

CLASS NOTES  
FOR  
ELEMENTARY STATISTICS  
LETU MATH-1423

Nicholas Capo  
[nicholas.capo@gmail.com](mailto:nicholas.capo@gmail.com)

September 12, 2012  
10:38am

This document comprises classroom notes from Statistics Class  
at [LeTourneau University](#), in the Fall of 2012.

Although the author will attempt to be complete and correct in these notes, it is the reader's responsibility to learn and understand the material. The author assumes no responsibility for the completeness or accuracy of this content.

If you have any suggestions or corrections feel free to email the author at  
[nicholas.capo@gmail.com](mailto:nicholas.capo@gmail.com)

The latest version of this document is available at:  
[https://bitbucket.org/nicholascapo/statisticsnotes/src/tip/  
StatisticsNotes.pdf](https://bitbucket.org/nicholascapo/statisticsnotes/src/tip/StatisticsNotes.pdf)

# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
1.1	Data . . . . .	5
1.1.1	Data Sets . . . . .	5
1.1.2	Types of Data . . . . .	5
1.2	Sample Mean and Median . . . . .	5
1.2.1	Definition . . . . .	5
1.3	Measures of Variability . . . . .	6
1.3.1	Standard Deviation . . . . .	6
1.4	Descriptive Statistics . . . . .	6
1.4.1	Quartiles . . . . .	6
1.4.2	Range and Interquartile Range . . . . .	6
1.4.3	Box and Whisker Plot . . . . .	7
1.5	Stem and Leaf Plots . . . . .	7
1.5.1	Key Notation . . . . .	8
1.5.2	Double Stem and Leaf . . . . .	8
1.6	Frequency Distribution . . . . .	8
1.6.1	Midpoint of a Class . . . . .	8
1.6.2	Relative Frequency . . . . .	8
1.7	Scatter Plots . . . . .	8
1.7.1	Example Scatter Plot . . . . .	8
1.8	Homework . . . . .	9
<b>2</b>	<b>Probability</b>	<b>11</b>
2.1	Experiments . . . . .	11
2.2	Sample Space . . . . .	11
2.2.1	Example . . . . .	11
2.2.2	Tree Diagrams . . . . .	11
2.2.3	Events . . . . .	11
2.3	Counting Sample Points . . . . .	12
2.3.1	Multiplication Rule . . . . .	12
2.3.2	Factorial . . . . .	12
2.3.3	Permutation . . . . .	12
2.3.4	Permutations at a Time . . . . .	12
2.3.5	Permutations in a Circle . . . . .	12

2.3.6	Permutations of a Kind . . . . .	13
2.3.7	Partitioning . . . . .	13
2.3.8	Combinations . . . . .	13
2.3.9	Partitioning and Combinations . . . . .	13
2.4	Probability of an Event . . . . .	13
2.4.1	Complimentary Probability . . . . .	14
2.4.2	Different Outcomes . . . . .	14
2.5	Homework . . . . .	14

# Chapter 1

## Introduction

### Definition of Statistics

“Statistics is the science of collecting, organizing, analyzing, and interpreting data in order to make decisions.”

### 1.1 Data

#### 1.1.1 Data Sets

**Population** The collection of all outcomes, responses, measurements, or counts, that are of interest.

**Sample** A subset of the population.

**Parameter** A number that describes a population characteristic.

**Statistic** A number that describes a sample characteristic.

#### 1.1.2 Types of Data

**Qualitative Data** Attributes, labels, or non-numerical entries.

**Quantitative Data** Numerical measurements or counts.

### 1.2 Sample Mean and Median

#### 1.2.1 Definition

**Sample Mean** The average of the sample data points, however it may not be a data point.

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} = \frac{x_1 + x_2 + x_3 \cdots x_n}{n}$$

**Sample Median** The middle value of the data.

$$\tilde{x} = \begin{cases} x_{(\frac{n+1}{2})} & \text{if } n \text{ is odd} \\ \frac{1}{2}(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}) & \text{if } n \text{ is even} \end{cases}$$

**Trimmed Mean** A trimmed mean is computed by trimming off the largest and smallest set of values. For example a 10% trimmed mean is found by eliminating the largest 10% and smallest 10% and computing the mean of the remaining values. This may be useful for data that contains possible outliers. Denoted by  $x_{tr(\text{percent})}$

## 1.3 Measures of Variability

### 1.3.1 Standard Deviation

**Sample Variance**

$$s^2 = \sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n-1}$$

**Sample Standard Deviation**

$$s = +\sqrt{s^2}$$

The standard deviation is 0 when all the data points are the same.

## 1.4 Descriptive Statistics

### 1.4.1 Quartiles

Quartiles approximately divide an ordered data set into four equal parts.

**First Quartile,  $Q_1$**  About 25% of the data fall on or below  $Q_1$

**Second Quartile,  $Q_2$**  About 50% of the data fall on or below  $Q_2$

**Third Quartile,  $Q_3$**  About 75% of the data fall on or below  $Q_3$

### 1.4.2 Range and Interquartile Range

**Range**

$$\text{range} = \text{max value} - \text{min value}$$

**Interquartile Range**

$$IQR = Q_3 - Q_1$$

To help find outliers, compute  $1.5 \times IQR$ , and any values that lie outside the interval  $[Q_1 - 1.5 \times IQR, Q_3 + 1.5 \times IQR]$  is a possible (and probable) outlier.

1.4.3 Box and Whisker Plot

Exploratory Data Analysis Tool

- Requires
  - Min
  - $Q_1$
  - Median
  - $Q_3$
  - Max

Example

Example Data	[1, 2, 3, 4, 5, 6, 11]
Min	1
Median	4.0
Max	6
Outlier	11

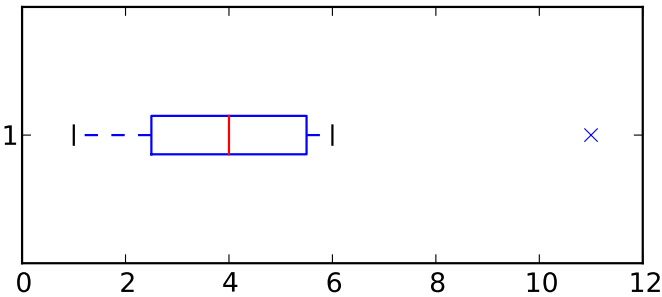


Figure 1.1: Example Box And Whisker Plot

1.5 Stem and Leaf Plots

These look like a sideways histogram

Data: [31, 21, 32, 33, 41, 42, 58, 25, 21]

Stem	Leaf	Key: $a b = ab$
2	1,1,5	
3	1,2,3	
4	1,2	
5	8	

### 1.5.1 Key Notation

Key: 4—5 = 45 Key: 4—5 = 4.5

### 1.5.2 Double Stem and Leaf

Separate the leaves into two groups, (0-4, and 5-9)

Data: [31, 21, 32, 33, 41, 42, 58, 25, 21]

Stem	Leaf	Key: $a b = ab$
2	1,1	
2	5	
3	1,2,3	
4	1.2	
4		
5		
5	8	

## 1.6 Frequency Distribution

A table that shows classes or intervals of data with a count of the number of entries in each class.

### 1.6.1 Midpoint of a Class

Average of the class limits.

$$\frac{(\text{lower class limit}) + (\text{upper class limit})}{2}$$

### 1.6.2 Relative Frequency

$$\frac{\text{class frequency}}{\text{sample size}} = \frac{f}{n}$$

## 1.7 Scatter Plots

Each entry in one data set corresponds to one entry in a second set, one-to-one mapping.

### 1.7.1 Example Scatter Plot

Data:

X: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]  
 Y: [10, 11, 1, 6, 5, 7, 3, 3, 6, 3, 7, 2]



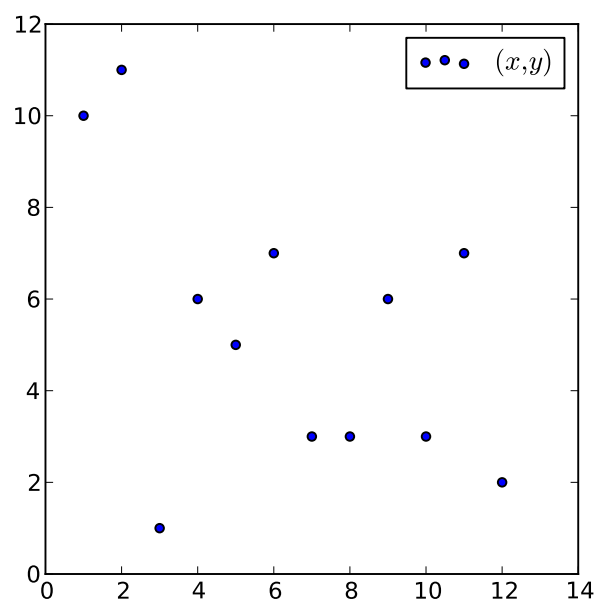


Figure 1.2: Example Scatter Plot

## 1.8 Homework

- Page 13 #'s 1.5, 1.6
- Page 17 #'s 1.11, 1.12
- Page 31 #'s 1.18, 1.19, 1.20, 1.29, 1.30



## Chapter 2

# Probability

### 2.1 Experiments

Any process that generates a set of data.

### 2.2 Sample Space

The set of all possible outcomes of a statistical experiment, denoted  $S$ . The sample space with no elements is the empty set or null set, denoted  $\emptyset$

#### 2.2.1 Example

$$S = \{3, 2, 1, 0\}$$

$$S = \{x | 0 < x < 25\}$$

$$S = \{x^2 | x \in \mathbb{R}\}$$

#### 2.2.2 Tree Diagrams

A Tree Diagram can be used to list all possible outcomes

#### 2.2.3 Events

An event is a subset of a sample space. The null set ( $\emptyset$ ) and the sample space ( $S$ ) are both subsets of the sample space  $S$ .

#### Intersection

The intersection of two events  $A$  and  $B$ , denoted  $A \cap B$ , is the event containing all elements that are common to  $A$  and  $B$ . If  $A \cap B = \emptyset$  then  $A$  and  $B$  are called mutually exclusive or disjoint.

**Union**

The union of two events  $A$  and  $B$ , denoted  $A \cup B$ , is the event containing all elements that belong to  $A$  or  $B$  or both.

**Compliment**

The compliment of an event  $A$  with respect to  $S$  is a subset of all elements of  $S$  not in  $A$ , denoted  $A'$

**2.3 Counting Sample Points****2.3.1 Multiplication Rule**

If an operation can be preformed in  $n_1$  ways and if for each of the ways a second operation can be preformed in  $n_2$  ways, then the two operations can be preformed together in  $n_1 n_2$  ways. This principle can be extended to more than two operations. See Example 2.14 in Walpole et al. [1, p. 45]

**2.3.2 Factorial**

For any non-negative integer  $n$ ,  $n!$  called “n factorial”, is defined as

$$n! = n(n-1) \cdots (2)(1)$$

with the special case  $0! = 1$ .

**2.3.3 Permutation**

A permutation is an arrangement of all or a part of a set objects. For permutations the order of objects matters. The number of permutations of  $n$  distinct objects is  $n!$ .

**2.3.4 Permutations at a Time**

The number of permutations of  $n$  distinct objects taken  $r$  at a time is

$${}_n P_r = \frac{n!}{(n-r)!}$$

This is called “ $n$  Permute  $r$ ”

**2.3.5 Permutations in a Circle**

The number of permutations of  $n$  objects arranged in a circle is  $(n-1)!$ .

### 2.3.6 Permutations of a Kind

The number of distinct permutations of  $n$  objects of which  $n_1$  are of one kind,  $n_2$  of a second kind,  $\dots$ ,  $n_k$  of a  $k$ th kind is

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

### 2.3.7 Partitioning

The number of way of partitioning a set of  $n$  objects into  $k$  cells with  $n_1$  elements in the first cell,  $n_2$  elements in the second cell, and so forth, is

$$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1!n_2!\cdots n_k!}$$

Where  $n_1 + n_2 + \cdots + n_k = n$

Notice that this is the same as the last example.

### 2.3.8 Combinations

The number of combinations of  $n$  distinct object taken  $r$  at a time, is:

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

This is partitioning with only two cells.

### 2.3.9 Partitioning and Combinations

Note that:

$$\binom{10}{5, 4, 1} = \binom{10}{5} \binom{5}{4} \binom{1}{1}$$

## 2.4 Probability of an Event

The probability of an event  $A$  is the sum of the weifhts of all sample points on  $A$ . Therefore

$$0 \leq P(A) \leq 1$$

$$P(\emptyset) = 0$$

$$P(S) = 1$$

Furthermore, if  $A_1, A_2, \dots$  is a set of mutually exclusive events, then

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

### 2.4.1 Complimentary Probability

If  $A$  and  $A'$  are complimentary events, then

$$P(A) + P(A') = 1$$

### 2.4.2 Different Outcomes

If an experiment can result in any one of  $N$  different equally likely outcomes, and of exactly  $n$  of those outcomes correspond to event  $A$ , the probability of event  $A$  is

$$P(A) = \frac{n}{N}$$

## 2.5 Homework

- Page 42 #'s 2.3, 2.6, 2.10, 2.11, 2.14, 2.16, 2.18
- Page 51 #'s 2.24, 2.25, 2.28, 2.29, 2.30, 2.33, 2.34, 2.38, 2.39, 2.40, 2.44, 2.46, 2.47, 2.48
- Page 59 #'s 2.52, 2.53, 2.56, 2.57, 2.58, 2.62, 2.67, 2.70

## Homework Overview

Chapter 1 Homework [1.8](#)

Chapter 2 Homework [2.5](#)





# Bibliography

- [1] R.E. Walpole et al. *Probability and Statistics for Engineers and Scientists*. Pearson Education, 2010. ISBN: 9780321629111. URL: <http://books.google.com/books?id=tzZxRQAACAAJ> (cit. on p. 12).