

# Registers and Counters

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2. Shift Registers
3. Design of Binary Counters
4. Counters for Other Sequences
5. Counter Design using S-R and J-K Flip-Flops
6. Derivation of Flip-Flop Input Equations - Summary

# Objectives

- Explain the operation of registers. Show how to transfer data between registers using a tri-state bus.
- Explain the operation of shift registers, show how to build them using flip-flops, and analyze their operation. Construct a timing diagram for a shift register.
- Explain the operation of binary counters, show how to build them using flip-flops and gates, and analyze their operation.
- Given the present state and desired next state of a flip-flop, determine the required flip-flop inputs.
- Given the desired counting sequence for a counter, derive the flip-flop input equations.
- Explain the procedures used for deriving flip-flop input equations.
- Construct a timing diagram for a counter by tracing signals through the circuit.

# Registers and Register Transfers

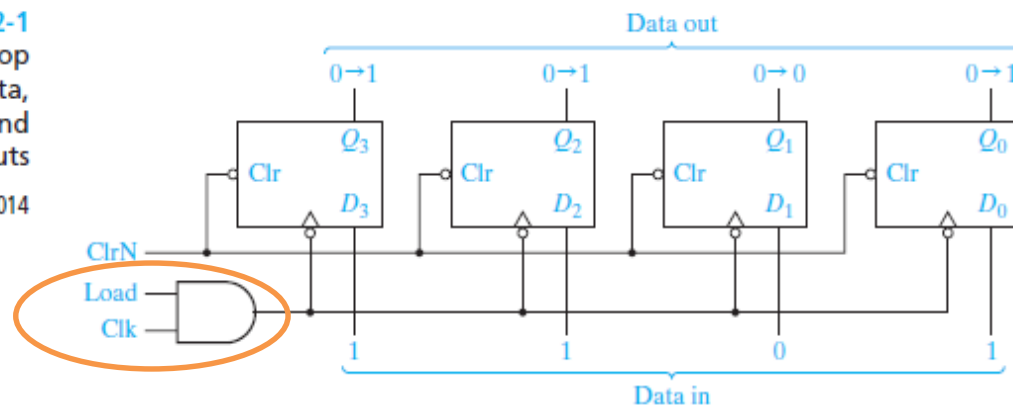
## Introduction:

- A register consists of a group of flip-flops with a common clock input. Registers are commonly used to store and shift binary data.
- A counter is usually constructed from two or more flip-flops which change states in a prescribed sequence when input pulses are received.

## 4-Bit D Flip-Flop Registers with Data, Load, Clear and Clock Inputs:

- When Load = 1, the clock signal (Clk) is transmitted to the flip-flop clock inputs and the data applied to the  $D$  inputs will be loaded into the flip-flop on the falling edge of the clock.
- The flip-flops in the register have asynchronous clear inputs that are connected to a common clear signal, ClrN. ClrN is normally 1, and if it is changed momentarily to 0, the  $Q$  outputs of all four flip-flops will become 0.
- Note that gating the clock with another signal can cause timing problems.

**FIGURE 12-1**  
4-Bit D Flip-Flop  
Registers with Data,  
Load, Clear, and  
Clock Inputs  
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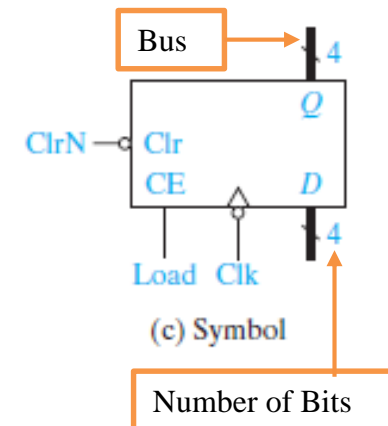
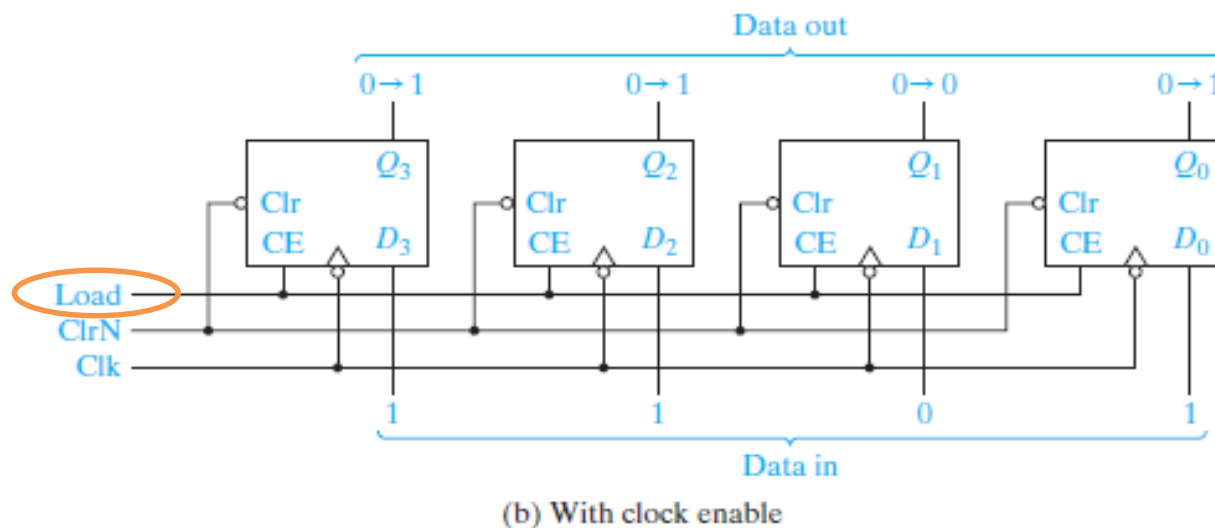


(a) Using gated clock

# Registers and Register Transfers

## 4-Bit D Flip-Flop Registers with Clock Enable:

- The load signal is connected to all four CE inputs. When Load = 0, the clock is disabled and the register holds its data. When Load is 1, the clock is enabled, and the data applied to the  $D$  inputs will be loaded into the flip-flops, following the falling edge of the clock.
- Figure (c) shows a symbol for the 4-bit register using bus notation for the  $D$  inputs and  $Q$  outputs. A group of wires that perform a common function is often referred to as a bus. A heavy line is used to represent a bus, and a slash with a number beside it indicates the number of bits in the bus.

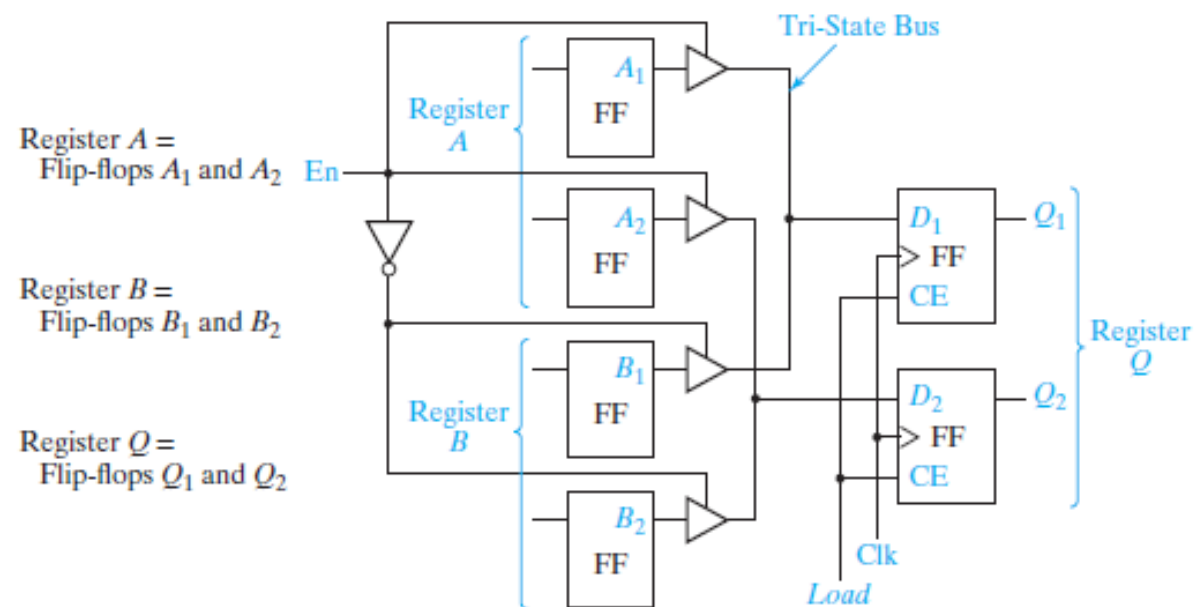


# Registers and Register Transfers

## Data Transfer Between Registers:

- If  $En = 1$  and  $Load = 1$ , the output of register *A* is enabled onto the tri-state bus and the data in register *A* will be stored in *Q* after the rising edge of the clock.
- If  $En = 0$  and  $Load = 1$ , the output of register *B* will be enabled onto the tri-state bus and stored in *Q* after the rising edge of the clock.

**FIGURE 12-2**  
Data Transfer  
Between Registers  
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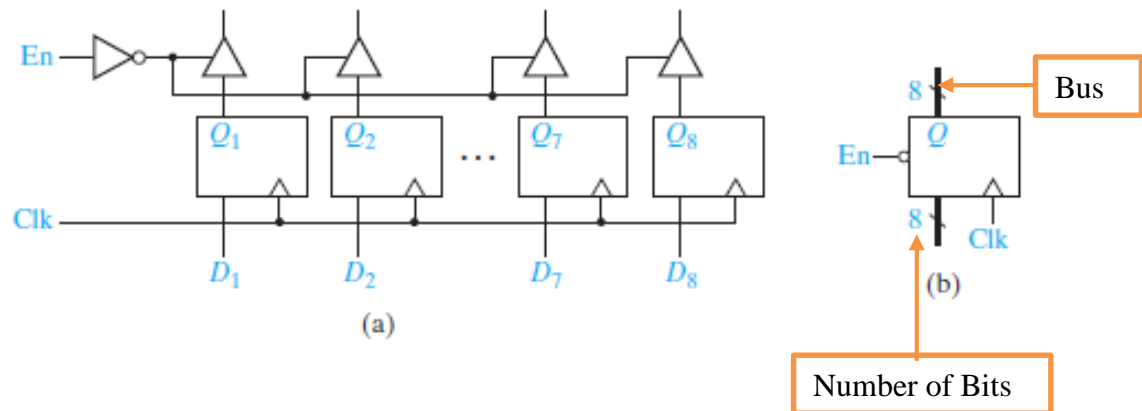


# Registers and Register Transfers

## 8-Bit Register with Tri-State Output:

- Figure 12-3(a) shows an integrated circuit register that contains eight  $D$  flip-flops with tri-state buffers at the flip-flop outputs.
- These buffers are enabled when  $En = 0$ . A symbol for this 8-bit register is shown in Figure 12-3(b).

**FIGURE 12-3**  
Logic Diagram for  
8-Bit Register with  
Tri-State Output  
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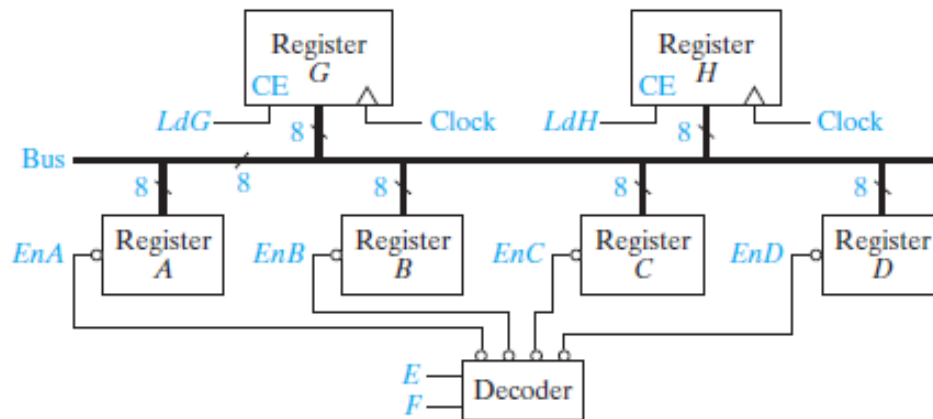


# Registers and Register Transfers

## Data Transfer Using a Tri-State Bus:

- When  $EnA = 0$ , the tri-state outputs of register  $A$  are enabled on to the bus.
- If  $LdG = 1$ , these signals on the bus are loaded into register  $G$  after the rising clock edge (or into register  $H$  if  $LdH = 1$ )
- If  $LdG = LdH = 1$ , both  $G$  and  $H$  will be loaded from the bus.

**FIGURE 12-4**  
Data Transfer Using  
a Tri-State Bus  
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If  $EF = 00$ ,  $A$  is stored in  $G$  (or  $H$ ).  
If  $EF = 01$ ,  $B$  is stored in  $G$  (or  $H$ ).  
If  $EF = 10$ ,  $C$  is stored in  $G$  (or  $H$ ).  
If  $EF = 11$ ,  $D$  is stored in  $G$  (or  $H$ ).



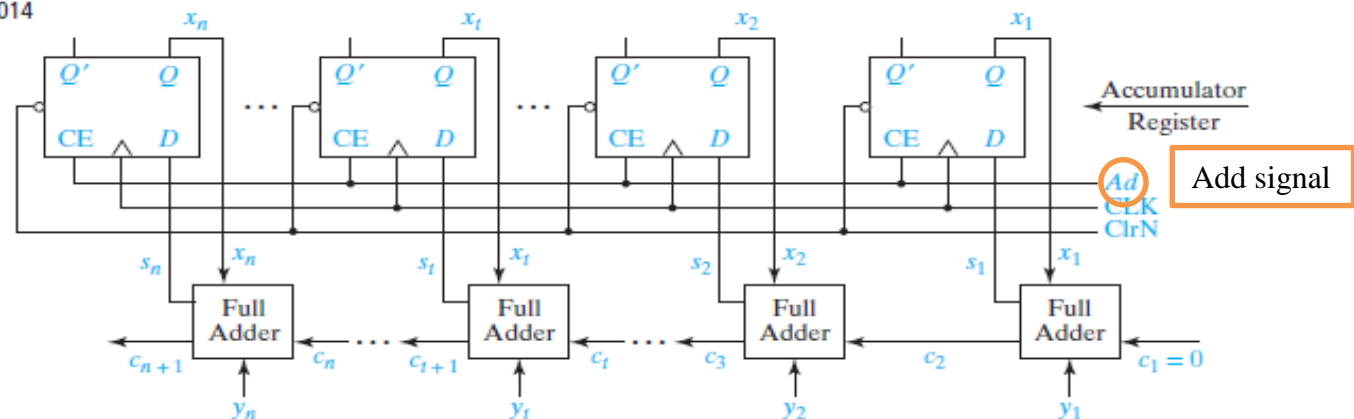
# Registers and Register Transfers

### Parallel Adder with Accumulator:

- It is frequently desirable to store one number in a register of flip-flops (called an **accumulator**) and add a second number to it, leaving the result stored in the accumulator.
- An add signal ( $Ad$ ) is used to load the adder outputs into the accumulator flip-flops on the rising clock edge.
- The next state of the flip-flop can be written as  $x_i^+ = s_i$ .
- If  $Ad = 1$ , the number  $X$  in the accumulator is replaced with the sum of  $X$  and  $Y$ , following the rising edge of the clock.

FIGURE 12-5 *n*-Bit Parallel Adder with Accumulator

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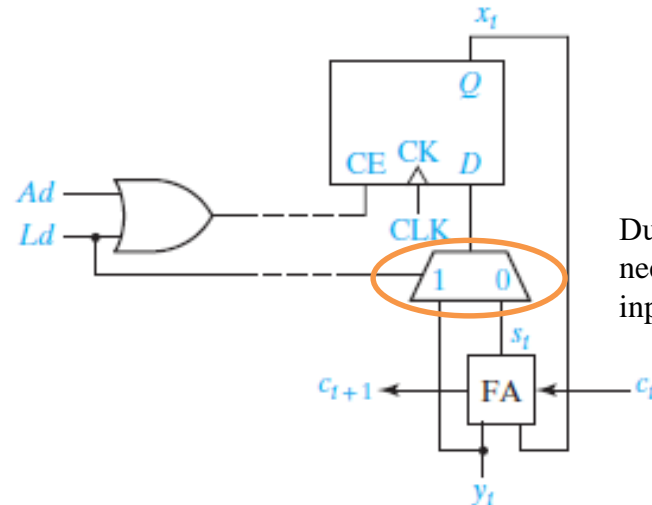
# Registers and Register Transfers

## Adder Cell with Multiplexer:

- Figure 12-6 shows a typical cell of the adder where the accumulator flip-flop can either be loaded directly from  $y_i$  or from the sum output  $s_i$  through the multiplexer.
- This would eliminate the extra step of clearing the accumulator but would add to the hardware complexity. (Otherwise clear the accumulator using the asynchronous clear inputs on the flip-flops)
- When  $Ld = 1$ , the multiplexer selects  $y_i$ , and  $y_i$  is loaded into the accumulator flip-flop ( $x_i$ ) on the rising clock edge.
- When  $Ad = 1$  and  $Ld = 0$ , the adder output ( $s_i$ ) is loaded into  $x_i$ .
- The  $Ad$  and  $Ld$  signals are ORed together to enable the clock when either addition or loading occurs.
- When  $Ad = Ld = 0$ , the clock is disabled and the accumulator outputs do not change.

**FIGURE 12-6**  
Adder Cell with  
Multiplexer

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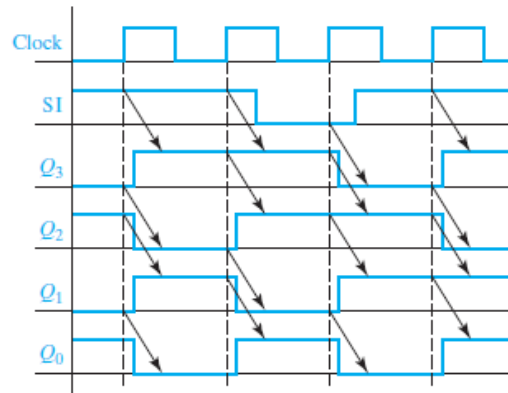
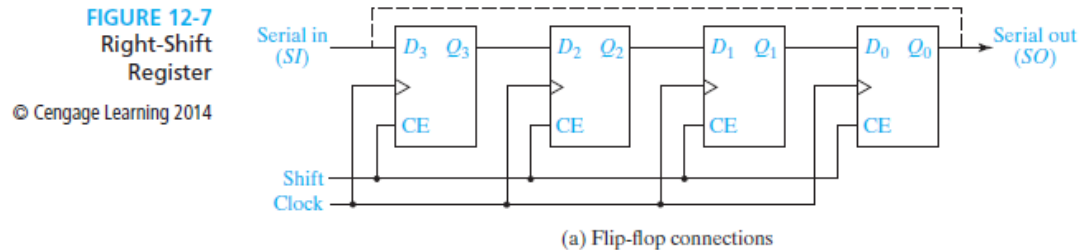


Due to this multiplexer, we don't need to use asynchronous clear inputs on the flip-flops

# Shift Registers

## Shift Registers:

- A shift register is a register in which binary data can be stored, and this data can be shifted to the left or right when a shift signal is applied.
- Shifts can be linear or cyclic.
- The figure shows a 4-bit right-shift register with serial input and output constructed from  $D$  flip-flops.
- When Shift = 1, the clock is enabled and shifting occurs on the rising clock edge. When Shift = 0, no shifting occurs and the data in the register is unchanged.



(b) Timing diagram

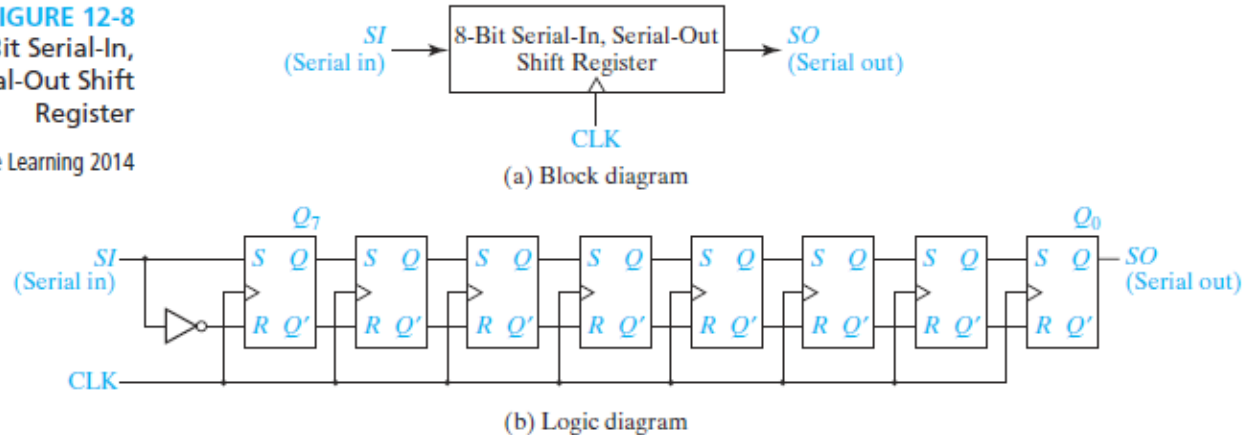
# Shift Registers

## Serial-In Serial-Out Shift Registers:

- **Serial in** means that data is shifted into the first flip-flop one bit at a time, and the flip-flops cannot be loaded in parallel.
- **Serial out** means that data can only be read out of the last flip-flop and the outputs from the other flip-flops are not connected to terminals of the integrated circuit.
- The inputs to the first flip-flops are  $S = SI$  and  $R = SI'$ .

**FIGURE 12-8**  
8-Bit Serial-In,  
Serial-Out Shift  
Register

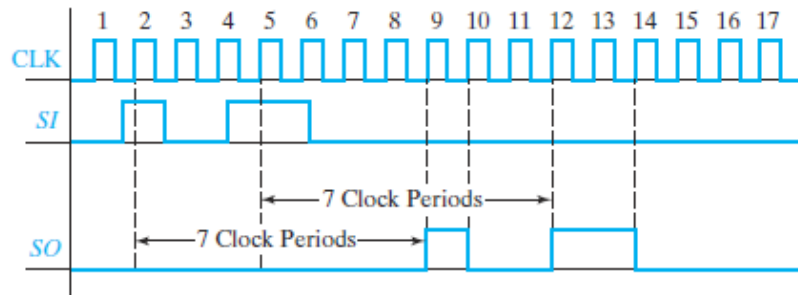
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- If  $SI = 1$ , a 1 is shifted into the register when it is clocked, and if  $SI = 0$ , a 0 is shifted in.

**FIGURE 12-9**  
Typical Timing  
Diagram for  
Shift Register of  
Figure 12-8

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# Shift Registers

## Parallel-In Parallel-Out Shift Register:

- Parallel-in implies that all four bits can be loaded at the same time, and parallel-out implies that all bits can be read out at the same time.
- The shift register has two control inputs, shift enable ( $Sh$ ) and load enable ( $L$ ).

- If  $Sh = 1$  (and  $L=1$ , or  $L=0$ ), clocking the register causes the serial input ( $SI$ ) to be shifted into the first flip-flop, while the data in flip-flops  $Q_3, Q_2, Q_1$  are shifted right.
- If  $Sh = 0$  and  $L = 1$ , clocking the shift register will cause the four data inputs ( $D_3, D_2, D_1, D_0$ ) to be loaded in parallel into the flip-flops.
- If  $Sh = L = 0$ , clocking the register causes no change of state.

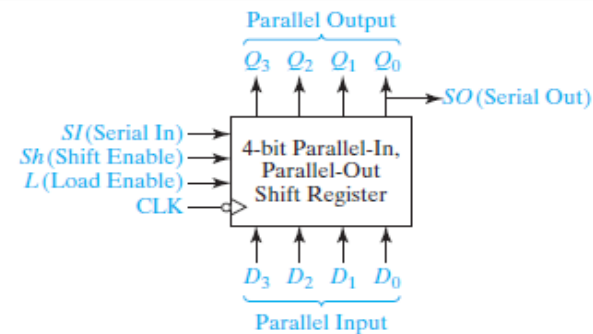
**TABLE 12-1**  
Shift Register  
Operation

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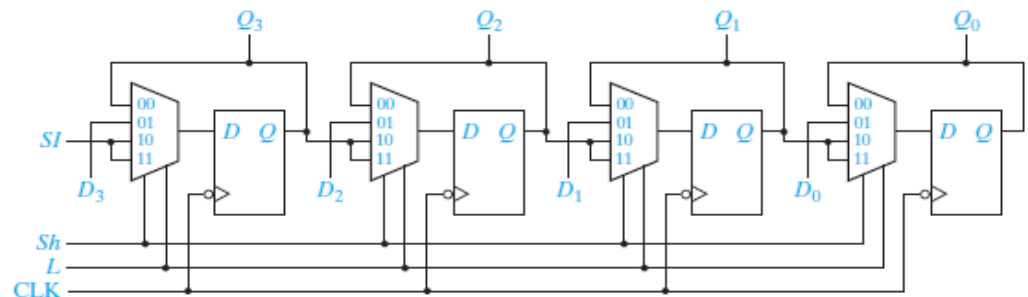
Inputs		Next State				Action
$Sh$ (Shift)	$L$ (Load)	$Q_3^+$	$Q_2^+$	$Q_1^+$	$Q_0^+$	
0	0	$Q_3$	$Q_2$	$Q_1$	$Q_0$	No change
0	1	$D_3$	$D_2$	$D_1$	$D_0$	Load
1	X	$SI$	$Q_3$	$Q_2$	$Q_1$	Right shift

**FIGURE 12-10**  
Parallel-In,  
Parallel-Out  
Right-Shift  
Register

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(a) Block diagram



(b) Implementation using flip-flops and MUXes

# Shift Registers

## Parallel-In Parallel-Out Shift Register:

- For the first flip-flop, when  $Sh = L = 0$ , the flip-flop  $Q_3$  output is selected by the MUX, so  $Q_3^+ = Q_3$  and no state change occurs.
- When  $Sh = 0$  and  $L = 1$ , the data input  $D_3$  is selected and loaded into the flip-flop.
- When  $Sh = 1$  and  $L = 0, 1$ ,  $SI$  is selected and loaded into the flip-flop. The second MUX selects  $Q_2$ ,  $D_2$  or  $Q_3$ , etc.
- The next-state equations for the flip-flops are

$$Q_3^+ = Sh' \cdot L' \cdot Q_3 + Sh' \cdot L \cdot D_3 + Sh \cdot SI$$

$$Q_2^+ = Sh' \cdot L' \cdot Q_2 + Sh' \cdot L \cdot D_2 + Sh \cdot Q_3$$

$$Q_1^+ = Sh' \cdot L' \cdot Q_1 + Sh' \cdot L \cdot D_1 + Sh \cdot Q_2$$

$$Q_0^+ = Sh' \cdot L' \cdot Q_0 + Sh' \cdot L \cdot D_0 + Sh \cdot Q_1$$

# Shift Registers

## Conversion of Parallel Data to Serial Data:

- The output from the last flip-flop ( $Q_0$ ) serves as a serial output as well as one of the parallel outputs.
  - ✓ The first clock pulse loads data into the shift register in parallel.
  - ✓ Assuming that the register is initially clear ( $Q_3Q_2Q_1Q_0 = 0000$ ), that the serial input is  $SI = 0$  throughout, and that the data inputs  $D_3D_2D_1D_0 = 1011$  during the load time  $t_0$ .
  - ✓ Shifting occurs at the end of  $t_1$ ,  $t_2$ , and  $t_3$  and the serial output can be read during these clock times. During  $t_4$ ,  $Sh = L = 0$ , so no state change occurs.

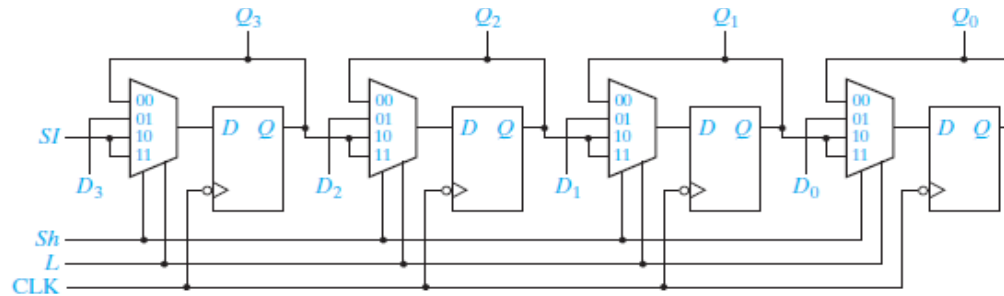
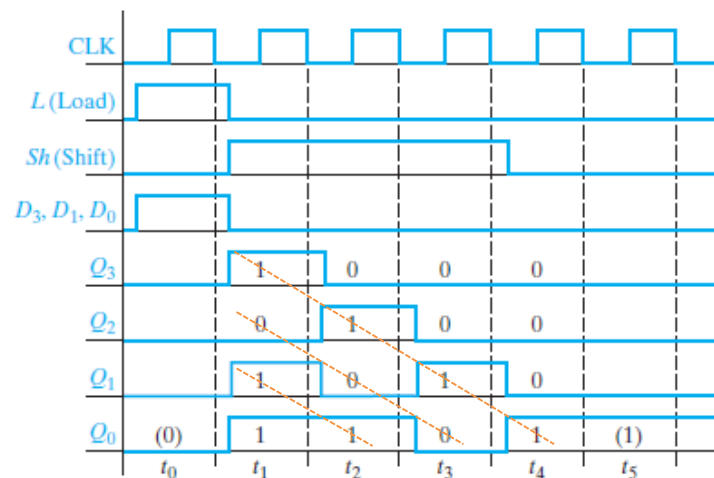


FIGURE 12-11  
Timing Diagram for  
Shift Register  
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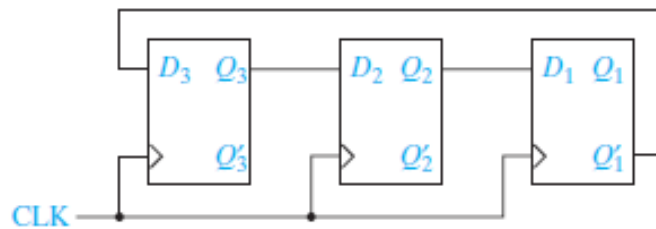
# Shift Registers

## Johnson Counter:

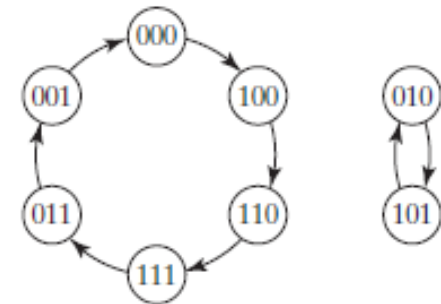
- A circuit that cycles through a fixed sequence of states is called a **counter**.
- A shift register with inverted feedback is called a **Johnson counter** or a **twisted ring counter**.
- If the register is in state 010, then a shift pulse takes it to 101 and vice versa; therefore, we have a secondary loop on the transition graph
- If the feedback is not converted (e.g.,  $Q_1$  connected to  $D_3$ ), the shift register counter is called a **ring counter**.

FIGURE 12-12  
Shift Register  
with Inverted  
Feedback

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(a) Flip-flop connections



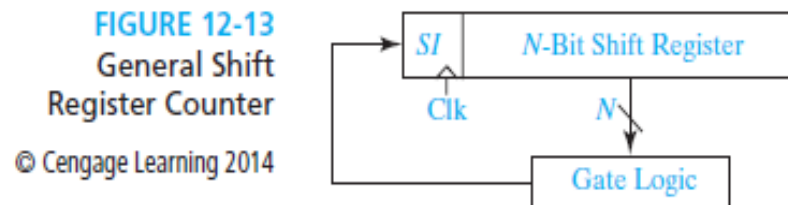
(b) Transition graph



# Shift Registers

## General Form of a Shift Register Counter:

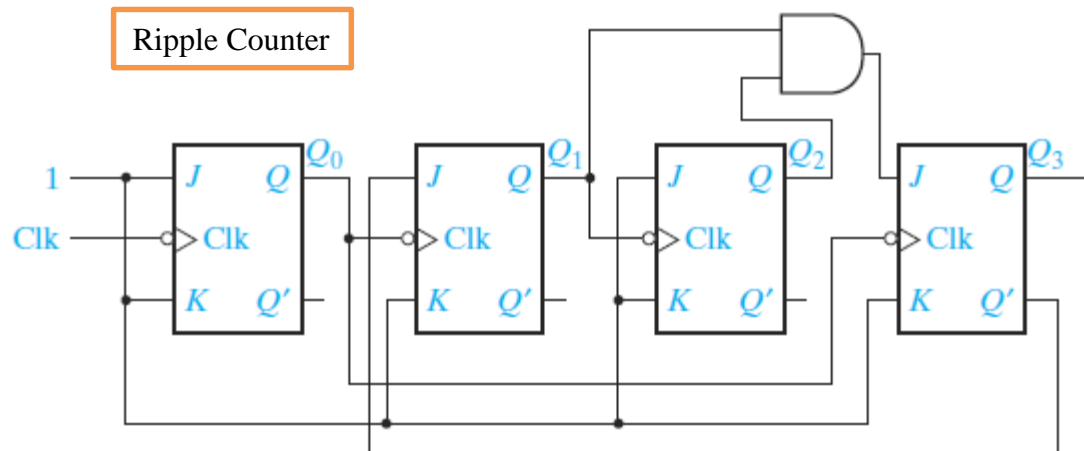
- The bit being shifted into the leftmost stage can be a general function of the shift register contents.
- If the gate logic only contains exclusive-OR gates, the counter is called a linear (feedback) shift register counter.
- It can be shown that, for each integer  $n$ , there exists a linear  $n$ -bit shift register counter that generates a count cycle of length  $2^n - 1$ ; all states are included except for the all 0's state.
- Linear shift register counters have many applications, including as random number generators and as encoders and decoders for linear error-correcting codes.



# Design of Binary Counters

## Synchronous and Ripple Counters:

- For synchronous counters, the operation of the flip-flops is synchronized by a common clock pulse so that when several flip-flops must change state, the state changes occur simultaneously.
- Ripple counters are those in which the state change of one flip-flop triggers another flip-flop.



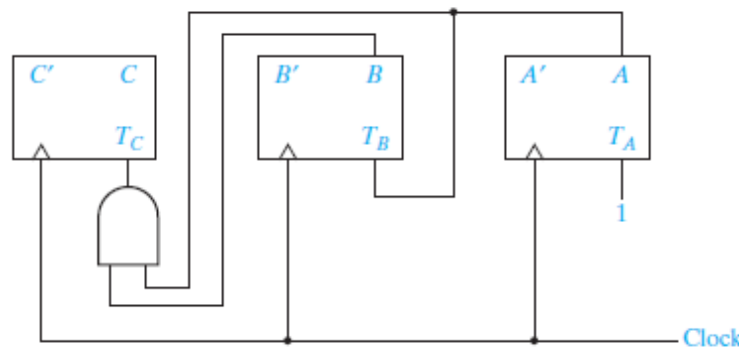
# Design of Binary Counters

## Binary Counters Using 3 T Flip-Flops to Count Clock Pulses:

- We assume that all the flip-flops change state a short time following the rising edge of the input pulse.
- The state of the counter is determined by the states of the individual flip-flops; for example, if flip-flop  $C$  is in state 0,  $B$  in state 1, and  $A$  in state 1, the state of the counter is 011.
- Initially, assume that all flip-flops are set to the 0 state. When a clock pulse is received, the counter will change to state 001; when a second pulse is received, the state will change to 010, etc. When 111 is reached, the counter resets to the 000 state.
- The sequence of flip-flop states is  $CBA = 000, 001, 010, 011, 100, 101, 110, 111, (\text{repeat}) 000, \dots$  (Design by inspection)
  - ✓ Because  $A$  changes state on every rising clock edge,  $T_A$  must equal 1.
  - ✓  $B$  changes state only if  $A = 1$ . Therefore,  $A$  is connected to  $T_B$ , so that if  $A = 1$ ,  $B$  will change state when a rising clock edge occurs.
  - ✓  $C$  changes state when a rising clock edge occurs only if  $B$  and  $A$  are both 1.

FIGURE 12-14  
Synchronous  
Binary Counter

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Present State			Next State		
$C$	$B$	$A$	$C^+$	$B^+$	$A^+$
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

# Design of Binary Counters

## Design of Binary Counter Using Transition Tables:

- This table shows the present state of flip-flops  $C$ ,  $B$ , and  $A$  (before a clock pulse is received) and the corresponding next state (after the clock pulse is received).
- A third column in the table is used to derive the inputs for  $T_C$ ,  $T_B$ , and  $T_A$ .
  - ✓ Whenever the entries in the  $A$  and  $A^+$  columns differ, flip-flop  $A$  must change state and  $T_A$  must be 1.
  - ✓ Similarly, if  $B$  and  $B^+$  differ,  $B$  must change state so  $T_B$  must be 1.

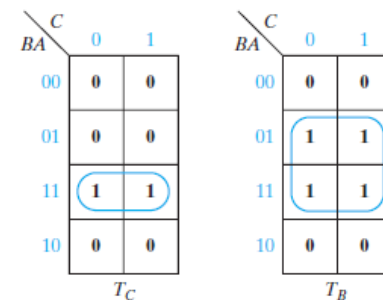
## Karnaugh Maps for Binary Counters:

- Figure 12-15 shows the Karnaugh maps for  $T_C$  and  $T_B$ , from which  $T_C = BA$  and  $T_B = A$ . These equations yield the same circuit derived previously for Figure 12-14.

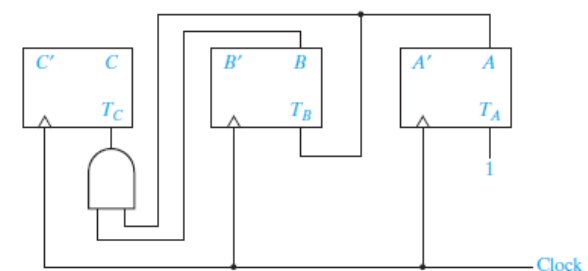
**TABLE 12-2**  
Transition Table for  
Binary Counter  
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Present State			Next State			Flip-Flop Inputs		
$C$	$B$	$A$	$C^+$	$B^+$	$A^+$	$T_C$	$T_B$	$T_A$
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

**FIGURE 12-15**  
Karnaugh Maps  
for Binary Counter  
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**FIGURE 12-14**  
Synchronous  
Binary Counter  
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# Design of Binary Counters

## Binary Counter with D Flip-Flops:

We can also derive the D flip-flop inputs for the binary counter starting with its transition table (Table 12-2). For a D flip-flop,  $Q^+ = D$ . By inspection of the table,  $Q_A^+ = A'$ , so  $D_A = A'$ . The maps for  $Q_B^+$  and  $Q_C^+$  are plotted in Figure 12-17. The D input equations derived from the maps are

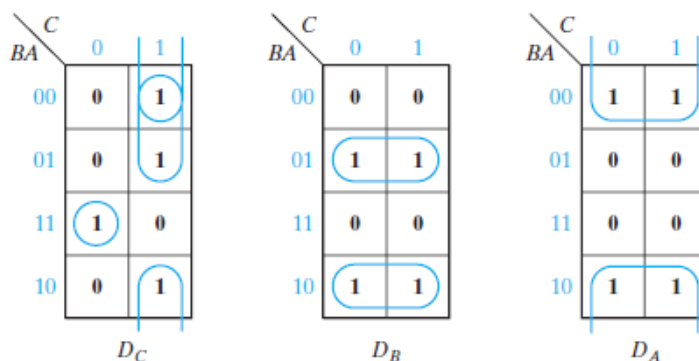
$$D_A = A^+ = A'$$

$$D_B = B^+ = BA' + B'A = B \oplus A \quad (12-2)$$

$$D_C = C^+ = C'BA + CB' + CA' = C'BA + C(BA)' = C \oplus BA$$

which give the same logic circuit as was obtained by inspection.

**FIGURE 12-17**  
Karnaugh Maps  
for D Flip-Flops  
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Present State			Next State		
C	B	A	C <sup>+</sup>	B <sup>+</sup>	A <sup>+</sup>
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

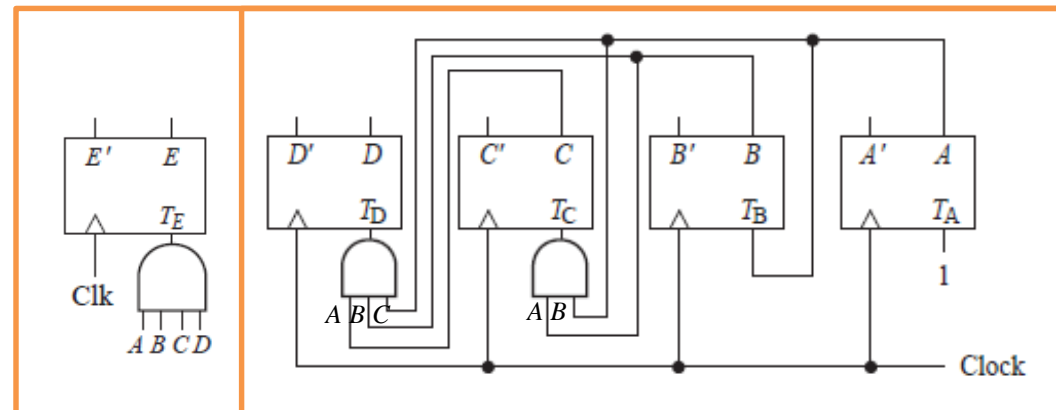
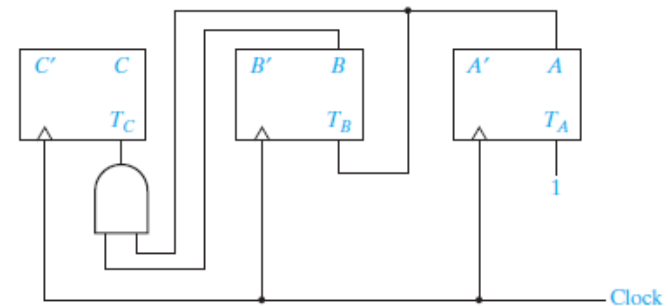
# Design of Binary Counters

## Binary Counter with T Flip-Flops:

- 3-bit, 4-bit, and 5-bit binary counters with T flip-flops

Present State $DCBA$	Next State $D^+C^+B^+A^+$	Flip-Flop Inputs $T_D T_C T_B T_A$
0000	0001	0001
0001	0010	0011
0010	0011	0001
0011	0100	0111
0100	0101	0001
0101	0110	0011
0110	0111	0001
0111	1000	1111
1000	1001	0001
1001	1010	0011
1010	1011	0001
1011	1100	0111
1100	1101	0001
1101	1110	0011
1110	1111	0001
1111	0000	1111

FIGURE 12-14  
Synchronous  
Binary Counter  
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5-bit binary counter

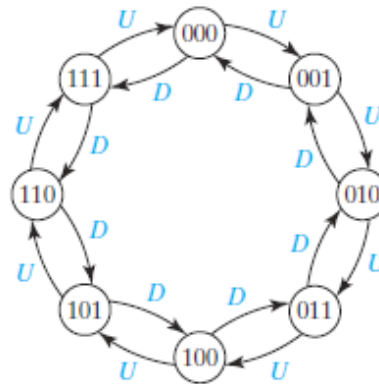
4-bit binary counter

# Design of Binary Counters

## Binary Up-Down Counter:

- The transition graph and table for up-down counter are shown below.

**FIGURE 12-18**  
Transition Graph  
and Table for  
Up-Down  
Counter  
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CBA	C+B+A+	
	U	D
000	001	111
001	010	000
010	011	001
011	100	010
100	101	011
101	110	100
110	111	101
111	000	110

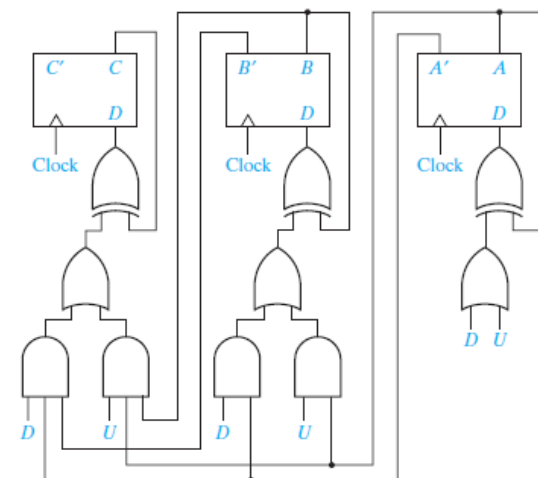
- The up-down counter can be implemented using D flip-flops and gates as shown below. The corresponding logic equations are

$$D_A = A^+ = A \oplus (U + D)$$

$$D_B = B^+ = B \oplus (UA + DA')$$

$$D_C = C^+ = C \oplus (UBA + DB'A')$$

**FIGURE 12-19**  
Binary Up-Down  
Counter  
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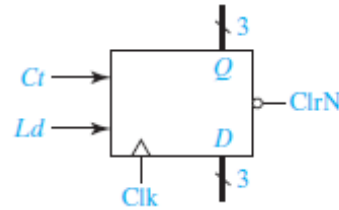


# Design of Binary Counters

## Loadable Counter with Count Enable:

- The loadable counter has two control signals  $Ld$  (load) and  $Ct$  (Count).

**FIGURE 12-20**  
Loadable Counter  
with Count Enable  
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(a)

ClrN	Ld	Ct	C <sup>+</sup>	B <sup>+</sup>	A <sup>+</sup>	
0	X	X	0	0	0	
1	1	X	$D_C$	$D_B$	$D_A$	(load)
1	0	0	C	B	A	(no change)
1	0	1	Present state + 1			

(b)

- The next-state equations for the counter of Figure 12-21 are

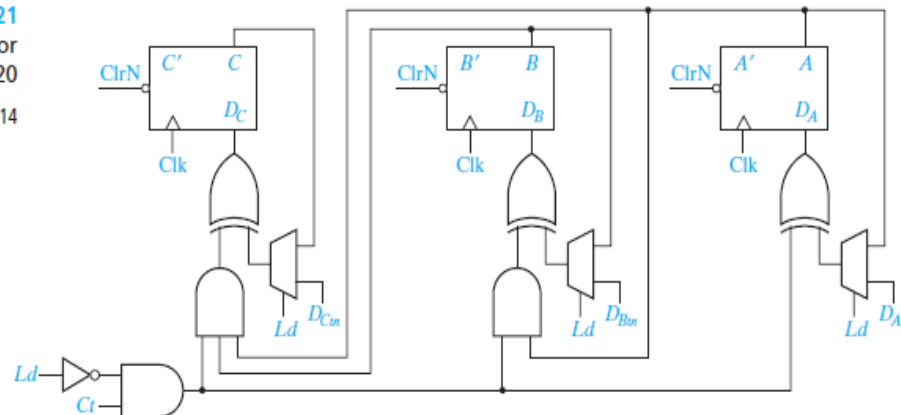
$$A^+ = D_A = (Ld' \cdot A + Ld \cdot D_{Ain}) \oplus Ld' \cdot Ct$$

$$B^+ = D_B = (Ld' \cdot B + Ld \cdot D_{Bin}) \oplus Ld' \cdot Ct \cdot A$$

$$C^+ = D_C = (Ld' \cdot C + Ld \cdot D_{Cin}) \oplus Ld' \cdot Ct \cdot B \cdot A$$

When  $Ld = 0$  and  $Ct = 1$ , these equations reduce to  $A^+ = A'$ ,  $B^+ = B \oplus A$ , and  $C^+ = C \oplus BA$ , which are the equations previously derived for a 3-bit counter.

**FIGURE 12-21**  
Circuit for  
Figure 12-20  
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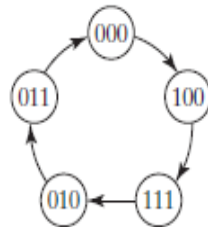
# Counters for Other Sequences

## Counter for Other Sequences (Example):

- We will design a counter for the transition table shown in Table 12-3 using T Flip-Flops.

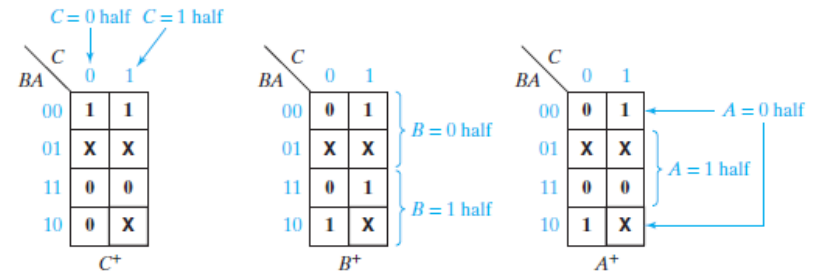
**FIGURE 12-22**  
Transition Graph  
for Counter

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**FIGURE 12-23**

© Cengage Learning 2014

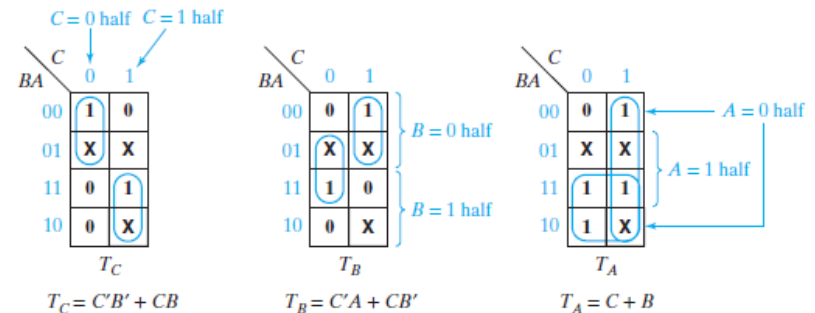


(a) Next-state maps for Table 12-3

**TABLE 12-3**  
Transition Table for  
Figure 12-22

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C	B	A	$C^+$	$B^+$	$A^+$
0	0	0	1	0	0
0	0	1	-	-	-
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	-	-	-
1	1	0	-	-	-
1	1	1	0	1	0



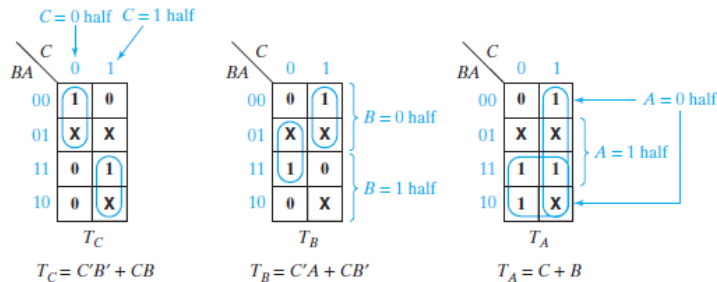
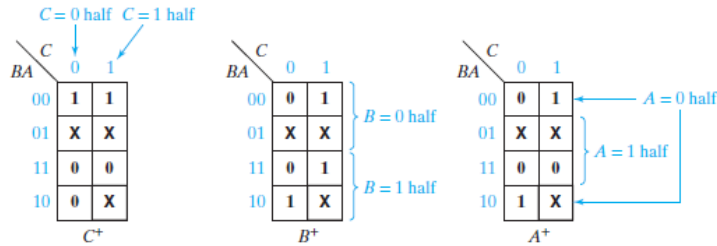
(b) Derivation of T inputs

# Counters for Other Sequences

## Determination of Flip-Flop inputs from Next-State Equations

FIGURE 12-23

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**TABLE 12-9**  
Determination of  
Flip-Flop Input  
Equations from  
Next-State  
Equations  
Using Karnaugh  
Maps

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Type of Flip-Flop	Input	$Q = 0$		$Q = 1$		Rules for Forming Input Map From Next-State Map*	
		$Q^+ = 0$	$Q^+ = 1$	$Q^+ = 0$	$Q^+ = 1$	$Q = 0$ Half of Map	$Q = 1$ Half of Map
Delay	$D$	0	1	0	1	no change	no change
Toggle	$T$	0	1	1	0	no change	complement
Set-Reset	$S$	0	1	0	X	no change	replace 1's with X's**
	$R$	X	0	1	0	replace 0's with X's**	complement
J-K	$J$	0	1	X	X	no change	fill in with X's
	$K$	X	X	1	0	fill in with X's	complement

$Q^+$  means the next state of  $Q$

X is a don't-care

\*Always copy X's from the next-state map onto the input maps first.

\*\*Fill in the remaining squares with 0's.

# Counters for Other Sequences

## Input for T Flip-Flop:

- $Q$  represents the present state of the flip-flop ( $C$ ,  $B$ , or  $A$ ) under consideration, and  $Q^+$  represents the next state ( $C^+$ ,  $B^+$ , or  $A^+$ ) of the same flip-flop.
- $T=1$  whenever a state of change is required.

TABLE 12-4

Input for  
T Flip-Flop

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$Q$	$Q^+$	$T$
0	0	0
0	1	1
1	0	1
1	1	0

$$T = Q^+ \oplus Q$$

## Counter Using T Flip-Flops and Timing Diagram:

FIGURE 12-24  
Counter Using  
T Flip-Flops

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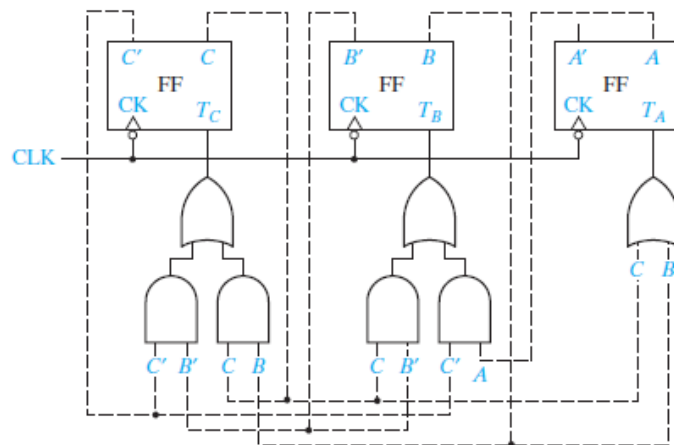
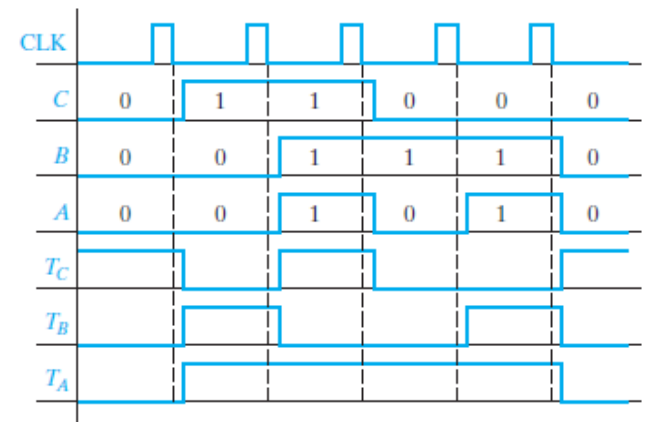


FIGURE 12-25  
Timing Diagram  
for Figure 12-24

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# Counters for Other Sequences

## Procedure for Designing a Counter Using T Flip-Flops:

1. Form a transition table which gives the next flip-flop states for each combination of present flip-flop states.
2. Plot the next-state maps from the table.
3. Plot a  $T$  input map for each flip-flop. When filling in the  $T_Q$  map,  $T_Q$  must be 1 whenever  $Q^+ \neq Q$ . This means that the  $T_Q$  map can be formed from the  $Q^+$  map by complementing the  $Q = 1$  half of the map and leaving the  $Q = 0$  half unchanged.
4. Find the  $T$  input equations from the maps and realize the circuit.

**TABLE 12-9**  
Determination of  
Flip-Flop Input  
Equations from  
Next-State  
Equations  
Using Karnaugh  
Maps

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Type of Flip-Flop	Input	$Q = 0$		$Q = 1$		Rules for Forming Input Map From Next-State Map*	
		$Q^+ = 0$	$Q^+ = 1$	$Q^+ = 0$	$Q^+ = 1$	$Q = 0$ Half of Map	$Q = 1$ Half of Map
Delay	$D$	0	1	0	1	no change	no change
Toggle	$T$	0	1	1	0	no change	complement
Set-Reset	$S$	0	1	0	X	no change	replace 1's with X's**
	$R$	X	0	1	0	replace 0's with X's**	complement
J-K	$J$	0	1	X	X	no change fill in with X's	fill in with X's
	$K$	X	X	1	0		complement

$Q^+$  means the next state of  $Q$

X is a don't-care

\*Always copy X's from the next-state map onto the input maps first.

\*\*Fill in the remaining squares with 0's.

# Counters for Other Sequences

## Counter Design using D Flip-Flops:

- For a D flip-flop,  $Q^+ = D$ , so the  $D$  input map is identical with the next-state map.

FIGURE 12-22  
Transition Graph  
for Counter

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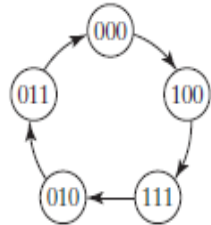
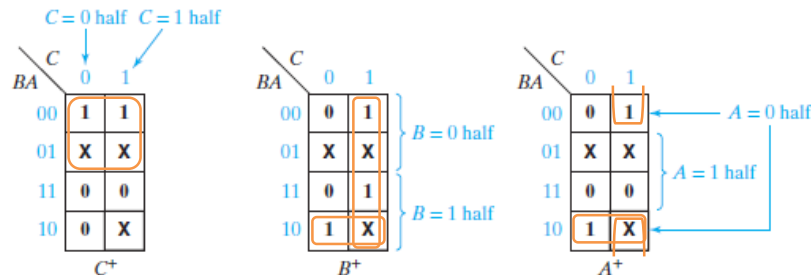


TABLE 12-3  
Transition Table for  
Figure 12-22

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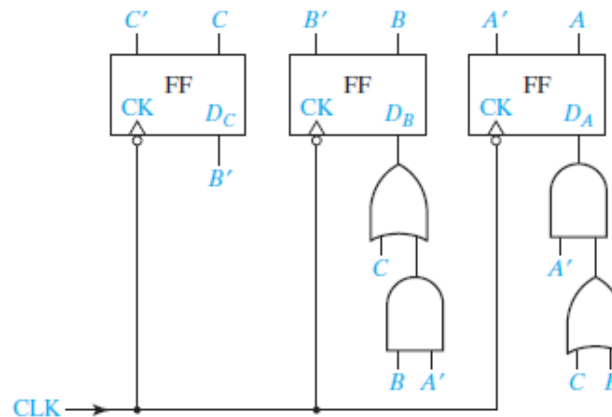
C	B	A	C <sup>+</sup>	B <sup>+</sup>	A <sup>+</sup>
0	0	0	1	0	0
0	0	1	-	-	-
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	-	-	-
1	1	0	-	-	-
1	1	1	0	1	0

FIGURE 12-23  
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$$\begin{aligned}
 D_C &= C^+ = B' \\
 D_B &= B^+ = C + BA' \\
 D_A &= A^+ = CA' + BA' = A'(C+B)
 \end{aligned}$$

FIGURE 12-27  
Counter of  
Figure 12-22  
Using D Flip-Flops  
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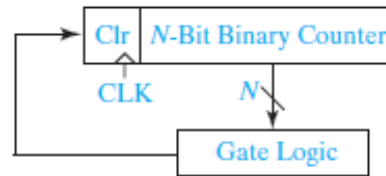


# Counters for Other Sequences

## Binary Counter with Clear:

- Counters and shift registers with clear, preset, or parallel load capability can also be used to generate non-binary count cycles.
- Consider a binary counter with a clear input as shown Figure 12-28.

**FIGURE 12-28**  
Binary Counter  
with Clear  
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- Synchronous Clear:  $\text{Clr} = Q_3Q_0$
- Asynchronous Clear:  $\text{Clr} = Q_3Q_1$

## Binary Counter with Parallel Load:

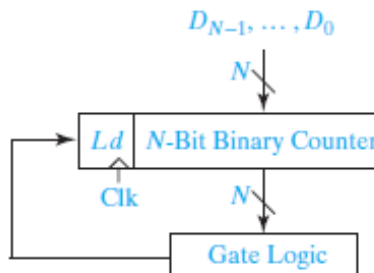
- A decimal counter that counts using the excess-3 code
  - The counter must cycle through states 3 to 12. The logic must cycle through 3 through 12.
  - The logic must generate  $Ld$  when the counter is in state 12 and the parallel inputs must be 0011.

$$D_3 = 0, D_2 = 0, D_1 = 1, D_0 = 1$$

$$Ld = Q_3Q_2$$

- States 0, 1, 2, 13, 14 and 15 are don't cares.

**FIGURE 12-29**  
Binary Counter  
with Parallel Load  
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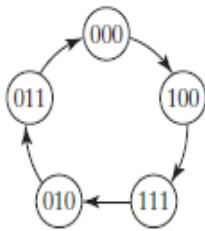
# Counter Design Using S-R and J-K Flip-Flops

## Procedure for Counter Design Using S-R Flip-Flops:

- Instead of deriving an input equation for each D or T flip-flop, the S and R input equations must be derived.

**FIGURE 12-22**  
Transition Graph  
for Counter

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**TABLE 12-5**  
S-R Flip-Flop  
Inputs

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(a)			
S	R	Q	Q <sup>+</sup>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	—
1	1	1	—

inputs not allowed

(b)			
Q	Q <sup>+</sup>	S	R
0	0	0	0
0	1	0	1
1	0	1	0
1	1	0	0

(c)			
Q	Q <sup>+</sup>	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

**TABLE 12-3**  
Transition Table for  
Figure 12-22

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C	B	A	C <sup>+</sup>	B <sup>+</sup>	A <sup>+</sup>
0	0	0	1	0	0
0	0	1	—	—	—
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	—	—	—
1	1	0	—	—	—
1	1	1	0	1	0

**TABLE 12-6**  
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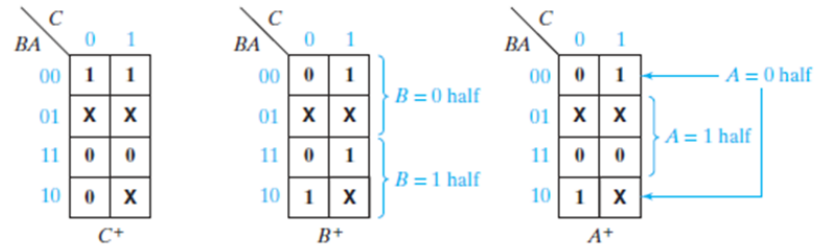
C	B	A	C <sup>+</sup>	B <sup>+</sup>	A <sup>+</sup>	S <sub>C</sub>	R <sub>C</sub>	S <sub>B</sub>	R <sub>B</sub>	S <sub>A</sub>	R <sub>A</sub>
0	0	0	1	0	0	1	0	0	X	0	X
0	0	1	—	—	—	X	X	X	X	X	X
0	1	0	0	1	1	0	X	X	0	1	0
0	1	1	0	0	0	0	X	0	1	0	1
1	0	0	1	1	1	X	0	1	0	1	0
1	0	1	—	—	—	X	X	X	X	X	X
1	1	0	—	—	—	X	X	X	X	X	X
1	1	1	0	1	0	0	1	X	0	0	1



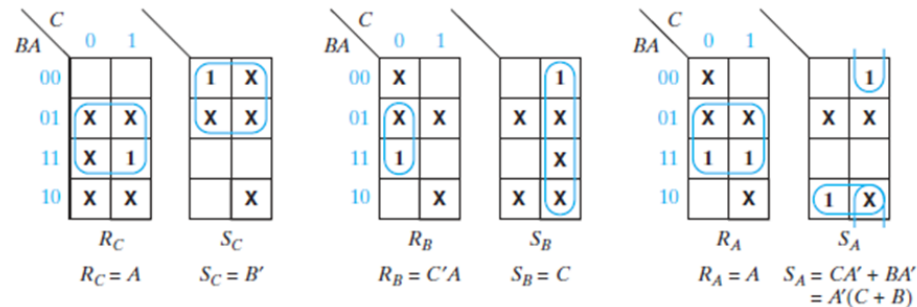
# Counter Design Using S-R and J-K Flip-Flops

## Counter Using S-R Flip Flops:

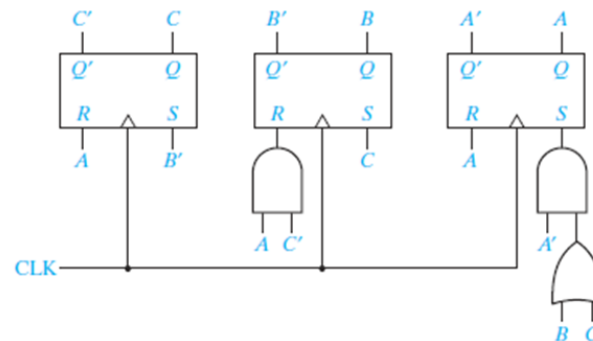
**FIGURE 12-30**  
Counter of  
Figure 12-22 Using  
S-R Flip-Flops  
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(a) Next-state maps



(b) S-R flip-flop equations



(c) Logic circuit

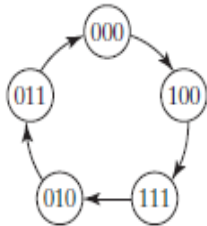
# Counter Design Using S-R and J-K Flip-Flops

## Procedure for Counter Design Using J-K Flip-Flops:

- The procedure used to design a counter with J-K flip-flops is very similar to that used for S-R flip-flops, except that J and K can be 1 simultaneously, in which case the flip-flop changes state.

**FIGURE 12-22**  
Transition Graph  
for Counter

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**TABLE 12-7**  
J-K Flip-Flop  
Inputs

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(a)				(b)				(c)			
J	K	Q	Q <sup>+</sup>	Q	Q <sup>+</sup>	J	K	Q	Q <sup>+</sup>	J	K
0	0	0	0	0	0	{ 0 0		0	0	{ 0 X	
0	0	1	1			{ 0 1				{ 1 X	
0	1	0	0	0	1	{ 1 0		1	0	{ X 1	
0	1	1	0			{ 1 1				{ X 0	
1	0	0	1	1	0	{ 0 1		1	0	{ 0 0	
1	0	1	1			{ 1 1				{ 1 0	
1	1	0	1	1	1	{ 0 0		1	1	{ 0 0	
1	1	1	0			{ 1 0				{ 1 0	

**TABLE 12-8**

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C	B	A	C <sup>+</sup>	B <sup>+</sup>	A <sup>+</sup>	J <sub>C</sub>	K <sub>C</sub>	J <sub>B</sub>	K <sub>B</sub>	J <sub>A</sub>	K <sub>A</sub>
0	0	0	1	0	0	1	X	0	X	0	X
0	0	1	—	—	—	X	X	X	X	X	X
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	0	0	0	0	X	X	1	X	1
1	0	0	1	1	1	X	0	1	X	1	X
1	0	1	—	—	—	X	X	X	X	X	X
1	1	0	—	—	—	X	X	X	X	X	X
1	1	1	0	1	0	X	1	X	0	X	1

**TABLE 12-3**  
Transition Table for  
Figure 12-22

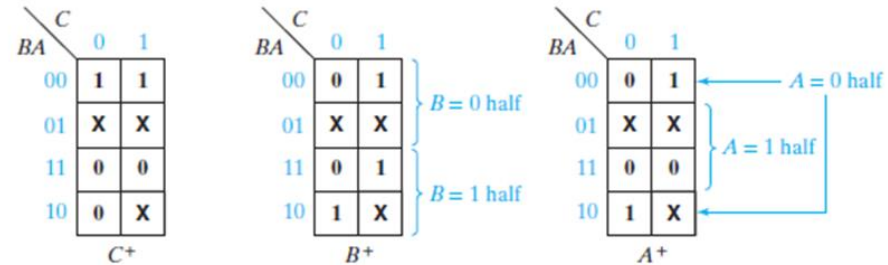
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C	B	A	C <sup>+</sup>	B <sup>+</sup>	A <sup>+</sup>
0	0	0	1	0	0
0	0	1	—	—	—
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	—	—	—
1	1	0	—	—	—
1	1	1	0	1	0

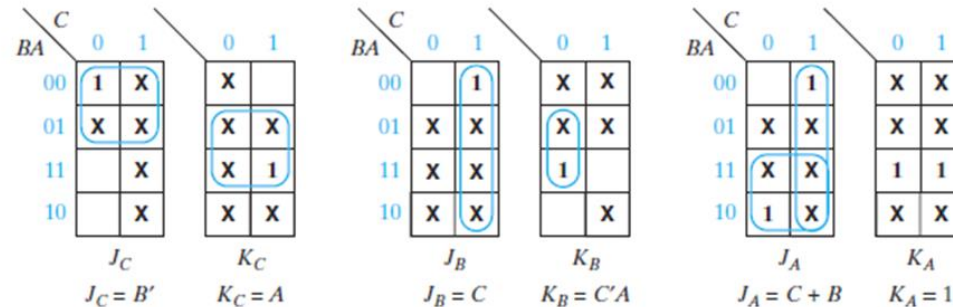
# Counter Design Using S-R and J-K Flip-Flops

## Counter Design Using J-K Flip-Flop:

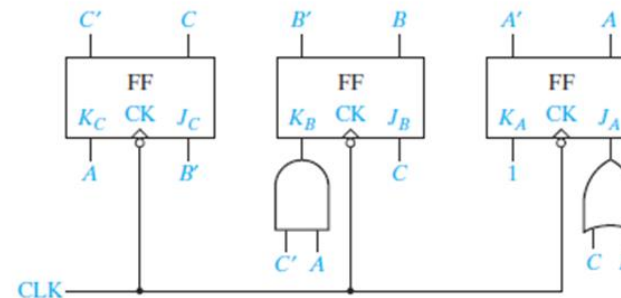
**FIGURE 12-31**  
Counter of  
Figure 12-22 Using  
J-K Flip-Flops  
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(a) Next-state maps



(b) J-K flip-flop input equations



(c) Logic circuit (omitting the feedback lines)

# Derivation of Flip-Flop Input Equation - Summary

## Summary:

- The input equation for the flip-flops in a sequential circuit may be derived from the next-state equations by using truth tables or by using Karnaugh maps.
- For the D flip-flop, the input is the same as the next state.
- For the T flip-flop, the input is 1 whenever a state change is required.
- For the S-R flip-flop,  $S$  is 1 whenever the flip-flop must be set to 1 and  $R$  is 1 when it must be reset to 0.
- For a J-K flip-flop, the J and K inputs are the same as S and R, respectively, except that when one input is 1 the other input is X.

Determination of Flip-Flop Input Equations from Next-State Equations Using Karnaugh Maps		Type of Flip-Flop	Input	$Q = 0$		$Q = 1$		Rules for Forming Input Map From Next-State Map*	
				$Q^+ = 0$	$Q^+ = 1$	$Q^+ = 0$	$Q^+ = 1$	$Q = 0$ Half of Map	$Q = 1$ Half of Map
© Cengage Learning 2014	Delay	$D$		0	1	0	1	no change	no change
	Toggle	$T$		0	1	1	0	no change	complement
	Set-Reset	$S$		0	1	0	X	no change	replace 1's with X's**
		$R$		X	0	1	0	replace 0's with X's**	complement
	J-K	$J$		0	1	X	X	no change	fill in with X's
		$K$		X	X	1	0	fill in with X's	complement

$Q^+$  means the next state of  $Q$

X is a don't-care

\*Always copy X's from the next-state map onto the input maps first.

\*\*Fill in the remaining squares with 0's.

# Derivation of Flip-Flop Input Equation - Summary

## Example 1:

Example  
(illustrating  
the use of  
Table 12-9)

AB \ Q		
	0	1
00	0	1
01	1	0
11	0	0
10	1	X

$Q^+$

Next-state map

AB \ Q		
	0	1
00	0	1
01	1	0
11	0	0
10	1	X

$D = Q'A'B + QB' + AB'$

D input map

AB \ Q		
	0	1
00	0	0
01	1	1
11	0	1
10	1	X

$T = A'B + AB' + QB$

T input map

AB \ Q		
	0	1
00	0	X
01	1	0
11	0	0
10	1	X

$S = AB' + Q'A'B$

S-R input maps

AB \ Q		
	0	1
00	X	0
01	0	1
11	X	1
10	0	X

$R = QB$

AB \ Q		
	0	1
00	0	X
01	1	X
11	0	X
10	1	X

$J = A'B + AB'$

J-K input maps

AB \ Q		
	0	1
00	X	0
01	X	1
11	X	1
10	X	X

$K = B$

For the S-R flip-flop, note that when  $Q = 0$ ,  $R = X$  if  $Q^+ = 0$ ; and when  $Q = 1$ ,  $R = 1$  if  $Q^+ = 0$ . Therefore, to form the  $R$  map from the  $Q^+$  map, replace 0's with X's on the  $Q = 0$  half of the map and replace 0's with 1's on the  $Q = 1$  half (and fill in 0's for the remaining entries). Similarly, to form the  $S$  map from the  $Q^+$  map, copy the 1's on the  $Q = 0$  half of the map, and replace the 1's with X's on the  $Q = 1$  half.