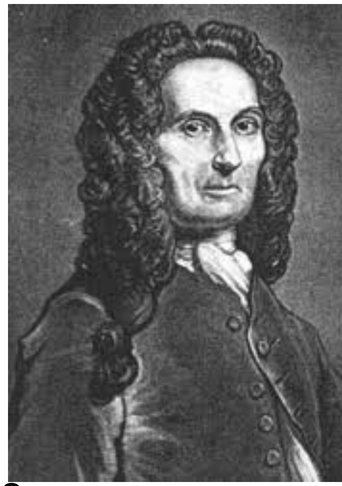
Analysis 2020-21

Complex #'s review



Quick go over Formative Assessment

Quick HW debrief

Complex #'s review

Geometric proof of Demoivre's Theorem

Start HW

1. For this problem you will be considering whether the set of irrational numbers and 0, under addition, forms a mathematical group under addition.

If + and 0:
Identity = 0
Inverse: No because only 0 has an inverse; no way to get from √2 to 0 by addition
closed: yes adding two positive irrat #s is another irrat#.

If ± and 0:
Identity = 0
Inverse: yes all elements have an inverse
closed: no because √2 + (1-√2) = 1 and 1 is not irrational!

2. Below are some of the elements of the 8-post snap group. Below each element, write down its period.

a)  b)  c)  d) 

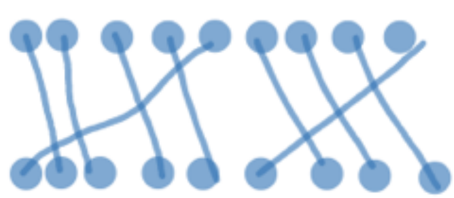
a) 2

b) 4

c) 3, 2 ⟶ 6

d) 5, 2 ⟶ 10

3. What is the **maximum** period of an element in the 9-post snap group? Draw the element.



Per=20

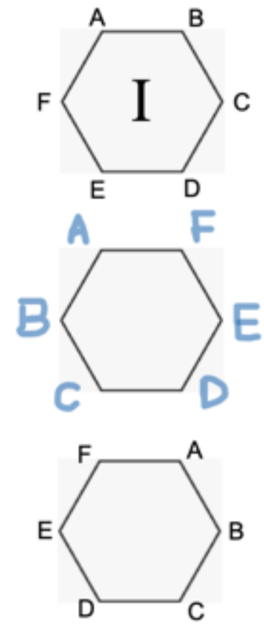
5. Consider the dihedral (reflection/rotation) group for the regular hexagon where the first element shown is the identity element.

Let "f" be defined as the operation "reflect over the horizontal symmetry line (for the identity this would be line FC). Let "r" be defined as the operation rotate counterclockwise by 60 degrees.

a) Label the blank middle element to represent $f \cdot r^2$

b) Using no more than three operations, name the third element (use r's and f's). (You can use the text box or write directly in the graphics box)

frf



6. Isomorphic

6. Consider the three groups:

Group A: Hexagonal Prism under Rotation
Group B: Hexagon under rotation
Group C: The 6-post snap group

For each group below, write the letter (A, B, or C) of its isomorphic group. If the given group is not isomorphic to any of the above groups, write "X".

a) _______ is isomorphic to the dihedral group D6
b) _______ is isomorphic to the group generated by a hexagonal pyramid under reflection
c) _______ is isomorphic to the group generated by a hexagon under reflection
d) _______ is isomorphic to the group generated by a hexagonal prism under reflection

a) A

b) A

c) A

d) X

Takeaway points

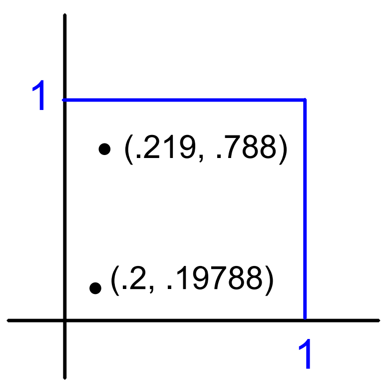
1. Uncountable infinities are always bigger than countable infinites.

2. Two countable infinite sets have the same cardinality.

3. Two uncountable sets........

Are there more points on a line or on a plane?

Let's start with points on a segment vs in a square...



How do I create a correspondence that would pair these 2 points with different points on the segment?

$$\left.\begin{array}{l}(.219,\ .788) \longrightarrow .219788 \\ (.2,\ .19788) \longrightarrow .219788\end{array}\right\}\ \textit{Not unique}$$

$$\left.\begin{array}{l}(.219,\ .788) \longrightarrow .271898 \\ (.20000,\ .19788) \longrightarrow .21090708080\end{array}\right\}\ \textit{Unique}$$

Are there more points on a line or on a plane?

Repeat with other segments and other squares.

The set of points on a line has the same cardinality as the set of points on a plane!

Compare the sets of real #s and complex #s.

use the same strategy as on previous slide.

4. Why are our original flip group (D3) and the rotation group of the hexagon NOT isomorphic despite having the same number of elements?

Flips: R120, R240, I, refA, refB, refC

Rotation of a hexagon:

R60, R120, R180, R240, R300, R360

Can Infinite groups be isomorphic?

9b and 9i are!

9b: integers under addition

inverse of 3 is -3

$1 + 2 = 3$

gets back to additive identity of 0

9i: integer powers of 2 under multiplication

$2^1 \times 2^2 = 2^3$

inverse of 2^3 is 2^{-3}

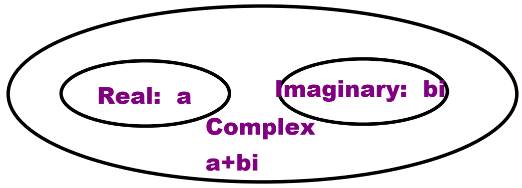
gets back to multiplicative identity of 1

Complex #'s

Complex Number basics:

i =

i^2=



complex #   z = a + bi

conjugates:

a + bi, a – bi

Notation: Re(z) = the real part of z   (a)

Im(z) = the imaginary part (b)

$\overline{z}$ = a - bi = the conjugate of z

arg(z) = angle that it makes with the x-axis

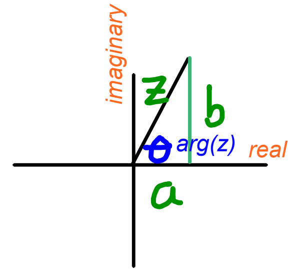
"argument"

$0 < \arg(z) \leq 180°$

Graphing complex numbers:

z = a + bi (rectangular form)

vector z with x and y components a and b



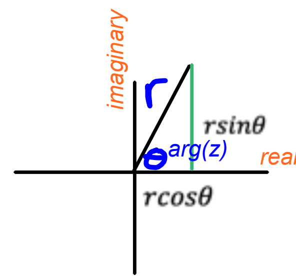
but note: a = $r\cos\theta$ b= $r\sin\theta$

polar (or trig) form of a complex number:

$$z = r\cos\theta + i(r\sin\theta)$$

$$r(\cos\theta + i\sin\theta)$$

or: z = $r cis\theta$



practice:

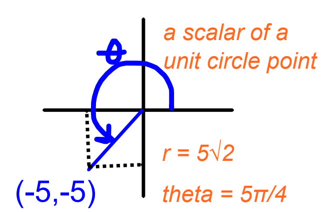
a) change -5 - 5i into cis form and

$$r = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{5}{5}\right) = \frac{5\pi}{4}$$

$$-5 - 5i = 5\sqrt{2}cis\frac{5\pi}{4}$$

or:



b) $6cis\left(\frac{11\pi}{6}\right)$ into rectangular form.

$$6cis\left(\frac{11\pi}{6}\right) = 6\left[cos\frac{11\pi}{6} + isin\frac{11\pi}{6}\right]$$

$$= 6\left[\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right]$$

$$= 3\sqrt{3} - 3i$$

Multiplying and Dividing complex numbers in trig form:

Given $z_1=r_1 cis(\Theta_1)$ $z_2=r_2 cis(\Theta_2)$

$$z_1 \cdot z_2 = r_1 cis(\theta_1) r_2 cis(\theta_2)$$

$$= r_1 r_2 [cos\theta_1 + isin\theta_1][cos\theta_2 + isin\theta_2]$$

$$= r_1 r_2 [cos\theta_1 cos\theta_2 + icos\theta_1 sin\theta_2 + isin\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2]$$

$$= r_1 r_2 [(cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2) + i(cos\theta_1 sin\theta_2 + sin\theta_1 cos\theta_2)]$$

$$= r_1 r_2 [cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)]$$

$$z_1 z_2 = r_1 r_2 [cis(\theta_1 + \theta_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2)$$

we'll show this geometrically soon.

DeMoivre's Theorem:

$(rcis\Theta)^n=$

$$(rcis\theta)^n = r^n (cis\theta)^n$$

$$= r^n [cis\theta \cdot cis\theta \cdots]$$

$$= r^n [cis2\theta \cdot cis\theta \cdots]$$

$$= r^n [cis3\theta \cdot cis\theta \cdots]$$

$$(rcis\theta)^n = r^n cisn\theta$$

Three questions:

1. evaluate: $(2+2i)^{10}$ using Demoivre's.

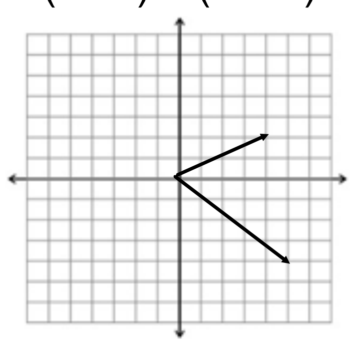
$$(2+2i)^{10} = \left(2\sqrt{2}cis\frac{\pi}{4}\right)^{10}$$

$$\left(2\sqrt{2}cis\frac{\pi}{4}\right)^{10} = \left(2\sqrt{2}\right)^{10} cis\frac{10\pi}{4} = 2^{15}i$$

2. Show that adding complex numbers is like adding vectors by doing

(4+2i) + (5 - 4i) geometrically:

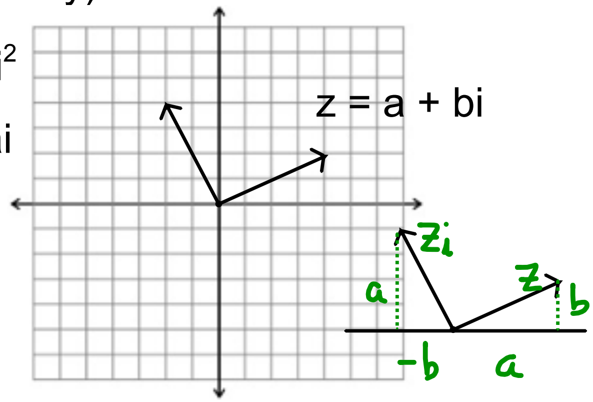
(4+2i) + (5 - 4i)



3. why must z and zi always be perpendicular (justify graphically)

$zi = ai + bi^2$

$= -b + ai$



Is the angle between the vectors a right angle?
How can you tell?

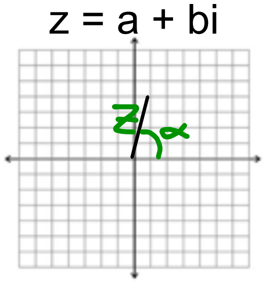
- dot product = 0   <a,b>•<-b,a>=0
- slope of z = v/a, slope of zi = -a/b; product is -1
- z=rcisT, i=1cis90; zi=rcisT times 1cis90 = rcis(T+90);

Compare zi to z, or, rcis(T+90) vs rcisT, note angles are 90° apart.

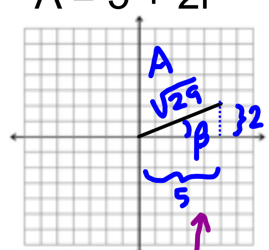
- You can also use congruent triangle theorems

Show geometrically **how** multiplying complex numbers works:

z = a + bi

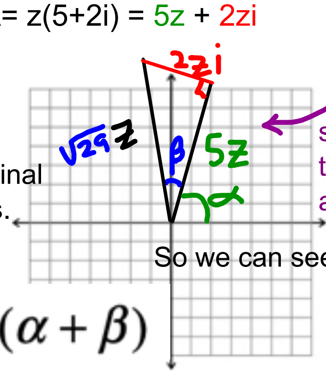


A = 5 + 2i



zA= z(5+2i) = 5z + 2zi

We can see the magnitude is the product of the original vector magnitudes.



similar triangles 2:5:√29; therefore both those angles are β

So we can see the angles get added.

$$zA = \sqrt{29}\, z\, cis(\alpha + \beta)$$

HW 1 - 15 from complex numbers