Complex #'s Day 2

HW debrief

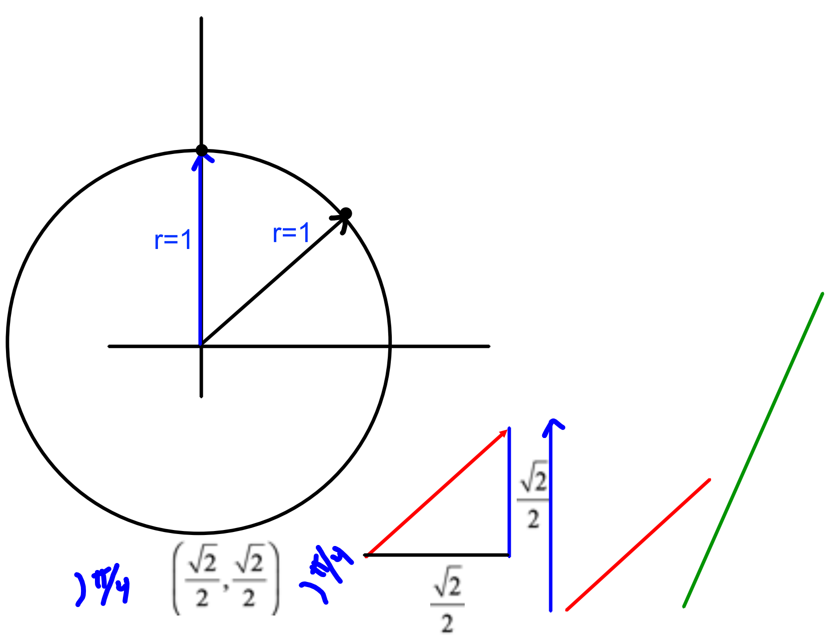
Demoivres -----> Trig formulas!

HW sneak preview

2020-21

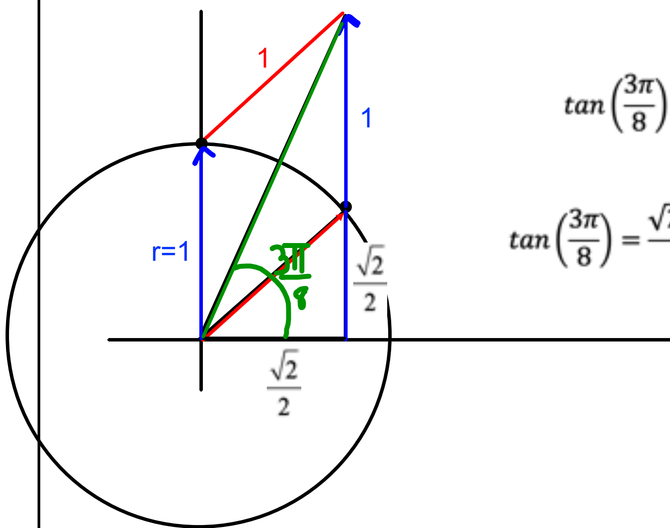
11. Draw $cis\left(\frac{\pi}{4}\right) + cis\left(\frac{\pi}{2}\right)$

Use your picture to prove that $tan\left(\frac{3\pi}{8}\right) = 1 + \sqrt{2}$



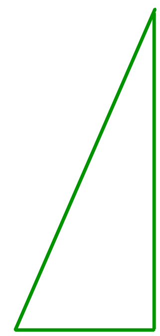
11. Draw $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) + cis\left(\frac{\pi}{2}\right)$

Use your picture to prove that $tan\left(\frac{3\pi}{8}\right) = 1 + \sqrt{2}$



$$tan\left(\frac{3\pi}{8}\right) = \frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \quad \frac{\sqrt{2}}{\sqrt{2}}$$

$$tan\left(\frac{3\pi}{8}\right) = \frac{\sqrt{2}+1}{1} = \sqrt{2}+1$$



#12 solve $z^3 = 1$ algebraically

old school way:

$z^3 - 1 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$

$(z-1)(z^2+z+1) = 0$

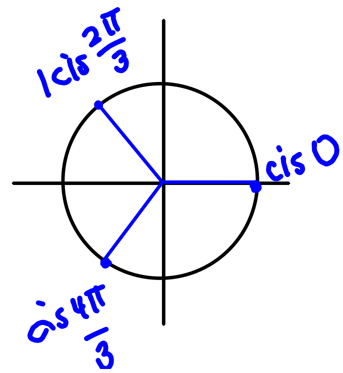
$$z = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2}$$

$$z = \frac{-1 \pm i\sqrt{3}}{2} \Rightarrow z = 1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$$

new school way:

$$z^3 = r^3 \operatorname{cis} 3\theta = 1$$
$$\Longrightarrow r = 1$$
$$\operatorname{cis} 3\theta = 1$$
$$\cos 3\theta = 1$$
$$3\theta = 2\pi k$$
$$\theta = \frac{2\pi k}{3}$$
$$\theta \in \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$$

$z^3 = 1$



$$\operatorname{cis} 0, \operatorname{cis} \frac{2\pi}{3}, \operatorname{cis} \frac{4\pi}{3}$$

This group is isomorphic to the rotation group of the equilateral triangle C3

So what group of numbers would be isomorphic to the rotation group of the regular nonagon (nine sides)

$$z^9 = 1$$

$1cis40$ $\qquad$ $360 \div 9 = 40$

$1cis80$

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Generating the double angle formulas using Demoivres! $cos2\theta, sin2\theta$

$$(cis\theta)^2 = (cos\theta + isin\theta)^2$$

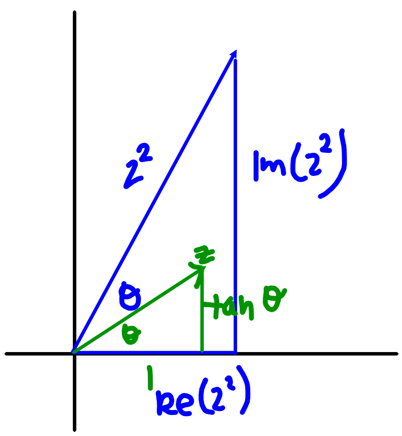
$$cis2\theta = cos^2\theta + 2icos\theta sin\theta - sin^2\theta$$

$$cos2\theta + isin2\theta = (cos^2\theta - sin^2\theta) + 2icos\theta sin\theta$$

$$cos2\theta = cos^2\theta - sin^2\theta$$

$$sin2\theta = 2cos\theta sin\theta$$

Tangent is a little trickier



$$z = 1 + itan\theta$$

$$z^2 = (1 + itan\theta)^2$$

$$= 1 - tan^2\theta + 2itan\theta$$

$$\tan 2\theta = \frac{Im(z^2)}{\mathrm{Re}(z^2)}$$

$$tan2\theta = \frac{2tan\theta}{1 - tan^2\theta}$$

Try sine (3θ), tan(3θ) from your homework, #'s 17, and 20

Friendly reminder: We didn't assign ALL of the Hproblems tonight. Check the assignment sheet!