac{1}{T_1}\left[\frac{F_1}{K_0}-1\right]^2 = \frac{1}{0.0683486}\left[\frac{1962.89996}{1960}-1\right]^2 = 0.00003203$$

$$\frac{1}{T_2}\left[\frac{F_2}{K_0}-1\right]^2 = \frac{1}{0.0882686}\left[\frac{1962.40006}{1960}-1\right]^2 = 0.00001699$$

Now calculate σ^2_1 and σ^2_2:

$$\sigma^2_1 = \frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_1 T_1} Q(K_i) - \frac{1}{T_1}\left[\frac{F_1}{K_0}-1\right]^2 = 0.018495 - 0.00003203 = \mathbf{0.01846292}$$

$$\sigma^2_2 = \frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_2 T_2} Q(K_i) - \frac{1}{T_2}\left[\frac{F_2}{K_0}-1\right]^2 = 0.\ 018838 - 0.\ 00001699 = \mathbf{0.01882101}$$

Step 3

Calculate the 30-day weighted average of σ^2_1 and σ^2_2. Then take the square root of that value and multiply by 100 to get the VIX value.

$$\mathbf{VIX = 100 \times} \sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2}-N_{30}}{N_{T_2}-N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{30}-N_{T_1}}{N_{T_2}-N_{T_1}}\right]\right\}\times\frac{N_{365}}{N_{30}}}$$

The inclusion of SPX Weeklys in the VIX Index calculation means that the near-term options will always have more than 23 days to expiration and the next-term options always have less than 37 days to expiration, so the resulting VIX value will always reflect an interpolation of σ^2_1 and σ^2_2 ; i.e., each individual weight is less than or equal to 1 and the sum of the weights equals 1.

Returning to the example…

N_{T1} = number of minutes to settlement of the near-term options (35,924)
N_{T2} = number of minutes to settlement of the next-term options (46,394)
N_{30} = number of minutes in 30 days (30 x 1,440 = 43,200)
N_{365} = number of minutes in a 365-day year (365 x 1,440 = 525,600)

$$\mathbf{VIX = 100 \times} \sqrt{\left\{0.0683486\times 0.0184629\times\left[\frac{46{,}394-43{,}200}{46{,}394-12{,}960}\right] + 0.0882686\times 0.018821\times\left[\frac{43{,}200-35{,}924}{53{,}280-35{,}924}\right]\right\}\times\frac{525{,}600}{43{,}200}}$$

VIX = 100 × 0.13685821 = 13.69

Related VIX Values

In addition to the VIX Index, Cboe publishes the Cboe VIX Indicative Bid Index ("VWB"), a VIX value based on SPX bid quotations, and the Cboe VIX Indicative Ask Index ("VWA"), a VIX value based on SPX option ask quotations. These values provide a market estimate of SPX option bid-ask "spreads" expressed in volatility terms. Cboe also publishes volatility information related to the near-term and next-term VIX "components", σ_1 and σ_2, under ticker symbols "VIN" (Cboe Near-Term VIX Index) and "VIF" (Cboe Far-Term VIX Index) every 15 seconds during each Cboe trading day.

Appendix 1: Complete SPX Option Data Used in Sample VIX Index Calculation

Option Series included in the VIX Index calculation are highlighted.

Near-Term Options				
Strike	Calls		Puts	
	Bid	Ask	Bid	Ask
800	1160.90	1164.40	0.00	0.10
900	1060.90	1064.50	0.00	0.10
1000	961.00	964.50	0.00	0.10
1050	911.00	914.50	0.00	0.10
1100	861.00	864.60	0.00	0.05
1125	836.00	839.60	0.00	0.05
1150	811.00	814.60	0.00	0.05
1175	786.10	789.60	0.00	0.05
1200	761.10	764.60	0.00	0.05
1220	741.10	744.60	0.00	0.10
1225	736.10	739.60	0.00	0.05
1240	721.10	724.60	0.00	0.10
1250	711.10	714.60	0.00	0.05
1260	701.10	704.60	0.00	0.10
1270	691.10	694.60	0.00	0.10
1275	686.10	689.60	0.00	0.10
1280	681.10	684.60	0.00	0.10
1290	671.10	674.70	0.00	0.10
1300	661.10	664.70	0.05	0.10
1305	656.10	659.70	0.00	0.10
1310	651.10	654.70	0.00	0.10
1315	646.10	649.70	0.00	0.10
1320	641.20	644.70	0.00	0.10
1325	636.20	639.70	0.05	0.10
1330	631.20	634.70	0.00	0.10
1335	626.20	629.70	0.00	0.15
1340	621.20	624.70	0.00	0.15
1345	616.20	619.70	0.00	0.15
1350	611.20	614.70	0.05	0.15
1355	606.20	609.70	0.05	0.35
1360	601.20	604.70	0.00	0.35
1365	596.20	599.70	0.00	0.35
1370	591.20	594.70	0.05	0.35
1375	586.20	589.70	0.10	0.15
1380	581.20	584.70	0.10	0.20
1385	576.20	579.70	0.10	0.35
1390	571.20	574.70	0.10	0.35
1395	566.20	569.70	0.10	0.15
1400	561.20	564.80	0.10	0.15
1405	556.20	559.80	0.00	0.35
1410	551.20	554.80	0.05	0.40
1415	546.20	549.80	0.00	0.40
1420	541.20	544.80	0.05	0.40
1425	536.30	539.80	0.15	0.20

Next-Term Options				
Strike	Calls		Puts	
	Bid	Ask	Bid	Ask
1225	735.90	738.80	0.00	0.10
1250	710.80	713.80	0.00	0.10
1275	686.00	688.70	0.05	0.10
1300	660.90	663.80	0.00	0.10
1325	635.90	638.60	0.10	0.20
1350	610.90	613.60	0.10	0.20
1375	585.90	588.70	0.10	0.25
1400	561.00	563.70	0.15	0.25
1425	536.00	538.80	0.20	0.30
1450	511.10	513.80	0.25	0.35
1475	486.10	488.90	0.30	0.40
1500	461.20	464.00	0.35	0.45
1510	451.30	454.00	0.35	0.50
1520	441.30	444.00	0.40	0.50
1525	436.30	439.10	0.40	0.55
1530	431.30	434.10	0.45	0.55
1540	421.40	424.10	0.45	0.60
1550	411.40	414.20	0.50	0.60
1555	406.40	409.20	0.50	0.65
1560	401.40	404.20	0.55	0.65
1565	396.50	399.20	0.55	0.70
1570	391.20	394.00	0.60	0.70
1575	386.50	389.30	0.60	0.75
1580	381.50	384.30	0.60	0.75
1585	376.60	379.30	0.65	0.75
1590	371.30	374.10	0.65	0.80
1595	366.60	369.40	0.70	0.80
1600	361.60	364.40	0.70	0.85
1605	356.70	359.40	0.75	0.85
1610	351.70	354.50	0.75	0.90
1615	346.70	349.50	0.80	0.90
1620	341.80	344.50	0.80	0.95
1625	336.80	339.50	0.85	0.95
1630	331.80	334.60	0.90	1.00

Near-Term Options (cont.)				
Strike	Calls		Puts	
	Bid	Ask	Bid	Ask
1430	531.30	534.80	0.05	0.40
1435	526.30	529.80	0.15	0.40
1440	521.30	524.80	0.05	0.30
1445	516.30	519.80	0.05	0.40
1450	511.30	514.80	0.15	0.25
1455	506.30	509.80	0.05	0.45
1460	501.30	504.80	0.05	0.45
1465	496.30	499.80	0.05	0.45
1470	491.30	494.80	0.05	0.45
1475	486.30	489.90	0.15	0.25
1480	481.30	484.90	0.05	0.45
1485	476.30	479.90	0.20	0.50
1490	471.30	474.90	0.05	0.30
1495	466.40	469.90	0.05	0.50
1500	461.40	464.90	0.25	0.40
1505	456.40	459.90	0.30	0.35
1510	451.40	454.90	0.05	0.55
1515	446.40	449.90	0.05	0.55
1520	441.40	445.00	0.10	0.60
1525	436.40	440.00	0.30	0.40
1530	431.40	435.00	0.05	0.60
1535	426.40	430.00	0.10	0.65
1540	421.40	425.00	0.10	0.65
1545	416.50	420.00	0.10	0.65
1550	411.50	415.00	0.30	0.70
1555	406.50	410.10	0.15	0.70
1560	401.50	405.10	0.15	0.70
1565	396.50	400.10	0.15	0.70
1570	391.50	395.10	0.20	0.75
1575	386.50	390.10	0.35	0.75
1580	381.50	385.10	0.25	0.80
1585	376.60	380.20	0.25	0.80
1590	371.60	375.20	0.25	0.80
1595	366.60	370.20	0.25	0.80
1600	361.60	365.20	0.50	0.85
1605	356.60	360.30	0.30	0.85
1610	351.60	355.30	0.35	0.90
1615	346.70	350.30	0.35	0.90
1620	341.70	345.30	0.35	0.90
1625	336.70	340.40	0.40	0.95
1630	331.70	335.40	0.40	0.95
1635	326.70	330.40	0.45	1.00
1640	321.80	325.40	0.45	1.00
1645	316.80	320.50	0.50	1.05
1650	311.80	315.50	0.50	0.85
1655	306.80	310.50	0.55	1.10
1660	301.90	305.60	0.55	1.10
1665	296.90	300.60	0.60	1.15

Next-Term Options (cont.)				
Strike	Calls		Puts	
	Bid	Ask	Bid	Ask
1635	326.90	329.60	0.90	1.05
1640	321.90	324.70	0.95	1.05
1645	316.90	319.70	0.95	1.10
1650	312.00	314.70	1.00	1.15
1655	307.00	309.80	1.05	1.15
1660	302.10	304.80	1.10	1.20
1665	297.10	299.90	1.15	1.25
1670	292.20	294.90	1.15	1.30
1675	287.20	289.90	1.20	1.35
1680	282.30	285.00	1.25	1.40
1685	277.30	280.10	1.30	1.45
1690	272.40	275.10	1.35	1.50
1695	267.40	270.20	1.40	1.55
1700	262.50	265.20	1.45	1.60
1705	257.50	260.30	1.50	1.70
1710	252.60	255.30	1.60	1.75
1715	247.70	250.40	1.65	1.80
1720	242.70	245.50	1.70	1.90
1725	237.80	240.60	1.75	1.95
1730	232.90	235.60	1.85	2.00
1735	228.00	230.70	1.90	2.10
1740	223.40	225.30	2.00	2.20
1745	218.50	220.40	2.10	2.25
1750	213.60	215.50	2.20	2.35
1755	208.70	210.60	2.30	2.45
1760	203.80	205.70	2.40	2.55
1765	198.90	200.80	2.50	2.65
1770	194.00	195.90	2.65	2.80
1775	189.20	191.10	2.75	2.90
1780	184.30	185.80	2.90	3.10
1785	179.40	180.90	3.00	3.20
1790	174.60	176.10	3.10	3.40
1795	169.70	171.20	3.30	3.60
1800	164.90	166.40	3.50	3.70
1805	160.10	161.60	3.70	3.90
1810	155.30	156.70	3.80	4.10
1815	150.50	152.00	4.10	4.30
1820	145.70	147.20	4.30	4.50
1825	140.90	142.40	4.50	4.80
1830	136.20	137.70	4.80	5.00
1835	131.50	132.90	5.00	5.30
1840	126.80	128.20	5.30	5.60
1845	122.10	123.50	5.60	5.90
1850	117.40	118.80	5.90	6.20
1855	112.80	114.20	6.30	6.60
1860	108.20	109.60	6.60	6.90
1865	103.60	105.00	7.00	7.30
1870	99.00	100.40	7.50	7.80

Near-Term Options (cont.)				
Strike	Calls		Puts	
	Bid	Ask	Bid	Ask
1670	291.90	295.70	0.60	1.15
1675	287.00	290.70	0.65	1.20
1680	282.00	285.70	0.70	1.25
1685	277.00	280.80	0.75	1.30
1690	272.10	275.80	0.75	1.30
1695	267.10	270.90	0.80	1.35
1700	262.10	265.90	0.85	1.40
1705	257.20	261.00	0.85	1.40
1710	252.20	256.00	0.90	1.45
1715	247.30	251.10	0.95	1.50
1720	242.30	246.10	1.00	1.55
1725	237.40	241.20	1.05	1.60
1730	232.40	236.30	1.10	1.65
1735	227.50	231.30	1.15	1.70
1740	222.50	226.40	1.20	1.75
1745	217.60	221.50	1.25	1.85
1750	212.60	216.60	1.30	1.90
1755	207.70	211.60	1.40	1.95
1760	202.80	206.70	1.45	2.05
1765	197.80	201.80	1.50	2.15
1770	192.90	196.90	1.60	2.20
1775	188.00	192.00	1.65	2.35
1780	183.10	187.10	1.75	2.40
1785	178.20	182.20	1.85	2.50
1790	173.30	177.30	1.90	2.60
1795	168.40	172.40	2.00	2.75
1800	163.50	167.50	2.15	2.90
1805	158.60	162.60	2.25	3.00
1810	153.80	157.80	2.35	3.20
1815	148.90	152.90	2.50	3.40
1820	144.10	148.10	2.65	3.50
1825	139.20	143.30	3.00	3.60
1830	134.40	138.40	3.00	3.90
1835	129.60	133.60	3.20	4.10
1840	124.80	128.80	3.40	4.40
1845	120.10	124.10	3.60	4.60
1850	115.40	119.30	3.80	4.90
1855	110.60	114.60	4.10	5.20
1860	105.90	109.90	4.40	5.50
1865	101.30	105.20	4.70	5.80
1870	96.60	100.50	5.00	6.20
1875	92.00	95.90	5.40	6.60
1880	87.40	91.30	5.80	7.00
1885	82.90	86.70	6.20	7.50
1890	78.40	82.20	6.70	8.00
1895	74.00	77.70	7.20	8.60
1900	69.60	73.20	7.80	8.80
1905	66.00	68.50	8.50	9.50

Next-Term Options (cont.)				
Strike	Calls		Puts	
	Bid	Ask	Bid	Ask
1875	94.50	95.90	8.00	8.30
1880	90.00	91.40	8.40	8.80
1885	85.50	86.90	9.00	9.40
1890	81.10	82.50	9.50	10.00
1895	76.80	78.10	10.20	10.60
1900	72.40	73.70	10.90	11.30
1905	68.20	69.40	11.60	12.00
1910	64.00	65.20	12.40	12.80
1915	59.80	61.10	13.20	13.70
1920	55.70	57.00	14.20	14.60
1925	51.70	53.00	15.20	15.60
1930	47.80	49.10	16.20	16.60
1935	44.60	45.10	17.40	17.80
1940	40.80	41.30	18.60	19.00
1945	37.20	37.70	20.00	20.40
1950	33.70	34.40	21.40	21.80
1955	30.30	30.90	23.00	23.40
1960	27.00	27.60	24.70	25.10
1965	23.80	24.50	26.50	27.30
1970	20.80	21.40	28.50	29.40
1975	18.00	18.60	30.50	31.60
1980	15.50	15.90	33.00	34.00
1985	13.10	13.50	35.50	36.60
1990	10.90	11.30	38.40	39.50
1995	9.00	9.30	41.30	42.50
2000	7.20	7.60	44.50	45.80
2005	5.70	6.00	48.10	49.30
2010	4.50	4.80	51.70	53.00
2015	3.40	3.70	55.80	57.00
2020	2.60	2.80	59.90	61.70
2025	1.95	2.15	64.10	66.10
2030	1.45	1.65	68.60	70.60
2035	1.05	1.25	73.30	75.20
2040	0.80	0.95	78.00	80.00
2045	0.60	0.75	82.00	84.80
2050	0.50	0.65	86.90	89.60
2060	0.30	0.40	96.60	99.40
2070	0.20	0.30	106.70	109.50
2075	0.15	0.25	111.70	114.50
2100	0.10	0.20	136.30	139.10
2125	0.05	0.15	161.50	164.30
2150	0.05	0.15	186.30	189.00
2175	0.00	0.10	211.30	214.00
2200	0.05	0.10	236.30	239.00
2225	0.00	0.10	261.30	264.00
2250	0.00	0.10	286.30	289.00

Near-Term Options (cont.)				
Strike	Calls		Puts	
	Bid	Ask	Bid	Ask
1910	61.60	64.10	9.10	10.20
1915	57.40	59.80	9.90	11.30
1920	53.30	55.60	10.70	12.10
1925	49.10	51.20	11.60	12.60
1930	45.20	47.30	12.50	14.00
1935	41.20	43.40	13.60	14.70
1940	37.40	39.50	14.70	15.80
1945	33.70	35.70	15.90	17.20
1950	30.10	32.10	17.70	18.80
1955	26.70	28.50	19.00	20.50
1960	23.40	25.10	20.60	22.00
1965	20.30	21.80	22.30	24.00
1970	17.40	18.80	24.30	25.80
1975	14.60	15.90	26.50	28.10
1980	12.20	13.30	28.90	30.60
1985	9.90	11.00	31.40	33.20
1990	7.90	9.00	34.30	36.50
1995	6.20	7.10	37.40	39.70
2000	4.70	5.20	40.70	43.20
2005	3.40	4.20	44.00	47.70
2010	2.65	3.10	48.00	51.40
2015	1.75	2.30	52.20	56.00
2020	1.20	1.70	56.60	60.40
2025	1.00	1.25	61.20	65.00
2030	0.45	1.00	65.90	69.70
2035	0.25	0.80	70.70	74.40
2040	0.35	0.65	75.60	79.30
2045	0.20	0.60	80.50	84.10
2050	0.20	0.30	85.40	89.00
2055	0.15	0.50	90.40	94.00
2060	0.15	0.30	95.30	98.90
2065	0.15	0.20	100.30	103.90
2070	0.10	0.20	105.30	108.90
2075	0.10	0.20	110.30	113.80
2080	0.05	0.45	115.30	118.80
2085	0.05	0.40	120.30	123.80
2090	0.05	0.15	125.30	128.80
2095	0.05	0.35	130.30	133.80
2100	0.05	0.15	135.30	138.80
2120	0.00	0.15	155.30	158.80
2125	0.05	0.15	160.30	163.80
2150	0.00	0.10	185.20	188.80
2175	0.00	0.05	210.20	213.70
2200	0.00	0.05	235.20	238.70
2225	0.05	0.10	260.20	263.70
2250	0.00	0.05	285.20	288.70

Appendix 2: Individual Contributions — K_0 = 1960

Near term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike
1370	Put	0.200	5	0.0000005328
1375	Put	0.125	5	0.0000003306
1380	Put	0.150	5	0.0000003938
1385	Put	0.225	5	0.0000005865
1390	Put	0.225	5	0.0000005823
1395	Put	0.125	5	0.0000003212
1400	Put	0.125	7.5	0.0000004783
1410	Put	0.225	10	0.0000011318
1420	Put	0.225	7.5	0.0000008369
1425	Put	0.175	5	0.0000004309
1430	Put	0.225	5	0.0000005502
1435	Put	0.275	5	0.0000006677
1440	Put	0.175	5	0.0000004220
1445	Put	0.225	5	0.0000005388
1450	Put	0.200	5	0.0000004756
1455	Put	0.250	5	0.0000005905
1460	Put	0.250	5	0.0000005864
1465	Put	0.250	5	0.0000005824
1470	Put	0.250	5	0.0000005785
1475	Put	0.200	5	0.0000004596
1480	Put	0.250	5	0.0000005707
1485	Put	0.350	5	0.0000007936
1490	Put	0.175	5	0.0000003941
1495	Put	0.275	5	0.0000006152
1500	Put	0.325	5	0.0000007222
1505	Put	0.325	5	0.0000007174
1510	Put	0.300	5	0.0000006579
1515	Put	0.300	5	0.0000006535
1520	Put	0.350	5	0.0000007575
1525	Put	0.350	5	0.0000007525
1530	Put	0.325	5	0.0000006942
1535	Put	0.375	5	0.0000007958
1540	Put	0.375	5	0.0000007906
1545	Put	0.375	5	0.0000007855
1550	Put	0.500	5	0.0000010406
1555	Put	0.425	5	0.0000008788
1560	Put	0.425	5	0.0000008732
1565	Put	0.425	5	0.0000008676
1570	Put	0.475	5	0.0000009635
1575	Put	0.550	5	0.0000011086
1580	Put	0.525	5	0.0000010515
1585	Put	0.525	5	0.0000010449
1590	Put	0.525	5	0.0000010384
1595	Put	0.525	5	0.0000010319
1600	Put	0.675	5	0.0000013184
1605	Put	0.575	5	0.0000011161
1610	Put	0.625	5	0.0000012056
1615	Put	0.625	5	0.0000011982

Near term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike
1275	Put	0.075	50	0.0000023069
1325	Put	0.150	37.5	0.0000032041
1350	Put	0.150	25	0.0000020577
1375	Put	0.175	25	0.0000023141
1400	Put	0.200	25	0.0000025511
1425	Put	0.250	25	0.0000030779
1450	Put	0.300	25	0.0000035673
1475	Put	0.350	25	0.0000040219
1500	Put	0.400	17.5	0.0000031112
1510	Put	0.425	10	0.0000018640
1520	Put	0.450	7.5	0.0000014608
1525	Put	0.475	5	0.0000010213
1530	Put	0.500	7.5	0.0000016020
1540	Put	0.525	10	0.0000022138
1550	Put	0.550	7.5	0.0000017170
1555	Put	0.575	5	0.0000011890
1560	Put	0.600	5	0.0000012328
1565	Put	0.625	5	0.0000012759
1570	Put	0.650	5	0.0000013185
1575	Put	0.675	5	0.0000013606
1580	Put	0.675	5	0.0000013520
1585	Put	0.700	5	0.0000013932
1590	Put	0.725	5	0.0000014339
1595	Put	0.750	5	0.0000014741
1600	Put	0.775	5	0.0000015137
1605	Put	0.800	5	0.0000015528
1610	Put	0.825	5	0.0000015914
1615	Put	0.850	5	0.0000016295
1620	Put	0.875	5	0.0000016671
1625	Put	0.900	5	0.0000017042
1630	Put	0.950	5	0.0000017878
1635	Put	0.975	5	0.0000018237
1640	Put	1.000	5	0.0000018591
1645	Put	1.025	5	0.0000018940
1650	Put	1.075	5	0.0000019743
1655	Put	1.100	5	0.0000020081
1660	Put	1.150	5	0.0000020867
1665	Put	1.200	5	0.0000021644
1670	Put	1.225	5	0.0000021963
1675	Put	1.275	5	0.0000022723
1680	Put	1.325	5	0.0000023474
1685	Put	1.375	5	0.0000024215
1690	Put	1.425	5	0.0000024947
1695	Put	1.475	5	0.0000025670
1700	Put	1.525	5	0.0000026385
1705	Put	1.600	5	0.0000027520
1710	Put	1.675	5	0.0000028642
1715	Put	1.725	5	0.0000029325

Individual Contributions (Cont.)				
Near term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike
1620	Put	0.625	5	0.0000011908
1625	Put	0.675	5	0.0000012781
1630	Put	0.675	5	0.0000012703
1635	Put	0.725	5	0.0000013561
1640	Put	0.725	5	0.0000013478
1645	Put	0.775	5	0.0000014320
1650	Put	0.675	5	0.0000012397
1655	Put	0.825	5	0.0000015060
1660	Put	0.825	5	0.0000014970
1665	Put	0.875	5	0.0000015782
1670	Put	0.875	5	0.0000015688
1675	Put	0.925	5	0.0000016485
1680	Put	0.975	5	0.0000017273
1685	Put	1.025	5	0.0000018051
1690	Put	1.025	5	0.0000017944
1695	Put	1.075	5	0.0000018709
1700	Put	1.125	5	0.0000019464
1705	Put	1.125	5	0.0000019350
1710	Put	1.175	5	0.0000020092
1715	Put	1.225	5	0.0000020825
1720	Put	1.275	5	0.0000021549
1725	Put	1.325	5	0.0000022265
1730	Put	1.375	5	0.0000022972
1735	Put	1.425	5	0.0000023670
1740	Put	1.475	5	0.0000024360
1745	Put	1.550	5	0.0000025452
1750	Put	1.600	5	0.0000026123
1755	Put	1.675	5	0.0000027192
1760	Put	1.750	5	0.0000028248
1765	Put	1.825	5	0.0000029292
1770	Put	1.900	5	0.0000030324
1775	Put	2.000	5	0.0000031740
1780	Put	2.075	5	0.0000032746
1785	Put	2.175	5	0.0000034132
1790	Put	2.250	5	0.0000035112
1795	Put	2.375	5	0.0000036856
1800	Put	2.525	5	0.0000038967
1805	Put	2.625	5	0.0000040286
1810	Put	2.775	5	0.0000042353
1815	Put	2.950	5	0.0000044776
1820	Put	3.075	5	0.0000046417
1825	Put	3.300	5	0.0000049541
1830	Put	3.450	5	0.0000051511
1835	Put	3.650	5	0.0000054200
1840	Put	3.900	5	0.0000057598
1845	Put	4.100	5	0.0000060224
1850	Put	4.350	5	0.0000063551
1855	Put	4.650	5	0.0000067568
1860	Put	4.950	5	0.0000071542

Individual Contributions (Cont.)				
Near term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike
1720	Put	1.800	5	0.0000030423
1725	Put	1.850	5	0.0000031087
1730	Put	1.925	5	0.0000032160
1735	Put	2.000	5	0.0000033221
1740	Put	2.100	5	0.0000034682
1745	Put	2.175	5	0.0000035715
1750	Put	2.275	5	0.0000037144
1755	Put	2.375	5	0.0000038556
1760	Put	2.475	5	0.0000039951
1765	Put	2.575	5	0.0000041330
1770	Put	2.725	5	0.0000043491
1775	Put	2.825	5	0.0000044834
1780	Put	3.000	5	0.0000047344
1785	Put	3.100	5	0.0000048648
1790	Put	3.250	5	0.0000050718
1795	Put	3.450	5	0.0000053539
1800	Put	3.600	5	0.0000055557
1805	Put	3.800	5	0.0000058319
1810	Put	3.950	5	0.0000060287
1815	Put	4.200	5	0.0000063750
1820	Put	4.400	5	0.0000066419
1825	Put	4.650	5	0.0000069808
1830	Put	4.900	5	0.0000073160
1835	Put	5.150	5	0.0000076474
1840	Put	5.450	5	0.0000080490
1845	Put	5.750	5	0.0000084461
1850	Put	6.050	5	0.0000088388
1855	Put	6.450	5	0.0000093724
1860	Put	6.750	5	0.0000097557
1865	Put	7.150	5	0.0000102785
1870	Put	7.650	5	0.0000109385
1875	Put	8.150	5	0.0000115914
1880	Put	8.600	5	0.0000121664
1885	Put	9.200	5	0.0000129463
1890	Put	9.750	5	0.0000136478
1895	Put	10.400	5	0.0000144809
1900	Put	11.100	5	0.0000153743
1905	Put	11.800	5	0.0000162582
1910	Put	12.600	5	0.0000172697
1915	Put	13.450	5	0.0000183386
1920	Put	14.400	5	0.0000195317
1925	Put	15.400	5	0.0000207797
1930	Put	16.400	5	0.0000220146
1935	Put	17.600	5	0.0000235035
1940	Put	18.800	5	0.0000249767
1945	Put	20.200	5	0.0000266989
1950	Put	21.600	5	0.0000284031
1955	Put	23.200	5	0.0000303512
1960	Put/Call Average	26.100	5	0.0000339711

Individual Contributions (Cont.)				
Near term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike
1865	Put	5.250	5	0.0000075471
1870	Put	5.600	5	0.0000080073
1875	Put	6.000	5	0.0000085335
1880	Put	6.400	5	0.0000090541
1885	Put	6.850	5	0.0000096393
1890	Put	7.350	5	0.0000102883
1895	Put	7.900	5	0.0000109999
1900	Put	8.300	5	0.0000114961
1905	Put	9.000	5	0.0000124003
1910	Put	9.650	5	0.0000132263
1915	Put	10.600	5	0.0000144526
1920	Put	11.400	5	0.0000154626
1925	Put	12.100	5	0.0000163269
1930	Put	13.250	5	0.0000177861
1935	Put	14.150	5	0.0000188962
1940	Put	15.250	5	0.0000202603
1945	Put	16.550	5	0.0000218745
1950	Put	18.250	5	0.0000239979
1955	Put	19.750	5	0.0000258376
1960	Put/Call Average	24.250	5	0.0000296432
1965	Call	21.050	5	0.0000272588
1970	Call	18.100	5	0.0000233198
1975	Call	15.250	5	0.0000195486
1980	Call	12.750	5	0.0000162614
1985	Call	10.450	5	0.0000132609
1990	Call	8.450	5	0.0000106691
1995	Call	6.650	5	0.0000083544
2000	Call	4.950	5	0.0000061876
2005	Call	3.800	5	0.0000047264
2010	Call	2.875	5	0.0000035582
2015	Call	2.025	5	0.0000024938
2020	Call	1.450	5	0.0000017768
2025	Call	1.125	5	0.0000013718
2030	Call	0.725	5	0.0000008797
2035	Call	0.525	5	0.0000006339
2040	Call	0.500	5	0.0000006007
2045	Call	0.400	5	0.0000004782
2050	Call	0.250	5	0.0000002974
2055	Call	0.325	5	0.0000003848
2060	Call	0.225	5	0.0000002651
2065	Call	0.175	5	0.0000002052
2070	Call	0.150	5	0.0000001750
2075	Call	0.150	5	0.0000001742
2080	Call	0.250	5	0.0000002889
2085	Call	0.225	5	0.0000002588
2090	Call	0.100	5	0.0000001145
2095	Call	0.200	5	0.0000002278
2100	Call	0.100	15	0.0000003401
2125	Call	0.100	25	0.0000005536

Individual Contributions (Cont.)				
Near term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike
1965	Call	24.150	5	0.0000312732
1970	Call	21.100	5	0.0000271851
1975	Call	18.300	5	0.0000234584
1980	Call	15.700	5	0.0000200240
1985	Call	13.300	5	0.0000168776
1990	Call	11.100	5	0.0000140152
1995	Call	9.150	5	0.0000114952
2000	Call	7.400	5	0.0000092502
2005	Call	5.850	5	0.0000072763
2010	Call	4.650	5	0.0000057550
2015	Call	3.550	5	0.0000043718
2020	Call	2.700	5	0.0000033086
2025	Call	2.050	5	0.0000024997
2030	Call	1.550	5	0.0000018807
2035	Call	1.150	5	0.0000013885
2040	Call	0.875	5	0.0000010513
2045	Call	0.675	5	0.0000008070
2050	Call	0.575	7.5	0.0000010262
2060	Call	0.350	10	0.0000008248
2070	Call	0.250	7.5	0.0000004376
2075	Call	0.200	15	0.0000006968
2100	Call	0.150	25	0.0000008504
2125	Call	0.100	25	0.0000005536
2150	Call	0.100	37.5	0.0000008113
2200	Call	0.075	50	0.0000007748

Sum of Individual Contributions 0.000831402

$$\frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)$$ 0.018838

$$\frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)$$

Sum of Individual Contributions 0.0006320516

$$\frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)$$ 0.018494953