10 min HW 20 min Quiz then INFINITIES!!

ps://www.youtube.com/watch?v=DeuejYoRbps

Infinite Groups

K-pop band formed in 2010.

ps://www.youtube.com/watch?v=NrFHEYIvkc0

Are all Infinities (the math kind) equally big?

$$\infty$$

Are there more natural numbers or even numbers?

Make some WILD guesses at # 7

GATM Chap 5 #7 **goto Padlet**

(a) natural numbers, N vs. positive even numbers, 2N

(b) natural numbers, N vs. positive rational numbers, Q+

(c) natural numbers, N vs. real numbers between zero and one, [0, 1)

(d) real numbers, R vs. complex numbers, C

(e) real numbers, R vs. points on a line

(f) points on a line vs. points on a line segment

(g) points on a line vs. points on a plane

Padlet https://padlet.com/dgleason10/n9zfk97qgvgap9ca

Two different types of infinities:

1. Countable infinities

e.g. the natural numbers. You can number them, and know that you haven't skipped one.

2. Uncountable infinities e.g. the real numbers between zero and 1 (there's always another number in between any two)

The "order" of a finite set is:

The "cardinality" of an infinite set is:

To show that two infinite groups are of equal size (**cardinality**):

Show that there is a one to one correspondence between group members.

Are there more natural numbers or even numbers?

1, 2, 3, 4, 5, 6, 7,...... a_n

same size!

2, 4, 6, 8, 10........b_n

both groups are ___countable___ infinities.

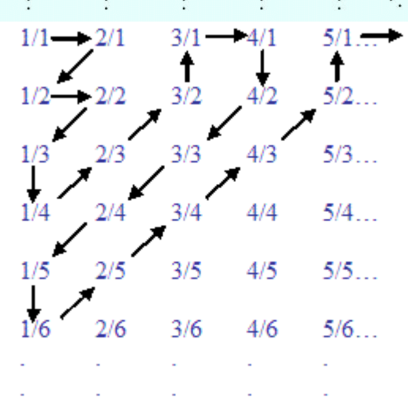
Can we find a one to one correspondence between the two sets?

$b_n = 2a_n$

Are there more positive rational numbers or natural numbers?

q \ p:	1	2	3	4	5	
1	1/1	2/1	3/1	4/1	5/1	...
2	1/2	2/2	3/2	4/2	5/2	...
3	1/3	2/3	3/3	4/3	5/3	...
4	1/4	2/4	3/4	4/4	5/4	...
5	1/5	2/5	3/5	4/5	5/5	...
⋮	⋮	⋮	⋮	⋮	⋮	⋱

Becaus
on dow
diagona



and go
forth

Are there more real numbers between 0 and 1 or natural numbers?

1 ⟷ .987546...
2 ⟷ .738458...
3 ⟷ .012452...
4 ⟷ .798541...
5 ⟷ .798534...
6

Connect all natural numbers to a real number between 0 and 1. All the integers are now paired up with a real number. We can easily see that we didn't skip any integers. Did we skip some real numbers?

ps://www.youtube.com/watch?v=A-QoutHCu4o&feature=youtu.be

Are there more real numbers between 0 and 1 or natural numbers?

We can write another real number which is not in our list. There are an infinitely large quantity of these lonely numbers. Therefore, the set of real numbers is bigger than the set of natural numbers.

1 ⟷ .987546...
2 ⟷ .738458...
3 ⟷ .012452...
4 ⟷ .798541...
5 ⟷ .798534...
6 ⟷ .79814...

I can find a "skipped" real number this way:

First digit not 9, so it can't be this first number;

2nd digit not 3, so it can't be this second number;

3rd digit not 2, so it can't be this third number;

4th digit not 5, so it can't be this fourth number;

Etc....

natural numbers:	decimal numbers:
1 $\longrightarrow$	$0.a_1\, a_2\, a_3\, a_4\, a_5\, a_6\, a_7\, a_8\, a_9\, a_{10}\, a_{11}\ldots$
2 $\longrightarrow$	$0.b_1\, b_2\, b_3\, b_4\, b_5\, b_6\, b_7\, b_8\, b_9\, b_{10}\, b_{11}\ldots$
3 $\longrightarrow$	$0.c_1\, c_2\, c_3\, c_4\, c_5\, c_6\, c_7\, c_8\, c_9\, c_{10}\, c_{11}\ldots$
4 $\longrightarrow$	$0.d_1\, d_2\, d_3\, d_4\, d_5\, d_6\, d_7\, d_8\, d_9\, d_{10}\, d_{11}\ldots$
5 $\longrightarrow$	$0.e_1\, e_2\, e_3\, e_4\, e_5\, e_6\, e_7\, e_8\, e_9\, e_{10}\, e_{11}\ldots$
6 $\longrightarrow$	$0.f_1\, f_2\, f_3 f_4\, f_5 f_6\, f_7\, f_8\, f_9 f_{10}\, f_{11}\ldots$
$\vdots$	$\vdots \qquad \vdots$

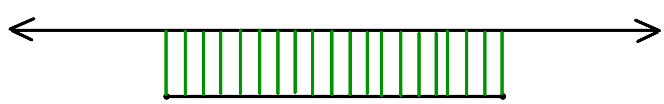
$$
\begin{array}{l}
0.\boxed{a_1}\, a_2\, a_3\, a_4\, a_5\, a_6\, a_7\, a_8\, a_9\, a_{10}\, a_{11}\ldots \\
0.b_1\, \boxed{b_2}\, b_3\, b_4\, b_5\, b_6\, b_7\, b_8\, b_9\, b_{10}\, b_{11}\ldots \\
0.c_1\, c_2\, \boxed{c_3}\, c_4\, c_5\, c_6\, c_7\, c_8\, c_9\, c_{10}\, c_{11}\ldots \\
0.d_1\, d_2\, d_3\, \boxed{d_4}\, d_5\, d_6\, d_7\, d_8\, d_9\, d_{10}\, d_{11}\ldots \\
0.e_1\, e_2\, e_3\, e_4\, \boxed{e_5}\, e_6\, e_7\, e_8\, e_9\, e_{10}\, e_{11}\ldots \\
0.f_1\, f_2\, f_3 f_4\, f_5 \boxed{f_6}\, f_7\, f_8\, f_9 f_{10}\, f_{11}\ldots \\
\vdots \qquad\qquad\qquad\qquad \vdots
\end{array}
$$

new number not in list:

$$0.!a_1!b_2!c_3!d_4!e_5!f_6!g_7!h_8!i_9!j_{10}!k_{11}\ldots$$

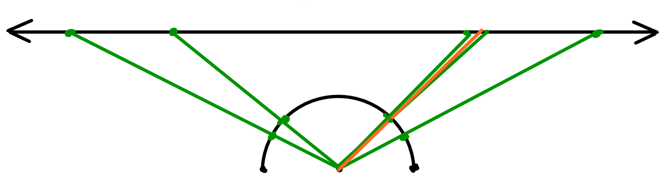
Are there more points on a line or a line segment? A geometric approach:

What we would think:



Every point on segment gets a partner on the line...

But, we could bend the line segment:

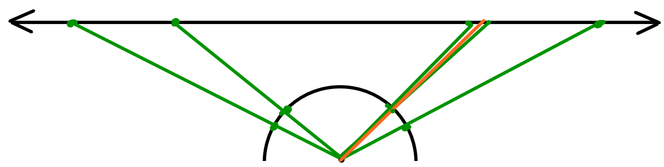


Every point on the line gets a partner on the "segment"...

We have a "bijection", so sets have same cardinality!

Are there more points on a line or a line segment? A geometric approach:

Bending the line segment:



It seems there are two extra points on the segment....We can "push" those endpoints into just inside the segment interval.

Takeaway points

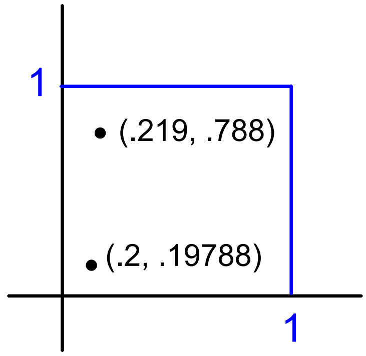
1. Uncountable infinities are always bigger than countable infinites.

2. Two countable infinite sets have the same cardinality.

3. Two uncountable sets........

Are there more points on a line or on a plane?

Let's start with points on a segment vs in a square...



How do I create a correspondence that would pair these 2 points with different points on the segment?

(.219, .788) → .219788
(.2, .19788) → .219788

Not unique

(.219, .788) → .271898
(.20000, .19788) → .21090708080

Unique

Are there more points on a line or on a plane?

Repeat with other segments and other squares.

The set of points on a line has the same cardinality as the set of points on a plane!

Compare the sets of real #s and complex #s.

use the same strategy as on previous slide.