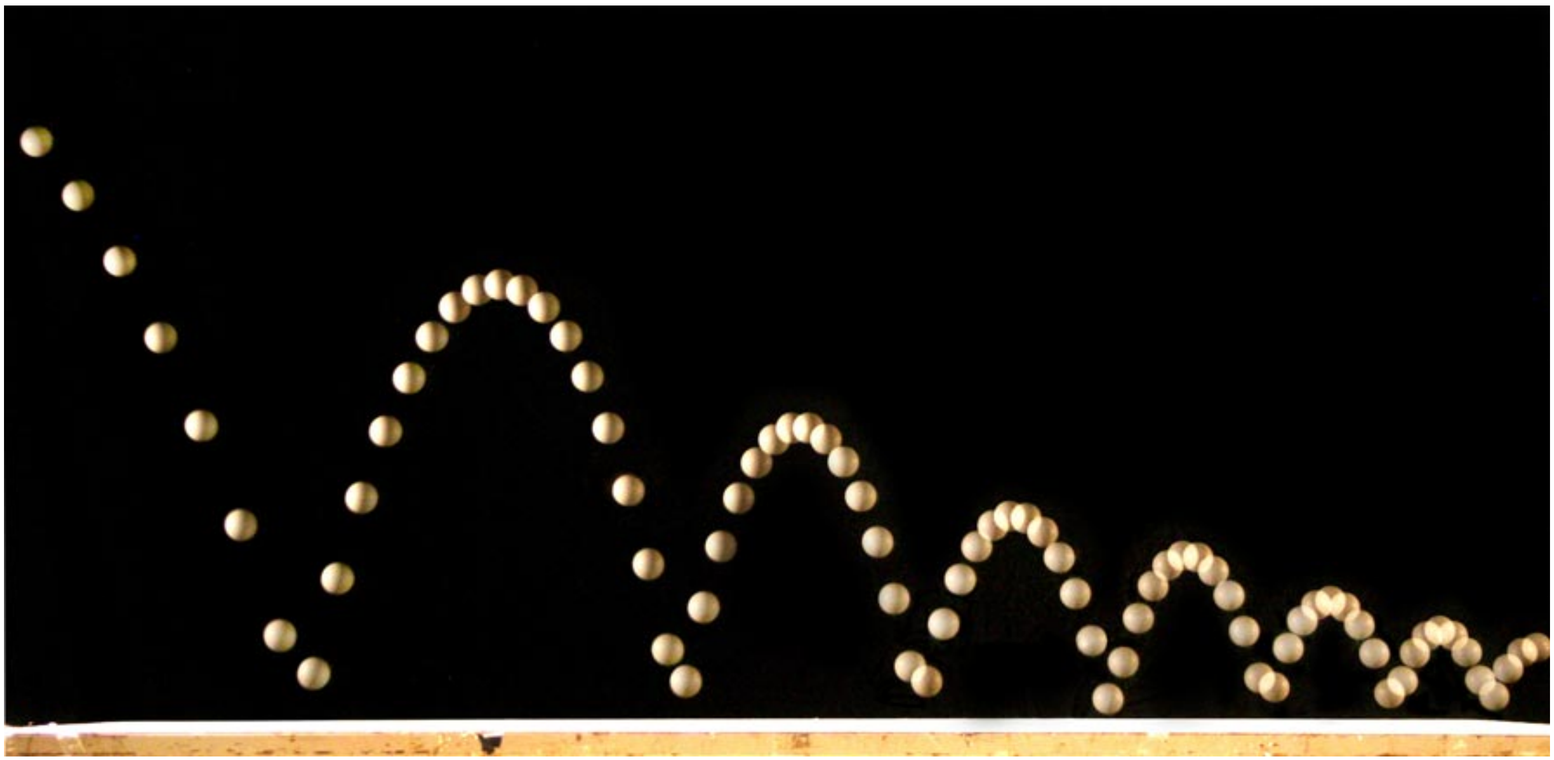
ASEN2003
Lab #2
Bouncing Ball Experiment



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ASEN2003 Lab #2 Objectives

- Gain physical understanding of direct central impact problems and coefficient of restitution.
- Derive expressions for coefficient of restitution.
- Perform experiments to explore variations of measured results.
- Assess experimental results.

ASEN2003 Lab #2 Problem Statement

When a ball that is not spinning is dropped vertically on a horizontal surface we model the dynamics as a direct central impact of a particle. If the ball is released from rest at an initial height h_0, the height of subsequent bounces and the total time the ball bounces may be used to estimate the coefficient of restitution of the ball and floor. In this lab assignment, your tasks are to derive the expressions for e as a function of ball heights, time between bounces, and time to stop bouncing, measure these values several times for different balls, compute and compare your derived estimates of e, and assess these results based upon experimental error.

ASEN2003 Lab #2 Theory

- A ball is released from rest at a height, h_0, above a horizontal surface. The height of the mass center of the ball after n subsequent bounces is termed h_n. The time required for a single bounce is t_n, and for the ball to stop bouncing is t_S (See Figure 1 below).

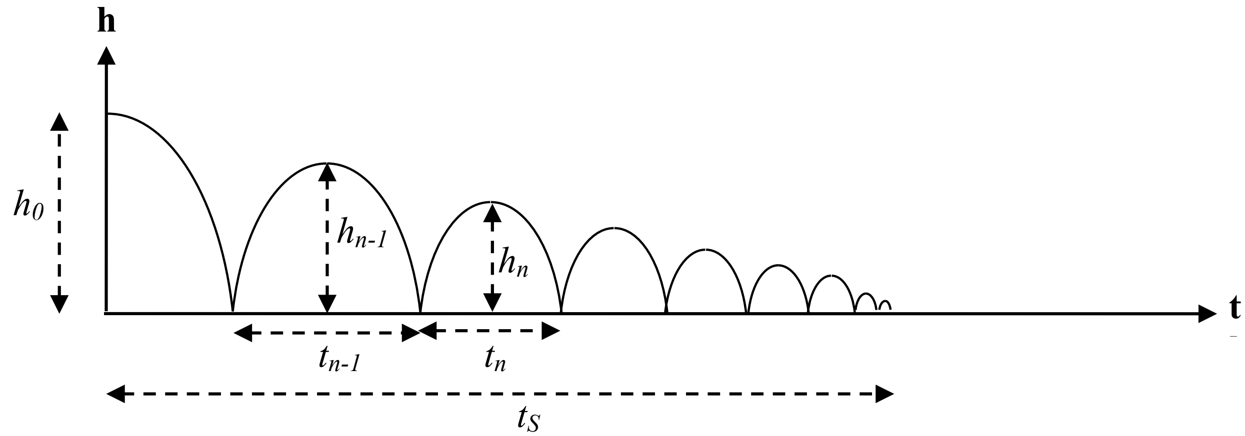

Figure 1. Ball height versus time for a sequence of bounces.

- For each experiment write up a clear and concise description of the experimental procedure you developed, including sketches as necessary. The description should be clear enough that other groups could perform your experiments from it. The description can take the form of numbered or bulleted list where each item clearly describes one step in the process. Describe how you came up with the experiment, justifying your choice of measuring devices and data collection methods. Also, describe important sources of error in your experiments.

ASEN2003 Lab #2 Data to be Collected

- Ten trials for method 1 with ping pong ball.
- Ten trials for method 2 with ping pong ball.
- Ten trials for method 3 with ping pong ball.
- Pick one of the methods, improve it, and collect 10 more trials with ping pong ball. Also collect 10 trials with a new ball (your choice).

ASEN2003 Lab #2 Methods

Starting with the principles of energy and momentum for direct central impacts, the coefficient of restitution can be estimated from three different methods. Derive the expressions given below for the coefficient of restitution for each method.

Method 1: Height of Bounce

Estimate of e_{height} from height of bounces:

$$e = \left(\frac{h_n}{h_{n-1}}\right)^{\frac{1}{2}} = \left(\frac{h_n}{h_0}\right)^{\frac{1}{2n}} \qquad [1]$$

Method 2: Time of Bounce

Estimate of e_{bounce} from time of 2 adjacent bounces

$$e = \left(\frac{t_n}{t_{n-1}}\right) \qquad [2]$$

Method 3: Time to Stop

Estimate of e_{stop} from time to stop bouncing:

$$e = \frac{t_s - \sqrt{\frac{2h_0}{g}}}{t_s + \sqrt{\frac{2h_0}{g}}} \qquad [3]$$

ASEN2003 Lab #2 Error Analysis

Error analysis is a significant component of any experimental study. What are the sources of experimental error in measuring the bounce heights and the stop time? For the experiments you designed, which have the greatest measurement error? Attempt to quantify the measurement error for each experiment. Error in the collected data creates error in the final estimate of the coefficient of restitution. Calculate the error you expect in the final estimate for the coefficient of restitution. Can you use this error analysis to suggest refinements to your experiment design?

For each method also calculate the sensitivity of the estimate to measurement error. We will be using propagation of errors, if you want to go back and read your notes from ASEN 2012. The sensitivity maps the errors in the measured quantities (i.e. the height or time) to errors in the final estimate of the coefficient of restitution. There is a handout on D2L that talks about error analysis

Error Analysis

$$\bar{x} = E[x_i] = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}$$

$$x = h(y_1, y_{2,} \ldots\ldots y_n)$$

e.g. coefficient of restitution

height, time, etc.

Measurements: $z_i = y_i + v_i$

measurement errors

The quantity we want to derive from the measurements is:

$$\bar{x} = h(z_1, z_{2,} \ldots\ldots z_n)$$

We want to understand the error in $\bar{x}$

Error Analysis

$$\tilde{x} = \bar{x} - x$$

estimated value of x   true value of x

Errors in y_i and their effect on x can be evaluated using a Taylor Series Expansion about the measurements z_i:

$$x = h(y_1, \ldots\ldots y_n) = h(z_1 - v_1, \ldots\ldots, z_n - v_n) =$$

$$\underbrace{h(z_1, \ldots\ldots z_n)}_{\bar{x}} - \frac{dh}{dy_1}\left(z_1, \ldots\ldots, z_n\right) \cdot v_1 - \ldots\ldots - \frac{dh}{dy_n}\left(z_1, \ldots\ldots, z_n\right) \cdot v_n + H.O.T.$$

Assume the errors are zero-mean, normally (Gaussian) distributed, and no cross-correlation:

$$E[v_i^2] = \sigma_i^2 \qquad E[v_i \cdot v_j] = 0 \quad i \neq j$$

Error Analysis

Then:

$$\sigma_x^2 = \sum_{i=1}^{n} \left(\frac{dh}{dy_i} \right)^2 \sigma_i^2$$

Measurement error

Determined from how the experiment was conducted and what the expected "noise" in the measurements are.

Can be computed from the statistics of v_i

How might you compute this?