1)

$$E[\hat{\Theta}] = E\left[\left(\frac{1}{n}\sum_{k=1}^{n} X_k\right)^2\right]$$

$$= \frac{1}{n^2} E\left[\sum_{k=1}^{n} X_k^2 + \sum_{i \neq j} X_i X_j\right]$$

$$= \frac{1}{n^2}\left(\sum_{k=1}^{n} E[X_k^2] + \sum_{i \neq j} E[X_i]E[X_j]\right)$$

$$= \frac{1}{n^2}\left(n(\sigma^2 + E[X_k]^2) + (n^2-n)E[X_k]^2\right)$$

$$= \frac{1}{n^2}\left(n(\sigma^2+\mu^2) + (n^2-n)\mu^2\right) = \frac{\sigma^2}{n} + \mu^2$$

$B(\hat{\theta}) = E[\hat{\theta}] - \theta = \frac{\sigma^2}{n}$ $B(\hat{\theta}) \neq 0$, $\hat{\theta}$ is a biased estimator of θ

2.

(a) $H_0: \theta = \theta_0 = 0.1$

$H_A: \theta < \theta_0$

(b)

$$\frac{\bar{X}_n - \theta_0 n}{\sqrt{n\theta_0(1-\theta_0)}} \sim N(0,1) \qquad \frac{21 - 0.1 \times 225}{\sqrt{225 \cdot 0.1 \cdot 0.9}} \approx -0.33$$

P-val $= \Phi(-0.33) \approx 0.37$

(c) at level of $\alpha = 0.05$ We cannot reject the null hypothesis