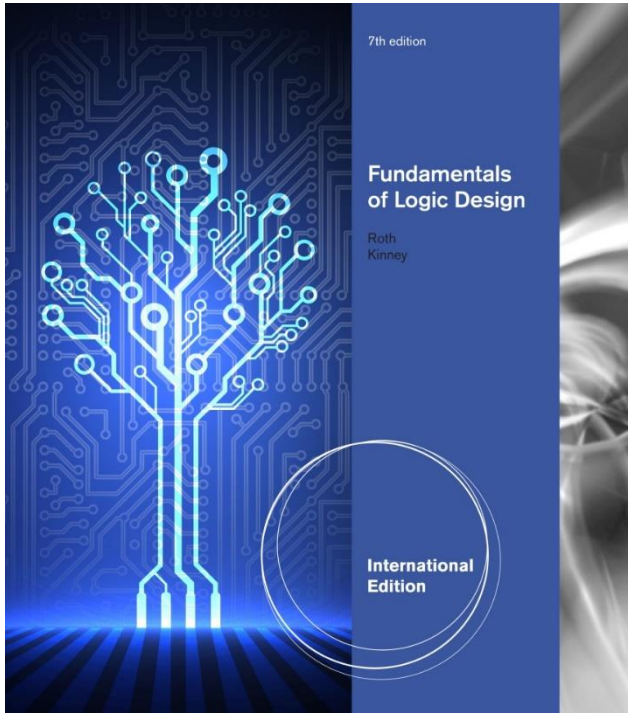


Introduction
Number Systems and Conversion

Contents

Objectives

Introduction:

- To be able to explain the difference between analog and digital systems, as well as why digital systems are capable of greater accuracy
- To be able to understand the difference between combinational and sequential circuits
- To be able to explain why two-valued signals and binary numbers are commonly used in digital systems

Number Systems and Conversion:

To be able to solve the following types of problems:

- Given a positive integer, fraction, or mixed number in any base (2 through 16); convert to any other base. Justify the procedure using a power series expansion for the number.
- Add, subtract, multiply, and divide positive binary numbers. Explain the addition and subtraction process in terms of carries and borrows.
- Write negative binary numbers in sign and magnitude, 1's complement, and 2's complement forms. Add signed binary numbers using 1's complement and 2's complement arithmetic. Justify the methods used and identify overflows.
- Represent a decimal number in binary-coded-decimal (BCD), 6-3-1-1 code, excess-3 code, etc. Given a set of weights, construct a weighted code.

Digital Systems and Switching Circuits

Digital and Analog Systems:

- **Digital systems** are capable of greater accuracy and reliability than analog systems.
- Digital systems are used extensively in computation and data processing, control systems, communications, and measurement.
- In a **digital system**, physical quantities can only assume discrete values.
- In an **analog system**, physical quantities or signals can vary continuously over a specified range.
- In many cases digital systems can be designed so that for a given input, the output is exactly correct.
- The output of an analog system might have an error ranging from a fraction of one percent to a few percent because these systems do not work with discrete quantities.

Analog to Digital Conversion

Digital Systems and Switching Circuits

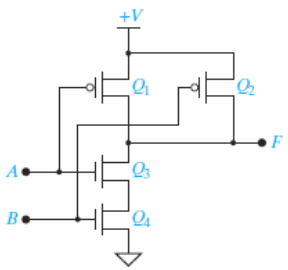
Advantages of Digital Systems over Analog Systems:

- Reproducibility of the results
- Accuracy of results
- More reliable than analog systems due to better immunity to noise
- Ease of design: No special math skills needed to visualize the behavior of small digital logic circuits
- Flexibility and functionality
- Programmability
- Speed: A digital logic element can produce an output in less than 10 nanosecond
- Economy: Due to the integration of millions of digital logic elements on a single miniature chip forming low cost integrated circuits (ICs)

Design of Digital Systems

- **System design**: breaking the overall system into subsystems and specifying the characteristics of each subsystem – the system design of a digital computer.
- **Logic design:** determining how to interconnect basic logic building blocks to perform a specific function – interconnection of logic gates and flip-flops for binary addition and so on → Focus on the logic design in this course
- **Circuit design:** specifying the interconnection of specific hardware components such as resistors, diodes, and transistors – to form a gate, flip-flop or other logic building block.

FIGURE A-5
CMOS NAND Gate



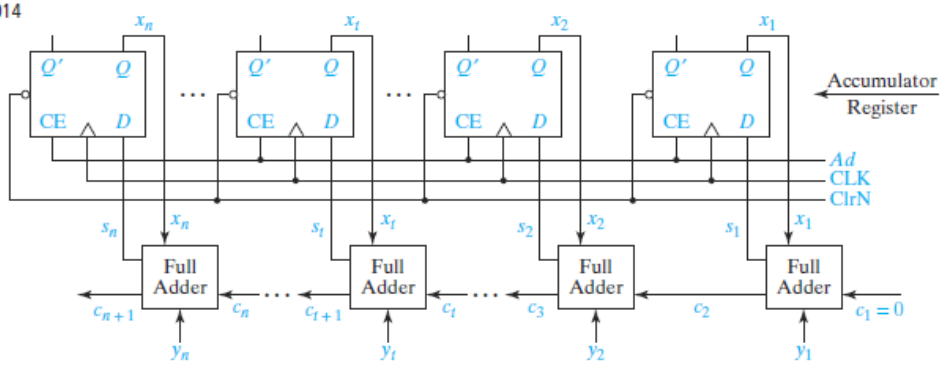
(a) Circuit diagram

A	B	F	Q_1	Q_2	Q_3	Q_4
0	0	+V	ON	ON	OFF	OFF
0	+V	+V	ON	OFF	OFF	ON
+V	0	+V	OFF	ON	ON	OFF
+V	+V	0	OFF	OFF	ON	ON

(b) Truth table

Circuit Design for NAND gate

FIGURE 12-5 *n*-Bit Parallel Adder with Accumulator



Logic Design for Adder

Digital Systems and Switching Circuits

Switching Circuits:

- A **switching circuit** has one or more inputs and one or more outputs which take on discrete values. Two types of switching circuits – combinational and sequential
- In **combinational circuits**, output values of circuit only depend on the present values of inputs, not past.
- In **sequential circuits**, output values of circuit depend on both past and present input values. For this reason, sequential circuits are said to have "memory", because they must remember past sequence of inputs.

Example of Switching Circuit:
- Full Adder
- Combinational Circuits

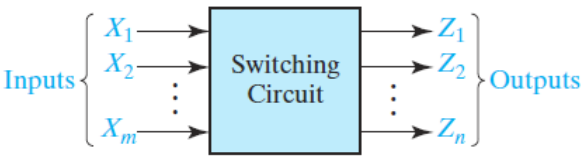
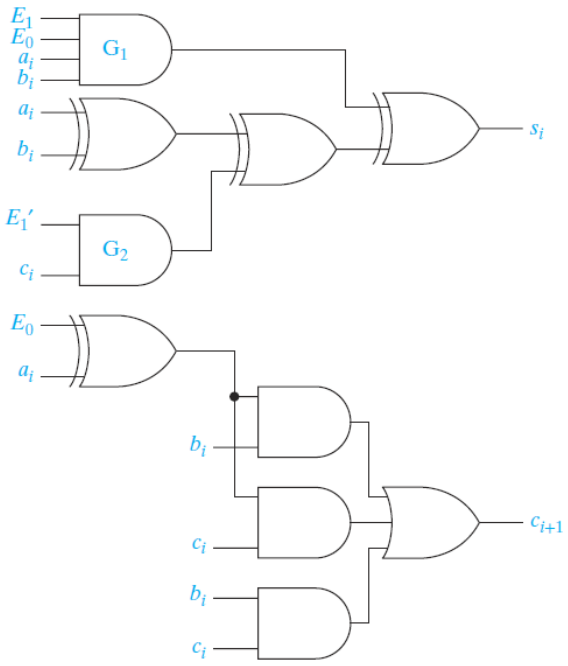


FIGURE 1-1
Switching Circuit

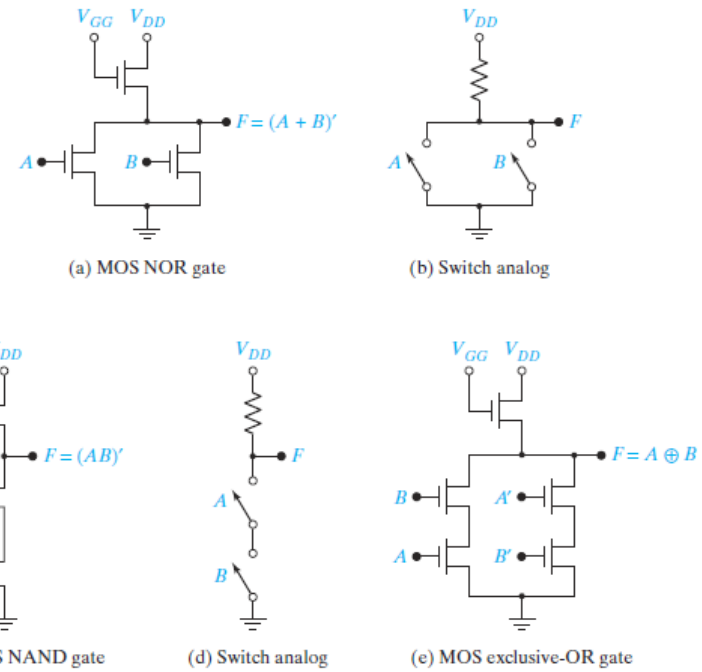


Digital Systems and Switching Circuits

Building Blocks of Switching Circuits:

- Basic building block of combinational circuits: **logic gate**
- Basic building block of sequential circuits: **flip-flop**
- **Switching devices** used in digital systems are usually two-state devices – relays (closed/open state), diodes (conducting/non-conducting state) and transistors (closed/open state).
- Because the outputs of most switching devices assume only two different values, it is natural to use **binary** numbers internally in digital systems.

FIGURE A-3
MOS Gates



Number Systems and Conversion

Explanation of decimal and binary numbers

- When we write decimal (base 10) numbers, we use a positional notation; each digit is multiplied by an appropriate power of 10 depending on its position in the number.

$$953.78_{10} = 9 \times 10^2 + 5 \times 10^1 + 3 \times 10^0 + 7 \times 10^{-1} + 8 \times 10^{-2}$$

- When we write binary (base 2) numbers, each digit is multiplied by an appropriate power of 2 on its position in the number.

$$1011.11_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$
$$= 8 + 0 + 2 + 1 + \tfrac{1}{2} + \tfrac{1}{4} = 11\tfrac{3}{4} = 11.75_{10}$$

Number Systems and Conversion

Power series expansion in any number system of base R:

Any positive integer R ($R > 1$) can be chosen as the *radix* or *base* of a number system. If the base is R, then R digits $(0, 1, \ldots, R-1)$ are used. For example, if $R = 8$, then the required digits are 0,1,2,3,4,5,6, and 7. A number written in positional notation can be expanded in a power series in R. For example,

$$\begin{aligned} N &= (a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3})_R \\ &= a_4 \times R^4 + a_3 \times R^3 + a_2 \times R^2 + a_1 \times R^1 + a_0 \times R^0 \\ &\quad + a_{-1} \times R^{-1} + a_{-2} \times R^{-2} + a_{-3} \times R^{-3} \end{aligned}$$

where a_i is the coefficient of R^i and $0 \leq a_i \leq R - 1$

Example: Conversion 147.3_8 to decimal number

$$\begin{aligned} 147.3_8 &= \overset{a_2}{1} \times \overset{R^2}{8^2} + \overset{a_1}{4} \times \overset{R^1}{8^1} + \overset{a_0}{7} \times \overset{R^0}{8^0} + \overset{a_{-1}}{3} \times \overset{R^{-1}}{8^{-1}} = 64 + 32 + 7 + \tfrac{3}{8} \\ &= 103.375_{10} \end{aligned}$$

Number Systems and Conversion

Conversion to other bases:

The power series expansion can be used to convert to any base. For example, converting 147_{10} to base 3 would be written as

$$147_{10} = 1 \times (101)^2 + (11) \times (101)^1 + (21) \times (101)^0$$

where all the numbers on the right-hand side are base 3 numbers. (*Note:* In base 3, 10 is 101, 7 is 21, etc.) To complete the conversion, base 3 arithmetic would be used.

Counting in Base 3

base 10	base 3	base 10	base 3	base 10	base 3
1	1	10	101	19	201
2	2	11	102	20	202
3	10	12	110	21	210
4	11	13	111	22	211
5	12	14	112	23	212
6	20	15	120	24	220
7	21	16	121	25	221
8	22	17	122	26	222
9	100	18	200	27	1000

Number Systems and Conversion

Bases greater than 10:

- For base greater than 10, more than 10 symbols are needed to represent the digits.

In this case, letters are usually used to represent digits greater than 9. For example, in hexadecimal (base 16), A represents 10_{10}, B represents 11_{10}, C represents 12_{10}, D represents 13_{10}, E represents 14_{10}, and F represents 15_{10}. Thus,

$$A2F_{16} = 10 \times 16^2 + 2 \times 16^1 + 15 \times 16^0 = 2560 + 32 + 15 = 2607_{10}$$

Decimal	4-Bit Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Conversion of a decimal integer to base *R* using the division method:

- The base *R* equivalent of a decimal integer *N* is as follows:

$$N = (a_n a_{n-1} \cdots a_2 a_1 a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \cdots + a_2 R^2 + a_1 R^1 + a_0$$

If we divide N by R, the remainder is a_0:

$$\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \cdots + a_2 R^1 + a_1 = Q_1, \text{ remainder } a_0$$

Then we divide the quotient Q_1 by R:

$$\frac{Q_1}{R} = a_n R^{n-2} + a_{n-1} R^{n-3} + \cdots + a_3 R^1 + a_2 = Q_2, \text{ remainder } a_1$$

Next we divide Q_2 by R:

$$\frac{Q_2}{R} = a_n R^{n-3} + a_{n-1} R^{n-4} + \cdots + a_3 = Q_3, \text{ remainder } a_2$$

This process is continued until we finally obtain a_n. Note that the remainder obtained at each division step is one of the desired digits and the least significant digit is obtained first.

Example 1: Conversion of decimal integer to binary

Example Convert 53_{10} to binary.

$2 \overline{)53}$		
$2 \overline{)26}$	rem. $= 1 = a_0$	The least significant digit
$2 \overline{)13}$	rem. $= 0 = a_1$	
$2 \overline{)6}$	rem. $= 1 = a_2$	$53_{10} = 110101_2$
$2 \overline{)3}$	rem. $= 0 = a_3$	
$2 \overline{)1}$	rem. $= 1 = a_4$	
0	rem. $= 1 = a_5$	

Number Systems and Conversion

Conversion of a decimal fraction *F* to base *R* using successive multiplications:

$$F = (.a_{-1}a_{-2}a_{-3}\cdots a_{-m})_R = a_{-1}R^{-1} + a_{-2}R^{-2} + a_{-3}R^{-3} + \cdots + a_{-m}R^{-m}$$

Multiplying by R yields

$$FR = a_{-1} + a_{-2}R^{-1} + a_{-3}R^{-2} + \cdots + a_{-m}R^{-m+1} = a_{-1} + F_1$$

where F_1 represents the fractional part of the result and a_{-1} is the integer part. Multiplying F_1 by R yields

$$F_1R = a_{-2} + a_{-3}R^{-1} + \cdots + a_{-m}R^{-m+2} = a_{-2} + F_2$$

Next, we multiply F_2 by R:

$$F_2R = a_{-3} + \cdots + a_{-m}R^{-m+3} = a_{-3} + F_3$$

This process is continued until we have obtained a sufficient number of digits. Note that the integer part obtained at each step is one of the desired digits and the most significant digit is obtained first.

Number Systems and Conversion

Example 2: Conversion of a decimal fraction *F* to base *R* using successive multiplications

Example

Convert 0.625_{10} to binary.

$F = .625$	$F_1 = .250$	$F_2 = .500$	
$\times \quad 2$	$\times \quad 2$	$\times \quad 2$	$.625_{10} = .101_2$
1.250	0.500	1.000	
$(a_{-1} = 1)$	$(a_{-2} = 0)$	$(a_{-3} = 1)$	

The most significant digit

Number Systems and Conversion

Example 3: Conversion of a decimal fraction *F* to base *R* using successive multiplications

- This successive multiplication process does not always terminated, but if it does not terminated, the result is repeating fraction.

Example Convert 0.7_{10} to binary.

$$
\begin{array}{r}
.7 \\
\times\ 2 \\
\hline
(1).4 \\
\times\ 2 \\
\hline
(0).8 \\
\times\ 2 \\
\hline
(1).6 \\
\times\ 2 \\
\hline
(1).2 \\
\times\ 2 \\
\hline
(0).4 \quad \longleftarrow \text{ process starts repeating here because 0.4 was previously obtained} \\
\times\ 2 \\
\hline
(0).8
\end{array}
$$

$0.7_{10} = 0.1\underline{0110}\ \underline{0110}\ \underline{0110}\ \ldots_2$

Number Systems and Conversion

Example 4: Conversion between two bases other than decimal

Example

Convert 231.3_4 to base 7.

$$231.3_4 = 2 \times 16 + 3 \times 4 + 1 + \tfrac{3}{4} = 45.75_{10}$$

$$\begin{array}{rll} 7\,/\!\!\underline{45} & & .75 \\ 7\,/\!\!\underline{\;6} & \text{rem. } 3 & \underline{\quad 7} \\ 0 & \text{rem. } 6 & (5).25 \\ & & \underline{\quad 7} \\ & & (1).75 \\ & & \underline{\quad 7} \\ & & (5).25 \\ & & \underline{\quad 7} \\ & & (1).75 \end{array}$$

$$45.75_{10} = 63.5151\ldots_7$$

Number Systems and Conversion

Conversion from binary to hexadecimal and binary to octal:
Conversion from binary to hexadecimal (and conversely) can be done by inspection because each hexadecimal digit corresponds to exactly four binary digits (bits).

$$1001101.010111_2 = \underbrace{0100}_{4}\ \underbrace{1101}_{D}.\underbrace{0101}_{5}\ \underbrace{1100}_{C} = 4D.5C_{16}$$

A similar conversion can be done from binary to octal, base 8 (and conversely), except each octal digit corresponds to three binary digits, instead of four.

$$1001011010110 10_2 = 100\ 101\ 101\ 011\ 010 = 45532_8$$

Hexadecimal	Binary
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Binary Arithmetic

Binary Addition:

For simplicity, arithmetic in digital systems is performed in binary (base 2).

Addition table for binary:

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 0 \quad \text{and carry 1 to the next column}$$

Carrying 1 to a column is equivalent to adding 1 to that column.

Example 5: Binary Addition

Example Add 13_{10} and 11_{10} in binary.

$$\begin{array}{rl} & 1111 \longleftarrow \text{carries} \\ 13_{10} = & 1101 \\ 11_{10} = & \underline{1011} \\ & 11000 = 24_{10} \end{array}$$

Binary Arithmetic

Binary Subtraction:

- Subtraction table for binary:

The subtraction table for binary numbers is

$0 - 0 = 0$
$0 - 1 = 1$ and borrow 1 from the next column
$1 - 0 = 1$
$1 - 1 = 0$

Borrowing 1 from a column is equivalent to subtracting 1 from that column.

Example 6: Subtractions in Binary

- An alternative to binary subtraction is the use of 2's complement arithmetic

Examples of Binary Subtraction

(a)
```
  1←— (indicates
 11101   a borrrow
-10011   from the
------   3rd column)
 1010
```

(b)
```
 1111←— borrows
 10000
-   11
------
 1101
```

(c)
```
 111←— borrows
 111001
-  1011
-------
 101110
```

Notice how in (b), the borrow propagates from column to column.

Binary Arithmetic

Binary Multiplication:

- Multiplication table for binary

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

Example 7: Binary Multiplication

- Note that each partial product is either the multiplicand (1101) shifted over the appropriate number of places or is zero.

13_{10} by 11_{10} in binary:

$$\begin{array}{r} 1101 \\ 1011 \\ \hline 1101 \\ 1101 \\ 0000 \\ 1101 \\ \hline 10001111 \end{array} = 143_{10}$$

Binary Arithmetic

Binary Multiplication:

- When doing binary multiplication, a common way to avoid carries greater than 1 is to add in the partial products one at a time as illustrated by the following example.

1111	multiplicand
1101	multiplier
1111	first partial product
0000	second partial product
(01111)	sum of first two partial products
1111	third partial product
(1001011)	sum after adding third partial product
1111	fourth partial product
11000011	final product (sum after adding fourth partial product)

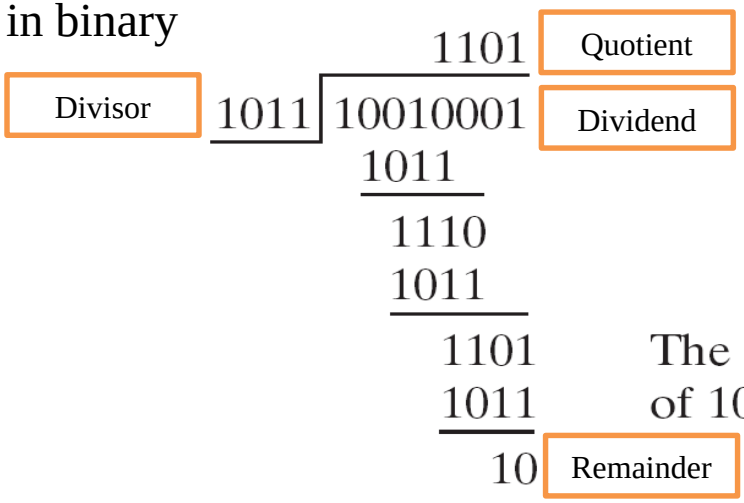
Binary Arithmetic

Binary Division:

- Binary division is similar to decimal division, except it is much easier because the only two possible quotient digits are 0 and 1.
- We start division by comparing the divisor with the upper bits of the dividend.
- If we cannot subtract without getting a negative result, we move one place to the right and try again.
- If we can subtract, we place a 1 for the quotient above the number we subtracted from and append the next dividend bit to the end of the difference and repeat this process with this modified difference until we run out of bits in the dividend.

Example 8: Binary Division

145_{10} by 11_{10} in binary



The quotient is 1101 with a remainder of 10.

Representation of Negative Numbers

Common methods of representing both positive and negative numbers are:
(if *n* bits are used to represent the numbers)

- Sign and Magnitude: $-(2^{(n-1)}-1)$ to $+(2^{(n-1)}-1)$, positive zero, negative zero
- 2's Complement: $-2^{(n-1)}$ to $+(2^{(n-1)}-1)$, positive zero
- 1's Complement: $-(2^{(n-1)}-1)$ to $+(2^{(n-1)}-1)$, positive zero, negative zero

In each of these methods, the leftmost bit of a number is 0 for positive numbers and 1 for negative numbers.

3 systems for representing negative numbers in binary: $+5_{10} = 0101_2$ for $n = 4$

Sign & Magnitude: Most significant bit is the sign
Ex: $-5_{10} = 1101_2$

2's Complement: $N^* = 2^n - N$
Ex: $-5_{10} = 2^4 - 5 = 16 - 5 = 11_{10} = 1011_2$

1's Complement: $\overline{N} = (2^n - 1) - N$
Ex: $-5_{10} = (2^4 - 1) - 5 = (16 - 1) - 5 = 10_{10} = 1010_2$

Representation of Negative Numbers

Negative Integers

- Sign and Magnitude: sign bit 0 for positive, 1 for negative
  - ✓ Most significant bit is the sign
  - ✓ Note that $-0 = 1000_2$
- 2's Complement: start at the right and leave any 0's on the right end and the first 1 unchanged, then complement all bits to the left of the first 1
  - ✓ Note that $-2^{(n-1)} = 1000_2$
- 1's Complement: simply complement *N* bit-by-bit
  - ✓ Note that $-0 = 1111_2$

TABLE 1-1
Signed Binary Integers (word length: $n = 4$)

			Negative Integers		
$+N$	Positive Integers (all systems)	$-N$	Sign and Magnitude	2's Complement N^*	1's Complement $\overline{N}$
+0	0000	−0	1000	——	1111
+1	0001	−1	1001	1111	1110
+2	0010	−2	1010	1110	1101
+3	0011	−3	1011	1101	1100
+4	0100	−4	1100	1100	1011
+5	0101	−5	1101	1011	1010
+6	0110	−6	1110	1010	1001
+7	0111	−7	1111	1001	1000
		−8	——	1000	——

Representation of Negative Numbers

2's Complement:

- Positive number, N, is represented by a 0 followed by the magnitude of N as in the sign and magnitude system
- For negative numbers, $-N$, 2's compliment is represented by N^*, as follows:

$$N^* = 2^n - N,$$

where the word length is n bits.

- N^* is obtained by complementing N bit-by-bit and then adding 1.
  - ✓ $N^* = 2^n - N = (2^n - 1 - N) + 1$
  - ✓ In binary, $2^n - 1$ consists of n 1's. Subtracting a number from all 1's does not produce any borrows and the subtraction can be done by replacing 0's with 1's and 1's with 0's (i.e., simply complement N bit-by-bit)
  - ✓ If n=7, N = 0101100,

$$\begin{array}{r} 2^n - 1 = 1111111 \\ - 0101100 \\ \hline 1010011 \\ + 0000001 \\ \hline N^* = 1010100 \end{array}$$

- Alternative way: start at the right and leave any 0's on the right end and the first 1 unchanged, then complement all bits to the left of the first 1.

Representation of Negative Numbers

Example 9: 2's Complement Addition

- When the word length is *n* bits, we say that an *overflow* has occurred if the correct representation of the sum (including sign) requires more than *n* bits.

1. Addition of two positive numbers, sum $< 2^{n-1}$

	0000	
+3	0011	
+4	0100	
+7	0111	(correct answer)

2. Addition of two positive numbers, sum $\geq 2^{n-1}$

	0100	
+5	0101	
+6	0110	
	1011	⟵ wrong answer because of overflow (+11 requires 5 bits including sign)

An overflow occurs if and only if the carry out of the sign position is not equal to the carry into the sign position

3. Addition of positive and negative numbers (negative number has greater magnitude)

	0000	
+5	0101	
−6	1010	
−1	1111	(correct answer)

4. Same as case 3 except positive number has greater magnitude

	1110	
−5	1011	
+6	0110	
+1	(1)0001	⟵ correct answer when the carry from the sign bit is ignored (this is *not* an overflow)

$-A+B$ (where $B > A$)
$A^*+B = (2^n - A) + B = 2^n + (B-A) > 2^n$
Throwing away the last carry is equivalent to subtracting 2^n

Representation of Negative Numbers

Example 9: 2's Complement Addition

5. Addition of two negative numbers, $|\text{sum}| \leq 2^{n-1}$

	1100	
−3	1101	
−4	1100	
−7	(1)1001	⟵ correct answer when the last carry is ignored (this is *not* an overflow)

$-A-B$ (where $B > A$)
$A^*+B^* = (2^n - A) + (2^n - B) = 2^n + 2^n - (A+B)$
Discarding the last carry yields $2^n - (A+B) = (A+B)^*$

6. Addition of two negative numbers, $|\text{sum}| > 2^{n-1}$

	1010	
−5	1011	
−6	1010	
	(1)0101	⟵ wrong answer because of overflow (−11 requires 5 bits including sign)

An overflow occurs if and only if the carry out of the sign position is not equal to the carry into the sign position

Representation of Negative Numbers

1's complement:

$$\overline{N} = (2^n - 1) - N$$

Ex: $-5_{10} = (2^4 - 1) - 5 = 16 - 1 - 5 = 10_{10} = 1010_2$

- The 1's complement of N can be obtained by complementing N bit-by-bit since $(2^n - 1)$ consists of all 1's and subtracting a bit from 1 is the same as complementing the bit.
- Note that the 1's complement of 0000 is 1111, which represents minus zero.
- **End around carry:** In 1's compliment, the last carry is not discarded as it is in 2's compliment, but rather added to the n-bit sum in the position furthest to the right.

Representation of Negative Numbers

Example 10: 1's complement Addition

3. Addition of positive and negative numbers (negative number with greater magnitude)

+5	0101	
−6	1001	
−1	1110	(correct answer)

4. Same as case 3 except positive number has greater magnitude

−5	1010	
+6	0110	
(1)	0000	
↳	1	(end-around carry)
	0001	(correct answer, *no* overflow)

$-A+B$ (where $B > A$)
$\overline{A}+B = (2^n - 1 - A) + B = 2^n - (B-A) - 1$
The end-around carry is equivalent to subtracting 2^n and adding 1

Representation of Negative Numbers

Example 10: 1's compliment Addition

5. Addition of two negative numbers, $|\text{sum}| < 2^{n-1}$

$$
\begin{array}{rll}
-3 & 1100 & \\
-4 & 1011 & \\
\hline
(1) & 0111 & \\
 & \hookrightarrow\ 1 & \text{(end-around carry)} \\
\hline
 & 1000 & \text{(correct answer, } no \text{ overflow)}
\end{array}
$$

-A-B (where $A+B < 2^{n-1}$)
$\overline{A}+\overline{B} = (2^n - 1 - A) + (2^n - 1 - B) = 2^n + [2^n - 1 - (A+B)] - 1$
After the end-around carry, the result is
$2^n - 1 - (A+B) = \overline{(A+B)}$ which is the correct representation for -$(A+B)$

6. Addition of two negative numbers, $|\text{sum}| \geq 2^{n-1}$

$$
\begin{array}{rll}
-5 & 1010 & \\
-6 & 1001 & \\
\hline
(1) & 0011 & \\
 & \hookrightarrow\ 1 & \text{(end-around carry)} \\
\hline
 & 0100 & \text{(wrong answer because of overflow)}
\end{array}
$$

Representation of Negative Numbers

Example 11: 1's complement Addition (n = 8)

1. Add −11 and −20 in 1's complement.

+11 = 00001011 +20 = 00010100

taking the bit-by-bit complement,

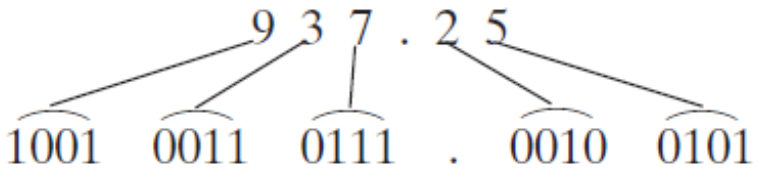
−11 is represented by 11110100 and −20 by 11101011

```
  11110100     (−11)
  11101011   +(−20)
(1) 11011111
  └──────→ 1     (end-around carry)
  11100000 = −31
```

- A general rule for detecting overflow when adding two n-bit signed binary numbers (1's or 2's complement) to get an n-bit sum is:
  - ✓ An overflow occurs if adding two positive numbers gives a negative answer or if adding two negative numbers gives a positive answer.
- An alternative method for detecting overflow in 2's complement addition is as follows:
  - ✓ An overflow occurs if and only if the carry out of the sign position is not equal to the carry into the sign position.

Binary Codes – BCD code

- Although most large computers work internally with binary numbers, the input-output equipment generally uses decimal numbers.
- Because most logic circuits only accept two-valued signals, the decimal numbers must be coded in terms of binary signals.
- In the simplest form of binary code, each decimal digit is replaced by its binary equivalent. For example, 937.25 is represented by:



- This representation is referred to as binary-coded-decimal (BCD) or more explicitly as 8-4-2-1 BCD.
- Note that 1010 through 1111 are not valid BCD codes because there are only ten decimal digits.

Binary Codes – Weighted vs. Non-Weighted

- Weighted Codes
  - ✓ $N = w_3a_3+w_2a_2+w_1a_1+w_0a_0$
  - ✓ 8-4-2-1 and 6-3-1-1 codes are weighted codes
- Non-weighted Codes
  - ✓ Exess-3 code has self complement property, i.e., complement 4 (0111) equals to 5 (1000), etc.
  - ✓ 2-out-of-5 code has error-checking property
  - ✓ A Gray code has a property that the codes for successive decimal digits differ exactly one bit: used for translating an analog quantity into a digital form

Decimal Digit	8-4-2-1 Code (BCD)	6-3-1-1 Code	Excess-3 Code	2-out-of-5 Code	Gray Code
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	1110
6	0110	1000	1001	10001	1010
7	0111	1001	1010	10010	1011
8	1000	1011	1011	10100	1001
9	1001	1100	1100	11000	1000

Binary Codes – ASCII code

- ASCII stands for American Standard Code for Information Interchange
- 7-bit code, 128 (2^7) different code combinations

Parity Bit	Zone Bits			Numeric Bits			
7	6	5	4	3	2	1	0
C	1	0	0	Characters A~O (0001~1111)			
	1	0	1	Characters P~Z (0000~1010)			
	0	1	1	Number 0~9 (0000~1001)			

TABLE 1-3 ASCII Code

Character	ASCII Code A_6	A_5	A_4	A_3	A_2	A_1	A_0
space	0	1	0	0	0	0	0
!	0	1	0	0	0	0	1
"	0	1	0	0	0	1	0
#	0	1	0	0	0	1	1
$	0	1	0	0	1	0	0
%	0	1	0	0	1	0	1
&	0	1	0	0	1	1	0
'	0	1	0	0	1	1	1
(	0	1	0	1	0	0	0
)	0	1	0	1	0	0	1
*	0	1	0	1	0	1	0
+	0	1	0	1	0	1	1
,	0	1	0	1	1	0	0
-	0	1	0	1	1	0	1
.	0	1	0	1	1	1	0
/	0	1	0	1	1	1	1
0	0	1	1	0	0	0	0
1	0	1	1	0	0	0	1
2	0	1	1	0	0	1	0
3	0	1	1	0	0	1	1
4	0	1	1	0	1	0	0
5	0	1	1	0	1	0	1
6	0	1	1	0	1	1	0
7	0	1	1	0	1	1	1
8	0	1	1	1	0	0	0
9	0	1	1	1	0	0	1
:	0	1	1	1	0	1	0
;	0	1	1	1	0	1	1
<	0	1	1	1	1	0	0
=	0	1	1	1	1	0	1
>	0	1	1	1	1	1	0
?	0	1	1	1	1	1	1

Character	ASCII Code A_6	A_5	A_4	A_3	A_2	A_1	A_0
@	1	0	0	0	0	0	0
A	1	0	0	0	0	0	1
B	1	0	0	0	0	1	0
C	1	0	0	0	0	1	1
D	1	0	0	0	1	0	0
E	1	0	0	0	1	0	1
F	1	0	0	0	1	1	0
G	1	0	0	0	1	1	1
H	1	0	0	1	0	0	0
I	1	0	0	1	0	0	1
J	1	0	0	1	0	1	0
K	1	0	0	1	0	1	1
L	1	0	0	1	1	0	0
M	1	0	0	1	1	0	1
N	1	0	0	1	1	1	0
O	1	0	0	1	1	1	1
P	1	0	1	0	0	0	0
Q	1	0	1	0	0	0	1
R	1	0	1	0	0	1	0
S	1	0	1	0	0	1	1
T	1	0	1	0	1	0	0
U	1	0	1	0	1	0	1
V	1	0	1	0	1	1	0
W	1	0	1	0	1	1	1
X	1	0	1	1	0	0	0
Y	1	0	1	1	0	0	1
Z	1	0	1	1	0	1	0
[	1	0	1	1	0	1	1
\	1	0	1	1	1	0	0
]	1	0	1	1	1	0	1
^	1	0	1	1	1	1	0
—	1	0	1	1	1	1	1

Character	ASCII Code A_6	A_5	A_4	A_3	A_2	A_1	A_0
'	1	1	0	0	0	0	0
a	1	1	0	0	0	0	1
b	1	1	0	0	0	1	0
c	1	1	0	0	0	1	1
d	1	1	0	0	1	0	0
e	1	1	0	0	1	0	1
f	1	1	0	0	1	1	0
g	1	1	0	0	1	1	1
h	1	1	0	1	0	0	0
i	1	1	0	1	0	0	1
j	1	1	0	1	0	1	0
k	1	1	0	1	0	1	1
l	1	1	0	1	1	0	0
m	1	1	0	1	1	0	1
n	1	1	0	1	1	1	0
o	1	1	0	1	1	1	1
p	1	1	1	0	0	0	0
q	1	1	1	0	0	0	1
r	1	1	1	0	0	1	0
s	1	1	1	0	0	1	1
t	1	1	1	0	1	0	0
u	1	1	1	0	1	0	1
v	1	1	1	0	1	1	0
w	1	1	1	0	1	1	1
x	1	1	1	1	0	0	0
y	1	1	1	1	0	0	1
z	1	1	1	1	0	1	0
{	1	1	1	1	0	1	1
|	1	1	1	1	1	0	0
}	1	1	1	1	1	0	1
~	1	1	1	1	1	1	0
delete	1	1	1	1	1	1	1