

$$11. M_X(s) = M_{X_1}(s) \cdot M_{X_2}(s) \cdots M_{X_n}(s)$$

$$= (M_{X_1}(s))^n = \left(\frac{\lambda}{\lambda-s}\right)^n$$

$$M_{X_1} = X \sim \text{Gamma}(n, \lambda) \rightarrow \text{Same}$$

MGF of Gamma(n, λ)

$$23. P(Y \geq a) \leq \min_{s > 0} \{ e^{-sa} M_X(s) \}$$

$$M_X(s) = \left(\frac{\lambda}{\lambda-s}\right)^n \quad s < \lambda$$

$$\frac{d}{ds} e^{-sa} \left(\frac{\lambda}{\lambda-s}\right)^n = -a e^{-sa} \left(\frac{\lambda}{\lambda-s}\right)^n + n e^{-sa} \left(\frac{\lambda}{\lambda-s}\right)^{n+1} \frac{1}{\lambda-s}$$

$$\therefore s = \lambda - \frac{n}{a} \quad \left(\frac{\lambda}{\lambda-s}\right)^n \left(-a e^{-sa} + n e^{-sa} \frac{1}{\lambda-s}\right)$$

$$a = \frac{n}{\lambda-s} \quad a\lambda - sa = n$$

$$P(Y \geq a) \leq e^{-\lambda a n \left(\frac{\lambda a n}{n}\right)}$$

$$e^{-\lambda a n \left(\frac{\lambda a n}{n}\right)} = e^{-\lambda a \left[1 + O\left(\frac{1}{n}\right)\right]} \left(\frac{1}{n}\right)^n (\lambda a)^n$$

$$= e^{-\lambda a} \left(\frac{\lambda a}{n}\right)^n + o\left(\frac{1}{n}\right)^{n+1}$$

\therefore goes to 0