

# Ray Tracing

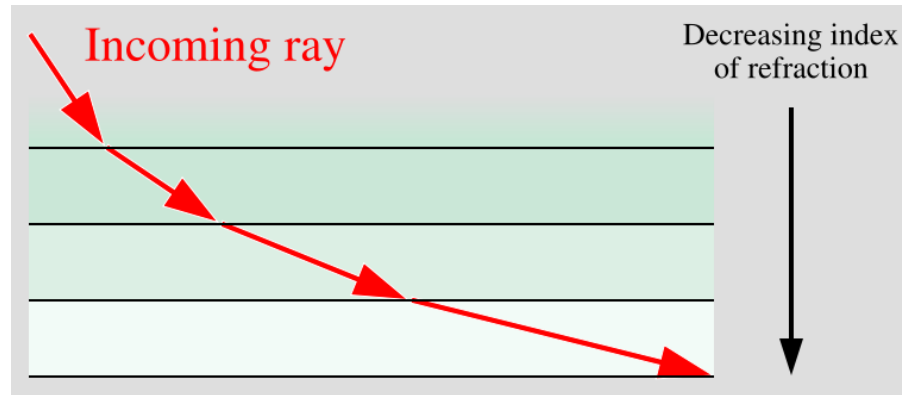
**Computer Graphics**  
**Instructor: Sungkil Lee**

# Overview

# Overview

- **Ray tracing in physics**

- a method for calculating the path of waves/particles through a system with regions of varying propagation velocity, absorption, and reflecting surfaces.
- Ray tracing solves the problem by **repeatedly advancing idealized narrow beams** called **rays** through the medium by discrete amounts.
- Simple problems can be analyzed using mathematics, but more detailed analysis can be performed by using a computer to propagate many rays.

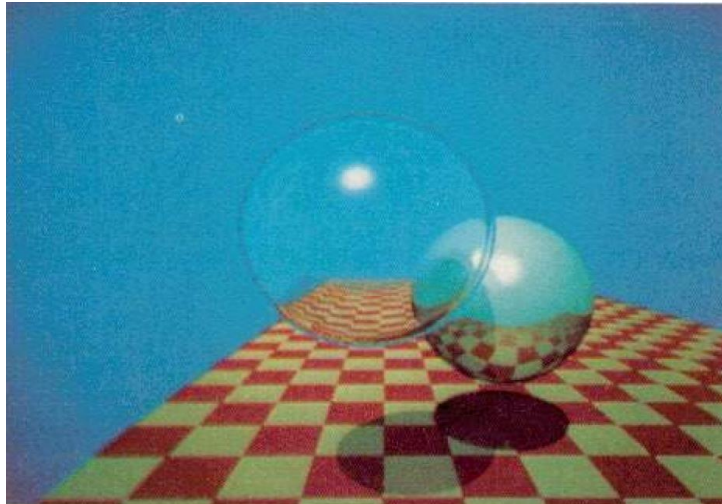


Ray tracing of a beam of light passing through a medium with changing refractive index.

# Overview

- **Ray tracing in computer graphics**

- The name stems from that of physics.
- The ray tracing indicates a **simple ray tracing** that recursively considers only **perfect reflection** and refraction.
- It is often referred to **Whitted ray tracing**.
  - An improved illumination model for shaded display [Turner Whitted, 1979]

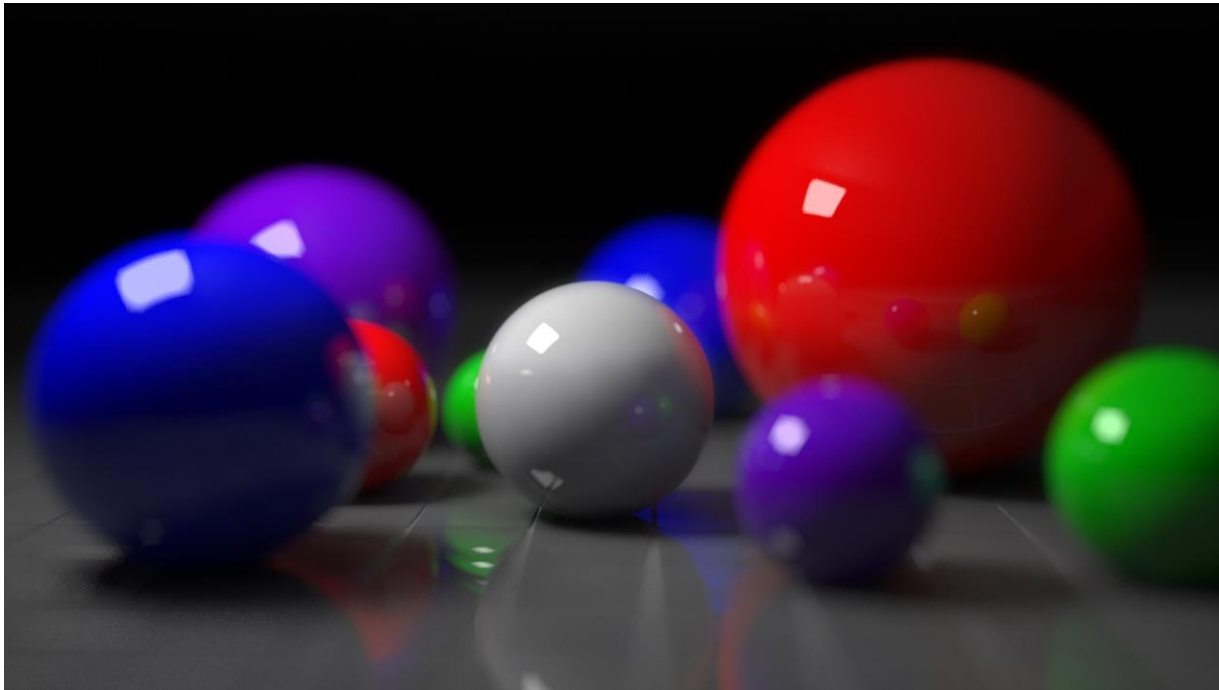


- More general ray tracing is referred to "**path tracing**," which integrates all the types of reflections [Kajiya, 1986].

# Overview

- **(Whitted) Ray tracing**

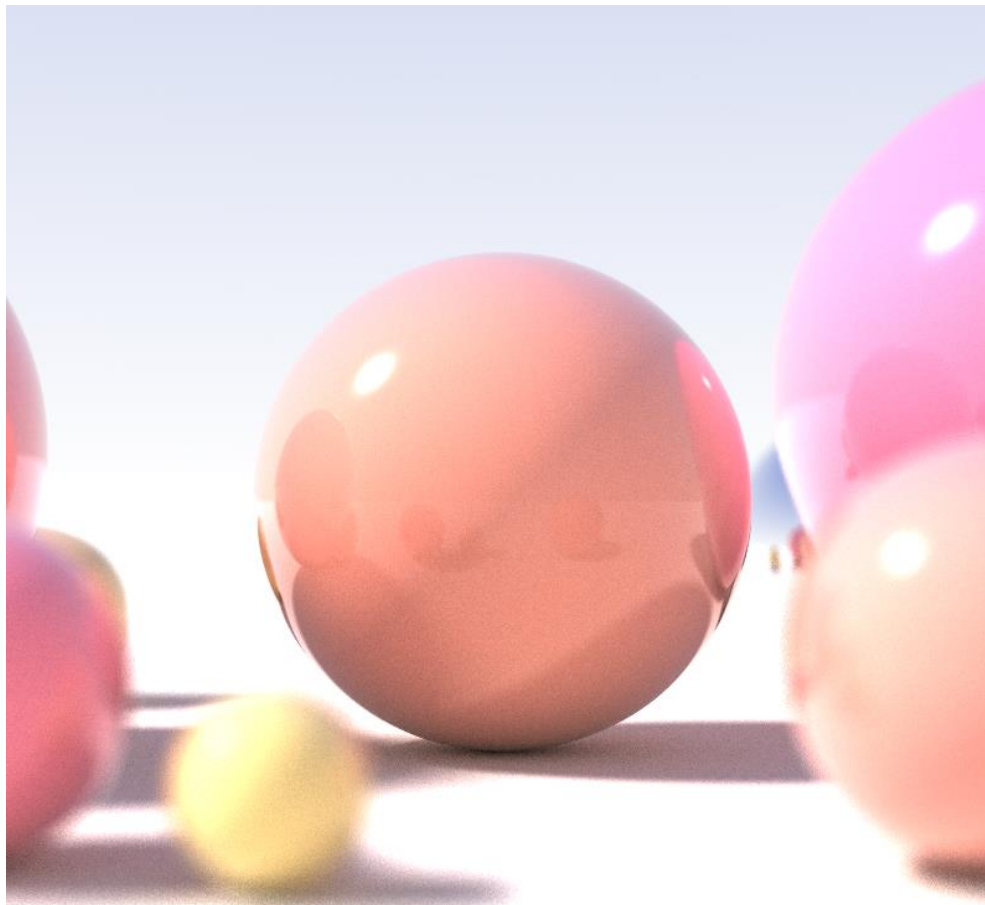
- The most basic step before starting global illumination techniques in CG.
- Approximate (in terms of illumination model; e.g., Phong model) but still better than pipeline/scanline approaches in quality.
- Scene description uses not only polygonal meshes but also implicit representations (e.g., analytical spheres, surfaces, ...)



# Overview

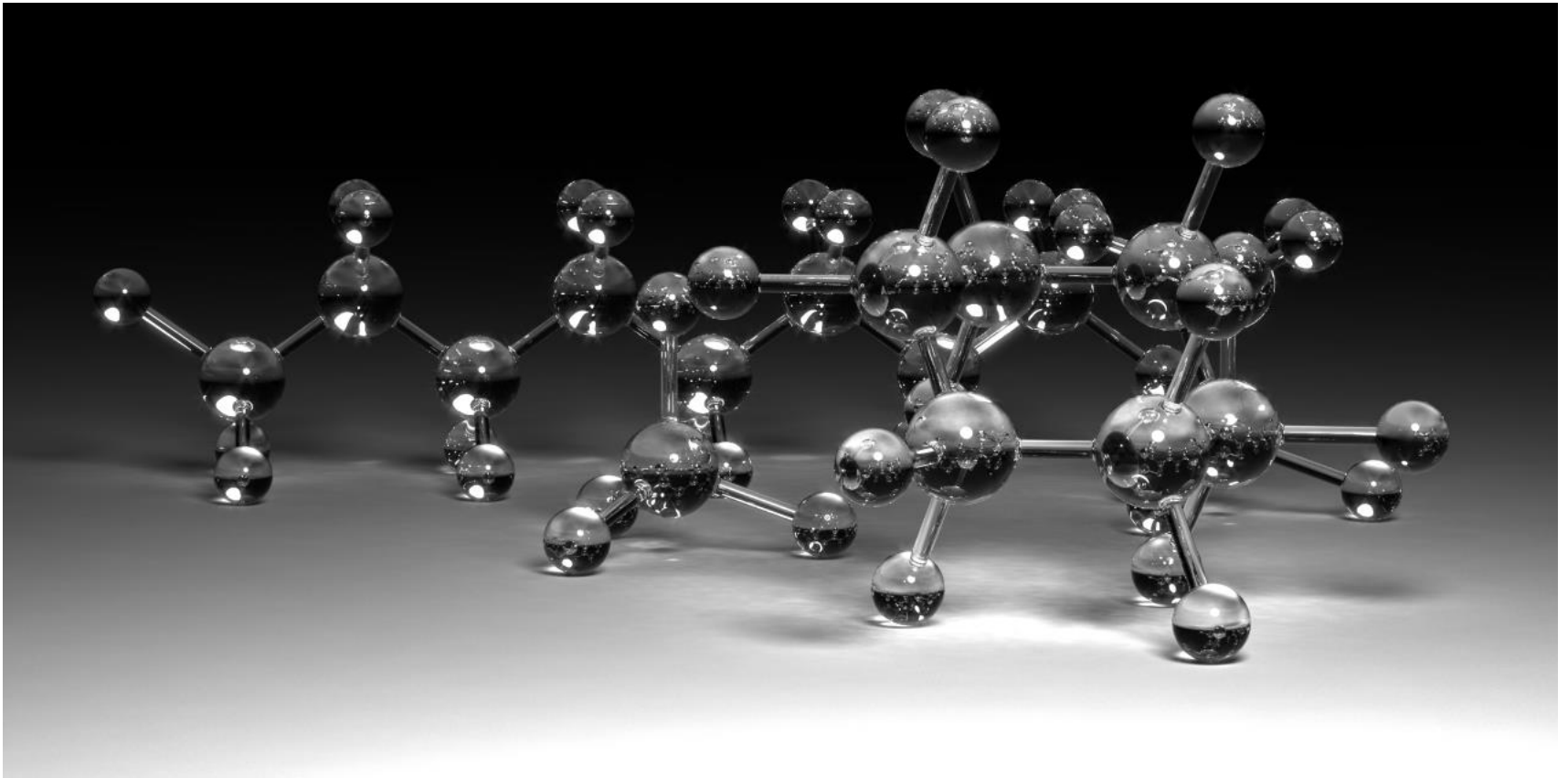
- **Effects in ray tracing (in Balls Example)**

- Observe inter-object (mirror) reflection, soft shadows and depth-of-field (defocus blur) effect



# Overview

- **Effects in ray tracing (in Molecules Example)**
  - Observe the [refraction](#) inside the glass atoms and [caustics](#) (the concentration of rays to a small bright area)

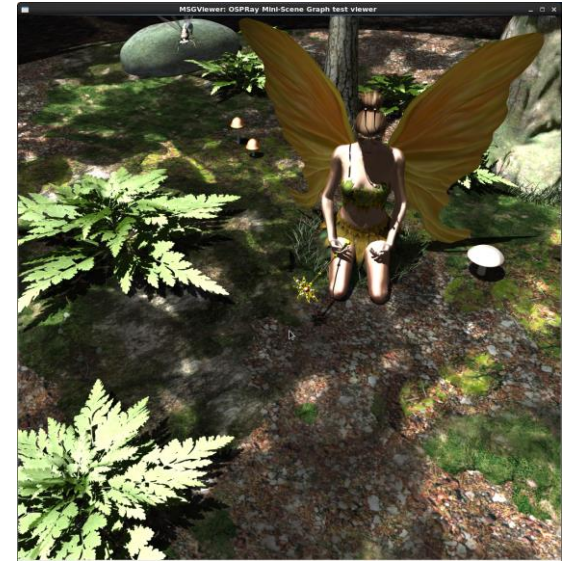
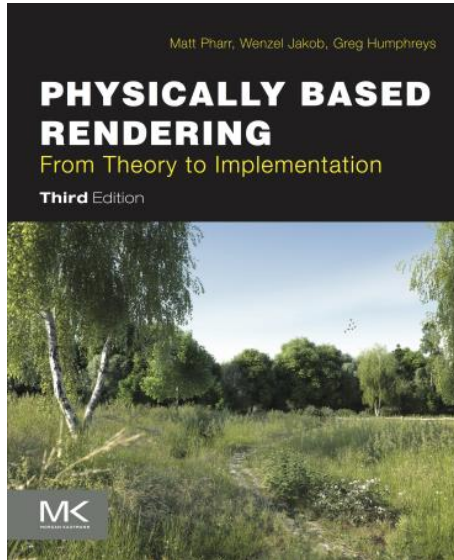




# Overview

- **Ray tracing implementation**

- Usually implemented in software: slow and batch rendering
- Common open-source software for ray tracing
  - PBRT, Povray, Intel Embree, Pixar's RenderMan, ...



PBRT book and the scenes from PovRay and Intel Embree



# Overview

- **Ray tracing implementation**

- Recent GPU implementations (e.g., NVIDIA RTX 2080)
  - DX12 RTX and Vulkan (since NVIDIA Volta arch.), which are the descendants of NVIDIA OptiX (written in CUDA) with ray tracing-specific APIs.

Unreal Engine demonstration with GPU Ray Tracing, *NVIDIA GTC 2018*



# **Ray Tracing Algorithms**

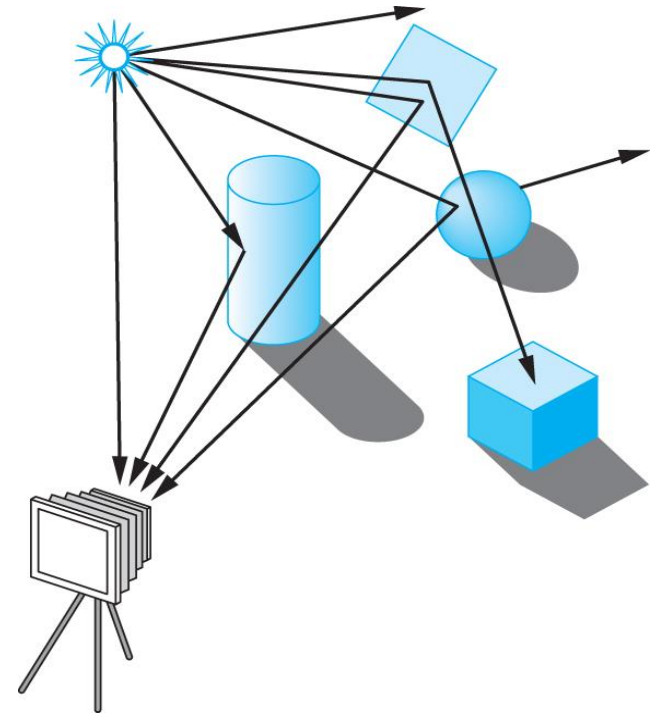
# Ray Tracing: Forward vs. Backward

- **Forward ray tracing:**

- Follow rays from a point light source
- Can account for reflections and transmission
- In general, computationally inefficient.
  - *most of rays are missing in camera*

- **An alternative for general cases:**

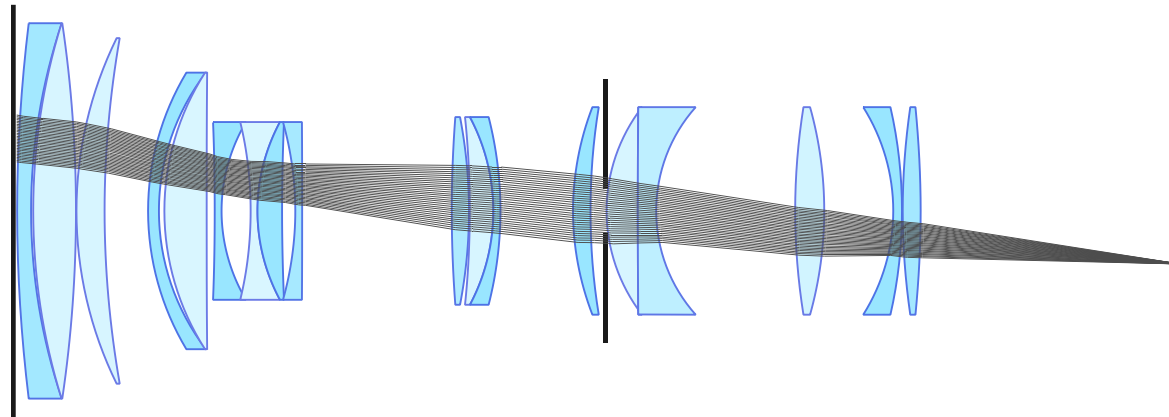
- **Backward** ray tracing (or **ray casting**)



# Ray Tracing: Forward vs. Backward

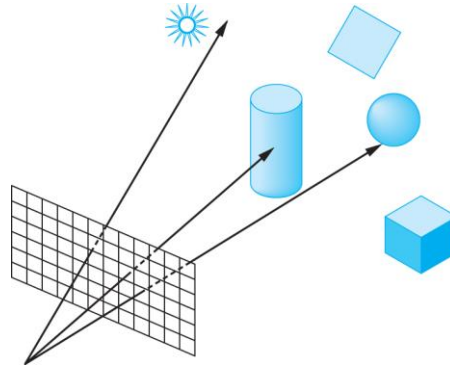
- **Forward ray tracing:**

- Given a known ray path, the forward tracing can be useful.
  - e.g., optical ray tracing (Hullin et al., 2011)

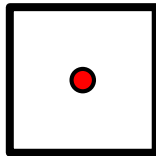


# Backward Ray Tracing

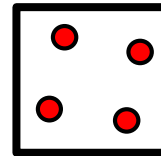
- **Only rays that reach the eye/camera matter.**
  - A ray goes in a *reverse* direction against the light path.



- **Need at least one ray/sample per pixel.**
  - More rays at different sampling locations suppress aliasing.



1 spp (samples per pixel)

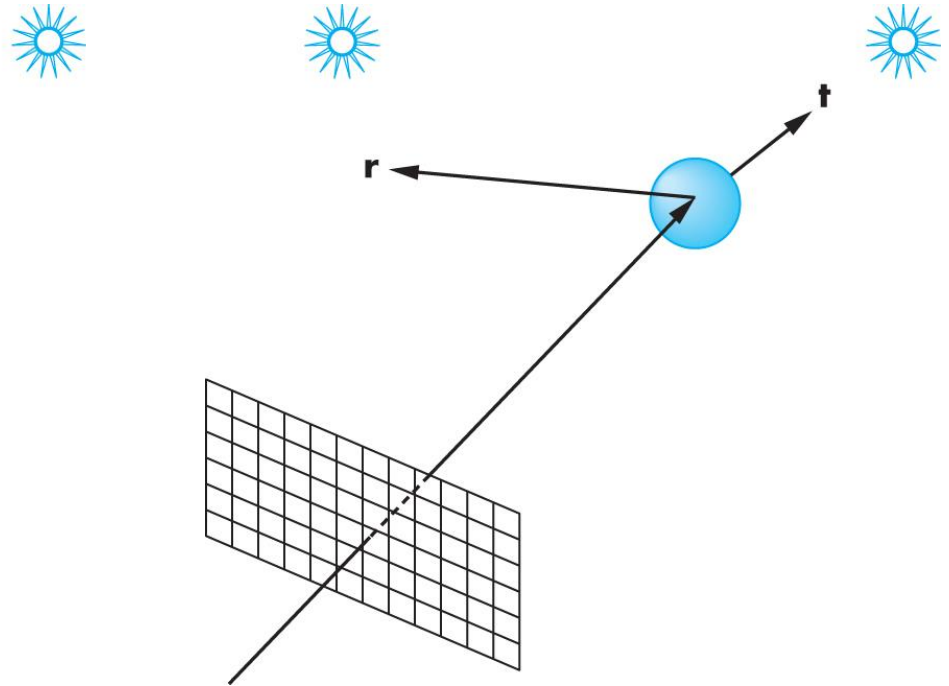
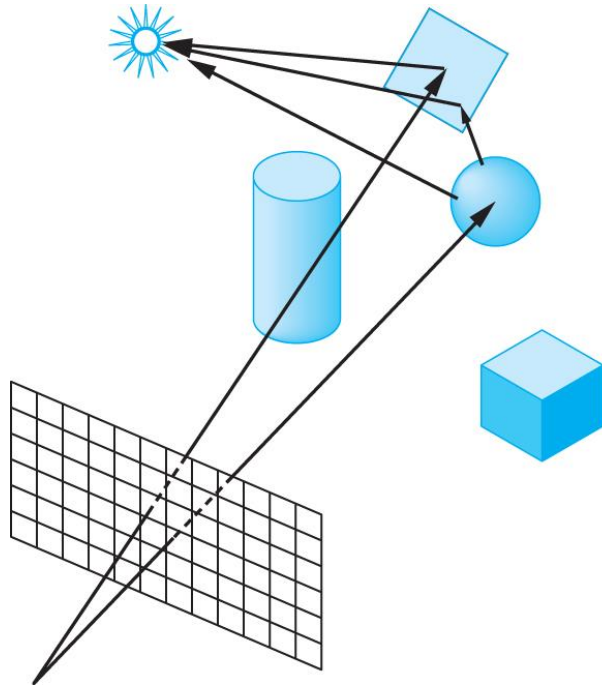


4 spp

# Reflection and Refraction

- **Reflection and refraction (transmission)**

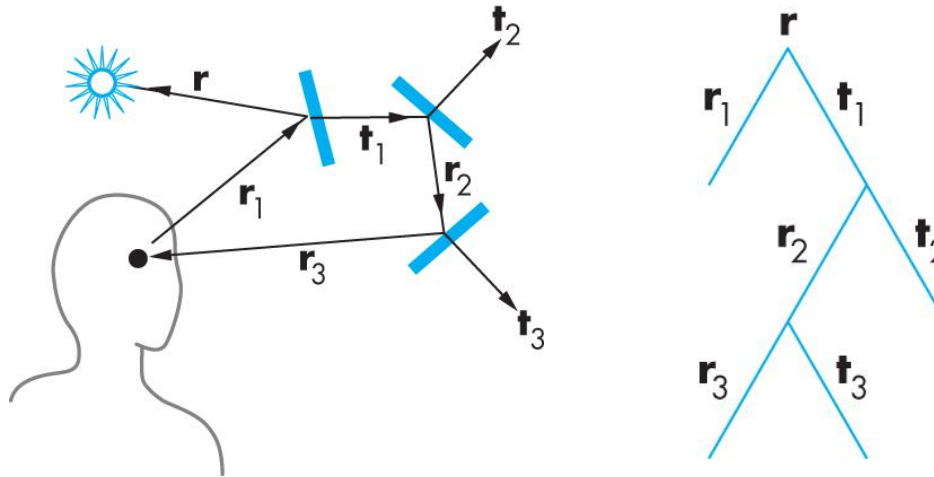
- In a simple ray tracing, only the mirror reflection is traced.



# Ray Tree

- **A binary tree can express the recursive ray tracing**

- Since there are only a single reflection, the binary tree is enough.
- The tree is simple, so, we can recursively traverse the tree.

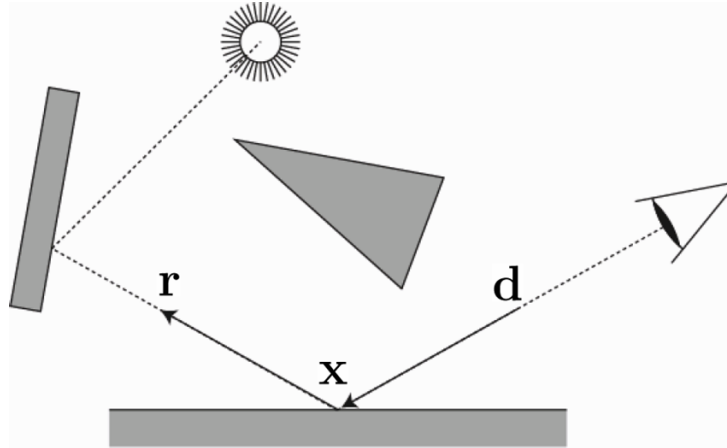


- For the path tracing, the tree can expand beyond the binary tree.
- Then, the recursive tracing might not fit with memory, which we need to come up with an alternative approach (e.g., Monte-Carlo solutions).



# Reflection

- **Mirror/perfect reflection**



- Recall how to derive a reflection vector in Phong illumination model.
- Given the incoming direction  $\mathbf{d}$  and the normal  $\mathbf{n}$ , the outgoing vector  $\mathbf{r}$  is:

$$\mathbf{r} = \mathbf{d} - 2(\mathbf{d} \cdot \mathbf{n})\mathbf{n}$$

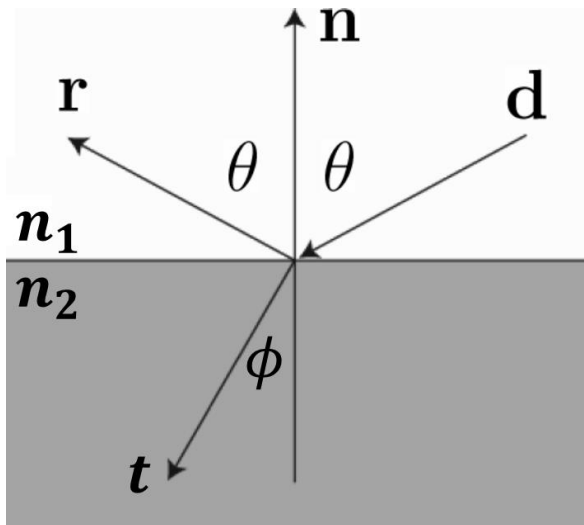
# Refraction

- **Dielectric materials**

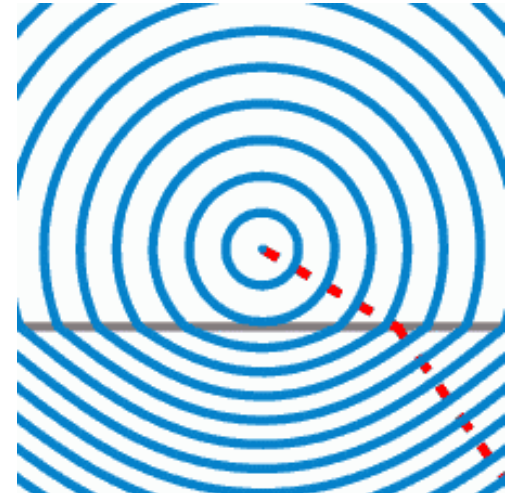
- Insulators that can be polarized by an electric field.
- Examples: diamond, glass, water, air

- **Snell's law**

- Light travels slower at a denser dielectric medium than the vacuum.
- Light is bent when it goes from one medium to another.
- Given the refractive indices  $n_1$  and  $n_2$ , the relationship is given by:



$$n_1 \sin \theta = n_2 \sin \phi$$



# **Ray Tracing Pseudocode**

# Pseudocode of Ray Tracing

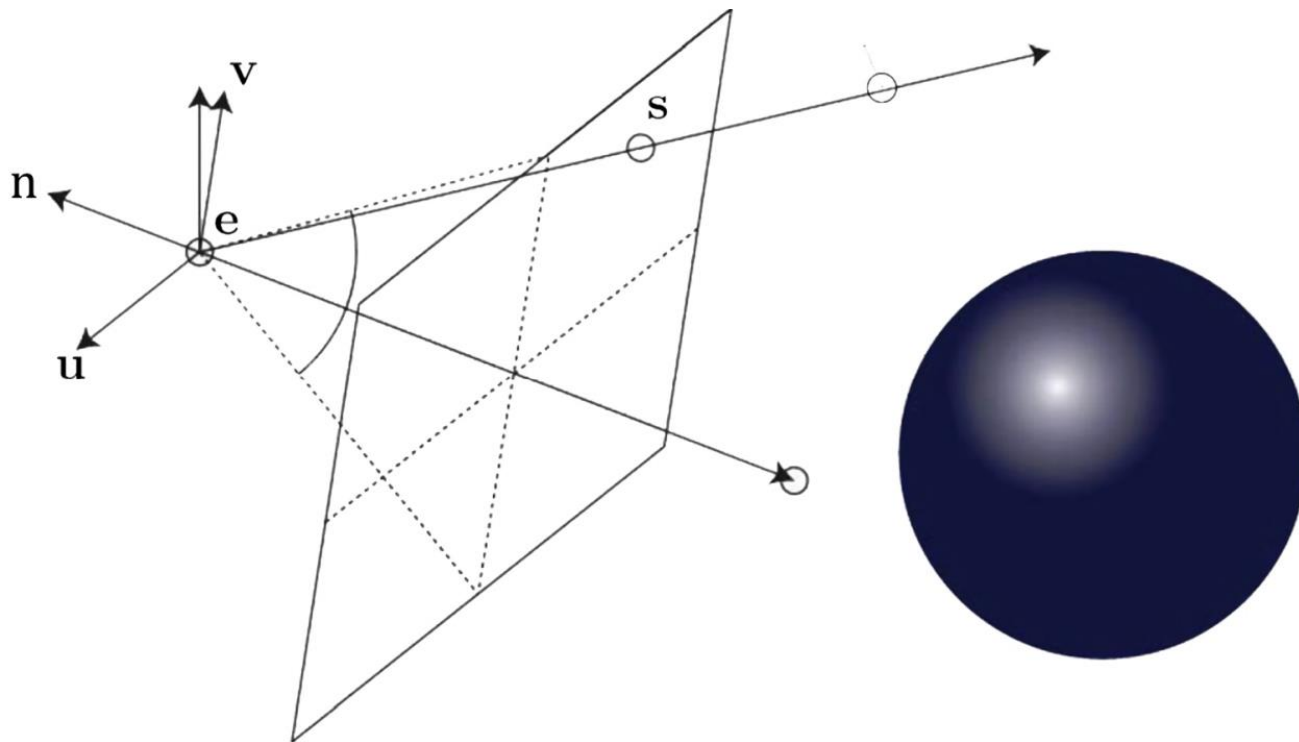
- **Given a scene representation, a main function can be:**

```
ray_trace()  
{  
    // acceleration structure (e.g., KD-tree or BVH)  
    construct scene representation  
  
    // main recursive loop  
    for each pixel  
    {  
        generate primary ray  
        color c = trace( primary ray, 0 )  
        assign c to the current pixel  
    }  
}
```

# Primary Rays

- **A primary ray  $p$ :**
  - emanates from  $e$  to  $s$  is defined as:

$$p(t) = e + t(s - e)$$



# Primary Rays

- **We derive primary rays in the camera frame, but transform them to the world frame.**

$$p(t) = e + t(s - e)$$

- Eye position  $e$ : simply the origin at the camera frame
- Given an screen coordinate  $(u, v) \in [0,1]^2$  for perspective viewing,
  - a 3D pixel position  $s$  at  $z = -1$  can be:

$$s(x, y, z) = (\tan(0.5 \cdot fovx) * (u - 0.5), \tan(0.5 \cdot fovy) * (v - 0.5), -1)$$

- Transform  $s$  and  $e$  to the world frame by multiplying inverse view matrix.
  - We prefer intersection tests in the [world frame](#) to in the [camera frame](#).
  - [Otherwise, all the objects require to be transformed to the camera frame](#) (as done in the pipeline), but it's too costly in ray tracing.

# Pseudocode

- **Given a scene representation, trace() can be:**
  - See the details in the following pages.

```
color trace( ray i, int step )
{
    if( step > max ) return background_color;

    status s = intersect(i,q); // q: output ray
    if(s==light_source) return light_source_color;
    if(s==no_intersection) return background_color;

    vec3 n = get_face_normal(q); // do not use a vertex normal
    vec3 r = reflect(q,n);
    vec3 t = refract(q,n);

    color local = phong_shade(q,n,r);
    color reflected = trace(ray(q,r), step+1);
    color refracted = trace(ray(q,t), step+1);

    return local + reflected + refracted;
}
```



# Stop Condition in Recursion

```
color trace( ray i, int step )
{
    if( step > max ) return background_color;
    ...
    status s = intersect(i,q); // q: output ray
    if(s==light_source) return light_source_color;
    if(s==no_intersection) return background_color;
}
```

- **When to stop:**

- Count steps (or recursion depth)
  - Typical option for implementation; e.g., 15 to 20
- Ignore rays that go off to infinity or to light sources
  - Put large sphere around problem
- Some light will be absorbed at each intersection
  - Track amount left

# Intersection

- **Rays are tested with primitives to find intersections.**

```
color trace( ray i, int step )
{
    ...
    status s = intersect(r,q);
    ...
}
```

- **There are many solutions on various primitives.**
  - Refer to "[Graphics Gems](#)" Books or other notes.
  - An acceleration will be handled in the acceleration structures.

```
point intersect( ray i, out int status )
{
    // implemented for plane, triangles, sphere, surfaces
}
```

# Ray-Triangle Intersection Test

- **Here, we see a way of testing intersection between a ray and a triangle.**

- Parametric description of a triangle with triangle vertices  $\mathbf{a}, \mathbf{b}, \mathbf{c}$ .

$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}), \text{ where } \beta > 0, \gamma > 0, \beta + \gamma < 1$$

- Ordering convention: counter-clockwise ordering of the vertices.
- Intersection condition for the primary ray :

$$\mathbf{e} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

- The condition can be written in matrix form, and solve for the three unknowns  $\beta, \gamma, t$ , in matrix form:

$$\begin{bmatrix} a_x - b_x & a_x - c_x & d_x \\ a_y - b_y & a_y - c_y & d_y \\ a_z - b_z & a_z - c_z & d_z \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} a_x - e_x \\ a_y - e_y \\ a_z - e_z \end{bmatrix}$$

# Reflection

- **Reflection is computed at the intersection**

```
color trace( ray i, int step )
{
    ...
    vec3 n = get_face_normal(q); // do not use a vertex normal
    vec3 r = reflect(q,n);
    vec3 t = refract(q,n);

    color local = phong_shade(q,n,r);
    color reflected = trace(ray(q,r), step+1);
    color refracted = trace(ray(q,t), step+1);
    return local + reflected + refracted;
}
```

- For triangle, face normal is computed by the cross product from vertices.

- **Example implementation of `reflect()`**

```
// I: incident vector, N: normal
vec3 reflect( vec3 I, vec3 N ){ return I-N*dot(I,N)*2.0f; }
```

# Transmission/Refraction

- **Some rays are refracted at intersection.**

```
color trace( ray i, int step )
{
    ...
    vec3 n = get_face_normal(q); // do not use a vertex normal
    vec3 r = reflect(q,n);
    vec3 t = refract(q,n); // need to provide refractive index
    color local = phong_shade(q,n,r);
    color reflected = trace(ray(q,r), step+1);
    color refracted = trace(ray(q,t), step+1);
    return local + reflected + refracted;
}
```

- **Example implementation of `refract()`**

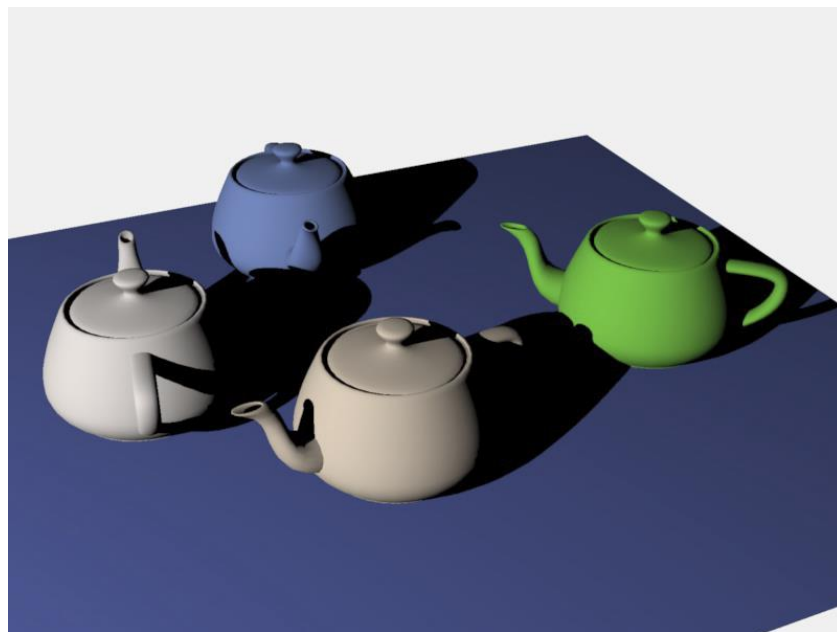
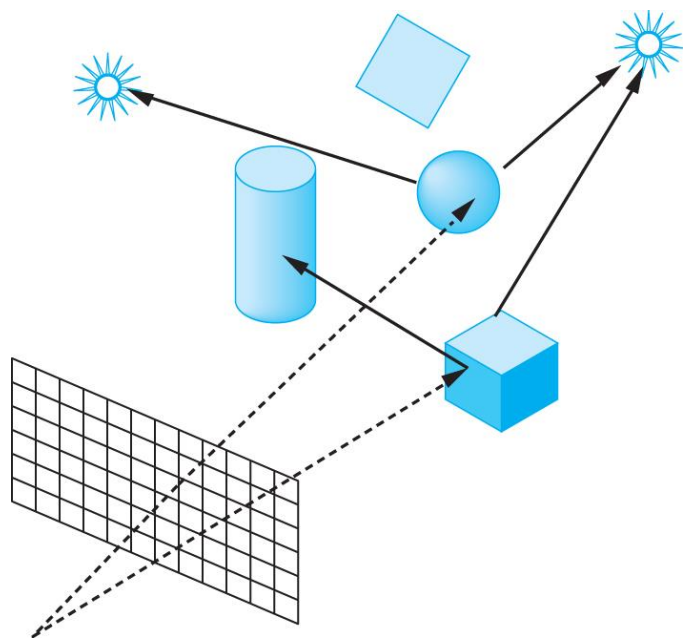
```
// I: incident vector, N: normal
vec3 refract( vec3 I, vec3 N, float eta /* = n0/n1 */ )
{
    float d=dot(I,N), k=1-eta*eta*(1-d*d);
    return k<0?0:(I*eta-N*(eta*d+sqrt(k)));
}
```

# Shadows

# Shadows

- **Shadow Rays**

- Even if a point is visible, it will not be lit unless we can see a light source from that point.
- Cast shadow rays to light sources for each intersection surface point.
  - For multiple light sources, we repeat the shadow ray casting.
  - Could be very slow with many light sources (i.e., soft shadows)



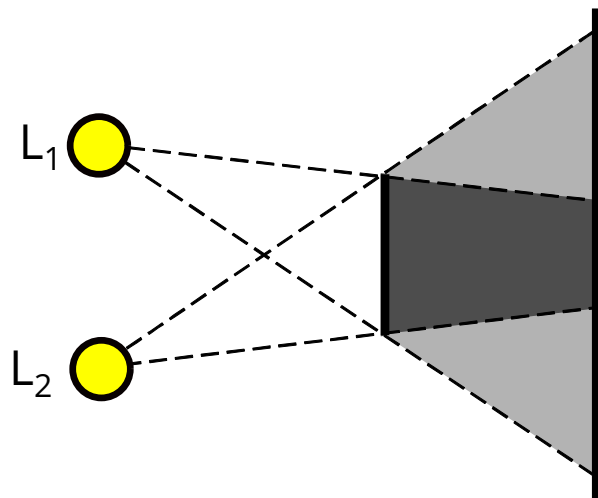
Example of hard shadows



# Soft Shadows

- **Soft shadows from multiple light sources**

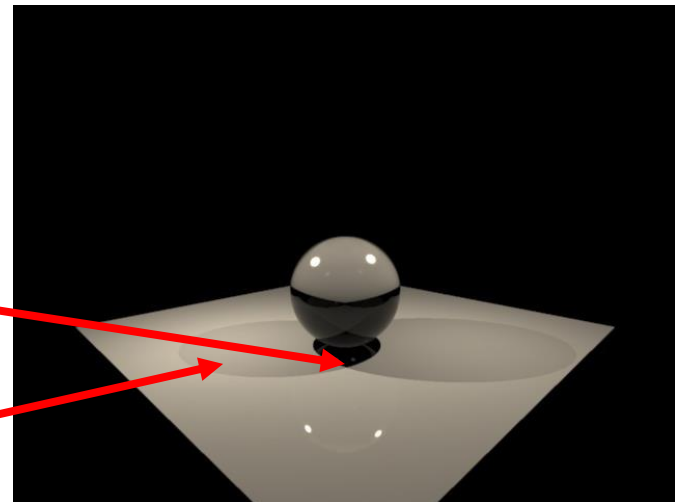
- **Umbra**: Innermost/darkest part of a shadow, completely occluded from the light source
- **Penumbra**: the region in which only a portion of the light source is obscured by the occluders



Penumbra

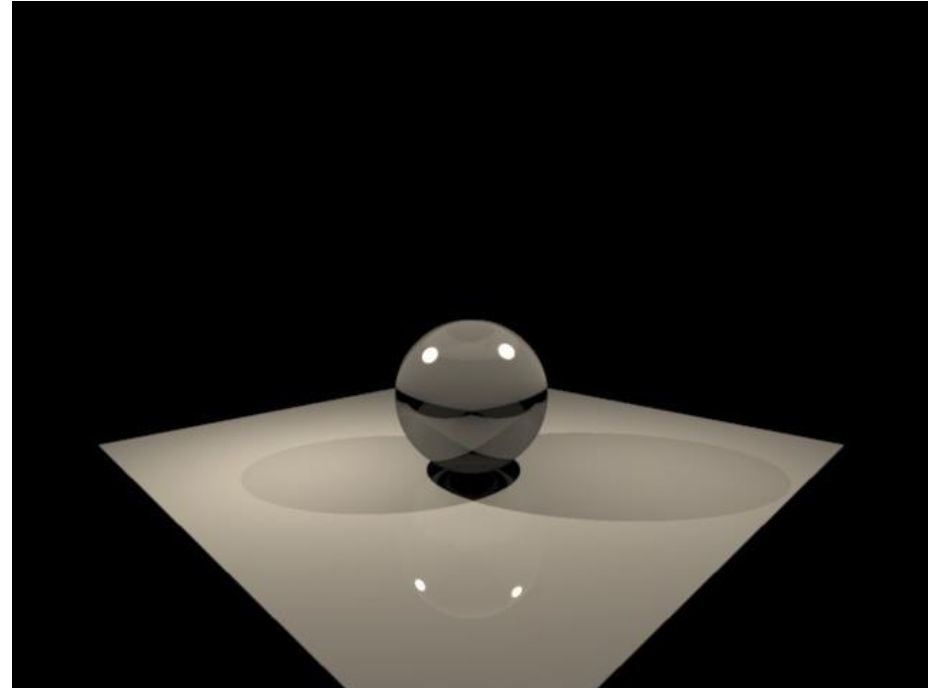
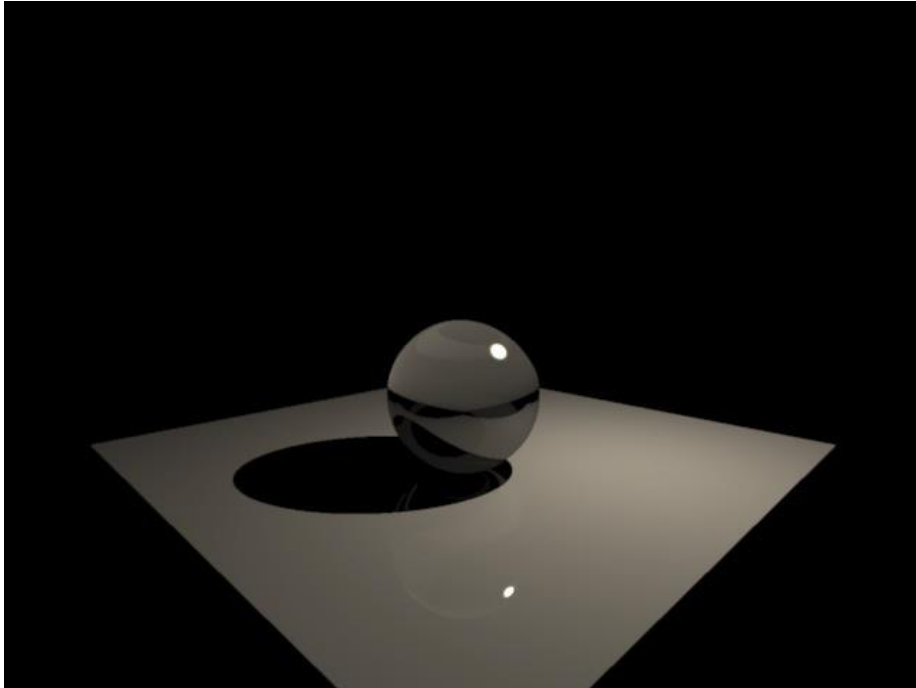
Umbra

Penumbra



# Examples

- **A glass sphere with one vs. two light sources**
  - Observe reflection and shadows (umbra and penumbra).

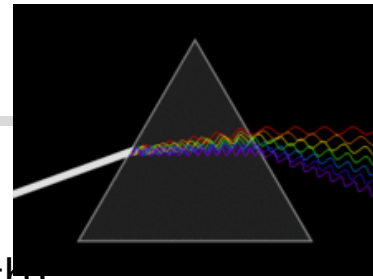


## **More on Reflection/Refraction**

# Dispersion

- **Refraction index varies with wavelength.**

- Typically, 1.5 for glass at  $\lambda = 587.6$  nm (reference wavelength)
  - but it may significantly change with other wavelengths.

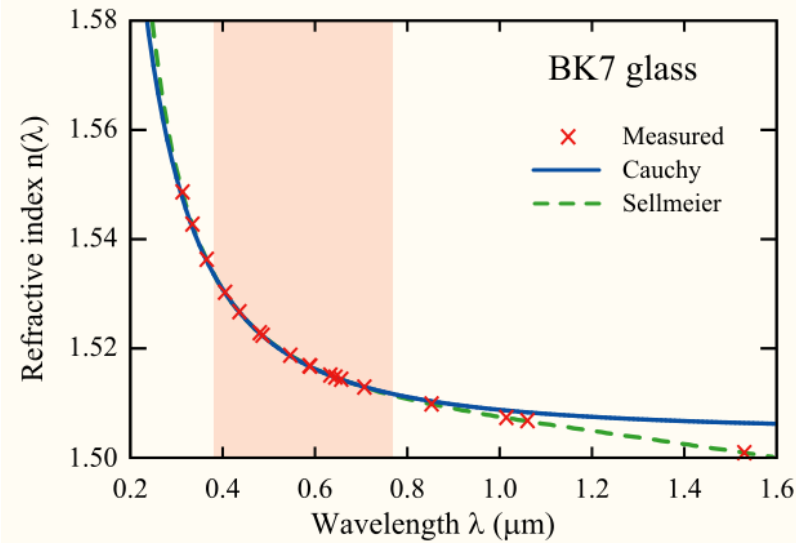


# Dispersion

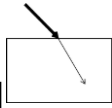
- **Refraction index varies with wavelength.**

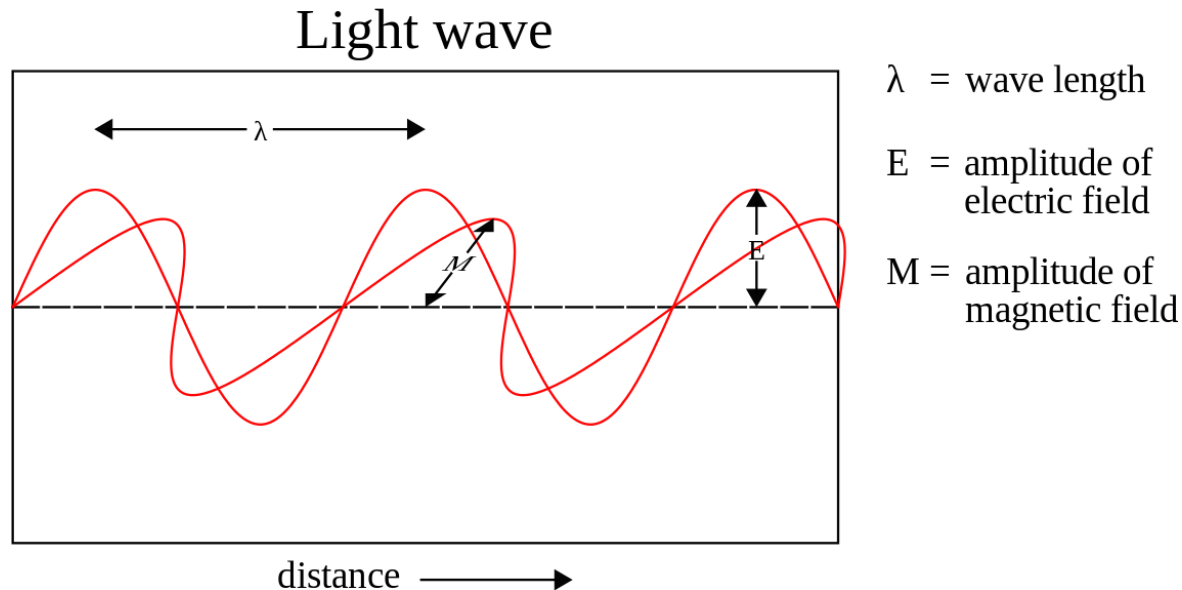
- Analytical formulae (Sellmeier or Cauchy equation) can be used to approximate dispersion.
- Example: Sellmeier's equation requiring 6 coefficients

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3}$$

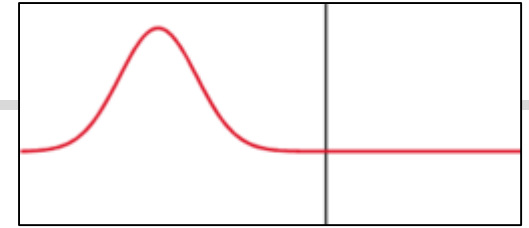


# Fresnel Equations

- **For specular refraction (  ), it computes how much light is reflected and refracted.**
  - Derived from the electromagnetic wave equations
  - Involved polarization of the waves
  - While natural light is partially polarized, direct sunlight or reflections from dielectrics are highly polarized.



# Fresnel Equations



- **Reflection of light is polarized**

- *Fresnel equations* describe how much light is reflected and transmitted.
- Reflectance for p- and s-polarized lights  $R_p$  and  $R_s$  can be derived as:

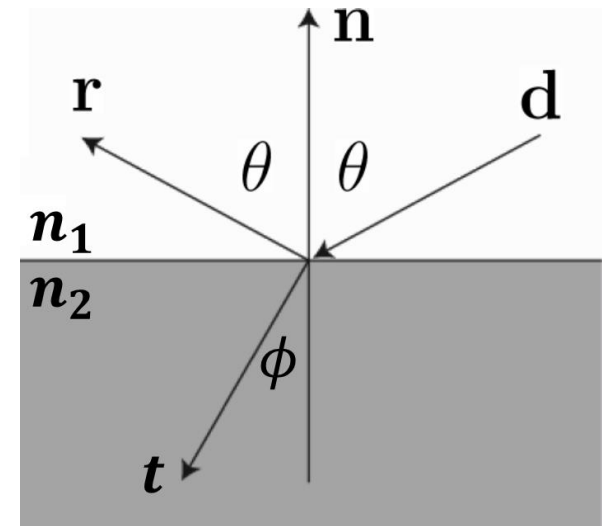
$$R_p = \left( \frac{n_2 \cos \theta - n_1 \cos \phi}{n_2 \cos \theta + n_1 \cos \phi} \right)^2$$

$$R_s = \left( \frac{n_1 \cos \theta - n_2 \cos \phi}{n_1 \cos \theta + n_2 \cos \phi} \right)^2$$

- The effective reflectance for natural lights can be the average of both:

$$R_{\text{effective}} = \frac{1}{2} (R_p + R_s)$$

$$T_{\text{effective}} = 1 - R_{\text{effective}}$$





# Fresnel Equations

- **Schlick approximation**

- The full equations are often too costly to evaluate, Schlick proposed the approximation below, using the reflectance at normal incidence.
- Given  $n_2$  and  $n_1$ , the reflectance at normal incidence (i.e.,  $\theta = \phi = 0$ ) is:

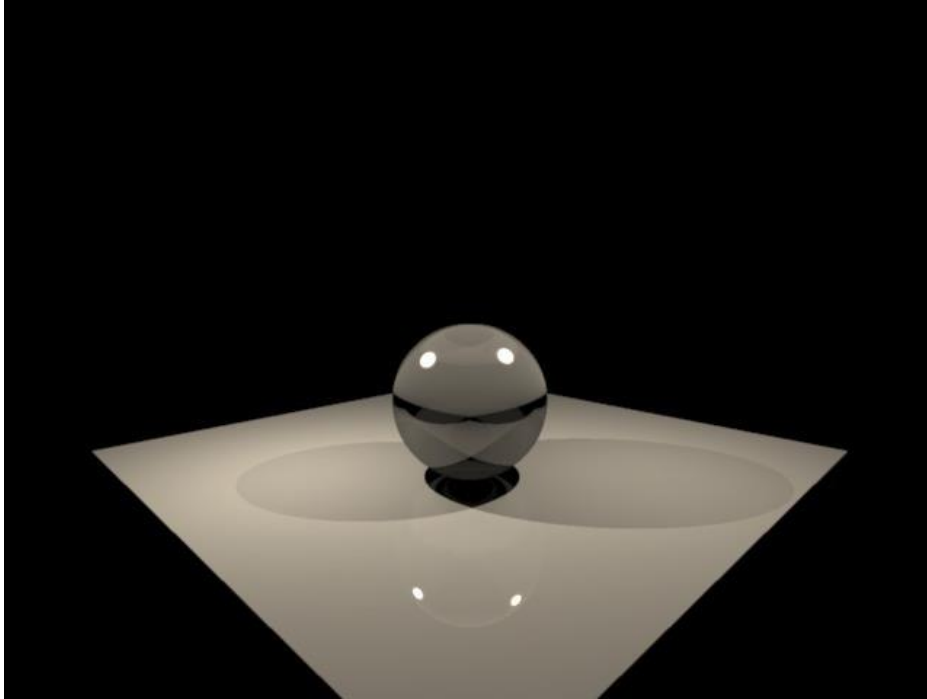
$$R_0 = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

- Then, a reflectance at angle  $\theta$  can be approximated as:

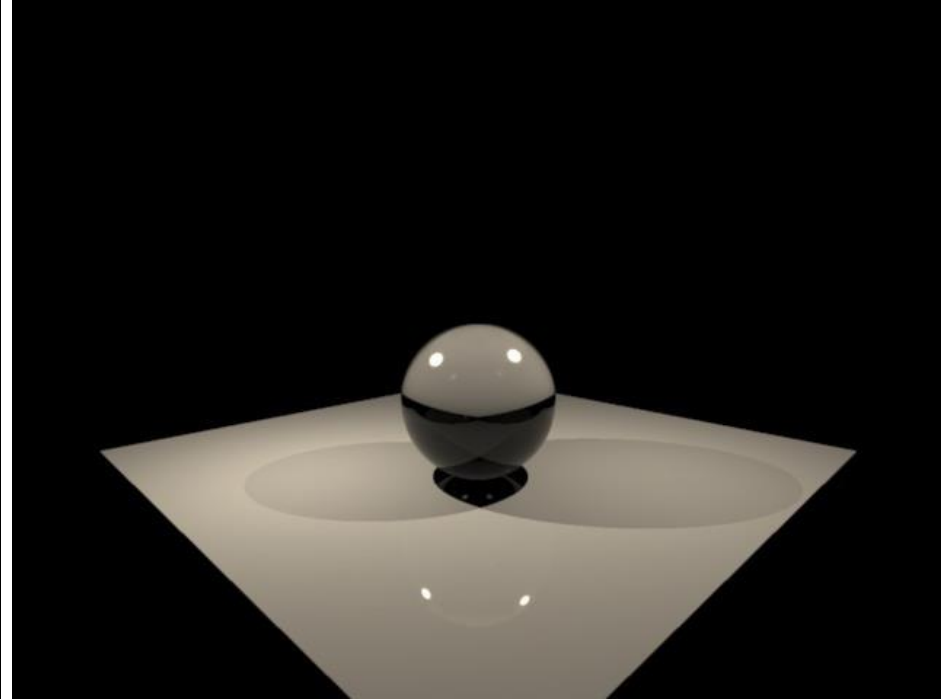
$$R(\theta) = R_0 + (1 - R_0)(1 - \cos\theta)^5$$

# Fresnel Equations

- **Examples**



Without Fresnel reflection



With Fresnel reflection