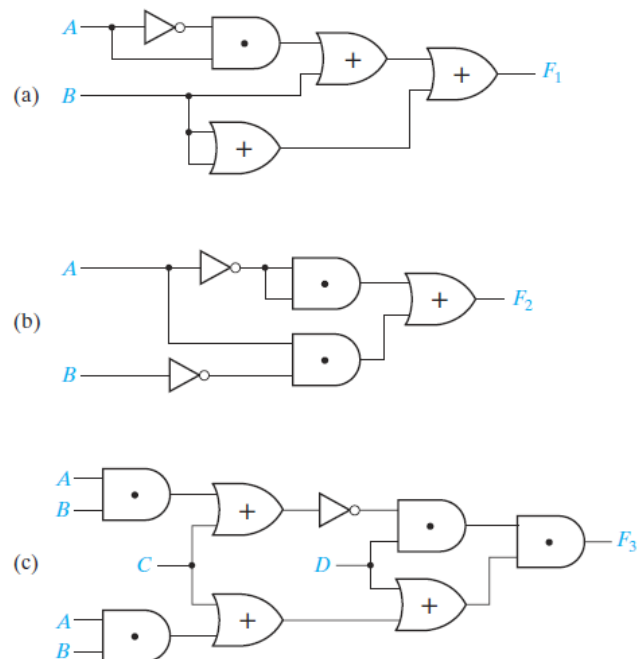


Name _____ Student ID _____ Colleges & Schools _____ Department _____

Homework Unit2 & Unit 3

- For each of the following circuits, find the output and design a simpler circuit that has the same output. (Hint: Find the circuit output by first finding the output of each gate, going from left to right, and simplifying as you go.)



Sol.)

$$a) F_1 = A'A + B + (B+B) = 0 + B + B = B.$$

$$b) F_2 = A'A' + AB' = A' + AB' = A' + B' \leftarrow (\text{by Elimination Theorem})$$

$$c) F_3 = [(AB+C)'D][(AB+C)+D]$$

$$= (AB+C)'D(AB+C) + (AB+C)'D \leftarrow (\text{by Absorption Theorem})$$

$$= (AB+C)'D$$

- Simplify the following expression to a minimum sum of products. Only individual variables should be complemented: $[(XY)'] + (X' + Y)'Z$

Sol.)

$$[(XY)'] + (X' + Y)'Z = (X' + Y) + (X' + Y)'Z = X' + Y + Z \leftarrow (\text{by Elimination Theorem})$$

- Use only DeMorgan's relationship and Involution to find the complements of the following

function: $F(A, B, C, D) = [A + (BCD)'][(AD)' + B(C' + A)]$

$$\begin{aligned}
 \text{Sol.) } F &= \{[A + (BCD)'][(AD)' + B(C' + A)]\}' = [A + (BCD)']' + [(AD)' + B(C' + A)]' \\
 &= A'(BCD)'' + (AD)''[B(C' + A)]' = A'BCD + AD[B' + (C' + A)'] \\
 &= A'BCD + AD(B' + C''A') = A'BCD + ADB' + ADCA' \\
 &= A'BCD + ADB'
 \end{aligned}$$

4. Reduce to a minimum sum of products:

$$F = WXY' + (W'Y' \equiv X) + (Y \oplus WZ)$$

$$\begin{aligned}
 \text{Sol.) } F &= WXY' + (W'Y' \equiv X) + (Y \oplus WZ) \\
 &= WXY' + (W'Y'X + (W'Y')'X') + (Y(WZ)' + Y'WZ) \\
 &= \underline{WXY'} + \underline{W'XY'} + (W+Y)X' + Y(W'+Z') + Y'WZ \leftarrow (\text{by Uniting Theorem}) \\
 &= \underline{XY'} + \underline{WX'} + YX' + YW' + YZ' + Y'WZ + \underline{Y'W} \leftarrow (\text{add } Y'W \text{ by Consensus Theorem}) \\
 &= XY' + \underline{WX'} + \underline{YX'} + \underline{YW'} + YZ' + Y'W \leftarrow (\text{remove } Y'WZ \text{ by Absorption Theorem}) \\
 &= XY' + WX' + W'Y + YZ' + WY' \leftarrow (\text{remove } YX' \text{ by Consensus Theorem}) \\
 &= \underline{XY'} + \underline{WX'} + W'Y + YZ' + \underline{WY'} \leftarrow (\text{remove } WY' \text{ by Consensus Theorem}) \\
 &= XY' + WX' + W'Y + YZ'
 \end{aligned}$$

Alternate Solutions: $F = W'Y + WX' + WZ' + XY'$, $F = YZ' + W'X + XY' + WY'$

$$F = W'X + X'Y + XZ' + WY', F = W'X + XY' + WZ' + WY'$$

5. Factor to obtain a product of four terms and then reduce to three terms by applying the consensus theorem: $X'Y'Z' + XYZ$

Sol.)

$$\begin{aligned}
 X'Y'Z' + XYZ &= (X+Y'Z')(X'+YZ) = (X+Y')(X+Z')(X'+Y)(X'+Z) \\
 &= (X+Y')\underline{(X+Z')(X'+Y)}\underline{(X'+Z)(Z'+Y)} \leftarrow (\text{add } (Z'+Y) \text{ by Consensus Theorem}) \\
 &= (X+Y')(X+Z')\underline{(X'+Y)(X'+Z)(Z'+Y)} \leftarrow (\text{remove } (X'+Y) \text{ by Consensus Theorem}) \\
 &= \underline{(X+Y')(X+Z')(X'+Z)}\underline{(Z'+Y)} \leftarrow (\text{remove } (X+Z') \text{ by Consensus Theorem}) \\
 &= (X+Y')(X'+Z)(Z'+Y)
 \end{aligned}$$

Alternative Solution: $(X'+Y)(Y'+Z)(X+Z')$ by adding $(Y'+Z)$ as consensus in the 2nd line.