Nicholas Livingstone HW10 Math-375 4/20/20

1.

```
F = @(u,v) [6*u^3+u*v-3*v^3-4;
          u^2-18*u*v^2+16*v^3+1];
DF = @(u,v) [18*u^2 + v, u-9*v^2;
            2*u - 18*v^2, -36*u*v+48*v^2];

newton1(20,20,1e-8,F, DF);
```

```
Error: 4.387450e-14
u: 0.86593892
v: 0.46216792
k: 12
```

```
newton1(1,-0.5,1e-8,F, DF);
```

```
Error: 5.965577e-15
u: 0.88680942
v: -0.29400704
k: 6
```

2.

a)

```
F = @(w1, w2, w3, c1, c2, c3)  [w1+w2+w3-1;
                                w1*c1+w2*c2+w3*c3-0.5;
                                w1*c1^2+w2*c2^2+w3*c3^2-(1/3);
                                w1*c1^3+w2*c2^3+w3*c3^3-0.25;
                                w1*c1^4+w2*c2^4+w3*c3^4-0.20;
                                w1*c1^5+w2*c2^5+w3*c3^5-(1/6)];

DF = @(w1, w2, w3, c1, c2, c3) [1, 1, 1, 0, 0, 0;
                                c1, c2, c3, w1, w2, w3;
                                c1^2, c2^2, c3^2, 2*w1*c1, 2*w2*c2, 2*w3*c3;
                                c1^3, c2^3, c3^3, 3*w1*c1^2, 3*w2*c2^2, 3*w3*c3^2;
                                c1^4, c2^4, c3^4, 4*w1*c1^3, 4*w2*c2^3, 4*w3*c3^3;
                                c1^5, c2^5, c3^5, 5*w1*c1^4, 5*w2*c2^4, 5*w3*c3^4];

[w1, w2, w3, c1, c2, c3] = newton2(1/3, 1/3, 1/3, 0, 0.5, 1, 1e-8, F, DF);
```

```
Error: 9.253193e-14
w1: 0.27777778
w2: 0.44444444
w3: 0.27777778
c1: 0.11270167
c2: 0.50000000
c3: 0.88729833
k: 6
```

b)

```
f_x = @(x) exp(x)*cos(5*x);
b = pi/4;
a = 0;
h = b - a;

integral_approx = h*(w1*f_x(a+c1*h) + w2*f_x(a+c2*h) + w3*f_x(a+c3*h))
```

```
integral_approx =
  -0.394919053276976
```

```
%see scratch paper for solving%
```

```
indef_integral = @(x) (5/26)*exp(x)*sin(5*x) + (1/26)*exp(x)*cos(5*x);
integral_exact = indef_integral(b) - indef_integral(a)
```

```
integral_exact =
  -0.396357660827237
```

The integral approximation is only accurate to two decimal places. In this case it doesn't appear to be an effective method for solving a single integral that is more efficiently and accurately done by hand.

```
function [u, v, err] = newton1(u0, v0,  tol, F, DF)
    u = u0;
    v = v0;
    k_max = 1000;
    err = 1000;
    k = 0;
    while(err > tol && k < k_max)
        u_prev = u;
        v_prev = v;
        Z = DF(u,v)\F(u,v);
        u = u - Z(1);
        v = v - Z(2);
        err = sqrt((u-u_prev)^2+(v-v_prev)^2);
        k = k+1;
    end

    fprintf("Error: %e\n", err);
    fprintf("u: %.8f\n", u);
    fprintf("v: %.8f\n", v);
    fprintf("k: %u", k);
end

function [w1, w2, w3, c1, c2, c3] = newton2(w01, w02, w03, c01, c02, c03, tol, F, DF)
    w1 = w01;
    w2 = w02;
    w3 = w03;
    c1 = c01;
    c2 = c02;
    c3 = c03;

    k_max = 1000;
    err = 1000;
    k = 0;

    while(err > tol && k < k_max)
        w1_prev = w1;
        w2_prev = w2;
        w3_prev = w3;
        c1_prev = c1;
        c2_prev = c2;
        c3_prev = c3;

        Z = DF(w1, w2, w3, c1, c2, c3)\F(w1, w2, w3, c1, c2, c3);
        w1 = w1 - Z(1);
        w2 = w2 - Z(2);
        w3 = w3 - Z(3);
        c1 = c1 - Z(4);
        c2 = c2 - Z(5);
        c3 = c3 - Z(6);

        err = sqrt((w1-w1_prev)^2+(w2-w2_prev)^2+(w3-w3_prev)^2+(c1-c1_prev)^2+(c2-c2_prev)^2+(c3-c3_prev)^2);
        k = k+1;
    end

    fprintf("Error: %e\n", err);
    fprintf("w1: %.8f\n", w1);
    fprintf("w2: %.8f\n", w2);
```

```
    fprintf("w3: %.8f\n", w3);
    fprintf("c1: %.8f\n", c1);
    fprintf("c2: %.8f\n", c2);
    fprintf("c3: %.8f\n", c3);
    fprintf("k: %u", k);

end
```

1.

$f_1(u,v) = 6u^3 + uv - 3v^3 - 4$

$\frac{df_1}{du} = 18u^2 + v$ $\qquad$ $\frac{df_1}{dv} = u - 9v^2$

$f_2(u,v) = u^2 - 18uv^2 + 16v^3 + 1$

$\frac{df_2}{du} = 2u - 18v^2$ $\qquad$ $\frac{df_2}{dv} = -36uv + 48v^2$

		dw_1	dw_2	dw_3	dc_1	dc_2	dc_3
2.	f_1	1	1	1	0	0	0
a)	f_2	c_1	c_2	c_3	w_1	w_2	w_3
	f_3	c_1^2	c_2^2	c_3^2	$2w_1c_1$	$2w_2c_2$	$2w_3c_3$
	f_4	c_1^3	c_2^3	c_3^3	$3w_1c_1^2$	$3w_2c_2^2$	$3w_3c_3^2$
	f_5	c_1^4	c_2^4	c_3^4	$4w_1c_1^3$	$4w_2c_2^3$	$4w_3c_3^3$
	f_6	c_1^5	c_2^5	c_3^5	$5w_1c_1^4$	$5w_2c_2^4$	$5w_3c_3^4$

b) $\int_0^{\pi/4} e^x \cos(5x)\,dx$ $\qquad$ $u = e^x$, $v' = \cos(5x)$

$= e^x \frac{1}{5}\sin(5x) - \frac{1}{5}\int e^x \cancel{\tfrac{1}{5}} \sin(5x)\,dx$

$e^x \frac{1}{5}\sin(5x) - \frac{1}{5}\left(-\frac{1}{5}e^x\cos(5x) \cancel{+} \left(-\frac{1}{5}\int e^x\cos(5x)\,dx\right)\right) = \int e^x \cos(5x)\,dx$

$e^x\frac{1}{5}\sin(5x) + \frac{1}{25}e^x\cos(5x) - \frac{1}{25}\int e^x\cos(5x) = \int e^x\cos(5x)\,dx$

$e^x\frac{1}{5}\sin(5x) + \frac{1}{25}e^x\cos(5x) = \frac{26}{25}\int e^x\cos(5x)$

$$\int e^x\cos(5x) = \frac{5e^x\sin(5x)}{26} + \frac{e^x\cos(5x)}{26}$$