Nicholas Livingstone HW9 Math-375 4/13/20

1.

a)

```
n = 6;
H = hilb(10);
A = H(1:10,1:n);
c = transpose(ones(1, n));
b = A*c;
[Q, R] = qr(A);
QtB = Q'*b;
c_apprx = R(:,:)\QtB(:)
```

```
c_apprx = 6�1
    0.999999999999910
    1.000000000001625
    0.999999999992104
    1.000000000015688
    0.999999999986252
    1.000000000004415
```

This method is correct to 10 decimal places

b)

```
n = 8;
H = hilb(10);
A = H(1:10,1:n);
c = transpose(ones(1, n));
b = A*c;
[Q, R] = qr(A);
QtB = Q'*b;
c_apprx = R(:,:)\QtB(:)
```

```
c_apprx = 8�1
    0.999999999998164
    1.000000000077673
    0.999999999193717
    1.000000003486769
    0.999999992488604
    1.000000008476126
    0.999999995232271
    1.000000001046109
```

In this case, QR factorization is correct to 8 decimal places. I hypothesize that our digits of accuracy are the machine epsilon's accuracy - n, where n is the number of colums of A i.e. in this matlab environment 16-6 = 10, 16-8 = 8. Compared to homework 8, this method provides much more accurate approximations even in the case of having a poorly conditioned matrix to begin with.

2.

```
A = [-4, -4; -2, 7; 4, -5];
b = [3; 9; 0];
[Q, R] = qr(A);

QtB = Q'*b;
c_apprx = rats(R(:,:)\QtB(:))
```

```
c_apprx = 2�14 char array
    '    -11/18    '
    '      4/9     '
```

```
Q = rats(Q)
```

```
Q = 3�42 char array
    '     -2/3           2/3            1/3      '
    '     -1/3          -2/3            2/3      '
    '      2/3           1/3            2/3      '
```

```
R = rats(R)
```

```
R = 3�28 char array
    '       6            -3      '
    '       0            -9      '
    '       0             0      '
```

Compared to the results produced by hand, The approximated x is the same in the matlab.

2. $$A = \begin{bmatrix} -4 & -4 \\ -2 & 7 \\ 4 & -5 \end{bmatrix} \qquad b = \begin{bmatrix} 3 \\ 9 \\ 0 \end{bmatrix}$$

$$y_1 = A_1 = \begin{bmatrix} -4 \\ -2 \\ 4 \end{bmatrix} \qquad r_{11} = 6 \qquad q_1 = \frac{y_1}{r_{11}} = \begin{bmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

$$y_2 = A_2 - q_1(q_1^T A_2) = \begin{bmatrix} -4 \\ 7 \\ -5 \end{bmatrix} - \begin{bmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}(-3) = \begin{bmatrix} -6 \\ 6 \\ -3 \end{bmatrix} \qquad r_{12} = -3 \qquad r_{22} = 9 \qquad q_2 = \frac{y_2}{r_{22}} = \begin{bmatrix} -\frac{2}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$q_3 = q_1 \times q_2 = \begin{bmatrix} i & j & k \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{bmatrix} \qquad Q = \begin{bmatrix} -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \qquad R = \begin{bmatrix} 6 & -3 \\ 0 & 9 \\ 0 & 0 \end{bmatrix}$$

$$Q^T B = \begin{bmatrix} -5 \\ 4 \\ -7 \end{bmatrix}$$

$$Rx = Q^T B \Rightarrow \left[\begin{array}{cc|c} 6 & -3 & -5 \\ 0 & 9 & 4 \\ \cancel{0} & \cancel{0} & \cancel{-7} \end{array}\right] \begin{matrix} \frac{1}{6} \\ \frac{1}{9} \end{matrix} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & -\frac{5}{6} \\ 0 & 1 & \frac{4}{9} \end{array}\right] + \tfrac{1}{2}R_2 \left[\begin{array}{cc|c} 1 & 0 & -\frac{11}{18} \\ 0 & 1 & \frac{4}{9} \end{array}\right]$$

$$x_{approx} = \begin{bmatrix} -\frac{11}{18} \\ \frac{4}{9} \end{bmatrix}$$

$$\text{Residual} = b - b_{approx}$$

$$= \begin{bmatrix} 3 \\ 9 \\ 0 \end{bmatrix} - A x_{approx} = \begin{bmatrix} 3 \\ 9 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ \frac{13}{3} \\ -\frac{14}{3} \end{bmatrix} = \begin{bmatrix} \frac{7}{3} \\ \frac{14}{3} \\ \frac{14}{3} \end{bmatrix}$$