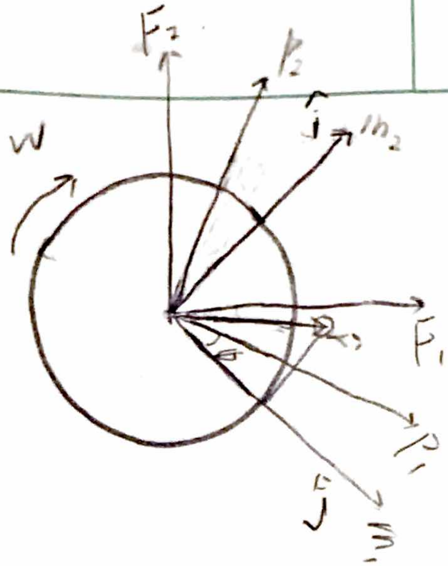



$\hat{r} = R\phi\,\hat{i} + R\hat{j}$ ①

$$\vec{V}_m = \vec{V}_{rel} + (w+\dot{\phi})k \times \hat{r} = \dot{r} + \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 0 & w+\dot{\phi} \\ R\phi & R & 0 \end{vmatrix} = \dot{r} + (w+\dot{\phi})R\phi\hat{j} - R(w+\dot{\phi})\hat{i}$$

$$= R\dot{\phi}\hat{i} + (w+\dot{\phi})R\phi\hat{j} - R(w+\dot{\phi})\hat{i} = -Rw\hat{i} + R\phi(w+\dot{\phi})\hat{j}$$ ②

Angular momentum of the mass

$$h = r \times mv = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R\phi & R & 0 \\ -mRw & mR\phi(w+\dot{\phi}) & 0 \end{vmatrix} = mR^2\phi^2(w+\dot{\phi})\hat{k} + mR^2w\hat{k}$$

$$= mR^2\left[w + \phi^2(w+\dot{\phi})\right]\hat{k}.$$ ③

Total angular momentum

$$H = Iw\hat{k} + h = \left\{Iw + mR^2\left[w+\phi^2(w+\dot{\phi})\right]\right\}\hat{k}$$ ④

System kinetic energy.

$$T = \frac{1}{2}Iw^2 + \frac{1}{2}mV_m^2 = \frac{1}{2}Iw^2 + \frac{1}{2}m\left[R^2w^2 + R^2\phi^2(w+\dot{\phi})^2\right]$$

$$= \frac{1}{2}Iw^2 + \frac{1}{2}mR^2\left[w^2 + \phi^2(w+\dot{\phi})^2\right]$$ ⑤

Using conservation of energy and momentum

$$H_o = I w_o = (I + mR^2) w_o = I w + mR^2 [ w + \phi^2 (w + \dot{\phi}) ] \quad (6)$$

$$\frac{I w_o + mR^2 w_o}{mR^2} = \frac{I w + mR^2 [ w + \phi^2 (w + \dot{\phi}) ]}{mR^2}$$

$$\left( \frac{I}{mR^2} + 1 \right) w_o = \left( \frac{I}{mR^2} + 1 \right) w + \phi^2 (w + \dot{\phi})$$

let $C = \frac{I}{mR^2} + 1$

$$C w_o = C w + \phi^2 (w + \dot{\phi}) \quad (7)$$

$$T_o = \frac{1}{2} I w_o^2 = \frac{1}{2} (I + mR^2) w_o^2 = \frac{1}{2} I w^2 + \frac{1}{2} mR^2 [ w^2 + \phi^2 (w + \dot{\phi})^2 ] \quad (8)$$

$$\Rightarrow \frac{\frac{1}{2} I w_o^2 + \frac{1}{2} mR^2 w_o^2}{mR^2} = \frac{\frac{1}{2} I w^2 + \frac{1}{2} mR^2 [ w^2 + \phi^2 (w + \dot{\phi})^2 ]}{mR^2}$$

$$\left( \frac{I}{mR^2} + 1 \right) w_o^2 = \left( \frac{I}{mR^2} + 1 \right) w^2 + \phi^2 (w + \dot{\phi})^2$$

$$C w_o^2 = C w^2 + \phi^2 (w + \dot{\phi})^2 \quad (9)$$

$(7) \rightarrow \quad C(w_o - w) = \phi^2 (w + \dot{\phi})$

$(9) \rightarrow \quad C(w_o^2 - w^2) = \phi^2 (w + \dot{\phi})^2$

$\rightarrow \quad C(w_o + w)(w_o - w) = \phi^2 (w + \dot{\phi})^2$

$$\frac{(9)}{(7)} \rightarrow w_o + w = w + \dot{\phi} \Rightarrow w_o = \dot{\phi} \Rightarrow w_o t = \phi \quad (10)$$

substitude (10) into (7)

$$C w_o = C w + w_o^2 t^2 (w + w_o) \Rightarrow (C + w_o^2 t^2) w = (C - w_o^2 t^2) w_o$$

$$\Rightarrow \boxed{w = \left( \frac{C - w_o^2 t^2}{C + w_o^2 t^2} \right) w_o}$$

$$w = w_0\left(\frac{C - w_0^2 t^2}{C + w_0^2 t^2}\right) \qquad L = \phi R = w_0 t R$$

$$\Rightarrow \boxed{w = w_0\left[\frac{C-(\frac{L}{R})^2}{C+(\frac{L}{R})^2}\right] = w_0\left(\frac{CR^2-L^2}{CR^2+L^2}\right)}$$

(b) $$w = w_0\left(\frac{C-w_0^2t^2}{C+w_0^2t^2}\right)$$

$$\alpha = \frac{dw}{dt} = w_0 \frac{(-2w_0^2 t)(C+w_0^2t^2) - (2w_0^2 t)(C - w_0^2t^2)}{(C+w_0^2t^2)^2}$$

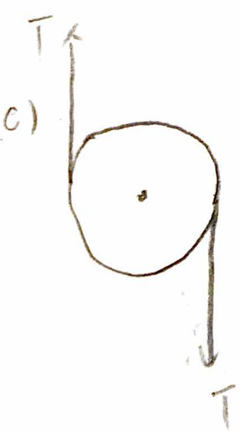
$$= w_0 \frac{(-2w_0^2t)(C+w_0^2t^2+C-w_0^2t^2)}{(C+w_0^2t^2)^2}$$

$$\Rightarrow \boxed{\alpha = \frac{-4Cw_0^3 t}{(C+w_0^2t^2)^2} \quad ①}$$

$$L=\phi R = w_0 t R$$

$$\Rightarrow \boxed{\alpha = \frac{-4Cw_0^2 \frac{L}{R}}{[C+(\frac{L}{R})^2]^2} \quad ②}$$

(c)




$$2T = \frac{I\alpha}{R} = \frac{I}{R}\cdot\frac{-4Cw_0^2\frac{L}{R}}{[C+(\frac{L}{R})^2]^2}$$

$$\boxed{T = \frac{-2ICw_0^2\frac{L}{R}}{R[C+(\frac{L}{R})^2]^2}}$$

$$\boxed{= \frac{-2ICw_0^3t}{R(C+w_0^2t^2)^2}}$$

(4) from the conservation of energy

$$\frac{1}{2} I w_0^2 + \frac{1}{2} m V_m^2 = \frac{1}{2} I w_f^2 + \frac{1}{2} m V_{mf}^2$$

$$\Rightarrow \frac{1}{2} I w_0^2 + \frac{1}{2} m \times w_0^2 R^2 = \frac{1}{2} m V_{mf}^2$$

$$V_{mf} = \sqrt{\frac{I w_0^2}{m} + w_0^2 R^2} = R\sqrt{\left(\frac{I}{mR^2} + 1\right) w_0^2} = R w_0 \sqrt{c}$$

from the conservation of angular momentum.

$$-I_{tot} w_0 = r \times m\vec{V}_{mf} + I w_f$$

$$-(I + mR^2) w_0 = -mR w_0 \sqrt{c}\,(R+L) + I w_f$$

$$\Rightarrow w_f = \frac{mR w_0 \sqrt{c}\,(R+L) - (I + mR^2) w_0}{I}$$

$$w_f = 0 \Rightarrow mR \cancel{w_0} \sqrt{c}\,(R+L) = (I + mR^2) \cancel{w_0}$$

$$mR^2\sqrt{c} + mR\sqrt{c}\,L = I + mR^2$$

$$mR\sqrt{c}\,L = I + mR^2 - mR^2\sqrt{c}$$

$$\boxed{L = \frac{I}{mR\sqrt{c}} + \frac{R}{\sqrt{c}} - R}$$