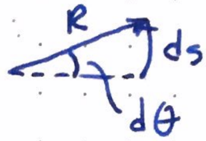
Problem 1



$$\oint_C f(\theta)\, ds \qquad ds = R\,d\theta$$

$$\Rightarrow \oint_C f(\theta)\, R\, d\theta = R \oint_C f(\theta)\, d\theta$$

$$L' = R \int_0^{2\pi} -P(\theta) \sin\theta \, d\theta \qquad C_p = \frac{P(\theta) - P_\infty}{q_\infty}$$

$$C_p = 1 - 4\sin^2\theta$$

$$\rightarrow \quad 1 - 4\sin^2\theta = \frac{P - P_\infty}{q_\infty} \qquad P = C_p q_\infty + P_\infty$$

$$P(\theta) = q_\infty - 4\sin^2\theta\, q_\infty + P_\infty \qquad \cancel{(C_p(\theta)\, q_\infty + P_\infty)}$$

$$L' = -R \int_0^{2\pi} (q_\infty - 4\sin^2\theta\, q_\infty + P_\infty) \sin\theta \, d\theta$$

Composite Simpson's Rule:

$$\int_a^b f(\theta)\, d\theta \approx \frac{h}{3} \sum_{k=1}^{N/2} \left[f(\theta_{2k-1}) + 4 f(\theta_{2k}) + f(\theta_{2k+1}) \right]$$

$$\theta_k = a + (k-1)h \quad , \quad h = \frac{b-a}{N}$$

$$\rightarrow \quad L' = -R \int_0^{2\pi} \overbrace{(q_\infty - 4\sin^2\theta\, q_\infty + P_\infty)\sin\theta}^{f(\theta)} \, d\theta$$

$$L' \approx -R\, \frac{h}{3} \sum_{k=1}^{N/2} \left[f(\theta_{2k-1}) + 4 f(\theta_{2k}) + f(\theta_{2k+1}) \right]$$

$$\text{w/ } f(\theta) = (q_\infty - 4\sin^2\theta\, q_\infty + P_\infty) \sin\theta$$

$$L' : f(\theta)\, (C_p(\theta)\, q_\infty + P_\infty) \sin\theta$$

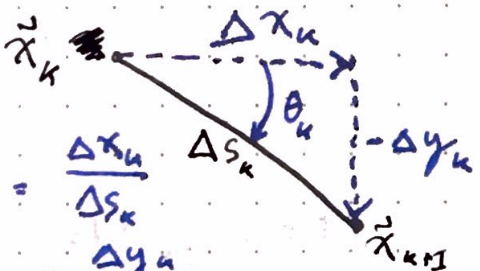
$$D' : f(\theta) = (C_p(\theta)\, q_\infty + P_\infty) \cos\theta$$

Lab 1 - Comp - Problem 2

$$N' = N_u' + N_l'$$

$$N_u' = -\int_{LE}^{TE} \left(P_u \cos\theta + \cancel{\tau_u s\theta}^{\,0}\right) dS_u$$

$$= -\sum_{k=1}^{N} |\vec{x}_{k+1} - \vec{x}_k| \left(\left[\frac{P_u(\vec{x}_{k+1}) + P_u(\vec{x}_k)}{2} \right] c\theta_k \right)$$



$$\cos\theta_k = \frac{\Delta x_k}{\Delta s_k}$$

$$\sin\theta_k = -\frac{\Delta y_k}{\Delta s_k}$$

$$\Delta s_k = |\vec{x}_{k+1} - \vec{x}_k|$$

$$\Delta x_k = x_{k+1} - x_k$$

$$\Delta y_k = y_{k+1} - y_k$$

$$N_u' \approx -\sum_{k=1}^{N} \Delta s_k \left(\frac{P_u(\vec{x}_{k+1}) + P_u(\vec{x}_k)}{2} \frac{\Delta x_k}{\Delta s_k} \right)$$

$$\approx -\sum_{k=1}^{N} \left(\frac{P_u(\vec{x}_{k+1}) + P_u(\vec{x}_k)}{2} \Delta x_k \right)$$

$$\therefore \quad N_l' \approx \sum_{k=1}^{N} \left(\frac{P_l(\vec{x}_{k+1}) + P_l(\vec{x}_k)}{2} \Delta x_k \right)$$

$$A' = A_u' + A_l' \Rightarrow A_u' = -\sum_{k=1}^{N} \left(\frac{P_u(\vec{x}_{k+1}) + P_u(\vec{x}_k)}{2} \Delta y_k \right)$$

$$A_l' = \sum_{k=1}^{N} \left(\frac{P_l(\vec{x}_{k+1}) + P_l(\vec{x}_k)}{2} \Delta y_k \right)$$

$$C_p = \frac{P - P_\infty}{q_\infty}$$

$$\Rightarrow \quad C_p q_\infty + P_\infty = P$$