Nicholas Livingstone

Math 3Z5

HW #7

1. Constructing interpolating polynomials

$D_3 = \{(0,-2),(2,1),(4,12)\}$

Monomial

$$P(x) = \sum_{i=1}^{3} c_i x^{i-1} = c_3 x^2 + x c_2 + c_1$$

$$\begin{aligned} c_1 & & &= -2 \\ c_1 &+ 2c_2 + 4c_3 &&= 1 \\ c_1 &+ 4c_2 + 16c_3 &&= 12 \end{aligned} \qquad V = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \end{bmatrix} \qquad y = \begin{bmatrix} -2 \\ 1 \\ 12 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 1 & 2 & 4 & 1 \\ 1 & 4 & 16 & 12 \end{array}\right] \begin{array}{c} \\ -R_1 \\ -R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 2 & 4 & 3 \\ 0 & 4 & 16 & 14 \end{array}\right] \begin{array}{c} \\ \tfrac{1}{2} \\ -2R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 2 & 3/2 \\ 0 & 0 & 8 & 8 \end{array}\right] \begin{array}{c} \\ \\ \tfrac{1}{8} \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 2 & 3/2 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{c} \\ -2R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow c = \begin{bmatrix} -2 \\ -\tfrac{1}{2} \\ 1 \end{bmatrix}$$

$$\boxed{P(x) = x^2 - \tfrac{1}{2}x - 2}$$

Newton

$$P(x) = c_0 + c_1(x - x_1) + c_2(x - x_1)(x - x_2)$$

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2-0 & 0 \\ 1 & 4-0 & 4\cdot 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 4 & 8 \end{bmatrix} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 1 & 2 & 0 & 1 \\ 1 & 4 & 8 & 12 \end{array}\right] \begin{array}{c} \\ -R_1 \\ -R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 2 & 0 & 3 \\ 0 & 4 & 8 & 14 \end{array}\right] \begin{array}{c} \\ \tfrac{1}{2} \\ -2R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 8 & 8 \end{array}\right] \begin{array}{c} \\ \\ \tfrac{1}{8} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow c = \begin{bmatrix} -2 \\ 3/2 \\ 1 \end{bmatrix}$$

$$\begin{aligned} P(x) &= -2 + \tfrac{3}{2}(x - 0) + (x - 0)(x - 2) \\ & -2 + \tfrac{3}{2}x + x^2 - 2x \\ & -2 + \tfrac{3}{2}x - \tfrac{4}{2}x + x^2 \end{aligned}$$

$$\boxed{p(x) = x^2 - \tfrac{1}{2}x - 2}$$

Same as monomial ✓

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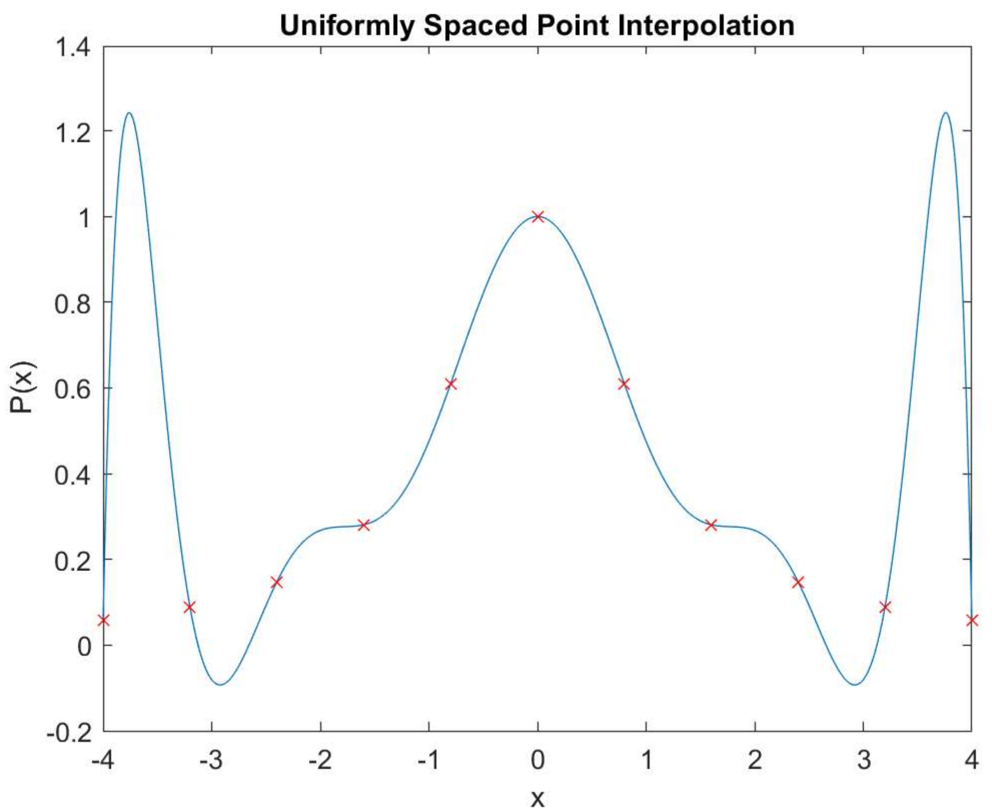
2.

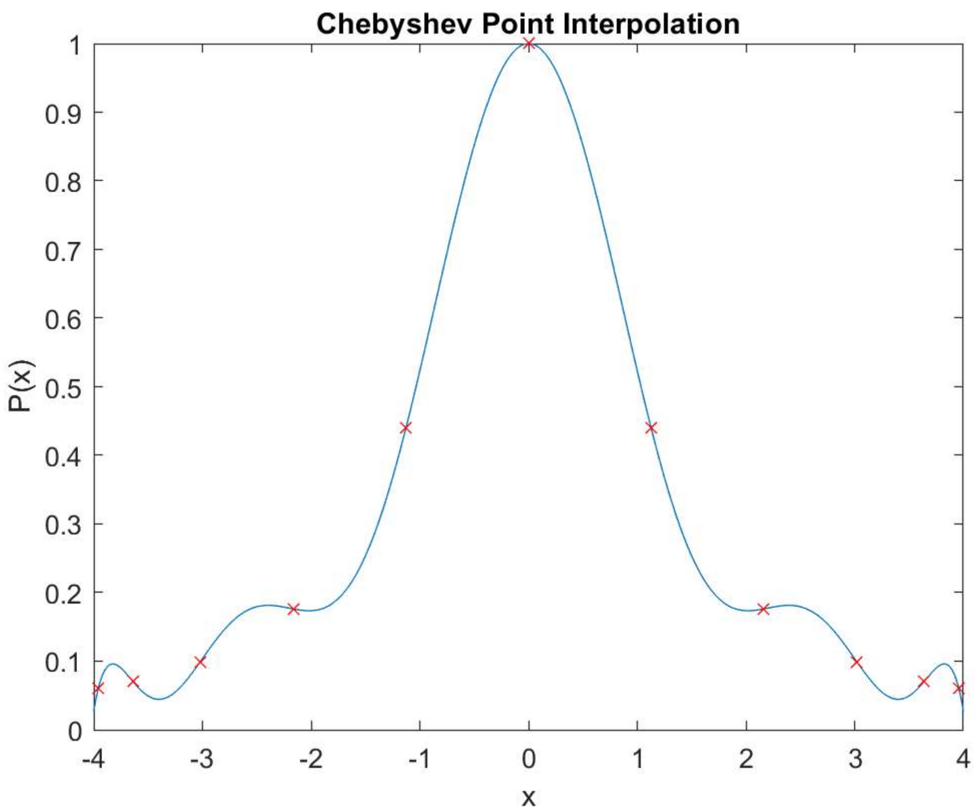
```
n = 11;
j = 1:n;
z = linspace(-4,4,500);

%Uniform Spaced Points
x = -4 + 8 .* (j - 1) ./(n-1);
y = 1./(x.^2 + 1);
c = interpnewt(x, y);
p = hornernewt(c, x, z);
plot(z, p);
hold on;
plot(x, y, 'rx');
title("Uniformly Spaced Point Interpolation");
ylabel("P(x)");
xlabel("x");
hold off;
```




```
%Chebyshev Points
x = 4 * cos((pi.*(2.*j-1))./(2*n));
y = 1./(x.^2 + 1);
c = interpnewt(x, y);
p = hornernewt(c, x, z);
```

```
plot(z, p);
hold on;
plot(x, y, 'rx');
title("Chebyshev Point Interpolation");
ylabel("P(x)");
xlabel("x");
hold off;
```




```
function c=interpnewt(x,y)
    % function c=interpnewt(x,y)
    % computes coefficients c of Newton interpolant through (x_k,y_k), k=1:length(x)
    n=length(x);
    for k=1:n-1
        y(k+1:n)=(y(k+1:n)-y(k))./(x(k+1:n)-x(k));
    end
    c=y;
end

function p = hornernewt(c,x,z)
    % function p = hornernewt(c,x,z)
    % Uses Horner method to evaluate in nested form a polynomial defined
    % by coefficients c and shifts x. Polynomial is evaluated at z.
    n = length(c); % 1 + degree of polynomial.
    p = c(n);
    for k = n-1:-1:1
        p = p.*(z-x(k))+c(k);
    end
end
```