HW 06A

Problem 7.1

1 Name Nicholas Hawse

2 Given

$C^* = 5000 \frac{ft}{s}$ $P_{c,i} = 500\ Psi$

$r_b = a P_c^{0.4} \frac{in}{s}$ $\rho_p = 0.05 \frac{lbm}{in^2}$

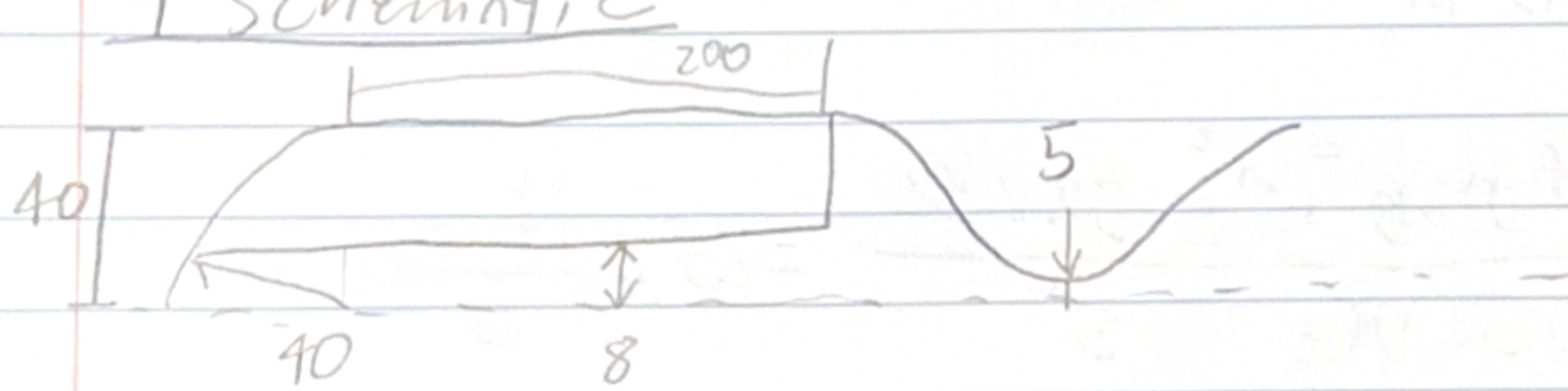
3 find

a) the constant a

b) P_c @ 10in

c) t @ 10in

4 schematic



5 Assumptions

- Steady State
- constant burn rate
- No throught erosion
- instant flame spread

6 basic equations

$$\dot{m} = \frac{P_c A_t}{C^*} = A_b \cdot r \cdot \rho_p$$

$$L^2 + 8^2 = 40^2$$

7 Analysis

$$A_{bi} = \pi 2 r_i \left(\sqrt{40^2 - r_i^2} + 200\right) + \pi\left(40^2 - r_i^2\right)$$

$$A_{bi} = 16850 \text{ in}^2$$

$$\dot{m} = \frac{500 \text{ lbf} \; \pi \cdot 5^2 \; \cancel{\text{in}^2} \; \text{s} \; 32.2 \; \cancel{\text{ft}}}{\cancel{\text{in}^2} \; 5000 \; \cancel{\text{ft}} \; 32.2 \; \text{s}^{\cancel{2}}} = 253 \frac{\text{lbm}}{\text{s}}$$

$$r = \frac{\dot{m}}{A_b \cdot \rho_p} = \frac{253 \; \cancel{\text{lbm}} \; \text{in}^{\cancel{3}}}{\text{s} \; 16850 \; \cancel{\text{in}^2} \; 0.05 \; \cancel{\text{lbm}}} = 0.3 \frac{\text{in}}{\text{s}}$$

$$r = a P_c^{0.4} \qquad a = \frac{r}{P_c^{0.4}} = \frac{0.3 \text{ in}}{\text{s} \cdot 500^{0.4}} = 0.025$$

$$A_{b,10} = \pi\left(2(r_i+10)\left(\sqrt{40^2-(r_i+10)^2}+200\right)+\left(40^2-(r_i+10)^2\right)\right)$$

$$A_b = 30668\ in^2$$

$$m_{dot} = 30668\,in^2 \cdot 0.3\,\frac{in}{s} \cdot 0.05\,\frac{lbm}{in^3} = 460\,\frac{lbm}{s}$$

$$P_c = \frac{\dot{m}\,c^*}{A_t} = \frac{460\,lbm\ 5000\,ft\ s^2}{s\ \pi \cdot 25\,in^2\ \ s\ 32.2\,ft} = 909\,\frac{lb_f}{in^2}$$

$$t = \frac{w}{r} = \frac{10\,in\ s}{0.3\,in} = 33\,s$$

8 Answers

a) $a = 0.025$
b) $P_c = 909$ Psi
c) $t = 33.\overline{3}\,s$

9 Comment

It seems like assuming that r is constant is not a great assumption. It might be better to use a computer

7.26

1 Name Nicholas

2 Given

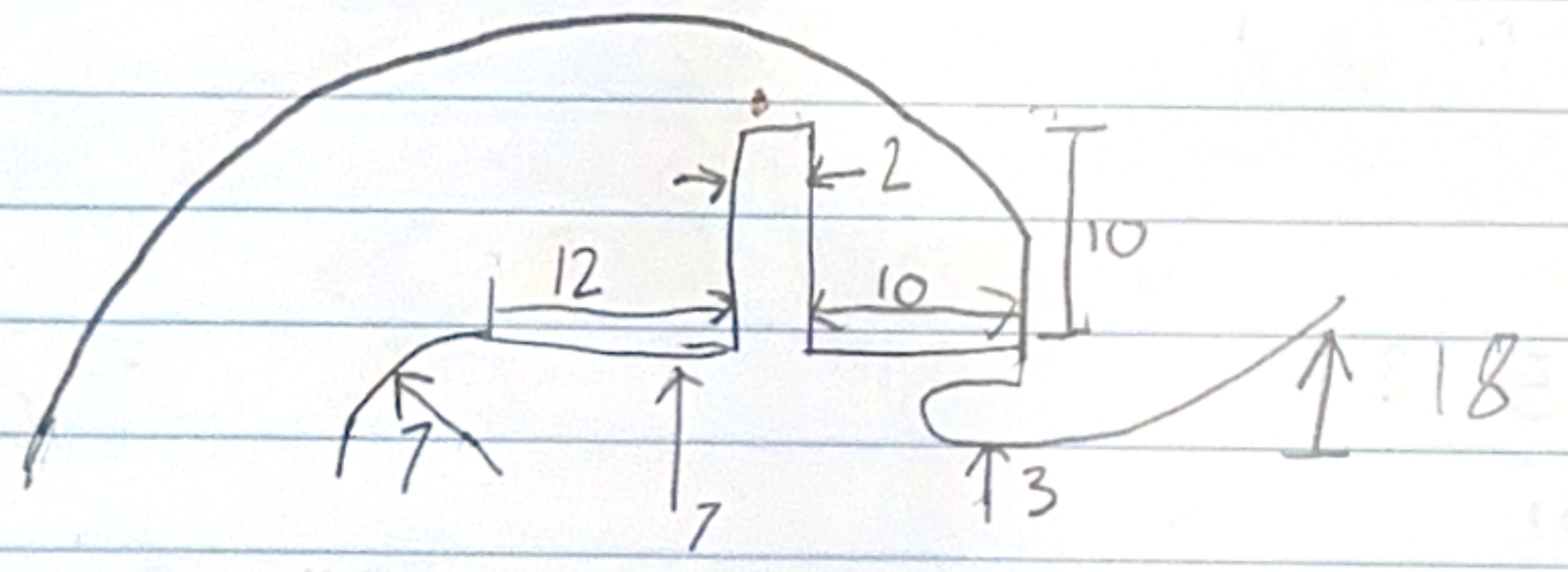
$\rho_p = 0.063 \frac{lb}{in^3}$ $c^* = 4900 \frac{ft}{s}$ $\gamma = 1.2$

$r_b = 0.06 P_c^{0.25}$

3 find

P_{ci} $F_{t,vac}$

4 Schematic



5 Assumptions

steady state
no throught erosion
instant flame transport

6 basic equations

$$\frac{P_c}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\dot{m} = \frac{P_c A_t}{C^*} = \rho r A_b$$

$$F = \dot{m} V_e + (P_e - P_a) A_e = P_c A_t \left[\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]^{1/2} + P_e A_e$$

$$\frac{A_e}{A_t} = \frac{1}{M_e}\left(\frac{2 + (\gamma-1) M_e^2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

7 Anlysis

$$\dot{m} = \frac{P_c A_t}{c^*} = \rho_p r A_b$$

$$\frac{P_c A_t}{c^*} = \rho_p \, 0.06 P_c^{1/4} A_b \qquad \frac{P_c}{P_c^{1/4}} = \frac{c^* \rho_p \cdot 0.06 A_b}{A_t}$$

$$P_c = \left(\frac{c^* \cdot \rho_p 0.06 A_b}{A_t}\right)^{4/3}$$

$$A_b = \frac{1}{2} 4\pi 7^2 + 12 \cdot 7 \cdot 2\pi + 2 \cdot \pi (17^2 - 7^2) + 2\pi \cdot 2 \cdot 17 + 2 \cdot \pi \cdot 10 \cdot 7$$

$$A_b = 3000 \text{ in}^2$$

$$A_t = \pi \cdot 3^2 = 28.27$$

$$P_c = \left(\frac{4900\,ft \cdot 0.063\,lb \cdot 0.06 \cdot 3000\,in^2\,ft\,in}{s\;\;in^3\;\;28.27\,in^2\;\;32.2\,ft\;\;s}\right)^{4/3}$$

$$P_c = \left(\frac{1493\,m \cdot 1743\,Kg \cdot 0.0254\,m \cdot 1.939\,m^2}{s\;\;m^3\;\;s\;0.0182\,m^2}\right)^{4/3}$$

$$P_c = 240\ Psi$$

$$\dot{m} = \frac{P_c A_t}{C^* \, 32.2} = \frac{240\,lb \; 9.\pi\,in^2\,s}{in^2\;4900\,ft} \, 32.2 \frac{ft}{s^2} = 44.6 \frac{lb}{s}$$

$$\frac{A_e}{A_t} \simeq \frac{18^2}{3^2} = 36 \quad M_e = 4.17$$

$$\frac{P_e}{P_c} = \left(1 + \frac{1.2-1}{2}(4.17)^2\right)^{\frac{-1.2}{1.2-1}} = 2.3 \times 10^{-4}$$

$$P_e = \frac{P_e}{P_c} \cdot P_c = 0.013$$

$$240(9\pi)\left(\frac{2 \cdot 1.2^2}{0.2}\left(\frac{2}{2.2}\right)^{\frac{2.2}{0.2}}\left(1-\left(2.3\times10^{-3}\right)^{\frac{0.2}{1.2}}\right)\right)^{1/2} + 0.013 \cdot \pi \cdot 81^2$$

$$F_{tvac} = 12725\ lb$$

8 Answers

$$\boxed{P_c = 240 \text{ Psi} \qquad F_{t,vac} = 12725 \text{ lb}}$$

9 Comment

It is imparitive to Pay attention to the units in the Pressure equaton to get the correct number

Sutton 12.1

1 Name Nicholas Hause

2 Given

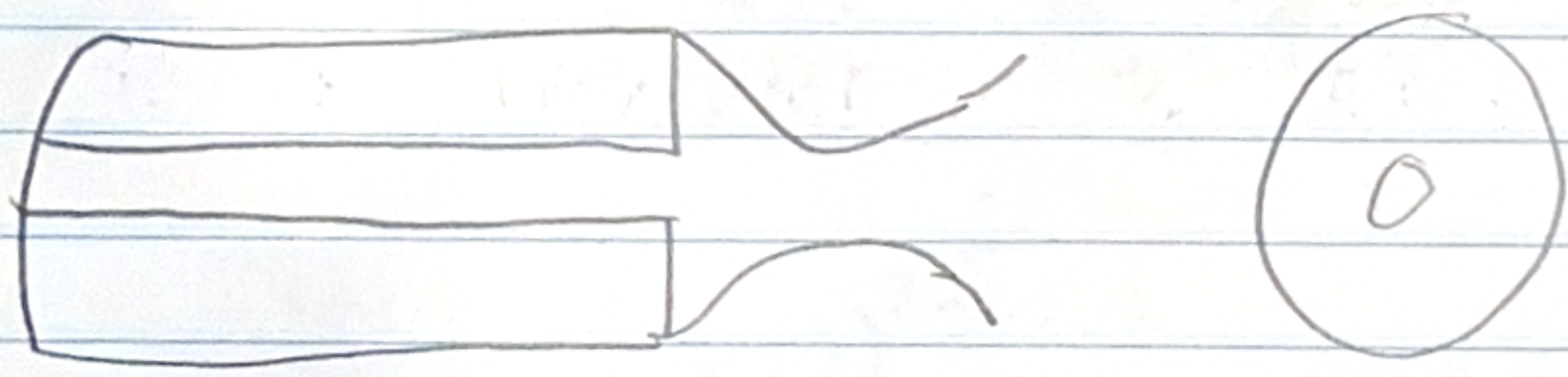
$P_c = 14\ MPa$ $T_c = 2220K$ $\gamma = 1.21$ $\mathfrak{M} = 23 \frac{kg}{Kmol}$

$r = 38 \frac{mm}{s}$ $\rho_p = 1710 \frac{kg}{m^3}$ $n = 0.3$ $\sigma_p = 0.007\ K^{-1}$

3 find

$\frac{A_b}{A_t}$, a, π_K

4 Schematic



5 Assumptions

Ideal rocket

Steady state

no thought cecations

6 Basic equations

$$\pi_K = \frac{\sigma_p}{1-n} \qquad c^* = \frac{\sqrt{\gamma R T_c}}{\gamma\sqrt{\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{\gamma-1}}}} \qquad a = \frac{r}{P_c^{\,n}}$$

$$\frac{A_b}{A_t} = \frac{P_c^{\,1-n}}{\rho_p a c^*}$$

7 Analysis

$$\pi_K = \frac{0.007}{K\ 0.3} = 0.023\ K^{-1}$$

$$c^* = \frac{\sqrt{\dfrac{1.27 \cdot 8314\ \cancel{kg}\,m^2 s^{-2} \cancel{K^{-1}} \cancel{kmol^{-1}}\ \cancel{kmol}\ 2220\cancel{K}}{23\ \cancel{kg}}}}{1.27\sqrt{\left(\frac{2}{2.27}\right)^{\frac{2.27}{0.27}}}}$$

$$c^* = 1354\ \frac{m}{s}$$

$$a = \frac{38\times10^{-3}\ m}{s\,(14\times10^6\,Pa)^{0.3}} = 2.7\times10^{-4}\ \frac{m}{s\ Pa^{0.3}}$$

$$\frac{A_b}{A_t} = \frac{(14\times10^{6}\,Pa)^{1-0.3}\;Pa^{-0.3}\;m^3\;s\;Pa^{0.3}\;s}{1710\,Kg\;2.72\times10^{-4}\,m\;1354\,m}$$

$$\frac{A_b}{A_t} = 159$$

8 Answer

$$\boxed{\frac{A_b}{A_t} = 159, \quad a = 2.72\times10^{-4}\,\frac{m}{s\,Pa^{0.3}}, \quad \pi_K = 0.023\,\frac{1}{K}}$$

9 comment

It seems the units will just work if everything is in base SI units. SI units are great.

Problem SP07-A1

1. **Name: Nicholas Hawse**
2. **Given**: Solid rocket motor with the following characteristics:

Propellant	**Motor**
$c^* = 5210\ ft/s$	$N = 4$ (Number of Grains)
$\gamma = 1.25$	$r_1 = 1.00\ in$ (Initial Grain Radius)
$a_0 = 0.030\left(\frac{in}{s}\right)\left[\left(\frac{lbf}{in^2}\right)^{-n}\right]$	$r_0 = 2.375\ in$ (Final Grain Radius)
$n = 0.35$	$L_0 = 8.00\ in$ (Initial Grain Length)
$\sigma_p = 0.001\ /F$	$A_{t0} = 1.00\ in^2$ (Initial Throat Area)
$T_{b,0} = 70\ °F$ *(ref temp)*	$\varepsilon_0 = 4.0$
$\rho_p = 0.065\frac{lbm}{in^3}$	

3. **Find:**
   a) Equation for Area of Burn Surface, $A_{b,i}$ as a function of web, w_i
   b) Calculate Area of Burn Surface value by hand at web = 0.50 in
   c) Equation for mass of propellant, $m_{p,i}$ as a function of web, w_i
   d) Calculate mass of propellant by hand at web = 0.50 in
   e) Equation for steady state chamber pressure, $p_{c,i}$
   f) Calculate steady state chamber pressure by hand at web = 0.50 in
   g) Calculate Vacuum Thrust Coefficient, $c_{f,v}$, at web = 0.50 in
   h) Calculate Vacuum Thrust, $F_{v,i}$ at web = 0.50 in
   i) Equation for propellant burning rate $r_{b,i}$, including temperature sensitivity terms
   j) Calculate propellant burning rate by hand at web = 0.50 in
   k) Equation for time $t_{i+1} - t_i$ step function of burning rate and web step ($w_{i+1} - w_i$)
   l) Calculate the time step by hand at web = 0.50 in
   m) Equation for total impulse as a function of F_{i+1}, F_i, t_{i+1}, and t_i
   n) Equation for Specific Impulse, I_{sp} based on total impulse

Summary:

Hand Calculations of (w = 0.50 in., T_b = 100 °F)

$A_{b,i}$ (in^2)	$m_{p,i}$ (lbm)	$P_{c,i}$ (lbf/in^2)	$c_{f,i}$ (-)	r_i (in/s)	$F_{vac,i}$ (lb_f)	($t_{i+1} - t_i$) sec
350	19.3	1450	1.6	0.395	2310	0.0253

TYPE Final equations here in symbolic form, using symbols from the end of this document.

$$A_{b,i} = -75.4w_i^2 + 101w_i + 318$$

$$m_{p,i} = 1.63w_i^3 - 3.27w_i^2 - 20.7w_i + 30.3$$

$$p_{c,i} = (-24.5w_i^2 + 32.8w_i + 103.4)^{1.538}$$

$$c_{f,v} = 1.6$$
$$F_{vac,i} = 1.6(-24.5w_i^2 + 32.8w_i + 103.4)^{1.538}$$
$$r_{b,i} = 0.031(-24.5w_i^2 + 32.8w_i + 103.4)^{0.538}$$
$$t_{i+1} - t_i = \frac{w_{i+1} - w_i}{r_{b,i}}$$
$$I_i = \left(\frac{F_{i+1} + F_i}{2}\right)(t_{i+1} - t_i)$$
$$I = \sum_{i=0}^{k} \left(\frac{F_{i+1} + F_i}{2}\right)(t_{i+1} - t_i)$$
$$I_{sp} = \frac{I_{total}}{m_{p,i}(0) g_e}$$

Problem SP07-A2

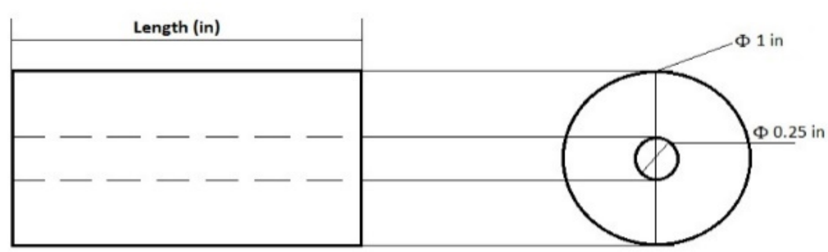
1. **Name: Nicholas Hawse**
2. **Given**: Solid propellant center perforated, cylindrical grain, Inner radius R_i = 0.25 *in*, Outer radius R_f = 1.0 *in*, length L_0 = 3.0 *in*, a_0 = 0.03 *(in/s)(psi^-n)*, n = 0.35, σ_p = 0.001/*F*, c^* = 5210 *ft/s*, ρ_p = 0.065 lb_m/in^3, A_t = 0.05 in^2, k=1.3; $T_{0,b}$ = 70*F*, area ratio ε =4.0, p_a =14.7 *psia*

3. **Find:**
   a) For a grain length of 3.0 in., plot of total burn surface area as a function of web distance burned if
      1. the ends are not inhibited.
      2. Both ends are inhibited.

   b) Repeat part (a) for a grain length of 1.0 in.

   c) Minimum values of L_0 and (R_f - R_i)

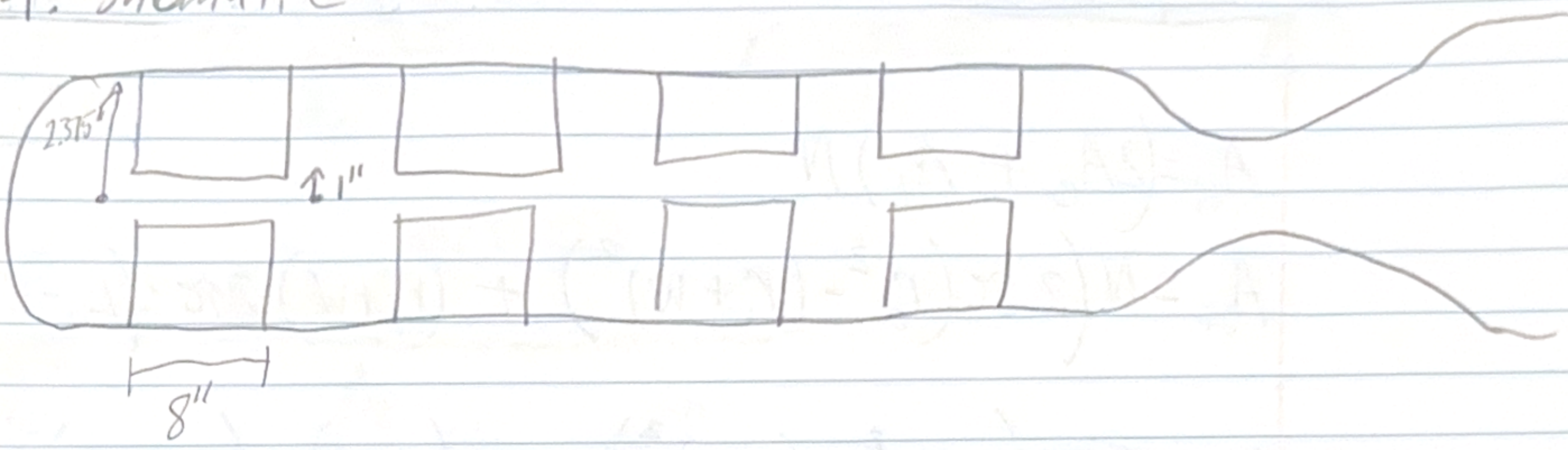
4. **Schematic:**



5. **Assumptions:**
   - Assume $T_b = T_{bo} = 70\ °F$
   - Steady state
   - Instant flame spread

SPOT-A1

4. Shematic



5 Assumptions

Ideal rocket
steady state
no throat erosion

6 Basic equations

$$A_c = \pi r^2 \quad A_{cy} = 2\pi r L$$

$$\dot{m} = \frac{P_c A_t}{c^*} = \rho_p r A_b \qquad r = a_0 e^{(\sigma_p (T_b - T_{b0}))} P_c^n$$

$$P_c = \frac{\rho_p r_i A_b c^*}{A_t} \qquad \frac{P_e}{P_c} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma-1}{\gamma}}$$

$$C_{f,v} = \left[\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right)\right] + \left(\frac{P_e}{P_c}\right)\varepsilon$$

$$\varepsilon = \frac{1}{M}\left(\frac{2+(\gamma-1)M^2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

7 Analysis

$$A_{b_i} = (2A_e + A_L)N$$

$$A_{b_i} = N\left(2\pi\left(r_o^2 - (r_i + w)^2\right) + (r_i + w)\cdot 2\pi\cdot(L_o - 2w)\right)$$

$$A_{b_i} = 8\pi\left(2.375^2 - (1+w)^2\right) + 8\pi(1+w)\cdot(8-2w)$$

$$= 8\pi(5.64) - 8\pi(1+2w+w^2) + 8\pi(8-2w+8w-2w^2)$$

$$= 8\pi(5.64 - 1 - 2w - w^2 + 8 - 2w + 8w - 2w^2)$$

a) $\boxed{= -75.4w^2 + 101w + 318}$

b)

$$-75.4(0.5)^2 + 101(0.5) + 318 = \boxed{350\ in^2}$$

$$m_p = \rho_p V$$

$$V = N\cdot L\cdot A_e = (8-2w)\cdot 4\cdot\pi\left(2.375^2 - (1+w)^2\right)$$

$$m_p = 0.065\frac{lbm}{in^3}\cdot 4\cdot(8-2w)\cdot\pi\cdot(2.375^2 - 1 - 2w - w^2)$$

c) $\boxed{m_p = 1.63w^3 - 3.27w^2 - 20.7w + 30.3}$

$$m_{p_{0.5}} = 1.63(0.5^3) - 3.27(0.5^2) - 20.7(0.5) + 30.3$$

d) $\boxed{m_p = 19.3\ lbm}$

$$P_c = \frac{\rho_p\, r_i\, A_b\, c^*}{A_t} \qquad r_i = a_o\, e^{(\sigma_p (T_b - T_{bo}))}\, P_c^{\,n}$$

$$\frac{P_c}{P_c^{\,n}} = \frac{\rho_p\, a_o\, e^{\sigma_p (T_b - T_{bo})}\, A_b\, c^*}{A_t}$$

$$P_c^{\,1-n} = \frac{0.065 \frac{lbm}{in^3} \cdot 0.03 \frac{in}{s\, in^{-2n}}\, lbf^{-n}\, e^{0.001 \frac{}{F} (100°F - 70°F)}\, (-75.4w^2 + 101w + 318)\, in^2\, 5210 \frac{ft}{s}}{1.00\, in^2 \cdot 32.2}$$

$$\frac{\cancel{lbm} \cdot \cancel{in} \cdot (psi)^{-n} \cdot \cancel{in^2} \cdot \cancel{ft}\, \cancel{s}\, lbf}{\cancel{in^3} \cdot in^2 \cdot \cancel{s} \cdot 32.2\, \cancel{ft}\, \cancel{lbm}}$$

$$\boxed{P_c = (-24.5w^2 + 32.8w + 103.4)^{1.538}} \quad e)$$

$$\boxed{P_{c\,0.5} = 1450\ psi} \quad f)$$

$$M = f(\varepsilon, \gamma) = 2.7$$

$$\frac{P_e}{P_c} = \left(1 + \frac{0.25}{2}(2.7)^2\right)^{\frac{0.25}{1.25}} = 0.0395$$

$$C_{f,v} = \left[\frac{2 \cdot 1.25^2}{0.25}\left(\frac{2}{2.25}\right)^{\frac{2.25}{0.25}}\left(1-\left(\frac{1}{1.13}\right)^{\frac{0.25}{1.25}}\right)\right]^{0.5} + \left(\frac{1}{1.13}\right)(4)$$

g) $\boxed{C_{f,v} = 1.6}$

$$F_{0.5} = 1.6 \cdot 1450 \cdot 1 =$$

h) $\boxed{F_{0.5} = 2310 \text{ lbf}}$

$$r_{b_i} = 0.03 e^{(0.03)}\left(\left(-24.5W^2 + 32.8W + 103.4\right)^{1.538}\right)^{0.35}$$

i) $\boxed{r_{b_i} = 0.031\left(-24.5W^2 + 32.8W + 103.4\right)^{0.538}}$

j) $\boxed{r_{b_i,0.5} = 0.395}$

k) $\boxed{\Delta t = \frac{W_{i+1} - W_i}{r_i}}$ $\quad t = \frac{d}{V}$

$$\Delta W = 0.01$$

$$\Delta t = \frac{0.01}{0.031\left(-24.5(0.5)^2+32.8(0.5)+103.4\right)^{0.538}}$$

l) $\boxed{\Delta t_{0.5} = 0.0253\ s}$

$$I = F \cdot t =$$

m) $\boxed{I = \sum_{i=0}^{n} \left(\frac{F_{i+1}+F_i}{2}\right)\left(t_{i+1}-t_i\right)}$

n) $\boxed{I_{sp} = \frac{I}{m_{p_t}(0)\, g_e}}$

9 Comment

It seems like this is a process best done only once. I has taken over 3 hours to complete.

6 Basic Equations

$A = \pi r^2 \quad A = 2\pi r L$

Z analysis

1) $A = 2\cdot\pi\left(R_f^2 - (R_i + w)^2\right) + 2\pi(R_i + w)(L_o - 2w)$

2) $A = 2\pi(R_i + w)(L_o - 2w)$

1) $A = 2\pi\left(1 - (0.25 + w)^2\right) + 2\pi(0.25 + w)(3 - 2w)$

2) $A = 2\pi(0.25 + w)(3 - 2w)$

$0.25 + w \leq 1$

$w \leq 0.75$

$0 \leq 3 - 2w$

$-3 \leq -2w \quad w \leq \frac{3}{2} = 1.5$

$0 \leq 1 - 2w$

$-1 \leq -2w$

$w \leq \frac{1}{2}$

8 Answers

a) See Plots below

b) See Plots below

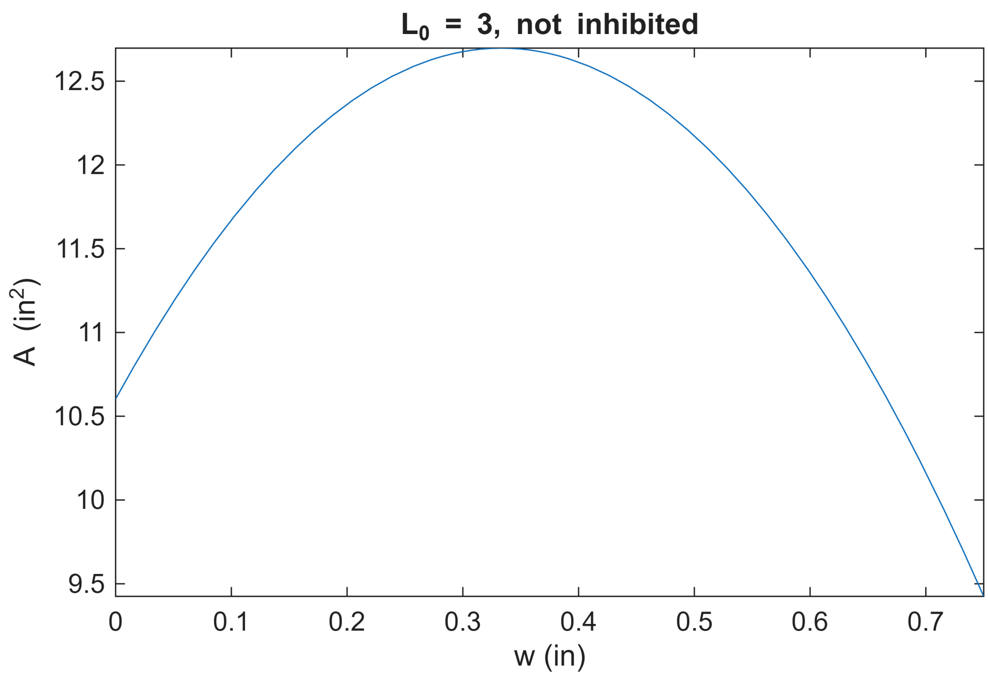
c)

$L_0 = 3$	Case 1	$\boxed{W \le 0.75}$
	Case 2	$W \le 1.5$
$L_0 = 1$	Case 1	$W \le 0.75$
	Case 2	$\boxed{W \le 0.5}$

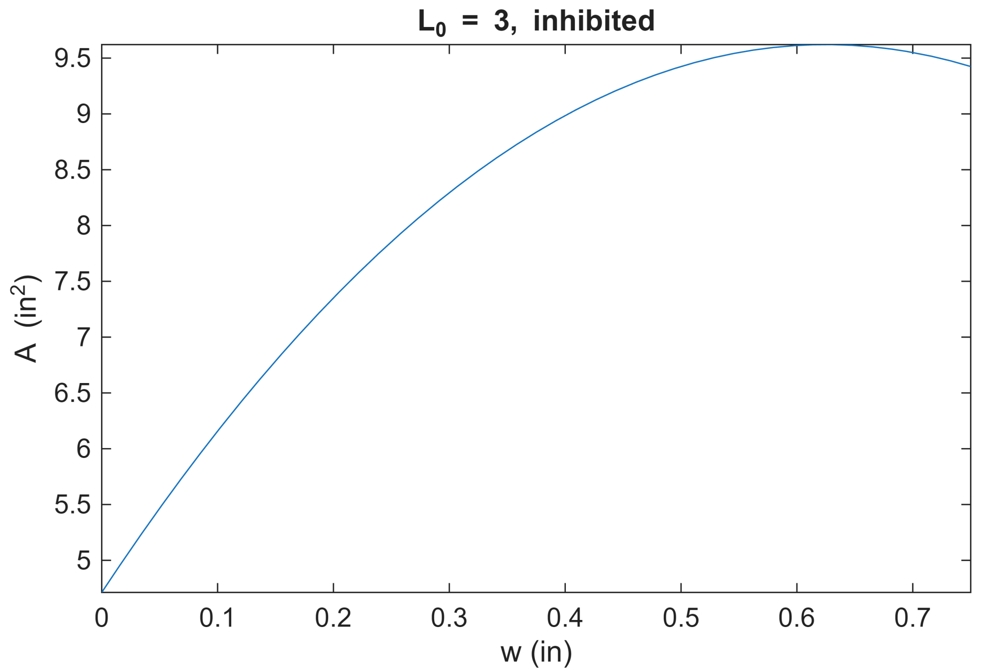
9 coment

It seems that it is nessesary to chek the web distance carefully before useing it to find area.

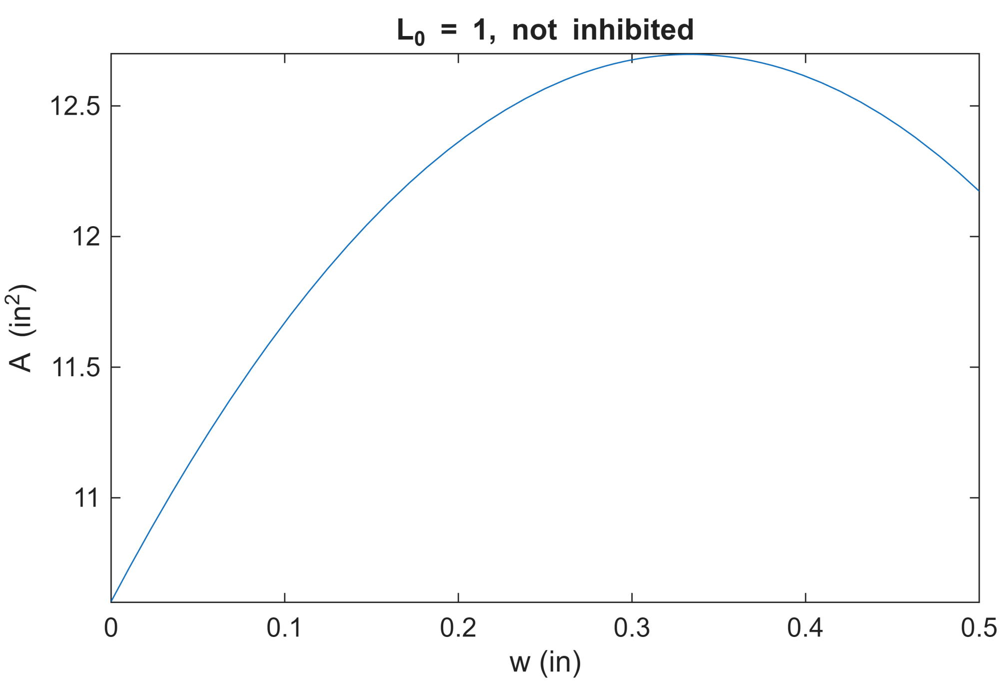
```
clear; clc; close all; clf;

r_f = 1;
r_i = 0.25;
l_0 = 3;


A_1 = @(w) 2*pi*( r_f^2 - (r_i+w).^2 + (r_i+w).*(l_0 - 2.*w));

A_2 = @(w) 2*pi.*(r_i+w).*(l_0 - 2.*w);

fplot(A_1,[0 0.75])
xlabel('w (in)')
ylabel('A (in^2)')
title('L_0 = 3, not inhibited')
```




```
fplot(A_2,[0 0.75])
xlabel('w (in)')
ylabel('A (in^2)')
title('L_0 = 3, inhibited')
```




```
l_0 = 1;


fplot(A_1,[0 0.5])
xlabel('w (in)')
ylabel('A (in^2)')
title('L_0 = 1, not inhibited')
```




```
fplot(A_2,[0 0.75])
```

```
xlabel('w (in)')
ylabel('A (in^2)')
title('L_0 = 1, inhibited')
```