MAE 284 Spring 2024 Homework Assignment #3 (100 pts)

1) Numerical integration methods

Consider the definite integral:

$$\int_1^4 (10x^5 + 2.2x^4 - 0.1x^3 - 0.8x^2 - 6)\ dx$$

Calculate the value of the integral using the methods below. You may only use `simpson13` and `simpson38` in part f. You may not use any other built-in or self-written functions

a) The exact value of the integral. You can either use the symbolic tool box or calculate using the following method. Create an anonymous function defined to be the indefinite integral. For example, for $f(x) = x^2$, the indefinite integral is $g(x) = \left(\frac{1}{3}\right)x^3$. Then the exact or true value of the integral can be computed as $I_{\text{true}} = g(b) - g(a)$ where a and b are the limits of integration.

b) A single application of the trapezoidal rule

c) A composite application of the trapezoidal rule with n=7

d) A single application of the Simpson's 1/3 rule

e) A single application of the Simpson's 3/8 method

f) Construct an algorithm in a function named simpsons7 that takes three inputs: a function handle, the lower bound of the integration and the upper bound of the integration. The function should compute and return the three different approximations of an integral using exactly seven segments. You may call simpson13 and simpson38 as sub-functions (you must include this in the header).

g) Create a table with the Method, Value, and MTPRE (magnitude of the true percent relative error) for each of the methods in part a-f. Include 4 digits to the right of the decimal point in all numbers. Create an anonymous function to calculate the MTPRE (Do NOT put a semicolon at the end). The table should look like the one below. Note that the "-" in the last three lines of the table is a dash (**not a minus sign**).

```
Method           Value      MTPRE
---------------- ---------- --------
Exact            xxxxx.xxxx xxx.xxxx
TrapSingle       xxxxx.xxxx xxx.xxxx
Simpsons 1/3     xxxxx.xxxx xxx.xxxx
Simpsons 3/8     xxxxx.xxxx xxx.xxxx
TrapComposite    xxxxx.xxxx xxx.xxxx
S13 - S13 - S38  xxxxx.xxxx xxx.xxxx
S38 - S13 - S13  xxxxx.xxxx xxx.xxxx
S13 - S38 - S13  xxxxx.xxxx xxx.xxxx
```

2) Integrating real data

The temperature in °F along a metal bar has been measured and the results tabulated in Table *1* where x is the distance from the left end of the bar in meters. Note that due to test limitations, the data was not collected at evenly spaced intervals.

Table 1: Measured temperature data of 5 m bar

x [m]	0	0.34	0.99	1.60	2.27	2.96	3.52	3.95	4.54	5
$T(x)$ [°F]	-6.29	-1.79	4.57	12.59	12.49	40.73	50.44	67.7	93.2	99.43

a) The theoretical temperature is given by $T(x) = 4x^2 + 3x - 7$ Plot the theoretical temperature as a smooth blue line and the measured data from the table (red asterisk) in a single plot.

b) Use one of the integration techniques we have studied to estimate the integral of the data. Display the result using an `fprintf` statement with 4 digits to the right of the decimal point.

c) Calculate the magnitude of the true percent relative error. The true value is the integral of the theoretical temperature function. Display the result using an `fprintf` statement with 2 digits to the right of the decimal point.

3) 2D integration

Consider the following double integral with $a = -12$, $b = -6.7$, $c = -10$, and $d = 4$.

$$\int_a^b \int_c^d 133 + \left(\frac{x^5}{32768} + \frac{y^5}{7776}\right) dydx$$

a) Use the mesh function to plot the function (do not use fmesh). Use the axis command to limit the range of x and y to the range specified by a, b, c, and d. Make sure the lower bound for the z axis is 0. Use the original upper bound for z (i.e. before using the axis command). You can save the original axes using: `>> v = axis;`

b) Calculate an estimate of the integral using a single application of Simpson's 3/8 rule in each dimension, starting with the x axis. Use an fprintf statement to print your answer.

c) Calculate an estimate of the integral using a single application of Simpson's 3/8 rule in each dimension, starting with the y axis. Use an fprintf statement to print your answer.

d) Calculate the true value of the integral using the int function. Use an `fprintf` statement to print your answer.

e) Are the results from b through d the same? Why or why not?

f) Extra Credit (6 points) Recreate the plot from part (a) in a new figure. Add two things to the plot: (i) a magenta plane corresponding to a height of 102 units above the x-y plane (i.e. z = 102) and (ii) plot all of the points used in part (c) as red asterisks. Use the view command to set the view to AZ = 0.3 and EL = 24.2. Note: AZ represents the azimuth angle in degrees, and EL represents the elevation angle in degrees.

4) Gauss-Legendre

Consider the following integral.

$$f(x) = \int_{-2}^{3} -0.5x^6 - 0.5x^5 + 1.6x^4 + 1.8x^3 + 1.9x^2 - 0.9x + 379$$

a) Plot the function (integrand) over the interval. Include a solid black line for the x axis (do not use the grid function). You may need to use the axis function to make sure that the x axis line is visible.

b) Calculate the exact value of the integral in MATLAB. Use an `fprintf` statement to print your answer with 10 digits after the decimal point.

c) Use the three-point Gauss-Legendre method to calculate the integral of the function. Show all the steps in the method in your code. Do not use any functions you have written or the built-in integration functions. Use an `fprintf` statement to print the result with 10 digits after the decimal point.

d) Why is the true error nonzero for the Gauss-Legendre method? Explain in text.

5) Romberg Integration

Consider the integral given in problem 1.

a) Use the Romberg integration technique to calculate the integral with an error of $O(h^8)$. Show all the steps in your code (no loop). You can use the `trap` function from the textbook but not the `romberg` function. Print the results in a table like the one below using `fprintf`.

```
j         k=1          k=2          k=3               k=4
- ----------- ----------- ----------- ----------------
1 xxxxx.xxxxx xxxxx.xxxxx xxxxx.xxxxx xxxxx.xxxxxxxxxx
2 xxxxx.xxxxx xxxxx.xxxxx xxxxx.xxxxx
3 xxxxx.xxxxx xxxxx.xxxxx
4 xxxxx.xxxxx
```

b) Use the `romberg` function from the textbook with default relative error tolerance and maximum iterations to verify your answer. Use a single `fprintf` statement to print the result with 10 digits after the decimal point as well as the number of iterations it took.