3.3

1. Name Nicholas Hawse

2 Given

$V_{boid} = 10000 \frac{ft}{s}$ $\quad M_{Pl} = 500\ lbm$

$\Delta V_1 = \Delta V_2$

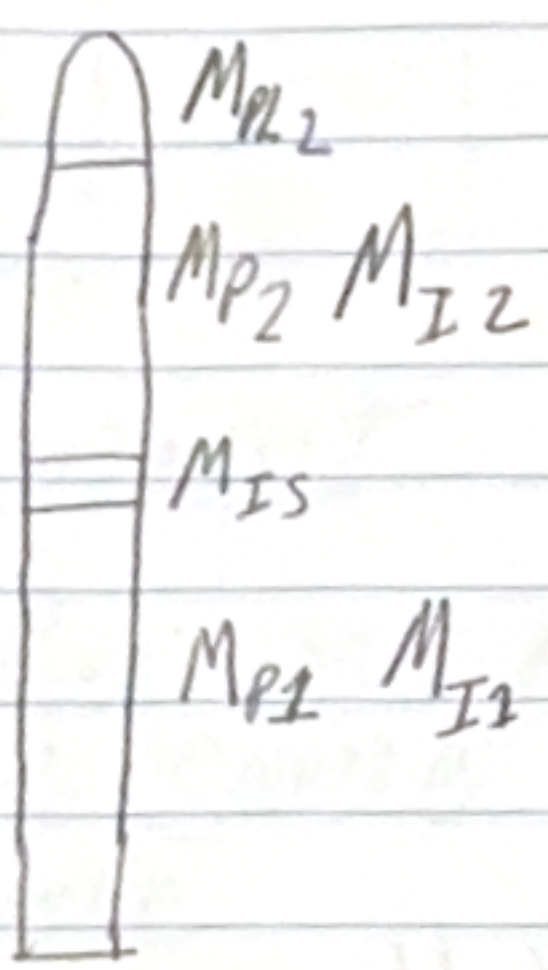
1st $I_{sp_1} = 270\,s$ $\quad \lambda_1 = 0.9$ $\quad M_{Is} = 200\,lbm$

2nd $I_{sp2} = 350\,s$ $\quad \lambda_2 = 0.8$

3 Find

a) M_{P1} M_{P2} M_{I1} M_{I2} b) GLOW

4 Schematic



5 Assumptions

V_{ideal}
no gravity
no drag
$\Delta V_1 = \Delta V_2$
constant thrust
constant $\dot{m}$

6 Basic Equations

$$\Delta V_{id} = g_e I_{sp} \ln\left(\frac{m_0}{m_f}\right) \qquad MR = \frac{m_0}{m_f} = \frac{m_{pl} + m_p + m_i}{m_{pl} + m_i} = \frac{\lambda m_{pl} + m_p}{\lambda m_{pl} + m_p(1-\lambda)}$$

$$\lambda = \frac{m_p}{m_p + m_i} \qquad MR = e^{\frac{\Delta V_{id}}{g_e I_{sp}}} \qquad m_p = m_{pl} \frac{MR - 1}{MR - (MR-1)\lambda^{-1}}$$

7 Analasis

$$MR_2 = e^{\frac{5000 ft s^2}{s\,3505\,32.2 ft}} = 1.558$$

$$m_{p2} = 500 lbm \left| \frac{(1.558-1)}{1.558-(1.558-1)\left(\frac{1}{0.8}\right)} \right. = \boxed{324.5 lbm}$$

$$(m_{p1}+m_i)MR = m_{p1}+m_p+m_i$$

$$MR\,m_i + MR\,m_{p1} = m_{p1}+m_p+m_i$$

$$m_i(MR-1) = m_{p1}(1-MR)+m_p$$

$$m_i = \frac{m_{p1}(1-MR)+m_p}{MR-1}$$

$$\boxed{m_{i2} = 81.1 lbm}$$

$$M_2 = 81.1+324.5+500 = 906 lbm$$

$$m_{PL1} = m_2 + m_{IS} = 1106 lbm$$

$$MR_1 = e^{\frac{5000}{270 \cdot 32.2}} = 1.78$$

$$m_{p1} = 1106 \cdot \frac{1.78-1}{1.78-(1.78-1)\cdot 0.9} = 940.7$$

$$\boxed{m_{i1} = 105\ lbm}$$

b) $GLOW = m_{p1,1} + m_{p,1} + m_{i,1}$

$= 2150\ lbm$

8 Answers

	1	2 (lbm)
M_p	940.7	324.5
M_i	105	81.1

GLOW = 2150 lbm

9 Comment

A Real rocket would likely not have the same ΔV in the upper and lower stage.

3.13

1) Name Nicholas Howse

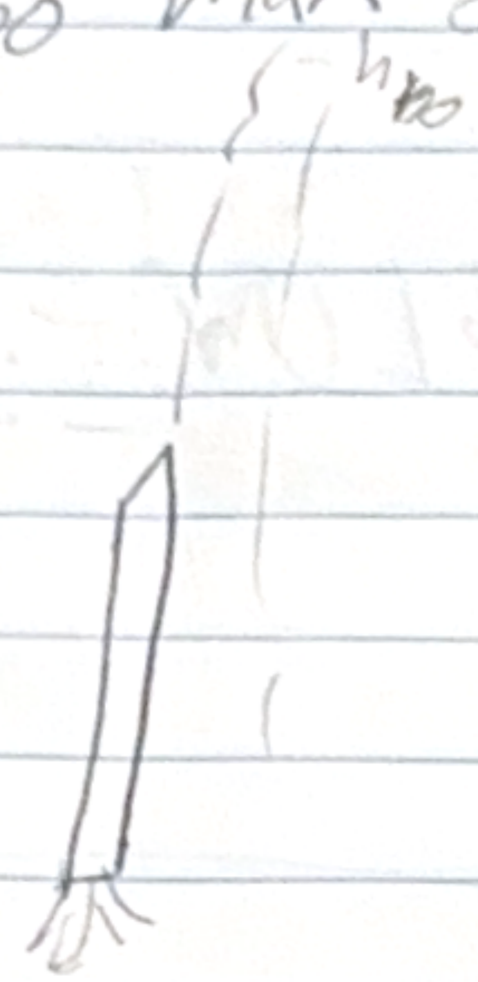
2) Given

$m_o = 50kg \quad m_{bo} = 20kg$

$I_{sp} = 250s$

3) h_{bo} max for t_b

4)



5 Assumptions

Point mass

no drag

const gravity

6) Basic equations

$$h_{bo} = g_e I_{sp} t_b \left(\frac{m_f}{m_o - m_f} \ln\left(\frac{m_f}{m_o}\right) + 1 \right) - \frac{g_e}{2} t_b^2$$

7) analysis

$$\frac{\partial t_b}{\partial} = 0$$

$$0 = g_e I_{sp}\left(\frac{m_f}{m_0 - m_f}\ln\left(\frac{m_f}{m_0}\right)+1\right) - g_e t_b$$

$$t_b = I_{sp}\left(\frac{m_f}{m_0 - m_f}\ln\left(\frac{m_f}{m_0}\right)+1\right) = 250\left(\frac{20}{50-20}\ln\left(\frac{2}{5}\right)+1\right)$$

$$t_b = 97.285\ s$$

$$h_{bo} = 9.81 \cdot 250 \cdot 97\left(\frac{20}{50-20}\ln\left(\frac{2}{5}\right)+1\right) - \frac{9.81}{2}(97)^2$$

$$h_{bo} = 46420\ m$$

8 Answer

$$\boxed{t_{bo} = 97.3\ s \quad h_{bo} = 46{,}420\ m}$$

9 comment

maximizeing the burnout height is not a good way to increse the overall apogee.

3.23

1 Name Nicholas House

2 Given

$\dot{m}_A = 150-5t \quad 0<t<30$

$\dot{m}_B = 12.5+10t \quad 0<t<20$

3 find

greator Isp between A, B

greator V_{bo} between A, B

4 Schematic

5 Assumptions

no drag

point mass

const Isp

6 Basic equations

$F=ma \qquad F_T = I_{sp} g_e \dot{m} \qquad \cancel{F_g = g_e m(t)}$

$\Delta V = I_{sp} g_e \ln\left(\frac{m_0}{m_f}\right)$

$V_{bo} = I_{sp} g_e \ln\left(\frac{m_0}{m_f}\right) - g_e t_b$

Z analysis

$$\frac{ma}{\cancel{m}} = \frac{I_{sp} g_e \dot{m}}{m} - \cancel{\frac{g_e m}{m}}$$

$$a = I_{sp} g_e \frac{\dot{m}(t)}{m(t)} - \cancel{g_e}$$

$$\frac{dV}{dt} = I_{sp} g_e \frac{\dot{m}}{m} - \cancel{g_e}$$

A) $dV = I_{sp} g_e \cancel{\frac{150-5t}{m_0-150+5t}}$

$$m(t) = \int_0^t \dot{m}(t) \qquad \Delta m_A(t) = \int_0^{30} 150-5t\,dt$$

$$\Delta m_A = \left[150t - \frac{5}{2}t^2\right]_0^{30} = 2250 \qquad \bar{\dot{m}}_{A} = 75$$

$$\Delta m_B = \left[12.5t + 5t^2\right]_0^{20} = 2250 \qquad \bar{\dot{m}}_B = 112.5$$

$$\Delta V_A = I_{sp} g_e \ln\left(\frac{m_0}{m_0-2250}\right) \qquad \Delta V_B = I_{sp} g_e \ln\left(\frac{m_0}{m_0-2250}\right)$$

$\Delta V_A = \Delta V_B$ Identical

b)

$$V_{boA} = I_{sp} g_e \ln\left(\frac{m_0}{m_0 - 2250}\right) - g_e(30)$$

$$V_{boB} = I_{sp} g_e \ln\left(\frac{m_0}{m_0 - 2250}\right) - g_e(20)$$

$$V_{boB} > V_{boA}$$

8 Answer

a) $\Delta V_A = \Delta V_B$

b) $V_{boB} > V_{boA}$

9 Comment

With no gravity the mass flow rate doesent matter assuming constant Isp, and with gravity you want as large of a $\dot{m}$ as possible

9803-B1

1 Name Nicholas Hawse

2 Given

$I_{sp} = 200\ s$

$m_0 = 53.0\ lbm$

$m_p = 10\ lbm$

$V_0 = h_0 = 0$

$t_b = 3.2\ s$

$A_f = 0.208987\ ft^2$

$C_D = 0.3$

$\rho\left(\frac{lbm}{ft^3}\right) = (1.255 \times 10^{-11})h^2 - (1.9453 \times 10^{-6})h + 0.67579$

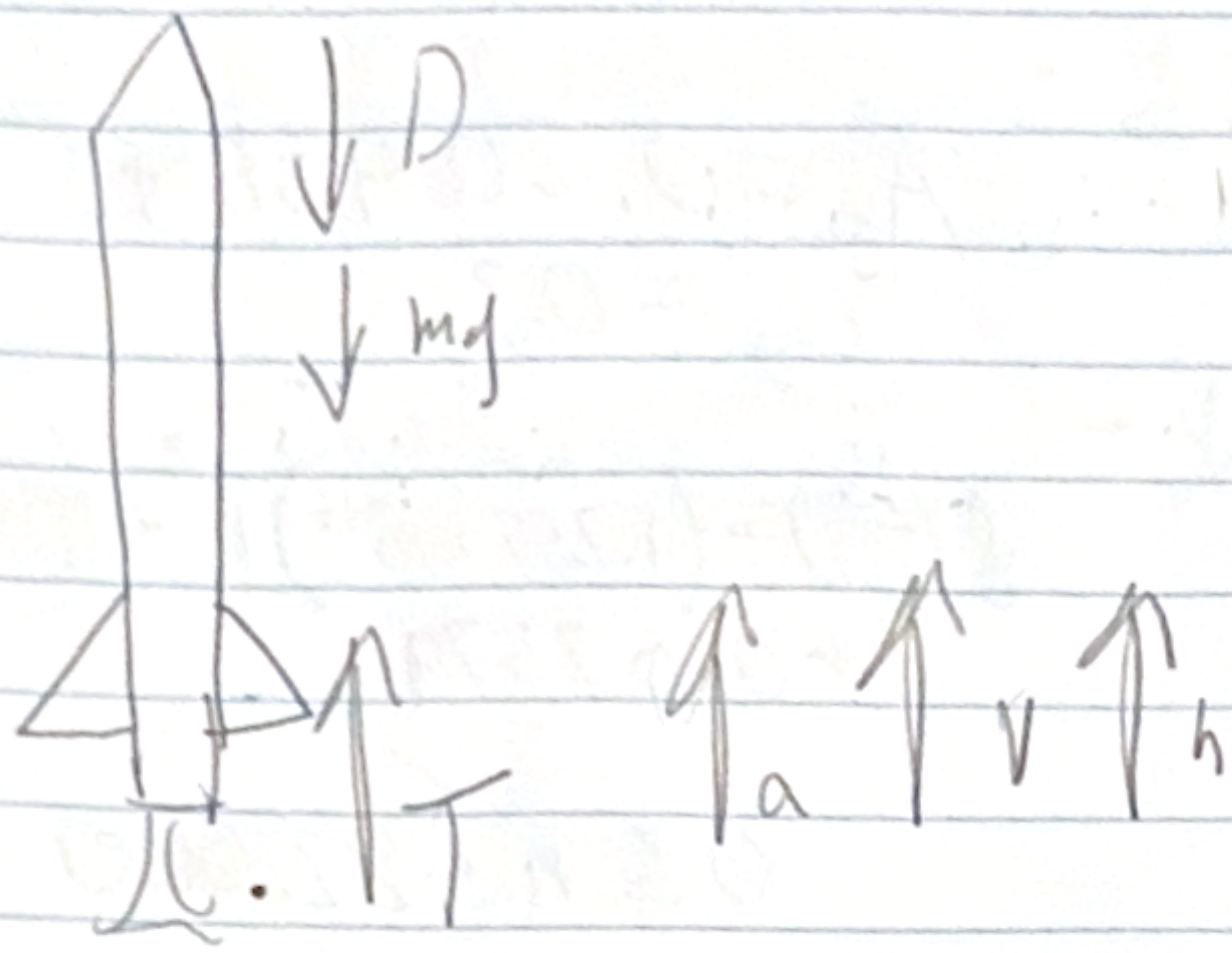
$0 \le h \le 82000$

3 Find

Plot of density of air between h=0, h=100000 ft
Show equations in symbolic form and
hand calculate the first 3 steps
Plot a vs t from t=0-10s with and without drag
do the same for V, h

	NO D	D
$a_0\ \frac{ft}{s^2}$		
$a_b\ \frac{ft}{s^2}$		
h_b ft		
$V_b\ \frac{ft}{s}$		
$V_{max}\ \frac{ft}{s}$		
h_{ap} ft		
t_{ap} s		

4 Schematic



5 assume

Vertical trajectory
Cost ISP
Cost $\dot{m}$
Cons $g = 32.2 \frac{ft}{s^2}$
$V_0 = h_0 = 0$
$\Delta t = 0.25$
Correct no drag solution
Cons $C_D = 0.3$
Given air density
Point mass

6 basic equations

$$m_{i+1} = m_i - \dot{m}_i (t_{i+1} - t_i)$$

$$a_i = \frac{I_{sp} g_e \dot{m}}{m_i} - g_e - \frac{\rho_{h_i} C_D V_i |V_i| A_f}{2 m_i} \qquad 0 \le t < t_b$$

$$a_i = -g_e - \frac{\rho_{h_i} C_D V_i |V_i| A_f}{2 m_i} \qquad t_b < t < \infty$$

$$V_{i+1} = V_i + a_i (t_{i+1} - t_i)$$

$$h_{i+1} = h_i + \frac{(V_{i+1} + V_i)}{2} (t_{i+1} - t_i)$$

$$F_t = \dot{m} g_e I_{sp} \qquad \dot{m} = \frac{m_p}{t_b}$$

7 analysis

$$\dot{m} = \frac{10}{3.2}$$

$t=0$ $m = 53\,lbm$ $h_0 = 0$ $V_0 = 0$

$$a_0 = \frac{32.2 \cdot 10 \cdot 200}{3.2 \cdot 53} - 32.2 - \frac{0.0758 \cdot 0.3 \cdot 0 \cdot 0 \cdot 0.209}{2 \cdot 53}$$

$$a_0 = 348.5 \frac{ft}{s^2}$$

$t = 0.2$

$$m_r = 53 - \frac{10}{3.2}(0.4 - 0.2) = 52.375$$

$$V_1 = 0 + 347.5(0.2 - 0) = 69.5 \frac{ft}{s}$$

$$h_1 = 0 + \frac{(69.5 + 0)}{2}(0.2 - 0) = 6.95\, ft$$

$$a_1 = \frac{32.2 \cdot 10 \cdot 200}{3.2 \cdot 52.375} - 32.2 - \frac{0.0758 \cdot 0.3 \cdot (69.5)(69.5) \cdot 0.209}{2 \cdot 52.375}$$

$$a_1 = 351.8 \frac{ft}{s^2}$$

$t=0.4$

$$m_2 = 52.375 - \frac{10}{3.2}(0.6-0.4) = 51.75 \text{ lbm}$$

$$V_2 = 69.5 + 351.8(0.4-0.2) = 139.9 \ \frac{ft}{s}$$

$$h_2 = 6.95 + \frac{(139.9+69.5)}{2}(0.4-0.2) = 27.9 ft$$

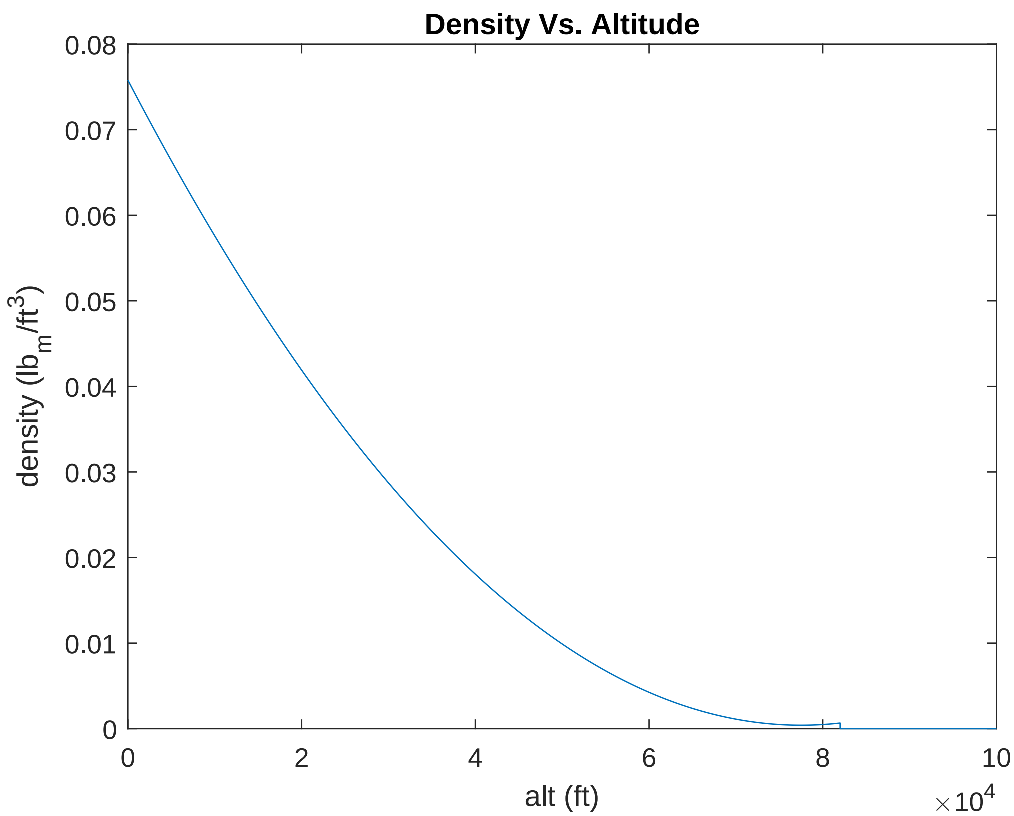
$$a_2 = \frac{32.2 \cdot 10 \cdot 200}{3.2 \cdot 51.75} - 32.2 - \frac{0.0757 \cdot 0.3 \cdot (139.9)^2 \cdot 0.208}{2 \cdot 51.75}$$

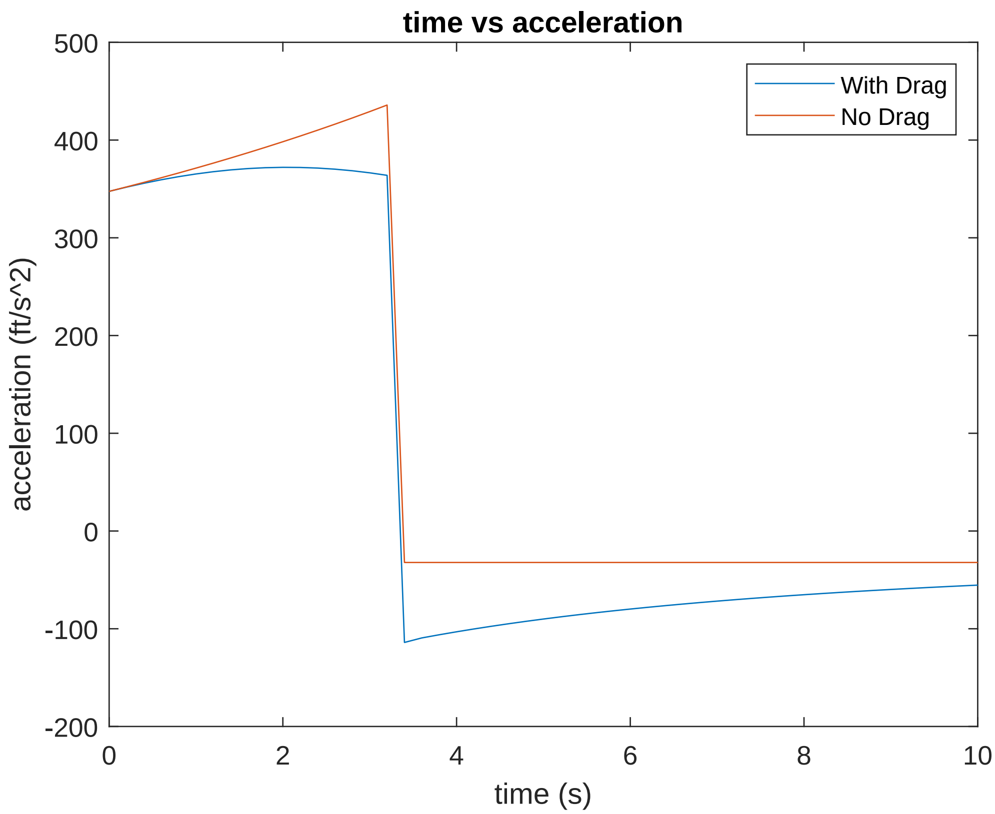
$$a_2 = 355.8$$

8 Answers

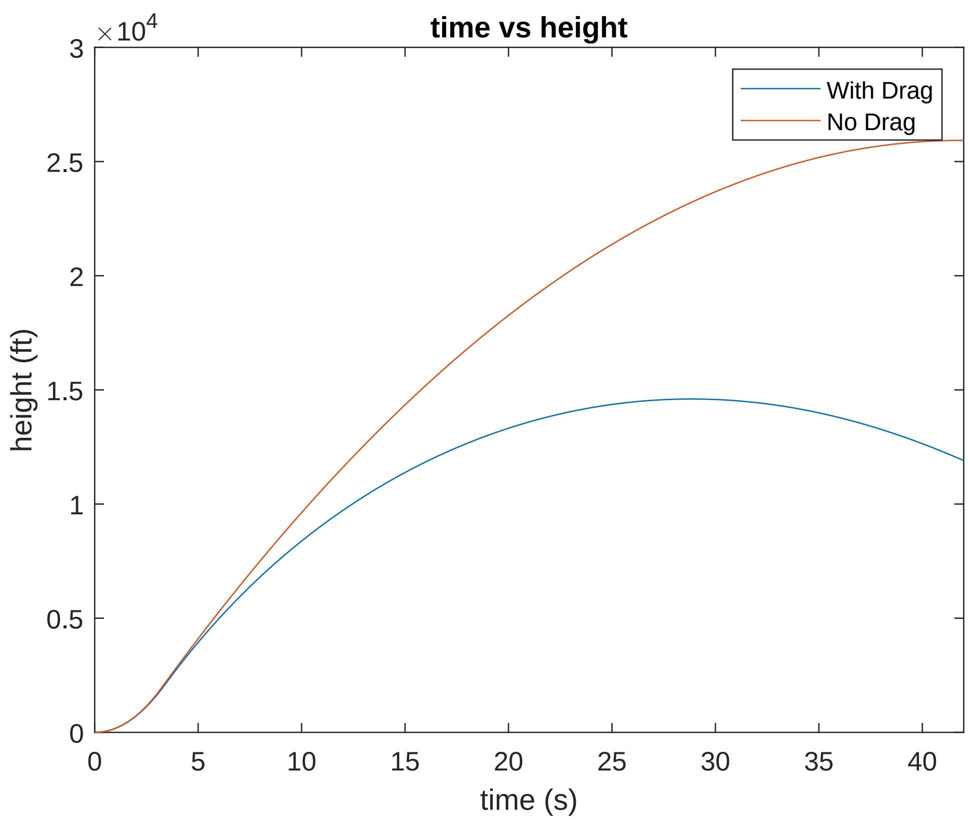
	NOD	D
a_0	347.5	347.5
a_b	435.8	363.9
h_b	1915	1852
V_b	1244	1169
V_{max}	1244	1169
h_{ap}	25925	14600
t_{ap}	42	29

t	m	a	V	h
0	53	348	0	0
0.2	52.375	352	69.5	6.95
0.4	51.75	356	140	27.9

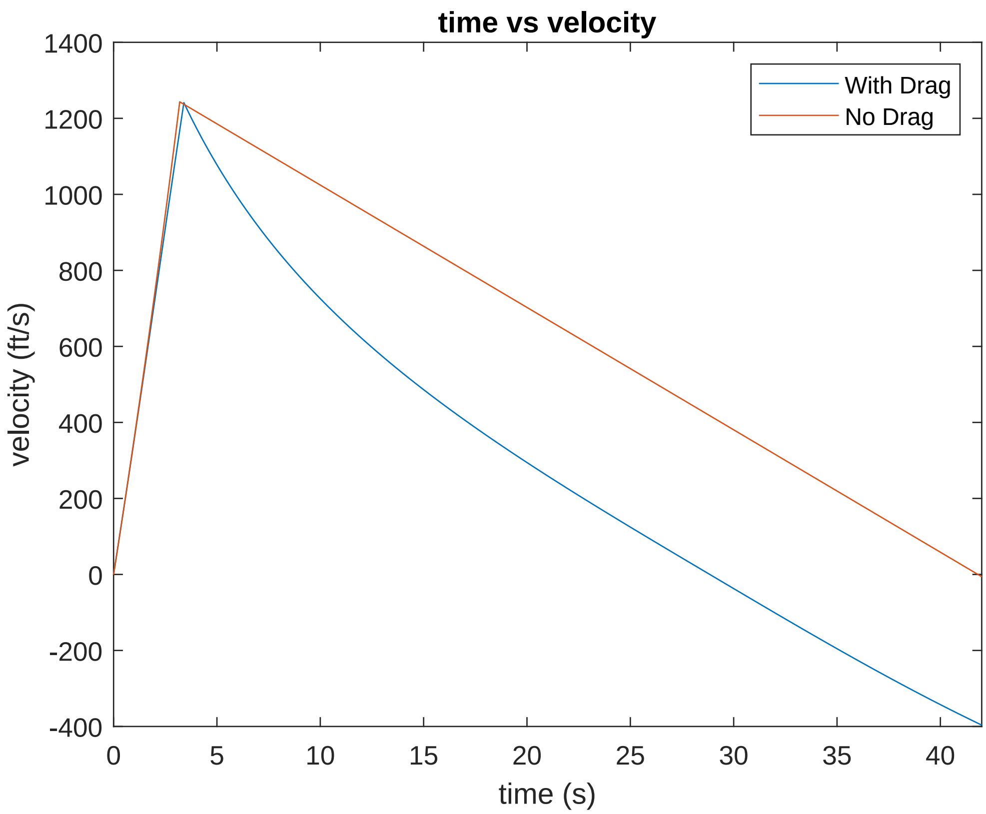

Density Vs. Altitude
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
density (lb_m/ft^3)
0
2
4
6
8
10
alt (ft)
$\times 10^4$



time vs acceleration
With Drag
No Drag
acceleration (ft/s^2)
time (s)
500
400
300
200
100
0
-100
-200
0
2
4
6
8
10



time vs height
$\times 10^4$
3
2.5
2
1.5
1
0.5
0
height (ft)
0
5
10
15
20
25
30
35
40
time (s)
With Drag
No Drag


time vs velocity
With Drag
No Drag
velocity (ft/s)
1400
1200
1000
800
600
400
200
0
-200
-400
0
5
10
15
20
25
30
35
40
time (s)

9 Comment

The eulers method code was no harder to implement and thus should basically allways be used.