Nicholas Howse | MAE 488-03 HW #2 | 1/25/2025

Problem 1

State with a reson, if the following ODEs are linear or Nonlinear, homogenious or nonhomogenious, constant or Varible coeficiant and what the order is.

a) $\ddot{y} + 3t^2\dot{y} + 5y^2 - e^{-2t} = 0$

i. this is a linear ODE because all dependent varible terms are linear

ii. this is a nonhomogenious ODE because when Put in Standard form it is not equal to zero

iii. this ODE is a varible coefficent because the first derivitve of the dependent varible is multiplied by a functon of the independent varible. $(3t^2\dot{y})$

iv. this is a second order ODE because the highest derivitive term is a second derivative

b) $\dot{z} + \sin z = 0$

i. this is a nonlinear ODE because the second term of the dependent varible is a nonlinear function. $(\sin z)$

ii. this is a homogenious ODE because in standard form it is equal to zero.

iii. this is a constant coeffient ODE because all coeffients are constant.

→ cont.

Problem 1 cont.

b) cont.

iv. this is a first order ODE because the highest derivative is a first derivative.

C) $y'' + 3yy' = 5t + y'$

i. this is a nonlinear ODE because the Second term is nonlinear. $(3yy')$

ii. this is a non homogenious ODE because when put in Standard form it is equal to a non zero term. $(5t)$

iii. this is a constant coefficant ODE because all terms of the dependent Varible have no functions of the independent Varible.

iv. the order of this ODE is second because the higest derivative is a second derivative.

D) $\dot{z} + \sin(t) = 0$

i. this is a linear ODE because all functions of the dependent varible are linear.

ii. this is a non homogenious ODE because in Standard form it is not equal to zero.

cont →

Problem 1 cont.

D) cont.

iii. this is a constant coefficient ODE because all functions of the dependent varible have only constant coefficient.

iv. the order of the ODE is first because the highest derivative is a first derivative.

E) $\dddot{x} + 3x\dot{x} = 2xt$

i. this is a nonlinear ODE because the Second term of the dependent varible is a nonlinear term. $(3x\dot{x})$

ii this is a homogenious ODE becouse when written in standard form it is equal to zero.

iii. this is a varible coeffient ODE because the third term of the dependent varible has a function of the independent varible.

iv. this is a third order ODE because the higest derivative term is the third derivative

F) $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} = e^{4x}$

i. this is a linear ODE because all terms of the dependent varible are linear functions.

cont. →

Problem 1 Cont.

F) cont

ii. this is a nonhomogenrous ODE because When in Standard form it is not equal to 0.

iii this is a varible coefficiant ODE because the second term of the dependent varible is multiplied by a function of the independent Varible.

iv. this is a second order ODE because the higest derivative term is a second deritative.

Problem 2.

Solve the ODE $2\ddot{x} = 60t$ with initial Conditions $\dot{x}(0) = 60$ and $x(0) = 5$ by method of direct integration then check your sol.

$$2\ddot{x} = 60t \rightarrow \ddot{x} = 30t \rightarrow \frac{d^2x}{dt^2} = 30t \rightarrow$$

$$\rightarrow d^2x = 30t\,d^2t \rightarrow \int d^2x = \int 30t\,d^2t \rightarrow$$

$$dx = 30t^2 \cdot \frac{1}{2} + c\,dt \rightarrow dx = 15t^2 + c_1 dt$$

$$\frac{dx}{dt}(0) = 60 \quad 60 = 15(0^2) + c_1 \rightarrow c_1 = 60 \rightarrow$$

$$\rightarrow dx = 15t^2 + 60\,dt \rightarrow \int dx = \int 15t^2 + 60\,dt \rightarrow$$

$$\rightarrow x = 15t^3 \frac{1}{3} + 60t + c_2 \rightarrow x(0) = 5 \rightarrow$$

$$5 = 5(0^3) + 60(0) + c_2 \rightarrow c_2 = 5 \rightarrow$$

$$\boxed{x = 5t^3 + 60t + 5} \rightarrow \frac{dx}{dt} = 15t^2 + 60 \rightarrow$$

$$\frac{d^2x}{dt^2} = 30t \rightarrow 2\ddot{x} = 60t \rightarrow 2(30t) = 60t \rightarrow$$

$$\rightarrow 60t = 60t \quad \checkmark$$

Problem 3

Solve the ODE $\frac{dy}{dx}+8xy=16x$ with initial Conditions $y(0)=4$ with the method of Seperation of Varibles then Check Sol.

$\frac{dy}{dx}+8xy=16x \rightarrow \frac{dy}{dx}\left(\frac{1}{x}\right)+8y=16 \rightarrow$

$\frac{1}{x}\frac{dy}{dx}=16-8y \rightarrow \frac{dy}{dx}=(16-8y)x \rightarrow$

$\frac{dy}{dx}\frac{1}{16-8y}=x \rightarrow \frac{1}{16-8y}dy=x\,dx \rightarrow$

$\frac{1}{8}\int\frac{1}{2-y}=\int x\,dx \rightarrow -\frac{1}{8}\ln(2-y)=\frac{1}{2}x^2+c_1 \rightarrow$

$\ln\left((2-y)^{-\frac{1}{8}}\right)=\frac{1}{2}x^2+c_1 \rightarrow e^{\ln\left((2-y)^{-\frac{1}{8}}\right)}=c_2e^{\frac{1}{2}x^2}$

$(2-y)^{-\frac{1}{8}}=c_2e^{\frac{1}{2}x^2} \rightarrow 2-y=c_2e^{-4x^2} \rightarrow$

$\rightarrow y=2-c_2e^{-4x^2} \rightarrow 4=2-c_2e^0 \rightarrow 4=2-c_2$

$c_2=-2$ $\boxed{y=2+2e^{-4x^2}}$ $\rightarrow \frac{dy}{dx}=2\left(e^{-4x^2}(-8x)\right)$

$\rightarrow \frac{dy}{dx}=-16xe^{-4x^2} \rightarrow -16xe^{-4x^2}+8xy=16x \rightarrow$

$-2e^{-4x^2}+y=2 \rightarrow y=2+2e^{-4x^2}=2+2e^{-4x^2}$

$\checkmark$

Problem 4

for each of the following functions find the Laplace transform from the tables and properties then write in simplest form. Specify the transform pair or property.

A) $X(t) = \sin(2t)u(t)$

$\rightarrow$ Transform Pair 8. $\rightarrow \boxed{X(s) = \frac{2}{s^2+2^2}}$

B) $X(t) = t\sin(2t)u(t)$

Part A & Property 7

$\rightarrow -\frac{d}{ds}\frac{2}{s^2+2^2} = X(s) \rightarrow -2(s^2+4)^{-1}\frac{d}{ds} \rightarrow$

$-2(-1)(s^2+4)^{-2}(2s) \rightarrow \boxed{\frac{4s}{s^2+4} = X(s)}$

C) $X(t) = e^{-5t}\,t\sin(2t)u(t)$

Part B & Property 6

$X(s) = \frac{4(s+5)}{(s+5)^2+4} \rightarrow \boxed{X(s) = \frac{4s+20}{s^2+10s+29}}$

D) $z(t) = t^4 u(t)$

Transform Pair 5

$Z(s) = \frac{4!}{s^5} \rightarrow \boxed{Z(s) = 24s^{-5}}$

cont $\rightarrow$

Problem 4 Cont.

E) $y(t) = [e^{-3t}t^3 + 3t^2]u(t)$

$\rightarrow y(t) = e^{-3t}t^3 u(t) \quad + \quad 3t^2 u(t)$

Property 1 & Transform Pair 5 & Property 6

$\mathcal{L}(e^{-3t}t^3 u(t)) = \frac{3!}{(s+3)^4} \quad \mathcal{L}(3t^2 u(t)) = 3\frac{2!}{s^3}$

$\rightarrow \boxed{\frac{6}{(s+3)^4} + \frac{6}{s^3} = Y(s)}$

F) $x(t) = e^{-4t}\cos(5t)u(t)$

Property 6 & Transform Pair 9

$X(s) = \frac{(s+4)}{(s+4)^2+5^2} \rightarrow \boxed{\frac{s+4}{s^2+8s+41} = X(s)}$

Problem 5

Find the inverse Laplace transform for each of the following functions, then Verify each answer with matlab. Use partial fraction decomposition.

A) $F(S) = \frac{32}{S(S+4)^2}$

$\rightarrow \frac{A}{S} + \frac{B}{S+4} + \frac{C}{(S+4)^2} = \frac{32}{S(S+4)^2}$

$\rightarrow A(S+4)^2 + B(S+4)S + CS = 32$

$S=0 \quad 16A = 32$

$S=1 \quad 25A+5B+C = 32$

$S=-4 \quad -4C = 32$

$C = -8 \quad A = 2 \quad B = -2$

$25(2)+5B-8 = 32$

$F(S) = \frac{2}{S} + \frac{-2}{S+4} + \frac{-8}{(S+4)^2}$

Transform Pair 2, 6, 7 Property 1

$f(t) = \left[2 - 2e^{-4t} - 8\frac{1}{1!}t^1 e^{-4t}\right]u(t)$

$\boxed{f(t) = (2 - 2e^{-4t} - 8te^{-4t})\,u(t)}$

see Matlab Code

Cont $\rightarrow$

Problem 5 Cont.

B) $F(s) = \dfrac{2s^2+10s+4}{s^2(s+2)}$

$\rightarrow \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s+2} = \dfrac{2s^2+10s+4}{s^2(s+2)} \rightarrow$

$A(s)(s+2) + B(s+2) + C(s^2) = 2s^2+10s+4$

$s=0 \rightarrow 2B = 4 \quad B=2$

$s=-2 \rightarrow 4C = 8-20+4 \quad C=-2$

$s=1 \rightarrow 3A+3B+C = 2+10+4 \rightarrow 3A=12 \quad A=4$

$F(s) = 4\frac{1}{s} + 2\frac{1}{s^2} - 2\frac{1}{s+2}$

Transform Pairs 2, 5, 6 & Property 1

$\boxed{f(t) = [4 + 2t - 2e^{-2t}]\,u(t)}$

c) $F(s) = \dfrac{4s+14}{s^2+4s+13}$

$\rightarrow \dfrac{-4 \pm \sqrt{16-4\cdot1\cdot13}}{2} \rightarrow \dfrac{-4 \pm \sqrt{-36}}{2} \rightarrow -2+3i \quad -2-3i$

$F(s) = \dfrac{4s+14}{(s+2-3i)(s+2+3i)} \rightarrow = \dfrac{A}{s+2-3i} + \dfrac{B}{s+2+3i}$

$A(s+2+3i) + B(s+2-3i) = 4s+14$

$s=-2-3i \rightarrow B(-6i) = -8-12i+14 \quad B = 1i+2 = 2+i$

$s=-2+3i \rightarrow A(6i) = -8+12i+14 \quad A = -1i+2 = 2-i$

$F(s) = \dfrac{2-i}{s+2-3i} + \dfrac{2+i}{s+2+3i}$

Cont →

Problem 5 Cont.

C) $F(s) = \dfrac{4s+14}{s^2+4s+13}$

$\rightarrow F(s) = \dfrac{4s+14}{s^2+4s+4+9} \rightarrow \dfrac{4s+14}{(s+2)^2+3^2}$

$\rightarrow \dfrac{4s+8+6}{(s+2)^2+3^2} \rightarrow 4\dfrac{s+2}{(s+2)^2+3^2} + 2\dfrac{3}{(s+2)^2+3^2}$

Transform Pair 10, 11, Property 1

$$\boxed{f(t) = \left[4e^{-2t}\cos(3t) + 2e^{-2t}\sin(3t)\right] u(t)}$$

D) $F(s) = \dfrac{46s+48}{s^3+11s^2+24s}$

$\rightarrow \dfrac{46s+48}{s(s^2+11s+24)} \rightarrow \dfrac{46s+48}{s(s+3)(s+8)} \rightarrow \dfrac{A}{s} + \dfrac{B}{s+3} + \dfrac{C}{s+8}$

$\rightarrow A(s+3)(s+8) + B(s)(s+8) + C(s)(s+3) = 46s+48$

$s=-3 \quad -15B = -90 \quad B = 6$

$s=-8 \quad -40C = -320 \quad C = 8$

$s=0 \quad 24A = 48 \quad A = 2$

$F(s) = 2\frac{1}{s} + 6\frac{1}{s+3} + 8\frac{1}{s+8}$

Transform Pair 2, 6 Property 1

$f(t) = \left[2 + 6e^{-3t} + 8e^{-3t}\right] u(t)$

$$\boxed{f(t) = \left[2 + 14e^{-3t}\right] u(t)}$$

See Matlab

```
% MAE488_Nicholas_Hawse_HW1
% MAE 488 03 Analisis of ANALY ENGINEERING SYSTEMS
% Homework 2
% Nicholas Hawse
% 1/26/2025

% This code checks the hand calculated partial fraction decompositions
% of the homework questions useing the residual function.


%
==============================================================================
% Problem 2
%
==============================================================================
% In this problem you find the patial fraction expasion of several
% functions then take the inverse lapace transform of each function.
% this code calculates the roots of the denomenator function and
% calculates the coefficents of the partial fraction expasion.
%
clc;clear;close

%problem 2 part a
fprintf('==============================\n')
fprintf('Problem 5 \n')
fprintf('==============================\n')
fprintf('This code determines the coefficents on the partial fraction\n')
fprintf(['decomposition of the laplace transform functions useing' ...
    ' the \nresidual function built into matlab\n'])

fprintf('==============================\n')
fprintf('Problem 5 Part a\n')
fprintf('==============================\n')

%

partATopFun = 32; % the coefficent of the function in the numerator of the
function

partABottomFun = [1 8 16 0]; % the coefficent of the function in the
denominator of the function

[partAR,~,~] = residue(partATopFun,partABottomFun); %finds the coefficents
for the PFD

%problem 2 part b

fprintf('The partial fraction decomposition has coefficents %2d %2d
%2d\n\n',partAR)

fprintf('==============================\n')
fprintf('Problem 5 Part b\n')
```

```
fprintf('==============================\n')


partBTopFun = [2 10 4]; % the coefficent of the function in the numerator of
the function
partBBottomFun = [1 2 0 0]; % the coefficent of the function in the
denominator of the function

[partBR,~,~] = residue(partBTopFun,partBBottomFun); %finds the coefficents
for the PFD

fprintf('The partial fraction decomposition has coefficents %2d %2d %2d\n\n'
...
    ,partBR)

%problem 2 part c

fprintf('==============================\n')
fprintf('Problem 5 Part c\n')
fprintf('==============================\n')


partCTopFun = [4 14]; % the coefficent of the function in the numerator of
the function
partCBottomFun = [1 4 13]; % the coefficent of the function in the
denominator of the function

[partCR,~,~] = residue(partCTopFun,partCBottomFun); %finds the coefficents
for the PFD

fprintf('The partial fraction decomposition has coefficents %4s %4s\n\n' ...
    ,num2str(partCR(1)),num2str(partCR(2)))

%problem 2 part d

fprintf('==============================\n')
fprintf('Problem 5 Part d\n')
fprintf('==============================\n')


partDTopFun = [46 48]; % the coefficent of the function in the numerator of
the function
partDBottomFun = [1 11 24 0]; % the coefficent of the function in the
denominator of the function

[partDR,~,~] = residue(partDTopFun,partDBottomFun); %finds the coefficents
for the PFD

fprintf('The partial fraction decomposition has coefficents %2.0f %2.0f
%2d\n' ...
    ,partDR)
```

```
==============================
Problem 5
```

```
==============================
This code determines the coefficents on the partial fraction
decomposition of the laplace transform functions useing the
residual function built into matlab
==============================
Problem 5 Part a
==============================
The partial fraction decomposition has coefficents -2 -8  2

==============================
Problem 5 Part b
==============================
The partial fraction decomposition has coefficents -2  4  2

==============================
Problem 5 Part c
==============================
The partial fraction decomposition has coefficents 2-1i 2+1i

==============================
Problem 5 Part d
==============================
The partial fraction decomposition has coefficents -8  6  2
```

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