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Homework Unit 4 Solutions

1. A combinational logic circuit has four inputs (A, B, C, and D) and one output Z. The output is 1 iff the input has three consecutive 0's or three consecutive 1's. For example, if A=1, B=0, C=0, and D=0, then Z=1, but if A=0, B=1, C=0, and D=0, then Z=0. Design the circuit using <u>one</u> four-input OR gate and <u>four</u> three-input AND gates.

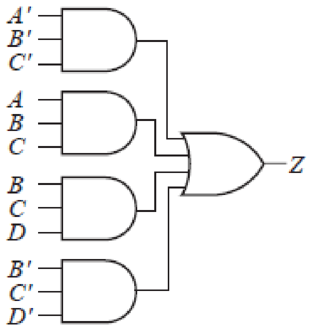
   Sol.)

$ABCD$	Z
0 0 0 0	1
0 0 0 1	1
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	0
1 0 1 0	0
1 0 1 1	0
1 1 0 0	0
1 1 0 1	0
1 1 1 0	1
1 1 1 1	1

$$Z = A'B'C'D' + A'B'C'D + AB'C'D' + ABCD' + ABCD + A'BCD$$
$$= A'B'C' + ABC + AB'C'D' + A'BCD$$
$$= A'B'C' + ABC + AB'C'D' + A'BCD + \underline{BCD} + \underline{B'C'D'}$$

(Added consensus terms)

$$\therefore Z = A'B'C' + ABC + BCD + B'C'D'$$



2. Given $F_1 = \prod M(0, 4, 5, 6)$ and $F_2 = \prod M(0, 4, 7)$, find the maxterm expansion for $F_1\ F_2$. State a general rule for finding the maxterm expansion of $F_1\ F_2$ given the maxterm expansions of F_1 and F_2. Prove your answer by using the general form of the maxterm expansion.

   Sol.) $F_1\ F_2 = \prod M(0, 4, 5, 6, 7)$. General rule: $F_1\ F_2$ is the product of all maxterms that are not present in either F_1 or F_2.

   Proof: Let $F_1 = \prod (a_i + M_i)$; $F_2 = \prod (b_j + M_j)$;

$$F_1\ F_2 = \prod (a_i + M_i) \prod (b_j + M_j) = (a_0 + M_0)(b_0 + M_0)(a_1 + M_1)(b_1 + M_1) \cdots$$
$$= (a_0 b_0 + M_0)(a_1 b_1 + M_1)(a_2 b_2 + M_2) \cdots = \prod (a_i b_i + M_i)$$

   Maxterm M_i is present in $F_1\ F_2$ iff $a_i b_i = 0$, i.e., if either $a_i = 0$ or $b_i = 0$. Maxterm M_i is present in F_1 iff $a_i = 0$. Maxterm M_i is present in F_2 iff $b_i = 0$. Therefore, maxterm M_i is present in $F_1\ F_2$

iff it is present in F_1 or F_2.

3. Given f(a, b, c) = a (b+c′).

Sol.) f(a,b,c) = a (b+c′) = ab+ac′ = ab(c+c′) + a(b+b′)c′ = abc + abc′ + abc′ + ab′c′.

= abc + abc′ + ab′c′ = m_7 + m_6 + m_4.

(a) Express f as a minterm expansion (use m-notation).

Sol.) $f = \sum m(4, 6, 7)$

(b) Express f as a maxterm expansion (use M-notation).

Sol.) $f = \prod M(0, 1, 2, 3, 5)$

(c) Express f' as a minterm expansion (use m-notation).

Sol.) $f' = \sum m(0, 1, 2, 3, 5)$

(d) Express f' as a maxterm expansion (use M-notation).

Sol.) $f' = \prod M(4, 6, 7)$

4. (a) If m_1 and m_2 are minterms of n variables, prove that $m_1 + m_2 = m_1 \oplus m_2$.

Sol.) $m_1 + m_2 = m_1(m_2' + m_2) + (m_1' + m_1)\, m_2 = m_1 m_2' + m_1 m_2 + m_1' m_2$

But $m_1 m_2 = 0$, so $m_1 + m_2 = m_1 m_2' + m_1' m_2 = m_1 \oplus m_2$.

(b) Prove that any switching function can be written as the exclusive OR sum of products where each product does not contain a complemented literal. (*Hint*: Start with the function written as a sum of minterms and use part (a).)

Sol.) Using part (a), any function can be written as the exclusive OR sum of its minterms. However, if a product contains a complemented literal, it can be written as the exclusive OR sum of two products without a complemented literal by using

$$x'p = (x \oplus 1)p = xp \oplus p.$$

By repeated application of the preceding relationship, all complemented literals can be removed from the products.