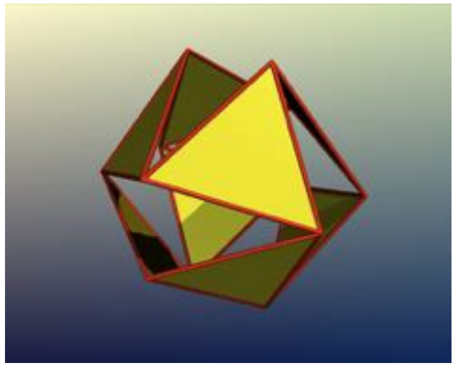
Other Rotation and Reflection Groups



Desmos questions

Recap

What are the four requirements of a group?

A set of elements under an operation

Identity element

Every element has an inverse (invertibility)

Associative Property

Closure

From Friday: Make up a set of exactly 4 numbers that is a group under multiplication if possible.

1, -1, i, -i

What is the maximum period possible for elements in the 10 post Snap group?

make components of 2,3,5. period 30

5 post

per
2

3

1 9

2 8

3 7

6 4 → 12

2 3 5

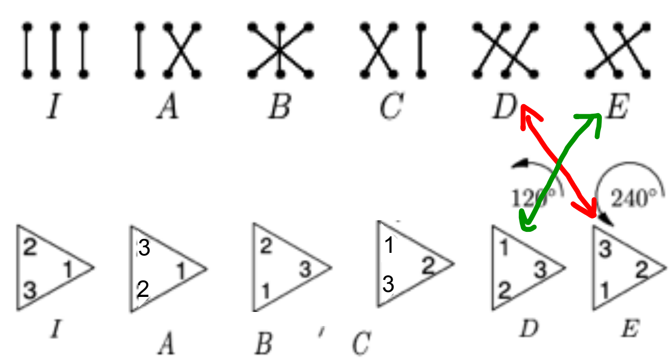
2 8 → 8

3

What is meant by **an isomorphic** group?

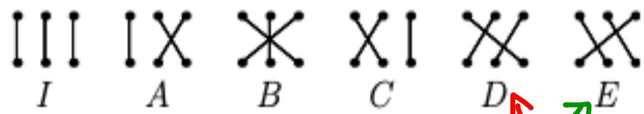
If you can find a one-to-one correspondence between the two groups' elements, where the results of each group's operation on corresponding elements also correspond.

Or: Every element in one, has a partner in the other that plays the same "role".

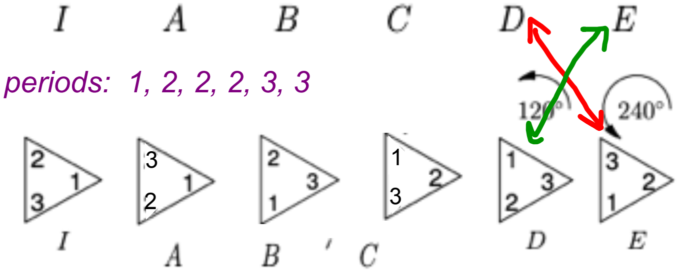


Isomorphic groups:

- same number of elements
- corresponding elements must have same period
- equivalent tables
- same inverses

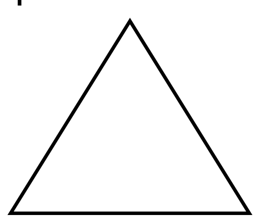


periods: 1, 2, 2, 2, 3, 3



periods: 1, 2, 2, 2, 3, 3

Yesterday you created the "flip" group, formally known as the **"rotation - reflection"** group of an equilateral triangle.



"rotate me, flip me!"

Desmos questions

1. What does it mean to be a generator?

2. Could you have generated the entire "flip" group with....

a) One rotation?

b) One reflection?

c) One reflection and one rotation?

d) Two rotations?

e) Two reflections?

1. What does it mean to be a generator?

all the elements in the group can be created by one element
or by two or more elements together.

2. Could you have generated the entire "flip" group with....

a) One rotation? No. The orientation will never change, therefore cannot generate the reflections.

b) One reflection? No. For example, the A reflection just toggles with I.

c) One reflection and one rotation? Yes. By combining the reflection and the rotation in various ways.

d) Two rotations? No. The orientation won't change.

e) Two reflections? Yes. Two different reflections generates a rotation.

So when we called our flip group a "reflection rotation" group, we really could have just called it a reflection group. (two reflections can create all the rotations). Herreshoff also calls them "dihedral" groups (two sides, like the front and back of our equilateral triangle). All regular polygons are dihedral.

These are all synonyms:

Reflection/Rotation Group

Reflection Group

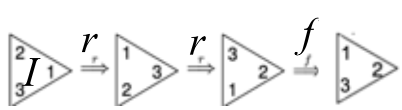
Dihedral Group; D3 is the group of symmetries of an equilateral triangle.

Starting from #8 in Chap3 From Snaps to Flips:

Since one reflection (flip) combined with one rotation can generate all the elements in the Flip group, let's use the element A and element D to generate the other four elements.

f **= element A, flip over horizontal axis**

r **= element D, 120° rotation**

$fr^2 =$  **= element C, flip over "C" axis**

Now use f and r to generate the other three elements....

A: f B: fr C: fr^2 D: r E: r^2 I: I

Starting from #8 in Chap3 From Snaps to Flips:

Now let's use the element A and element D to generate the D3 table.

f = element A, flip over horizontal axis

r = element D, 120° rotation

A: f
B: fr
C: fr^2
D: r
E: r^2
I: I

note order is different on table

	$\cdot$	I	r (D)	r^2 (E)	f (A)	fr (B)	fr^2 (C)
	I						
D	r			fr^2			
E	r^2						
A	f						
B	fr						
C	fr^2						

Figure 6: Unfilled alternate D_3 table.

$\cdot$	I	r	r^2	f	fr	fr^2
I						
r				fr^2		
r^2						
f						
fr						
fr^2						

A: f

B: fr

C: fr^2

D: r

E: r^2

I: I

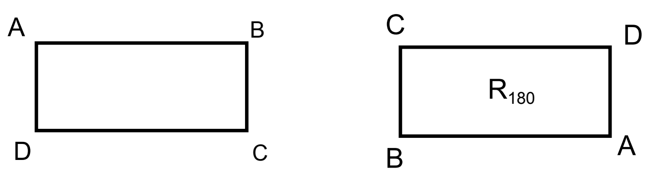
$\cdot$	I	r	r^2	f	fr	fr^2
I	I	r	r^2	f	fr	fr^2
r	r	r^2	I	fr^2	f	fr
r^2	r^2	I	r	fr	fr^2	f
f	f	fr	fr^2	I	r	r^2
fr	fr	fr^2	f	r^2	I	r
fr^2	fr^2	f	fr	r	r^2	I

Figure 8: Completed alternate D_3 table.

Objective today: investigate other geometric groups using rotations and/or reflections.

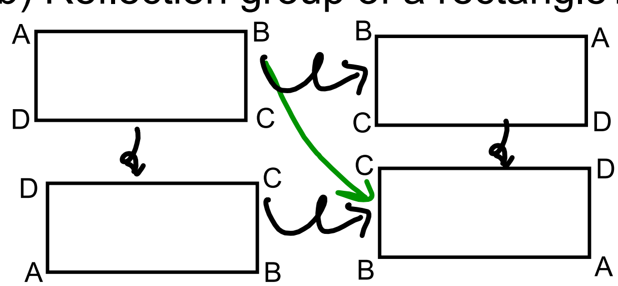
How many elements would be in the....

a) Rotation group of a rectangle?



generator:
R_{180}

b) Reflection group of a rectangle?

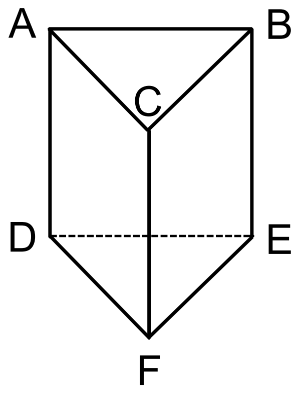


generator:

2 reflections or

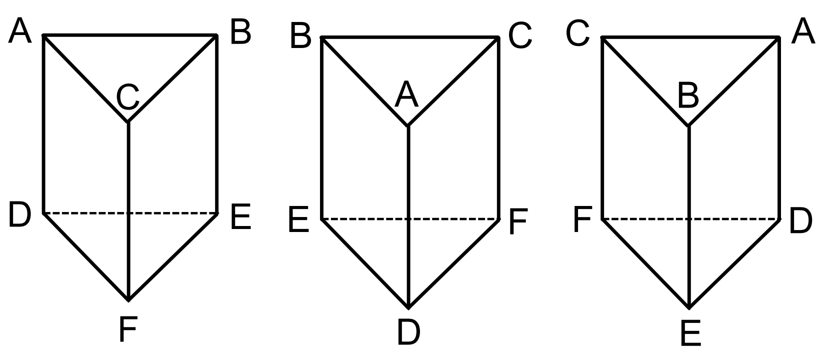
R_{180} and 1 reflection

How many elements are in the rotation group of an equilateral triangular prism?



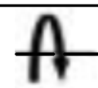
Only rotate me!

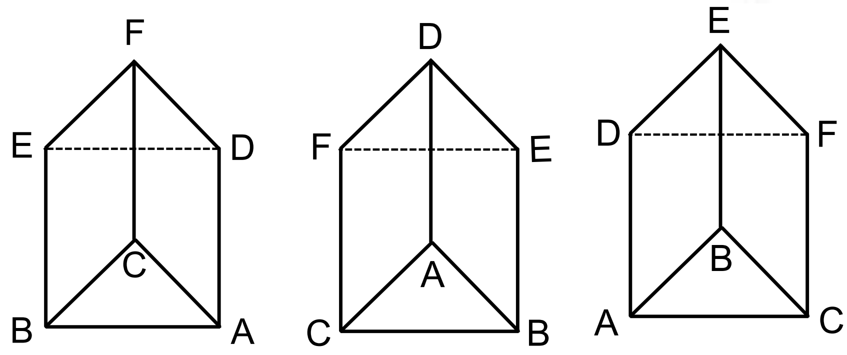
Horizontal Rotations



Identity

2 horizontal rotations

and now each with a Vertical Rotation 



3 vertical rotations

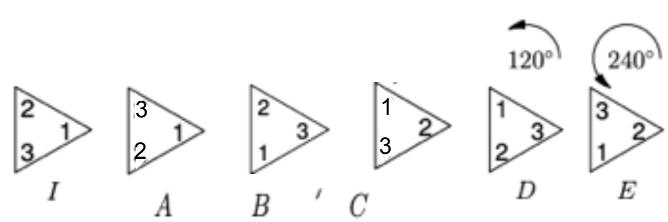
Note that the orientation of the vertices remains the same.

Identity

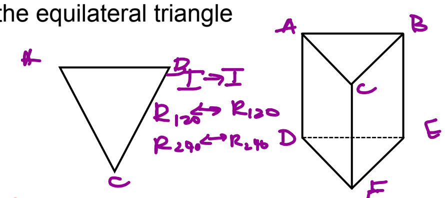
2 horizontal rotations

3 vertical rotations

What other group (that we've seen) is it isomorphic to?



Rotation group of an equilateral triangular prism is isomorphic to the Reflection group of the equilateral triangle



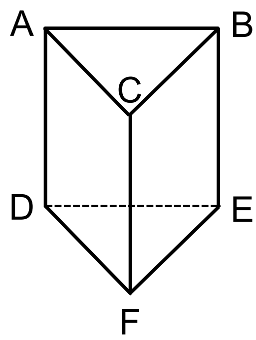
reflect and rotate me!

Only rotate me!

So the **reflection** group in 2-d is....

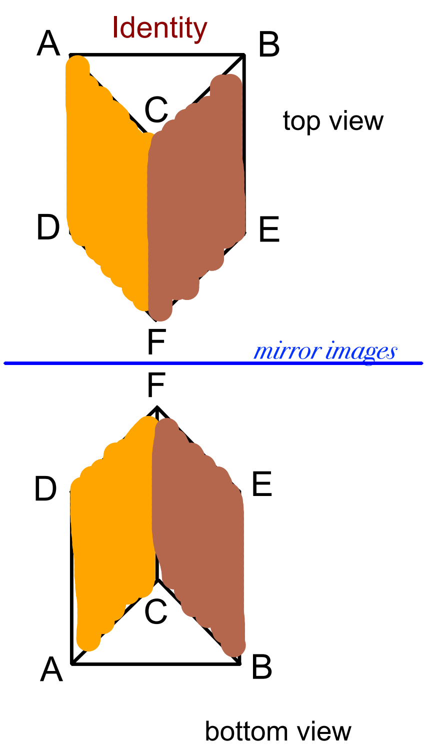
isomorphic to the **rotation** group of the corresponding 3-d prism.

What about the REFLECTION group of the triangular prism? There should be more than 6, but how many more?



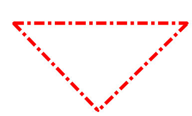
12

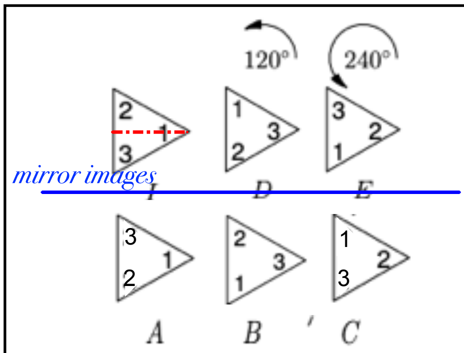
The 6 from before and those same 6 with reversed orientation.



Reflection of the triangular prism across a horizontal plane.

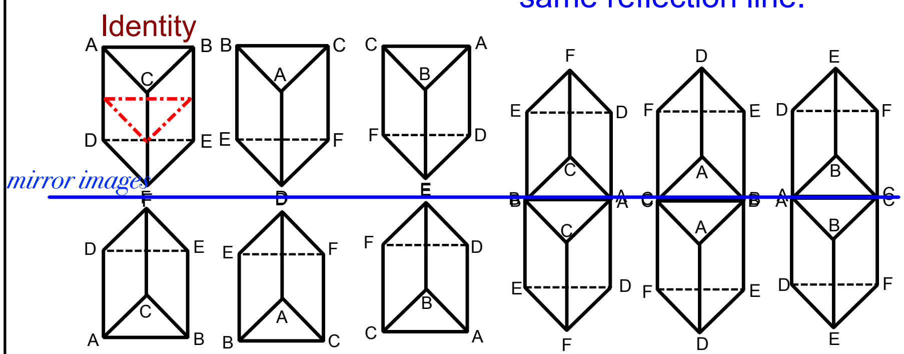
Note that the orientation of the vertices is not the same as the Identity.





To quickly find the reflections, simply reflect the Identity over 1 axis, then do the rotations on this reflection;

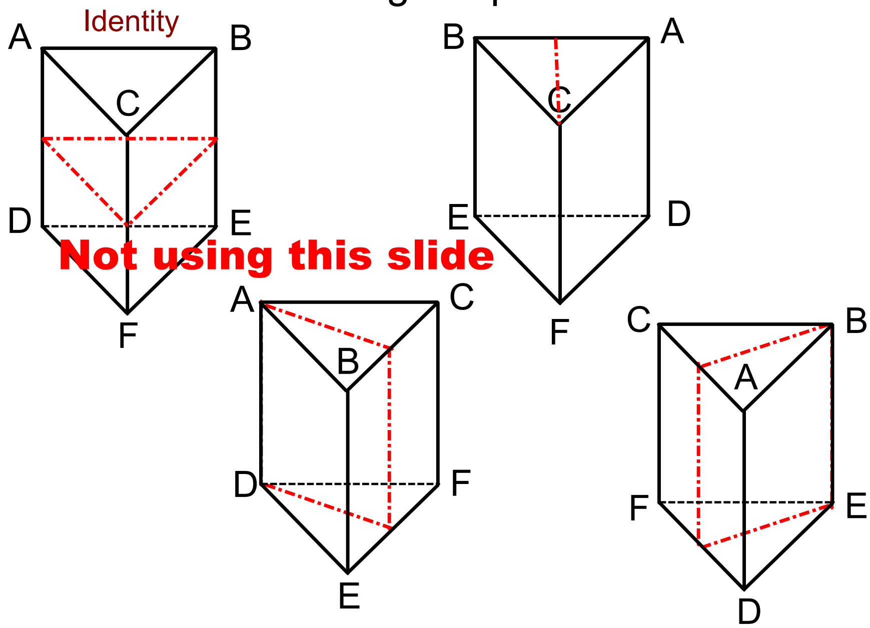
or reflect the Identity and its rotations all across the same reflection line.



The answer is twice as many as the rotate only group. 12 total! (your 6 from before and those same 6 with reversed orientation.

Will reflection groups ALWAYS be twice as many as their rotation group counterpart????????????????

Reflections of the triangular prism.



Note that the orientation of the vertices is not the same as the Identity.