problem 1

$$f_{X|Y}(x|2) = \frac{P_{Y|X}(2|x) f_X(x)}{\int P_{Y|X}(2|x) f_X(x)\, dx}$$

$$P_{Y|X}(2|x) = x(1-x)$$

$$\frac{x^2(1-x)^2}{\int_0^1 x^2(1-x)^2\, dx} = 30x^2(1-x)^2$$

The posterior integrates to 1

problem 3

$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} \propto f_{XY}(x,y)$$

$$\hat{x}_{MAP} = \underset{x \in [0,1]}{\operatorname{argmax}} \{ f_{XY}(x,y) \}$$

$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{f_{XY}(x,y)}{\int f_{XY}(x,y)\, dy}$$

$$\hat{x}_{ML} = \underset{x \in [0,1]}{\operatorname{argmax}} \left\{ \frac{f_{XY}(x,y)}{\{\int f_{XY}(x,y)\, dy\}} \right\}$$

$x = 1$, $\hat{x}_{MAP} = 1$

$$f_{Y|X}(y|x) = \frac{x + \frac{3}{2}y^2}{\int x + \frac{3}{2}y^2\, dy} = 1 + \frac{3y^2 - 1}{2x+1}$$

$$\hat{x}_{ML} = \begin{cases} 1 & \text{for } y \le \frac{1}{\sqrt{3}} \\ 0 & \text{otherwise} \end{cases}$$