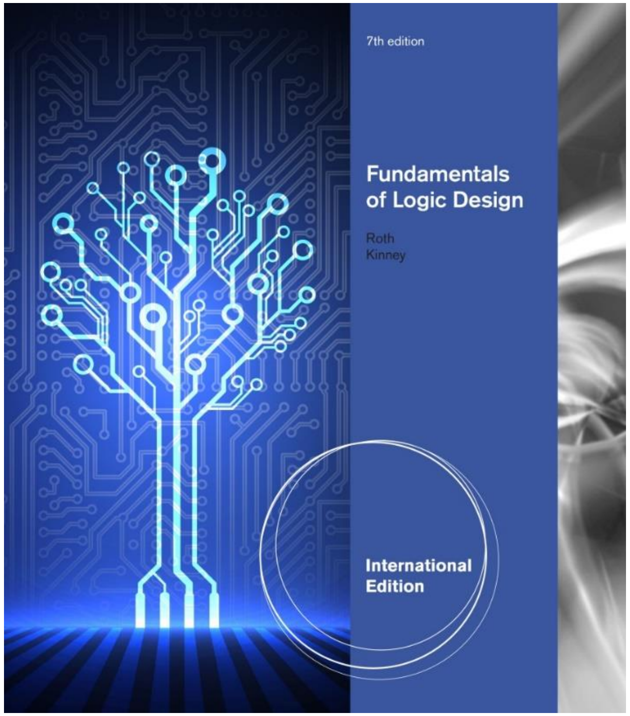

Quine-McCluskey Method

Quine and McCluskey

Willard Van Orman Quine

- Received his B.A in mathematics from Oberlin College in 1930, and his Ph.D. in Philosophy from Harvard University in 1932.
- He was affiliated with Harvard University, first as a student, then as a professor of Philosophy and a teacher of logic and set theory.
- He was an American Philosopher and logician in the analytic tradition.

Willard Van Orman Quine (1908~2000)

Edward J. McCluskey

- Graduated Bowdoin College in Maine in 1953, earning honors in mathematics and physics, then went on to study electrical engineering at MIT, where he earned his doctorate in 1956.
- Worked on electronic switching systems at the Bell Telephone Laboratories from 1955 to 1959.
- In 1959, he moved to Princeton University, where he was Professor of Electrical Engineering.
- n 1966, he joined Stanford University, where he was Emeritus Professor of Electrical Engineering and Computer Science.
- Developed the first algorithm for designing combinational circuits – the Quine-McCluskey logic minimization procedures as a doctoral student at MIT in 1956.

Edward J. McCluskey (1929~2016)

Review on minimum sum-of-products (SOP) expression

- Mission: Find minimum sum-of-products expression for a function *F*

TABLE 4-1 Minterms and Maxterms for Three Variables

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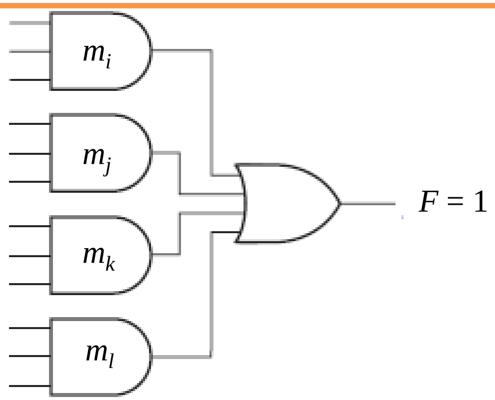
Row No.	A B C	Minterms
0	0 0 0	$A'B'C' = m_0$
1	0 0 1	$A'B'C = m_1$
2	0 1 0	$A'BC' = m_2$
3	0 1 1	$A'BC = m_3$
4	1 0 0	$AB'C' = m_4$
5	1 0 1	$AB'C = m_5$
6	1 1 0	$ABC' = m_6$
7	1 1 1	$ABC = m_7$

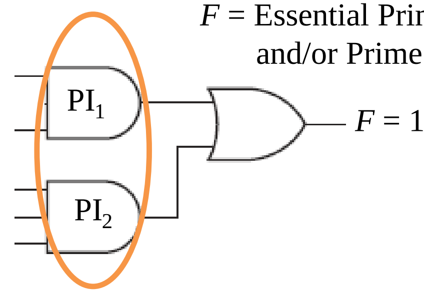
To find minimum sum of products expression:
1. Algebraic simplification using Boolean algebra
2. Karnaugh Maps (1954) 3~5 variables
3. Quine-McCluskey Method(1956) up to 15 or more
   - ✓ Find all of the prime implicants
4. Petrick's Method (1956)
   - ✓ Determining all minimum SOP solutions

Other factors to be considered:
1. Maximum number of inputs for a gate (fan-in)
2. Maximum number of output for a gate (fan-out)
3. Propagation delay
4. The number of interconnections in the circuit
5. Circuit layout on a PCB (Printed Circuit Board)



Minterm Expansion
- Two level
- 4 AND gates and 1 OR gate
- 16 inputs

F = Essential Prime Implicants and/or Prime Implicants



Minimum SOP Expression
- Two level (propagation delay is same)
- 2 AND gates and 1 OR gate (minimum number of product terms or gates)
- 7 inputs (minimum number of literals)

Contents

Objectives

- Find the prime implicants of a function by using the Quine-McCluskey method. Explain the reasons for the procedures used.
- Define *prime implicant and essential prime implicant.*
- Given the prime implicants, find the essential prime implicants and a minimum sum-of-products expression for a function, using a prime implicant chart and using Petrick's method.
- Minimize an incompletely specified function, using the Quine-McCluskey method.
- Find a minimum sum-of-products expression for a function, using the method of map-entered variables.

Introduction

Chapter Introduction:

- The Quine-McCluskey method presented in this unit provides a systematic simplification procedure which can be readily programmed for a digital computer.
- The Quine-McCluskey method reduces the minterm expansion (standard sum-of-products form) of a function to obtain a minimum sum of products.

- The Quine-McCluskey procedure consists of two main steps:
  1. Eliminate as many literals as possible from each term by systematically applying the theorem $XY + XY' = X$. *The resulting terms are called prime implicants*.
  2. Use a prime implicant chart to select a minimum set of prime implicants which, when ORed together, are equal to the function being simplified and which contain a minimum number of literals.

Definition of Prime Implicants

Forming Prime Implicants:

- In the first part of the Quine-McCluskey method, all of the prime implicants of a function are systematically formed by combining minterms.
- The minterms are represented in binary notation and combined using $XY+XY'=X$ where X represents a product of literals and Y is a single variable.
- Two minterms will combine if they differ in exactly one variable.

Determining Prime Implicants:

- To reduce the required number of comparisons, the binary minterms are sorted into groups according to the number of 1's in each term. Thus,

$$f(a, b, c, d) = \Sigma\, m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14) \qquad (6\text{-}2)$$

is represented by the following list of minterms:

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Definition of Prime Implicants

Determination of Prime Implicants (Column I):

- The term in group 0 has zero 1's, the terms in group 1 have one 1, the terms in group 2 have two 1's, and the terms in group 3 have three 1's.
- Two terms can be combined of they differ in exactly one variable ($XY + XY' = X$). Only terms in adjacent groups must be compared. (group 0 and group 1, group 1 and group 2, group 2 and group 3 in Column I)
  - ✓ Comparison of terms in nonadjacent groups is unnecessary because such terms will always differ in at least two variables.
  - ✓ Comparison of terms within a group is unnecessary because two terms with the same number of 1's must differ in at least two variables.

TABLE 6-1
Determination of Prime Implicants

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Group	Column I			Column II			Column III	
group 0	0	0000	✓	0, 1	000–	✓	0, 1, 8, 9	–00–
group 1	1	0001	✓	0, 2	00–0	✓	0, 2, 8, 10	–0–0
	2	0010	✓	0, 8	–000	✓	~~0, 8, 1, 9~~	~~–00–~~
	8	1000	✓	1, 5	0–01		~~0, 8, 2, 10~~	~~–0–0~~
group 2	5	0101	✓	1, 9	–001	✓	2, 6, 10, 14	-- 10
	6	0110	✓	2, 6	0–10	✓	~~2, 10, 6, 14~~	~~-- 10~~
	9	1001	✓	2, 10	–010	✓		
	10	1010	✓	8, 9	100–	✓		
group 3	7	0111	✓	8, 10	10–0	✓		
	14	1110	✓	5, 7	01–1			
				6, 7	011–			
				6, 14	–110	✓		
				10, 14	1–10	✓		

Definition of Prime Implicants

Determination of Prime Implicants (Column II):

- Whenever two terms combine, the corresponding decimal numbers differ by a power of 2 (1, 2, 4, 8, etc.).
- The terms in Column II have been divided into groups, according to the number of 1's in each term
- Apply $XY + XY' = X$ to combine pairs of terms in Column II.
  - ✓ It is necessary only to compare terms which have dashes (missing variables) in corresponding places and which differ by exactly one in the number of 1's
  - ✓ Terms in the first group in Column II need only be compared with terms in the second group which have dashes in the same places. Then, compare terms from the second and third groups.

TABLE 6-1
Determination of Prime Implicants

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	Column I	Column II	Column III
group 0	0 0000 ✓	0, 1 000– ✓	0, 1, 8, 9 –00–
group 1	1 0001 ✓	0, 2 00–0 ✓	0, 2, 8, 10 –0–0
	2 0010 ✓	0, 8 –000 ✓	~~0, 8, 1, 9 –00–~~
	8 1000 ✓	1, 5 0–01	~~0, 8, 2, 10 –0–0~~
group 2	5 0101 ✓	1, 9 –001 ✓	2, 6, 10, 14 --10
	6 0110 ✓	2, 6 0–10 ✓	~~2, 10, 6, 14 --10~~
	9 1001 ✓	2, 10 –010 ✓	
	10 1010 ✓	8, 9 100– ✓	
group 3	7 0111 ✓	8, 10 10–0 ✓	
	14 1110 ✓	5, 7 01–1	
		6, 7 011–	
		6, 14 –110 ✓	
		10, 14 1–10 ✓	

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Definition of Prime Implicants

Determination of Prime Implicants (Column III):

- Delete three pairs of duplicate terms in Column III.
- Because no further combination is possible, the process terminates.
- The terms which have not been checked off because they cannot be combined with other terms are called *prime implicants*. Because every minterm has been included in at least one of the prime implicants, the function is equal to the sum of its prime implicants.

$$f = a'c'a' + a'bd + a'bc + b'c' + b'd' + cd'$$

$$(1,5) \quad (5,7) \quad (6,7) \quad (0,1,8,9) \quad (0,2,8,10) \quad (2,6,10,14)$$

- In this expression, each term has a minimum number of literals, but the number of terms is not minimum. Using the consensus theorem to eliminate redundant terms yields

$$f = a'bd+b'c'+cd'$$

which is the minimum sum-of-products expression for *f*.

TABLE 6-1 Determination of Prime Implicants

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	Column I		Column II			Column III	
group 0	0 0000	✓	0, 1	000–	✓	0, 1, 8, 9	–00–
group 1	1 0001	✓	0, 2	00–0	✓	0, 2, 8, 10	–0–0
	2 0010	✓	0, 8	–000	✓	~~0, 8, 1, 9~~	~~–00–~~
	8 1000	✓	1, 5	0–01		~~0, 8, 2, 10~~	~~–0–0~~
group 2	5 0101	✓	1, 9	–001	✓	2, 6, 10, 14	-- 10
	6 0110	✓	2, 6	0–10	✓	~~2, 10, 6, 14~~	~~-- 10~~
	9 1001	✓	2, 10	–010	✓		
	10 1010	✓	8, 9	100–	✓		
group 3	7 0111	✓	8, 10	10–0	✓		
	14 1110	✓	5, 7	01–1			
			6, 7	011–			
			6, 14	–110	✓		
			10, 14	1–10	✓		

Definition of Prime Implicants

Implicant and Prime Implicant:

Given a function F of n variables, a product term P is an *implicant* of F iff for every combination of values of the n variables for which $P = 1$, F is also equal to 1.

A *prime implicant* of a function F is a product term implicant which is no longer an implicant if any literal is deleted from it.

- The Quine-McCluskey method finds all of the product term implicants of a function. The implicants which are nonprime are checked off in the process of combining terms so that the remaining terms are *prime implicants*.

Definition of Prime Implicants

Definition of Implicant:

Given a function F of n variables, a product term P is an *implicant* of F iff for every combination of values of the n variables for which $P = 1$, F is also equal to 1.

- For example, consider the function

$$F(a, b, c) = a'b'c'+ab'c'+ab'c+abc = b'c'+ac = 1.$$

- If $a'b'c' = 1$, then $F = 1$; if $ac = 1$, then $F = 1$; etc. Hence, the terms $a'b'c'$, ac, etc. are implicants of F.
- In this example, bc is not an implicant of F because when $a = 0$ and $b = c = 1$, $bc = 1$ and $F = 0$.
- In general, if F is written in sum-of-products form, every product term is an implicant.
- Every minterm of F is also an implicant of F, and so is any term formed by combining two or more minterms.
- Thus, all implicants of the function F can be written as

$$a'b'c',\ ab'c',\ ab'c,\ abc,\ b'c',\ ab',\ ac$$

$bc \backslash a$	0	1	
00	1	1	
01		1	
11	0	1	bc
10			

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The Prime Implicant Chart

Prime Implicant Chart:

- The prime implicant chart can be used to select a minimum set of prime implicants.
- The minterms of the function are listed across the top of the chart, and the prime implicants are listed down the side.
- A prime implicant is equal to a sum of minterms, and the prime implicant is said to cover these minterms.
- If a prime implicant covers a given minterm, an *X* is placed at the intersection of the corresponding row and column.
- Table 6-2 shows the prime implicant chart derived from Table 6-1. All of the prime implicants (terms which have not been checked off in Table 6-1) are listed on the left.
- If a given column contains only one *X*, then the corresponding row is an *essential prime implicant.*

TABLE 6-2
Prime Implicant Chart

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	⊗		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	⊗
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

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Prime Implicant Chart:

- If a minterm is covered by only one prime implicant, then that prime implicant is called an *essential prime implicant* and must be included in the minimum sum of products.
- Each time a prime implicant is selected for inclusion in the minimum sum, the corresponding row should be crossed out. After doing this, the columns which correspond to all minterms covered by that prime implicant should also be crossed out.
- Table 6-3 shows the resulting chart when the essential prime implicants and the corresponding rows and columns of Table 6-2 are crossed out.
- The resulting minimum sum of products is expressed as

$$f = a'bd+b'c'+cd',$$

where $b'c'$, cd' are *essential prime implicants* and $a'bd$ is a *prime implicant*.

TABLE 6-3

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		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	X		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	X
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

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The Prime Implicant Chart

Example 1:

A prime implicant chart which has two or more X's in every column is called a *cyclic* prime implicant chart. The following function has such a chart:

$$F = \Sigma\, m(0, 1, 2, 5, 6, 7) \tag{6-6}$$

Derivation of prime implicants:

0	000 ✓		0, 1	00–
1	001 ✓		0, 2	0–0
2	010 ✓		1, 5	–01
5	101 ✓		2, 6	–10
6	110 ✓		5, 7	1–1
7	111 ✓		6, 7	11–

Table 6-4 shows the resulting prime implicant chart. All columns have two X's, so we will proceed by trial and error. Both (0, 1) and (0, 2) cover column 0, so we will try (0, 1). After crossing out row (0, 1) and columns 0 and 1, we examine column 2, which is covered by (0, 2) and (2, 6). The best choice is (2, 6) because it covers two of the remaining columns while (0, 2) covers only one of the remaining columns. After crossing out row (2, 6) and columns 2 and 6, we see that (5, 7) covers the remaining columns and completes the solution. Therefore, one solution is $F = a'b' + bc' + ac$.

TABLE 6-4

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			0	1	2	5	6	7
① →	(0, 1)	$a'b'$	X	X				
	(0, 2)	$a'c'$	X		X			
	(1, 5)	$b'c$		X		X		
② →	(2, 6)	bc'			X		X	
③ →	(5, 7)	ac				X		X
	(6, 7)	ab					X	X

The Prime Implicant Chart

Example 1 (Continued) :

However, we are not guaranteed that this solution is minimum. We must go back and solve the problem over again starting with the other prime implicant that covers column 0. The resulting table (Table 6-5) is

TABLE 6-5

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			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Finish the solution and show that $F = a'c' + b'c + ab$. Because this has the same number of terms and same number of literals as the expression for F derived in Table 6-4, there are two minimum sum-of-products solutions to this problem. Compare these two minimum solutions for Equation (6-6) with the solutions obtained in Figure 5-9 using Karnaugh maps. Note that each minterm on the map can be covered by two different loops. Similarly, each column of the prime implicant chart (Table 6-4) has two X's, indicating that each minterm can be covered by two different prime implicants.

Petrick's Method:

- S. R. Petrick. *A direct determination of the irredundant forms of a Boolean function from the set of prime implicants*. Technical Report AFCRC-TR-56-110, Air Force Cambridge Research Center, Cambridge, MA, April, 1956.
- A more systematic way of finding all minimum solutions from a prime implicant chart.
- Before applying Petrick's method, all essential prime implicants and the minterms they cover should be removed from the chart.
- We will illustrate Petrick's method using Table 6-5.
  - ✓ Label the rows of the table P_1, P_2, P_3, etc.
  - ✓ Form a logic function, P, which is true when all of the minterms in the chart have been covered.
  - ✓ In order to cover minterm 0, the expression P_1+P_2 must be true
  - ✓ In order to cover minterm 1, the expression P_1+P_3 must be true, etc.
  - ✓ Since we must cover all of the minterms, the following function must be true:

$$P = (P_1+P_2)\,(P_1+P_3)\,(P_2+P_4)\,(P_3+P_5)\,(P_4+P_6)\,(P_5+P_6) = 1$$

TABLE 6-5

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			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Petrick's Method:

- We will illustrate Petrick's method using Table 6-5.
  - ✓ Since we must cover all of the minterms, the following function must be true:

$$P = (P_1+P_2)(P_1+P_3)(P_2+P_4)(P_3+P_5)(P_4+P_6)(P_5+P_6) = 1$$

  - ✓ Using $(X+Y)(X+Z) = X+YZ$

$$\begin{aligned} P &= (P_1+P_2P_3)(P_4+P_2P_6)(P_5+P_3P_6) \\ &= (P_1P_4+P_1P_2P_6+P_2P_3P_4+P_2P_3P_6)(P_5+P_3P_6) \\ &= P_1P_4P_5+P_1P_2P_5P_6+P_2P_3P_4P_5+P_2P_3P_5P_6+P_1P_3P_4P_6 \\ &\quad +P_1P_2P_3P_6+P_2P_3P_4P_6+P_2P_3P_6 \end{aligned}$$

  - ✓ Using $X+XY = X$

$$P = \underline{P_1P_4P_5}+P_1P_2P_5P_6+P_2P_3P_4P_5+P_1P_3P_4P_6+\underline{P_2P_3P_6}$$

  - ✓ Only two solutions with the minimum number of rows, $P_1P_4P_5$ and $P_2P_3P_6$.
  - ✓ The two solutions with the minimum number of prime implicants are obtained by choosing rows P_1, P_4, and P_5 ($F = a'b'+bc'+ac$) or rows P_2, P_3, and P_6 ($F=a'c'+b'c+ab$).

TABLE 6-5

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			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Patrick's Method

General Procedure:

1. Reduce the prime implicant chart by eliminating the essential prime implicant rows and the corresponding columns.
2. Label the rows of the reduced prime implicant chart P_1, P_2, P_3, etc.
3. Form a logic function P which is true when all columns are covered. P consists of a product of sum terms, each sum term having the form $(P_{i0} + P_{i1} + \cdots)$, where $P_{i0}, P_{i1} \ldots$ represent the rows which cover column i.
4. Reduce P to a minimum sum of products by multiplying out and applying $X + XY = X$.
5. Each term in the result represents a solution, that is, a set of rows which covers all of the minterms in the table. To determine the minimum solutions (as defined in Section 5.1), find those terms which contain a minimum number of variables. Each of these terms represents a solution with a minimum number of prime implicants.
6. For each of the terms found in step 5, count the number of literals in each prime implicant and find the total number of literals. Choose the term or terms which correspond to the minimum total number of literals, and write out the corresponding sums of prime implicants.

Patrick's Method

Example 2: Patrick's method for finding minimum cost solution

- Package arrive at the stockroom and are delivered on carts to offices and laboratories by student employees. The carts and packages are various sizes and shapes. The students are paid according to the carts used.
- There are five carts and the pay for their use is C1=\$2, C2=\$1, C3=\$4, C4=\$2, C5=\$2.
- On a particular day, seven packages arrive, and they can be delivered using the five carts as follows: C_1 for P_1, P_3, P_4, C_2 for P_2, P_5, P_6 , C_3 for P_1, P_2, P_5, P_6, P_7, C_4 for P_3, P_6, P_7, C_5 for P_2, P_4.
- The stockroom manager wants the packages delivered at minimum cost. Find the minimum cost solution.

Packages arrived

5 carts with different costs

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
C_1 (\$2)	X		X	X			
C_2 (\$1)		X			X	X	
C_3 (\$4)	X	X			X	X	X
C_4 (\$2)			X			X	X
C_5 (\$2)		X		X			

Patrick's Method

Example 2: Patrick's method for finding minimum cost solution

- $P = \underbrace{(C_1+C_3)(C_2+C_3+C_5)}_{(X+Y)(X+Z)\,=\,X+YZ}\ \underbrace{(C_1+C_4)(C_1+C_5)}_{(X+Y)(X+Z)\,=\,X+YZ}\ \underbrace{(C_2+C_3)\cancel{(C_2+C_3+C_4)}\ (C_3+C_4)}_{X(X+Y)\,=\,X}$

  $= (C_1C_2+C_1C_5+C_3)(C_1+C_4C_5)(C_2C_4+C_3)$

  $= (C_1C_2+C_1C_5+C_1C_3+\cancel{C_1C_2C_4C_5}+\cancel{C_1C_4C_5}+C_3C_4C_5)(C_2C_4+C_3) \longleftarrow X+XY = X$

  $= (C_1C_2+C_1C_5+C_1C_3+C_3C_4C_5)(C_2C_4+C_3)$

  $= C_1C_2C_4+C_1C_3+C_3C_4C_5$

- Each product term specifies a nonredundant combination of carts that can be used to deliver the packages. The minimal solution is using the three carts C_1, C_2, C_4 , and it costs only \$5.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
C_1 (\$2)	X		X	X			
C_2 (\$1)		X			X	X	
C_3 (\$4)	X	X			X	X	X
C_4 (\$2)			X			X	X
C_5 (\$2)		X		X			

Simplification of Incompletely Specified Functions

Handling "Don't Cares":

- In this section, we will show how to modify the Quine-McCluskey method in order to obtain a minimum solution when don't-care terms are present.
- In the process of finding the prime implicants, we will treat the don't-care terms as if they were required minterms.
- When forming the prime implicant chart, the "don't cares" are not listed at the top.

Example 3:

$$F(A, B, C, D) = \Sigma\, m(2, 3, 7, 9, 11, 13) + \Sigma\, d(1, 10, 15)$$

(the terms following d are don't-care terms)

The don't-care terms are treated like required minterms when finding the prime implicants:

1	0001	✓
2	0010	✓
3	0011	✓
9	1001	✓
10	1010	✓
7	0111	✓
11	1011	✓
13	1101	✓
15	1111	✓

(1, 3)	00–1	✓
(1, 9)	–001	✓
(2, 3)	001–	✓
(2, 10)	–010	✓
(3, 7)	0–11	✓
(3, 11)	–011	✓
(9, 11)	10–1	✓
(9, 13)	1–01	✓
(10, 11)	101–	✓
(7, 15)	–111	✓
(11, 15)	1–11	✓
(13, 15)	11–1	✓

(1, 3, 9, 11)	–0–1
(2, 3, 10, 11)	–01–
(3, 7, 11, 15)	- -11
(9, 11, 13, 15)	1- -1

Simplification of Incompletely Specified Functions

Example 3 (Continued) :

The don't-care columns are omitted when forming the prime implicant chart:

	2	3	7	9	11	13
(1, 3, 9, 11)		X		X	X	
*(2, 3, 10, 11)	X	X			X	
*(3, 7, 11, 15)		X	X		X	
*(9, 11, 13, 15)				X	X	X

*Indicates an essential prime implicant.

$$F = B'C + CD + AD$$

Note that although the original function was incompletely specified, the final simplified expression for F is defined for all combinations of values for $A, B, C,$ and D and is therefore completely specified. In the process of simplification, we have automatically assigned values to the don't-cares in the original truth table for F. If we replace each term in the final expression for F by its corresponding sum of minterms, the result is

$$F = (m_2 + m_3 + m_{10} + m_{11}) + (m_3 + m_7 + m_{11} + m_{15}) + (m_9 + m_{11} + m_{13} + m_{15})$$

Because m_{10} and m_{15} appear in this expression and m_1 does not, this implies that the don't-care terms in the original truth table for F have been assigned as follows:

for $ABCD = 0001, F = 0$; for $1010, F = 1$; for $1111, F = 1$

By using map-entered variables, Karnaugh map techniques can be extended to simplify functions with more than four or five variables.

- Figure 6-1(a) shows a four-variable map with two additional variables entered in the squares in the map.

FIGURE 6-1 Use of Map-Entered Variables

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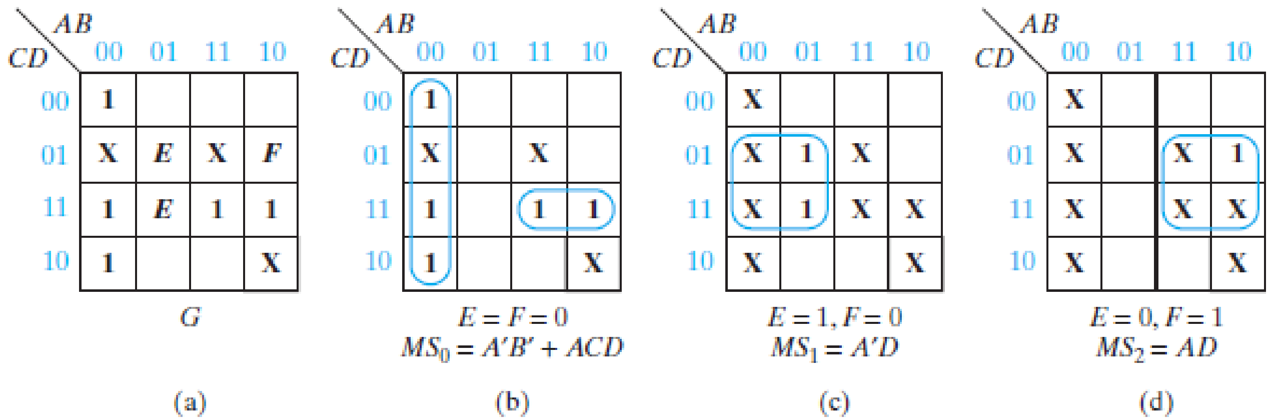

- When *E* appears in a square, this means that if *E*=1, the corresponding minterm is present in the function *G*, and if *E* = 0, the minterm is absent. Thus the map represents the six-variable function

$$G(A, B, C, D, E, F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15}$$
$$(+\text{don't-care terms})$$

  where the minterms are the minterms of variables *A*, *B*, *C*, and *D*. Note that m_9 is only present in *G* when *F*=1.

- The minimum sum-of-products for G can be expresses as

$$G = \underbrace{(A'B' + ACD)}_{MS_0} + E\,\underbrace{(A'D)}_{MS_1} + F\,\underbrace{(AD)}_{MS_2} = MS_0 + E\,MS_1 + F\,MS_2$$

Simplification Using Map-Entered Variables

- To simplify the following function,

$$F(A, B, C, D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C)$$

Don't care term

D is chosen as the map-entered variable.

FIGURE 6-2
Simplification Using a Map-Entered Variable

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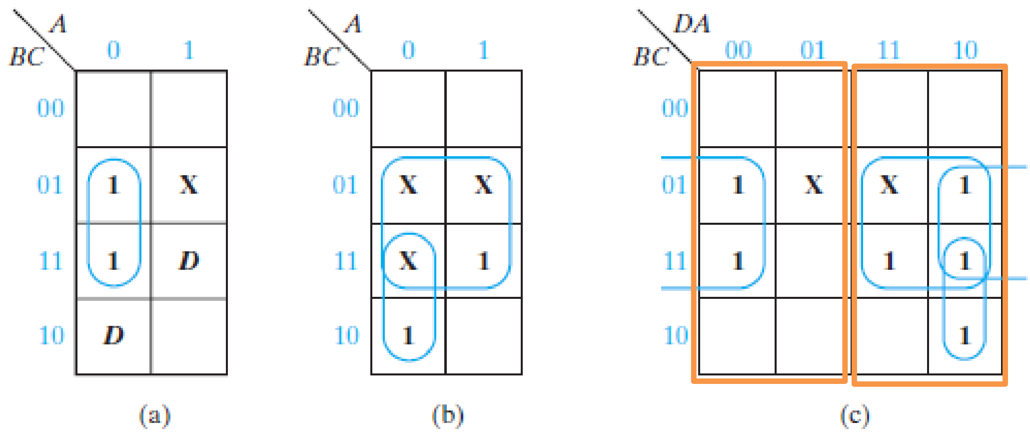

D = 0, $MS_0 = A'C$

D = 1, $MS_1 = C + A'B$

$D = 0$, $D = 1$

- First set $D = 0$ on the map, and F reduces to $A'C$.
- Setting $D = 1$, then the two 1's on the original map have already been covered by the term $A'C$, so they are changed to X's because we do not care whether they are covered again or not.
- The minimum expression for F can be represented as

$$F = \underbrace{A'C}_{MS_0} + D\,\underbrace{(C + A'B)}_{MS_1} = MS_0 + D\,MS_1$$

Simplification Using Map-Entered Variables

General Method of Simplifying Expressions Using Map-Entered Variables:

Find a sum-of-products expression for F of the form

$$F = MS_0 + P_1 MS_1 + P_2 MS_2 + \cdots$$

where

MS_0 is the minimum sum obtained by setting $P_1 = P_2 = \cdots = 0$.

MS_1 is the minimum sum obtained by setting $P_1 = 1$, $P_j = 0$ $(j \neq 1)$, and replacing all 1's on the map with don't-cares.

MS_2 is the minimum sum obtained by setting $P_2 = 1$, $P_j = 0$ $(j \neq 2)$ and replacing all 1's on the map with don't-cares.

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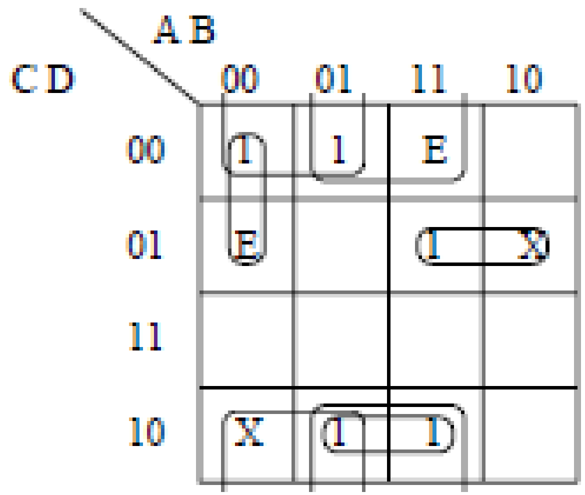
Simplification Using Map-Entered Variables

Example 4: Map-Entered Variables

- Using the method of map-entered variables, use four-variable maps to find a minimum sum-of-products expression for the function

$$F\,(A, B, C, D, E) = \sum m(0, 4, 6, 13, 14) + \sum d(2, 9) + E(m_1 + m_{12})$$



$$F = \mathrm{MS}_0 + E\,\mathrm{MS}_1 = \underbrace{(A'D'+BCD'+AC'D)}_{\mathrm{MS}_0} + E\,\underbrace{(A'B'C'+BD')}_{\mathrm{MS}_1}$$