Bend Testing and Weibull Statistics for Non-Metallic Composite Materials

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Abstract

The use of Weibull statistical analysis on bend testing results of non-metallic samples (composites) is shown to be useful in determining material strength variance and for analysis of the relationship between microstructure and variability.

Introduction

Knowing the real behavior of engineering materials is a vital part of engineering design for almost every materials application imaginable. However, real material properties are subject to unavoidable statistical variations due to tiny differences in microstructure. This fact is most easily seen in the dependence of strength on the length or volume of a sample, where more volume or length increases the statistical likelihood of defect inclusion in a sample (for the same processing/defect distribution). This variation of strength can make it difficult to determine a safe load to put onto a part, making it important to examine the variance there is in the fracture stress of a material with a given volume. The Weibull failure probability is the way that the variance of strength of a material is characterized, and is defined as:

$$P = 1 - e^{-\left(\frac{\sigma}{\sigma 0}\right)^{m}}$$

Where m is the Weibull modulus and σ0 is the reference stress. The Weibull modulus gives information about the 'reliability' of the material (its variance in ultimate strength), and the reference stress gives statistical information about the percentage of samples that will fail at a stress equal to σ0. It is possible to calculate both m and σ0 using specific data analysis on elastic testing data such as a 3-point bend test.

Experimental Procedure

In order to derive the Weibull constants for the non-metallic samples used, it is important to use a 3-point bend test, as tensile bars cannot be machined out of materials like concrete composites. For such a bend test, it is important to note the sample dimensions, so the eventual Weibull constants are a function of sample stress and can be more readily used for engineering applications. Finally, an ideal statistical test usually requires an adequate sample size (n~30) to be valid, but this condition can be relaxed somewhat for more reliable materials. From the collection of bend test results for a given material, rank the failure stresses from smallest to largest, and assign each a failure probability as follows:

$$P = \frac{n - 0.5}{N}$$

Where n is the rank (i.e. smallest fracture stress n=1) and N is the total number of samples. This assigns a failure probability as a function of failure stress to each test result. Next, plot the following linearized version of the Weibull failure probability equation using the calculated P values and measured stress (σ) values:

$$\ln(\ln(\frac{1}{1-P})) = m*\ln(\sigma) - m*\ln(\sigma o)$$

The slope of the best-fit line will give you the Weibull modulus and the intercept will give you the Weibull modulus multiplied by the natural log of the reference stress (σ0). Simple algebra is used to calculate σ0 from the intercept.

Results

The average elastic modulus for the wood sample was 1.698 GPa, and the average stress at fracture was 20.5 MPa. Using the data from the best-fit line equation from figure 1b, the Weibull failure probability equation is:

$$P = 1 - e^{-(\frac{\sigma}{14.79})^{1.1}}$$

Using the equation above, it is possible to determine the maximum stress allowed on the sample such that one sample in every one million fail (i.e. P = 10^-6). Doing so results in an allowed stress of 51.93 Pa.

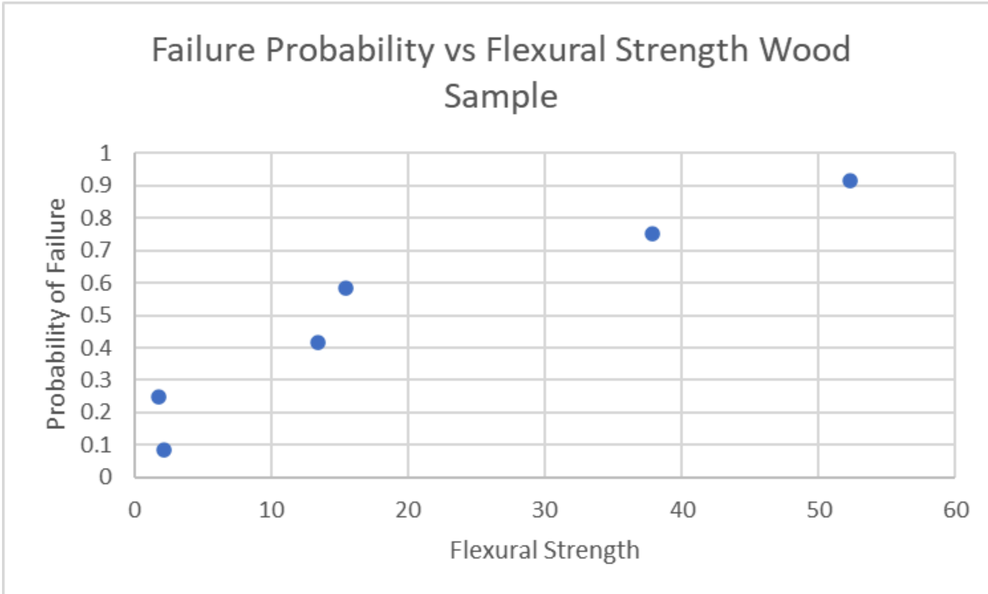

Figure 1a. Wood Failure Probability vs Stress

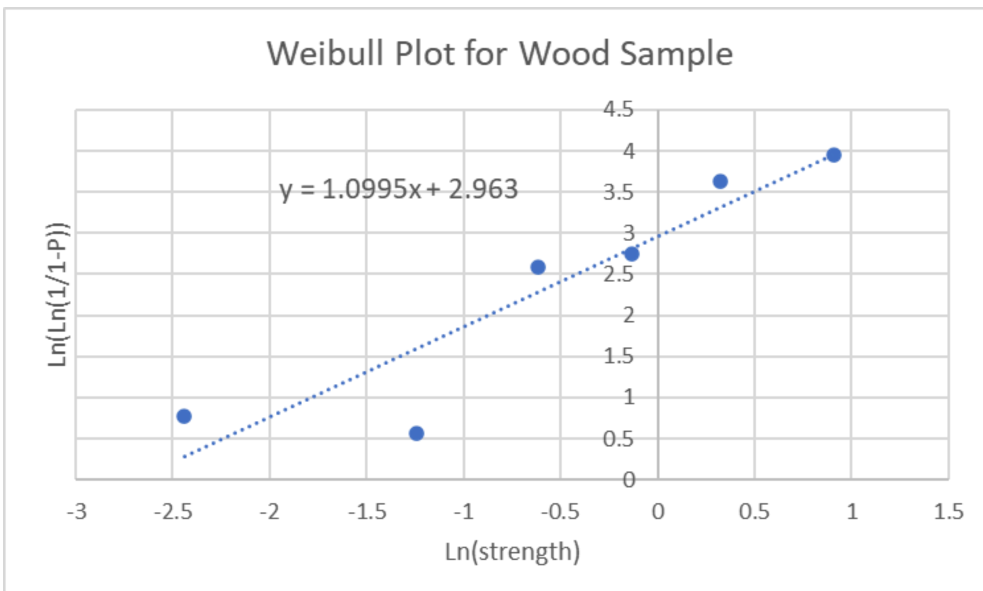

Figure 1b. Weibull Plot for Wood Sample

The average elastic modulus for the glass sample was 31.59 GPa, and the average stress at fracture was 92.158 MPa. Using the data from the best-fit line equation from figure 2b, the Weibull failure probability equation is:

$$P = 1 - e^{-\left(\frac{\sigma}{5185.4}\right)^{0.541}}$$

Using the equation above, it is possible to determine the maximum stress allowed on the sample such that P = 10^-6. Doing so results in a maximum allowed stress of 0.042 Pa.

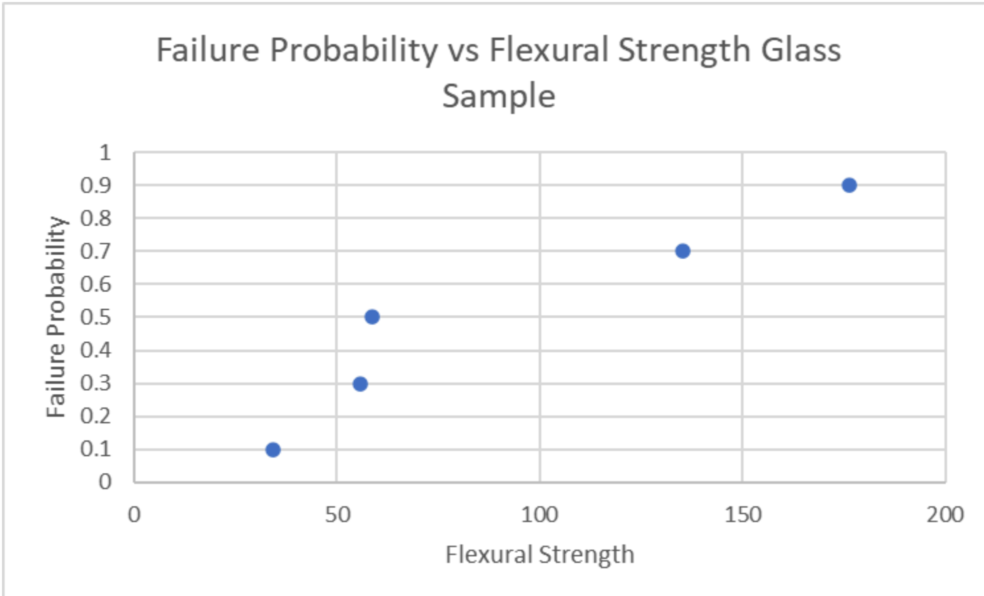

Figure 2a. Glass Failure Probability vs Stress

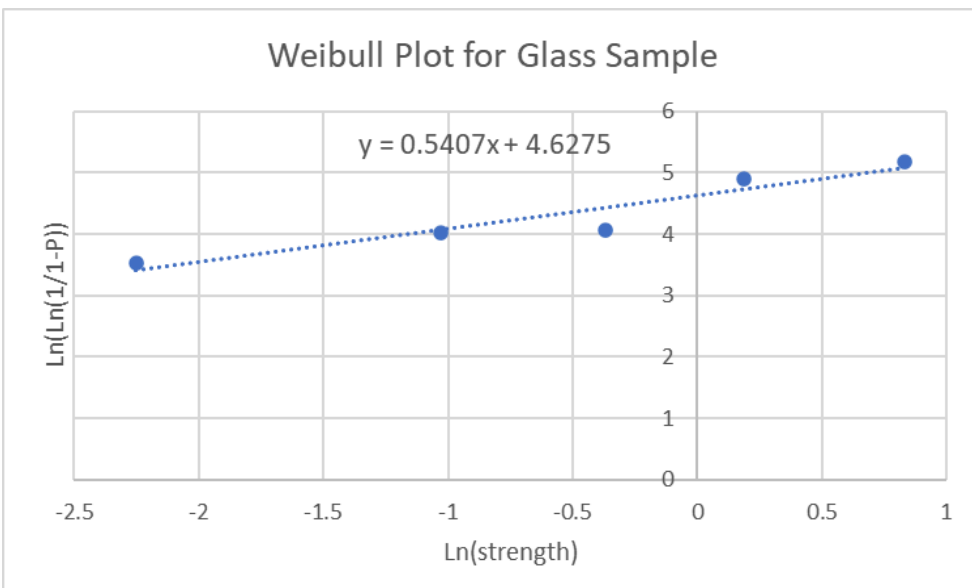

Figure 2b. Weibull Plot for Glass Sample

The average elastic modulus for the cement composite sample was 4.73 GPa, and the average stress at fracture was 20.33 MPa. Using the data from the best-fit line equation from figure 3b, the Weibull failure probability equation is:

$$P = 1 - e^{-\left(\frac{\sigma}{35.9}\right)^{0.857}}$$

Using the equation above, it is possible to determine the maximum stress allowed on the sample such that P = 10^-6. Doing so results in a maximum allowed stress of 3.59 Pa.

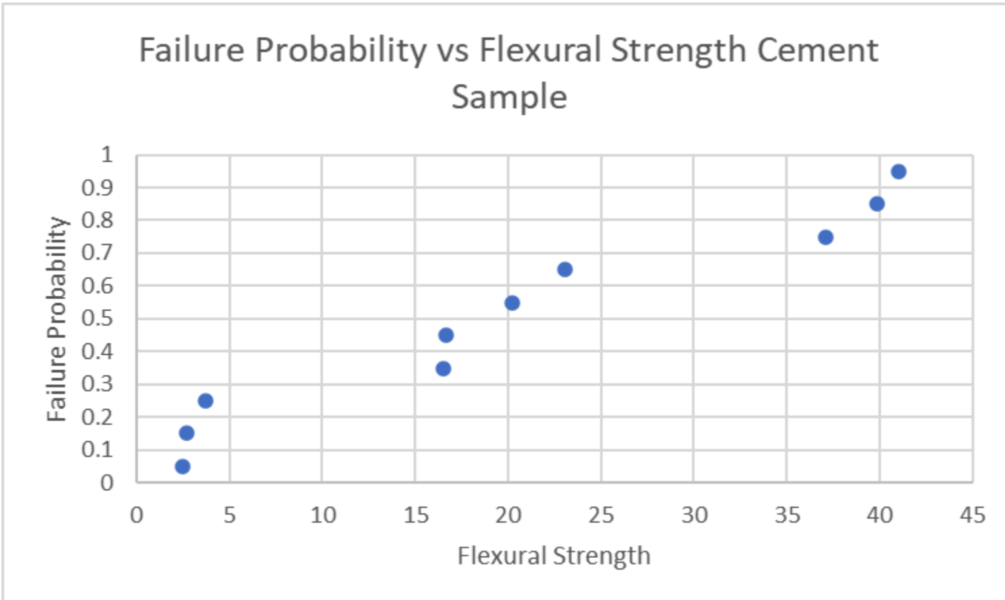

Figure 3a. Cement Failure Probability vs Stress

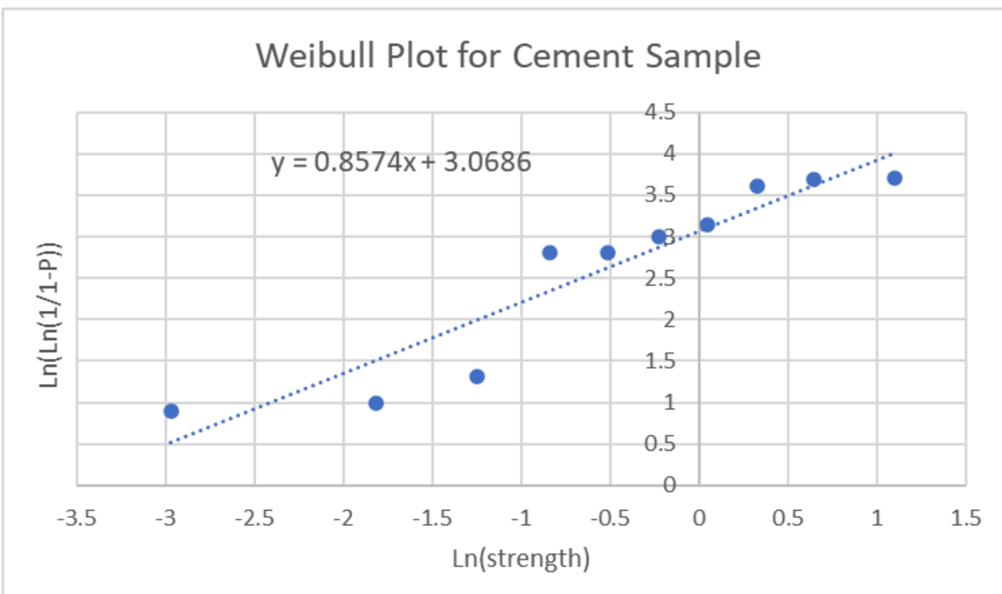

Figure 3b. Weibull Plot for Cement Sample

The average elastic modulus for the carbon fiber composite sample was 4.77 GPa, and the average stress at fracture was 18.16 MPa. Using the data from the best-fit line equation from figure 4b, the Weibull failure probability equation is:

$$P = 1 - e^{-\left(\frac{\sigma}{191.02}\right)^{0.565}}$$

Using the equation above, it is possible to determine the maximum stress allowed on the sample such that P = 10^-6. Doing so results in a maximum allowed stress of 0.0046 Pa.

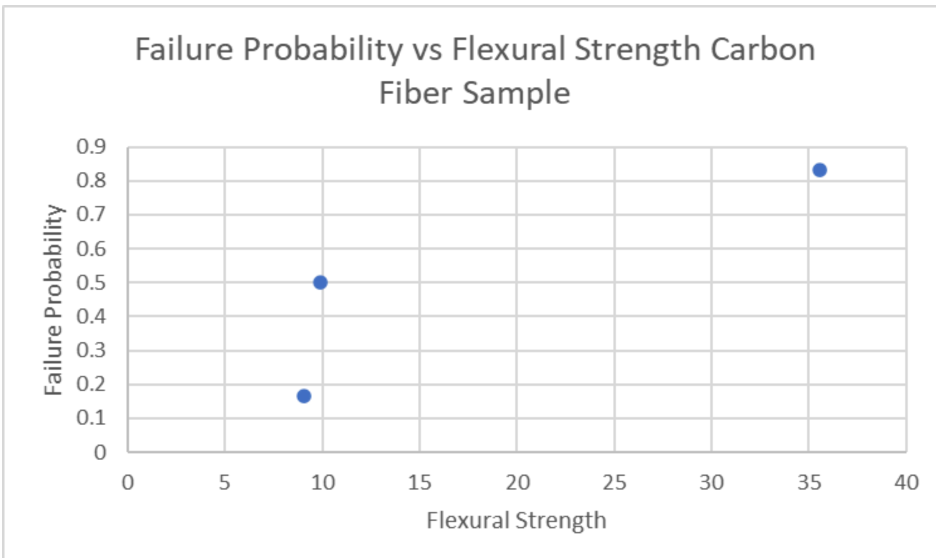

Figure 4a. Carbon Fiber Failure Probability vs Stress

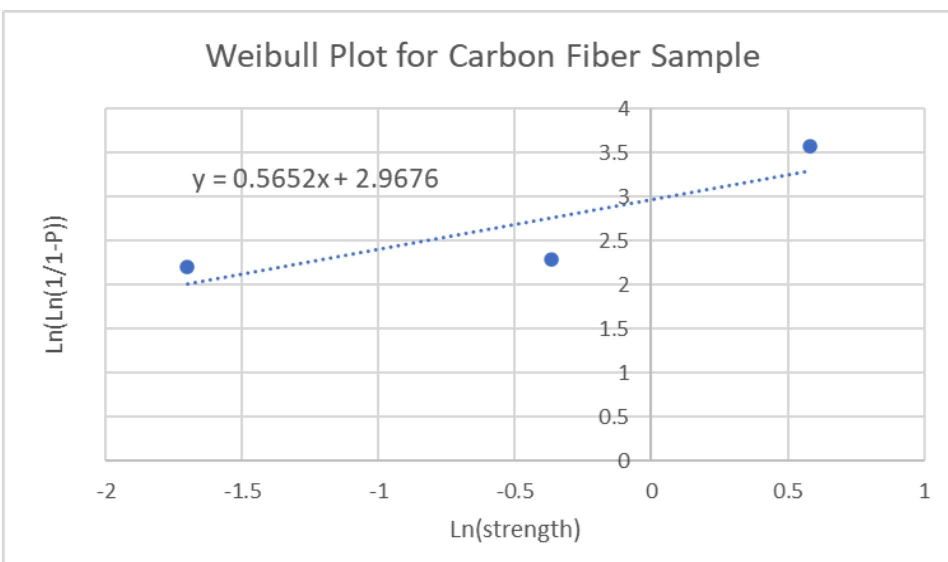

Figure 4b. Weibull Plot for CF Sample

It is important to note that the allowed stresses calculated for all of the samples in this experiment are extremely low. This is due to the fact that each sample has an unusually low Weibull modulus. This could be due to unreliable data (and in the case of the carbon fiber sample not enough data) and poor calculation of maximum flexural stress. Another reason for the unusual m values could be the assumption made regarding the inclusion of sample dimension in the data given by the bend test apparatus, since sample dimension has an impact on failure. In any case, the low reliability of the samples should be seen as highly unusual. However, assuming all of the samples fell victim to the same analytical error, it is still possible to relate material microstructure to its Weibull modulus (excluding the overly sparse dataset of the carbon composite).

The Weibull modulus seems to be connected to the fracture strength (and elastic modulus) of the samples with sufficient data. For example, the wood sample had a lower elastic modulus and ultimate strength than the glass sample, but had a significantly higher Weibull modulus, strange results notwithstanding. Similarly, the cement sample had an elastic modulus in between that of wood and glass, and also had a Weibull modulus in between those values as well. The general trend of elasticity and reliability seems to suggest that more ductile materials (less stiff in the case of this experiment concerning composites) are more reliable materials. This matches well with theory and experiment, as ductile materials do not fail suddenly and catastrophically due to statistically included defects.

Conclusion

The use of Weibull statistics on 3-point bend test data shows the relationship between stiffness and reliability. Notably, very stiff materials (i.e. ones that have no real ductile or soft deformation) are much less reliable due to their tendency to fail suddenly. This is an important fact for consideration when designing systems relying on stiff composite components, as the Weibull failure probability criterion may be far more limiting than a simple factor of safety.

References

[1] R. Smith (2021). MATE 350 Lab Notes 5: Bend testing and Weibull [PowerPoint slides].

[2] R. Smith (2023). MATE 350 Structural Materials Lab: Three-point bend test measurement [PowerPoint slides].