Secondary School Math

Algebra

Rules of Algebra

Addition on Both Sides $a - b = c \rightarrow a = c + b$
Subtraction on Both Sides $a + b = c \rightarrow a = c - b$ or $b = c - a$
Multiplication on Both Sides $\frac{a}{b} = c \rightarrow a = cb$
Division on Both Sides $ab = c \rightarrow a = c/b$ or $b = c/a$
Removing Square Root $a = \sqrt{c} \rightarrow a^2 = c$
Factoring $a(b + c) = ab + ac$
Outside factors go on top $a \cdot \frac{b}{c} = \frac{ab}{c}$
Finding ones in fractions $\frac{a(b+c)}{ad} = \frac{b+c}{d}$
F.O.I.L. (First, Outside, Inside, Last)
$(a + b)(c + d) = ac + ad + bc + bd$

Fractions

$\frac{a}{\frac{b}{c}} = \frac{ac}{b}$ $\quad \frac{\frac{a}{b}}{c} = \frac{a}{bc}$ $\quad \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ad}{bc}$ $\quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$

Exponents

$a^n a^m = a^{n+m}$ $\quad (ab)^n = a^n b^n$ $\quad \frac{a^n}{a^m} = a^{n-m}$

Radicals

$\sqrt[c]{ab} = \sqrt[c]{a} \cdot \sqrt[c]{b}$ $\quad \sqrt[c]{a^b} = a^{\frac{b}{c}}$

Logarithms

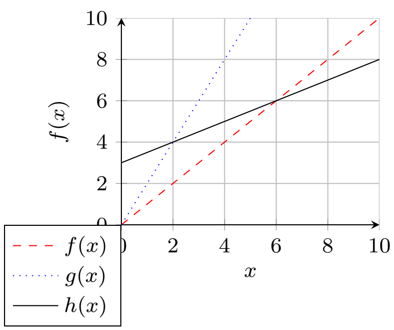
$y = log_b x \iff x = b^y$
$log_b b^x = x$ $\quad b^{log_b x} = x$ $\quad log_b x^c = c log_b x$
$log_b xy = log_b x + log_b y$ $\quad log_b(\frac{x}{y}) = log_b x - log_b y$
$ln = log_e$ $\quad ln(e) = 1$ $\quad ln(e^x) = x$

Linear Equations

Slope Intercept Form: $y = mx + b$
Point Slope Form $y = y_1 + m(x - x_1)$

Functions

A function is an operation which transforms a number x into a corresponding $f(x)$. The following lines are three linear equations - a simple type of function. If these lines are of the form $y = mx + b$, $f(x)$ has $m = 1, b = 0$, $g(x)$ has $m = 2, b = 0$, and $h(x)$ has $m = \frac{1}{2}, b = 3$



Inequalities

If you think of the lines above as "equalities" (equations), then the area on above or below these lines are inequalities. For example, if you shaded in the region below the line $f(x)$, that would be equivalent to the inequality $y \leq x$ or $f(x) \leq x$. If you shaded in the region above the black line, that would be equivalent to the inequality $y \geq \frac{1}{2}x + 3$

Domain and Range

Domain is simply all of the possible values that x can take for a given function, while range is all of the possible f(x) values. They are commonly expressed either as intervals or as inequalities.
If you want to express that x can be anywhere between or equal to 0 and 3, you could write

$x \in [0, 3]$ or $0 \leq x \leq 3$

To express that x can be between 0 and 3, but not equal to 0 or 3,

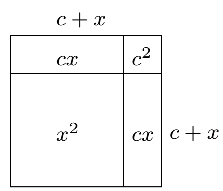
$x \in (0, 3)$ or $0 < x < 3$

There are certain functions which are only defined for certain values. For example, the function that rounds down to the nearest integer (the floor function) is only defined for integers. For example, given the function $f(x) = \lfloor x \rfloor$, and domain $x \in [0, 3]$ The range is

$f(x) \in \{0, 1, 2, 3\}$

Completing the Square

Completing the square is useful when you want to find the X intercepts (places where the line crosses the x-axis) for a given parabola. Completing the square is analogous to the following drawing, where you are trying to find the three portions of the drawing given a variable x and a constant a - c^2, $2 \cdot cx$, and x^2. This is a graphical way of thinking about factoring
$(x + c)(x + c) = x^2 + 2cx + c^2$



There are two general ways to solve for any quadratic equation (an equation that has only constants, a term multiplied by x, and a term multiplied by x^2) - using factoring and algebra, or using the quadratic formula.

Completing the Square

1. Arrange your equation into the form $ax^2 + bx + c = 0$
2. Divide out a to leave x^2 alone - $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
3. Move the constant to the right side $x^2 + \frac{b}{a}x = -\frac{c}{a}$
4. Set up left side for factoring by taking $\frac{1}{2}b$, squaring it, adding it to both sides
   $x^2 + \frac{b}{a}x + (\frac{b}{2})^2 = -\frac{c}{a} + (\frac{b}{2})^2$
5. Factor the left side
   $(x + \frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2})^2$
6. Take the square root
   $x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + (\frac{b}{2})^2}$
7. Solve for x
   $x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + (\frac{b}{2})^2}$

Quadratic Formula

The quadratic formula is neccesary when the numbers involved make it too complicated to solve the quadratic by hand, and comes from the procedure of completing the square.

1. Arrange your equation into the form $ax^2 + bx + c = 0$
2. Solve for x using quadratic formula
   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Parametric Equations

Parabolas

Vertex Form
$f(x) = a(x - h)^2 + k$ has a vertex at (h,k)
opens up if $a > 0$, down if $a < 0$
Standard Form
$f(x) = ax^2 + bx + c$ has a vertex at $(f(-\frac{b}{2a}), -\frac{b}{2a})$

Circle

A circle with center (h,k) and radius r has equation
$(x - h)^2 + (y - k)^2 = r^2$

Ellipse

An ellipse with center (h,k) and vertices at $\pm a$ and $\pm b$ has equation
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

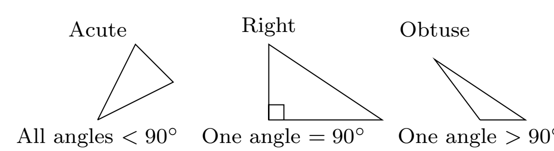
Hyperbola

A hyperbola with center (h,k) and vertices at $\pm a$ has equation
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
And asymptotes that go through the center with slope $\pm\frac{b}{a}$

Geometry

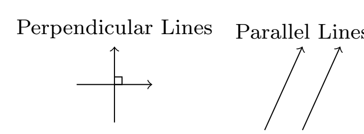
Triangles

The interior angles of a Triangle always add up to 180°



$a^2 + b^2 \geq c^2$ $\quad a^2 + b^2 = c^2$ $\quad a^2 + b^2 \leq c^2$
An **isosceles** triangle has two equal angles.
An **equilateral** triangle has three equal (60°) angles.

Perpendicular & Parallel



Congruent, Supplementary, Complementary

Two angles are **congruent** if they have the same measure
Two angles are **supplementary** if they add to 180°
Two angles are **complementary** if they add to 90°

1, 2 are **Vertical Angles** which are **Always Congruent**

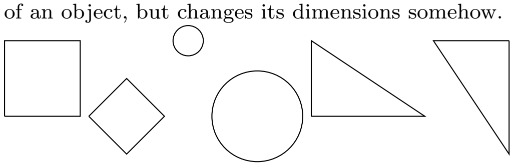
Two line segments are **congruent** if they have the same length.
Two triangles are **congruent** if they have the same angles and side lengths.
There are several ways to prove that two triangles are **congruent**.

SAS(Side Angle Side) A congruent angle in between two congruent sides
SSS(Side Side Side) Three congruent sides
ASA(Angle Side Angle) A congruent side in between two congruent angles
AAS(Angle Angle Side) Two congruent angles and a congruent side
RHS(Right Angle Hypotenuse Side) A congruent hypotenuse, side, and a right angle

Two common mistakes are to try to use **ASS**(Angle Side Side), which doesn't prove anything, and **AAA**(Angle Angle Angle, which only proves that two triangles are **similar**

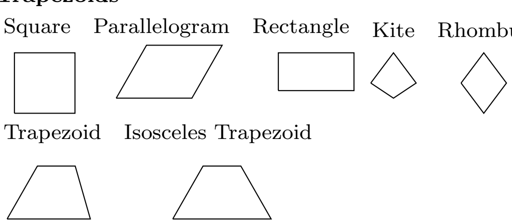
Similarity

Two shapes are **similar** if they have the same shape, but a different orientation or size.
A **transformation** is an operation that maintains the shape of an object, but changes its dimensions somehow.



Quadrilaterals

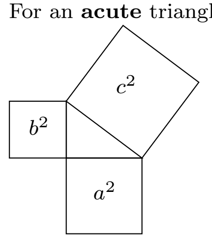
The interior angles of a Quadrilateral always add up to 360°
There are six special types of Quadrilaterals, **Squares, Rectangles, Parallelograms, Kites, Rhombuses, and Trapezoids**



A **square** has four equal sides and four right angles.
A **parallelogram** has two sets of parallel sides.
A **rectangle** has four right angles.
A **kite** has two sets of equal length sides.
A **rhombus** has two sets of equal angles.
A **trapezoid** has one set of parallel lines. An **isosceles trapezoid** also has two sets of equal angles.

Right Angles - Pythagorean Theorem

The **Pythagorean theorem** states that $a^2 + b^2 = c^2$ if a and b are the sides, and c the hypotenuse, of a right triangle.
You can imagine that if the right angle in the figure below becomes wider or shorter, the length of c will change, but a and b will remain the same. As a result, c^2 will become larger or smaller, while $a^2 + b^2$ is constant.
For an **obtuse** triangle, $a^2 + b^2 < c^2$
For an **acute** triangle, $a^2 + b^2 > c^2$

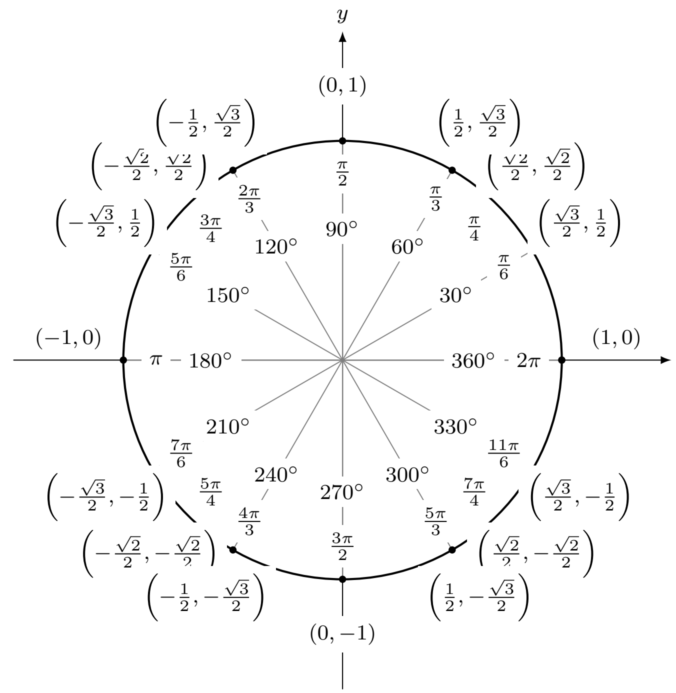


Perimeter, Area, Volume

The **perimeter** is a one-dimensional measurement of length around the sides of an object
The **surface area** is a two-dimensional measurement of area on the face of an object
The **volume** is a three-dimensional measurement of the space inside of an object
The **Perimeter** of a...
Circle:$2\pi r$ **Polygon:**$sides \cdot length$
The **Surface Area** of a...
Sphere:$4\pi r^2$ **Circle:**$r^2 \cdot \pi$
The **Volume** of a...
Rectangular Pyramid:$\frac{lwh}{3}$ **Right Circular Cone:**$\pi \cdot r^2 \frac{h}{3}$

Trigonometry

Unit Circle



SOH, CAH, TOA

Sin = $\frac{opp}{hyp}$ Cos = $\frac{adj}{hyp}$ Tan = $\frac{opp}{adj}$ Csc = $\frac{1}{Sin}$ Sec = $\frac{1}{Cos}$ Cot = $\frac{1}{Tan}$

Trigonometric Identities

Pythagorean Identity
$sin^2x + cos^2x = 1$ $sin^2x + cos^2x = 1$
Quotient Identities
$tanx = \frac{sinx}{cosx}$ $cotx = \frac{cosx}{sinx}$
Double Angle Identities
$sin(2x) = 2sinxcosx$
$cos(2x) = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x$
$tan(2x) = \frac{2tanx}{1-tan^2x}$
Sum to Product Identities
$sinx + siny = 2sin(\frac{x+y}{2})cos(\frac{x-y}{2})$
$sinx - siny = 2cos(\frac{x+y}{2})sin(\frac{x-y}{2})$
$cosx + cosy = 2cos(\frac{x+y}{2})cos(\frac{x-y}{2})$
$cosx - cosy = -2sin(\frac{x+y}{2})sin(\frac{x-y}{2})$
Product to Sum Identities
$sinxsiny = \frac{1}{2}[cos(x-y) - cos(x+y)]$
$cosxcosy = \frac{1}{2}[cos(x-y) + cos(x+y)]$
$sinxcosy = \frac{1}{2}[sin(x+y) + sin(x-y)]$
$cosxsiny = \frac{1}{2}[sin(x+y) - sin(x-y)]$

Precalculus

Proofs, Induction

A **proof** is simply a manipulation of assumptions (givens) into a conclusion.
Some methods of proof include direct proof, proof by contrapositive, truth table proofs, and proof by induction.
Induction is one of the most common methods of proof. If you can prove a given mathematical assumption for case of an abstract number n, and for the case of 1, you can assume that it is true for any n, because you can reach any integer by incrementing in steps of one from n.

Complex Numbers

Complex numbers are numbers that include the imaginary number $i = \sqrt{-1}$. Imaginary numbers are those which have no real part. A complex number is usually expressed in the form $a + bi$ where a is the real part, and b is the imaginary part.
$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, a \geq 0$
The **Complex Conjugate** has the same real part, and opposite value imaginary part. Where the original complex number is $a + bi$, the complex conjugate is $a - bi$.
The **Complex Modulus** is the absolute value of a complex number $|a + bi| = \sqrt{a^2 + b^2}$
$(a + bi) + (c + di) = a + c + (b + d)i$
$(a + bi)(c + di) = ac - bd + (ad + bc)i$
$(a + bi)(a - bi) = a^2 + b^2$

Polar Coordinates

Polar coordinates are another way of describing a flat plane or higher dimensional space. Whereas the two coordinates of a traditional plane represent the distance in two directions from the origin, the polar form uses an angle and a distance from the origin.
In 2 dimensional space, the conversion is given by the following:
Polar to Cartesian: $x = r \cdot cos(\theta), y = r \cdot sin(\theta)$
Cartesian to Polar: $r = \sqrt{x^2 + y2}, \theta = tan^{-1}(\frac{y}{x})$

Sum and Product Notation

Abstract sums and products are denoted with the capital Greek letters sigma (Σ) and pi (Π) respectively.
The numbers and letters on the top and bottom of these Greek letters represent the values for which the function should be executed. The process of finding the sum or product is as simple as completing the operation for each of these numbers, and then adding them (for a sum) or multiplying them together (for a product).
$\sum_{i=1}^{n} \frac{1}{n!} = \frac{1}{1} + \frac{1}{2*1} + \frac{1}{3*2*1} + \frac{1}{4*3*2*1} + ...$
$\prod_{n=1}^{5} 2n = 2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 = 4480$

Vectors and Matrices

A **vector** is an algebraic object which contains multiple pieces of data representing magnitudes of different dimensions.
$\vec{a} = (a_1 ... a_n)$
A **matrix** is a collection of vectors. A matrix is written as a grid of values. In the following matrix, there are three **rows** and four **columns**, representing three four dimensional vectors, or possibly four three dimensional vectors.
(I can't figure out why, but when i try to insert a matrix it screws up all the formatting - fix later)

Sequences and Series

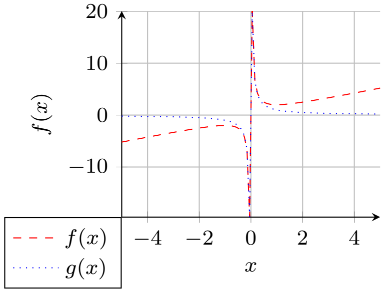
A **sequence** is an ordered list of numbers, where the natural numbers N=1,2,3,... each have a function applied in turn. The sequence has a corresponding **series**, which is simply the sum of the values of the sequence.
One example is the **arithmetic** sequence, which is any sequence where the terms are separated by a given constant. For an arithmetic sequence, the nth term is given by $a_n = a_m + (n - m)d$
A **geometric** sequence is one in which the terms are separated by a given ratio. For example, each term might be 2 times the previous term as in the sequence 2, 4, 8, 16,... In a geometric sequence, the nth term is given by $a_n = ar^{n-1}$, where r is the **common ratio**.
The **Binomial Theorem** is an important trick with applications in probability: $\sum \frac{n!}{k!(n-k)!} a^{n-k} b^k = (a + b)^n$

Probability

The **probability** of of a given event is the ratio of outcomes which lead to the event, over the number of all possible outcomes. $P(E) = \frac{event}{possibilities}$
Sometimes, probabilities are expressed as **odds**. Odds are expressed as the ratio of the number of ways the event can occur against the number of ways the event can not occur. $Odds = \frac{event}{notevent}$. To find the probability given odds, simply add together the probability of the event occuring and the probability of the event not occuring in the denominator.
$P(E) = \frac{event}{not+event}$
The fundamental theorem of counting says that if there are n ways for event 1 to occur, and there are m ways for event 2 to occur, then there are $n \cdot m$ ways for both events to occur. (draw tree later)
Combinations are a set of simultaneous events in which order doesn't matter. For example, the set {1,2,3} is considered the same as {3,2,1} The number of ways to choose k events out of n possibilities is given by $nCk = \frac{n!}{k!(n-k)!}$
Permutations are the set of simultaneous events for which order does matter. The number of permutations of k events chosen from n possibilities is given by $nPk = \frac{n!}{(n-k)!}$

Asymptotes

An **Asymptote** is a line which represents a limit which a function gets infinitesimally close to, but never reaches. Some examples of functions with asymptotes are as follows:
$f(x)$ has a horizontal asymptote at $f(x) = 0$ and a vertical asymptote at $x = 0$.The function $g(x)$ has a **skew** asymptote on the line $f(x) = x$.



When a function has an asymptote, there is at least some part of the function that eventually goes to zero. For example, the function $(x^3 + 2x^2 + 3x + 4)/x$ can be simplified into $x^2 + 2x + 3 + \frac{4}{x}$. In this form, we can see that the $\frac{4}{x}$ part will go to zero, and the $x^2 + 2x + 3$ is the **curvilinear asymptote**, or the line that the function will generally follow.

Introduction to Limits