ASEN2003
Lab #1
Roller Coaster Design

R. S. Nerem

ASEN2003 Lab #1 Objectives

- Use your knowledge of particle dynamics to design and analyze the performance of a new roller coaster.
- Gain design experience using an example in particle dynamics.
- Practice using free body diagrams (FBDs) and energy methods to set up and solve problems in dynamics.
- Document your design and analysis in a professional technical report.
- This is a “paper lab” – there is no lab apparatus.

ASEN2003 Lab #1 Problem Statement

Roller coasters are one of the main attractions for amusement and theme parks and vary considerably in their design. Trains on the coaster are brought to the top of a hill by some kind of lifting mechanism and from then on they coast for the remainder of the ride. Although things like friction, air resistance, mass distribution in the cars, etc. complicate things, we will do a first-cut design of a roller coaster by ignoring all that and treating the train as a point mass moving on a frictionless rail through space.

The primary tasks for this design project are to

- 1) analyze the dynamics of typical coaster track elements (hills, valleys, turns, loops, twists);
- 2) design specific track elements meeting the project requirements;
- 3) assemble a track design;
- 4) analyze overall track performance;
- 5) document the design and analysis in a group lab report.

ASEN2003 Lab #1 Problem Statement

- The features that make a roller coaster ride exciting are novelty, speed, and G's experienced, so your task is to optimize the experience in terms of these parameters. We will quantify novelty in terms of the number of different elements incorporated in the design.
- Maximum speed is limited by the initial height of the coaster, and you can adjust the speed entering and leaving each element of the track by selecting its height. G's experienced must be defined carefully.
- The number of G's the particle experiences is equal to the normal force (N) exerted by the track on it, divided by the particle weight (mg). The normal force will also be a function of m, so the mass of the car should not affect the final "G" calculation.
- Note that the normal force is a vector quantity so we can express the number of G's felt in each of three directions relative to the train car (up, forward, left for example). The human body is more sensitive to G's in some directions than others, so we will set the design requirements to make the ride comfortable (and safe) for the riders.

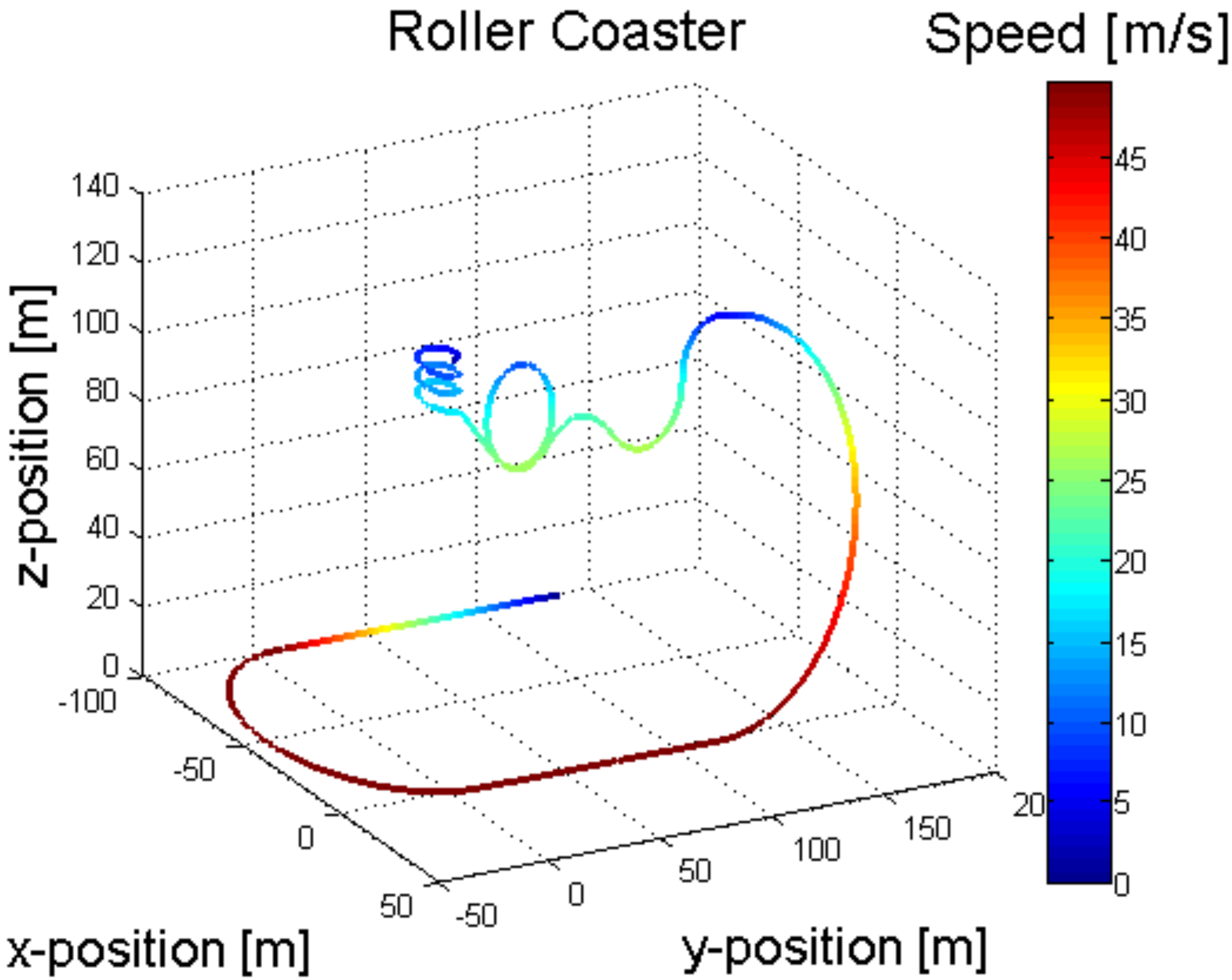
ASEN2003 Lab #1 Assumptions

- Assume that the roller coaster train and people inside it may be treated as a particle or point mass.
- The track is frictionless (except for any braking sections).
- The train is initially brought to the top of a 125 m (h_0) hill where it has zero velocity. The speed at any point on the track can be found based on the height compared to the initial height. .
- The train must remain above ground (i.e. the height must always be greater than or equal to zero).
- The train is locked to the track so that the force exerted on the train by the track can act in any direction orthogonal to the track (i.e. you can be held in your seat by the lap bar and pushed right or left by the side of the seat).
- A track element will refer to one of the following: circular or parabolic hill, circular or parabolic valley, loop, and helix.
- Note that neither the horizontal banked turn nor the braking section are considered a "track element".

ASEN2003 Lab #1 Design Requirements

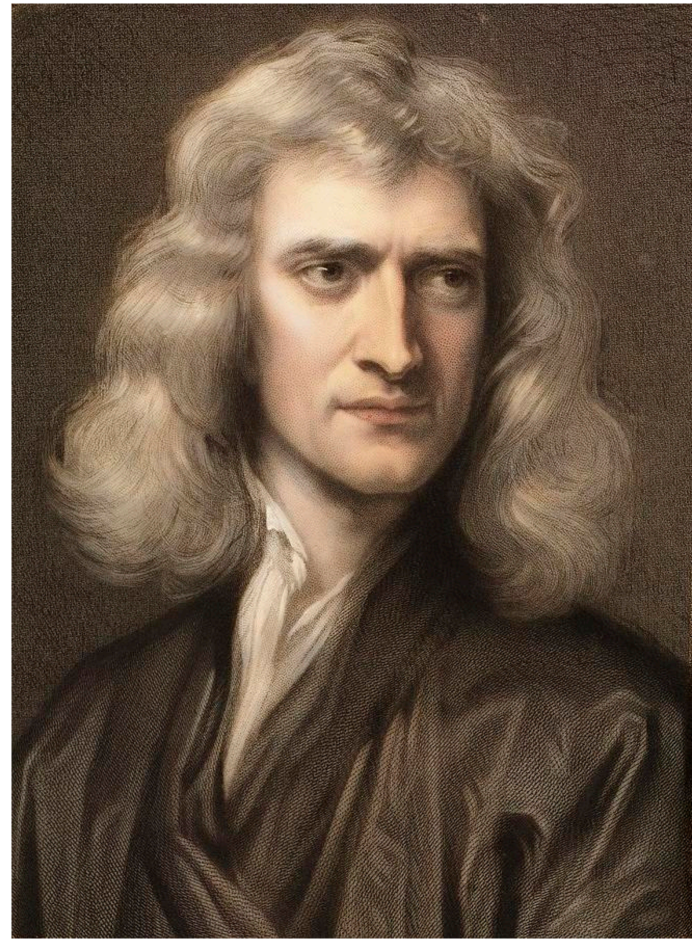
- The total linear distance of the track must be less than 1250 m with the train coming to rest (using a braking mechanism) at a final height of 0 m.
- The coaster must include at least three different types of track elements with transitions between them. All transitions must be smooth.
- The coaster must include at least one section that produces zero g throughout the ENTIRE element (not just at one point).
- The coaster track must contain at least one banked turn at a constant or changing altitude (i.e. the track cannot remain in a single plane.)
- The G's experienced by the passengers must be within the following ranges defined in a coordinate system fixed to the train:
  - forward (back of seat pushing on rider) < 5 G
  - back (seat restraint pushing back the rider) < 4 G
  - up (i.e. pushing up through the rider's seat) < 6 G
  - down (i.e. pushing down on the rider through the lap bar) < 1 G
  - lateral (pushing to the left or right on the rider) < 3 G

Sample Roller Coaster Design



Newton's 2nd Law

In an inertial reference frame, the vector sum of the forces, F, acting on an object is equal to the mass, m, of the object times the acceleration, a.



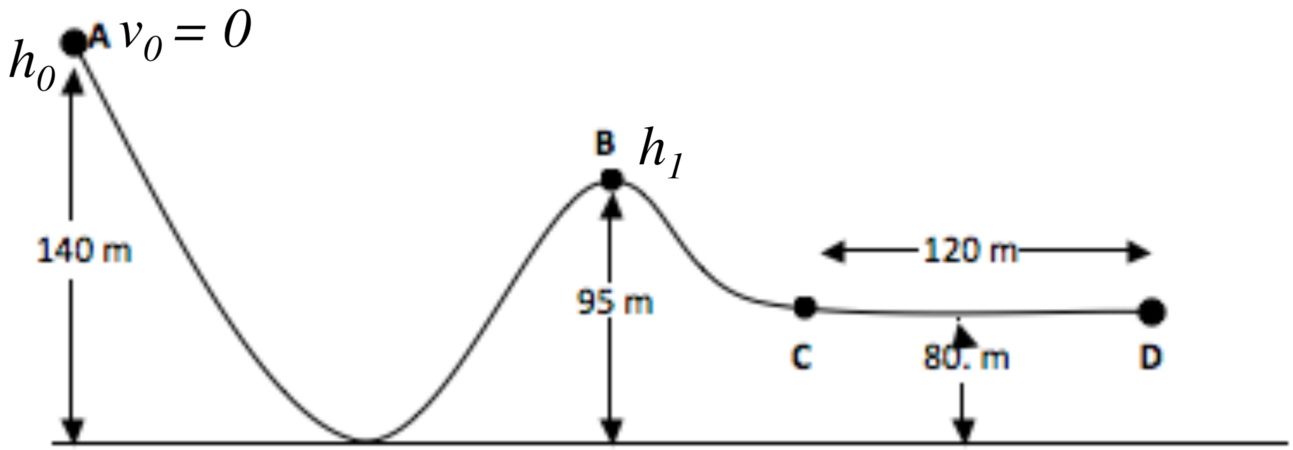
$$\boxed{\vec{F} = m\vec{a}}$$

Roller Coasters and Energy

- Energy is conserved in the absence of friction, drag, etc.

$$T = \text{kinetic energy} = \frac{1}{2}mv^2$$

$$V = \text{potential energy} = mgh \qquad h = \text{height}, \quad g = 9.81 \text{ m/s}^2$$

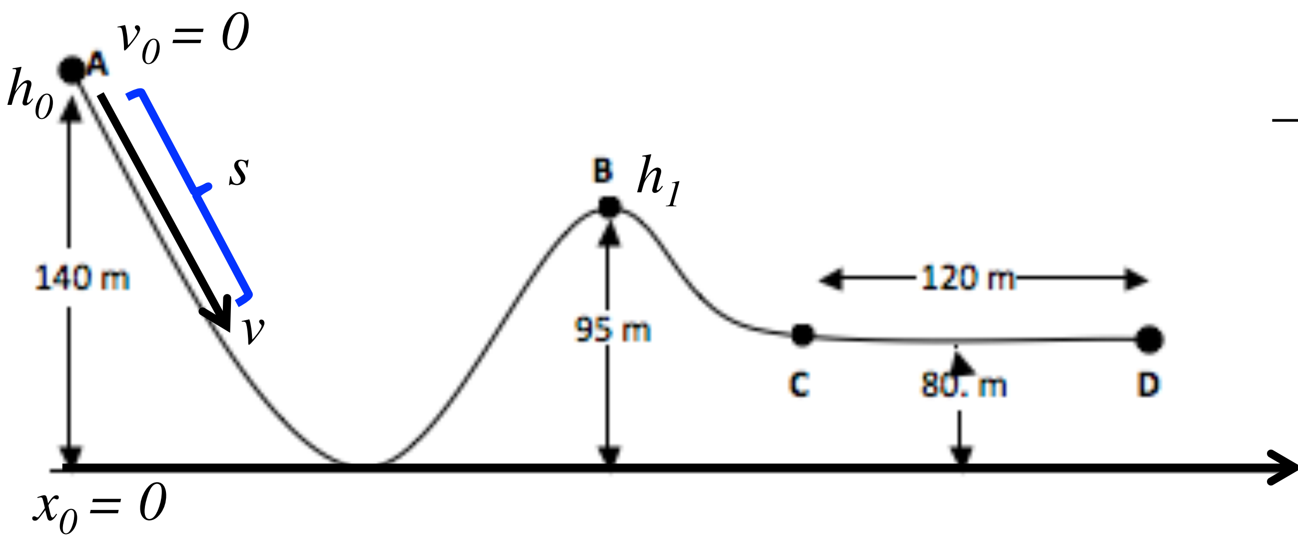


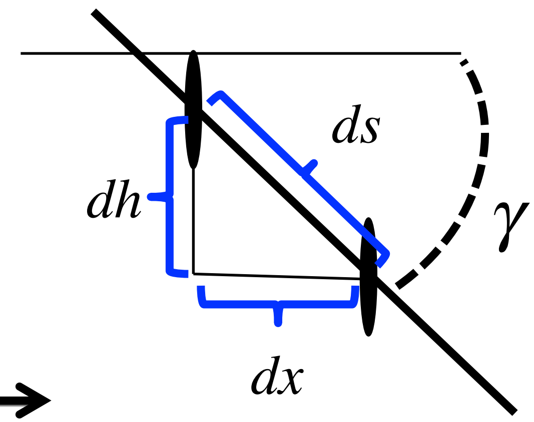
$$E = T + V = \frac{1}{2}mv^2 + mgh \qquad E_0 = mgh_0 \qquad E_1 = \frac{1}{2}mv_1^2 + mgh_1$$

$$mgh_0 = \frac{1}{2}mv_1^2 + mgh_1 \Rightarrow v = \sqrt{2g(h_0 - h_1)}$$

Can $h_1 > h_0$? Only if additional energy added.

Path Length, Altitude, Range





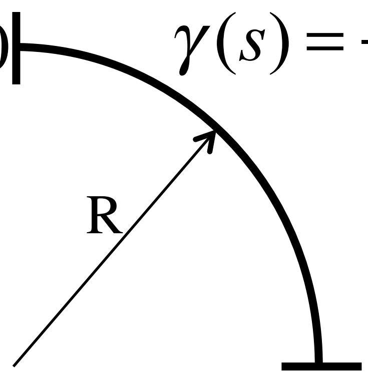
$$dh = \sin\gamma\, ds$$

$$dx = \cos\gamma\, ds$$

Can define path by stipulating $\gamma(s)$

$s = 0$

$$\gamma(s) = -\frac{s}{R}$$



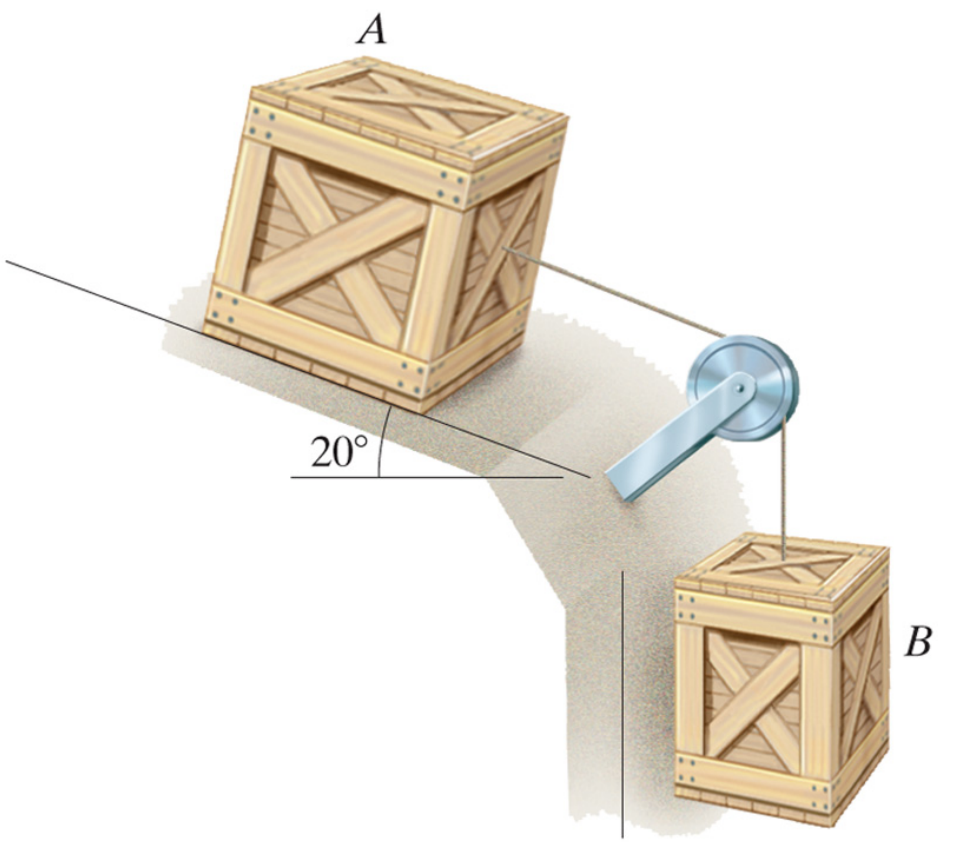
$$s = \frac{\pi}{2} R$$

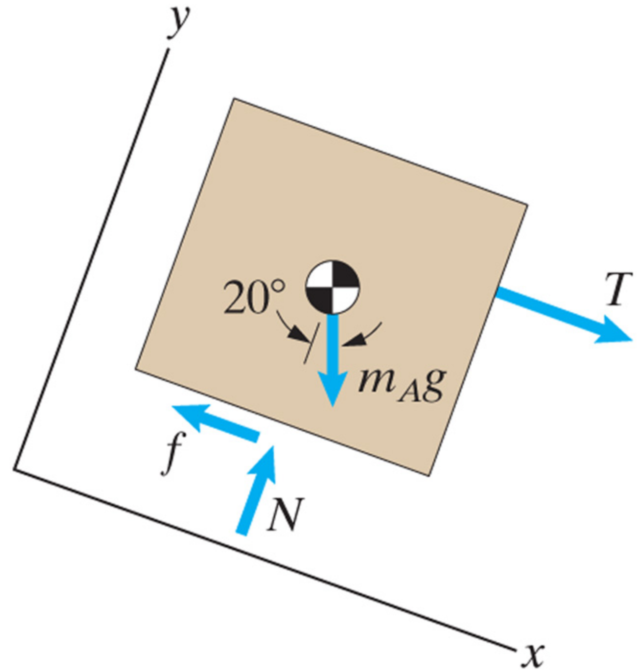
$$h = h_0 + \int_0^s \sin\gamma(s)\, ds \qquad x = x_0 + \int_0^s \cos\gamma(s)\, ds$$

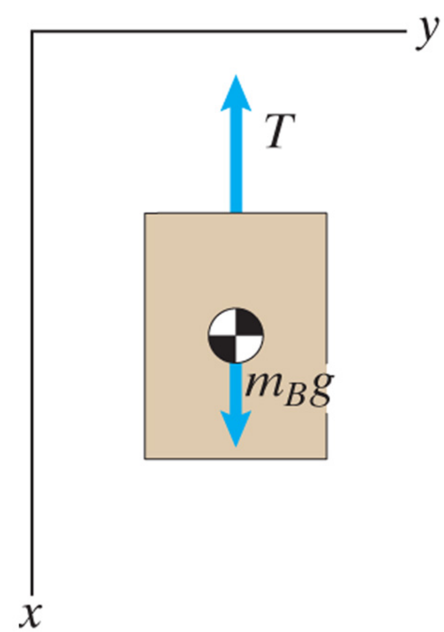
Can also invert procedure and stipulate $\gamma(h)$

$$ds = \frac{1}{\sin\gamma(h)} dh \qquad s - s_0 = \int_{h_0}^{h} \frac{dh}{\sin\gamma(h)}$$

Free Body Diagrams (FBDs)



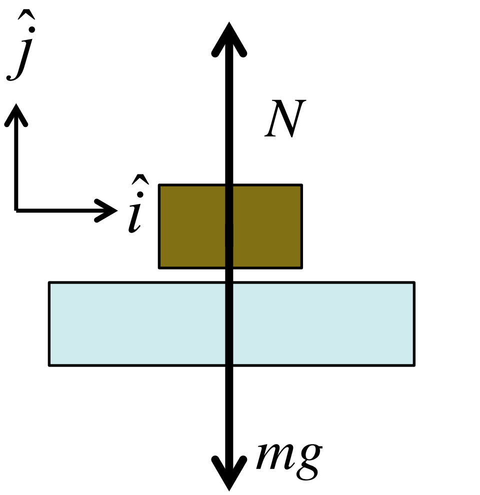




Roller Coasters and G Forces

- "Gs" are measured in units of the Earth's gravity g. An object in free fall under the influence of only gravity does not experience any G-force (called zero-g or weightlessness). However, when an object is pushed on by another object, such as when the track pushes on the roller coaster car, then you will have a G-force. This can be calculated by taking the normal force (N) and dividing by mg.

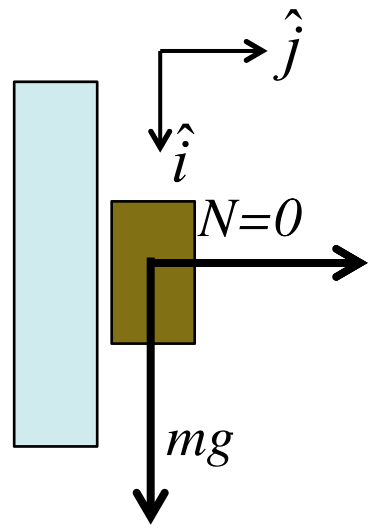
What is the G-force?



$$\sum F_y = 0 = N - mg$$

$$N = mg$$

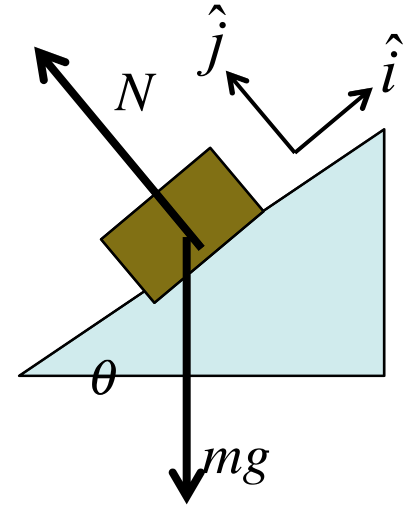
$$\text{G-force} = 1$$



$$\sum F_y = 0 = N$$

$$N = 0$$

$$\text{G-force} = 0$$

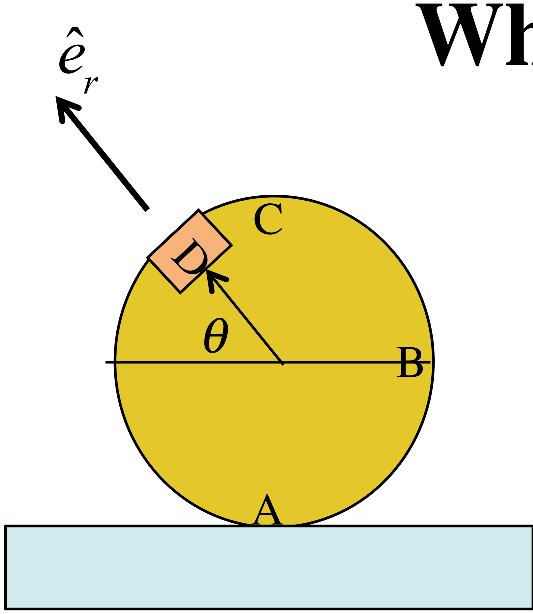


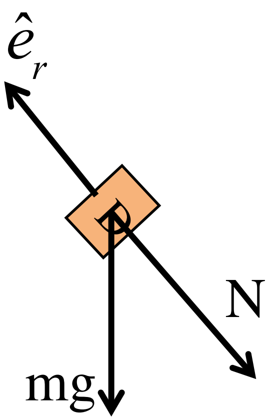
$$\sum F_y = 0 = N - \cos\theta mg$$

$$N = \cos\theta mg$$

$$\text{G-force} = \cos\theta$$

What is the G-force?





(A)

$$\sum F_r = -m\frac{v^2}{r} = -N + mg$$

$$N = m\frac{v^2}{r} + mg$$

$$\text{G-force} = \frac{v^2}{gr} + 1$$

(B)

$$\sum F_r = -m\frac{v^2}{r} = -N$$

$$N = m\frac{v^2}{r}$$

$$\text{G-force} = \frac{v^2}{gr}$$

(C)

$$\sum F_r = -m\frac{v^2}{r} = -N - mg$$

$$N = m\frac{v^2}{r} - mg$$

$$\text{G-force} = \frac{v^2}{gr} - 1$$

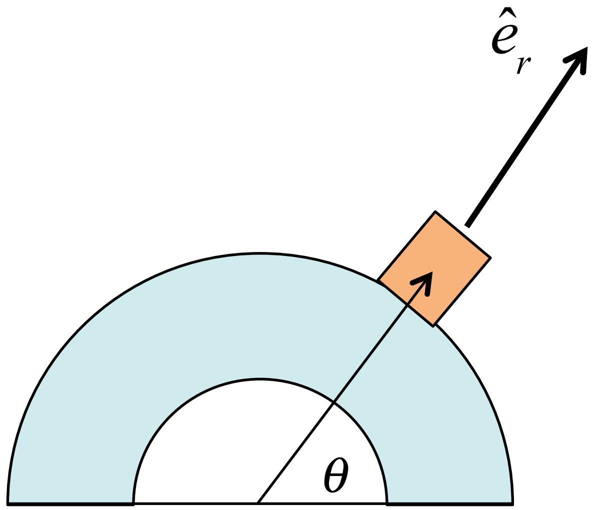
(D)

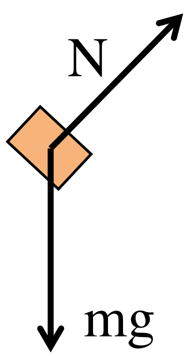
$$\sum F_r = -m\frac{v^2}{r} = -N - mg\sin\theta$$

$$N = m\frac{v^2}{r} - mg\sin\theta$$

$$\text{G-force} = \frac{v^2}{gr} - \sin\theta$$

What is the G-force?





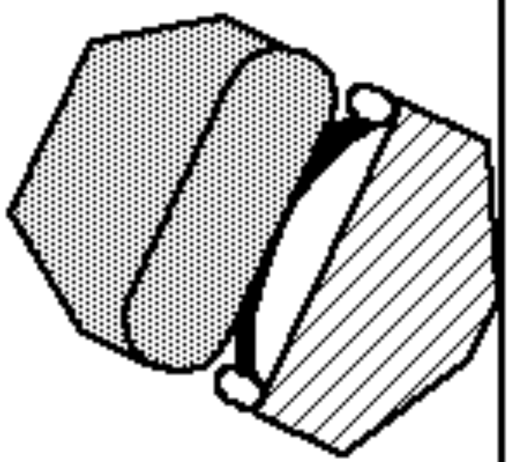
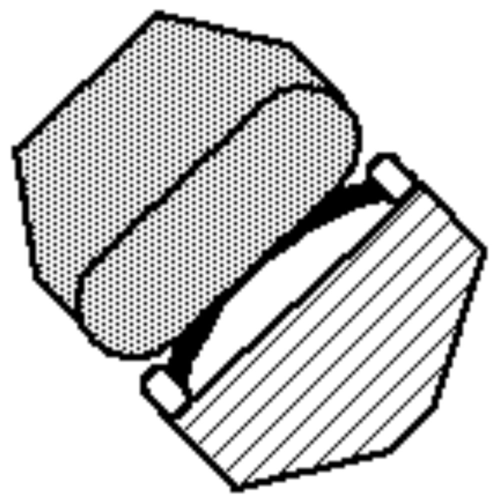
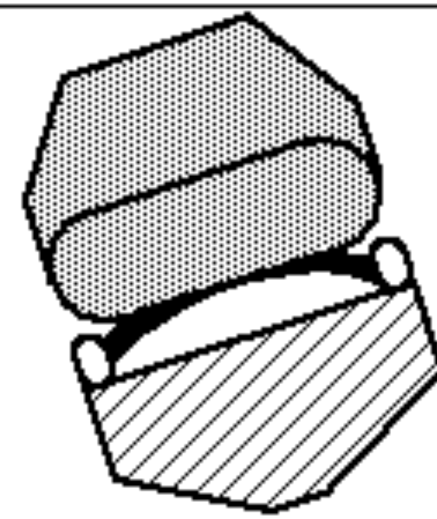
$$\sum F_r = -m\frac{v^2}{r} = N - mg\sin\theta$$

$$N = -m\frac{v^2}{r} + mg\sin\theta$$

$$\text{G-force} = -\frac{v^2}{gr} + \sin\theta$$

Banked Turns

A banked turn reduces the rider's sensation of being thrown sideways by turning the car sideways. The trick is to tilt it just the right amount. The ideal banked turn is one where no outside forces are needed to keep the car on the track.

HIGH SPEED BANK	MEDIUM SPEED BANK	LOW SPEED BANK
		
Too much bank for the car's velocity. the car could tip to the inside. The undercarriage wheels are holding the car on. The rider fells a force pushing himself down. Friction is needed to keep the car on the track.	At just the right bank for the car's velocity, the car does not need any type of undercarriage to stay on the track. The rider feels a force pushing his bottom into the seat. This is the optimum position where no friction is needed to keep the car on the track.	Not enough bank for the car's velocity. The car could tip to the outside. The undercarriage wheels are holding the car on. The rider fells a force pushing himself to the outside of the curve -sideways. Friction is needed to keep the car on the track.

Banked Turns

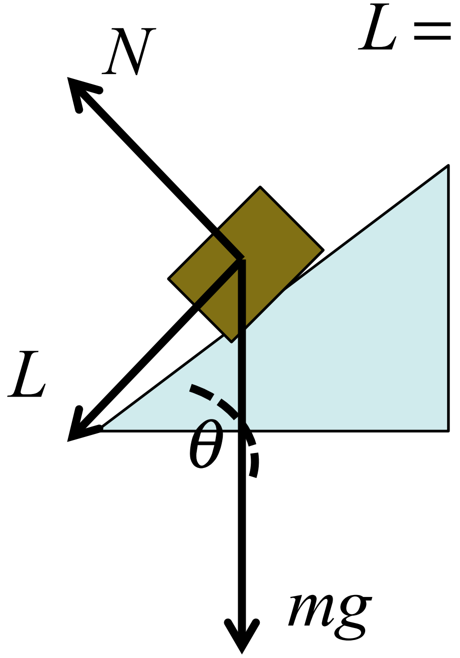
$r =$ radius of the turn

$v =$ velocity (constant)

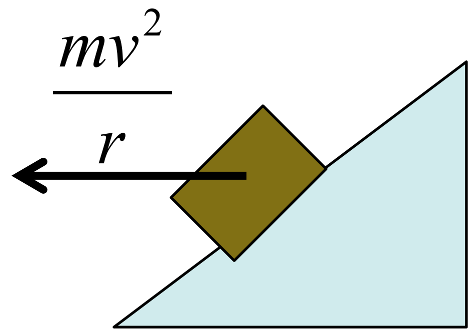
$\theta =$ bank angle

$N =$ normal force (6 Gs max)

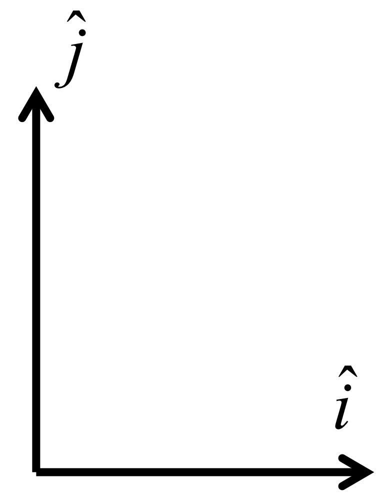
$L =$ lateral force (3 Gs max)



Forces
(back view)



Mass-acceleration
(back view)



Banked Turns

$$\left.\begin{aligned}\sum F_y &= N\cos\theta + L\sin\theta - mg = 0 \\ \sum F_x &= -N\sin\theta - L\cos\theta = -\frac{mv^2}{r}\end{aligned}\right\}\text{Solve for } \theta \text{ and } r \text{ or } N \text{ and } L$$

What if L = 0? (no lateral force applied)

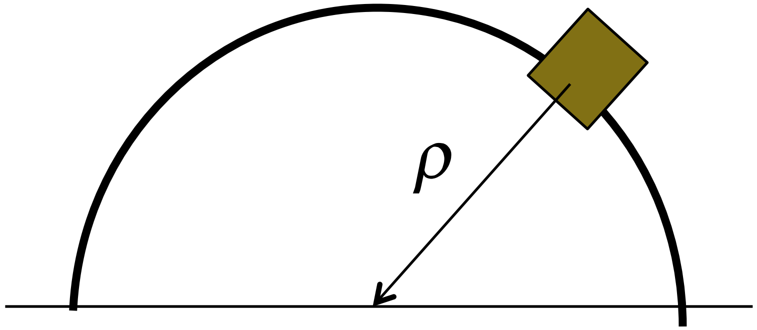
$$N = \frac{mv^2}{\sin\theta r} \qquad N = \frac{mg}{\cos\theta}$$

$$\frac{mg}{\cos\theta} = \frac{mv^2}{\sin\theta r} \Rightarrow r = \frac{v^2}{g}\cot\theta$$

$$\text{Gs felt} = \frac{N}{mg} = \frac{1}{\cos\theta}$$

Ideal banked turn with no lateral forces

What about motion on a parabola?



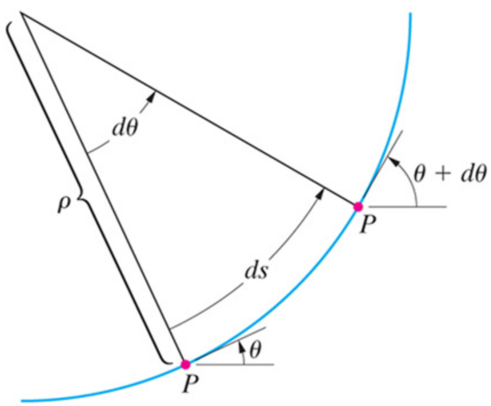
$$\vec{a} = \left[\ddot{r} - r\dot{\theta}^2\right]\hat{e}_r + \left[r\ddot{\theta} + 2\dot{r}\dot{\theta}\right]\hat{e}_\theta$$

$$V_\theta = r\dot{\theta} \quad \text{so} \quad -r\dot{\theta}^2 = -V_\theta\dot{\theta} = -\frac{V_\theta^2}{r}$$

$\rho =$ radius of curvature

$=$ radius of the approximating circle at that point.

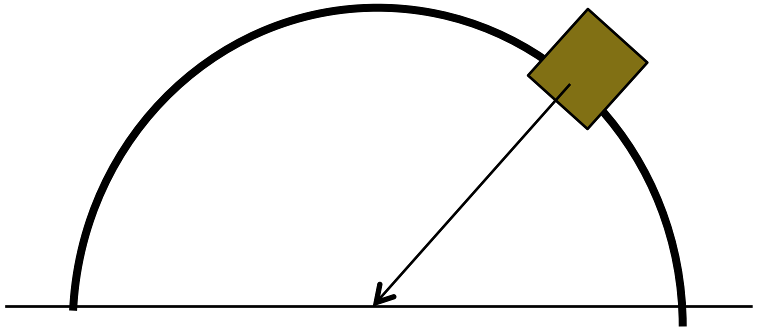
$$\sum F_r = \frac{-mv^2}{\rho} = N - \sin\theta mg$$



The parameter ρ is the *instantaneous radius of curvature* of the path. θ is the angle between a fixed reference direction and the path.

$$\rho = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}$$

What about motion on a parabola?



$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2 y}{dx^2}\right|}$$

Example: $y = 2x^3 - x + 3$ Find ρ at $x = 1$

$$\frac{dy}{dx} = 6x^2 - 1 \qquad \left(\frac{dy}{dx}\right)^2 = 36x^4 - 12x^2 + 1$$

$$\frac{d^2 y}{dx^2} = 12x \quad \rho = \frac{\left[1 + 36x^4 - 12x^2 + 1\right]}{|12x|} = 11.0478 @ x = 1$$

Common Mistakes

- Calculating number of Gs. The number of Gs is not acceleration/g, it is normal force/mg. It is a measure of the force on an object compared to its weight.
- Gs and acceleration are vectors. That means they have 3 components. Many people ignore this and simply take Gs at a point to be the number in whichever direction is greatest.
- Right-handed coordinate system. Using a RH coordinate system is very important – do not switch halfway through a loop.
- Linear distance. The linear distance of track is the same thing at the length of the track. It is not the same as the x position.

Common Mistakes

- Transitions. Roller coaster tracks should be piece-wise continuous and smooth. In order to be smooth, the derivatives must be equal where the two sections meet (i.e., must have the same slope).
- Banked Turns. Remember the track can push on the bottom and sides of a wheel. So the total normal force need not be normal to the track.