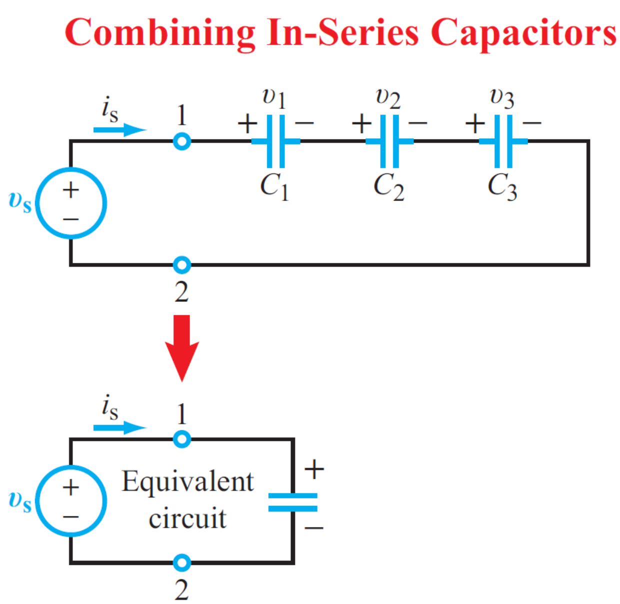
Capacitors in Series and Parallel

Capacitors in Series



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

The capacitors on the top left share the same current, i_s, which is related to their individual voltages by

$$i_s = C_1 \frac{d\upsilon_1}{dt} = C_2 \frac{d\upsilon_2}{dt} = C_3 \frac{d\upsilon_3}{dt}$$

Note also,

$$\upsilon_s = \upsilon_1 + \upsilon_2 + \upsilon_3$$

Now for the circuit on the bottom, it must be true that

$$\begin{aligned} i_s &= C_{eq} \frac{d\upsilon_s}{dt} \\ &= C_{eq} \left(\frac{d\upsilon_1}{dt} + \frac{d\upsilon_2}{dt} + \frac{d\upsilon_3}{dt}\right) \\ &= C_{eq} \left(\frac{i_s}{C_1} + \frac{i_s}{C_2} + \frac{i_s}{C_3}\right) \end{aligned}$$

which leads to

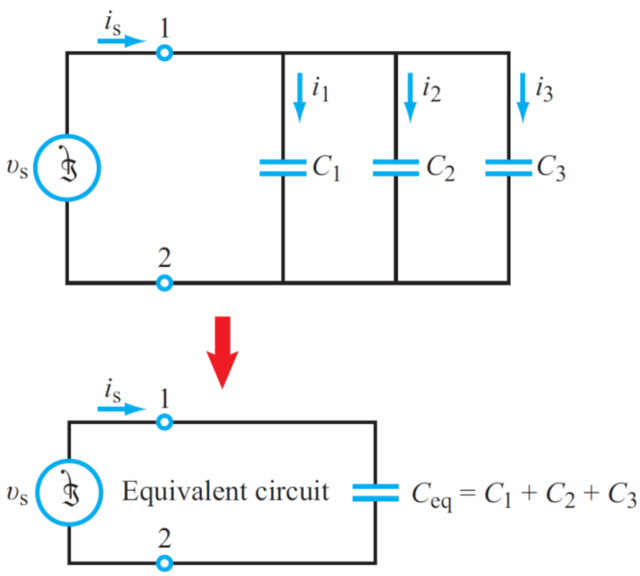
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For the general case of N capacitors in series:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

(capacitors in series).

Capacitors in Parallel



The three capacitors on the top left are connected in parallel. Hence, they share the same voltage υ_s, and the source current i_s is equal to the sum of their currents,

$$i_s = i_1 + i_2 + i_3$$
$$= C_1 \frac{d\upsilon_s}{dt} + C_2 \frac{d\upsilon_s}{dt} + C_3 \frac{d\upsilon_s}{dt}.$$

Now for the circuit on the bottom, it must be true that

$$i_s = C_{eq} \frac{d\upsilon_s}{dt}$$

Equating the expressions leads to

$$C_{eq} = C_1 + C_2 + C_3$$

For the general case of N capacitors in parallel:

$$C_{eq} = \sum_{i=1}^{N} C_i \quad \textbf{(capacitors in parallel)}.$$