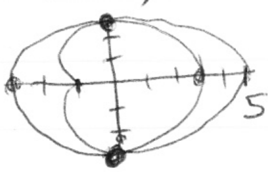
Shanksbook

HW 17 POLAR p. 493 #1-15 odd

1. $r=3$, $r=3+2\cos\theta$



$3=3+2\cos\theta$

$2\cos\theta=0$

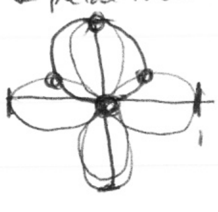
$\cos\theta=0$

$\theta=\pi/2,\ 3\pi/2$ and $r=3$

$\boxed{(3,\pi/2),\ (3,3\pi/2)}$

3. $r=\sin\theta$, $r=\cos 2\theta$

1 petal rose   4 petal rose



$\sin\theta=\cos 2\theta$ ← double angle formula

$\sin\theta=\cos^2\theta-\sin^2\theta$

$=(1-\sin^2\theta)-\sin^2\theta$. pythagorean identity

$=1-2\sin^2\theta$

$2\sin^2\theta+\sin\theta-1=0$ factor

$(2\sin\theta-1)(\sin\theta+1)=0$

$\sin\theta=\frac{1}{2}$

$\theta=\pi/6,\ 5\pi/6$

$\sin\theta=-1$

$\theta=3\pi/2$

$r=\sin\frac{3\pi}{2}=-1$

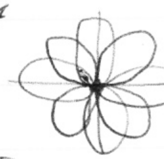
$(-1,3\pi/2)$

$r=\sin(\pi/6)=1/2$ $\boxed{(\tfrac{1}{2},\pi/6)(\tfrac{1}{2},5\pi/6),\ (1,\pi/2)}$

$(0,0)$

5. $r=\sin 2\theta$, $r=\cos 2\theta$ did in class

$\tan 2\theta=1$



$2\theta=\pi/4,\ 5\pi/4$

$\theta=\pi/8,\ 5\pi/8$ keep going in $[0,2\pi]$: $9\pi/8,\ 13\pi/8$.

Center point $(0,\text{any }\#)$

One Way: To get additional 4 points use geometry & symmetry.

So add $\frac{2\pi}{8}$ to $\pi/8$ contenually in $[0,2\pi]$ to get all 8 points.

another way: $(r,\theta)\to(-r,\theta+\pi)$

$r=\sin 2\theta$ $-r=\cos[2(\theta+\pi)]$

$\Longleftrightarrow r=-\cos(2\theta)$

$\tan 2\theta=-1$

$2\theta=3\pi/4,\ 7\pi/4$

$\theta=3\pi/8,\ 7\pi/8$ keep going in $[0,2\pi]$: $11\pi/8$ $15\pi/8$.

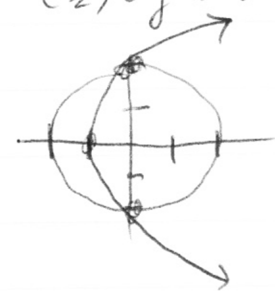
Points: $(0,0)$, $(\frac{\sqrt{2}}{2},\frac{\pi}{8})$, $(\frac{\sqrt{2}}{2},\frac{3\pi}{8})$, $(\frac{\sqrt{2}}{2},\frac{5\pi}{8})$ $(\frac{\sqrt{2}}{2},\frac{7\pi}{8})$ $(\frac{\sqrt{2}}{2},\frac{9\pi}{8})$ $(\frac{\sqrt{2}}{2},\frac{11\pi}{8})$ $(\frac{\sqrt{2}}{2},\frac{13\pi}{8})$ $(\frac{\sqrt{2}}{2},\frac{15\pi}{8})$

7. $r = 2, \quad r = \sec^2\left(\frac{\theta}{2}\right)$

We have not graphed $r = \sec^2\left(\frac{\theta}{2}\right)$ by hand.

$2\cos^2\left(\frac{\theta}{2}\right) = 1$

$\cos\left(\frac{\theta}{2}\right) = \pm\frac{\sqrt{2}}{2}$



$\frac{\theta}{2} = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$\theta = \pi/2, 3\pi/2$ [stay inside [0, 2π]]

points: $(2, \pi/2), (2, 3\pi/2)$

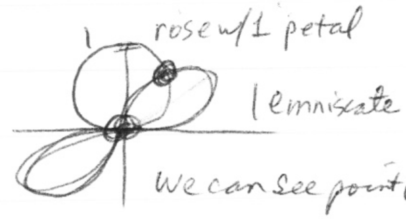
9. $r^2 = \sin 2\theta, \quad r = \sin\theta$

double angle identity

$r^2 = 2\sin\theta\cos\theta$

$\sin^2\theta - 2\sin\theta\cos\theta = 0$

$\sin\theta(\sin\theta - 2\cos\theta)$

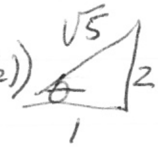


$\sin\theta = 0$ | $\sin\theta = 2\cos\theta$

$\theta = \pi, 0$ | $\tan\theta = 2$

not on unit circle

$\theta = \tan^{-1}(2) = 1.107$ $\rightarrow r = \sin(\tan^{-1}(2)) = \frac{2}{\sqrt{5}}$

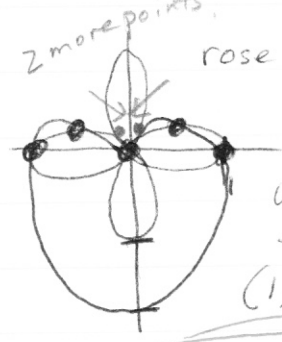


points: $(0, 0), \left(\frac{2}{\sqrt{5}}, \tan^{-1}(2)\right)$

11. $r = 1 - \sin\theta, \quad r = \cos 2\theta$

$1 - \sin\theta = \cos 2\theta$

Double angle, etc, like #3



$1 - \sin\theta = \cos^2\theta - \sin^2\theta$

$(1 - \sin^2\theta) - \sin^2\theta$

$1 - \sin\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta \cdot (2\sin\theta - 1) = 0$

more on next page

$\theta = 0, \pi$ | $\sin\theta = 1/2$

$r = 1 - 0 = 1$ | $\theta = \pi/6, 5\pi/6$

$r = \cos\pi/3 = \frac{1}{2}$

points: $(1, 0), (1, \pi), \left(\frac{1}{2}, \pi/6\right), \left(\frac{1}{2}, 5\pi/6\right), (0, 0)$

13. $r = \dfrac{-1}{\sin\theta + \cos\theta}$ lines $r = \dfrac{2}{3\sin\theta - 2\cos\theta}$

$$\frac{-1}{\sin\theta + \cos\theta} = \frac{2}{3\sin\theta - 2\cos\theta}$$

$$-3\sin\theta + 2\cos\theta = 2\sin\theta + 2\cos\theta$$

$$0 = 5\sin\theta$$

$$\sin\theta = 0$$

$$\theta = 0, \pi$$

when $\theta = 0$, $r = \dfrac{-1}{\sin 0 + \cos 0} = -1$ point $(-1, 0)$

when $\theta = \pi$, $r = \dfrac{-1}{\sin\pi + \cos\pi} = 1$ point $(1, \pi)$

same point.

15. $r = \theta$ is an example.

(r, θ) ✓ but $(-r, \theta + k\pi)$

$-r = \theta + \pi$ when $k = 1$

$-r = \pi$

$r = -\pi$ not the same as $r = \theta$

11 con't $(r, \theta) \to (-r, \theta + \pi)$

2 more points.

$1 - \sin\theta = -\cos 2(\theta + \pi)$ see ideas in #11

$1 - \sin\theta = -1 + 2\sin^2\theta$

$2\sin^2 x + \sin x - 2 = 0$ For Quad formula: $a = 2, b = 1, c = -2$

$\sin^{-1}\left(\dfrac{-1 + \sqrt{17}}{4}\right) = .8959$

other angle: $\pi - .8959 = 2.245$

$r = -.2192$

$(-.2192, .8959), (-.2192, 2.245)$