Grokking Tickets – An Analysis

By Nicholas Chen

Introduction

Nanda et. al (2023) (PROGRESS MEASURES FOR GROKKING VIA MECHANISTIC INTERPRETABILITY) hypothesized that phase transitions are inherent to composition largely due to the lottery ticket hypothesis – That early on, the network is a superposition of approximate circuits which improve slowly as each component is weak, but the moment any one component develops, the other components become more useful and generalization accelerates.

In a very recent paper, Minegishi et. al (2023) (GROKKING TICKETS: LOTTERY TICKETS ACCELERATE GROKKING) derived lottery tickets from various phases of grokking (memorization, circuit formation, cleanup) and showed that the later the stage of grokking, the more effective the lottery ticket is in terms of generalization speed, supporting the hypothesis in Nanda et. al. (2023).

This brings up some questions:

- What will the progress measures show if we use these lottery tickets? Naively, we expect the progress measures to be accelerated as we already gave the network better initialization which might allow it to skip grokking phases. At the same time, this is also a test of the progress measures' ability to explain the underlying phenomena. This was not studied in Minegishi et. al (2023).
- What can we say about the universality of circuit formation from these experiments? Recall that in lottery tickets, subnetworks are trained – These networks do not have the flexibility to explore the full optimization landscape compared to a base model. If a network is initialized from a lottery ticket derived from the memorization phase, will it be constrained to not be able to transit to the circuit formation phase? In Minegishi et. al (2023), this was not explored as the number of epochs was limited, so we do not know if some of the accuracies will plateau or continue improving.

Contributions

In this mini project, we make the following contributions:

- We reproduce the core result of Minegishi et. al (2023) by training lottery tickets derived from the 3 phases of grokking and observing the loss curves, and confirm that the later the stage of grokking, the faster the lottery ticket generalizes. One small difference is

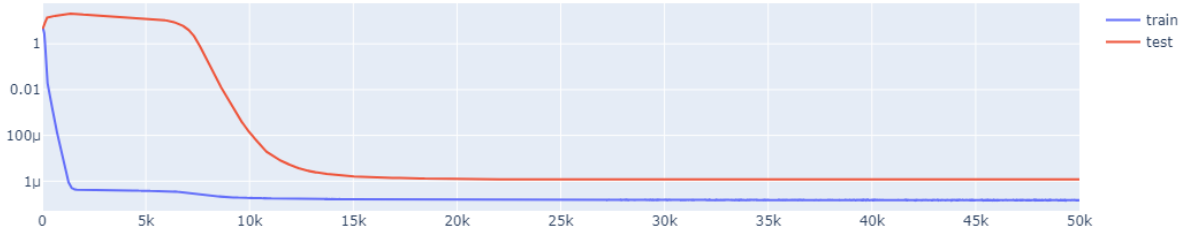
that in Minegishi et. al (2023), biases were not used in the MLP while our work implements it to allow a better comparison to Nanda et. al (2023)
- We show that grokking tickets derived from the circuit formation and cleanup phase display progress measures deviating from the standard phases in Nanda et. al (2023), suggesting that grokking tickets indeed accelerate the grokking phases

Approach

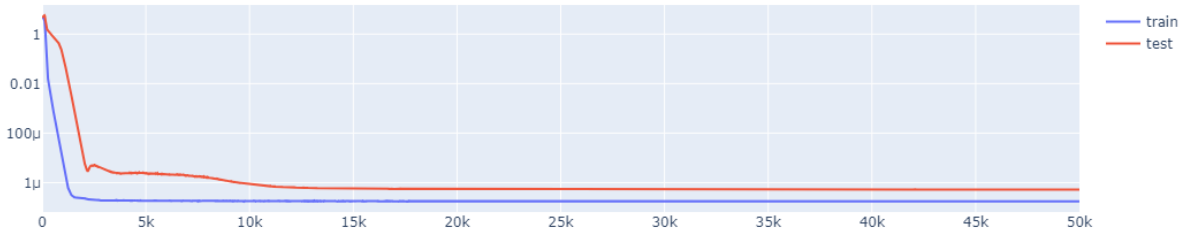
- Obtain the network weights in Nanda et. al (2023) at the various stages:
    - Epoch 0: Initialization
    - Epoch 1.4k: Memorization
    - Epoch 9k: Circuit formation
    - Epoch 14k: Cleanup
- For each of the networks above (excluding epoch 0), derive a mask which prunes away the bottom 40% of weights by magnitude.
- For each mask derived, apply it to the network at epoch 0 and train it via the approach in Nanda et. al (2023)

Grokking Tickets Speed Up Generalization

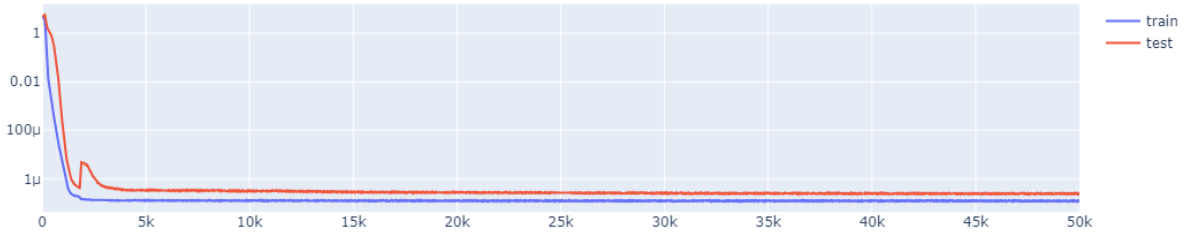
Memorization (derived from lottery ticket at 1.4k epochs):



Circuit formation (derived from lottery ticket at 9k epochs):



Cleanup (derived from lottery ticket at 14k epochs):

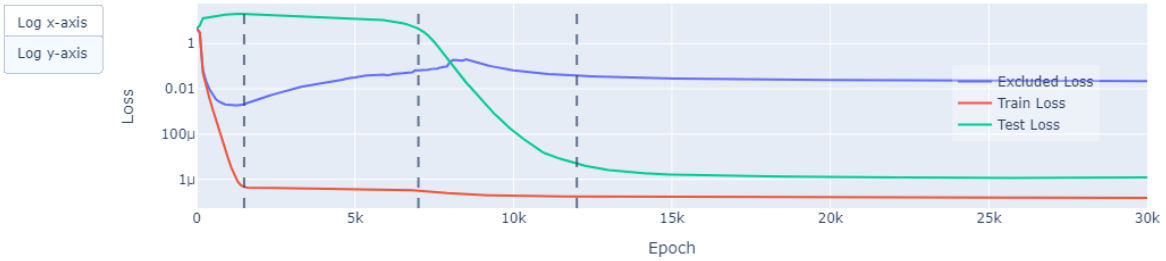


Observations:

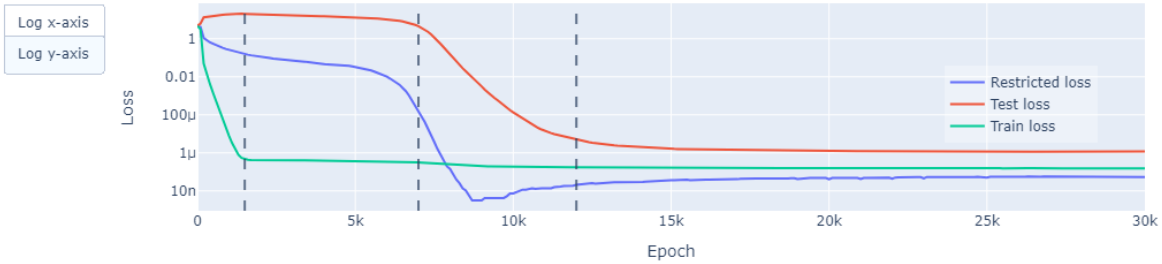
- Test loss always plateaus, and this value improves with the stage of grokking the lottery ticket was initialized with. Does this suggest that circuits do have a certain space of ‘fixed structures’? I.e. if the network topology space is constrained, then some circuits may not form that well and there are no alternatives to reconfigure the same circuit under a different network topology.
- The grokking stage affects the rate of improvement of test loss
- We observe the standard grokking curve in the memorization lottery ticket (train loss improves quickly but test loss increases and plateaus first before rapidly improving)
- In the circuit formation and cleanup ticket, we do not observe an initial test loss plateau, suggesting that these tickets really help ‘skip’ the memorization phase
- In the circuit formation and cleanup ticket, the test loss follows a pattern of ‘negative gradient’ -> ‘even more negative gradient’ -> double descent -> plateau, with the cleanup ticket having a more distinct double descent pattern. Why is this so?

Progress Measures (Memorization Lottery Ticket)

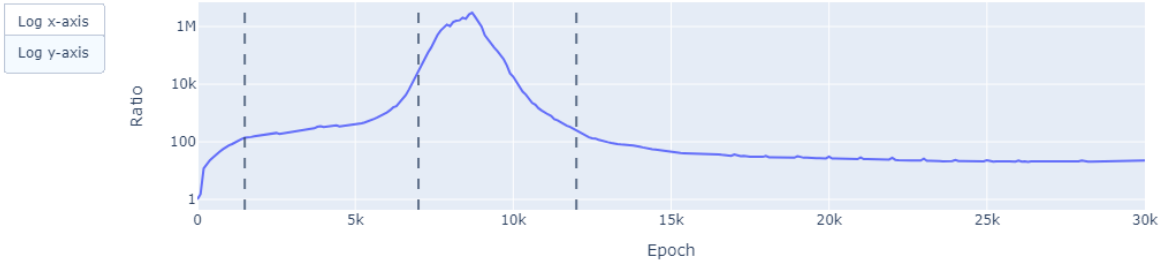
Excluded Loss Over All Frequencies (memorization)

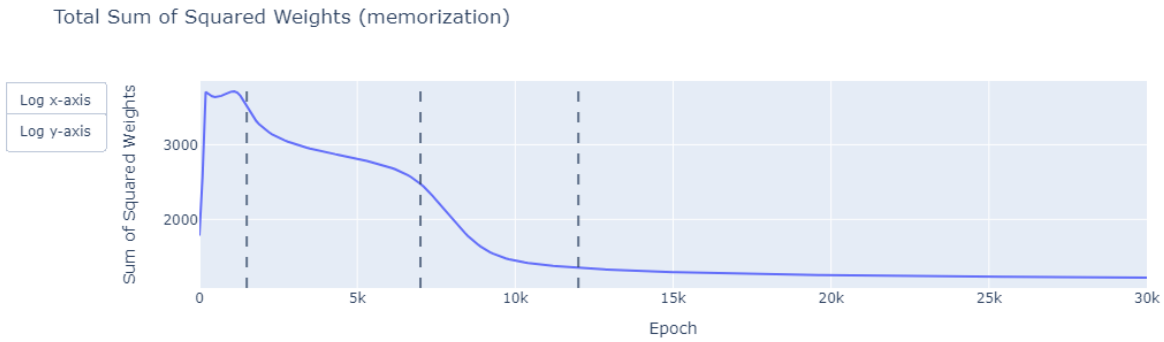


Pure Restricted Loss (memorization)



Ratio of Test Loss/Restricted Loss (memorization)

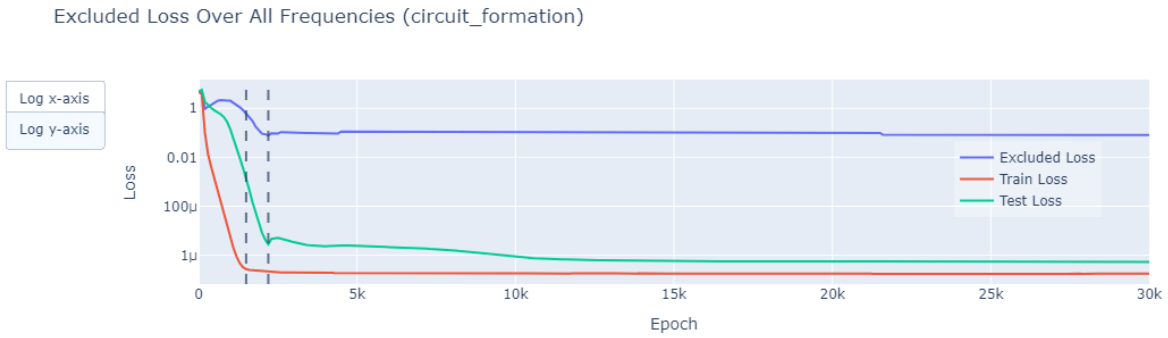


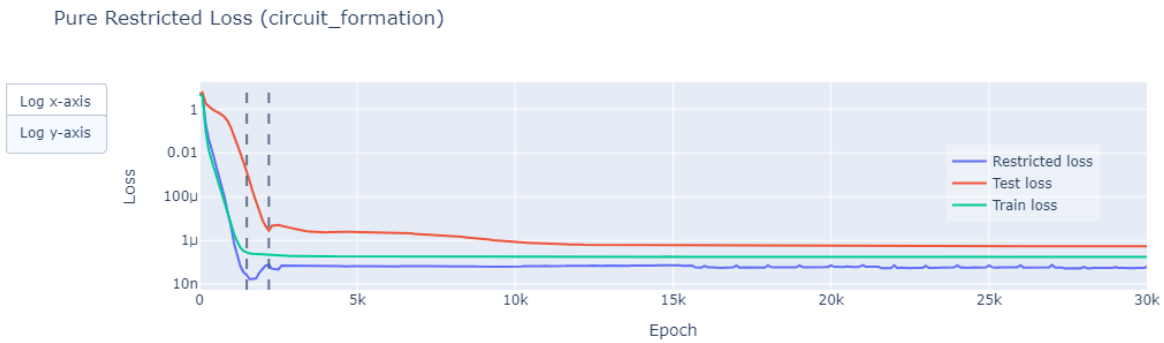


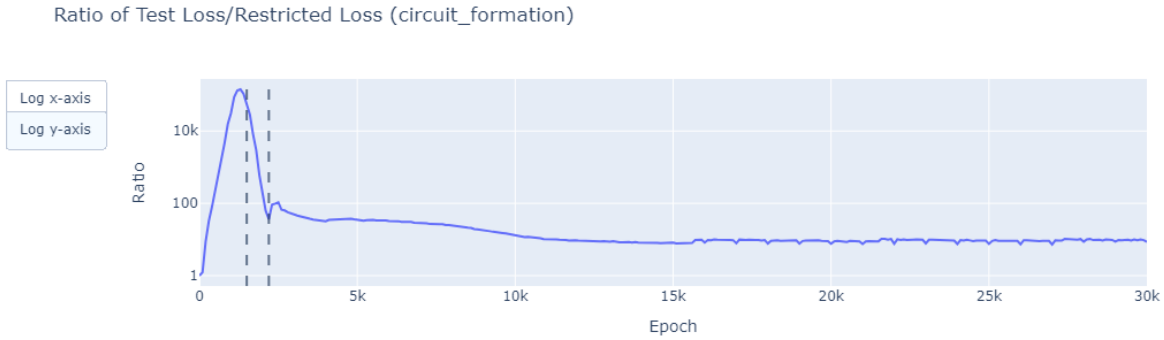
Observations:

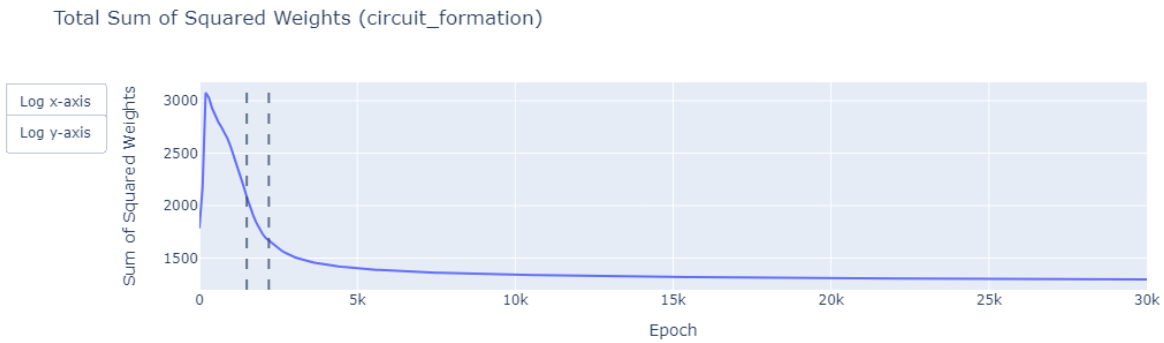
- The overall change in progress measures are consistent with the findings in Nanda et. al (2023)

Progress Measures (Circuit Formation Lottery Ticket)





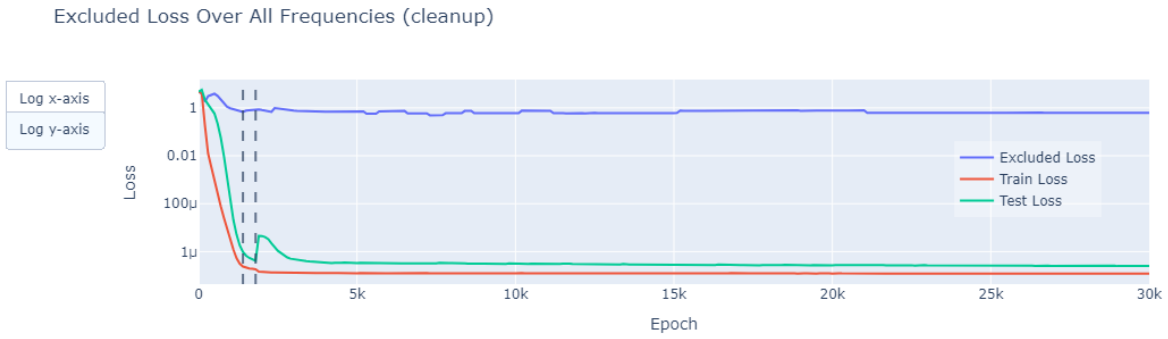


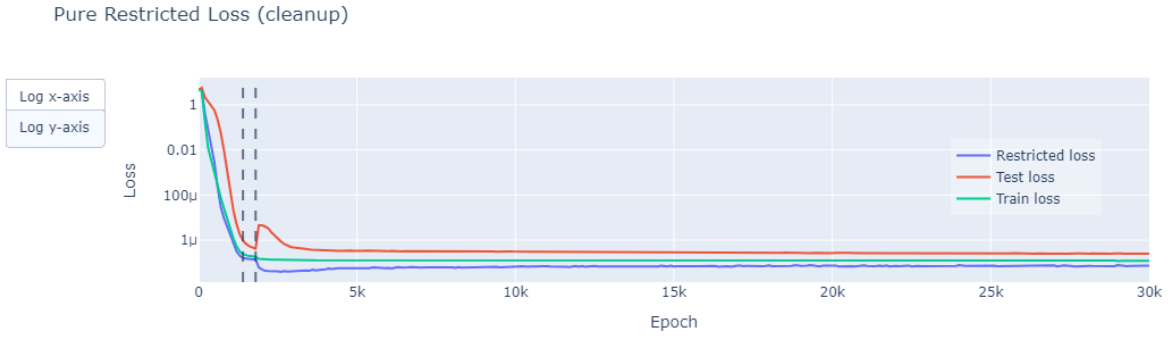


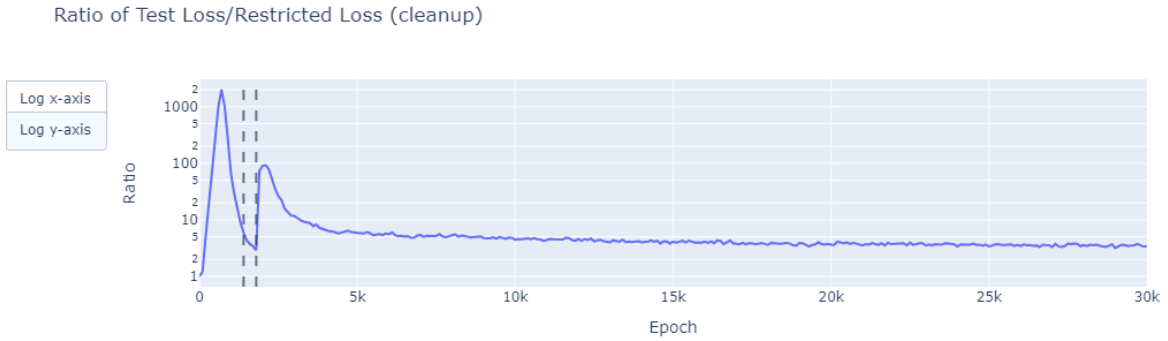
Observations:

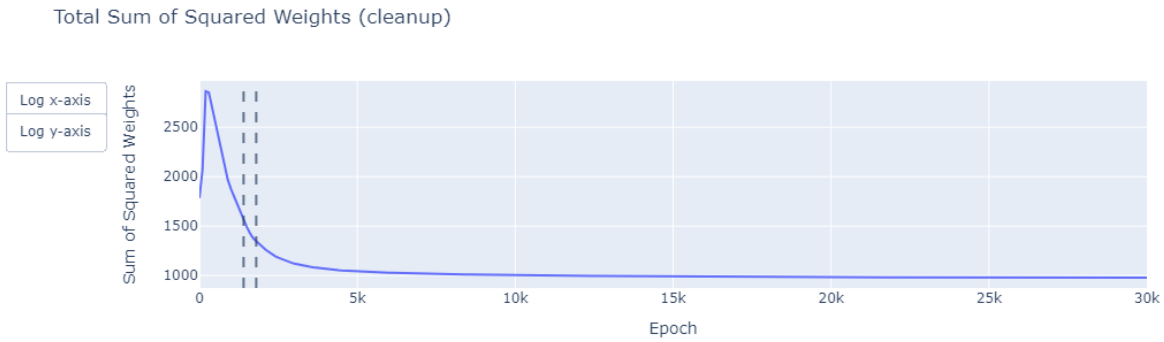
- Test loss does not plateau at the start; It appears that circuit formation overlaps with memorization
- Excluded loss decreases, increases slightly, decreases and then plateaus. This is quite different from the original pattern of decreasing during memorization, increasing during circuit formation then plateauing, suggesting that the grokking phases are indeed accelerated
- Restricted loss decreases immediately and sharply, skipping the ‘mild decline’ phase during circuit formation
- Total sum of squared weights only sees 1 sharp decline instead of 2

Progress Measures (Cleanup Lottery Ticket)









Observations:

- Exact same observations from the Circuit Formation Lottery Ticket, suggesting phases are skipped

Discussion & Conclusions

- Test loss of tickets from different phases plateau at different values – Might this suggest that circuits are universal, in the sense that it requires a certain topology?
- Indeed, progress measures behave differently (accelerated) under grokking tickets, supporting the hypothesis that the lottery ticket hypothesis is a factor behind phase transitions.
- Are there other progress measures that work well with such ‘accelerated phases’ of grokking tickets?

Future Work

- Run with multiple seeds for robustness
- Use the full excluded loss instead of the mean excluded loss, and calculate Gini coefficients as in Nanda et. al (2023) (could not work on this due to time constraints)
- Run with larger networks: If circuits are universal, constraining the topology of a larger network might be less effective at increasing the plateau test loss as the network can explore a larger network topology space despite the constraint