If **F** is a constant force in any direction, then the **work** W done by **F** in moving an object from A to B is

$$W = \mathbf{F} \cdot \overrightarrow{AB}$$
$$= |\mathbf{F}||\overrightarrow{AB}| \cos \theta$$

where θ is the angle between **F** and $\overrightarrow{AB}$. Except for the sign, the work is the magnitude of the effective force in the direction of $\overrightarrow{AB}$ times $\overrightarrow{AB}$.

EXAMPLE 7 Finding Work

Find the work done by a 10 pound force acting in the direction $\langle 1, 2\rangle$ in moving an object 3 feet from $(0, 0)$ to $(3, 0)$.

SOLUTION The force **F** has magnitude 10 and acts in the direction $\langle 1, 2\rangle$, so

$$\mathbf{F} = 10 \frac{\langle 1, 2\rangle}{|\langle 1, 2\rangle|} = \frac{10}{\sqrt{5}} \langle 1, 2\rangle.$$

The direction of motion is from $A = (0, 0)$ to $B = (3, 0)$, so $\overrightarrow{AB} = \langle 3, 0\rangle$. Thus, the work done by the force is

$$\mathbf{F} \cdot \overrightarrow{AB} = \frac{10}{\sqrt{5}} \langle 1, 2\rangle \cdot \langle 3, 0\rangle = \frac{30}{\sqrt{5}} \approx 13.42 \text{ foot-pounds.}$$

Now try Exercise 53.

.TS FOR WORK

'ork is usually measured in foot-
ounds or Newton-meters. One
Newton-meter is commonly referred to
as one Joule.

QUICK REVIEW 6.2 *(For help, go to Section 6.1.)*

In Exercises 1–4, find $|\mathbf{u}|$.

1. $\mathbf{u} = \langle 2, -3\rangle$ **2.** $\mathbf{u} = -3\mathbf{i} - 4\mathbf{j}$

3. $\mathbf{u} = \cos 35°\,\mathbf{i} + \sin 35°\,\mathbf{j}$

4. $\mathbf{u} = 2(\cos 75°\mathbf{i} + \sin 75°\mathbf{j})$

In Exercises 5–8, the points A and B lie on the circle $x^2 + y^2 = 4$. Find the component form of the vector $\overrightarrow{AB}$.

5. $A = (-2, 0), B = (1, \sqrt{3})$ **6.** $A = (2, 0), B = (1, \sqrt{3})$

7. $A = (2, 0), B = (1, -\sqrt{3})$

8. $A = (-2, 0), B = (1, -\sqrt{3})$

In Exercises 9 and 10, find a vector **u** with the given magnitude in the direction of **v**.

9. $|\mathbf{u}| = 2, \mathbf{v} = \langle 2, 3\rangle$ **10.** $|\mathbf{u}| = 3, \mathbf{v} = -4\mathbf{i} + 3\mathbf{j}$

SECTION 6.2 EXERCISES

In Exercises 1–8, find the dot product of **u** and **v**.

1. $\mathbf{u} = \langle 5, 3\rangle, \mathbf{v} = \langle 12, 4\rangle$

2. $\mathbf{u} = \langle -5, 2\rangle, \mathbf{v} = \langle 8, 13\rangle$

3. $\mathbf{u} = \langle 4, 5\rangle, \mathbf{v} = \langle -3, -7\rangle$

4. $\mathbf{u} = \langle -2, 7\rangle, \mathbf{v} = \langle -5, -8\rangle$

5. $\mathbf{u} = -4\mathbf{i} - 9\mathbf{j}, \mathbf{v} = -3\mathbf{i} - 2\mathbf{j}$

6. $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j}, \mathbf{v} = -8\mathbf{i} + 7\mathbf{j}$

7. $\mathbf{u} = 7\mathbf{i}, \mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$

8. $\mathbf{u} = 4\mathbf{i} - 11\mathbf{j}, \mathbf{v} = -3\mathbf{j}$

In Exercises 9–12, use the dot product to find $|\mathbf{u}|$.

9. $\mathbf{u} = \langle 5, -12\rangle$ **10.** $\mathbf{u} = \langle -8, 15\rangle$

11. $\mathbf{u} = -4\mathbf{i}$ **12.** $\mathbf{u} = 3\mathbf{j}$

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In Exercises 13–22, find the angle θ between the vectors.

13. $\mathbf{u} = \langle -4, -3\rangle$, $\mathbf{v} = \langle -1, 5\rangle$

14. $\mathbf{u} = \langle 2, -2\rangle$, $\mathbf{v} = \langle -3, -3\rangle$

15. $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle -3, 5\rangle$ **16.** $\mathbf{u} = \langle 5, 2\rangle$, $\mathbf{v} = \langle -6, -1\rangle$

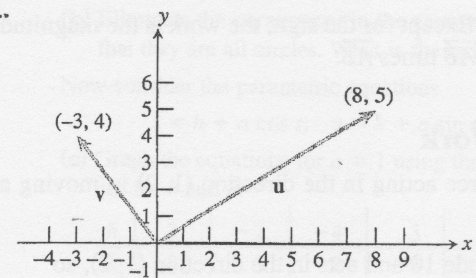
17. $\mathbf{u} = 3\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + 2\sqrt{3}\mathbf{j}$

18. $\mathbf{u} = -2\mathbf{i}$, $\mathbf{v} = 5\mathbf{j}$

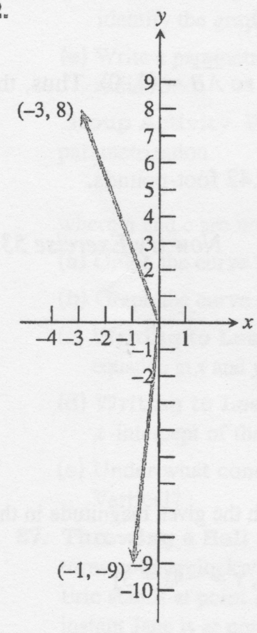
19. $\mathbf{u} = \left(2\cos\frac{\pi}{4}\right)\mathbf{i} + \left(2\sin\frac{\pi}{4}\right)\mathbf{j}$, $\mathbf{v} = \left(\cos\frac{3\pi}{2}\right)\mathbf{i} + \left(\sin\frac{3\pi}{2}\right)\mathbf{j}$

20. $\mathbf{u} = \left(\cos\frac{\pi}{3}\right)\mathbf{i} + \left(\sin\frac{\pi}{3}\right)\mathbf{j}$, $\mathbf{v} = \left(3\cos\frac{5\pi}{6}\right)\mathbf{i} + \left(3\sin\frac{5\pi}{6}\right)\mathbf{j}$

21.



22.



In Exercises 23–24, prove that the vectors **u** and **v** are orthogonal.

23. $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle 3/2, -1\rangle$

24. $\mathbf{u} = \langle -4, -1\rangle$, $\mathbf{v} = \langle 1, -4\rangle$

In Exercises 25–28, find the vector projection of **u** onto **v**. Then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.

25. $\mathbf{u} = \langle -8, 3\rangle$, $\mathbf{v} = \langle -6, -2\rangle$ **26.** $\mathbf{u} = \langle 3, -7\rangle$, $\mathbf{v} = \langle -2, -6\rangle$

27. $\mathbf{u} = \langle 8, 5\rangle$, $\mathbf{v} = \langle -9, -2\rangle$ **28.** $\mathbf{u} = \langle -2, 8\rangle$, $\mathbf{v} = \langle 9, -3\rangle$

In Exercises 29 and 30, find the interior angles of the triangle with given vertices.

29. $(-4, 5)$, $(1, 10)$, $(3, 1)$ **30.** $(-4, 1)$, $(1, -6)$, $(5, -1)$

In Exercises 31 and 32, find $\mathbf{u}\cdot\mathbf{v}$ satisfying the given conditions where θ is the angle between **u** and **v**.

31. $\theta = 150°$, $|\mathbf{u}| = 3$, $|\mathbf{v}| = 8$ **32.** $\theta = \frac{\pi}{3}$, $|\mathbf{u}| = 12$, $|\mathbf{v}| = 40$

In Exercises 33–38, determine whether the vectors **u** and **v** are parallel, orthogonal, or neither.

33. $\mathbf{u} = \langle 5, 3\rangle$, $\mathbf{v} = \left\langle -\frac{10}{4}, -\frac{3}{2}\right\rangle$

34. $\mathbf{u} = \langle 2, 5\rangle$, $\mathbf{v} = \left\langle \frac{10}{3}, \frac{4}{3}\right\rangle$

35. $\mathbf{u} = \langle 15, -12\rangle$, $\mathbf{v} = \langle -4, 5\rangle$

36. $\mathbf{u} = \langle 5, -6\rangle$, $\mathbf{v} = \langle -12, -10\rangle$

37. $\mathbf{u} = \langle -3, 4\rangle$, $\mathbf{v} = \langle 20, 15\rangle$

38. $\mathbf{u} = \langle 2, -7\rangle$, $\mathbf{v} = \langle -4, 14\rangle$

In Exercises 39–42, find

(a) the x-intercept A and y-intercept B of the line.

(b) the coordinates of the point P so that $\overrightarrow{AP}$ is perpendicular to the line and $|\overrightarrow{AP}| = 1$. (There are two answers.)

39. $3x - 4y = 12$ **40.** $-2x + 5y = 10$

41. $3x - 7y = 21$ **42.** $x + 2y = 6$

In Exercises 43 and 44, find the vector(s) **v** satisfying the given conditions.

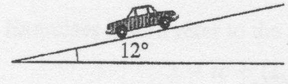
43. $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{u}\cdot\mathbf{v} = 10$, $|\mathbf{v}|^2 = 17$

44. $\mathbf{u} = \langle -2, 5\rangle$, $\mathbf{u}\cdot\mathbf{v} = -11$, $|\mathbf{v}|^2 = 10$

45. Sliding Down a Hill Ojemba is sitting on a sled on the side of a hill inclined at 60°. The combined weight of Ojemba and the sled is 160 pounds. What is the magnitude of the force required for Mandisa to keep the sled from sliding down the hill?

46. Revisiting Example 6 Suppose Juan and Rafaela switch positions. The combined weight of Rafaela and the sled is 125 pounds. What is the magnitude of the force required for Juan to keep the sled from sliding down the hill?

47. Braking Force A 2000 pound car is parked on a street that makes an angle of 12° with the horizontal (see figure).



(a) Find the magnitude of the force required to keep the car from rolling down the hill.

(b) Find the force perpendicular to the street.

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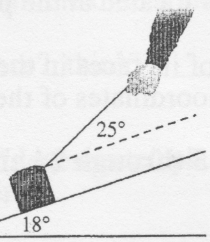
48. Effective Force A 60 pound force $\mathbf{F}$ that makes an angle of 25° with an inclined plane is pulling a box up the plane. The inclined plane makes an 18° angle with the horizontal (see figure). What is the magnitude of the effective force pulling the box up the plane?



49. Work Find the work done lifting a 2600 pound car 5.5 feet.

50. Work Find the work done lifting a 100 pound bag of potatoes 3 feet.

51. Work Find the work done by a force $\mathbf{F}$ of 12 pounds acting in the direction $\langle 1, 2\rangle$ in moving an object 4 feet from $(0, 0)$ to $(4, 0)$.

52. Work Find the work done by a force $\mathbf{F}$ of 24 pounds acting in the direction $\langle 4, 5\rangle$ in moving an object 5 feet from $(0, 0)$ to $(5, 0)$.

53. Work Find the work done by a force $\mathbf{F}$ of 30 pounds acting in the direction $\langle 2, 2\rangle$ in moving an object 3 feet from $(0, 0)$ to a point in the first quadrant along the line $y = (1/2)x$.

54. Work Find the work done by a force $\mathbf{F}$ of 50 pounds acting in the direction $\langle 2, 3\rangle$ in moving an object 5 feet from $(0, 0)$ to a point in the first quadrant along the line $y = x$.

55. Work The angle between a 200 pound force $\mathbf{F}$ and $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j}$ is 30°. Find the work done by $\mathbf{F}$ in moving an object from A to B.

56. Work The angle between a 75 pound force $\mathbf{F}$ and $\overrightarrow{AB}$ is 60°, where $A = (-1, 1)$ and $B = (4, 3)$. Find the work done by $\mathbf{F}$ in moving an object from A to B.

57. Properties of the Dot Product Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors and let c be a scalar. Use the component form of vectors to prove the following properties.

(a) $\mathbf{0} \cdot \mathbf{u} = 0$

(b) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

(c) $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$

(d) $(c\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$

58. Group Activity Projection of a Vector Let $\mathbf{u}$ and $\mathbf{v}$ be nonzero vectors. Prove that

(a) $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$

(b) $(\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u}) \cdot (\text{proj}_{\mathbf{v}}\mathbf{u}) = 0$

59. Group Activity Connecting Geometry and Vectors Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

60. If $\mathbf{u}$ is any vector, prove that we can write $\mathbf{u}$ as

$$\mathbf{u} = (\mathbf{u} \cdot \mathbf{i})\mathbf{i} + (\mathbf{u} \cdot \mathbf{j})\mathbf{j}.$$

Standardized Test Questions

61. True or False If $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u}$ and $\mathbf{v}$ are perpendicular. Justify your answer.

62. True or False If $\mathbf{u}$ is a unit vector, then $\mathbf{u} \cdot \mathbf{u} = 1$. Justify your answer.

In Exercises 63–66, you may use a graphing calculator to solve the problem.

63. Multiple Choice Let $\mathbf{u} = \langle 1, 1\rangle$ and $\mathbf{v} = \langle -1, 1\rangle$. Which of the following is the angle between $\mathbf{u}$ and $\mathbf{v}$?

(A) 0° (B) 45° (C) 60° (D) 90° (E) 135°

64. Multiple Choice Let $\mathbf{u} = \langle 4, -5\rangle$ and $\mathbf{v} = \langle -2, -3\rangle$. Which of the following is equal to $\mathbf{u} \cdot \mathbf{v}$?

(A) -23 (B) -7 (C) 7 (D) 23 (E) $\sqrt{7}$

65. Multiple Choice Let $\mathbf{u} = \langle 3/2, -3/2\rangle$ and $\mathbf{v} = \langle 2, 0\rangle$. Which of the following is equal to $\text{proj}_{\mathbf{v}}\mathbf{u}$?

(A) $\langle 3/2, 0\rangle$ (B) $\langle 3, 0\rangle$ (C) $\langle -3/2, 0\rangle$

(D) $\langle 3/2, 3/2\rangle$ (E) $\langle -3/2, -3/2\rangle$

66. Multiple Choice Which of the following vectors describes a 5 lb force acting in the direction of $\mathbf{u} = \langle -1, 1\rangle$?

(A) $5\langle -1, 1\rangle$ (B) $\dfrac{5}{\sqrt{2}}\langle -1, 1\rangle$ (C) $5\langle 1, -1\rangle$

(D) $\dfrac{5}{\sqrt{2}}\langle 1, -1\rangle$ (E) $\dfrac{5}{2}\langle -1, 1\rangle$

Explorations

67. Distance from a Point to a Line Consider the line L with equation $2x + 5y = 10$ and the point $P = (3, 7)$.

(a) Verify that $A = (0, 2)$ and $B = (5, 0)$ are the y- and x-intercepts of L.

(b) Find $\mathbf{w}_1 = \text{proj}_{\overrightarrow{AB}}\overrightarrow{AP}$ and $\mathbf{w}_2 = \overrightarrow{AP} - \text{proj}_{\overrightarrow{AB}}\overrightarrow{AP}$.

(c) **Writing to Learn** Explain why $|\mathbf{w}_2|$ is the distance from P to L. What is this distance?

(d) Find a formula for the distance of any point $P = (x_0, y_0)$ to L.

(e) Find a formula for the distance of any point $P = (x_0, y_0)$ to the line $ax + by = c$.

Extending the Ideas

68. Writing to Learn Let $\mathbf{w} = (\cos t)\,\mathbf{u} + (\sin t)\,\mathbf{v}$ where $\mathbf{u}$ and $\mathbf{v}$ are not parallel.

(a) Can the vector $\mathbf{w}$ be parallel to the vector $\mathbf{u}$? Explain.

(b) Can the vector $\mathbf{w}$ be parallel to the vector $\mathbf{v}$? Explain.

(c) Can the vector $\mathbf{w}$ be parallel to the vector $\mathbf{u} + \mathbf{v}$? Explain.

69. If the vectors $\mathbf{u}$ and $\mathbf{v}$ are not parallel, prove that

$$a\mathbf{u} + b\mathbf{v} = c\mathbf{u} + d\mathbf{v} \Rightarrow a = c,\ b = d.$$