Name_________ Student ID___________ Colleges & Schools__________ Department___________

Homework Unit 1 Solutions

1. (a) Show how to represent each of the numbers (7-1), (7^2-1), and (7^3-1) as base 7 numbers.

   Ans.) (7-1) = 6_7, (7^2-1) = 66_7, (7^3-1) = 666_7

   (b) Generalize your answers to part (a) and show how to represent (k^n-1) as a base *k* number, where *k* can be any integer larger than 1 and *n* any integer larger than 0. Give a mathematical derivation of your result.

   Ans.) $(k^n-1) = (k - 1)\ k^{n-1} + k^{n-1} - 1$

   $= (k - 1)\ k^{n-1} + (k - 1)\ k^{n-2} + k^{n-2} - 1$

   $= (k - 1)\ k^{n-1} + (k - 1)\ k^{n-2} + k^{n-2} + \cdots (k-1)k + (k-1)$

   This expression is the polynomial expansion for the base k number with *n* digits (k-1)(k-1) ⋯(k-1)

2. (a) It is possible to have negative weights in a weighted code for the decimal digits, e.g., 8, 4, -2, and -1 can be used. Construct a table for this weighted code.

   Ans.)

	8 4 -2 -1
0	0 0 0 0
1	0 1 1 1
2	0 1 1 0
3	0 1 0 1
4	0 1 0 0
5	1 0 1 1
6	1 0 1 0
7	1 0 0 1
8	1 0 0 0
9	1 1 1 1

   (b) If *x* is a decimal digit in this code, how can the code for 9-*x* be obtained?

   Ans.) The 9's complement of a decimal number represented with this weighted code can be obtained by replacing 0's with 1's and 1's with 0's (bit-by-bit complement)

3. An alternative algorithm for converting a base 20 integer, $d_{n-1}d_{n-2} \cdots d_1d_0$, into a base 10 integer is stated as follows: Multiply d_i by 2^i and add i 0's on the right, and then add all of the results.

(a) Use this algorithm to convert $GA7_{20}$ to base 10. (G_{20} is 16_{10}.)

Ans.)

($G_{20}=16_{10}$) x ($2^2=4$) = 64, add 2 0's on the right to get 6,400

($A_{20}=10_{10}$) x ($2^1=2$) = 20, add 1 0's on the right to get 200

($7_{20}=7_{10}$) x ($2^0=1$) = 7

Thus, 6,400+200+7 = 6607

(b) Prove that this algorithm is valid.

Ans.)

$$(d_{n-1}d_{n-2}\cdots d_1d_0)_{20} = d_{n-1}20^{n-1}+ d_{n-2}20^{n-2}+\cdots+d_1 20^1+d_0 20^0$$

$$= d_{n-1}2^{n-1}10^{n-1}+ d_{n-2}2^{n-2}10^{n-2}+\cdots+d_1 2^1 10^1+d_0 2^0 10^0.$$

The general term in the expansion is $d_i 2^i 10^i$. The multiplication by 10^i adds i 0's on the right of $d_i 2^i$.

(c) Consider converting a base 20 fraction, $0.d_{-1}d_{-2}\cdots d_{-n+1}d_{-n}$ into a base 10 fraction. State an algorithm to the one above for doing the conversion.

Ans.) The algorithm would be divide d_{-i} by $2i$ and shift the result i places to the right, then add all terms.

(d) Apply your algorithm of part (c) to $0.FA7_{20}$.

Ans.) 15/2 = 7.5 shifted right 1 place is 0.75

10/4 = 2.5 shifted right 2 places is 0.025

7/8 = 0.875 shifted right 3 places is 0.000875. Thus, 0.75+0.025+0.000875 = 0.775875.

4. Let $B=b_{n-1}b_{n-2}\cdots b_1b_0$ be an n-bit 2's complement integer. Show that the decimal value of B is $-b_{n-1}2^{n-1}+b_{n-2}2^{n-2}+b_{n-3}2^{n-3}+\cdots+b_1 2+b_0$. (Hint: Consider positive (b_{n-1}=0) and negative (b_{n-1}=1) numbers separately, and note that the magnitude of a negative number is obtained by subtracting each bit from 1 (i.e., complementing each bit) and adding 1 to the result.)

Ans.)

If $b_{n-1}=0$, then $b_{n-2}2^{n-2}+b_{n-3}2^{n-3}+\cdots+b_1 2+b_0$ is the value of the positive number.

If $b_{n-1}=1$, then

$$-2^{n-1}+b_{n-2}2^{n-2}+b_{n-3}2^{n-3}+\cdots+b_1 2+b_0 = -(2^{n-2}+2^{n-3}+\cdots+2+1+1)+b_{n-2}2^{n-2}+b_{n-3}2^{n-3}+\cdots+b_1 2+b_0$$

$$= -[(1-\ b_{n-2})2^{n-2}+(1-\ b_{n-3})2^{n-3}+(1-\ b_1)2^1\ +(1-b_0)+1]$$

This expression in brackets has each bit complemented and 1 added to the result.