3. $X=1$ head $X=0$ otherwise

$P_{XY}(x,y) = P(X=x, Y=y \mid A)P(A) +$
$P(X=x, Y=y \mid A^c)\,P(A^c)$ $P_{\frac{2}{3}}(x)P_{\frac{1}{2}}(y)(\frac{1}{2})$
$= P_{\frac{1}{3}}(x)P_{\frac{2}{3}}(y)(\frac{1}{2}) +$

$Y \to$ Bernoulli

$P_{XY}(x,y) = (\frac{1}{2})^x(\frac{2}{3})^y(\frac{1}{3})^{1-y} + (\frac{1}{2})^x(\frac{2}{3})^x(\frac{1}{3})^{1-x}$

	$Y=0$	$Y=1$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$
$X=1$	$\frac{1}{4}$	$\frac{1}{3}$

$P(X=0) = \frac{5}{12}$ $P(Y=0) = \frac{5}{12}$

$P(X=0)\cdot P(Y=0) = \frac{25}{144} \neq P(X=0, Y=0) \Rightarrow \frac{1}{6}$

X independent

15. $N \sim \text{Poisson}(\beta)$ $P(N=k) = \dfrac{e^{-\beta}\beta^k}{k!}$

$E(N) = \beta$ $VAR(N) = \beta$

$X_i \sim \text{Exponential}(\lambda)$

$E(X) = \frac{1}{\lambda}$ $VAR(X) = \frac{1}{\lambda^2}$ $E(X^2) = \frac{2}{\lambda^2}$

$E(Y) = E(N)\cdot E(X) = \dfrac{\beta}{\lambda}$

$VAR(Y) = E(Y^2) - E[Y]^2 = E(Y^2) - \dfrac{\beta^2}{\lambda^2}$

$E(Y^2) = \sum_{n=0}^{\infty}\left[\left(\sum_{i=1}^{n} X_i\right)^2\right]P_N(n)$

$= \sum_{n=0}^{\infty}\left[\dfrac{n^2+n}{\lambda^2}\right](P_N(n)) = \dfrac{e^{-\beta}\beta^n}{n!}$

$= \dfrac{\beta^2+2\beta}{\lambda^2}$

$VAR(Y) = \dfrac{2\beta}{\lambda^2}$