HW 04 B

Problem 4.36

1 Name Nicholas Howse

2 Given

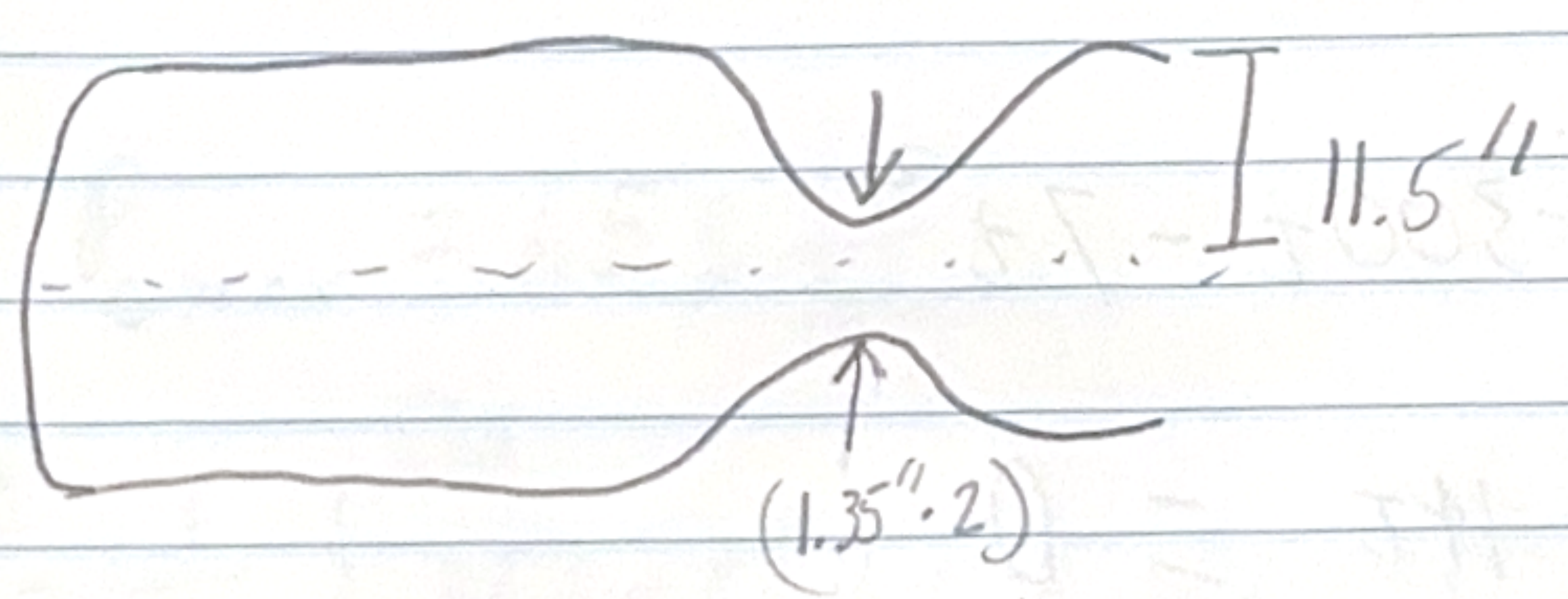
$T_c = 6000°R \quad \gamma = 1.2 \quad \mathcal{M} = 22 \frac{lb}{lb\,mol} \quad P_a = 0$

$\rho_p = 0.0635 \frac{lbm}{in^3} \quad T(t) = 4000 + 300t - 7t^2 \quad 0 < t < 53.5$

3 find

a) T_{max} b) $P_{c,max}$ c) $\overline{I_{sp}}$ d) m_p

4 Schematic



5 Assumptions

Isentropic

1Dimentional flow

circular nozzel

6 Basic Equations

$$F = \dot{m} V_e + (P_e - P_a) A_e$$

$$F = P_c A_t \left[\frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(1 - \left(\frac{P_e}{P_c} \right)^{\frac{(\gamma-1)}{\gamma}} \right) \right]^{1/2} + (P_c - P_a) A_e$$

$$\frac{A_e}{A_t} = \frac{1}{M_e} \left(\frac{2 + (\gamma - 1) M_e^2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{P_c}{P_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

7 Analysis

$$T(t) = 4000 + 300t - 7t^2$$

$$\frac{dT}{dt} = 300 - 14t = 0$$

$$t = \frac{300}{14} \approx \boxed{21.43\ s}$$

$$\frac{A_e}{A_t} = \frac{11.5^2}{1.35^2} = \frac{1}{M_e}\left(\frac{2+(0.2)M_e^2}{2.2}\right)^{\frac{2.2}{0.4}}$$

$$M_e \approx 4.6596$$

$$\frac{P_c}{P_e} = \left(1 + \frac{0.2}{2} 4.6596^2\right)^{\frac{1.2}{0.2}} \approx 1017$$

$$F = 4000 + 300t - 7t^2$$

$$= P_c \cdot \pi(1.35^2)\left[\frac{2 \cdot 1.2^2}{0.2}\left(\frac{2}{2.2}\right)^{\frac{2.2}{0.2}}\left(1-\left(1017^{-1}\right)^{\frac{0.2}{1.2}}\right)\right]^{1/2} + \left(\frac{P_c}{1017} - 0\right)\pi(11.5^2$$

$$= 10.64 P_c + 0.409 P_c = 11.05 P_c$$

$$4000 + 300t - 7t^2 = 17.5 P_c$$

$$P_c = 362 + 27.15t - 0.633t^2$$

$$\frac{dP_c}{dt} = 27.15 - 1.27t \qquad t = 21.43\ s$$

$$P_{c\,max} = 653 \frac{lb}{in^2}$$

$$\bar{P}_c = \frac{1}{53.5}\int_0^{53.5} 362 + 27.15t - 0.633t^2\,dt = 483.8\ Psi$$

$$\bar{P}_e = \frac{\bar{P}_c}{1017} = 0.476\ Psi$$

$$\bar{F} = \frac{1}{53.5}\int_0^{53.5} 4000 + 300t - 7t^2\,dt = 5346.42\ lbf$$

$$V_e = M_e\sqrt{\gamma R T_e} = 4.6596 \cdot \sqrt{1.2 \cdot \frac{1545 \cdot 32.2}{22} \cdot \frac{1}{3.17} \cdot 6000} = 10561\ \frac{ft}{s}$$

$$\frac{T_c}{T_e} = \left(1 + \frac{0.2}{2}(4.6596)^2\right)$$

$F = \dot{m} V_e + P_e A_e$

$5346.42 = \dot{m}\, 10561 + 0.476 \cdot \pi\, 11.5^2$

$\dot{m} = 0.488 \frac{slugs}{s}$

$I_{sp} = \frac{F}{\dot{m}\, g_c} = \frac{5346.42\ lb}{0.488 \cdot 32.2} = 340$

$m_p = \dot{m}\, t_b = 0.488 \cdot 53.5$

$m_p = 26.1\ slugs$

8 Answer

a) $T_{max} = 21.43$

b) $P_{cmax} = 653 \frac{lb}{in^2}$

c) $\overline{I_{sp}} = 340$

d) $m_p = 26.1\ slugs$

9 comment

lb-mol is the derived unit of mol in the imperial system. Useing this unit of kmol, it is nessary to multply by g_c.

Problem 4.38

1 Name Nicholas Hawse

2 Given

$P_c = 982\ Psi$ $d_t = 36in$ $\frac{A_t}{A_e} = \varepsilon = 16$ $\mathfrak{M} = 22 \frac{lb}{lb\text{-}mol}$

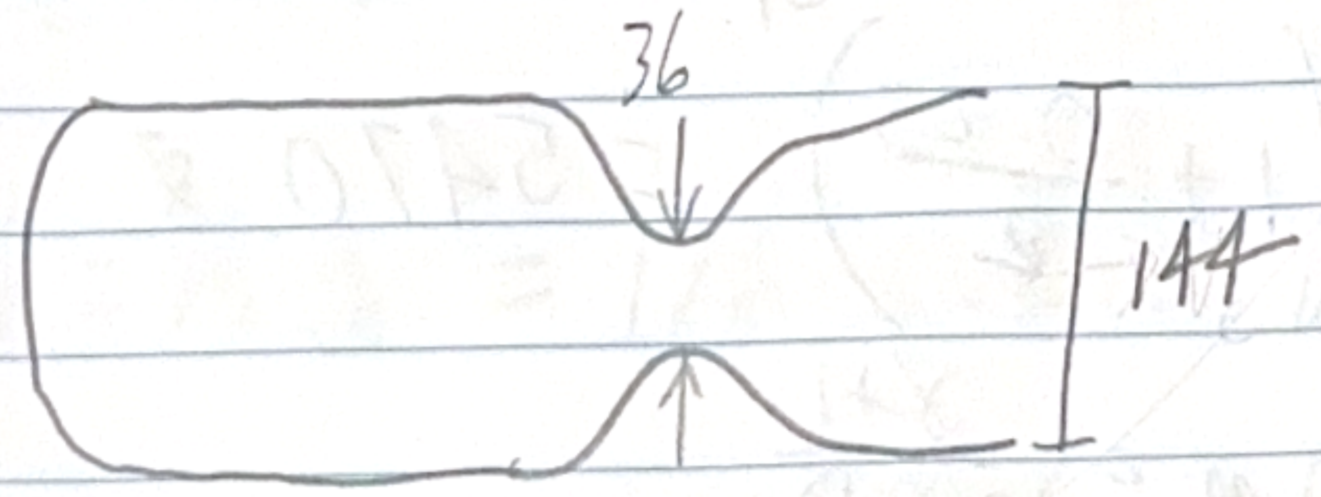
$\gamma = 1.23$ $T_c = 5640\ °F$ $I_{sp_a} = 265s$

3 Find @ sea level

a) momentum thrust b) Pressure thrust c) total thrust

d) V_e e) V_{eff} f) c^* g) I_{sp} h) $\dot{W}$ I) C_f J) I_{sp} eff

4 Schematic



5 Assumptions
Isentropic flow
1D flow
Sea level conditions

6 Basic Equations

$$F = \dot{m} V_e + (P_e - P_a) A_e$$

$$\frac{A_e}{A_t} = \frac{1}{M_e}\left(\frac{2 + (\gamma-1) M_e^2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad c^* = \frac{P_c A_t}{\dot{m}}$$

$$\frac{P_c}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \qquad \dot{m} = \frac{\gamma P_t A_t}{\sqrt{\gamma R T_t}}$$

$$\frac{T}{T_c} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \qquad V_e = \frac{M_e}{\sqrt{\gamma R T}}$$

7 Analysis

$$\dot{m} = \frac{\gamma P_t A_t}{\sqrt{\gamma R T_t}}$$

$$P_t = P_c \frac{P_t}{P_c} = 982 \cdot \left(1 + \frac{0.23}{2}\right)^{\frac{-1.23}{0.23}} = 548.6 \text{ psi}$$

$$\dot{m} = \frac{1.23 \cdot 548.6 \cdot \pi \cdot 0.25 (36^2)}{\sqrt{1.23 \frac{1545.3}{22} 32.2 \cdot 5470.8}} = 176 \frac{\text{slugs}}{s}$$

$$T_t = T_c \frac{T_t}{T_c} = 6100 \cdot \left(1 + \frac{0.23}{2}\right)^{-1} = 5470.8$$

$$\frac{A_e}{A_t} = \frac{1}{M_e} \left(\frac{2 + (\gamma - 1) M_e^2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} = 16$$

$$M_e = 3.718$$

$$V_e = \sqrt{1.23 \cdot \frac{1545.3}{22} \cdot 32.2 \cdot 2346} \cdot 3.718 = 9503 \frac{ft}{s}$$

$$\dot{m}_t = 1672528 \text{ lbf}$$

$$P_e = P_c \frac{P_e}{P_c} = 982 \cdot \left(1 + \frac{0.23}{2}(3.718^2)\right)^{\frac{-1.23}{0.23}}$$

$$P_e = 6.05 \ Psi$$

$$P_t = (6.05 - 14.7) 16 \cdot \pi \cdot 0.25 \cdot 36^2$$

$$P_t = -140791 \ lbf$$

$$T_{SL} = 1531737 \ lbf$$

$$I_{sp} = \frac{T_{SL}}{\dot{m} \ g_c} = \frac{1531737}{176 \cdot 32.2} = 270 \ s$$

$$V_{ec} = I_{sp} \cdot g_c = 8694 \ \frac{ft}{s}$$

$$\dot{W} = \dot{m} \ g_c = 176 \cdot 322 = 5667 \ lbf$$

$$c^* = \frac{P_c A_t}{\dot{m}} = \frac{982 \cdot \pi \ 0.25 \cdot 36^2}{176} = 567 \ \frac{ft}{s}$$

$$C_f = \frac{F}{P_c A_t} = \frac{1531737}{982 \cdot \pi \cdot 0.25 \cdot 36^2} = 1.53$$

$$eff_{Isp} = \frac{265}{270} = 98.1\%$$

8 Answers

a) $T_m = 1672528$ lbf

b) $T_p = -140800$ lbf

c) $T_t = 1531737$ lbf

d) $V_e = 9503 \frac{ft}{s}$

e) $V_{e,eff} = 8694 \frac{ft}{s}$

f) $c^* = 567 \frac{ft}{s}$

g) $Isp = 270s$

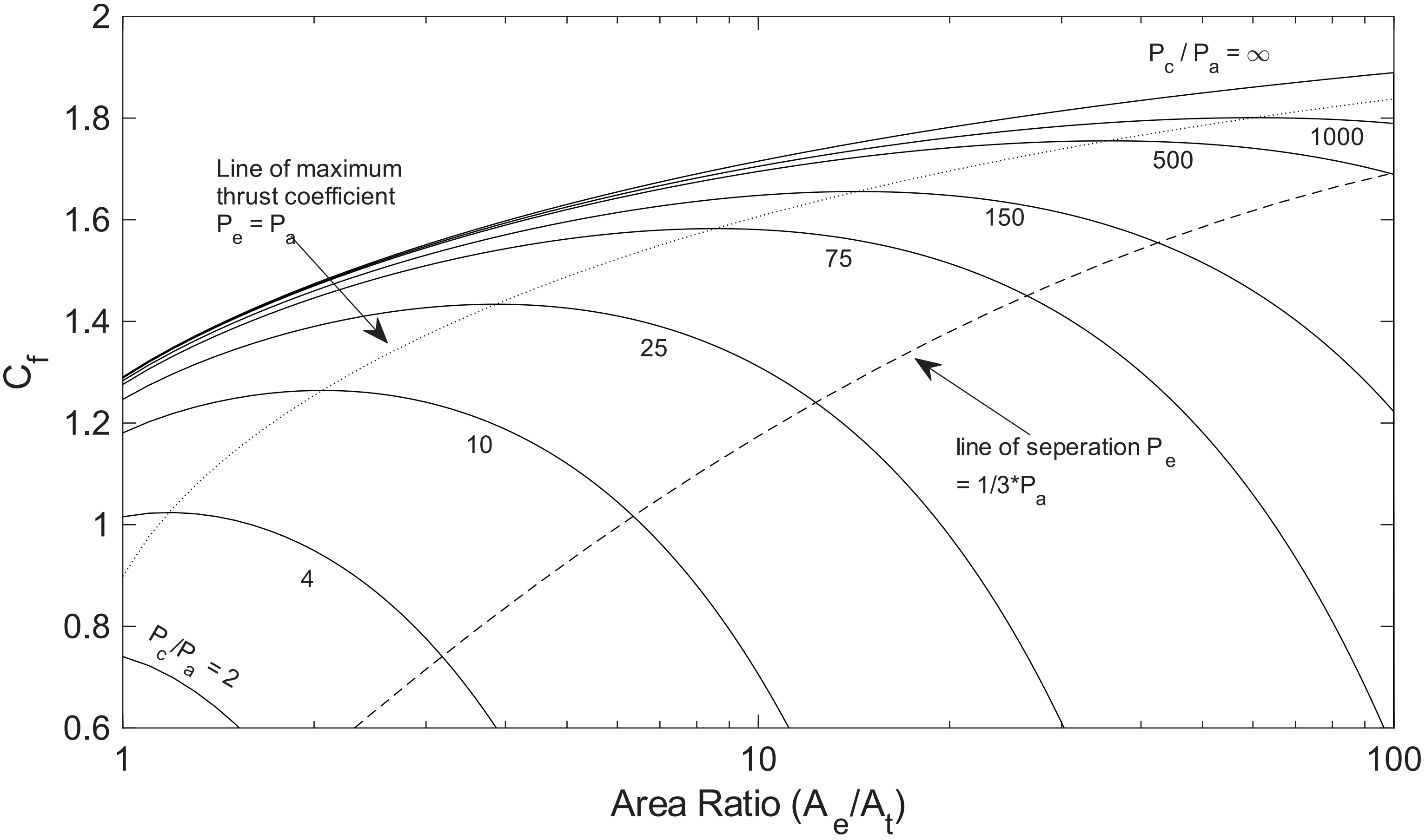
h) $\dot{W} = 5667 \frac{lbf}{s}$

I) $C_f = 1.53$

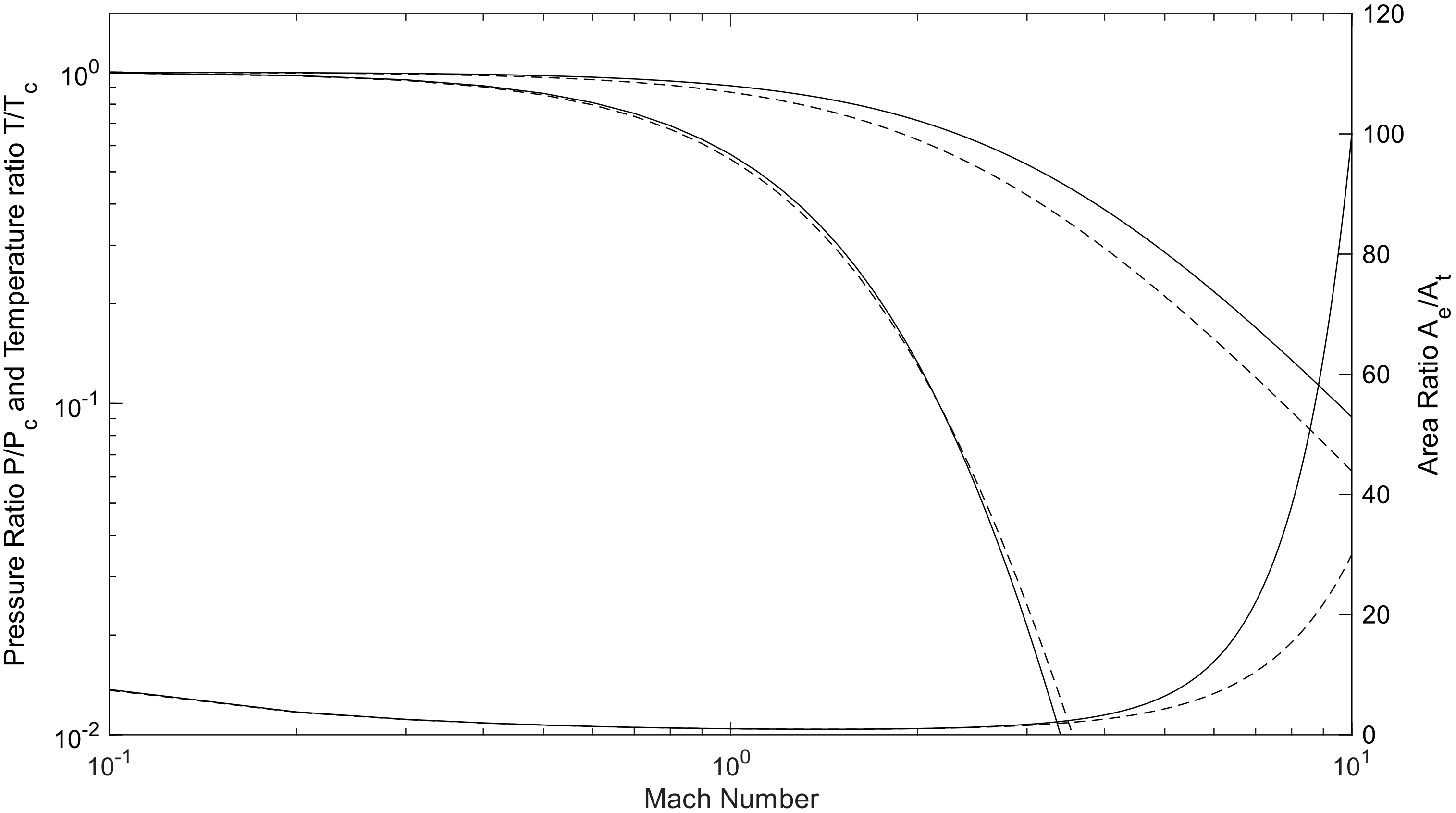
J) Isp eff. = 98.1%

9 Comment

This engine seems to have a very high ratio of true Isp to Ideal Isp. weight flow rate is worse than mass flow rate.



2
1.8
1.6
1.4
1.2
1
0.8
0.6
C_f
1
10
100
Area Ratio (A_e/A_t)
$P_c / P_a = \infty$
1000
500
150
75
25
10
4
$P_c/P_a = 2$
Line of maximum thrust coefficient $P_e = P_a$
line of seperation $P_e = 1/3*P_a$

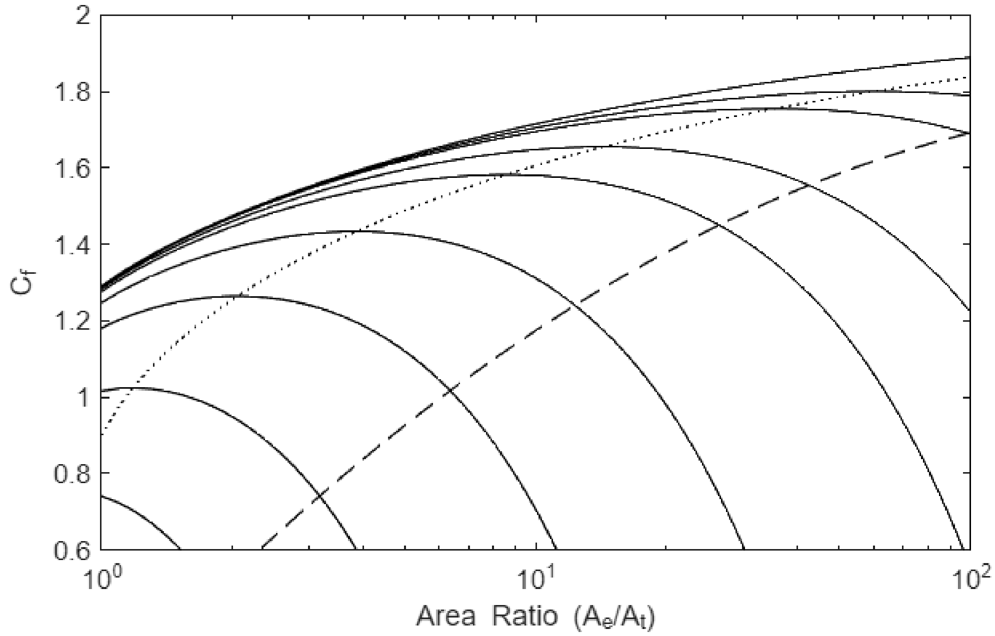


Pressure Ratio P/P_c and Temperature ratio T/T_c
Area Ratio A_e/A_t
Mach Number
10^0
10^-1
10^-2
10^-1
10^0
10^1
120
100
80
60
40
20
0

```
clear; close all; clf; clc;
AR = 0:0.1:100;
curve1 = zeros(length(AR),1);
curve2 = zeros(length(AR),1);
curve3 = zeros(length(AR),1);
curve4 = zeros(length(AR),1);
curve5 = zeros(length(AR),1);
curve6 = zeros(length(AR),1);
curve7 = zeros(length(AR),1);
curve8 = zeros(length(AR),1);
curve9 = zeros(length(AR),1);
curve10 = zeros(length(AR),1);
ratioOfTheSpecificHeats = 1.25;

for index = 1:1000
    ratio = index/10;
    curve1(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,2);
    curve2(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,4);
    curve3(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,10);
    curve4(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,25);
    curve5(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,75);
    curve6(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,150);
    curve7(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,500);
    curve8(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,1000);
    curve9(index) = thrust_coefficient(ratioOfTheSpecificHeats,ratio,100000000);
    curve10(index) = thrust_coefficient_special(ratioOfTheSpecificHeats,ratio);
end

cfmin = @(ar) -0.0445*log(ar).^2 + 0.5324.*log(ar) + 0.1843;
figure(1);
semilogx(AR,curve1,AR,curve2,AR,curve3,AR,curve4,AR,curve5,AR,curve6,AR,curve7,AR,cu
rve8,AR,curve9,'Color','k')
hold on;
semilogx(AR,curve10,':','Color','k')
fplot(cfmin,'--','Color','k');
ylim([0.6 2])
xlim([1 100])
xlabel('Area Ratio (A_e/A_t)')
ylabel('C_f')
hold off;
```




```
cf_proj4a = thrust_coefficient(1.25,10,10);
fprintf('the thrust coefficient at gamma = 1.25, epsilon = 10, P_c/P_a = 10 is C_f
= %f',cf_proj4a)
```

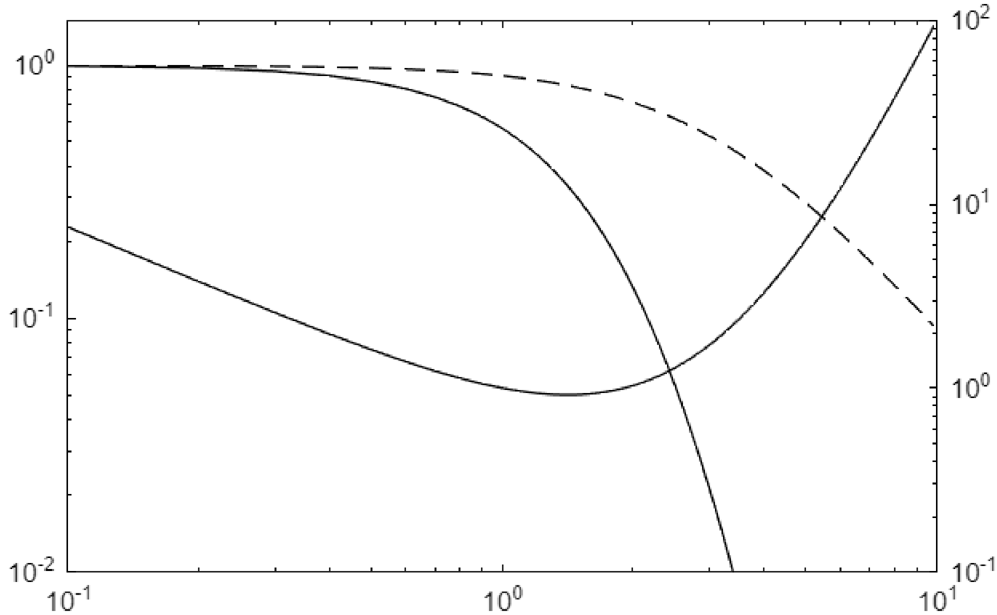
```
the thrust coefficient at gamma = 1.25, epsilon = 10, P_c/P_a = 10 is C_f = 0.714154
```

```

mach = 1:100;
mach = mach./10;

for jndex = 1:100
    machl = jndex/10;
    [pr1(jndex),tr1(jndex),ar1(jndex)] = isentropicRatios(1.2,machl);
    [pr2(jndex),tr2(jndex),ar2(jndex)] = isentropicRatios(1.2,machl);
end
figure(2);
colororder({'k','k'})
yyaxis left;
loglog(mach,pr1,mach,tr1)
ylim([0.01 1.5])
yyaxis right;

loglog(mach,ar1,mach,ar2)
xlim([0 10])
```




```
function [PPc,TTc,AreaRatio] = isentropicRatios(gamma,MachNumber)
% finds the isentropic relasionships pressure, temperature, and area

    PPc = (1 + (gamma - 1)/2 * MachNumber * MachNumber) ^ (-gamma/(gamma-1));

    TTc = (1 + (gamma - 1)/2 * MachNumber * MachNumber)^(-1);

    AreaRatio = MachNumber^-1 * (( 2 + (gamma-1) * MachNumber * MachNumber)/
(gamma+1))^(gamma/(2*gamma-2));

end


function [cf] = thrust_coefficient(gamma,areaRatio,pcpa)
% Calculate the thrust coefficient using the given parameters
    [~,mach] = machNumbers(areaRatio,gamma);

    pcpe = (1+ (gamma-1)/2*mach^2)^(gamma/(gamma-1));

    pepc = pcpe^(-1);

    papc = pcpa^(-1);
```

```
    cf = (2*gamma^2/(gamma-1)*(2/(gamma+1))^((gamma+1)/(gamma-1))*(1-
pepc^((gamma-1)/gamma)))^0.5 + (pepc - papc)*areaRatio;
end

function [cf] = thrust_coefficient_special(gamma,areaRatio)
% Calculate the thrust coefficient using the given parameters
    [~,mach] = machNumbers(areaRatio,gamma);

    pcpe = (1+ (gamma-1)/2*mach^2)^(gamma/(gamma-1));

    pepc = pcpe^(-1);

    papc = pepc;

    cf = (2*gamma^2/(gamma-1)*(2/(gamma+1))^((gamma+1)/(gamma-1))*(1-
pepc^((gamma-1)/gamma)))^0.5 + (pepc - papc)*areaRatio;
end


function [M_sub, M_super] = machNumbers(areaRatio, gamma)
% machNumbers - Computes the subsonic and supersonic Mach numbers
% corresponding to a given area ratio A/A*

    f = @(M) (1./M) .* ( (2/(gamma+1)) .* (1 + (gamma-1)/2 .* M.^2) ) ...
        .^((gamma+1)/(2*(gamma-1))) - areaRatio;

    try
        M_sub = fzero(f, [1e-6, 0.999]);
    catch
        M_sub = NaN;
    end

    try
        M_super = fzero(f, [1.0001, 50]);
    catch
        M_super = NaN;
    end
end
```