

2. a) $\frac{d}{dx} \frac{\ln x}{x^2} = \frac{1 - 2x \ln x}{x^4}$

$$\frac{d}{dx} \frac{u}{v} = \frac{u'v - uv'}{v^2}$$

b) $\frac{d}{du} \sqrt{3 \sin^2 u + 2}$

$$\frac{1}{2 \sqrt{3 \sin^2 u + 2}} \cdot -6 \sin u \cos u$$

$$= - \frac{3 \sin u \cos u}{\sqrt{3 \sin^2 u + 2}}$$

c) $\left. \frac{d^n}{dx^n} e^{kx} \right|_{x=0}$ k constant

$$= k^n e^{kx} = k^n$$

4. $f(x) = \begin{cases} x^2 + x + a, & x \leq 0 \\ bx + 2, & x > 0 \end{cases}$ a & b are constant.

$$bx + 2 = x^2 + x + a$$

@ $x=0$

$$2 = a$$

$$a = 2$$

$$2x + 1 = b$$

$$b = 1$$

$$bx + 2 = x^2 + x + 2$$

$$bx = x^2 + x$$

$$b = x + 1$$

$$\boxed{a=2, b=1}$$

where

7. $y = y(x)$

slope of normal line $\frac{dy}{dx} = \frac{y-1}{x}$

$$y' = \frac{y-1}{x}$$

but this is \perp to it, so $y' = \frac{x}{1-y}$

$$b) \frac{dy}{dx} = \frac{1-x}{y}$$

$$y \, dy = (1-x) \, dx$$

$$\int y \, dy = \int (1-x) \, dx$$

$$\frac{1}{2}y^2 + C_y = x - \frac{1}{2}x^2 + C_x$$

$$x^2 + y^2 + 2x + C = 0$$

$$(x+1)^2 + y^2 = 1$$

if ~~th~~

c) Circle w/ origin at $(-1, 0)$ and radius 1.

$$10. \quad F(x) = \int_0^x e^{-t^2} \, dt$$

$$\begin{aligned} a) \quad F'(1) &= \frac{d}{dx} \int_0^1 e^{-t^2} \, dt = \frac{d}{dx} \left[\int_0^1 \frac{1}{e^{t^2}} \, dt \right] \\ &= \int_0^1 -2t e^{-t^2} \, dt \\ &= -\frac{2}{e} \end{aligned}$$

$$F''(1) =$$

$$13. \quad \int_0^1 \frac{dx}{(x^2+1)^2}$$

$$\begin{aligned} x &= \tan u \\ dx &= \sec^2 u \, du \end{aligned}$$

$$\int_0^{\pi/4} \frac{\sec^2 u \, du}{\sec^4 u}$$

$$\int_0^{\pi/4} \cos^2 u \, du$$

$$\left. \frac{1 + \cos 2x}{2} \right|_0^{\pi/4} = \frac{1}{2} + 1 = \boxed{\frac{3}{2}}$$

$$16. \int_1^{\infty} \frac{dx}{x^{3/2}}$$

$$= \lim_{b \rightarrow \infty} -\frac{2}{\sqrt{b}} + 2 = \boxed{2}$$

$$19. \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \dots$$

$$\int \frac{1}{1+x^2} = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 - \dots$$