Probability and Random Process (SWE3026)

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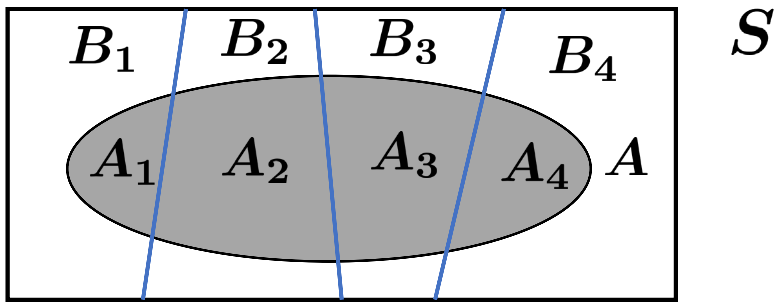
H. Pishro-Nik, "Introduction to probability, statistics, and random processes", available at https://www.probabilitycourse.com, Kappa Research LLC, 2014.

Law of Total Probability

Let $B_1, B_2, B_3, \cdots$ be a partition of the sample space S with $P(B_i) > 0$.

For any event A we have

$$P(A) = \sum_{i=1}^{m} P(A \cap B_i) = \sum_{i=1}^{m} P(A|B_i)P(B_i).$$



Law of Total Probability

$$A_i = A \cap B_i$$

$$P(A) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

$$P(A) = \sum_{i=1}^{m} P(A_i) = \sum_{i=1}^{m} P(A \cap B_i).$$

$$P(A|B_i) = \frac{P(A \cap B_i)}{P(B_i)} \;\Rightarrow\; P(A \cap B_i) = P(A|B_i)P(B_i).$$

Law of Total Probability

Example:

Three coins are in a bag:

a) **Coin 1: probability of heads is 0.9.** $P(H|C_1) = 0.9$

b) **Coin 2: probability of heads is 0.6.** $P(H|C_2) = 0.6$

c) **Coin 3: probability of heads is 0.3.** $P(H|C_3) = 0.3$

I draw a coin at random and toss it. What is the probability of heads?

Bayes' Rule

For any two events A and B, where $P(A) \neq 0$, we have

$$\boldsymbol{P(B|A) = \frac{P(A|B)P(B)}{P(A)}}.$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \;\Rightarrow\; P(A \cap B) = P(A|B)P(B).$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Bayes' Rule

If $B_1, B_2, B_3, \cdots$ is a partition of the sample space S, and A is any event with $P(A) > 0,$ we have

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_i P(A|B_i)P(B_i)}.$$

Bayes' Rule

Example.

In the previous problem, suppose that we know the result is heads; what is the probability that Coin 1 was chosen?

Bayes' Rule

Example.

A certain disease affects about 1 out of 10,000 people. There is a test to check whether the person has the disease. The test is quite accurate. In particular, we know that

- **the probability that the test result is positive (suggesting the person has the disease), given that the person does not have the disease, is only 2 percent;**
- **the probability that the test result is negative (suggesting the person does not have the disease), given that the person has the disease, is only 1 percent.**

A random person gets tested for the disease and the result comes back positive. What is the probability that the person has the disease?

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Conditional Independence

Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B), \quad \text{or equivalently, } P(A|B) = P(A).$$

Two events A and B are conditionally independent given an event C if and only if

$$P(A \cap B|C) = P(A|C)P(B|C).$$