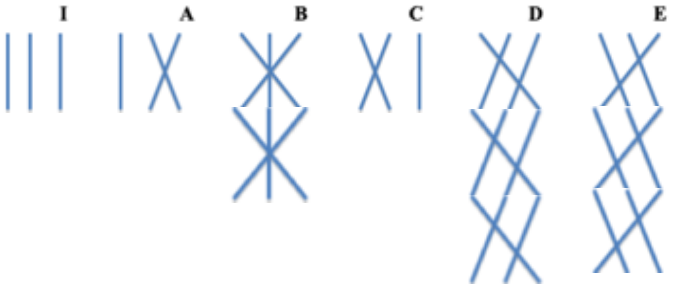
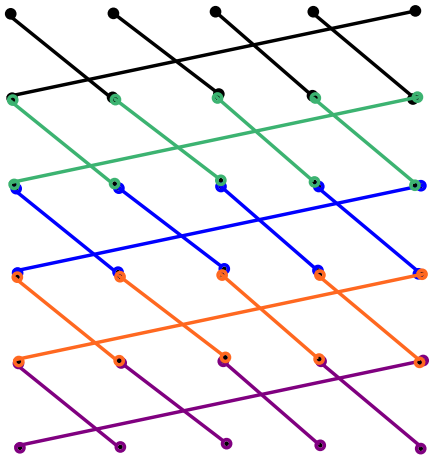
The Three Post Group

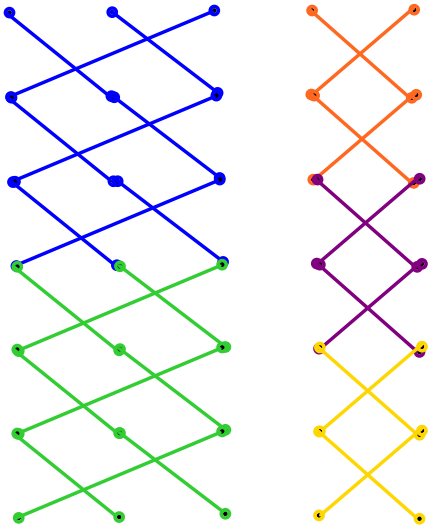


The Five Post Group



All post groups have elements with this kind of diagonal arrangement. The period for these elements is the number of posts.

The Five Post Group



Here we have one of the 5post elements, and we see it has two components: one component with 3 posts and one with 2 posts.

It takes two iterations of the 3 post component and three interations of the 2 post component before both components give us the Identity. So the period of this 5 post element is 6.

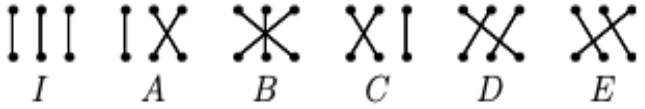
A group's generators are a combination of elements that can be used repeatedly to create all the elements of a group.

Two reflections will generate a rotation.

Isomorphic groups:

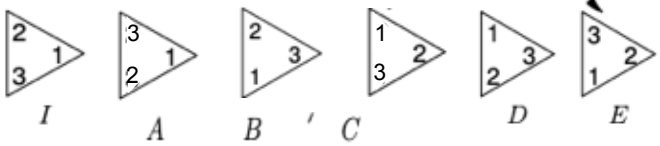
- same number of elements
- corresponding elements must have same period
- equivalent tables (transpose is fine; maybe have to switch some rows or columns)
- same inverses

post snap group:



periods: 1, 2, 2, 2, 3, 3
inverse: itself itself itself itself E D

geometric reflection group:



periods: 1, 2, 2, 2, 3, 3
inverse: itself itself itself itself E D

3 post Snap group is isomorphic to the reflection group of the equilateral triangle

Isomorphic groups:

- same number of elements
- corresponding elements must have same period
- equivalent tables
- same inverses

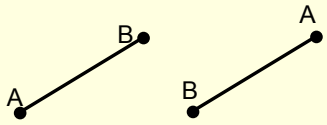
post snap group:

periods: 1
inverse: itself

periods: 1 2
inverse: itself itself

geometric reflection group:

periods: 1
inverse: itself



periods: 1 2
inverse: itself itself

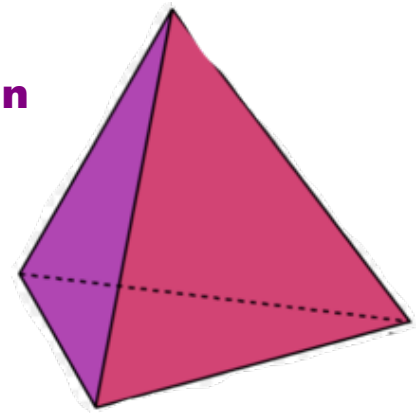
1 post Snap group is isomorphic to the reflection group of the point

2 post Snap group is isomorphic to the reflection group of the line segment

Note the dimension of the post snap group vs its geometric isometry

So, what geometric shape's reflection group will be isomorphic to the 4 post snap group?

tetrahedron



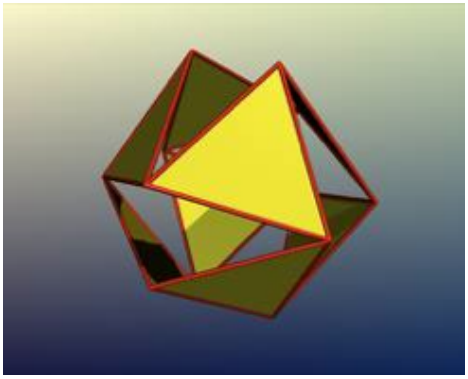
How many elements are there in the 4post snap group?

How many elements are there in the reflection group of the tetrahedron?

Do all geometric reflection groups have an isometry with a Post Snap group?

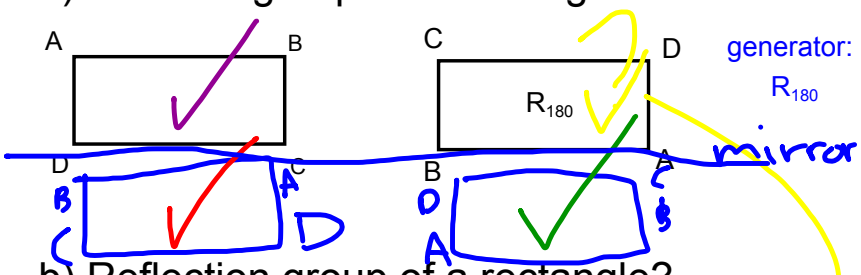
Chapter 4 Objective "Other Rotation and Reflection Groups":

investigate other geometric groups using rotations and/or reflections.

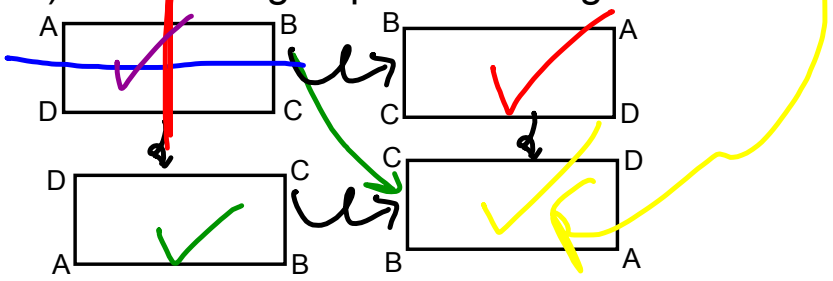


How many elements would be in the....

a) Rotation group of a rectangle?

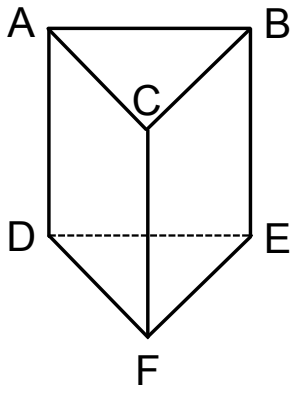


b) Reflection group of a rectangle?



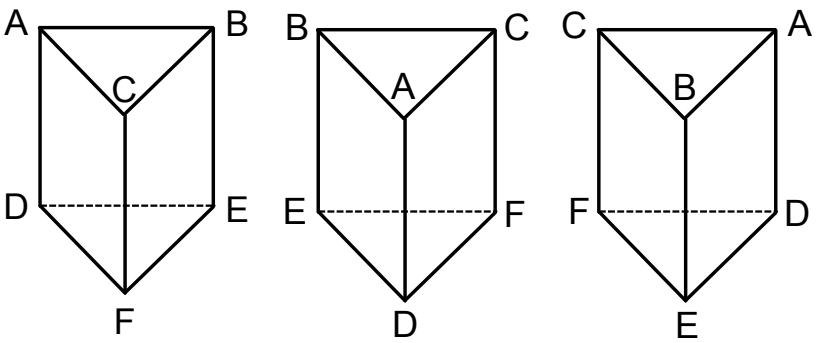
generator: 2 reflections or R_{180} and 1 reflection

How many elements are in the rotation group of an equilateral triangular prism?



Only rotate me!

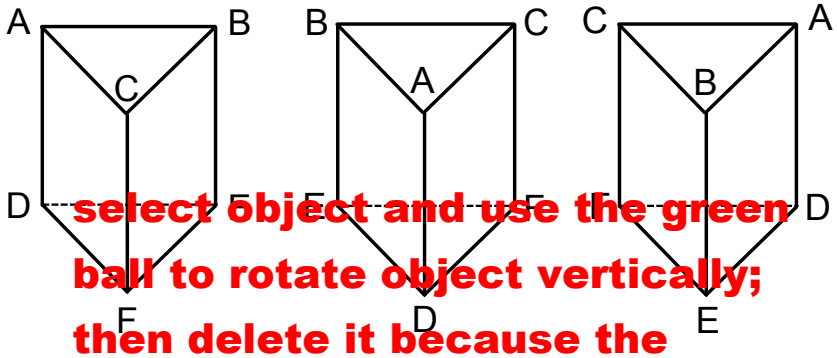
Horizontal Rotations



Identity

2 horizontal rotations

and now each with a Vertical Rotation

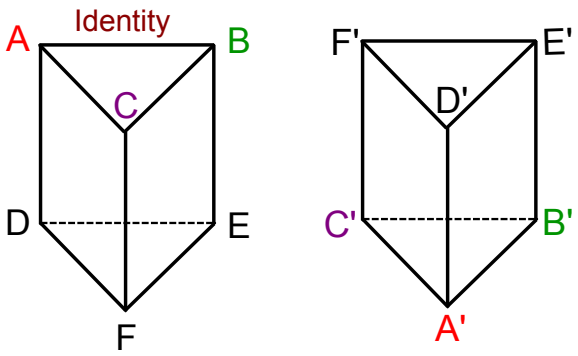


select object and use the green ball to rotate object vertically; then delete it because the version with rightside up letters is below.

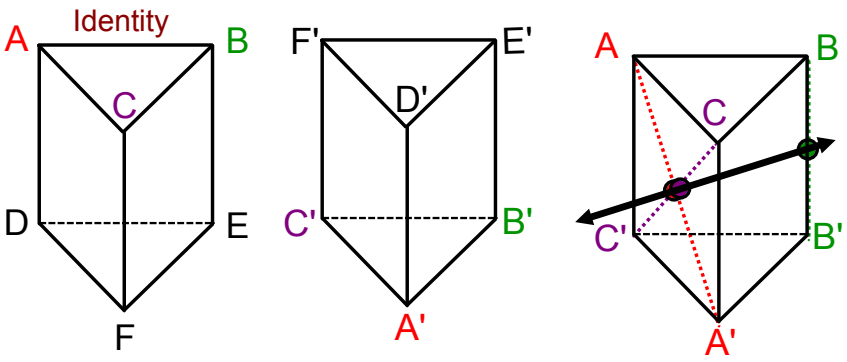
3 vertical rotations

Note that the orientation of the vertices remains the same.

and now each with a Vertical Rotation



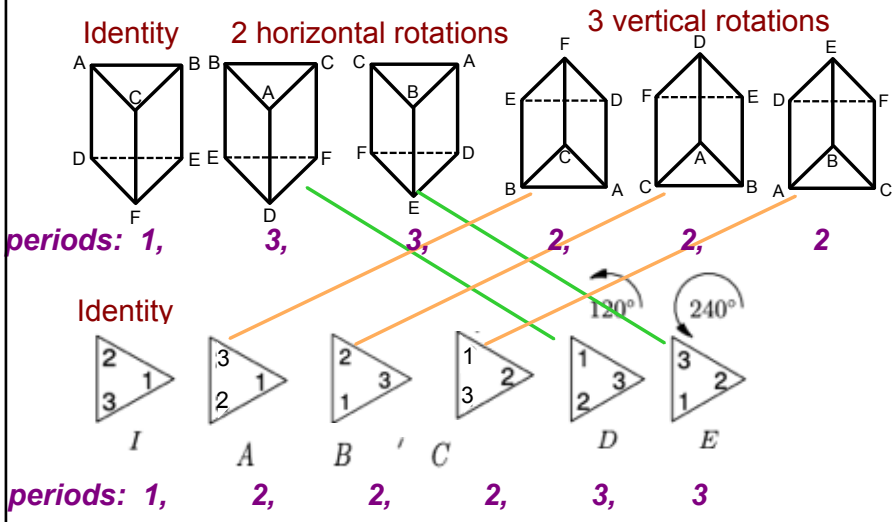
This element on the right can be generated by rotating Identity horizontally 120° then rotating vertically 180°, which we did on the previous slides. There is a single rotation that will generate it, which would be the name of this element.



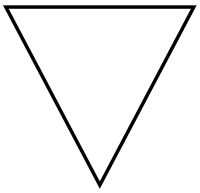
So here on the right I am showing vertices A,B, and C and their images in the same footprint. This single rotation is a rotation over the line shown above. This line is the median from the B edge thru the A,C,A',C' face, which is actually the A,C,F,D face on the Identity.

What is this element's period??

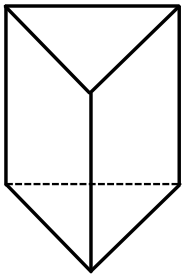
What other group (that we've seen) is the rotation group of an equilateral triangular prism isomorphic to?



Rotation group of an equilateral triangular prism is isomorphic to the Reflection group of the equilateral triangle



reflect and rotate me!

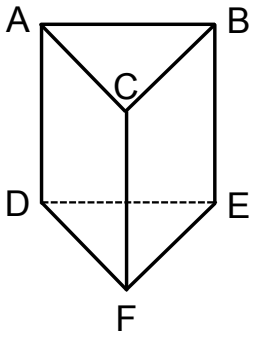


Only rotate me!

So the **reflection** group in 2-d is....

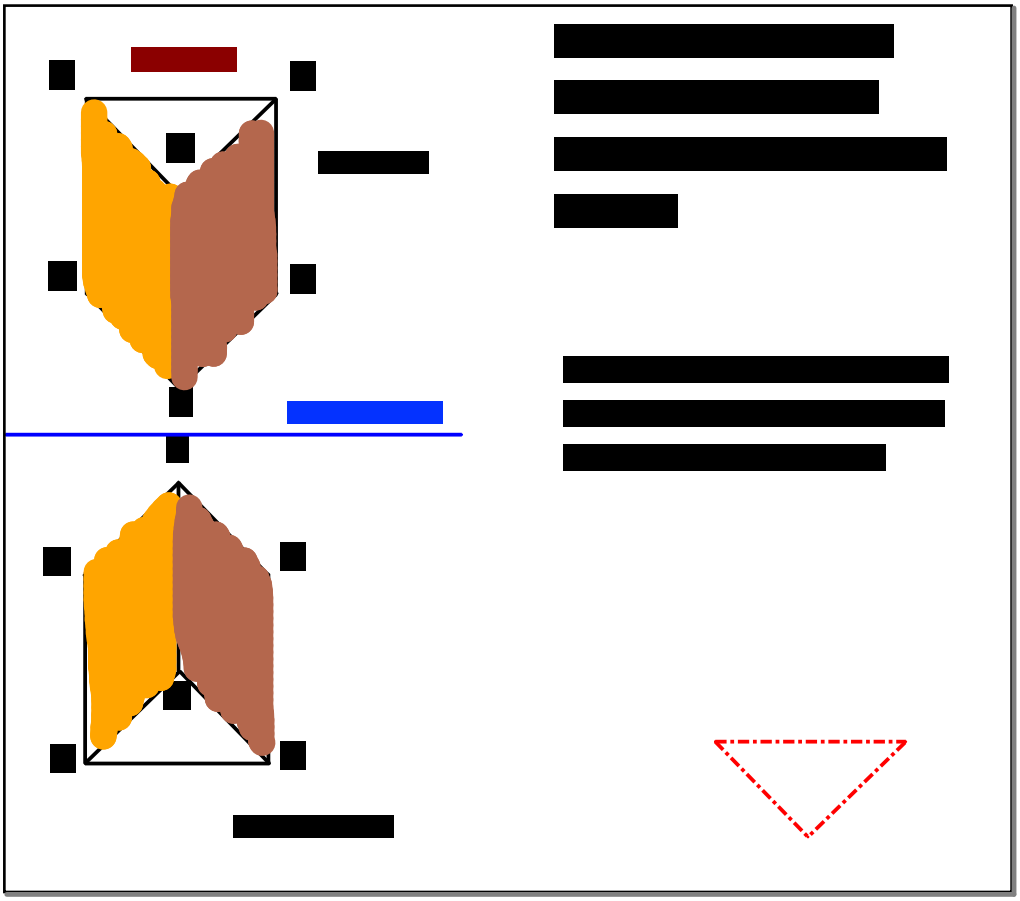
isomorphic to the **rotation** group of the corresponding 3-d prism.

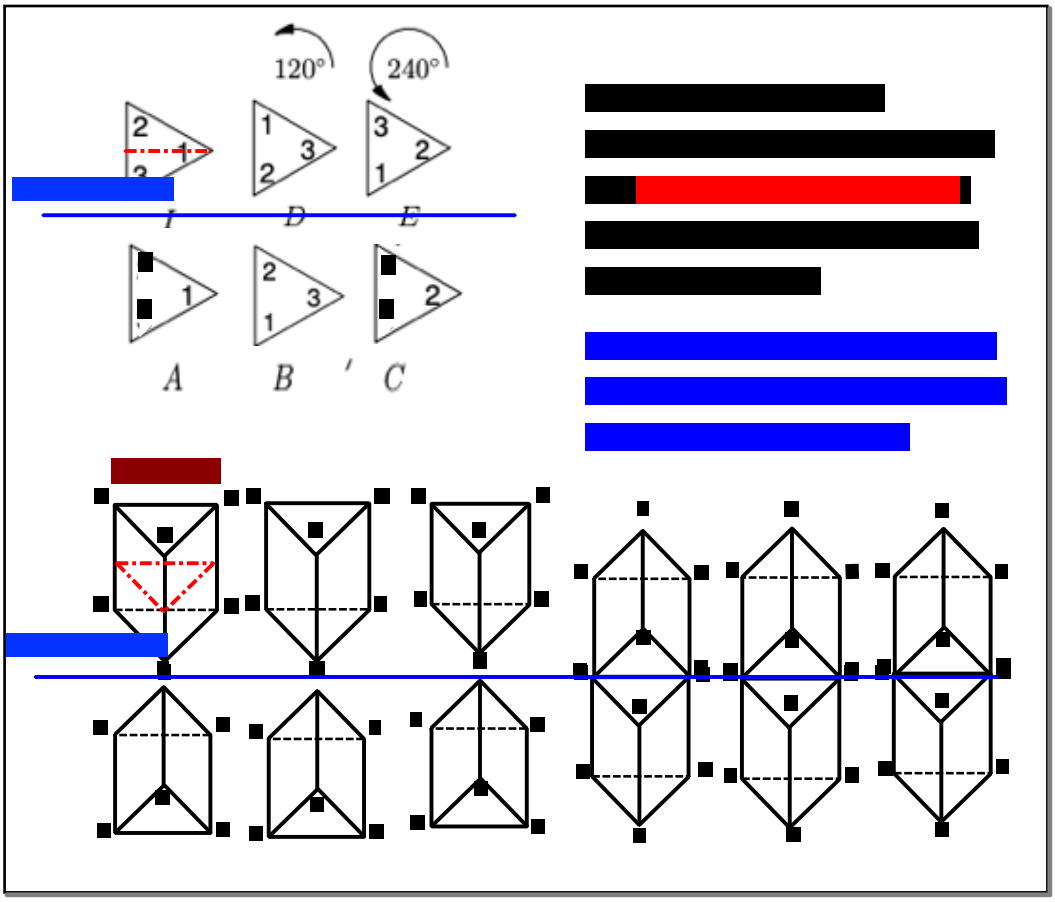
What about the REFLECTION group of the triangular prism? There should be more than 6, but how many more?



12

The 6 from before and those same 6 with reversed orientation.




120°
240°
2
1
3
1
2
3
1
2
3
1
2
3
1
2
3
1
2
3
1
2
3
1
2
3
1
2
3
A
B
C

The answer is twice as many as the rotate only group. 12 total! (your 6 from before and those same 6 with reversed orientation.

Will reflection groups ALWAYS be twice as many as their rotation group counterpart??????????????

Chapter 4 link

https://nichodon.github.io/gatm/textbook/chapters/rrg.pdf