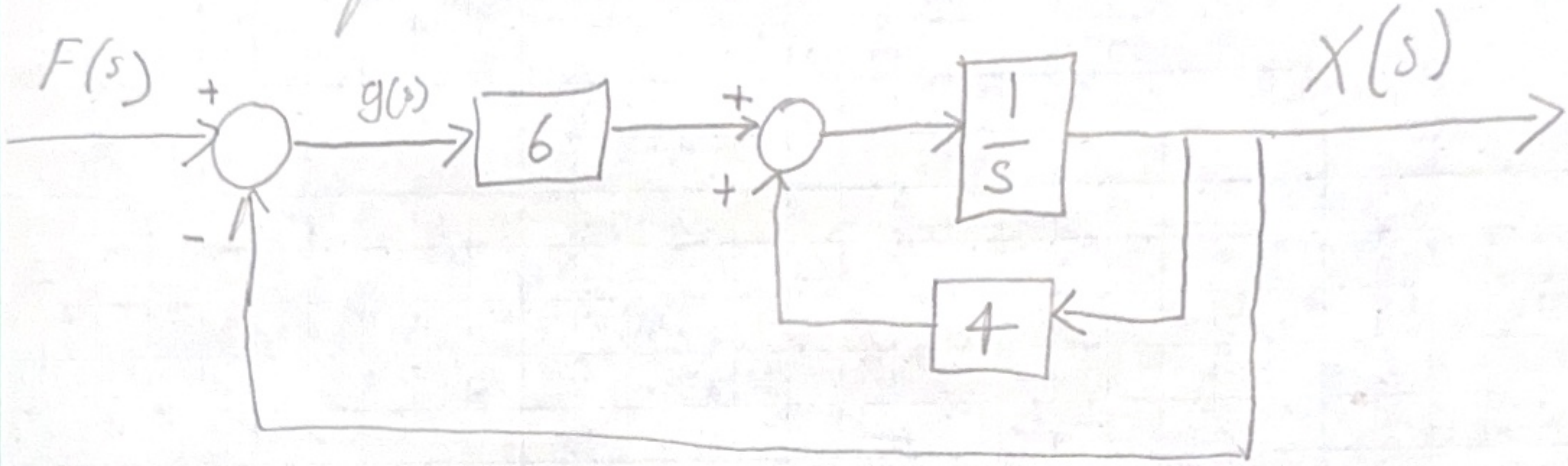
Nicholas Hawse | MAE488 HW 6 | 03/15/2025

Problem 1 (text 5.1) find the transfer function $\frac{X(s)}{F(s)}$ that the block diagram below represents.




$$\frac{6g(s) + 4X(s)}{s} = X(s)$$

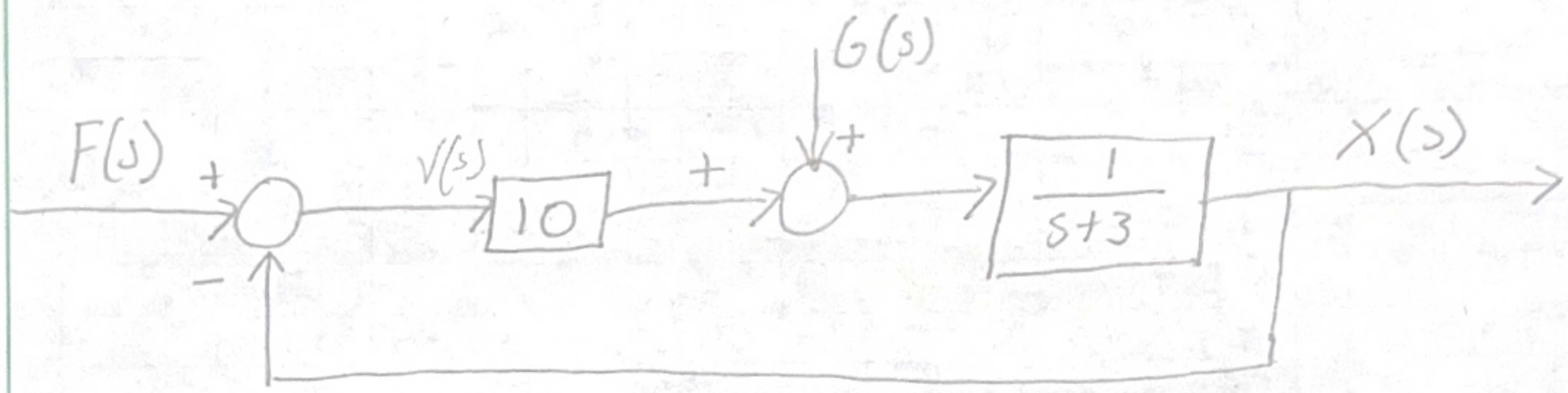
$$F(s) - X(s) = g(s)$$

$$6[F(s) - X(s)] + 4X(s) = sX(s)$$

$$6F(s) - 6X(s) + 4X(s) = sX(s) \qquad 6F(s) = (s+2)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{6}{s+2}}$$

Problem 2 (text 5.2) find the transfer function represented by the block diagram below $\frac{X(s)}{F(s)}$.




G(s) goes away

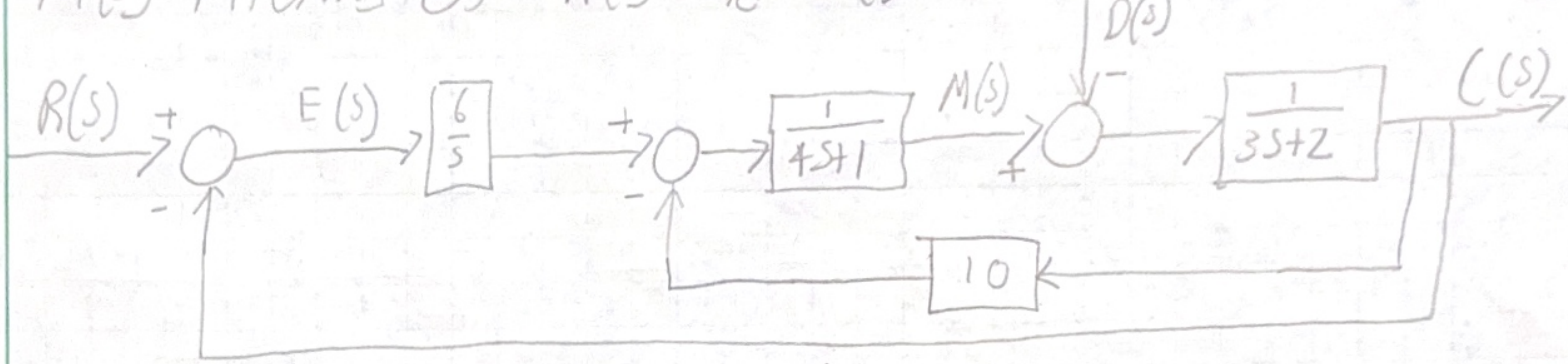
$$10\,V(s)\frac{1}{s+3} = X(s) \qquad V(s) = F(s) - X(s)$$

$$10\,F(s) - 10\,X(s) = (s+3)\,X(s)$$

$$10\,F(s) = (s+13)\,X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{10}{s+13}}$$

Problem 3 (text 5.7) useing the block diagram below derive expressions for C(s), E(s), and M(s) interms of R(s) and D(s)




$$E(s) = R(s) - C(s) \qquad M(s) = \frac{\frac{6}{s}E(s) - 10\,C(s)}{4s+1}$$

$$C(s) = \frac{M(s) - D(s)}{3s+2}$$

a) $$(3s+2)\,C(s) = \frac{\frac{6}{s}E(s) - 10\,C(s)}{4s+1} - D(s)$$

$$(3s+2)C = \frac{\frac{6}{s}R - \frac{6}{s}C - 10C}{4s+1} - D$$

$$(4s+1)(3s+2)C + (4s+1)D = \frac{6}{s}R - \frac{6}{s}C - 10C$$

$$(4s+1)(3s+2)C + \frac{6}{s}C + 10C = \frac{6}{s}R - (4s+1)D$$

$$\left[12s^2 + 11s + 2 + \frac{6}{s} + 10\right]C = \frac{6}{s}R - (4s+1)D$$

$$\left[12s^3 + 11s^2 + 12s + 6\right]C = 6R - (4s^2+s)D$$

$$C(s) = \frac{6R(s) - (4s^2+s)\,D(s)}{12s^3 + 11s^2 + 12s + 6}$$

Cont. →

Problem 3 text (5.7) cont.

b) $E(s) = R(s) - C(s)$

$$\boxed{E(s) = R(s) - \frac{6R(s) - (4s^2+s)D(s)}{12s^3+11s^2+12s+6}}$$

c) $M(s)$

$$C(s) = \frac{M(s) - D(s)}{3s+2}$$

$$M(s) = (3s+2)C(s) + D(s)$$

$$\boxed{M(s) = \frac{6R(s) - (4s^2+s)D(s)}{12s^3+11s^2+12s+6}(3s+2) + D(s)}$$

Problem 5 (text 5.17) find the state space model for the set of equations of motion Given below.

$$15\ddot{X}_1 + 7\dot{X}_1 - 4\dot{X}_2 + 30X_1 - 15X_2 = 0$$

$$6\ddot{X}_2 - 15X_1 + 15X_2 - 4\dot{X}_1 + 4\dot{X}_2 = f(t)$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ \dot{X}_1 \\ \dot{X}_2 \end{bmatrix}$$

$$15\ddot{X}_1 = -7\dot{X}_1 + 4\dot{X}_2 - 30X_1 + 15X_2$$

$$\ddot{X}_1 = \frac{-7}{15}\dot{X}_1 + \frac{4}{15}\dot{X}_2 - 2X_1 + X_2$$

$$6\ddot{X}_2 = 15X_1 - 15X_2 + 4\dot{X}_1 - 4\dot{X}_2 + f(t)$$

$$\ddot{X}_2 = \frac{15}{6}X_1 - \frac{15}{6}X_2 + \frac{2}{3}\dot{X}_1 - \frac{2}{3}\dot{X}_2 + \frac{1}{6}f(t)$$

$$\dot{z}_1 = z_3 \quad \dot{z}_2 = z_4 \quad \dot{z}_3 = \ddot{X}_1 \quad \dot{z}_4 = \ddot{X}_2$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -\frac{7}{15} & \frac{4}{15} \\ \frac{5}{2} & -\frac{5}{2} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{bmatrix} \begin{bmatrix} f(t) \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} f(t) \end{bmatrix}$$

Problem 6 (text 5.20) find the Matricies A, B, C, and D of the state space Models given below.

a) $\dot{X}_1 = -7X_1 + 4X_2$
$\dot{X}_2 = -3X_2 + 8u$

$$\rightarrow \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -7 & 4 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \end{bmatrix} [u]$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [u]$$

$$\boxed{A = \begin{bmatrix} -7 & 4 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 8 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

b) $\dot{X}_1 = -7X_1 + 5X_2 + 3u_1$
$\dot{X}_2 = -9X_2 + 2u_2$

$$\rightarrow \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} -7 & 5 \\ 0 & -9 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$$

Problem 7 (text 5.25 M) find the State Varible models for the following D.E.s by first findind the transfer functions then useing matlab to Convert the model.

a) $5\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 6y = f(t)$

Propertys 1, 2, 3, 4

$5s^3Y(s) + 7s^2Y(s) + 3sY(s) + 6Y(s) = F(s)$

$$\frac{Y(s)}{F(s)} = \frac{1}{5s^3+7s^2+3s+6}$$

b) $$\frac{Y(s)}{F(s)} = \frac{5}{s^2+2s+4}$$

Problem 8 (text 5.35M) find the numerical Solution to the odE below for the given inPut given below with the Matlab ode solver. Plot the solution.

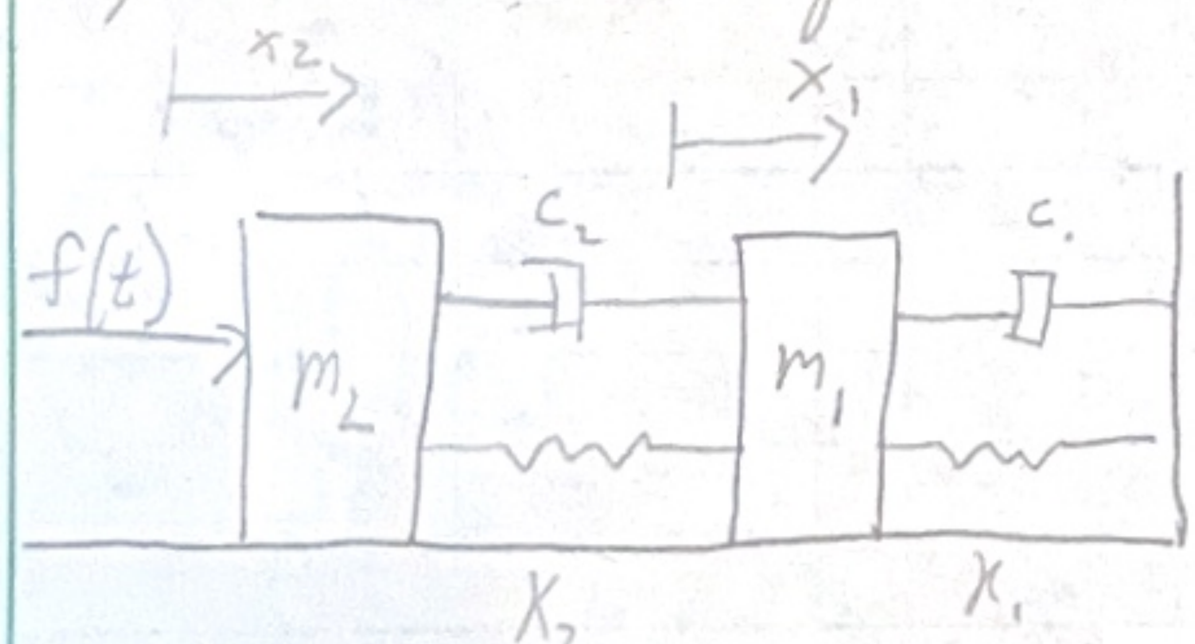
$$\dot{y} + 2y = f(t) \quad y(0) = 0$$

$$f(t) = \begin{cases} 3t & 0 \le t \le 2 \\ 6 & 2 \le t \le 5 \\ -3(t-5)+6 & 5 \le t \le 7 \end{cases}$$

Problem 9 (text 5.48 S)

find the solution to problem 5.20a in the text assuming zero initial conditions and $u = 3\sin(2t)$ useing simulink plot the solution

Problem 10 text 5.60S a) find a Simulink model of the system below b) Plot the system responce to the input below.




$$f(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 < t < 2 \\ 0 & t \ge 2 \end{cases}$$

$$m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 + (k_1+k_2)x_1 - c_2\dot{x}_2 - k_2x_2 = 0$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_2\dot{x}_1 - k_2x_1 = f(t)$$

$m_1 = m_2 = 1 \quad c_1 = 3 \quad c_2 = 1 \quad k_1 = 1 \quad k_2 = 4$

Cont →

$$\ddot{x}_1 m_1 = -(c_1 + c_2)\dot{x}_1 - (k_1 + k_2)x_1 + c_2\dot{x}_2 + k_2 x_2$$

$$\ddot{x}_2 m_2 = -c_2\dot{x}_2 - k_2 x_2 + c_2\dot{x}_1 + k_2 x_1 + f(t)$$

$$\ddot{x}_1 = -4\dot{x}_1 - 5x_1 + \dot{x}_2 + 4x_2$$

$$\ddot{x}_2 = -\dot{x}_2 - 4x_2 + \dot{x}_1 + 4x_1 + f(t)$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \quad \begin{matrix} \dot{z}_1 = z_3 & \dot{z}_2 = z_4 \\ \dot{z}_3 = \ddot{x}_1 & \dot{z}_4 = \ddot{x}_2 \end{matrix}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & 4 & -4 & 1 \\ 4 & -4 & 1 & -1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} f(t) \end{bmatrix}$$

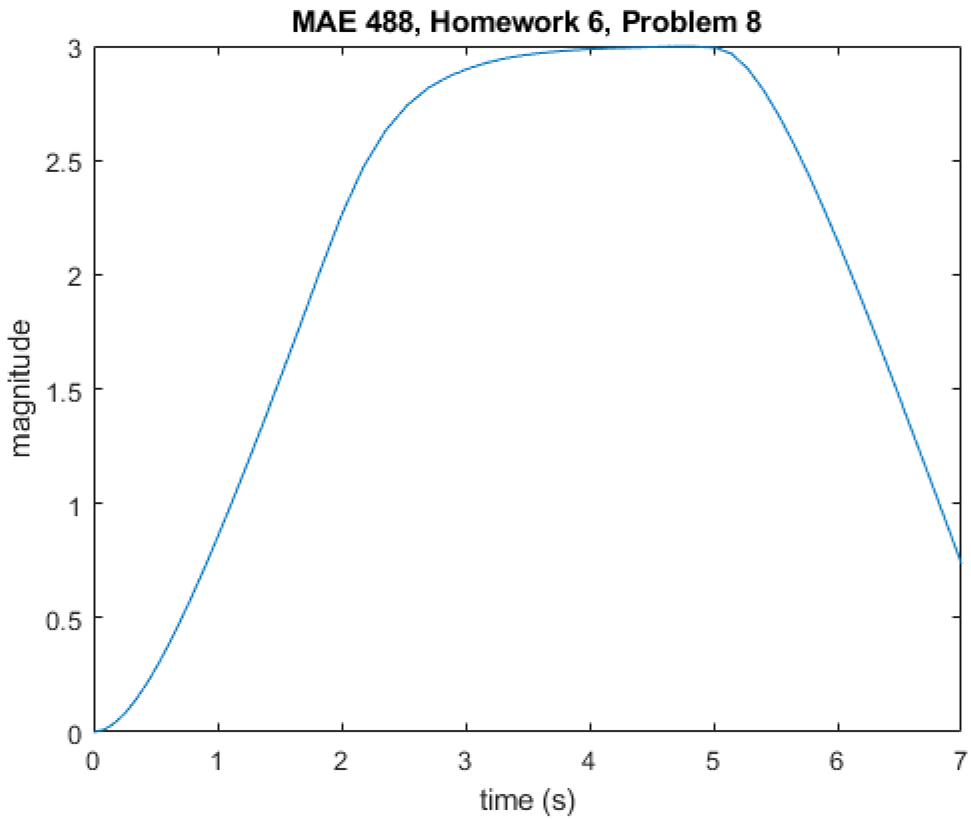
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} f(t) \end{bmatrix}$$

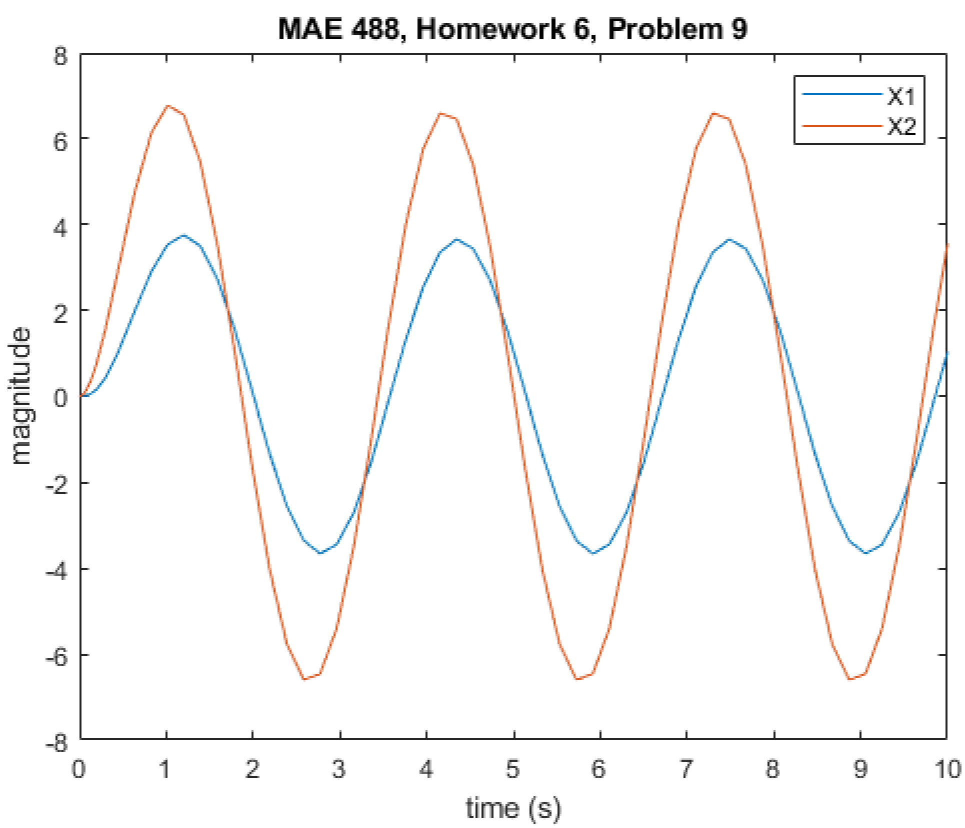
```
% MAE488_Nicholas_Hawse_HW6
% MAE 488 03 Analisis of ANALY ENGINEERING SYSTEMS
% Homework 6
% Nicholas Hawse
% 03/15/2025
% This code finds and plots solutions to the problems in HW 6

clear;
clc;
clf;
close all;

fprintf('==================================================================\
n')
fprintf('MAE 488, Homework # 1, Spring 2025\n')
fprintf('==================================================================\
n')
fprintf('\n\n')

%
============================================================================
% Problem 8
%
============================================================================
% find the numerical solutions to the ode given below and the responce to
% the input f(t)
%
% dy/dt + 2y = f(t)
% y(0) = 0
fprintf('=============================\n')
fprintf('Problem 8 \n')
fprintf('=============================\n')
fprintf('This code finds the numerical solutions to the ode given ')
fprintf('below and the responce to the input f(t)\n')
fprintf('\n')
fprintf('see the figure below\n\n\n\n\n')

function fval = fot(t) % input function f(t)
    if (t >= 0) && (t <= 2)
        fval = 3*t;

    elseif (t > 2) && (t < 5)
        fval = 6;

    elseif (t >= 5) && (t <= 7)
        fval = -3*(t-5)+6;
    else
        fval = 0;
    end
end
```

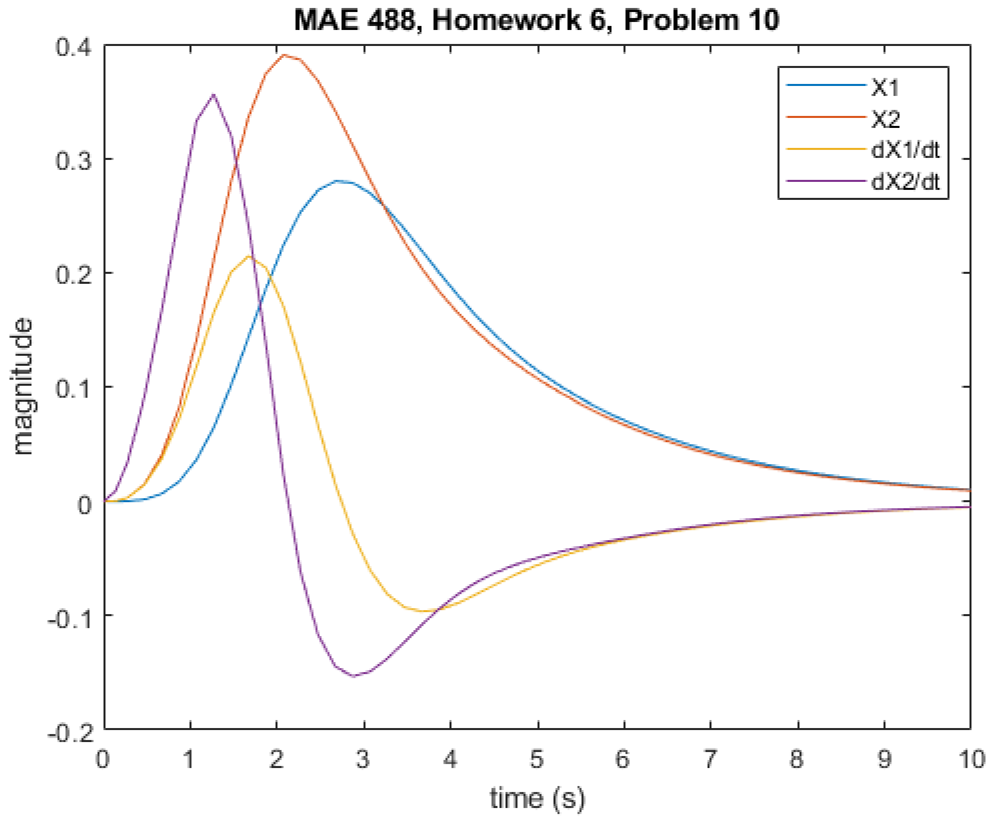
```
y0 = 0; % initial conditions

dydt = @(time,y) fot(time) - 2*y; % defines the ode

[t,y] = ode45(dydt,[0 7],y0); % solves the ode

figure(1)

plot(t,y) %plots the ode
title('MAE 488, Homework 6, Problem 8')
xlabel('time (s)')
ylabel('magnitude')


%
================================================================================
% Problem 9
%
================================================================================
% find the solutions to the ssm given below and the responce to
% the input u
%
%
%
fprintf('==============================\n')
fprintf('Problem 9 \n')
fprintf('==============================\n')
fprintf('This code finds the responces of the state space model to the')
fprintf('input of the function u(t)')
fprintf('\n')
fprintf('see the figure below\n\n\n\n\n')

A9 = [-7 4; 0 -3];
B9 = [0; 8];
C9 = [1 0; 0 1];
D9 = [0; 0];

t = 0:0.001:10;

uot = [t; 3*sin(2*t)]';

out1 = sim('mdl548');

time = out1.tout;
val1 = out1.out548;

figure(2)

plot(time,val1)
title('MAE 488, Homework 6, Problem 9')
xlabel('time (s)')
ylabel('magnitude')
legend('X1','X2')
```

```
%
================================================================================
% Problem 10
%
================================================================================
% find the solutions to the ssm of the spring mass system and the
% responce to the input of a force f(t)
%
%
fprintf('==============================\n')
fprintf('Problem 10 \n')
fprintf('==============================\n')
fprintf('find the solutions to the ssm of the spring mass system and the')
fprintf('responce to the input of a force f(t)')
fprintf('\n')
fprintf('see the figure below\n\n\n\n\n')

% the matricies that define the ss model
A10 = [0 0 1 0; 0 0 0 1; -5 4 -4 1; 4 -4 1 -1];
B10 = [0; 0; 0; 1];
C10 = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1];
D10 = [0; 0; 0; 0];


% creates the sections of the piecewise function
t1 = 0:0.0001:(1-0.0001);
t2 = 1:0.0001:(2-0.0001);
t3 = 2:0.0001:10;

t = [t1 t2 t3];

% creates the function values of the piecewise fun
fot1 = t1;
fot2 = 2 - t2;
fot3 = t3./t3.*0;

fot10 = [fot1 fot2 fot3];% composes the parts of the piecewises fun

fot10 = [t; fot10]'; % creates the input for the sim

out2 = sim('mdl560'); % runs the simulation

time = out2.tout; % gets the time out of the simulink
val2 = out2.out560; % gets the array of the outputs of the ss model

figure(3) % new figure

plot(time,val2) % plots the values of the outputs of the ss model
title('MAE 488, Homework 6, Problem 10')
xlabel('time (s)')
ylabel('magnitude')
legend('X1', 'X2', 'dX1/dt', 'dX2/dt')
```

```
==========================================================================
MAE 488, Homework # 1, Spring 2025
==========================================================================


==============================
Problem 8
==============================
This code finds the numerical solutions to the ode given below and the
responce to the input f(t)

see the figure below




==============================
Problem 9
==============================
This code finds the responces of the state space model to theinput of the
function u(t)
see the figure below




==============================
Problem 10
==============================
find the solutions to the ssm of the spring mass system and theresponce to
the input of a force f(t)
see the figure below
```


MAE 488, Homework 6, Problem 8
magnitude
time (s)
0
0.5
1
1.5
2
2.5
3
0
1
2
3
4
5
6
7


MAE 488, Homework 6, Problem 9
X1
X2
magnitude
time (s)
-8
-6
-4
-2
0
2
4
6
8
0
1
2
3
4
5
6
7
8
9
10

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