

You have a network with two layers of weights, \mathbf{W} and \mathbf{V} , corresponding to the first and second layers, respectively. The non-linear function through which the output of \mathbf{W} goes is the rectified linear function, $\max(0, x)$.

In pseudo-code, the forward pass of the network is as follows:

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z1 = W x
a1 = max(0, z1)
z2 = V a1
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The shapes and values of the weight matrices are:

$$\begin{aligned}\mathbf{W} &\in \mathbb{R}^{1 \times 1} & [0.20] \\ \mathbf{V} &\in \mathbb{R}^{1 \times 1} & [0.40]\end{aligned}$$

Given input $\mathbf{x} \in \mathbb{R}^{1 \times 1} = [0.10]$ and a target of 1, and mean squared error (LMS in our lectures) as the loss function,

- What are the values of z_1 , a_1 , z_2 ?

$$\begin{aligned}z_1 &= 0.2 \cdot 0.1 = 0.02 \\ a_1 &= \max(0, 0.02) = 0.02 \\ z_2 &= 0.4 \cdot 0.02 = 0.008\end{aligned}$$

- What is the value of the error?

$$\text{error} = l = (1 - z_2)^2 = (1 - 0.008)^2 = 0.992^2 = 0.984$$

- What are the gradients of \mathbf{W} and \mathbf{V} ? (The gradient of $\max(0, x)$ is 1 if $x > 0$, 0 otherwise).

$$\begin{aligned}\frac{dl}{dv} &= \frac{dl}{dz_2} \cdot \frac{dz_2}{dv} = -2(1 - z_2)a_1 = -2 \cdot 0.992 \cdot 0.02 = -0.0397 \\ \frac{dl}{da_1} &= \frac{dl}{dz_2} \cdot \frac{dz_2}{da_1} = -2(1 - z_2)v \\ \frac{dl}{dz_1} &= \frac{dl}{a_1} \cdot \frac{da_1}{dz_1} = -2(1 - z_2)v \cdot \mathbf{1}_{z_1 > 0} \\ \frac{dl}{dw} &= \frac{dl}{dz_1} \cdot \frac{dz_1}{dw} = -2(1 - z_2)v \cdot x = -2 \cdot 0.992 \cdot 0.4 \cdot 0.1 = -0.079\end{aligned}$$