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Improved Stochastic Models for Surfing Climate

William R. Dally†

† Surfbreak Engineering Sciences, Inc.
1010 Atlantic Street, Suite A-2
Melbourne Beach, FL 32951, U.S.A.
wdally@surfbreakengineering.com

ABSTRACT

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A new model for the joint probability density function (pdf) of surfer board-speed and wave break-rate is developed for use in making quantitative assessments of surfing wave climate. Maximum sustainable board speed is modeled upon the results of a companion study (DALLY, 2001) that relates board speed directly to breaker height. Starting from a Rayleigh distribution for wave height in deep water, the distribution of breaker heights is developed by a rigorous transformation of random variable. A strictly empirical, yet utilitarian, pdf for the sine of the peel angle is fashioned from the Beta Distribution. Assuming that breaker height and peel angle are independent random variables, the product of the two pdf's is analytically transformed into the joint distribution of board speed and break rate. The proportion of surfable waves is determined by integrating the joint pdf in the domain where the board speed is greater than the break rate, for the largest 10% of breakers. Results of exercising the model indicate that it is the distribution of peel angle, as opposed to the breaker height or wave period, that most affects the proportion of surfable waves.

ADDITIONAL INDEX WORDS: *Surfing, wave climate, stochastic modeling.*

BACKGROUND

In a coastal engineering survey report of popular surfing beaches in Hawaii, WALKER (1974) first identified and studied many of the parameters material to the evaluation of waves for recreational surfing. Perhaps the two most important are the 'wave velocity', *i.e.* the wave celerity at the point of incipient breaking (c_b), and the 'peel angle' (α_b), which is the angle between the wave crest and the path of the point of incipient breaking, as shown in Figure 1. With these two parameters, the speed of the point of incipient breaking, herein called the 'break rate' (P) can be computed using the relationship:

$$P = \frac{c_b}{\sin(\alpha_b)} \tag{1}$$

To clarify semantics, in DALLY (1990a) the 'peel rate' was defined as Equation (1). WALKER (1974) originally defined the peel rate as the speed of the break point along the wave crest, which in the present context would be $c_b/\tan \alpha_b$. Herein the term "break rate" is adopted so as not to contradict Walker's original definition of peel rate, and to be consistent with the usage of others in this special issue. Finally, WALKER (1974) suggested computing the breaker celerity based upon:

$$c_b = 1.25 \sqrt{gH_b} \tag{2}$$

in which g is gravity and H_b is the wave height at incipient breaking.

DALLY (1990a) applied the basic concepts of Walker in the development of stochastic models for evaluating the 'surfability' of random waves. In this regard, the term "board speed" (S) was defined as the maximum speed that a surfer could sustain on a particular wave, and so a wave was deemed 'surfable' if its board speed was greater than its break (peel) rate. If not, the wave was classified as a 'close-out'.

Using aerial photographs of the Hawaiian surfing sites, Walker made estimates of the break rates of individual waves, computed from the photographed peel angles and estimates of wave height at breaking. However in regard to the maximum board speed, no data have been available until recently. For several waves that were ridden by surfers, Walker estimated the actual surfer speed (*i.e.* the average speed required to stay with the breakpoint, not the maximum possible sustainable board speed), by triangulating the surfer's position using shore-based surveying transits. The greatest average speed esti-

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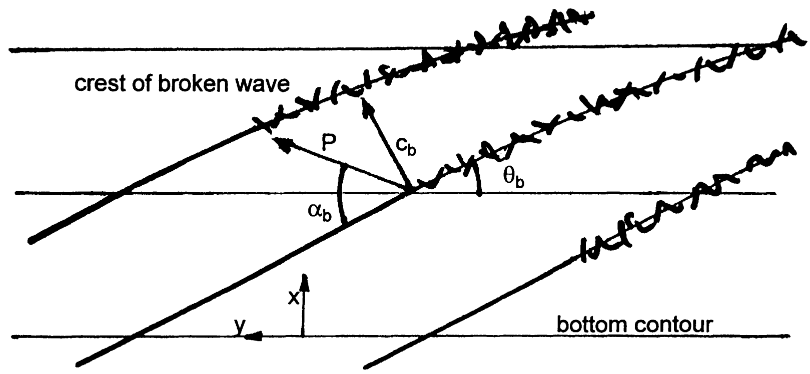

Figure 1. Definition sketch of parameters used in surf climate modeling.

mated was nearly 12 m/s, and at least a few of the rides documented are believed to have been at maximum speeds, *i.e.* the surfable limit.

Because of the lack of data on maximum sustainable board speed, and noting the importance of breaker type (*i.e.* plunging, spilling or collapsing) to the quality of surf, DALLY (1990a) used the Iribarren Number (I_o) instead. I_o is given by:

$$I_o = \frac{m}{(H_o/L_o)} = \frac{mgT^2}{2\pi H_o} \quad (3)$$

where m is the bottom slope, H_o is the deepwater wave height, and T is the wave period. The probability distribution for I_o was derived from the theoretical model for the joint distribution of wave height and period of LONGUET-HIGGINS (1983) and an analytical transformation of random variables. For the determination of break rate (1) was used, with the peel angle given by:

$$\tan\alpha_b = \tan\theta_b + \frac{-\left(\dfrac{\partial H_b}{\partial y}\right)}{\dfrac{5}{4}\kappa\, m \cos^2\theta_b} \quad (4)$$

in which θ_b is the angle between the wave crest and the bottom contours (see Figure 1), $\partial H_b/\partial y$ is the longshore gradient in wave height associated with finite-crested waves, and κ is the ratio of wave height to water depth at incipient breaking. In developing (4), shallow water linear wave theory was applied, and it was assumed that gradients in θ_b are small. Two joint probability density functions (pdf) for Iribarren Number and peel rate were devised. The first was for the special case of longshore-uniform wave heights (*i.e.* $\partial H_b/\partial y \equiv 0$, and so $\alpha_b = \theta_b$), and assumed that θ_b had a cosine-squared distribution. Numerical integration of wave period established the marginal pdf(I_o,P). The second pdf assumed normally incident waves ($\theta_b = 0$) and a single characteristic value for $\partial H_b/\partial y$. Although the models were heuristic, results indicated that the longshore gradient in wave height enhanced the surfbreak significantly more than the obliqueness of the incident waves. The major weaknesses of the stochastic models of surfing climate that have been developed thus far are 1) due to the lack of data on maximum sustainable board speeds, Iribarren Number has been used as its surrogate, and 2) the fact that equation (4) requires information on $\partial H_b/\partial y$ that is presently unavailable and that accurate measurements of $\partial H_b/\partial y$ would be extremely difficult to obtain. Also, shallow water linear wave theory has been utilized throughout most of the model development, whereas near incipient breaking (the point sought by surfers) the wave is highly nonlinear. In the present study, improvements in stochastic surf climate modeling have been achieved by 1) incorporating the results of a study of maximum sustainable board speed, and 2) introduction of simpler, direct modeling of the distribution of the peel angle. In addition, results of higher order wave theory (cnoidal), as well as findings from laboratory wave channel studies have been utilized near incipient breaking.

PROBABILITY DENSITY OF BREAKER HEIGHT

In a companion study described elsewhere in this issue (DALLY, 2001), video analysis techniques were utilized to study the board-speed of surfers. To establish the maximum sustainable board speed, the rides selected for analysis were limited to those in which the surfer rode directly along the face of the breaking wave for extended periods of time. That is, if the surfer had tried to make repeated turns up-and-down the wave (carving), he would have been overtaken by the breakpoint. The data strongly suggested a relationship between maximum sustainable board speed and breaker height given by:

$$S = \beta \sqrt{gH_b} \tag{5}$$

where $\beta = 1.93$. Although the data were dominated by plunging breakers, the limited number of spilling breakers displayed no special trend of their own. Consequently, it can be assumed for the purposes herein that board speed is dependent only on wave height, and not breaker type. Although breaker type, and therefore Iribarren Number, may be important to the amount of enjoyment extracted from the wave, they do not appear to be directly consequential to board speed. Therefore if the pdf(H_b) is known, the pdf(S) can be determined easily.

A model for the pdf(H_b) can be derived as follows. Starting from a Rayleigh distribution for wave height in deep water:

$$\mathrm{pdf}(H_o) = \frac{2}{H_{rmso}{}^2} H_o \exp^{-}\left(\frac{H_o}{H_{rmso}}\right)^2 \tag{6}$$

in which H_{rmso} is the root-mean-squared height in deep water, the waves can be shoaled using the recommendations of SHUTO (1974):

$$H = H_o\sqrt{\frac{C_{go}}{C_g}} \text{ (linear theory)}$$

$$\text{for } \frac{gHT^2}{h^2} \leq 30 \tag{7a}$$

and:

$$H^2h^{4/7} = \text{const. (cnoidal theory)}$$

$$\text{for } \frac{gHT^2}{h^2} > 30 \tag{7b}$$

where C_g is the local group velocity, C_{go} is the deepwater value, and h is the local water depth. In principle, linear theory (7a) is used to shoal each wave up to the associated water depth where gHT^2/h^2 equals 30. Thereafter, cnoidal theory (7b) is used to shoal each wave up to incipient breaking. The recommendations of SHUTO (1974) have been validated previously using field data for shoaling random waves (DALLY, 1992).

In applying (7) to make the transformation of random variable from H_o to H_b, although a probability distribution could be adopted for wave period (*e.g.* LONGUET-HIGGINS, 1983), for simplicity period is assumed uniform. If the depth where the transition between linear and cnoidal theory occurs is denoted as h_{30}, applying (7a) yields:

$$h_{30}{}^2 = \frac{gHT^2}{30} = \frac{gT^2}{30} H_o \sqrt{\frac{C_{go}(T)}{C_g(T, h_{30})}} \tag{8}$$

which can be solved for H_o:

$$H_o = \frac{30}{gT^2} h_{30}{}^2 \sqrt{\frac{C_g(T, h_{30})}{C_{go}(T)}} \tag{9}$$

Unfortunately, because the dispersion relation from linear theory is transcendental with respect to wave length, the group velocity cannot be expressed explicitly in terms of T and h_{30}. To resolve this problem, it is noted that surfers typically wait for the very largest waves, which break in the outermost surf zone. A single value for h_{30}, denoted h_*, is therefore computed using the deepwater wave whose height demarcates the highest 1/10 waves. For the Rayleigh distribution this height is approximately 1.52 H_{rmso}, and (8) becomes:

$$h_*{}^2 = \frac{gT^2}{30} 1.52\, H_{rmso} \sqrt{\frac{C_{go}(T)}{C_g(T, h_*)}} \tag{10}$$

which is solved for h_* by fixed-point iteration. Recalling that $\kappa = H_b/h_b$, then from (7b):

$$H = \frac{H_b{}^{9/7}}{\kappa^{2/7} h_*{}^{2/7}} \tag{11}$$

and substituting (11) into (7a) and applying (10) to provide the radical of the ratio of group velocities yields:

$$H_o = \frac{H_b{}^{9/7}}{\kappa^{2/7} h_*{}^{16/7}} \frac{gT^2}{30} 1.52\, H_{rmso} \tag{12}$$

Finally, to include the influence of wave steepness and bottom slope on κ, the formula of WEGGEL (1972) is utilized:

$$\kappa = b(m) - a(m)\frac{H_b}{gT^2} \tag{13a}$$

in which:

$$a(m) = 43.75[1 - \exp(-19m)]$$

$$b(m) = \frac{1.56}{1 + \exp(-19.5m)} \tag{13b}$$

Equation (13) is inserted into (12) to complete the relationship needed to perform a transformation of random variable from H_o to H_b:

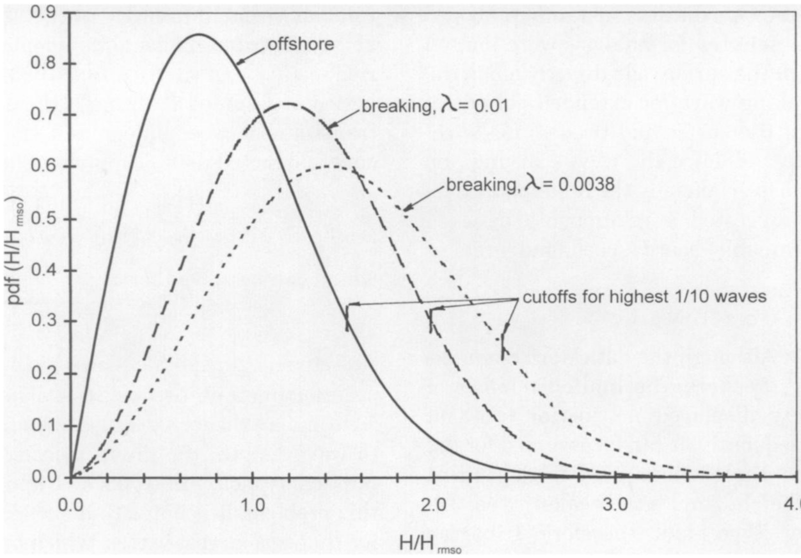

Figure 2. Probability density function of dimensionless breaker heights given by (19) for two values of deepwater wave steepness; $\lambda = 0.01$ (T = 8 s) and $\lambda = 0.0038$ (T = 13 s). Distribution of wave height for initial offshore conditions given by the Rayleigh distribution (Equation 16) is also shown for reference.

$$H_o = 1.52\ H_{rmso} \frac{gT^2}{30 h_*^{16/7}} \left(b - a \frac{H_b}{gT^2} \right)^{-2/7} H_b^{9/7} \qquad (14)$$

Before transforming the pdf(H_o), it is helpful to cast the model into dimensionless form using:

$$\hat{H}_o = \frac{H_o}{H_{rmso}} \qquad \hat{H}_b = \frac{H_b}{H_{rmso}} \qquad \hat{h}_* = \frac{2\pi h_*}{gT^2}$$

$$\lambda = \frac{2\pi H_{rmso}}{gT^2} \qquad (15)$$

Equation (6) becomes:

$$\mathrm{pdf}(\hat{H}_o) = 2\ \hat{H}_o \exp^-(\hat{H}_o^2) \qquad (16)$$

whereas (10) becomes:

$$\hat{h}_*^2 = \frac{1.52}{30} 2\pi\ \lambda \sqrt{\frac{Cg_o(T)}{Cg(T, \hat{h}_*)}} \qquad (17)$$

and (14) becomes:

$$\hat{H}_o = 2\pi \frac{1.52}{30} \lambda^{9/7}\ \hat{h}_*^{-16/7} \left(b - a \frac{\lambda}{2\pi} \hat{H}_b \right)^{-2/7} \hat{H}_b^{9/7} \qquad (18)$$

The probability density function for dimensionless breaker height is now computed from:

$$\mathrm{pdf}(\hat{H}_b) = \mathrm{pdf}(\hat{H}_o)_{\hat{H}_0 = f(\hat{H}_b)} \cdot \left| \frac{d\hat{H}_o}{d\hat{H}_b} \right| \qquad (19)$$

where $f(\hat{H}_b)$ is given by (18) and the Jacobian is given by:

$$\left| \frac{d\hat{H}_o}{d\hat{H}_b} \right| = \left| 2\pi \frac{1.52}{30} \lambda^{9/7}\ \hat{h}_*^{-16/7} \times \left\{ \left[\frac{9}{7} \hat{H}_b^{2/7} \left(b - a \frac{\lambda}{2\pi} \hat{H}_b \right)^{-2/7} \right] + \left[\frac{2}{7} a \frac{\lambda}{2\pi} \hat{H}_b^{9/7} \left(b - a \frac{\lambda}{2\pi} \hat{H}_b \right)^{-9/7} \right] \right\} \right| \qquad (20)$$

Sample results from (19) are shown in Figure 2, which displays the pdf($\hat{H}_b$) for a beach slope of 1/50 and two different values of deepwater steepness (λ), in contrast to the original pdf($\hat{H}_o$). It is clear that, if H_{rmso} is fixed (1.0 m in this case), longer period waves attain higher heights than shorter period waves, and will consequently give faster rides according to (5). The highest 1/10 breakers, *i.e.* the waves generally sought by surfers, are also identified in the figure. Each breaker cutoff, $\hat{H}_{bc}$, is determined using fixed-point iteration and the expression:

$$\hat{H}_{bc} = \left(\frac{30}{2\pi}\right)^{7/9} \hat{h}_{*}^{16/9} \lambda^{-1} \left(b - a\frac{\lambda}{2\pi}\hat{H}_{bc}\right)^{2/9} \quad (21)$$

which has been developed from (18) with $\hat{H}_{oc} = 1.52$.

PROBABILITY DENSITY OF PEEL ANGLE

As noted in the background section, attempts to model the distribution of the break rate are hindered by the inability to analytically predict the peel angle, due mostly to the difficulty in establishing any longshore gradient in wave height ($\partial H_b/\partial y$) for each wave. In DALLY (1990a) the plane wave case ($\partial H_b/\partial y = 0$), is rigorously developed and should be useful in situations where wave obliquity is truly the source of significant peel angles. However, the assumption of plane waves in nature is often weak, especially when characterizing the break on sandy beaches that have smooth and parallel bottom contours. For example on the central Atlantic coast of Florida, due to the broad continental shelf, long-period swell breaks nearly normally incident to the beach. However, the waves are usually sufficiently finite-crested (*i.e.* $\partial H/\partial y \neq 0$) to produce large peel angles and good surfing conditions. There exist several potential sources of this finite-crestedness, the most likely being the interaction of waves from two or more directions. Wave focusing by nearshore bathymetry is also a likely contributor to finite-crestedness at some beaches. Regardless of the source, the longshore gradient in the height of a particular wave is generally ephemeral and small, and therefore difficult to measure accurately in the field.

Conceding the trouble in deterministically predicting peel angles, for the purposes of stochastic modeling of surfing climate it appears that direct empirical/intuitive development of the distribution of peel angle is justified. The distribution proposed herein is the Beta Distribution, with the sine of the peel angle as the random variable. With the constraint of $-1 \leq \sin \alpha_b \leq 1$, this distribution is (see *e.g.* BENJAMIN and CORNELL, 1970):

$$\mathrm{pdf}(\sin \alpha_b) = \frac{1}{B(r, t)\, 2^{t-1}} (\sin \alpha_b + 1)^{r-1} (1 - \sin \alpha_b)^{t-r-1}$$

$$-1 \leq \sin \alpha_b \leq 1 \quad (22a)$$

in which B(r, t) is the Beta function defined as:

$$B(r, t) = \frac{(r-1)!\,(t-r-1)!}{(t-1)!}$$

for r and t integers

$$B(r, t) = \frac{\Gamma(t)\,\Gamma(t-r)}{\Gamma(t)}$$

for r and t non-integers. (22b)

The mean and variance are given by:

$$\overline{\sin \alpha_b} = -1 + 2\frac{r}{t} \qquad \sigma_{\sin \alpha_b}{}^2 = 4\frac{r(t-r)}{t^2(t+1)} \quad (23)$$

Figure 3 presents graphs of this pdf for three different hypothetical cases that have physical relevance. The first is a symmetrical distribution with t = 6 and r = 3 ($\sigma^2 = 0.143$), which would represent surfing conditions dominated by slight peel angles, resulting in close-outs. The second case is for t = 6 and r = 2 ($\sigma^2 = 0.127$) and would represent waves peeling predominantly to the surfer's left, and the third is for t = 6 and r = 4 ($\sigma^2 = 0.127$) and would be for waves peeling to the right. Figure 4 presents graphs with t = 8, showing that decreasing the variance further reduces the number of surfable waves for the close-out dominated case ($\sigma^2 = 0.111$), but increases the proportion of surfable waves for the left and right cases ($\sigma^2 = 0.083$). Thus with only two parameters, a broad range of peel conditions due to obliquely incident and/or finite-crested waves can be represented with this model. Although not shown, the distribution of peel angle for a 'point break', *i.e.* the situation with both lefts and rights but few close-outs, can be constructed by adding the distributions for lefts and for rights, and re-normalizing.

SURFABILITY AND THE JOINT DISTRUBTION OF BOARD SPEED AND BREAK RATE

In estimating break rate based upon peel angle, Equation (1) requires a reliable estimate of the breaker celerity, c_b. Although a wave at incipient breaking is highly nonlinear, numerous studies have used wave channel data to adjust the results of linear, shallow-water wave theory to produce simple yet accurate prediction formulas for c_b. Equation (2), used by WALKER (1974), is one such formula. It is interesting to note that if the breaker celerity is indeed given by (2), then from (1) and (5) a single critical peel angle is given by:

$$\sin \alpha_b \geq \frac{1.25}{\beta} \cong 0.6477 \quad (24)$$

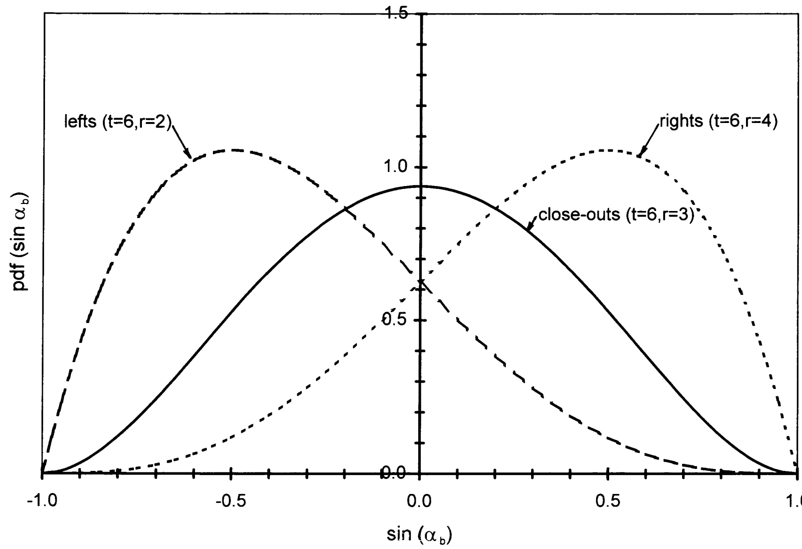

Figure 3. Hypothetical probability density function of the sine of the peel angle given by the Beta distribution (Equation 22, with t = 6), representing surfing climate dominated by close-outs (r = 3), left-directed rides (r = 2), and right-directed rides (r = 4).

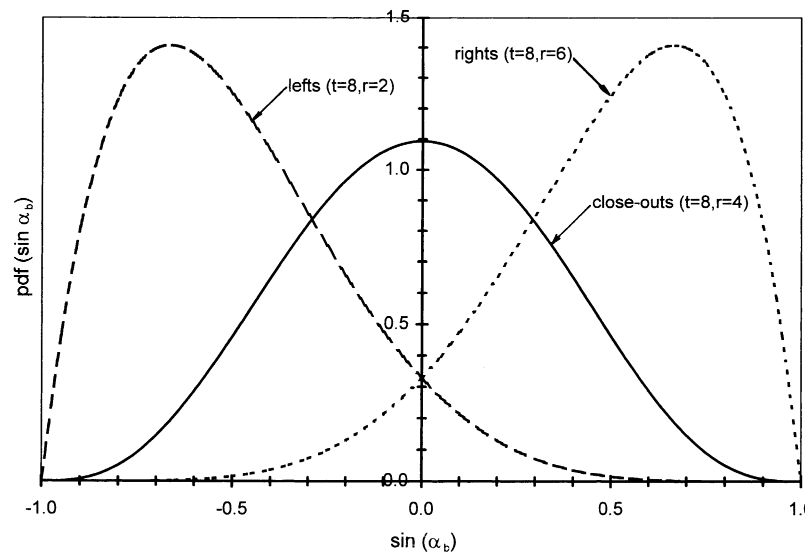

Figure 4. Same as Figure 3, but with reduced variance (t = 8): close-outs (r = 4), left-directed rides (r = 2), and right-directed rides (r = 6). In comparison to Figure 3, waves become less surfable for the close-out case, but more surfable for the left-directed and right-directed cases.

That is with this celerity formula and $\beta = 1.93$, the peel angle must be greater than about 40° for a wave to be surfable. Thus, if (2) were strictly correct, the proportion of surfable waves could be determined from the pdf ($\sin \alpha_b$) alone, *i.e.* without the need for wave height information. Herein, the celerity formula recommended by SCHAFFER, MADSEN, and HANSEN (1993) is utilized, which was validated using laboratory data from a broad range of breaking wave conditions:

$$c_b = 1.3\sqrt{gh_b} \tag{25}$$

and consequently:

$$c_b = \frac{1.3}{\sqrt{\kappa}}\sqrt{gH_b} \tag{26}$$

which from (1) and (5) yields:

$$\sin \alpha_b \geq \frac{1.3}{\beta\sqrt{\kappa}} \tag{27}$$

From (13), κ ranges between 0.78 and 1.56, so the critical peel angle ranges between 33° and 50°, with lower values of κ requiring higher peel angles. In other words, lower steepness waves and/or steep beaches (which promote plunging breakers), do not require as great a peel angle to have surfable conditions as do higher steepness waves and/or gently sloping beaches (which promote spilling breakers).

If it is assumed that breaker height and peel angle are independent random variables, their joint distribution is found simply by multiplying (19) and (22). However, this assumption is more appropriate for some types of surfbreak than for others. With H_b and h_b being somewhat directly proportional, one would expect the assumption to be strongest in situations where the peel angle at incipient breaking is relatively independent of water depth. This would be true, for example, for beach-break comprised of swell waves interacting with local wind-seas, expecting the resulting short-crested process to be only weakly dependent on incipient breaking depth. Peel angle and water depth at incipient breaking would be independent also in the situation of waves breaking on a reef characterized by V-shaped bathymetric contours (*i.e.* point break). Here, for unidirectional waves, all breakers encounter the reef at the same angle regardless of height. However for more smoothly shaped reefs/bathymetry, the smaller waves would penetrate to regions with less oblique contours before breaking, and the incipient peel angle would therefore decrease with breaking depth. This same type of dependence is also expected for beach-break with waves obliquely incident on straight and parallel bottom contours, because refraction has more opportunity to align the smaller waves with the contours before breaking is initiated.

Although their independence is debatable, the otherwise required development of a conditional pdf for the two variables H_b and $\sin(\alpha_b)$ would further complicate the surfbreak modeling problem. Therefore the simpler approach will be pursued, and the ramifications of making this assumption subsequently discussed in light of the modeling results.

Before constructing the joint pdf, it is useful to define dimensionless board speed and break rate as:

$$\hat{S} = \frac{S}{\sqrt{gH_{rmso}}} \tag{28}$$

$$\hat{P} = \frac{P}{\sqrt{gH_{rmso}}} \tag{29}$$

and, because the break rate is singular at $\sin \alpha_b = 0$, cast the model in terms of the reciprocal of the break rate. Using (1) and (26), the old variables can now be expressed in terms of the new:

$$\hat{H}_b = \frac{\hat{S}^2}{\beta^2} \tag{30}$$

$$\sin \alpha_b = \frac{1}{\hat{P}}1.3\sqrt{\frac{\hat{S}^2}{\beta^2 b - a\frac{\lambda}{2\pi}\hat{S}^2}} \tag{31}$$

The transformation of the joint pdf is given by:

$$\mathrm{pdf}\left(\hat{S}, \frac{1}{\hat{P}}\right) = \mathrm{pdf}(\hat{H}_b)_{\hat{H}_b=f_1(\hat{S})} \cdot \mathrm{pdf}(\sin \alpha_b)_{\sin \alpha_b = f_2(1/\hat{P})} \times \begin{vmatrix} \dfrac{\partial \hat{H}_b}{\partial \hat{S}} & \dfrac{\partial \hat{H}_b}{\partial \frac{1}{\hat{P}}} \\ \dfrac{\partial \sin \alpha_b}{\partial \hat{S}} & \dfrac{\partial \sin \alpha_b}{\partial \frac{1}{\hat{P}}} \end{vmatrix} \tag{32}$$

where $f_1(\hat{S})$ is given by (30), $f_2(1/\hat{P})$ is given by (31), and the Jacobian is:

$$\begin{vmatrix} \dfrac{\partial \hat{H}_b}{\partial \hat{S}} & \dfrac{\partial \hat{H}_b}{\partial \frac{1}{\hat{P}}} \\ \dfrac{\partial \sin \alpha_b}{\partial \hat{S}} & \dfrac{\partial \sin \alpha_b}{\partial \frac{1}{\hat{P}}} \end{vmatrix} = \frac{2.6\,\hat{S}}{\beta^2}\sqrt{\frac{\hat{S}^2}{\beta^2 b - a\frac{\lambda}{2\pi}\hat{S}^2}} \tag{33}$$

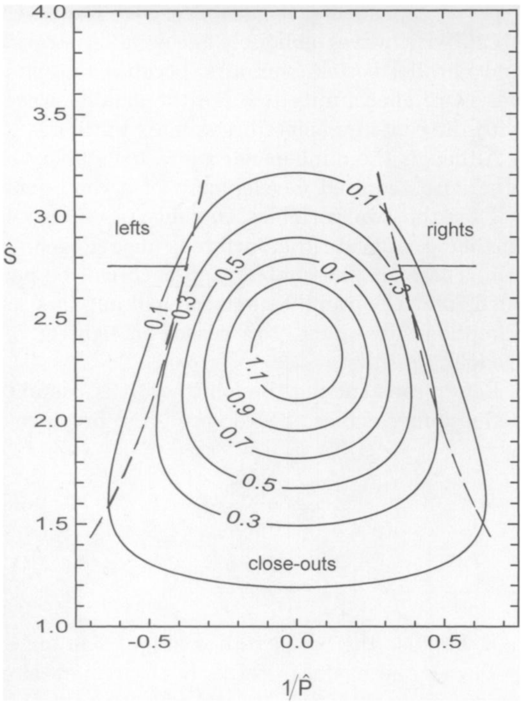

Figure 5. Joint probability density function for dimensionless board speed ($\hat{S} = S/\sqrt{gH_{rmso}}$) and reciprocal of break rate ($1/\hat{P} = \sqrt{gH_{rmso}}/P$) given by (32) for beach slope of 1/50, deepwater steepness $\lambda = 0.01$ (T = 8s), and close-out dominated surfbreak (t = 6, r = 3). Dashed lines separate surfable from unsurfable waves. Solid lines denote the cutoff for the highest 1/10th breakers.

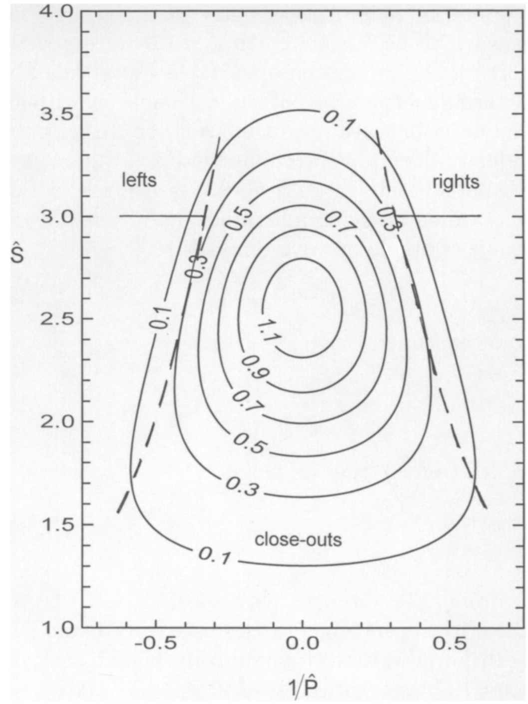

Figure 6. Same as Figure 5 (close-out dominated surfbreak), but for reduced deepwater steepness, $\lambda = 0.0038$ (T = 13 s).

The limits on the pdf are $0 \leq \hat{S} \leq \infty$ and $-1/\hat{P}_{max} \leq 1/\hat{P} \leq 1/\hat{P}_{max}$ where, from (31), the limit on the reciprocal of the peel rate is given by:

$$\frac{1}{\hat{P}_{max}} = \left(1.3\sqrt{\frac{\hat{S}^2}{\beta^2 b - a\dfrac{\lambda}{2\pi}\hat{S}^2}}\right)^{-1} \qquad (34)$$

Sample results of the model, in the form of contour plots of the pdf($\hat{S}$, $1/\hat{P}$), are presented in Figures 5–8 for conditions typical of the Atlantic coast of Florida (H_{rmso} = 1.0 m, T = 8 s and 13 s, m = 1/50). Figures 5 and 6 are for cases dominated by close-outs, whereas Figures 7 and 8 are for cases dominated by lefts. The effect of changing the wave period is clear, with the longer period causing the pdf to stretch in the $\hat{S}$ direction and contract in the $1/\hat{P}$ direction. The curve separating surfable from unsurfable waves is also shown, and indicates that for both the close-out-dominated and left-dominated cases, increasing the wave period does not appear to have a significant effect on the total proportion of surfable waves. This will be investigated in greater detail below.

As noted previously, surfers generally wait for the very largest breakers at the outer edge of the surf zone, not only because they present the greater challenge and excitement, but because in pursuing smaller waves one can get 'caught inside' by the already-broken largest waves. For this reason surfability analysis should focus on the highest waves, judged herein to be the highest 10%. This places an additional constraint on the domain of the pdf($\hat{S}$, $1/\hat{P}$), which is also drawn in Figures 5–8. Numerical integration indicates that for the close-out dominated cases of Figures 5 and 6, less than 2% of the highest waves are surfable. For the left-dominated cases, 15.4% for Figure 7 and 16.3% for Figure 8 are surfable. Thus for these cases for which the chosen wave conditions and bottom slope are typical of Florida, although the longer-period waves have larger breaker heights

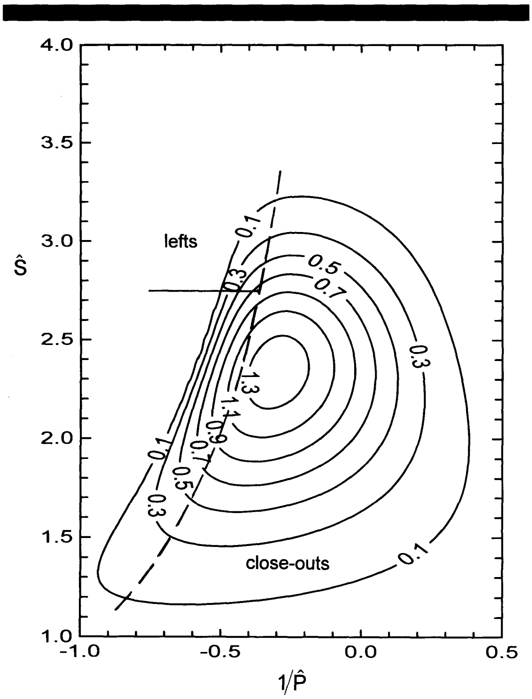

Figure 7. Same as Figure 5 but for left-dominated surfbreak (t = 6, r = 2). 15.4% of the largest 1/10th breakers are surfable.

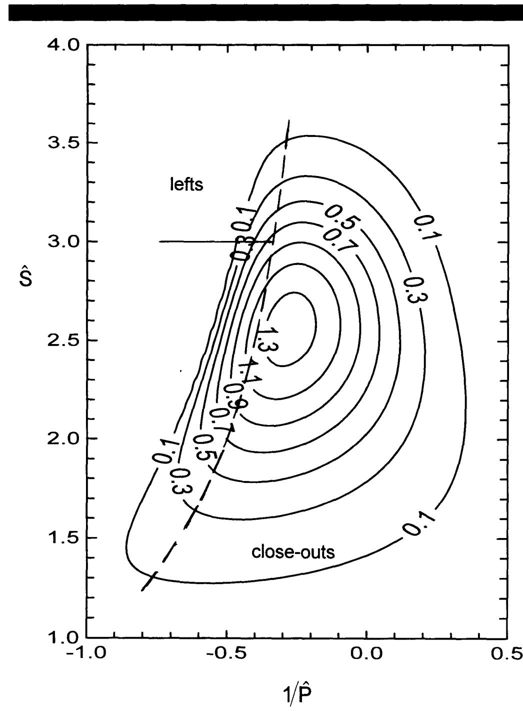

Figure 8. Same as Figure 7 (left-dominated surfbreak), but for reduced deepwater steepness, $\lambda = 0.0038$ (T = 13 s). 16.3% of the largest 1/10th breakers are surfable.

and may provide more enjoyment, in limiting the assessment to the highest 1/10th breakers the proportion of surfable waves remains relatively unchanged. On the other hand, if one were to base the assessment on an absolute cutoff value, *e.g.* $\hat{H}_{bc} = 2.75$, the longer period conditions would have a greater proportion of surfable waves.

DISCUSSION AND CONCLUSIONS

The model for the joint probability density function of maximum sustainable board speed and peel rate developed herein constitutes an improvement over earlier attempts for two reasons. Firstly, because maximum sustainable board speed has been established as a function of breaker height (DALLY, 2001), the derivation of the pdf(S) starts from a proven foundation (*i.e.* that H_o is Rayleigh distributed) and proceeds by utilizing well-established wave theory and empirical formulas. Also, the separation between surfable waves and close-outs is now more precisely defined. If Equation (5) is adopted for board speed, the model for the pdf(S) can be improved only by refining the value of β based on more data, or by refining the analytical prediction of H_b.

The second advancement in this study is that the distribution of peel angle has been modeled in a direct, empirical/intuitive fashion (Equation 22), rather than attempting to predict it deterministically based on parameters that in themselves are difficult to measure and to model stochastically, *i.e.* $\partial H_b/\partial y$. Conceivably, finite-crested waves could be studied deterministically using the state-of-the-art in nonlinear, time-dependent numerical wave models (see *e.g.* SCHAFFER *et al.,* 1993; JENKINS and DALLY, 1999), and statistics/distributions for $\partial H_b/\partial y$ developed from the results. Three-dimensional physical modeling of surf, based on undistorted Froude relations (see *e.g.* DALLY, 1990b), is another potentially valuable tool for the study of peel angles and break rates. In fact, numerical and/or physical model studies may be an essential element of the design of site-specific, manmade surfbreak improvements. However, WALKER (1974) has already shown that the peel angle of a wave can easily be determined from overhead aerial photography, and so development of databases for α_b for different surfing sites is feasible (see *e.g.* HUTT, BLACK, and MEAD, 2001, this issue). Acquiring such data is essential to the calibration and testing of (22), and to overall validation of the stochastic surfabilty model.

Because breaker height has been computed using reliable and robust shoaling formulas, and the versatile Beta Distribution has been adopted to describe the peel angle, the surf climate model presented is applicable to a wide variety of surfbreak types, such as reef break, beach break, and point break. There are no computational limitations on deepwater wave conditions, although in developing (18) other wave transformation processes such as refraction and diffraction have been neglected. It is noted that simple refraction effects on breaker height could be included analytically in the random variable transformation.

Results of exercising the stochastic model with wave/beach input conditions typical for Florida indicate that the distribution of peel angle is of greater importance than the distribution of wave height or the value of deepwater steepness in determining the proportion of surfable waves. This is because κ, although important to the critical peel angle (see Equation 27), is only weakly dependent on H_b (see Equation 13). Consequently for beach slope m fixed, there is not much variation in the critical peel angle. However, κ is highly dependent on m. Both of these findings are fortunate in regard to the idea of manmade surfbreak enhancement, as it is more practicable to modify the peel angle and/or the bottom slope with artificial shoals or reefs than to affect incoming wave height or steepness.

Because the proportion of surfable waves does not depend a great deal on breaker height, the assumed independence of H_b and α_b is not of primary consequence in the approach adopted. At some surf sites though, the fitting parameters r and t in the Beta Distribution may themselves be dependent on breaker height/depth. With sufficient data from a site, functional relationships $r(H_b)$ and $t(H_b)$ could conceivably be developed and applied to (22), thereby providing the conditional pdf(sin α_b: H_b) needed.

With the intent of rationally guiding the design of new surf-sites or the mitigation of the loss of existing surfbreak, although the model developed herein is a potentially useful tool in quantitatively rating surf climate, several important aspects of surfing have not been taken into account. The ease/difficulty in catching the otherwise surfable wave, the length of the ride, and the ease/difficulty in paddling back out are all additional issues material to the study and design of surfbreak.

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