04 HW - A

Problem 4.1

1 Name Nicholas Hawse

2 Given

$P_c = 5 \times 10^6$ Pa $T_c = 3000K$

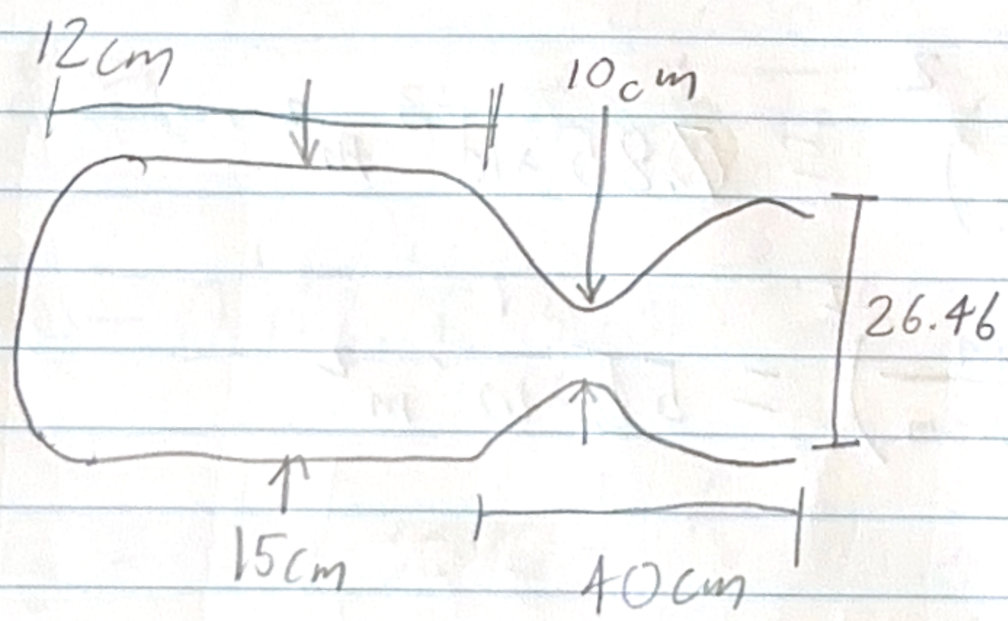
$m = 15 \frac{Kg}{Kgmol}$ $\gamma = 1.2$

3 Find

a) $F_{Ta}$, $Isp_a \rightarrow M_e = 0$

b) $F_{Tb}$, $Isp_b \rightarrow M_e \neq 0$

4 Schematic




5 Assumptions

Isentropic 1D flow

Somtimes chamber conditons are stagnant

Steady state

Choked

No flow separations

6 Basic Equations

$$\frac{A}{A_t} = \frac{1}{M}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right)^{\frac{\gamma+1}{2\gamma-2}}$$

$\dot{m} = \rho A V$

$$\frac{P_e}{P_c} = \left[1 + \frac{1}{2}(\gamma - 1)M_e^2\right]^{\left(-\frac{\gamma}{\gamma-1}\right)}$$

$$\frac{T_e}{T_c} = \left[1 + \frac{1}{2}(\gamma - 1)M_e^2\right]^{-1}$$

$$A = \pi R^2$$

$$F = \dot{m} V_e + (P_e - P_a) A_e$$

$$P = \rho \left(\frac{R_u}{\mathcal{M}}\right) T \qquad R_u = 8.314 \frac{N\,m}{Kg\ mol\ K}$$

$$M = \frac{V}{\sqrt{\gamma R T}} \qquad R = \frac{R_u}{\mathcal{M}}$$

## 7 Analysis

$$A_t = \frac{\pi}{4}\left(10\,cm \frac{1\,m}{100\,cm}\right)^2 = 7.85 \times 10^{-3}\ m^2$$

$$A_e = \frac{\pi}{4}\left(26.46\,cm \frac{1\,m}{100\,cm}\right)^2 = 55 \times 10^{-3}\ m^2$$

$$M_e = 3.03$$

$$\frac{P_e}{P_c} = \left[1 + \frac{1}{2}(1.2 - 1)(3.03^2)\right]^{\left(\frac{-1.2}{0.2}\right)} = 0.0201$$

$$P_e = 5 \times 10^6 \cdot 0.0201 = 101{,}000\,Pa$$

$$V_e = M_e \cdot \sqrt{\gamma R T}$$

$$\frac{T_e}{T_c} = \left[1 + 0.5(0.2)(3.03)^2\right]^{-1} = 0.521$$

$$T_e = 3000K \cdot 0.521 = 1564\ K$$

$$V_e = 3.03 \cdot \sqrt{1.2 \cdot \frac{8314\ \cancel{N}\ m\ \cancel{Kg}\ \cancel{Kmol}\ 1000\ \cancel{mol}\ 1564\ K\ 1\ \cancel{Kg}\ m}{\cancel{Kg}\ \cancel{mol}\ \cancel{K}\ 15\ \cancel{Kg} \cdot 1\ \cancel{Kmol}\ 1\ \cancel{N}\ s^2}}$$

$$V_e = 3090\ \frac{m}{s}$$

$$\dot{m} = \rho \cdot A_e \cdot V_e$$

$$\rho = \frac{P\,\mathcal{M}}{R_u \cdot T} = \frac{101000\ Pa\ \ 15}{8.314\ \ 1564} = 0.117\ \frac{Kg}{m^3}$$

$$\dot{m} = 0.117\ \frac{Kg}{m^3} \cdot 55 \times 10^{-3}\ m^2 \cdot 3090\ \frac{m}{s} = \boxed{19.8\ \frac{Kg}{s}}$$

$$F_t = 19.8 \cdot 3090 + \cancel{(101000 - 101000)}\ 55 \times 10^{-3}\ m^2$$

$$F_t = 61185\ N$$

$$I_{sp} = \frac{F_t}{\dot{m} \cdot g_c} = 315\ s$$

$$\frac{A_c}{A_t} = \frac{(15^2)}{(10)^2} = 2.25$$

$\hookrightarrow M_c = 0.2742$

$$\frac{P_c}{P_0} = \left[1 + \frac{1}{2}(1.2-1)(0.2742)^2\right]^{-\frac{1.2}{1.2-1}}$$

$$\frac{P_c}{P_0} = 0.956$$

$$P_0 = \frac{1}{0.956} 5\times10^6 \text{ Pa} = 5.23\times10^6 \text{ Pa}$$

$$\frac{P_e}{P_0} = 0.020081 \quad P_e = 0.020081 \cdot 5.23\times10^6 \text{ Pa}$$

$$P_e = 105 \text{ KPa}$$

$$\frac{T_c}{T_0} = \left[1 + \frac{1}{2}(1.2-1)(0.2742)^2\right]^{-1} = \frac{1}{0.993}$$

$$T_c = 3021$$

$$\frac{T_e}{T_0} = \left[1 + \frac{1}{2}(1.2-1)(3.03^2)\right]^{-1} = 0.521$$

$T_e = 1575\ K$

$$V_e = 3.03\sqrt{1.2 \cdot \frac{8.314}{15} \cdot 1575} = 3100\ \frac{m}{s}$$

$$\rho = \frac{105000 \quad 15}{8314 \quad 1575} = 0.12\ \frac{Kg}{m^3}$$

$$\dot{m} = \rho A V = 0.12 \cdot 55 \times 10^{-3} \cdot 3100 = 20.5\ \frac{Kg}{s}$$

$$F_t = 20.5 \cdot 3100 + \left(105 \cancel{\times 10^{3}} - 101 \cancel{\times 10^{3}}\right) 55 \cancel{\times 10^{-3}}$$

$$F_t = 63770 \qquad I_{sp} = \frac{63770}{20.5 \cdot 9.81} = 317$$

8 Answer

Slow is perfectly expanded

assume $V_c = 0$ $F_t = 61185\ N$ $I_{sp} = 315\ s$

assume $V_c \neq 0$ $F_t = 63770\ N$ $I_{sp} = 317\ s$

9 Comment

Its seems that assuming chamber conditions are stagnation conditions is a reasonable assumption that makes a small difference.

4.11

1 Name: Nichols

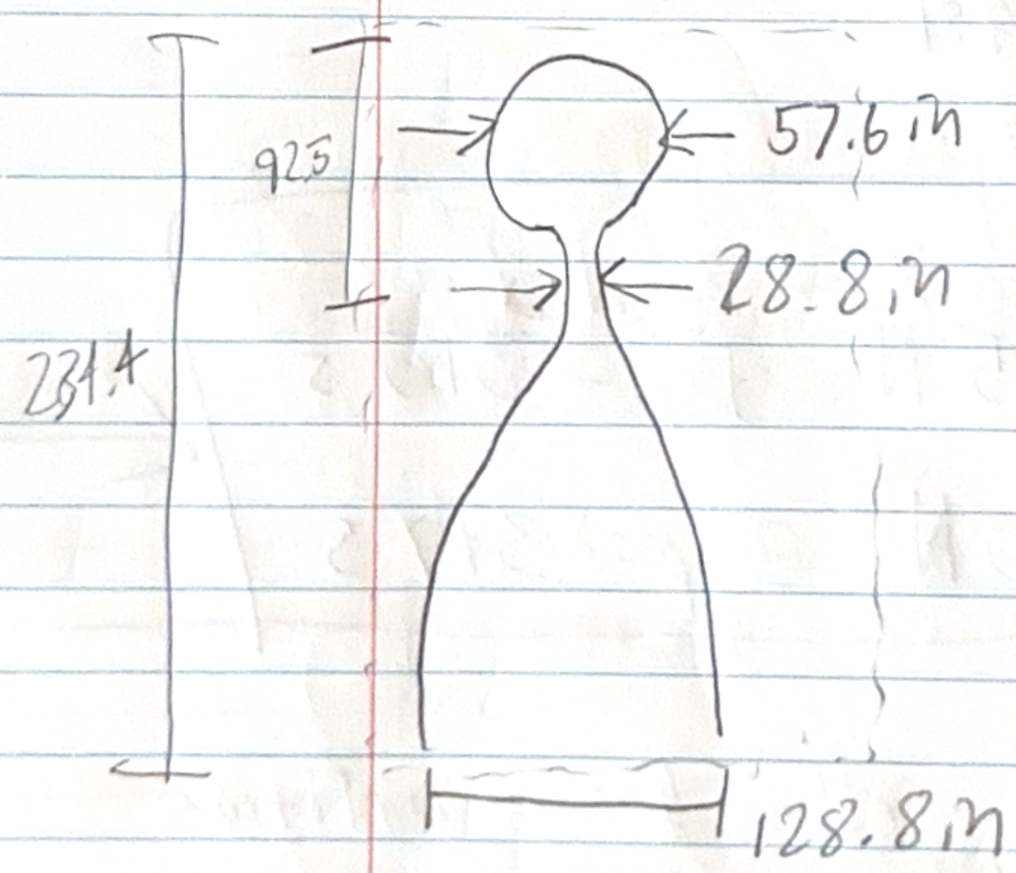
2 Given:

$P_c = 700$ psi $\quad C^* = 7800 \frac{ft}{s}$

$O_f = .6 \quad \gamma = 1.2$

3 find: $\dot{m}$, $F_t$, Jet thrust, Pressure thrust
Vaccum thrust

4 Schematic




5 Assumptions
Isentropic
Steady State
1D flow
Choked
No flow seperations

# 6 Basic Equations

$$M = \frac{V}{\sqrt{\gamma R T}} \qquad \frac{P}{P_c} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}}$$

$$\frac{T}{T_c} = \left[1 + \frac{1}{2}(\gamma - 1) M^2\right]^{-1} \qquad \rho = \frac{P \mathcal{M}}{T R_u}$$

$$\frac{A}{A_t} = \frac{1}{M}\left(\frac{2 + (\gamma - 1) M^2}{(\gamma + 1)}\right)^{\frac{\gamma + 1}{2\gamma - 2}}$$

$$F = \dot{m} V_e + (P_e - P_a) A_e$$

$$I_{sp} = \frac{C_F C^*}{g_e} \qquad F = C_F A_t P_c$$

$$C^* = \frac{P_c A_t}{\dot{m}} \qquad \dot{m} = \frac{P_c A_t}{C^*}$$

$$F = P_c A_t \left( \left[ \frac{2\gamma^2}{\gamma - 1} \left( \frac{2}{\gamma - 1} \right)^{\frac{\gamma + 1}{\gamma - 1}} \left( 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma - 1}{\gamma}} \right) \right]^{1/2} \right) + (P_e - P_a) A_e$$

# 7 Anaysis

$$\dot{m} = \frac{P_c A_t}{c^*} = \frac{700\ \cancel{lbf}\ \cancel{s}\ \pi\ 28.8^2\ \cancel{in^2}\ slug\ \cancel{ft}}{\cancel{in^2}\ 7800\ \cancel{ft}\ 4\ \cancel{lbf}\ s^2}$$

$$\boxed{\dot{m} = 58.46\ \frac{slug}{s}}$$

$$\frac{128.8^2}{28.8^2} = \frac{1}{M_e}\left(\frac{2+0.2M^2}{2.2}\right)^{\frac{2.2}{0.4}}$$

$$M_e = 3.759 = 3.76$$

$$\frac{P_e}{P_c} = \left(1+\frac{0.2}{2}(3.76)^2\right)^{\frac{-1.2}{0.2}} = 0.00506$$

$$P_e = 3.54\ Psi \qquad P_a = 14.7\ Psi$$

$$F = 700 \cdot 28.8^2 \cdot \frac{\pi}{4}\left(\left[\frac{2\cdot 1.2^2}{0.2}\left(\frac{2}{0.2}\right)^{\frac{2.2}{0.2}}\left(1-\left(0.00506\right)^{\frac{0.2}{1.2}}\right)\right]^{\frac{1}{2}}\right) +$$

$$+\left(3.54-14.7\right)128.8^2\frac{\pi}{4}$$

$$= 638593\ lbf$$

0

Jet thrust = 784000 lbf

Pressure thrust = -145407 lbf

Vaccum thrust = 830124 lbf

8 Answer

$\dot{m} = 58.16 \frac{slugs}{s}$

$F_t = 638593$ lbf Sea level

Jet thrust = 784000 lbf Sea level

Pressure thrust = -145407 lbf Sea level

Vaccume thrust = 830124 lbf

9 Comment

It appears that this engine is designed to opperate in vaccume and gets much more thrust without atmospheric pressure

4.39

1 Name Nicholas

$R_u = 10.731 \frac{psi\ ft^3/lb}{R\ mol}$

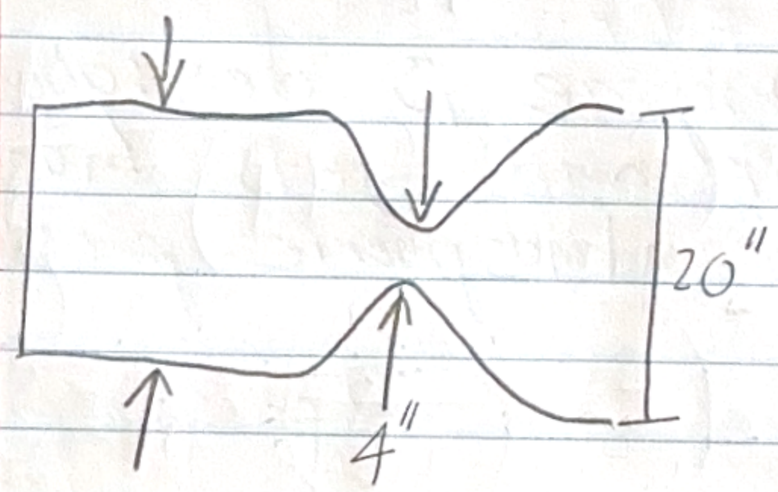
2 Given

$F_m = 18000\ lbf$ $P_c = 860\ psi$ $\mathcal{M} = 20 \frac{lb}{lb\ mol}$

$m_p = 6400\ lbm$ $t_b = 110\ s$ $\gamma = 1.2$

3 Find $I_{sp}$, $C^*$, $C_f$, $I_{sp,vac}$, $T_c$, $\frac{F_m}{F_{Id}}$

4 Schematic




5 Assumptions

Istrotopic

Qusi 1D flow

Choked

Steady State

6 Basic equations

$$C^* = \frac{P_c A_t}{\dot{m}} \qquad \dot{m} = \frac{m}{t_b}$$

$$\frac{A}{A_t} = \frac{1}{M}\left(\frac{2 + (\gamma-1)M^2}{\gamma+1}\right)^{\frac{\gamma+1}{2\gamma-2}}$$

$$\frac{P}{P_c} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$F = \dot{m} V_e + (P_e - P_a) A_e$$

$$A = \frac{\pi}{4} d^2 \qquad C_f = \frac{F}{P_c A_t} \qquad \varepsilon = \frac{A_e}{A_t}$$

$$I_{sp} = \frac{F}{\dot{m} g_c} \qquad C_f = C_{f,v} - \left(\frac{P_a}{P_c}\right)\varepsilon \qquad C^* = \sqrt{\frac{\gamma m}{R T_c}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2\gamma-2}}$$

$$P = \rho R T \qquad I_{sp} = \frac{C_f C^*}{g_c}$$

$$1\,\text{slug} = 1\,\text{lbf}\,\frac{s^2}{ft}$$

## 7 Analysis

$$\dot{m} = \frac{6400\,\text{lbm}}{110\,s}\,\frac{1\,\text{slug}}{32.2\,\text{lbm}} = 1.81\,\frac{\text{slug}}{s}$$

$$I_{sp} = \frac{18{,}000\,\text{lbf}\,s\,s^2\,\text{slug}\,ft}{1.80\,\text{slug}\,32.2\,ft\,\text{lbf}\,s^2} = 309.375\,s$$

$$C^* = \frac{860\,\text{lbf}\,\pi\,4^2\,in^2\,s\,\text{slug}\,ft}{in^2\,4\,1.81\,\text{slug}\,\text{lbf}\,s^2} = 5970\,\frac{ft}{s}$$

$$C_f = \frac{18000\,\text{lbf}\,in^2\,4}{860\,\text{lbf}\,4^2\,\pi\,in^2} = 1.67 \qquad P_a = 0.406\,Psi$$

$$1.67 = C_{fv} - \left(\frac{0.406}{860}\right)\left(\frac{20^2}{4^2}\right)$$

$$C_{f,v} = 1.682$$

$$I_{sp,v} = \frac{C_{f,v} \cdot c^*}{g_e} = \frac{1.68 \cdot 5920\ ft/s}{32.2\ ft} = 311\ s$$

$$\frac{20^2}{4^2} = \frac{1}{M_e}\left(\frac{2+(\gamma-1)M_e^2}{\gamma+1}\right)^{\frac{\gamma+1}{2\gamma-2}}$$

$$M_e = 3.91$$

$$\frac{P_e}{P_c} = \left(1+\frac{\gamma-1}{2}(3.6^2)\right)^{\frac{-1.2}{0.2}} = 0.00677$$

$$\left(5920\frac{ft}{s}\left(\frac{2}{1.2+1}\right)^{\frac{1.2+1}{2\cdot 1.2-2}}\right)^2 = \frac{1.2 \cdot 20\ lb\ in^2\ R\ mol}{lb\ mol\ 10.73\ lbf\ ft^3\ lb\ T_c}$$

$$T_c = 5585\ °R$$

$$\frac{T_e}{T_c} = \left(1+\frac{1.2-1}{2}3.91^2\right)^{-1} = 0.3915$$

$$T_e = 2206\ °R$$

$$V_e = 3.91 \cdot \sqrt{1.2 \cdot 2206 \frac{10 \cdot 2 \cdot 1000}{120}} = 8015\ \frac{ft}{s}$$

$P_e = 5.82$

$$eff = \frac{C_{fv}}{C_f} = \frac{1.67}{1.682} = 0.993$$

7 Answers

$I_{sp} = 309\ s$

$C^* = 5970\ \frac{ft}{s}$

$C_f = 1.67$

$I_{sp,vac} = 311\ s$

$T_c = 5585\ °R$

$\frac{F_m}{F_{ID}} = 0.993$

9 Comment

This engine preformed at near its peak efficiancy because vacume and pressure at 80000 ft.

SP04 - A1

1 Name Nicholas

2 Given : $P_c = 1000 \frac{lbf}{in^2}$ $T_c = 5000\,F$

$\gamma = 1.3$ $P_c = 500 \frac{lbf}{in^2}$

3 find

a) Piecewise $f(x)$ for cross sectional area

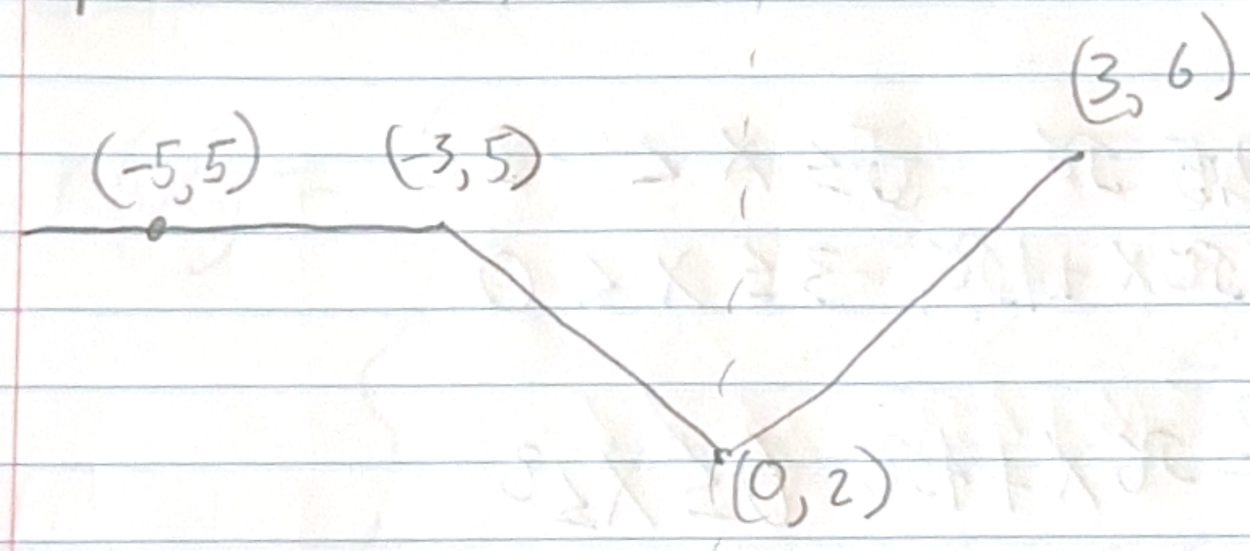
b) a Plot of radius from $x = -5 \rightarrow x = 3$ at 0.1 in increments

c) a Plot of mach number as a function of $x$ from -5 to 3

d) a graph of static temp as a function of $x$

e) a graph of static pressure as a function of $x$ for both chamber pressures

4 Schematic




5 Assumption
Isentropic
steady state
1D flow
choked

6 Basic Equations

$$\frac{T}{T_c} = \left[1 + 0.5(\gamma-1)M^2\right]^{-1} \qquad \frac{P}{P_c} = \left[1 + 0.5(\gamma-1)M^2\right]^{\frac{-\gamma}{\gamma-1}}$$

$$\frac{A}{A_t} = \frac{1}{M}\left(\frac{2+(\gamma-1)M^2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad A = \pi r^2$$

## Zanalysis

$$r(x) = \begin{cases} 5 & -5 < x < -3 \\ -x+2 & -3 < x < 0 \\ \frac{4}{3}x+2 & 0 < x < 6 \end{cases}$$

$$\frac{5-2}{-3-0} = -1 \qquad \frac{6-2}{3-0} = \frac{4}{3}$$

$$y-2 = -x$$

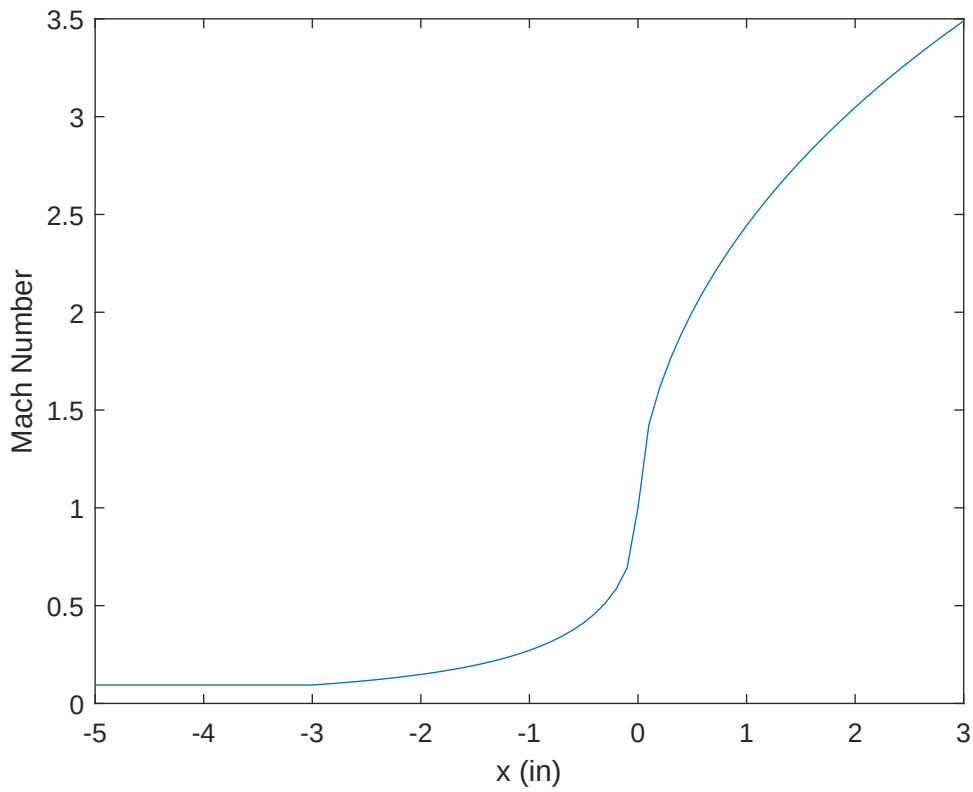
$$A = \pi r^2$$

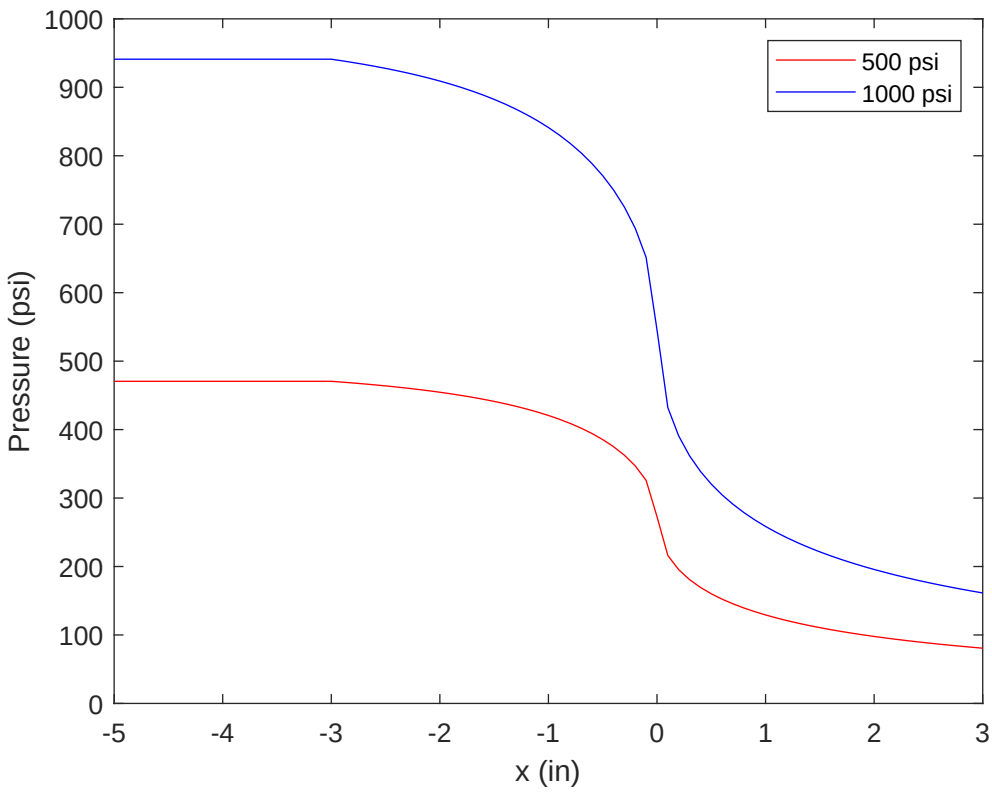
$$A = \begin{cases} 25\pi & -5 < x < -3 \\ \pi(-x+2)^2 & -3 < x < 0 \\ \pi\left(\frac{4}{3}x+2\right)^2 & 0 < x < 3 \end{cases}$$

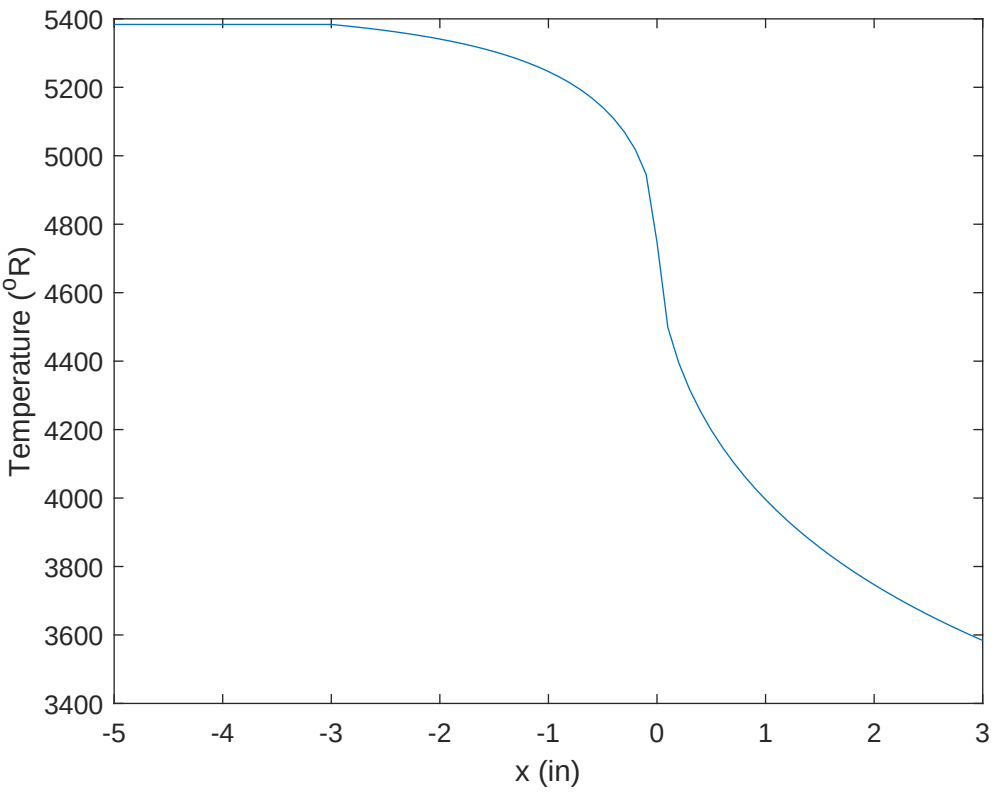
8 Answers

a)

$$\boxed{A = \begin{cases} 25\pi & -5 < x < -3 \\ \pi(-x+2)^2 & -3 < x < 0 \\ \pi\left(x\frac{3}{4}+2\right)^2 & 0 < x < 3 \end{cases}}$$


3.5
3
2.5
2
1.5
1
0.5
0
Mach Number
-5
-4
-3
-2
-1
0
1
2
3
x (in)


1000
900
800
700
600
500
400
300
200
100
0
Pressure (psi)
-5
-4
-3
-2
-1
0
1
2
3
x (in)
500 psi
1000 psi


5400
5200
5000
4800
4600
4400
4200
4000
3800
3600
3400
Temperature (°R)
-5
-4
-3
-2
-1
0
1
2
3
x (in)

| | Self assessment of submission | | | Self assessment after reviewing solution | | | |
|---|---|---|---|---|---|---|---|
| Hw Problem | G | A | P | G | A | P | Specific questions That you would like answering |
| | | | | | | | |
| SP-01A | | | | | | | |
| Prob. 2-1 | | | | | | | N/A |
| Prob. 2-5 | | | | | | | N/A |
| Prob. 2-11 | | | | | | | N/A |
| Prob. 2-13 | | | | | | | N/A |
| Prob. 2-18 | | | | | | | N/A |
| SP02 | | | | | | | N/A |
| Prob. 3-4 | | | | | | | N/A |
| Prob. 3-10 | | | | | | | N/A |
| Prob. 3.16 | | | | | | | N/A |
| SP03-A | | | | | | | N/A |
| Prob. 3.3 | | | | | | | N/A |
| Prob. 3-23 | | | | | | | N/A |
| Prob. 3.36 | | | | | | | N/A |
| SP03-B | | | | | | | N/A |