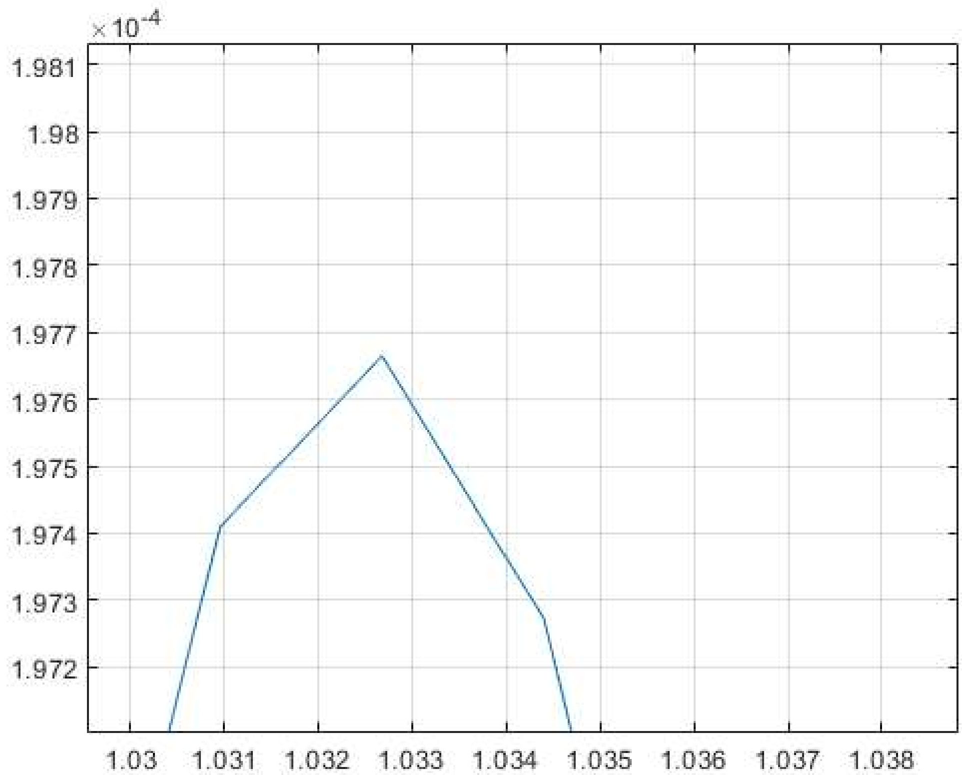
Nicholas Livingstone HW8 Math-375 4/6/20

1.

```
d = 14;
x = linspace(1,exp(1),d+1);
z = linspace(1, exp(1), 1000);
Psi = ones(size(z));
for k = 1:length(x)
    Psi = Psi.*(z-x(k));
end
plot(z,abs(Psi))
grid on
xlim([1.02956 1.03881])
ylim([0.000197105 0.000198132])
```



Max is roughly 1.97.

2.

a)

```
n = 6;
H = hilb(10);
A = H(:,1:n);
c = ones(n, 1);
b = A*c;
```

```
AtA = transpose(A)*A;
Atb = transpose(A)*b;
xbar = linsolve(AtA,Atb)
```

```
xbar = 6�1
   0.999999647667513
   1.000008953248857
   0.999944339764056
   1.000135689800381
   0.999857739453121
   1.000053752672782
```

```
cond(A)
```

```
ans =
     2.450593200883974e+06
```

b)

```
n = 8;
H = hilb(10);
A = H(:,1:n);
c = ones(n, 1);
b = A*c;
AtA = transpose(A)*A;
Atb = transpose(A)*b;
xbar = linsolve(AtA,Atb)
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  7.082899
xbar = 8�1
   1.003860638767132
   0.794397655895357
   3.661192372897510
 -13.272450414901732
  39.099065214798493
 -52.488821267944189
  38.796092799597936
  -9.595698513220729
```

```
cond(A)
```

```
ans =
     3.565872492514215e+09
```

A large condition number implies a large magnification of error even with small changes to the initial conditions. And considering the very large condition number of the matrix, it makes sense that such a large variation in accuracy occurs from n=6, to n=8. Using the normal equations, in the case of n=6, the approximated solution is accurate to 3 decimal places. N=6 results in no accuracy beyong the first element in the approximated solution.

1. $\max_{x \in [1,e]} |f(x) - Q_d(x)| \le \max_{z \in [1,e]} \frac{|f^{d+1}(z)|}{(d+1)!\,2^d} \frac{(e-1)^{d+1}}{2^{d+1}}$

$$= \max_{z \in [1,e]} \frac{d!}{z^{d+1}(d+1)!\,2^d} \frac{(e-1)^{d+1}}{2^{d+1}} = \frac{(e-1)^{d+1}}{(d+1)\,2^{2d+1}}$$

d	error
10	$.000017 = 1.7 \times 10^{-5}$
12	3×10^{-6}
13	10^{-6}
$\boxed{14}$	4×10^{-7}

$d = 14$