Name_________ **Student ID**___________ **Colleges & Schools**__________ **Department**___________

Homework Unit 6 Solutions

1. Find all of the prime implicants using the Quine-McCluskey method, and then using a prime implicant chart, find all mimimum sum-of-products solutions for the function *f*.

$$f(a, b, c, d) = \sum m(2, 4, 5, 6, 9, 10, 11, 12, 13, 15)$$

Sol.)

2	0010✓	2, 6	0-10	a'cd'	4, 5, 12, 13	-10-	bc'
4	0100✓	2, 10	-010	b'cd'	~~4, 12, 5, 13~~	~~-10-~~	
5	0101✓	4, 5	010-✓		9, 11, 13, 15	1--1	ad
6	0110✓	4, 6	01-0	a'bd'	~~9, 13, 11, 15~~	~~1--1~~	
9	1001✓	4, 12	-100✓				
10	1010✓	5, 13	-101✓				
12	1100✓	9, 11	10-1✓				
11	1011✓	9, 13	1-01✓				
13	1101✓	10, 11	101-	ab'c			
15	1111✓	12, 13	110-✓				
		11, 15	1-11✓				
		13, 15	11-1✓				

Prime implicants: bc', ad, a'cd', b'cd', a'bd', ab'c

		2	4	5	6	9	10	11	12	13	15
2, 6	a'cd'	×			×						
2, 10	b'cd'	×					×				
4, 6	a'bd'		×		×						
10, 11	ab'c						×	×			
4, 5, 12, 13	bc'		×	⊗					⊗	×	
9, 11, 13, 15	ad					⊗		×		×	⊗

Miminum sum-of-products solutions:

f = bc' + ad + a'cd' + b'cd'

f = bc' + ad + a'cd' + ab'c

f = bc' + ad + a'bd' + b'cd'

2. Packages arrive at the stockroom and are delivered on carts to offices and laboratories by student employees. The carts and packages are various sizes and shapes. The students are paid according to the carts used. There are five carts and the pay for their use is

Cart C1: \$2, Cart C2: \$1, Cart C3: \$4, Cart C4: \$2, Cart C5: \$2

On a particular day, seven packages arrive, and they can be delivered using five carts as follows:

C1 can be used for packages P1, P3, and P4.

C2 can be used for packages P2, P5, and P6.

C3 can be used for packages P1, P2, P5, P6, and P7.

C4 can be used for packages P3, P6, and P7.

C5 can be used for packages P2 and P4.

The stockroom manager wants the packages delivered at minimum cost. Using minimization techniques described in this unit, present a systematic procedure for finding the minimum cost solution.

Sol.) The carts and packages can be represented as the following table.

	Package						
	1	2	3	4	5	6	7
1	X		X	X			
2		X			X	X	
Cart 3	X	X			X	X	X
4			X			X	X
5		X		X			

Using Petrick's method:

P = (C1+C3)(C2+C3+C5)(C1+C4)(C1+C5)(C2+C3)(C2+C3+C4)(C3+C4)

= (C1C2+C1C5+C3)(C1+C4C5)(C2C4+C3)

= (C1C2+C1C5+C1C3+C3C4C5)(C2C4+C3)

= C1C2C4+C1C3+C3C4C5

Each product term specifies a non-redundant combination of carts that can be used to deliver the packages. The minimal cart solution, using carts C1 and C3, costs \$6. However, using the three carts C1, C2 and C4 costs only \$5 so it is the minimal cost solution desired by the stockroom manager.

3. Shown below is the prime implicant chart for a completely specified four-variable combinational logic function $U(w, x, y, z)$.

(a) Algebraically express U as a product of maxterms.

Sol.) All of the minterms of the function U can be expressed as (0, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15). Thus, all of the maxterms of the function U turns out to be (1, 2, 3, 12).

$U = \prod M(1, 2, 3, 12) = (w+x+y+z')\,(w+x+y'+z)\,(w+x+y'+z')\,(w'+x'+y+z)$

(b) Give algebraic expression for the prime implicants labeled A, C, and D in the table.

Sol.) Prime implicant A: (0, 4) = $w'y'z'$

Prime implicant C: (6, 7, 13, 15) = xy

Prime implicant D: (8, 9, 10, 11) = wx'

(c) Find all minimal sum-of-product expressions for *U*. You do not have to give algebraic expressions; instead just list the prime implicants (A, B, C, etc.) required in the sum(s).

Sol.) Using Petrick's method

P = (A+G)(A+H)(B+H)(C+H)(B+C+H)(D+G)(D+F)(D+E)(D+E+F)(B+F)(C+E)(B+C+E+F)

= (A+G)(ABC+H)(D+EFG)(B+F)(C+E)

= (ABC+AH+GH)(BCD+CDF+BDE+DEF+EFG)

= ABCD + ABCDF + ABCDE + ABCDEF + ABCEFG

+ ABCDH + ACDFH + ABDEH + ADEFH + AEFGH

+ BCDGH + CDFGH + BDEGH + DEFGH + EFGH

Thus, the minimum sum-of-products expressions for the function U turn out to be ABCD and EFGH.

	0	4	5	6	7	8	9	10	11	13	14	15
A	×	×										
B			×		×					×		×
C				×	×						×	×
D						×	×	×	×			
E								×	×		×	×
F							×		×	×		×
G	×					×						
H		×	×	×	×							

4. Find all prime implicants of the following function, and then find all minimum solutions using Petrick's method:

$F(A, B, C, D) = \sum m(7, 12, 14, 15) + \sum d(1, 3, 5, 8, 10, 11, 13)$

Sol.) Prime implicants: AC, AD', AB, CD, BD, $A'D$

Minimum solutions: $(AD'+CD)$; $(AD'+BD)$; $(AB+BD)$; $(AB+CD)$; $(AB+A'D)$