ASEN 2003 Lab 3: Locomotive Crank Shaft

Group 48:
02.28.2017
Luke Tafur*
Nicholas Renninger†
Pierre Guillaud‡
University of Colorado Boulder, Boulder, CO, 80309, U.S.

The purpose of this lab is to gain an understanding of linear to rotational planar motion by evaluating a locomotive crank shaft. The group tested different voltages of the motor to see the effect of increased rotational velocity on the linear motion. The team discovered that as linear velocity was increased, the crankshaft was easier to more around the disc and didn't appear to stall visually. The model was relatively similar to the experimental data taken. The discrepancies between the model and experimental data were calculated and they appear to fit a Gaussian distribution, finding a mean residual value of $9.08E-4$ for $7V$, $-2.17E-4$ for $9V$, and $-2.61E-3$ for $11V$.

Nomenclature

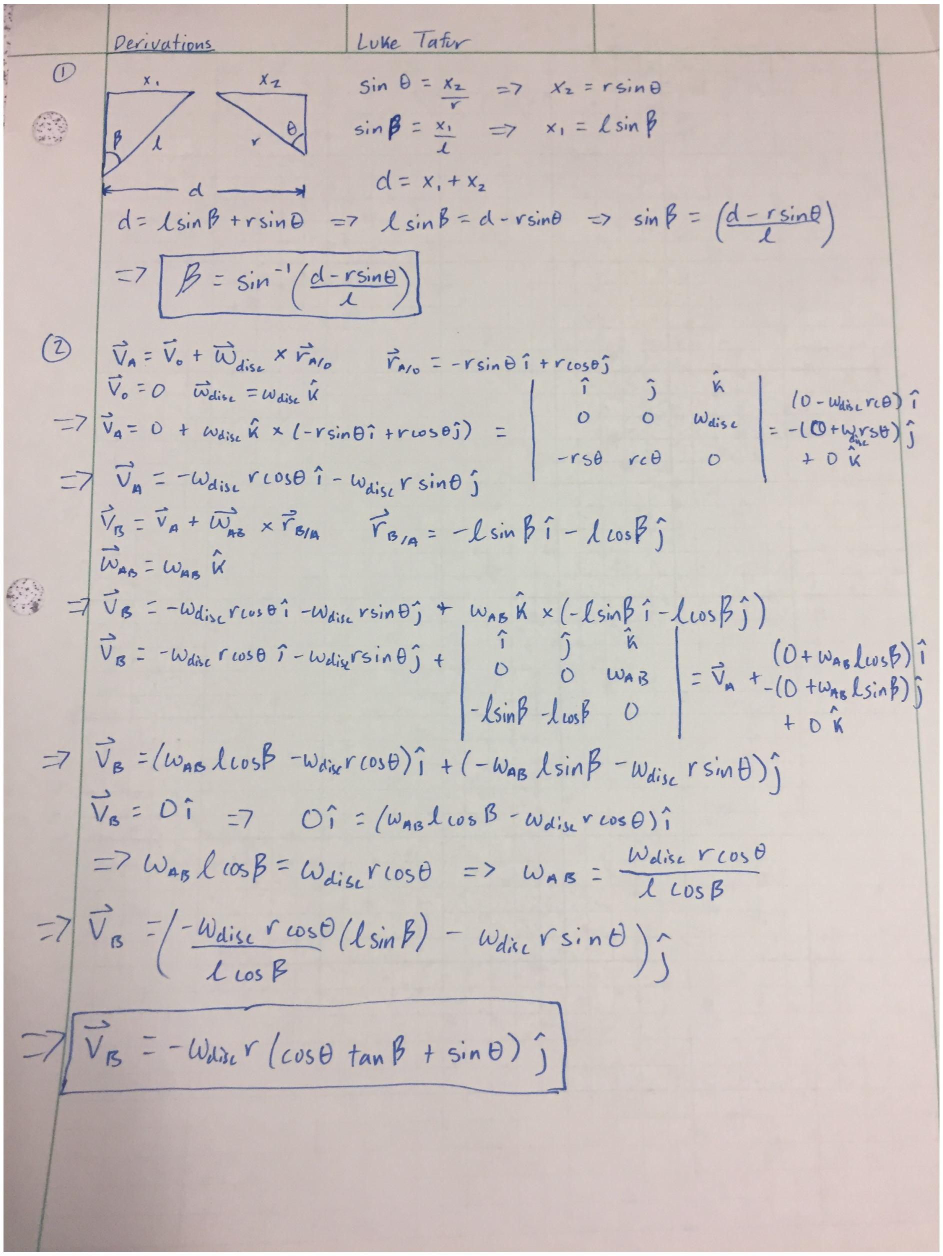
β	= Angular displacement of the connecting bar and the vertical shaft [°]
ω_{AB}	= Angular velocity of connecting bar from point A to point B [°]
ω_{Disc}	= Angular velocity of the disc [°]
θ	= Angular displacement of the connecting bar and y axis [°]
d	= Horizontal distance from vertical shaft and center of the disc [m]
l	= Length of connecting bar from point A to point B [m]
r	= Distance from point O to point A [m]

*104923506
†105492876
‡104426060

Contents

List of Figures

I. Model



(1)

$\sin\theta = \frac{x_2}{r} \Rightarrow x_2 = r\sin\theta$

$\sin\beta = \frac{x_1}{l} \Rightarrow x_1 = l\sin\beta$

$d = x_1 + x_2$

$d = l\sin\beta + r\sin\theta \Rightarrow l\sin\beta = d - r\sin\theta \Rightarrow \sin\beta = \left(\frac{d - r\sin\theta}{l}\right)$

$\Rightarrow \boxed{\beta = \sin^{-1}\left(\frac{d - r\sin\theta}{l}\right)}$

(2)

$\vec{V}_A = \vec{V}_o + \vec{\omega}_{disc} \times \vec{r}_{A/O} \qquad \vec{r}_{A/O} = -r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j}$

$\vec{V}_o = 0 \qquad \vec{\omega}_{disc} = \omega_{disc}\hat{k}$

$\Rightarrow \vec{V}_A = 0 + \omega_{disc}\hat{k} \times (-r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega_{disc} \\ -rs\theta & rc\theta & 0 \end{vmatrix} = (0 - \omega_{disc} rc\theta)\,\hat{i} - (0 + \omega_{disc} rs\theta)\,\hat{j} + 0\,\hat{k}$

$\Rightarrow \vec{V}_A = -\omega_{disc} r\cos\theta\,\hat{i} - \omega_{disc} r\sin\theta\,\hat{j}$

$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} \qquad \vec{r}_{B/A} = -l\sin\beta\,\hat{i} - l\cos\beta\,\hat{j}$

$\vec{\omega}_{AB} = \omega_{AB}\hat{k}$

$\Rightarrow \vec{V}_B = -\omega_{disc} r\cos\theta\,\hat{i} - \omega_{disc} r\sin\theta\,\hat{j} + \omega_{AB}\hat{k} \times (-l\sin\beta\,\hat{i} - l\cos\beta\,\hat{j})$

$\vec{V}_B = -\omega_{disc} r\cos\theta\,\hat{i} - \omega_{disc} r\sin\theta\,\hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega_{AB} \\ -l\sin\beta & -l\cos\beta & 0 \end{vmatrix} = \vec{V}_A + (0 + \omega_{AB} l\cos\beta)\,\hat{i} - (0 + \omega_{AB} l\sin\beta)\,\hat{j} + 0\,\hat{k}$

$\Rightarrow \vec{V}_B = (\omega_{AB} l\cos\beta - \omega_{disc} r\cos\theta)\,\hat{i} + (-\omega_{AB} l\sin\beta - \omega_{disc} r\sin\theta)\,\hat{j}$

$\vec{V}_B = 0\,\hat{i} \Rightarrow 0\,\hat{i} = (\omega_{AB} l\cos\beta - \omega_{disc} r\cos\theta)\,\hat{i}$

$\Rightarrow \omega_{AB} l\cos\beta = \omega_{disc} r\cos\theta \Rightarrow \omega_{AB} = \frac{\omega_{disc} r\cos\theta}{l\cos\beta}$

$\Rightarrow \vec{V}_B = \left(-\frac{\omega_{disc} r\cos\theta}{l\cos\beta}(l\sin\beta) - \omega_{disc} r\sin\theta\right)\hat{j}$

$\Rightarrow \boxed{\vec{V}_B = -\omega_{disc} r(\cos\theta\tan\beta + \sin\theta)\,\hat{j}}$

Figure 1. Derivations

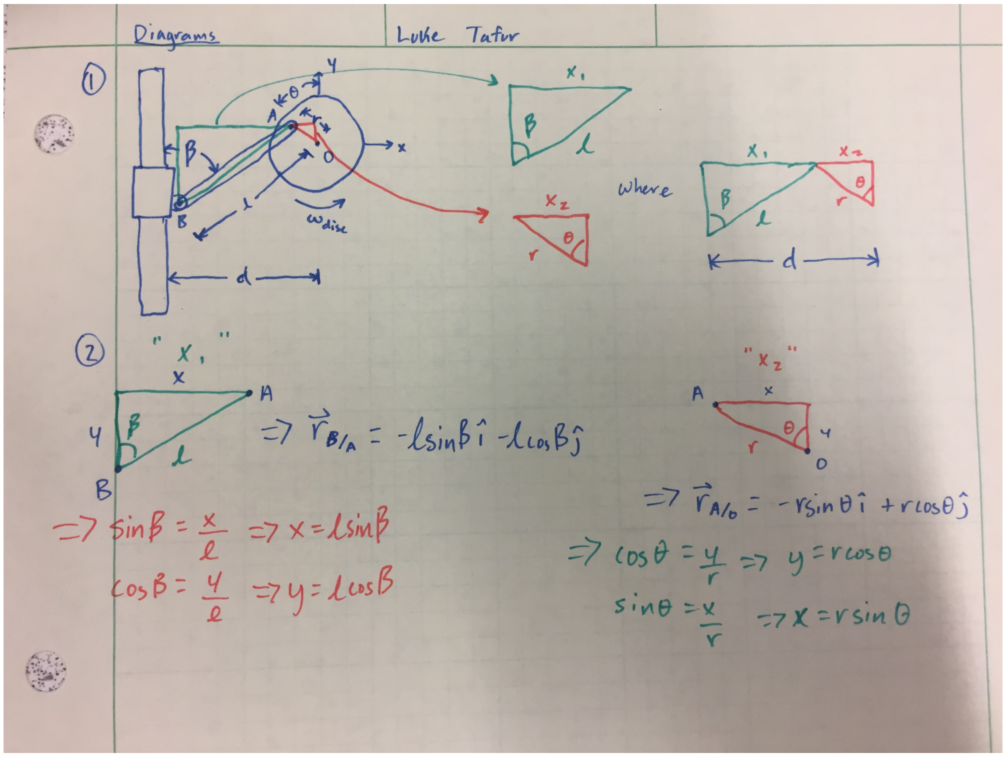


Figure 2. Supporting Diagrams for the Derivations.

II. Experiment

The lab was set up using a vertical shaft and disc that are connected using a rod. The vertical shaft was connected to a vertical rod that came from a motor giving it linear motion. The linear motion of moving the vertical shaft caused the disc to spin and the bar to move around.

The measurements acquired from the set up experiment were as follows: $7.7 \pm .1$ [cm] for r, $25.4 \pm .1$ [cm] for l and $15.3 \pm .1$ [cm] for d. The voltages that the data was measured at was 7, 9, and 11 volts. The lowest position of the collar was 74 ± 1 [mm] and the highest position was 274 ± 1 [mm].

At lower speeds, qualitatively the crank shaft appears to stall and slow down whenever the shaft needs to be brought upwards vertically. As the speed increases, the crank shaft appears to have an easier time moving and does not seem to stall.

The steps for performing this experiment were as follows[1]

1. Ensure crank shaft is set up properly
2. Make initial measurements
3. Turn on voltage box and run LabVIEW^{TM} Virtual Instrument
4. Set voltage box to desired voltage
5. Record data using LabVIEW^{TM} Virtual Instrument
6. Repeat steps IV and V until all desired voltages are tested
7. Evaluate data in post-processing

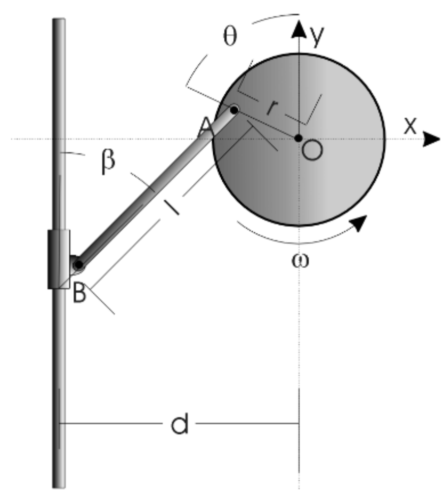


Figure 3. Sketch of lab setup

III. Results and Analysis

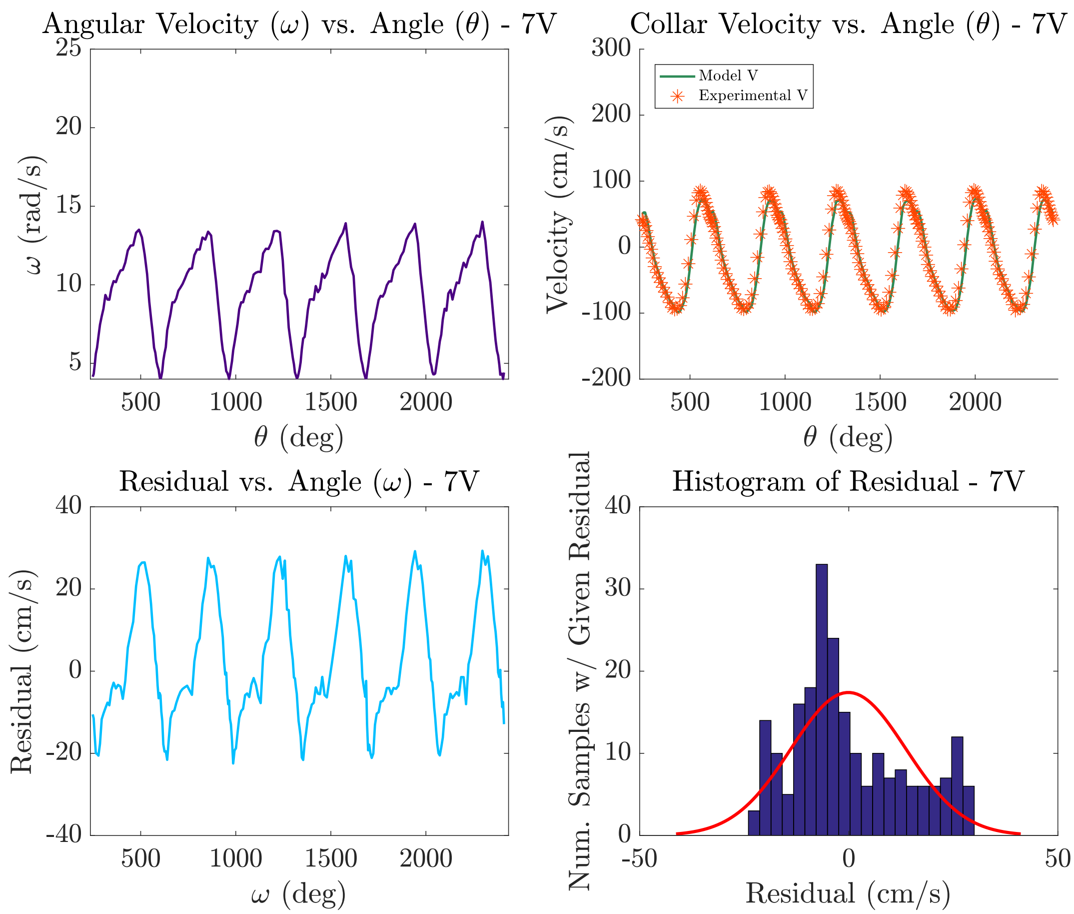


Figure 4. Data taken while running the motor at $7V$. *The residual here exhibits sinusoidal behavior, but the overall the model matches experimental data extremely well.*

```
7V-Statistics:

|  sigma  |  mean residual  | sigma/sqrt(N) |    N    | num. residuals > 3*sigma  |

|-------------------------------------------------------------------------------|

|000000.1 |    -0.000908    |   0000.009    |  0222   |          000              |
```

Figure 5. Calculated statistics during the $7V$ **trial.** *Due to the sinusoidal nature of the residual, the mean residual is very close to zero. This might lead to the false conclusion that the model was perfectly accurate, when in reality it was an artifact of statistics. None of the trials exceeded* 3σ*, which is a good sign for the accuracy of the model.*

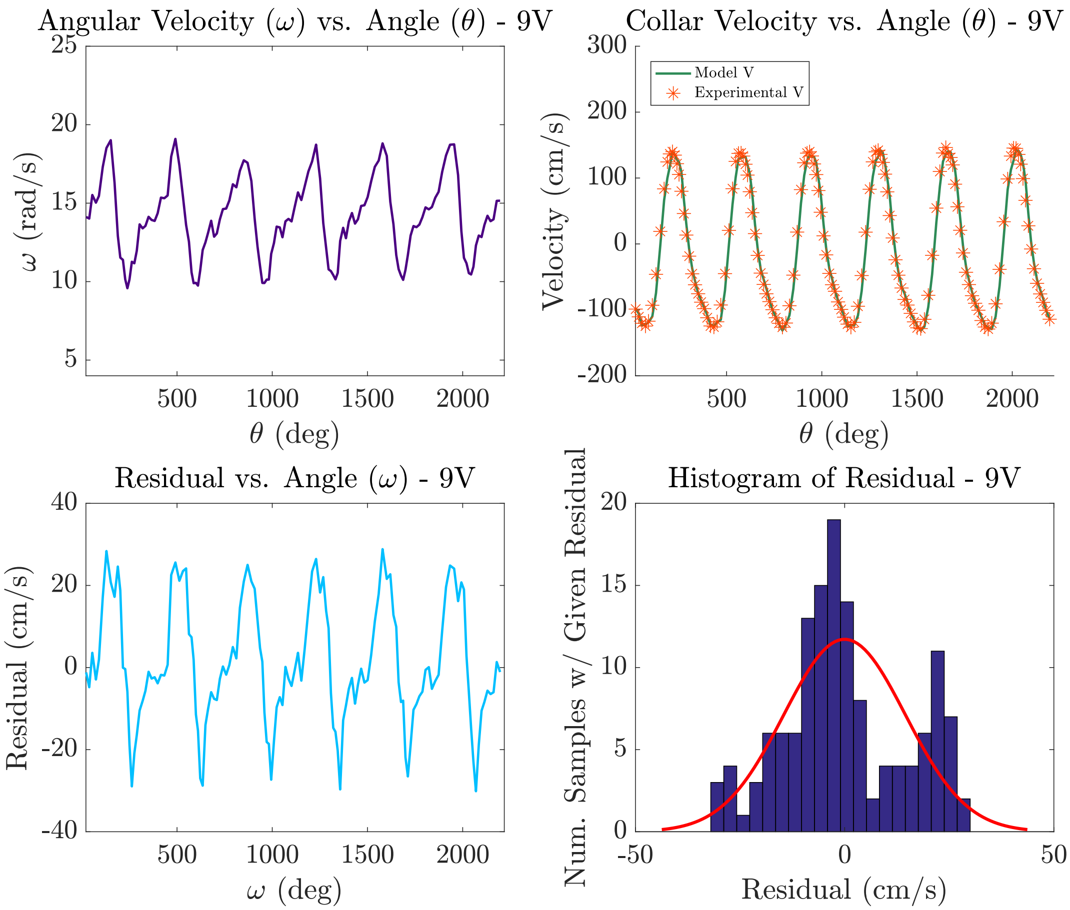


Figure 6. Data taken while running the motor at $9V$**.** *The residual here exhibits sinusoidal behavior, but the overall the model matches experimental data extremely well.*

```
9V-Statistics:

|  sigma  |  mean residual  | sigma/sqrt(N) |    N    | num. residuals > 3*sigma  |

|---------------------------------------------------------------------------------|

|000000.1 |    -0.000217    |   00000.01    |  0138   |           000             |
```

Figure 7. Calculated statistics during the $9V$ **trial.** *Due to the sinusoidal nature of the residual, the mean residual is very close to zero. This might lead to the false conclusion that the model was perfectly accurate, when in reality it was an artifact of statistics. None of the trials exceeded* 3σ*, which is a good sign for the accuracy of the model.*

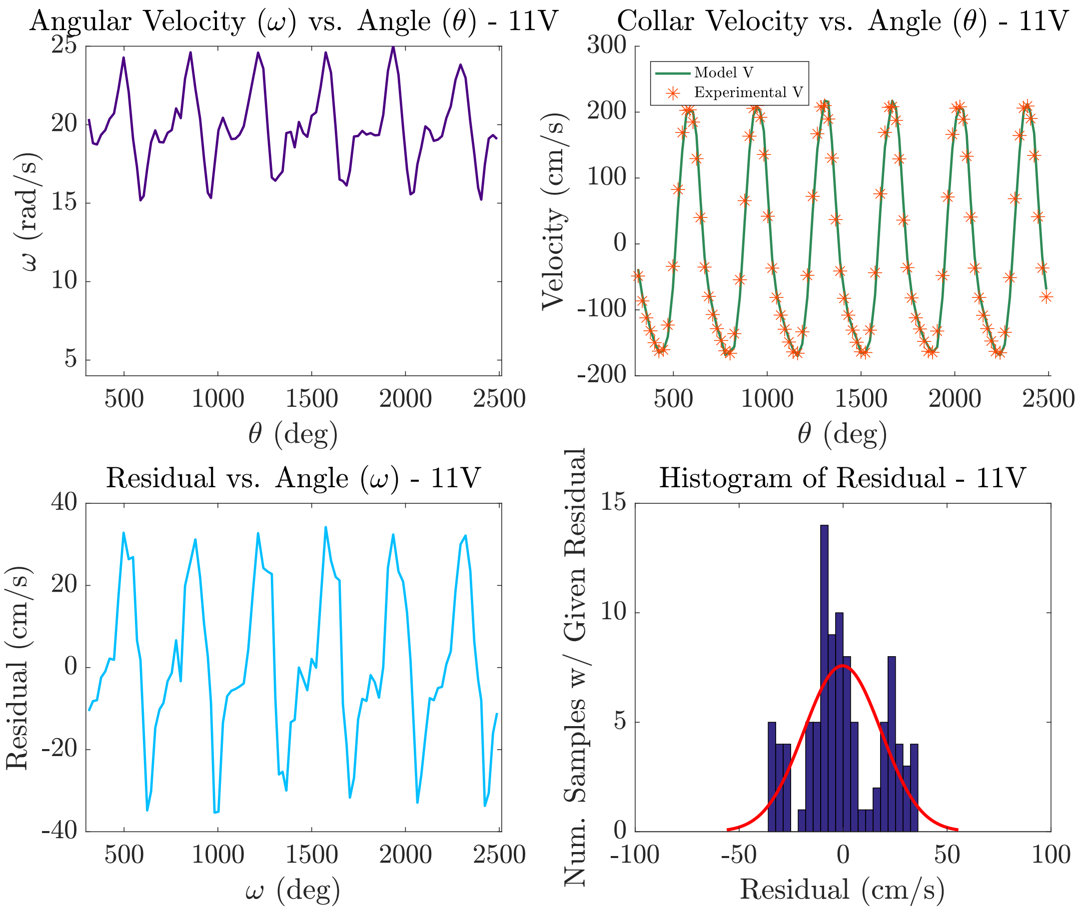


Figure 8. Data taken while running the motor at $11V$. *The residual here exhibits sinusoidal behavior, but the overall the model matches experimental data extremely well.*

```
11V-Statistics:

|  sigma   |  mean residual  | sigma/sqrt(N) |    N    | num. residuals > 3*sigma  |

|-------------------------------------------------------------------------------|

|000000.2 |    -00.00261    |    00000.02    |  0098   |           000             |
```

Figure 9. Calculated statistics during the $11V$ **trial.** *Due to the sinusoidal nature of the residual, the mean residual is very close to zero. This might lead to the false conclusion that the model was perfectly accurate, when in reality it was an artifact of statistics. None of the trials exceeded* 3σ*, which is a good sign for the accuracy of the model.*

The model matched the experimental results very well, as can be seen in Figs. 4, 6, & 8. The different possible natures of error are noise, gravity, data acquisition latency, frictional losses not accounted for by the model, and measurement uncertainties. With these errors in mind, our data still matched the model very well, with mean residual value of $9.08E-4$ for $7V$, $-2.17E-4$ for $9V$, and $-2.61E-3$ for $11V$. We also had zero residuals, across all trials, where the residual exceed three times its standard deviation, as can be seen in Figs. 5, 7, & 9. With no measured data points with this high of a residual, we can be very confident that our model accurately model the linear motion of the collar.

The fact that friction was excluded from the model, and the uncertainty in our measurements of the geometry of the system most likely caused the majority of the error. LabVIEW^{TM} also can create an uncertainty in the measurement of θ on account of the delay between the measurement query and time-stamping of the data. The final major source of error would be in the measurements, as all constants were measured by hand using a ruler with limited precision. Changing these values by less than 5% caused the mean residual and the standard deviation of the residual to increase by 100% in preliminary analysis.

IV. Conclusions and Recommendations

The team learned that linear motion can be translated into rotational motion and that the motion can be easily modeled. We also learned how much of an effect gravity can have on the actual experiment even though it's not accounted for in the model. Team learned what and how to use multiple functions in MATLAB and how to implement them properly.

The team believes that more could be learned if the experiment was done vertically as well as horizontally. Possibly seeing if allowing gravity to influence the system would make a measurable difference. The team would also like to increase the voltage to see how quickly the disc could rotate from the linear motion. Also, making the entire assembly out of wet ice would allow the effects of friction to be minimized.

Making the lab more related to aerospace (in the sense of using linear to rotational motion) in an actual aerospace application would improve the lab. For example could use the rotational motion of a motor to extend science experiments on a tray out into the vacuum space using gears on the motor and a bar with grooves that the gears could fit in. Could have the lab explaining how that would be possible and how much the hypothetical bars could hold of weight so that the number of science experiments to be tested could be determined. Also, designing and writing the LabVIEW^{TM} driver would give us better control over the latency and would make us learn how to use DAQ software and LabVIEW^{TM}-specific program architecture.

References

[1] Axelrad, P., "ASEN 2003_L3_LocomotiveCrankshaft_2017," Desire2Learn Available: https://learn.colorado.edu/d2l/le/content/193590/viewContent/2871036/View

Acknowledgments

Shout out to Felix for being there to help us out when we needed it and answering all our questions.

A. Code

A.A. lab3.m

```
    %%% lab3.m
    %%%
    %%% This script is the main function of this lab. It defines the
    %%% constants and finds the filenames of the files where the data sets
    %%% desired will be found. It then calls in the analyze.m function
    %%% which will calculate and plot everything.
    %%%
    %%% Authors: Pierre Guillaud, Nicholas Renninger, Luke Tafur
    %%% Date Created: 2/16/17
    %%% Last Modified: 2/28/17

    %% HOUSEKEEPING
    close all
    clearvars
    clc

    %% CONSTANTS
    L = 0.254; % m
    R = 0.077; % m
    D = 0.153; % m

    %% Create Directory of Testing Data
    dir_name = '../Data/Actual Data/';
    dir_data = dir(dir_name);

    %% Data Analysis, Plotting, and Statistics
    for i = 1:length(dir_data)

        % generate the filename of data to be loaded and analyzed
        filename = [dir_name, dir_data(i).name];

        if strfind(filename, 'Test') % if data is test data

            volt = str2double(dir_data(i).name(7:8));
            analyze(L, R, D, filename, volt);
        end
    end
```

A.B. analysis.m

```
function analyze(L, R, D, filename, volt)

    %%% analyze(L, R, D, filename, volt)
    %%%
    %%% This function takes multiple inputs, including constants and the
    %%% name of the file to study. This function then calls in three
    %%% functions: load_lcs.m, LCSMODEL.m, and calcStatistics.m, that loads
    %%% in the data from the desired file, calculates the analytical data,
    %%% and then finds the statistics of the error. This function also
    %%% creates plots for each data set.
    %%%
```

```
%%%
%%% Inputs:
%%%         - R: distance between the origin (rotation axis) and the
%%%              attachment point A. [m]
%%%
%%%         - D: horizontal distance between the vertical shaft and the
%%%              center of the disk. [m]
%%%
%%%         - L: length of the connecting bar from A to B. [m]
%%%
%%%         - filename: name of the file to load the data from
%%%
%%%         - volt: voltage of the data set studied
%%%
%%% Authors: Pierre Guillaud, Nicholas Renninger
%%% Date Created: 2/16/17
%%% Last Modified: 2/28/17

%% DATA LOADING
% Loads in the data from the desired file
[theta_exp, w_exp, v_exp] = load_lcs(filename);


%% CALCULATE
% Calls in the LCSMODEL.m function to find an analytical
v_model = LCSMODEL(R, D, L, theta_exp, w_exp);


%% ERROR
% Calculate the difference between the experimental and analytical
% data
residuals = v_exp - v_model;


%% Plots
% Puts in subplots:
% 1. The angular velocity vs angle,
% 2. the linear velocity vs angle
% 3. The Residual vs angle,
% 4. The histogram of the Residual plotted previously

%%%%%%%%%%%%%%%%%%%%%%%% Plot Setup %%%%%%%%%%%%%%%%%%%%%%%%
set(0, 'defaulttextinterpreter', 'latex');
titleString = sprintf('Analysis-%dV', volt);
saveLocation = '../Figures/';
saveTitle = cat(2, saveLocation, sprintf('%s.pdf', titleString));
LINEWIDTH = 1.5;
FONTSIZE = 17;
colorVecs = [0.294118 0 0.509804; % indigo
             0.180392 0.545098 0.341176; % sea green
             1 0.270588 0; % orange red
             0 0.74902 1]; % deep sky blue

hFig = figure('name', titleString);
scrz = get(groot, 'ScreenSize');
```

```
    set(hFig, 'Position', scrz)


    %%%%%%%%%%%%%%%%%%%%%%%% Plot Each Data Set %%%%%%%%%%%%%%%%%%%%%%%%

    % Subplot (1)
    subplot(2,2,1)

    x_data = theta_exp;
    xmin = min(x_data) * 0.95;
    xmax = max(x_data) * 1.01;

    y_data = w_exp;
    ymin = 4;
    ymax = 25;

    plot(x_data, y_data, 'linewidth', LINEWIDTH, ...
                          'Color', colorVecs(1, :));

    xlim([xmin, xmax])
    ylim([ymin, ymax])
    title(sprintf(['Angular Velocity ($\\omega$) vs. ', ...
                   'Angle ($\\theta$) - %dV'], volt))
    xlabel('$\theta$ (deg)')
    ylabel('$\omega$ (rad/s)')
    set(gca,'FontSize', FONTSIZE)
    set(gca, 'defaulttextinterpreter', 'latex')
    set(gca, 'TickLabelInterpreter', 'latex')


    % Subplot (2)
    subplot(2,2,2)

    x_data = theta_exp;
    xmin = min(x_data) * 0.95;
    xmax = max(x_data) * 1.01;

    y1_data = v_model;
    y2_data = v_exp;
    ymin = -2;
    ymax = 3;

    hold on

    plot(x_data, y1_data, 'linewidth', LINEWIDTH, ...
                             'Color', colorVecs(2, :));
    plot(x_data, y2_data, '*', 'MarkerSize', LINEWIDTH + 7, ...
                               'Color', colorVecs(3, :));

    xlim([xmin, xmax])
    ylim([ymin, ymax])
    title(sprintf('Collar Velocity vs. Angle ($\\theta$) - %dV', volt))
    xlabel('$\theta$ (deg)')
    ylabel('Velocity (m/s)')
    leg = legend({'Model V', 'Experimental V'}, 'location', ...
```

```matlab
                    'northwest', 'interpreter', 'latex');
    set(gca, 'FontSize', FONTSIZE)
    set(leg, 'FontSize', round(FONTSIZE * 0.6))
    set(gca, 'defaulttextinterpreter', 'latex')
    set(gca, 'TickLabelInterpreter', 'latex')


    % Subplot (3)
    subplot(2,2,3)
    x_data = theta_exp;
    xmin = min(x_data) * 0.95;
    xmax = max(x_data) * 1.01;

    ymin = -0.4;
    ymax = 0.4;

    plot(theta_exp,residuals, 'linewidth', LINEWIDTH, ...
                              'Color', colorVecs(4, :));

    xlim([xmin, xmax])
    ylim([ymin, ymax])
    title(sprintf('Residual vs. Angle ($\\omega$) - %dV', volt))
    xlabel('$\omega$ (deg)')
    ylabel('Residual (m/s)')
    set(gca,'FontSize', FONTSIZE)
    set(gca, 'defaulttextinterpreter', 'latex')
    set(gca, 'TickLabelInterpreter', 'latex')


    % Subplot (4)
    subplot(2,2,4)
    histfit(residuals, 20)
    title(sprintf('Histogram of Residual - %dV',volt))
    xlabel('Residual')
    ylabel('Num. Samples w/ Given Residual')
    set(gca,'FontSize', FONTSIZE)
    set(gca, 'defaulttextinterpreter', 'latex')
    set(gca, 'TickLabelInterpreter', 'latex')

    % setup and save figure as .pdf
    curr_fig = gcf;
    set(curr_fig, 'PaperOrientation', 'landscape');
    set(curr_fig, 'PaperUnits', 'normalized');
    set(curr_fig, 'PaperPosition', [0 0 1 1]);
    [fid, errmsg] = fopen(saveTitle, 'w+');
    if fid < 1 % check if file is already open.
       error('Error Opening File in fopen: \n%s', errmsg);
    end
    fclose(fid);
    print(gcf, '-dpdf', saveTitle);

    %% Calculate Statistics
    calcStatistics(residuals, volt)

end
```

A.C. load lcs.m

```matlab
function [theta_exp,w_exp,v_exp] = load_lcs(filename)

    %%% [theta_exp,w_exp,v_exp] = load_lcs(filename)
    %%%
    %%% This function stores the data from a file with an inputted name
       and
    %%% outputs the values required
    %%%
    %%% Inputs:
    %%%         - filename: name of the file to load the data from
    %%%
    %%% Outputs:
    %%%         - theta_exp: angular position of the disk [deg]
    %%%
    %%%         - w_exp: angular velocity of the disk [rad/s]
    %%%
    %%%         - v_exp: the vertical speed of the collar. [m/s]
    %%%
    %%% Author: Pierre Guillaud
    %%% Date Created: 2/16/17
    %%% Last Modified: 2/23/17

    % Loads data from file 'filename' using the path 'dir_name'.
    data = load(filename);

    % Sets initial angle to  and adjusts the rest.
    diff = mod(data(1,2),360);
    data(1:end,2) = data(1:end,2) - (data(1,2)-diff);

    % Finds all the data in the first 6 revolutions
    desired_data = find( data(1:end,2) < (6*360 + data(1, 2)) );
    index = desired_data(end)+1;

    % Defines the function outputs
    theta_exp = data(1:index, 2); % theta
    w_exp = data(1:index,4);     % deg/s
    w_exp = w_exp * pi / 180;   % converting to rad/s
    v_exp = data(1:index,5)*0.001;      % m/s

end
```

A.D. LCSMODEL.m

```matlab
function [v_model] = LCSMODEL(r, d, L, theta, w)

    %%% [v_mod] = LCSMODEL(r, d, l, theta, w)
    %%%
    %%% Computes the vertical speed of the collar based on the geometry of
    %%% the locamotive, its current angle theta, and the angular velocity
    %%% of the disk.
    %%%
    %%% Inputs:
    %%%         - r: distance between the origin (rotation axis) and the
    %%%              attachment point A. [m]
```

```
    %%%
    %%%        - d: horizontal distance between the vertical shaft and the
    %%%             center of the disk. [m]
    %%%
    %%%        - L: length of the connecting bar from A to B. [m]
    %%%
    %%%        - theta: angular position of the disk [deg]
    %%%
    %%%        - w: angular velocity of the disk [rad/s]
    %%%
    %%% Outputs:
    %%%         - v_model: the vertical speed of the collar. [m/s]
    %%%
    %%% Author: Nicholas Renninger
    %%% Date Created: 2/16/17
    %%% Last Modified: 2/20/17


    %% Implement Model in the most EXCELLENT way

    % Calculate the angle between the connecting rod to the vertical shaft
    beta = asind( (d - r .* sind(theta)) ./ L ); % [deg]

    % Calculate the linear velocity of the slider
    v_model = -w .* r .* (cosd(theta) .* tand(beta) + sind(theta)); % [m/s
       ]




end
```

A.E. calcStatistics.m

```
function calcStatistics(residuals, volt)

    %%% calcStatistics(residuals, volt)
    %%%
    %%% Calculates the statistics of the calculated residuals, and prints
    %%% the results in a table to the command window and to a file in the
    %%% following directory:
    %%%                    ../Data/Data Statistics/
    %%%
    %%% Inputs:
    %%%        - residuals: the vector of differences between experimental
    %%%                     results and the values predicted by the model.
    %%%
    %%%        - volt: which voltage input to the motor was being tested.
    %%%
    %%% Outputs:
    %%%         - writes statistics to command window and to an
    %%%         appropriately named file.
    %%%
    %%% Author: Nicholas Renninger
    %%% Date Created: 2/23/17
    %%% Last Modified: 2/28/17
```

```matlab


%% Calc Standard Deviation of the Residuals for Collar Velocity14
sigma = std(residuals);


%% Mean of the Residuals
meanResidual = mean(residuals);


%% Uncertainty of the Mean Residual
numResiduals = length(residuals);
sigmaMean = sigma / sqrt(numResiduals);


%% Number of Observations
numObservations = numResiduals;


%% Number of Residuals that are greater than 3*simga
num3SigmaResiduals = length(residuals(residuals > 3*sigma));


%% Print to File

% Creating File and Labeling what data is being Analyzed
testName = sprintf('%dV-Statistics', volt);
filename = sprintf('../Data/Data Statistics/%s.txt', testName);
fid = fopen(filename, 'w+');

%%%%%%%%%%%%%% print to file %%%%%%%%%%%%%

numBars = 82;

% header
fprintf(fid, '%s:\r\n\r\n', testName);
fprintf(fid, ['|  sigma   |  mean residual   | sigma/sqrt(N) |', ...
              '    N     | num. residuals > 3*sigma  |\r\n|']);

% print horiz bars
for i = 1:numBars
    fprintf(fid, '-');
end

% rest of data
fprintf(fid,['|\r\n|%08.1g |     %09.3g     |    %08.1g     |    %04d    ', ...
        '|            %03d             |\r\n'], sigma,...
        meanResidual, sigmaMean, numObservations, num3SigmaResiduals);

fclose(fid);


%%%%%%%%%%%%%% print to command window %%%%%%%%%%%%%
```

```
    numBars = 82;

    % header
    fprintf('\n%s:\n', testName);
    fprintf(['|  sigma   |  mean residual   | sigma/sqrt(N) |', ...
             '    N     | num. residuals > 3*sigma  |\n|']);

    % print horiz bars
    for i = 1:numBars
        fprintf('-');
    end

    % rest of data
    fprintf(['|\n|%08.1g |    %09.3g    |    %08.1g    |    %04d    ', ...
             '|           %03d            |\n'], sigma,...
             meanResidual, sigmaMean, numObservations, num3SigmaResiduals);



end
```