*Editor remarks*

*Editorially, we consider the following points key:*

*Both reviewers request a qualitative evaluation of the model fit that demonstrates how well the model emulates human data.*

We have leveraged the relatively large sample size in Study 3 to perform additional model validity tests and added these to the Supplementary materials.

We respond directly to the first reviewer’s question *“How does the BP model's prediction of participant behavior differ from the CS model?”* and the second reviewers request to “*see how well the models recreate the data*“ by supplying scatterplots showing predictions of sampling rates of individual participants, where the Biased Prior model and other models are validated by high predictive accuracy.

We also provide an analysis of decision threshold as a function of sequence position, using the Lee (2006) threshold-fitting method for both our human and model choices. Again, all the models perform similarly when predicting human performance.

These analyses contribute to the evidence that all our models / heuristics well-predict and fit human data to some degree. Indeed, our analyses of frequency of participants best fit by each model shows that a sizable portion of our samples may even be best fit by other models, especially Cost to Sample. Some participants may use these strategies. In our revised General Discussion, we have further developed this point.

The reviewers are not specific about which analyses they would like to see, aside from the analysis we have provided in response to Reviewer 1’s comments. We would be happy to consider further validation analyses and agree that learning more about when prediction diverge is an interesting research question. Given the apparently subtle nature of these divergences, however, perhaps this might be ideal to address this question as a separate research project using paradigms specifically designed to maximise divergent performance. We have also taken up this consideration in our revised General Discussion.

*Second, the reviewers raise questions about the (cognitive) feasibility of the modelled processes.*

We have added some text raising this issue in the General Discussion and (as advised by Reviewer 1) expanded the description of backwards induction, with additional text in the Supplementary Materials.

We also have added a comment to the General Discussion related to our uncertainty regarding the neural architectures that might be implementing these algorithms and their computational capacities. And, we discuss evidence from previous studies that brain responses can correlate with quantities computed by these types of backward-induction based optimal stopping models showing brain responses.

*Finally, they raise the question of the consequences of (non)optimal behaviour on the task.*

We now raise this issue in a new paragraph the General Discussion. It may appear, given the number of methods we have varied here, that we have thoroughly explored the potential space of factors that affect the sampling rate. Nevertheless, in our view, we have not. Indeed, we show (like Costa and Averbeck did before us in 2016) that the undersampling bias grows with larger sequence lengths. We have examined only sequences of 8, 12 and 14 here. We expect that undersampling bias likely continues to grow at least to some degree but more data in other studies. There are likely many parameters involved in real world searches, including longer sequence lengths, that we did not examine here in our relatively tightly controlled studies.

*The referees also highlight the potential for more model comparisons (additional models). While this may be interesting for future work, we ask you to not place an emphasis on this issue, and rather focus on the requests outlined above that will aid to demonstrate the characteristics and plausibility of the models currently applied.*

We understand. With respect to this proposal of new models, have focussed our response to new text in the General Discussion concerning the utility of the framework we have introduced here for future studies. We discuss the importance of models and heuristics that might be built and tested using this novel modelling framework.

Reviewer #1 (Remarks to the Author):  
  
*I thank the authors for addressing my previous comments and for the considerable effort invested in revising the manuscript. In this revised version of the manuscript, the authors have added several theoretical models aimed at explaining the behavior of the participants. Generally, the addition of these models improved the manuscript. However, since the article has undergone considerable changes, I have a few additional comments regarding the new version of the manuscript, both in relation to its clarity and in relation to the newly added models.*

*• Clarity and presentation - The introduction and paradigm description remain relatively clear. However, the model description and results sections are relatively complex and less intuitive. I suggest several improvements:*

We appreciate the reviewer’s appreciation of the new modelling contributions! And we of course pleased to implement these constructive comments to help improve clarity.  
  
 *o Ideal observer model - The basic model is explained in a relatively abstract way. A simple numerical example or visualization (e.g., one that illustrates the backwards induction) could help readers who are not familiar with this model to understand it better.*

We have added a section to the supplementary materials that takes the reader through an intuitive step by step description of backwards induction.

*o Abbreviations - Throughout the article, the authors used abbreviations such as CO, CS, etc. Using the full names of the models could enhance readability and flow of the text.*

We now spell out these abbreviations.

*o A table summarizing the key features of each model could help.*

We have created a new Table 1 for this purpose.

*o Figure 1 - The font size is relatively small, making it difficult to see what is written.*

As these are screen shots of the actual paradigms, we are not able to change the font sizes without misrepresenting how the paradigm was presented. Nevertheless, we have reorganised the figure to enlarge the screen images to make the text more visible. Unfortunately, this means individual paradigms cannot be separate panels. So we have created multiple figures, one for each paradigm, enlarged to full page size, in the supplementary materials. We have kept the baseline paradigm (previously panel A) as Figure 1 in the main text to keep one illustration of how a paradigm of this kind generally appears.

o Figures 2-7 - i. Adding a legend explaining the color coding would assist in understanding the Figure more easily. ii. Some of the BIC values look almost identical, and it is very difficult to distinguish between them. Adding numerical BIC values would facilitate easier comparisons between the models. iii. Figure 3 - Column headings would make the figure clearer.

• Qualitative versus quantitative comparison - While the BP model's superior BIC values indicate a better fit, it would be helpful to understand the specific qualitative aspects of the data that contribute to this superiority. For example, how does the BP model's prediction of participant behavior differ from the CS model (i.e., both models predict under-sampling, so where are their prediction differ)? Providing a more detailed comparison of the predictions and behavior patterns could illuminate why the BP model outperforms others.

*• Additional model considerations - The models implemented provide valuable insights, but exploring additional models could yield even richer findings.  
o For example, did the authors examine a combined BP and CS model? Since the mechanisms underlying these two models are different, a combined model might be able to achieve better results.  
o Another possible model is one similar to BP/CS but assumes time-varying parameters (for example, that the sampling cost increases over time). This idea is similar to the urgency signal/collapsing boundaries in sequential sampling models.  
o The CS and BP models are essentially versions of the IO model. There may be models from different families that might explain the participants' undersampling. For example, a model that assumes that the participants scanned the space of possibilities partially (e.g., only for 3-4 levels), and therefore undersampled.  
o Exploring some of these models (or at least discussing them as potential avenues in the discussion section) could strengthen the manuscript.*

We agree with the reviewer that the introduction of the modelling framework in this paper opens many new avenues for theory development and we share enthusiasm for these possibilities. We have added text in our General Discussion outlining these interesting new research directions, as advised by the reviewer.  
  
*• Model feasibility - In the context of the previous comment, the BP model outperformed the other models in terms of BIC results. However, the feasibility of the calculation required by it raises questions about its psychological plausibility. How realistic is it for participants to perform such complex calculations? Could simpler heuristics underlie what appears to be complex computation?*

We have added to the General Discussion some text addressing this issue, including citation of previous evidence from fMRI that neural responses may encode quantities computed by these kinds of models. We note also that we have already included in our model comparison a simpler heuristic (the cut off model) and the model fitting evidence appears to be against this heuristic and more in favour of the models based on the ideal observer.

“We have introduced a framework whereby optimality solutions including that of the Secretary Problem and that of the full information problem have been leveraged to explain accurate performance on optimal stopping tasks. And our framework has taken the approach of parameterizing these models to explain systematic sources of suboptimal performance. These models span theories that are relatively complex to compute (i.e., relying on backwards induction) as well as a simpler heuristic, the cut off heuristic, which has been previously proposed. As the models we consider here all specify how thresholds are to be computed, we have not fitted models where thresholds are estimated directly during parameter estimation from human participant data. Instead, we used this method to estimate and compare threshold values associated with our fitted models to those of human participants, as it can be estimate thresholds for any agent. Given that we have proposed these framework and demonstrated its utility, we expect that future research can refine the models we have proposed or build improved models that may better fit participant data. For example, more complex models that combine multiple bias-related free parameters (e.g., the cost to sample parameter and a constant added to the prior mean as fitted parameters at the same time) might be considered. We also note that the Bayesian ideal observer solution to the full information problem has computationally complex elements, including the backwards induction algorithm. Future research might explore models that compute the value of sampling again using more heuristic or computationally simpler solutions. Nevertheless, the computational and neural architecture used to solve such decision problems remains unknown, and so the constraints on the algorithmic capabilities of models remains unknown. Nevertheless, provisional evidence comes rom Costa & Averbeck (2016), who demonstrated possible neural responses that correlate trial-by-trial with quantities derived from the Cost to Sample model, after fitting to human choice data. Similar neural correlates have been observed in a study of the beads task, using a similar optimal stopping model that also employed backwards induction (Furl & Averbeck, 2011). Thus, neurons may be computing quantities like those in these kinds of models.”  
  
• *Optimality - Given that human performance closely matched the optimal model in term of rewards, what are the practical implications of under-sampling? And whether, given a sampling strategy similar to that of the Bayesian model, the subjects would indeed achieve better results or reach a plateau?*

We have expanded on our existing text in the General Discussion regarding the comparison of ranks of chosen options between participants and the Ideal Observer.

“Our study alone cannot explain whence this biased prior arises and this also opens a new question for future research. It is possible that participants, in economic contexts, might adopt a “safe” or conservative strategy (i.e., response bias) that protects against getting stuck with an especially poor outcome. Indeed, inspection of the ranks that participants achieved with their choices (a measure of their choice accuracy), shown in the first rows of Figures S4, S6, S7, S8 and S9 suggests that the quality of participants’ choices closely approximated those of the Ideal Observer’s choices, despite their suboptimally low sampling rates. Consequently, one can adopt a pessimistic stance that protects from the uncertainty of a poor outcome and still “satisfices”; that is, perform at near-optimal levels …

… We have already mentioned several ways that the modelling framework we propose here and the results we’ve obtained raise new research questions and open new research lines. One last issue that we also feel deserves further study relates to the extent to which systematic bias translates into real losses for participants. We have already proposed above an interesting theoretical possibility that biases like Biased Prior strategies might have an adaptive function, so long as they produce near-optimal performance. Indeed, within the narrow range of sequence lengths and domains (i.e., smartphones prices) that we have examined here, participants’ biased choices largely satisficed, and produced performance that, while not optimal, did not cause a striking loss for participants, when measured as rank of chosen option. Nevertheless, we also show herein that factors such as sequence length and incentivisation can affect the size of bias (e.g., longer sequences increase undersampling bias, as the Ideal Observer adjusts to the longer sequences but participants less so). Just how large biases can eventually become and the extent to which significant losses might accrue for agents due to ever larger undersampling biases cannot be answered directly by our data and would benefit from more direct investigations. Ideally, studies could better approximate real-world conditions of decision making, or even use field studies, to assess what kinds of losses might (or might not) occur under more ecologically valid conditions, as opposed to the more tightly controlled studies we report here.”

*• Minor comments  
o How did the researchers correct the comparisons for multiple corrections?*

We have clarified that the corrections were Bonferroni.

*o There is a typo in line 548*

We have corrected this.

*o Lines 582-583 are different from what the figure shows.*

Thank you for identifying this typo. We have corrected “undersampling” to “oversampling”.  
  
  
  
Reviewer #2 (Remarks to the Author):  
  
*This revision is a substantial revision from the previous version. The new version addresses nearly all of the issues I raised in the last round:  
- providing evidence that their studies could shift sampling rate  
- using statistical inference methods that allow one to collect evidence in support of the null (i.e.,Bayes factors)  
- using models to get at a more mechanistic explanation  
  
The paper and research is well executed and I found the results on undersampling and explanation to be informative.*

We are pleased the revision addresses this reviewer’s concerns and for this positive response.  
  
*One comment that would help give me more confidence is to see model fits to the actual data. How well do the models here recreate the behavior. I may have missed this, but the modeling results are relative model comparisons. It would be nice to see how well the models recreate the data.*

The upper panels of figures 3-7 show the sampling rates of the models and how they compare (favourably) relative to participants’ mean sampling rates, suggesting that they do reproduce the results. The Supplementary figures show the models’ mean ranks of chosen options alongside that of participants. With the exception of the Cut Off heuristic (which consistently and considerably undershoots participants’ performance), the other models perform similarly to participants.

We have now added scatterplots and correlation values, showing that sampling rates are also well-predicted by all models at the level of individual participants.

We hope these collectively demonstrate how well all the models included in the model comparison have some value for predicting participants’ sampling choices.