# Ecological forecasting in R

## Lecture 2: dynamic GLMs and GAMs

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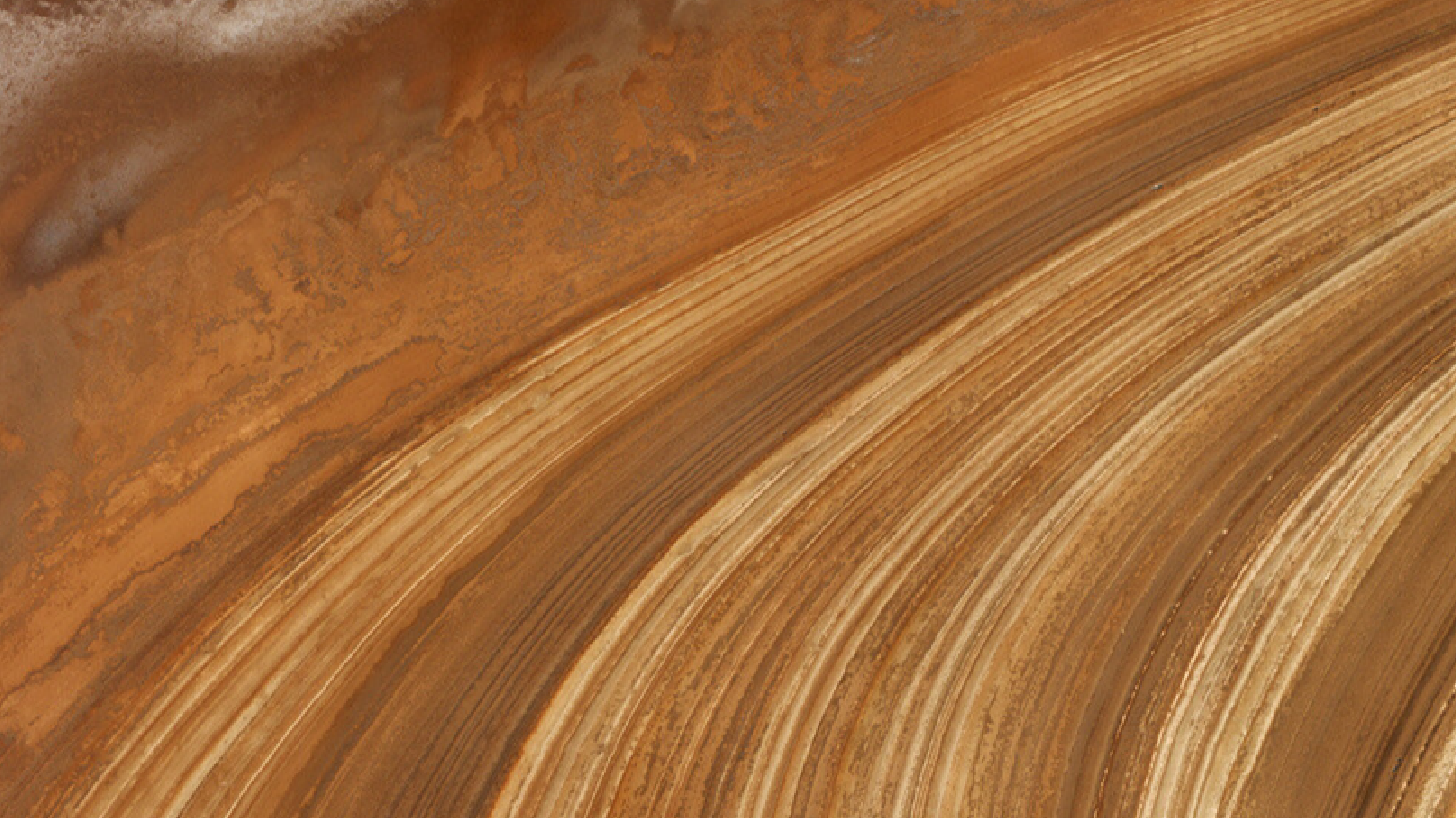
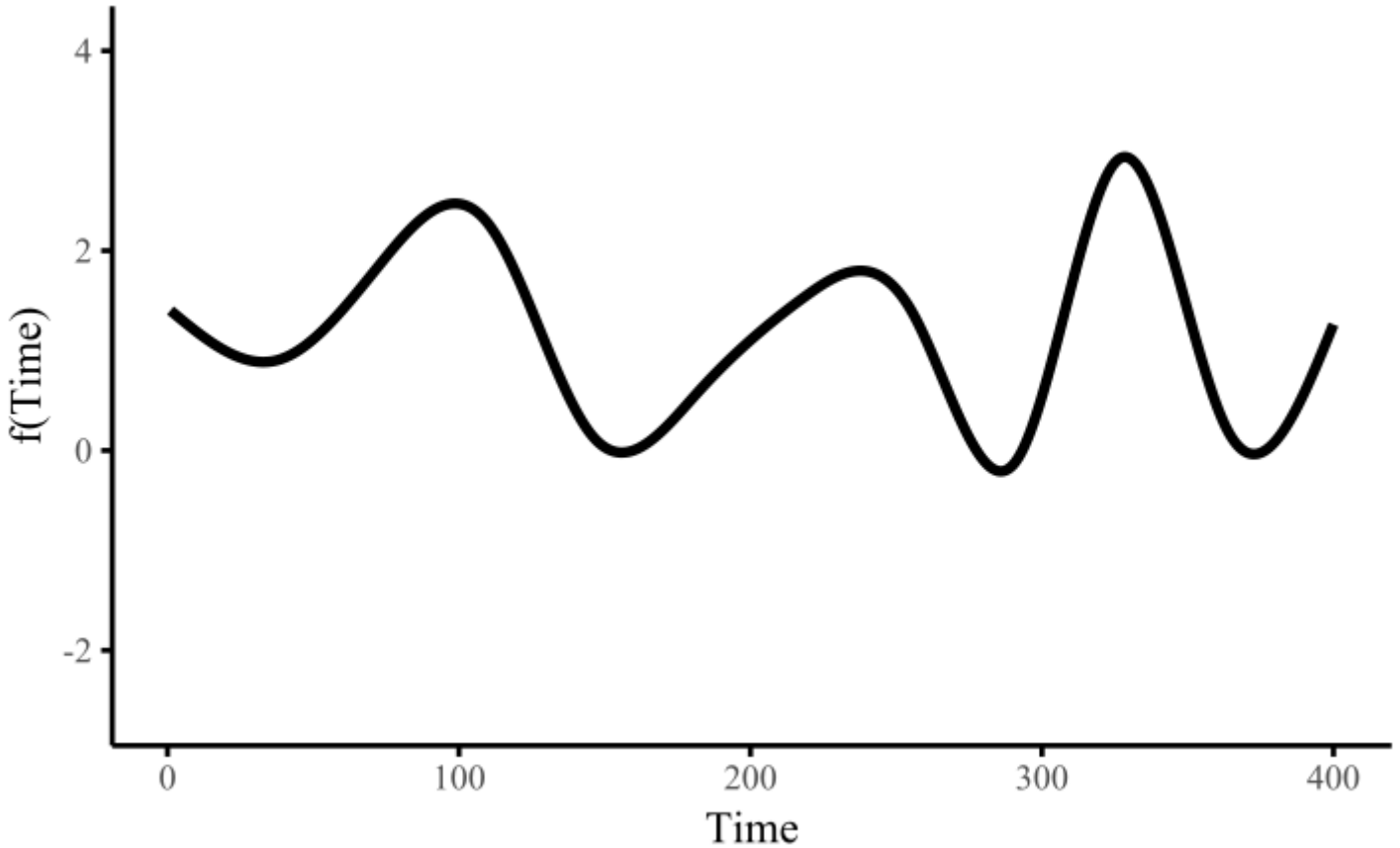
**0900–1200 CET Monday 4th September, 2023**




4
2
0
-2
f(Time)
0
100
200
300
400
Time

# Workflow

Press the "o" key on your keyboard to navigate among slides

Access the tutorial html here

Download the data objects and exercise  script from the html file

Complete exercises and use Slack to ask questions

Relevant open-source materials include:

An introduction to Bayesian multilevel modeling with `brms`

Introduction to Generalized Additive Models with  and `mgcv`

Statistical Rethinking 2023 - 04 - Categories & Curves

Statistical Rethinking 2023 - 12 - Multilevel Models

# This lecture's topics

Useful probability distributions for ecologists

Generalized Linear and Additive Models

Temporal random effects

Temporal residual correlation structures

**When applying statistical modelling to a time series, we aim to estimate parameters for a collection of probability distributions**

**These distributions are indexed by *time* (i.e. the observations are random draws from a set of time-varying distributions)**

**Usually we allow the mean of these distributions to vary over time. But what kinds of distributions are available to us?**

# Useful probability distributions

# Normal (Gaussian)

$$\boldsymbol{Y_t} \sim \text{Normal}(\mu_t, \sigma)$$

Properties

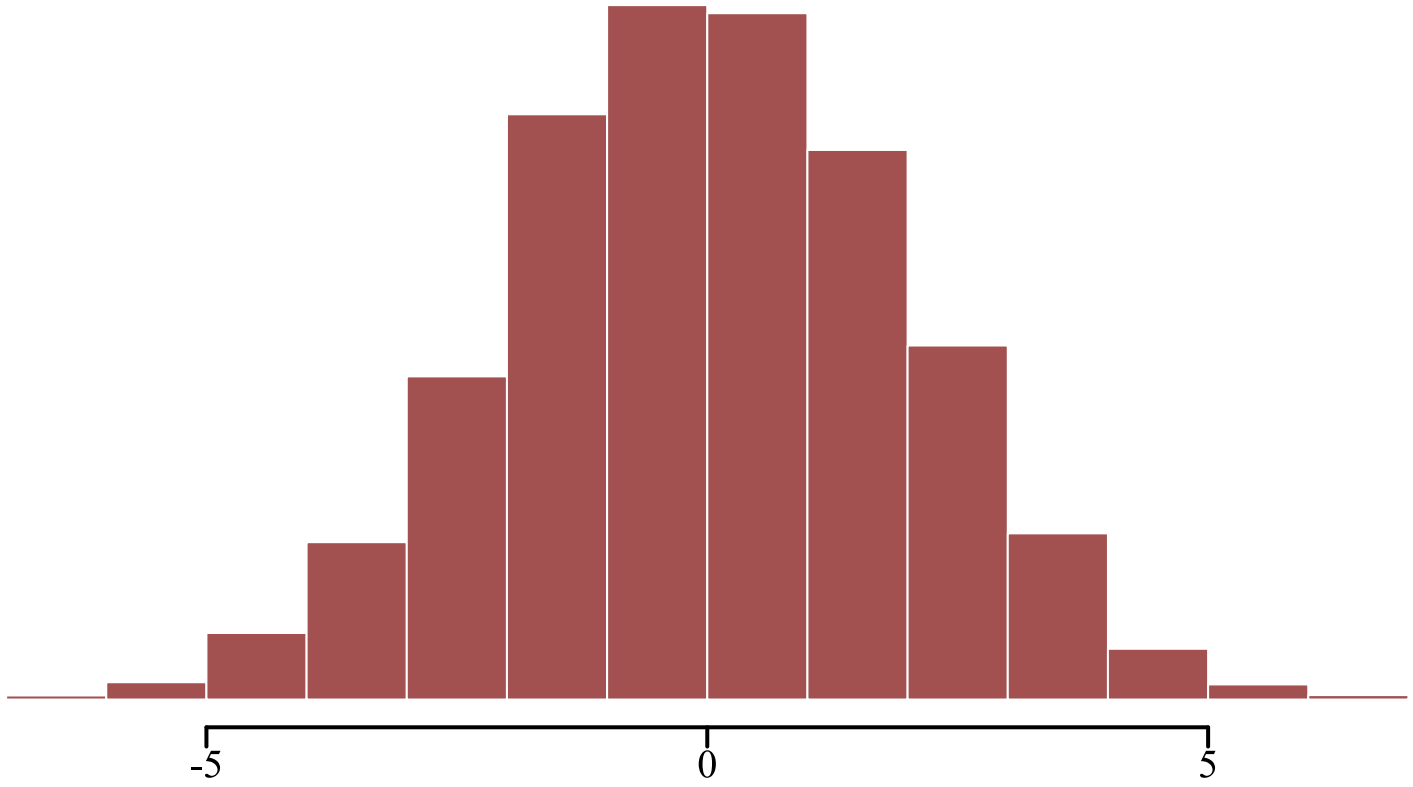
- Real-valued continuous observations (including any decimal)
- Unbounded (supports $-\infty$ to $\infty$)
- Symmetric spread, controlled by $\sigma$, about the mean $(\mu_t)$

Nearly all common time series models assume this data distribution

- RW, AR, and ARIMA
- ETS and TBATS
- Meta's Prophet 📦

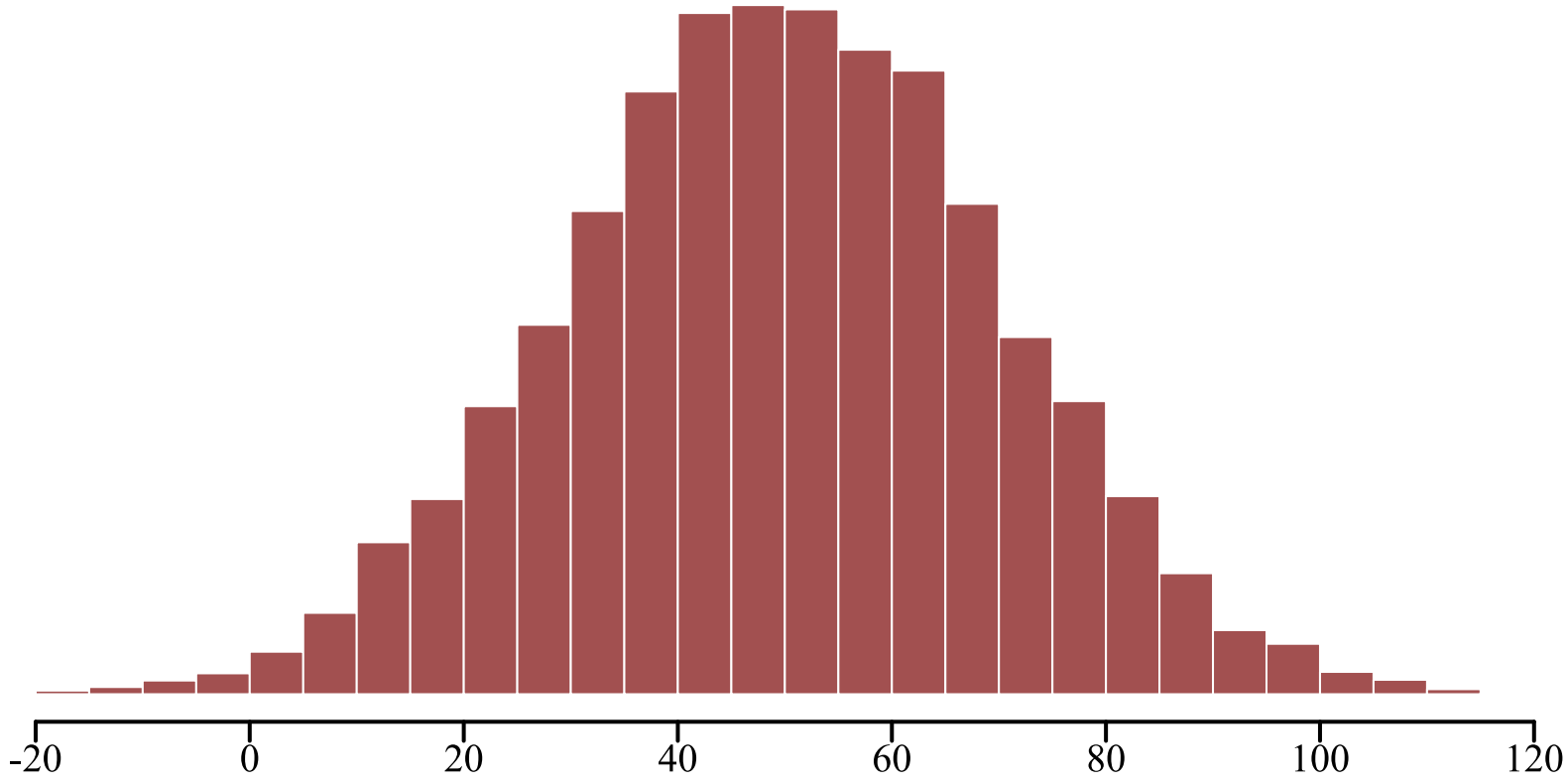
# Normal (Gaussian)

$$\boldsymbol{Y_t} \sim \text{Normal}(0, 2)$$



# Normal (Gaussian)

$$Y_t \sim \text{Normal}(50, 20)$$



# Linear regression

It is common to estimate linear predictors of $\mu$ with ***regression***

$$\boldsymbol{Y_t} \sim \text{Normal}(\alpha + \beta * \boldsymbol{X_t}, \sigma)$$

Where:

- $\boldsymbol{X_t}$ represents a design matrix of covariates that contribute linearly to variation in $\mu_t$
- $\alpha$ is an intercept coefficient
- $\beta$ is a vector of regression coefficients
- $\sigma$ controls the spread of the errors about $\mu_t$

# ETS(A,A,A)

Exponential smoothing with additive components for trend, seasonality and error assumes a Normal (Gaussian) distribution

$$\boldsymbol{Y}_t \sim \text{Normal}(l_{t-1} + b_{t-1} + \boldsymbol{s}_{t-m}, \sigma)$$

Where:

$l$ gives the value of the level

$b$ gives the value of the trend

$s$ gives the value of the seasonality

$m$ represents the seasonal period

# ARMA($p$, $q$)

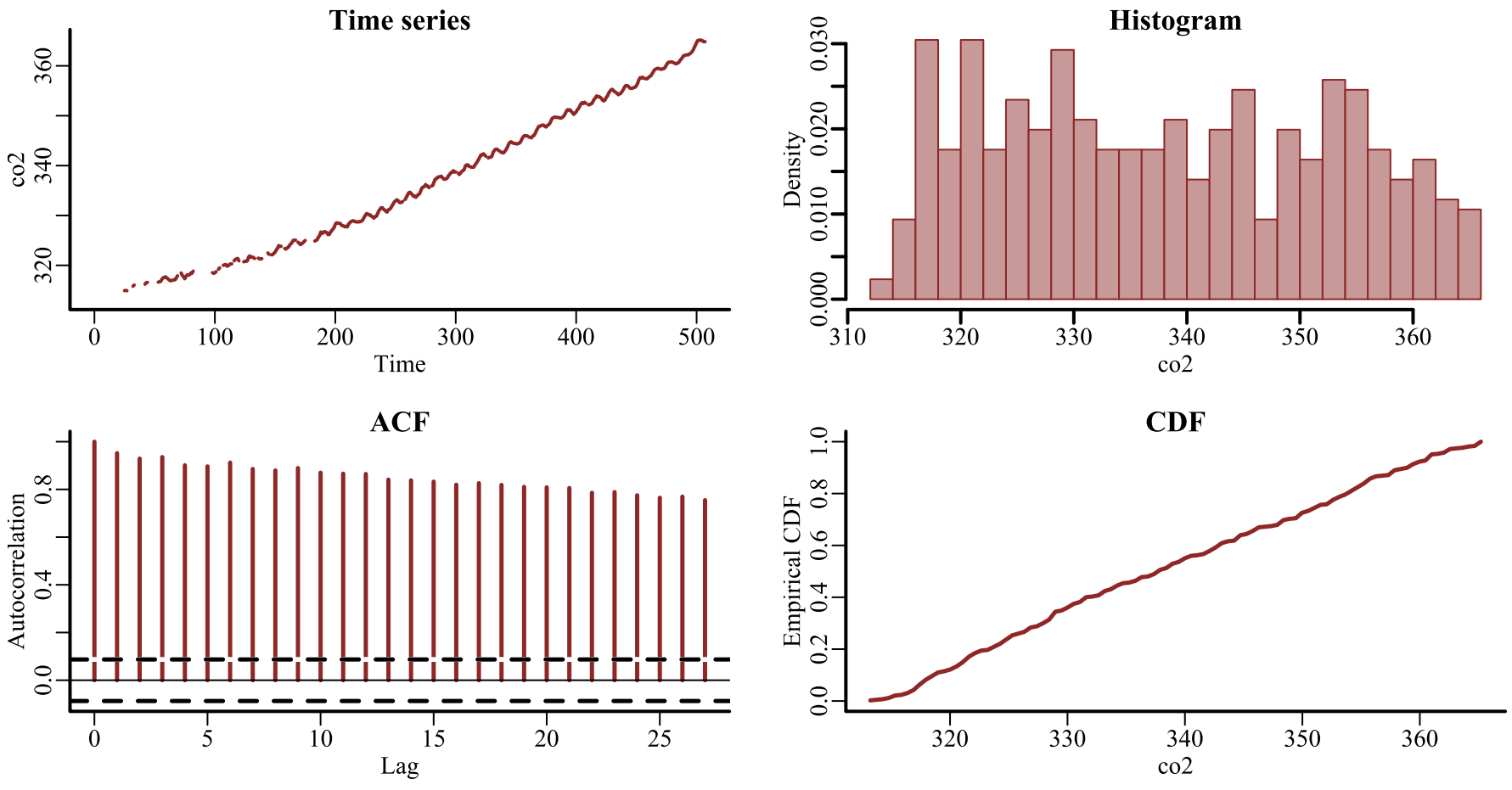
ARMA processes also assume Normality

$$\boldsymbol{Y}_t \sim \text{Normal}(c + \sum_{k=1}^{p} \phi_k(\boldsymbol{Y}_{t-k} - c) + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}, \sigma)$$

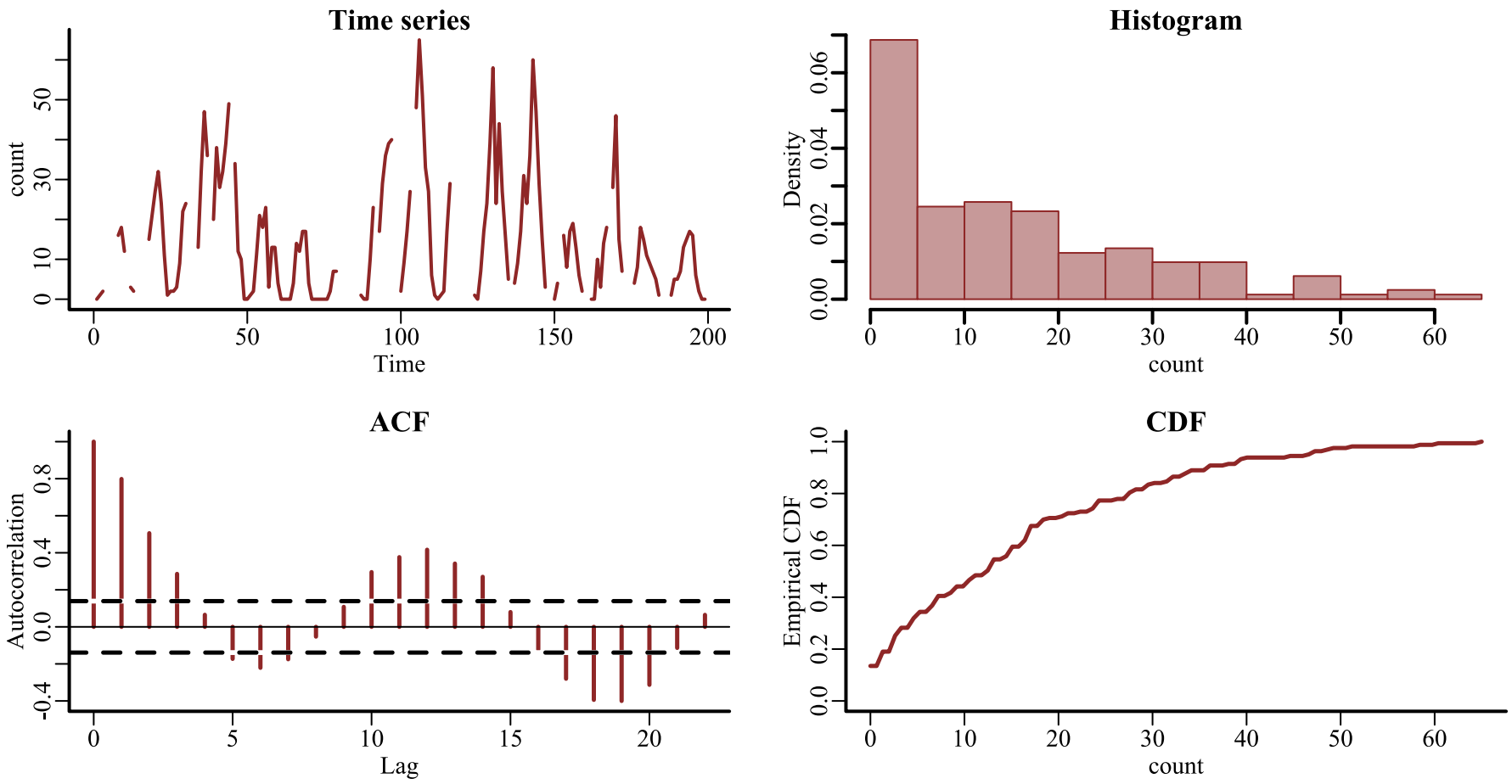
Where:

- $c$ is a constant (drift parameter)
- $p$ and $q$ gives orders of AR and MA processes
- $\phi$ and $\theta$ are AR and MA coefficients
- $\epsilon$ are historical errors (which are $\text{Normal}(0, \sigma)$)

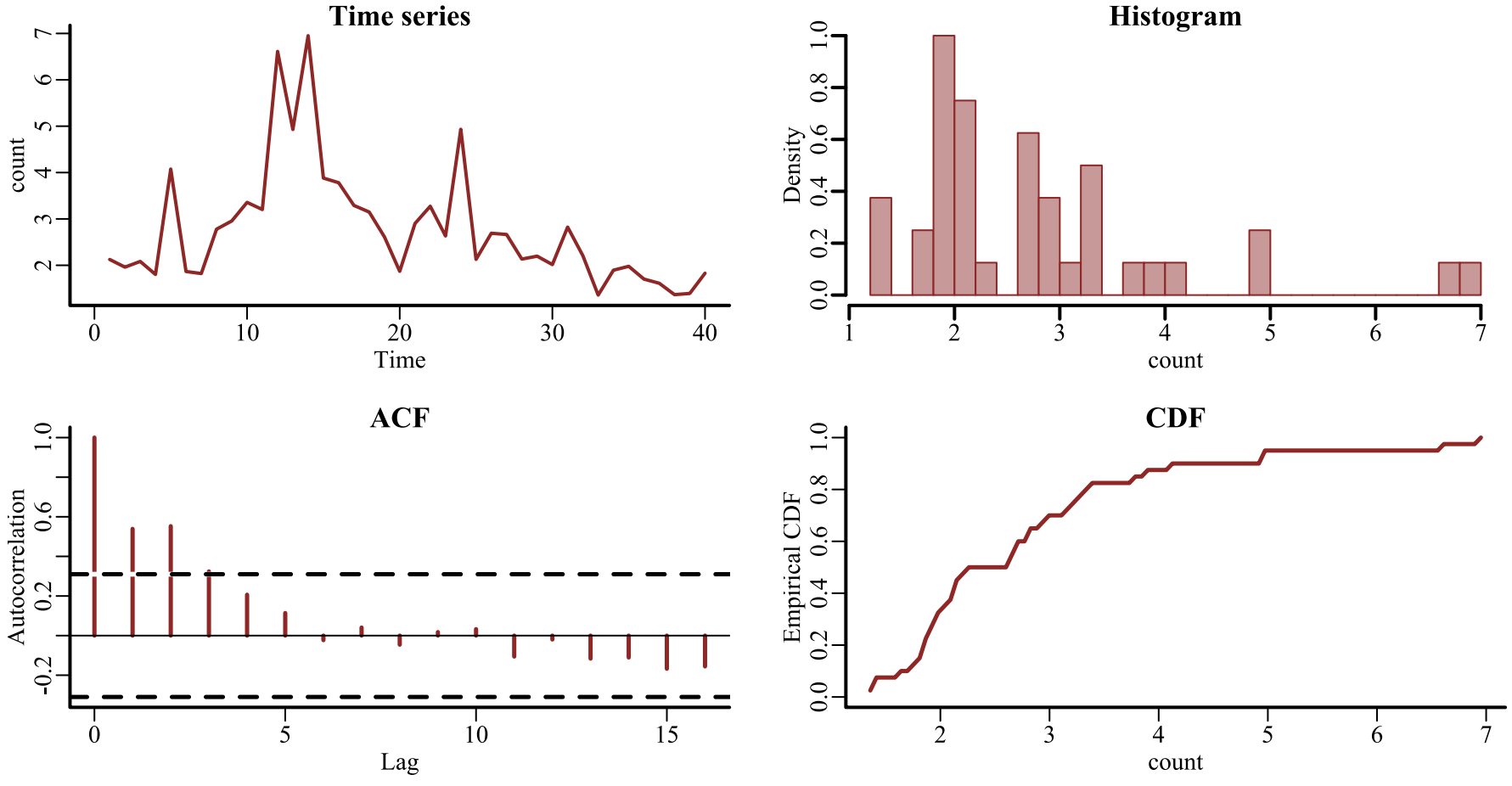
**But most real-world ecological observations, including time series, *are not Gaussian***




Properties of monthly CO2 measurement time series at the South Pole




Properties of lunar monthly Desert Pocket Mouse capture time series from a long-term monitoring study in Portal, Arizona, USA




Properties of annual American kestrel abundance time series in British Columbia, Canada

**“If our data contains small counts (0,1,2,...), then we need to use forecasting methods that are more appropriate for a sample space of non-negative integers.**

***Such models are beyond the scope of this book”***

Hyndman and Athanasopoulos, Forecasting Principles and Practice

# Ok. So now what?

# Poisson

$$\boldsymbol{Y_t} \sim \text{Poisson}(\lambda_t)$$

Properties

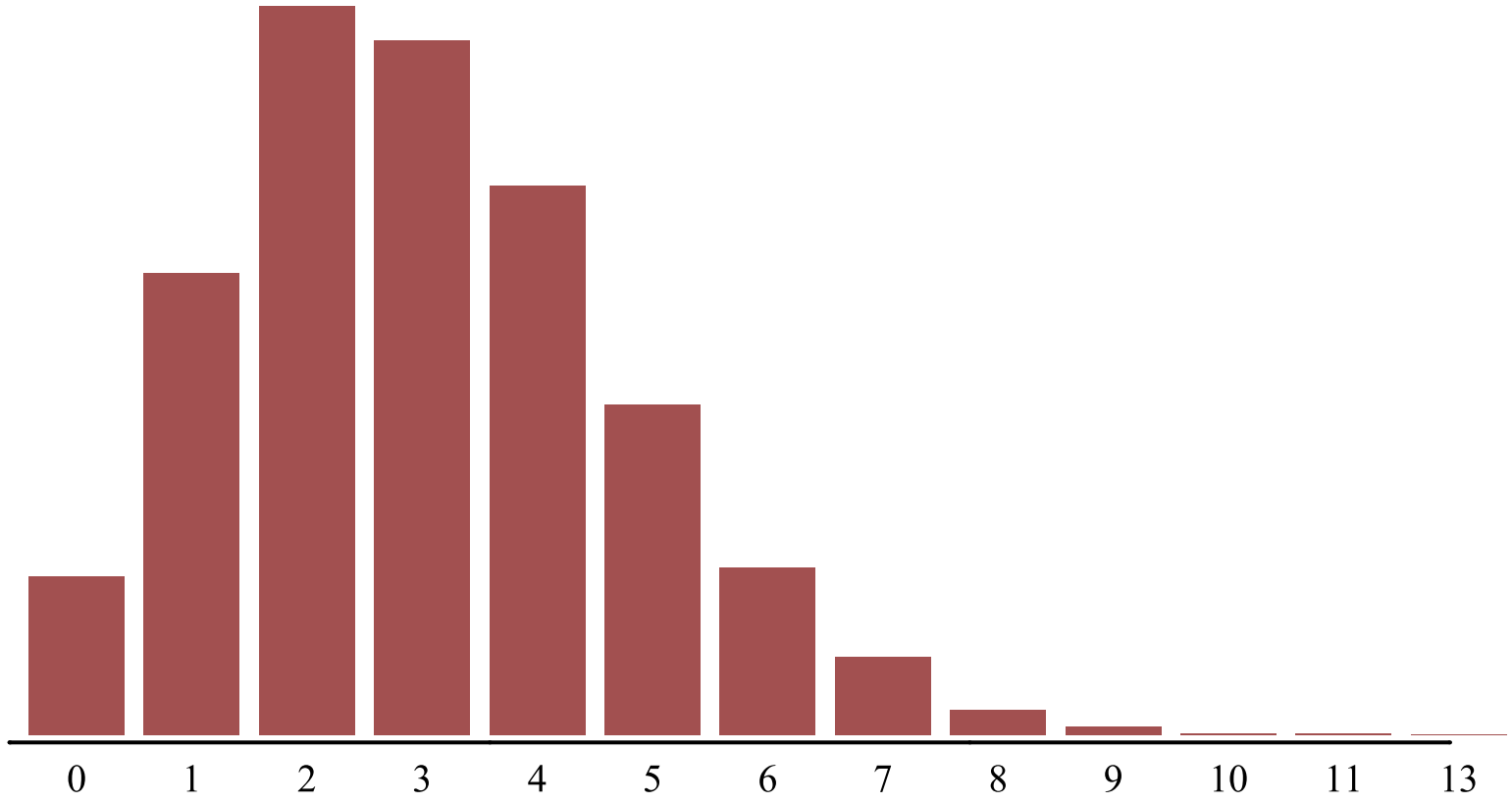
- Discrete, integer-valued observations (including $0$)
- Lower bound (supports $0$ to $\infty$)
- mean = variance = $\lambda_t$

***Virtually no time series models support this distribution***

- Most analysts use `log` or `Box-Cox` transformation
- But see the `tscount` 📦

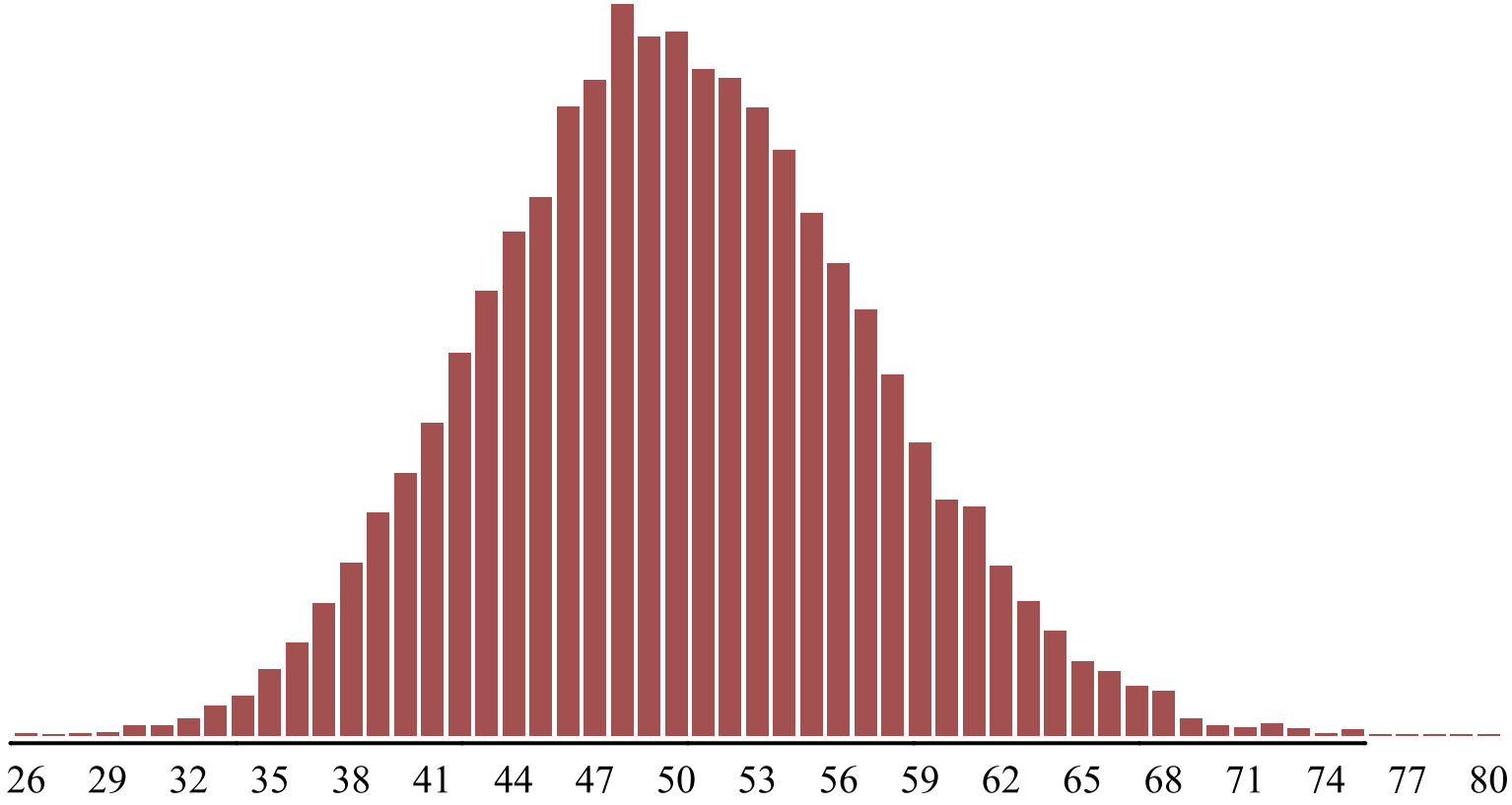
# Poisson

$$Y_t \sim \text{Poisson}(3)$$



# Poisson

$$Y_t \sim \text{Poisson}(50)$$



# How can we model non-Normal data using regression?

# Generalized linear models

Linear regression can't be trusted to give sensible predictions for non-negative count data (or other types of bounded / discrete / non-Normal data)

We can do better by choosing distributions that obey the constraints on our outcome variables

The idea is to ***generalize*** the linear regression by replacing parameters from other probability distributions with linear models

This requires a ***link function*** that transforms from the unbounded scale of the linear predictor to a scale that is appropriate for the parameters being modeled

# Modelling the mean

Most GLMs are used to model the conditional mean $(\mu_t)$

$$\mathbb{E}(\boldsymbol{Y_t}|\boldsymbol{X_t}) = \mu_t = g^{-1}(\alpha + \boldsymbol{X_t}\beta)$$

Where:

- $\mathbb{E}_t$ is the *expected value* of $\boldsymbol{Y_t}$ conditional on $\boldsymbol{X_t}$
- $g^{-1}$ is the *inverse* of the link function
- $\alpha$ is an intercept coefficient
- $\beta$ is a vector of regression coefficients

# Poisson GLM

A Poisson GLM models the conditional mean with a $log$ link

$$
\begin{aligned}
\boldsymbol{Y}_t &\sim \text{Poisson}(\lambda_t) \\
log(\lambda_t) &= \boldsymbol{X}_t\beta \\
&= \alpha + \beta_1\boldsymbol{x}_{1t} + \beta_2\boldsymbol{x}_{2t} + \cdots + \beta_j\boldsymbol{x}_{jt}
\end{aligned}
$$

Where:

- $\boldsymbol{X}_t$ is the matrix of predictor values at time $t$
- $\alpha$ is an intercept coefficient
- $\beta$ is a vector of regression coefficients
- $\mathbb{E}(\boldsymbol{Y}_t|\boldsymbol{X}_t) = exp(\alpha + \boldsymbol{X}_t\beta)$

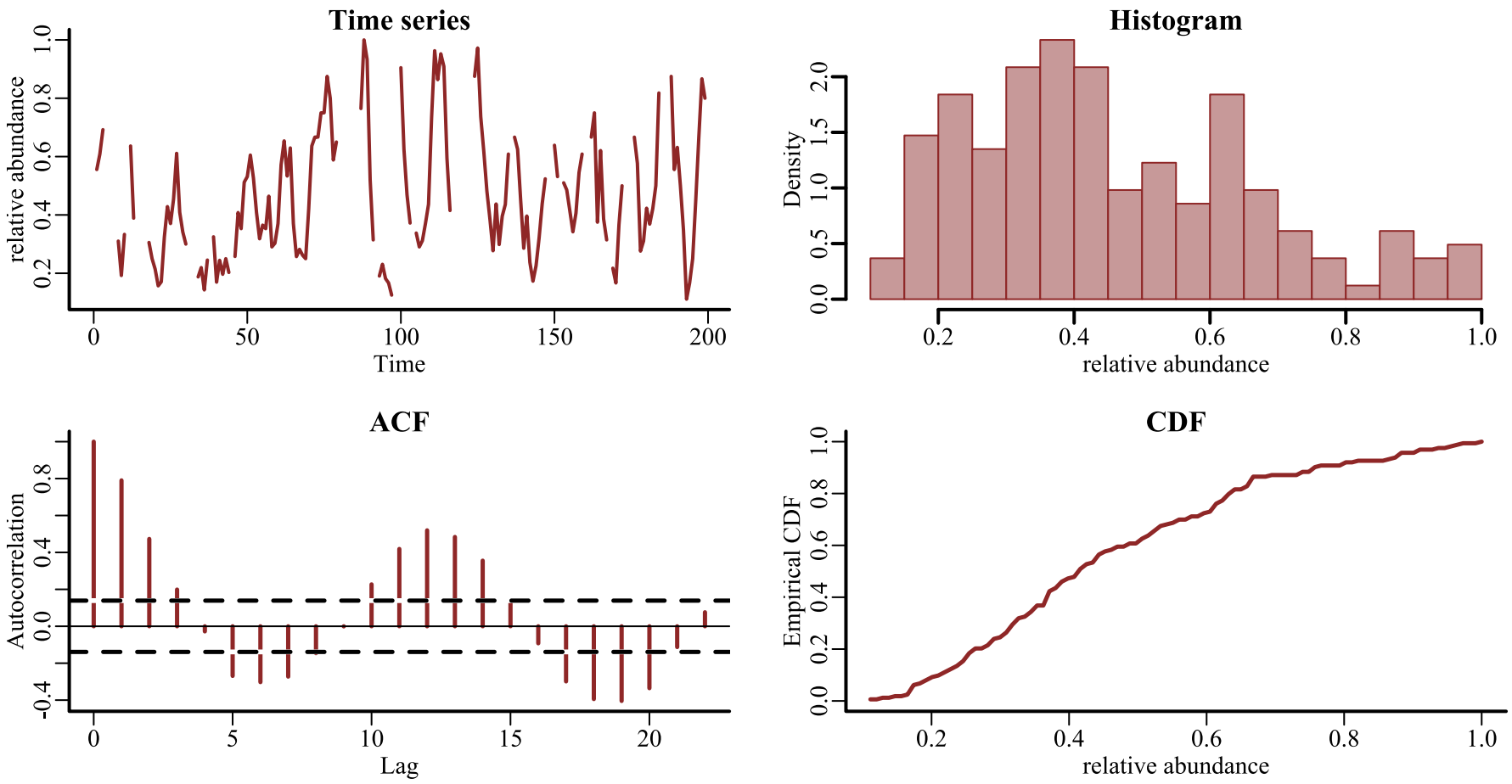
# Poisson GLM

A Poisson GLM models the conditional mean with a $log$ link

$$\begin{aligned} \boldsymbol{Y}_t &\sim \text{Poisson}(\lambda_t) \\ log(\lambda_t) &= \boldsymbol{X}_t\beta \\ &= \alpha + \beta_1\boldsymbol{x}_{1t} + \beta_2\boldsymbol{x}_{2t} + \cdots + \beta_j\boldsymbol{x}_{jt} \end{aligned}$$

The ***linear predictor component can be hugely flexible***, as we will see in later slides

# What if our data are proportional instead?




Properties of Merriam's kangaroo rat relative abundance time series from a long-term monitoring study in Portal, Arizona, USA

# Beta

$$\boldsymbol{Y_t} \sim \text{Beta}(\mu_t, \phi)$$

Properties

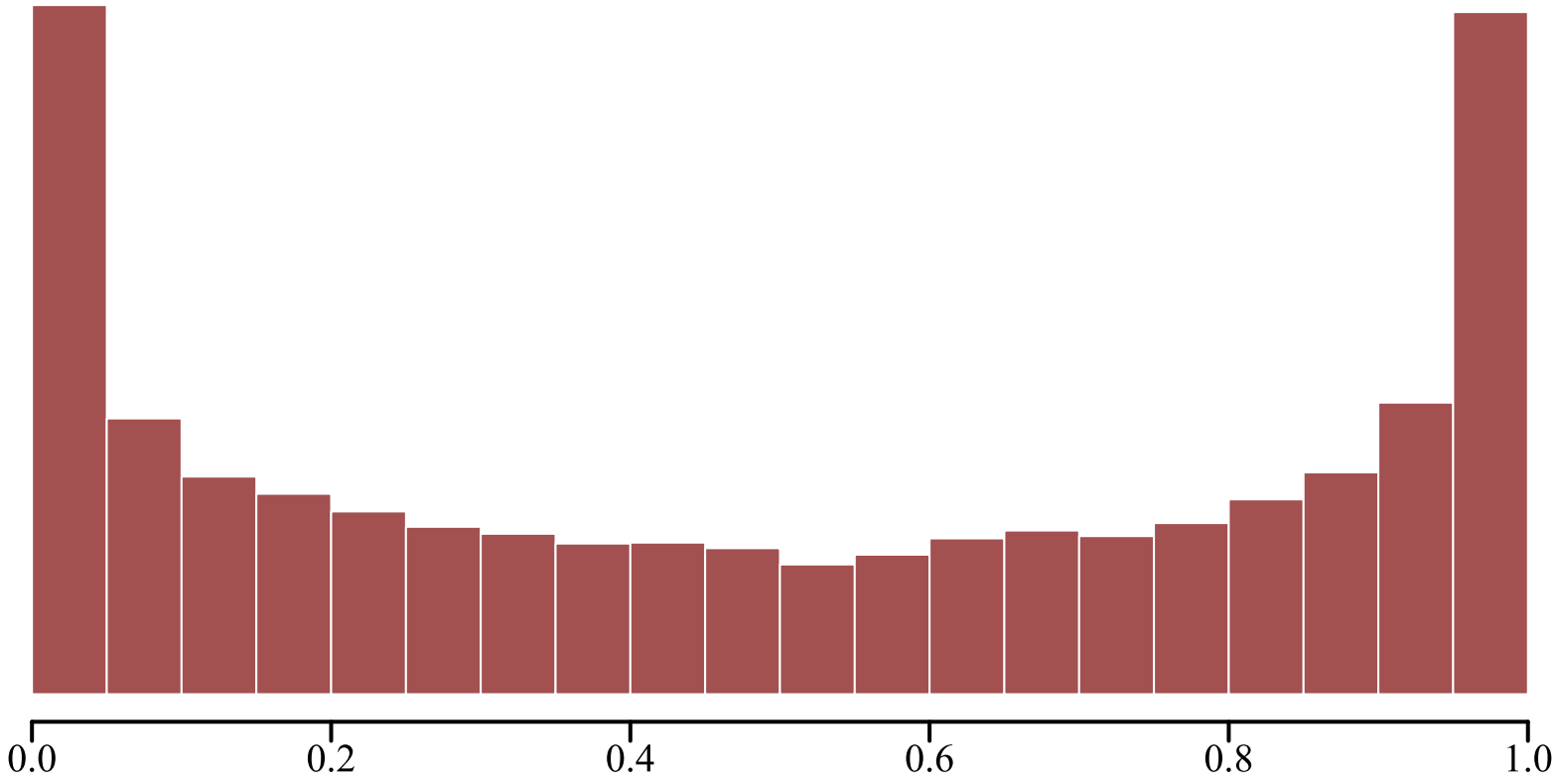
- Real-valued continuous observations (including any decimal)
- Both lower and upper bounds (supports $0$ to $1$, noninclusive)
- mean = $\mu_t$
- precision = $\phi$ (higher $\phi$ = a tighter spread about $\mu_t$)

***Virtually no time series models support this distribution***

- Most analysts would use `logit` transformation

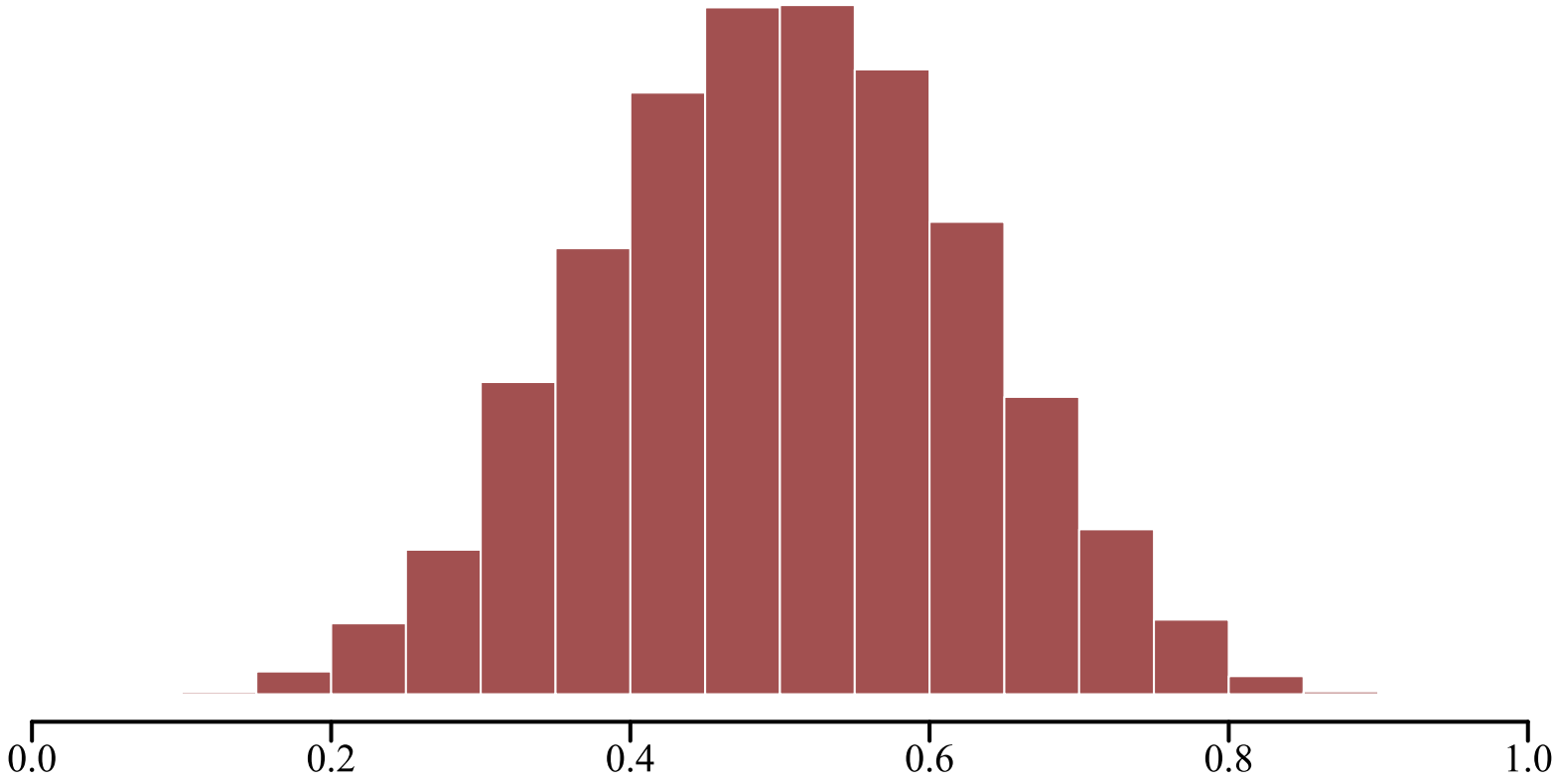
# Beta

$$\boldsymbol{Y_t} \sim \text{Beta}(0.5, 1)$$



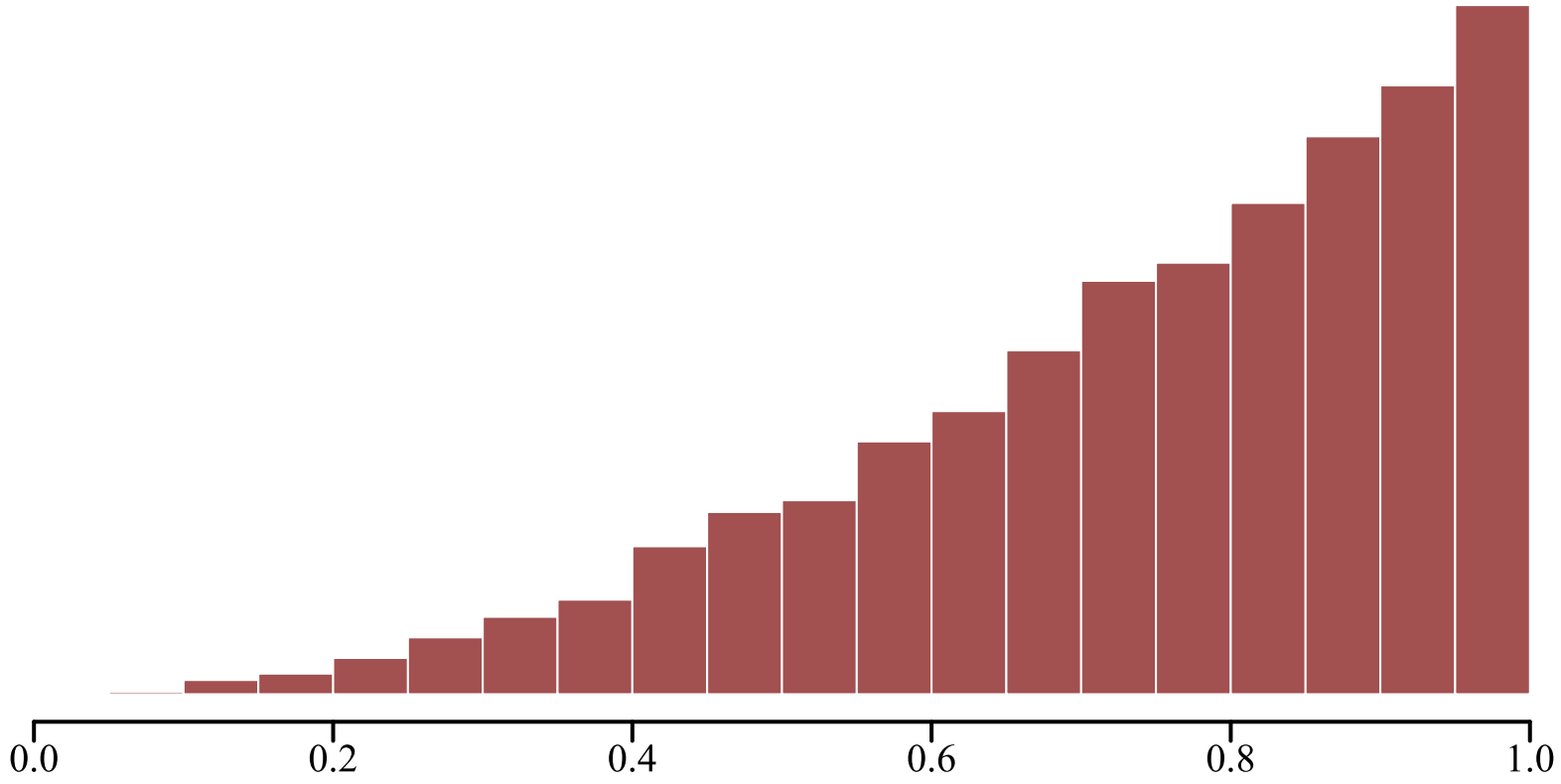
# Beta

$$\boldsymbol{Y_t} \sim \text{Beta}(0.5, 15)$$



# Beta

$$\boldsymbol{Y_t} \sim \text{Beta}(0.75, 4)$$



# Beta GLM

A Beta GLM models the conditional mean with a $logit$ link

$$
\begin{aligned}
\boldsymbol{Y}_t &\sim \text{Beta}(\mu_t, \phi) \\
logit(\mu_t) &= \boldsymbol{X}_t\beta \\
&= \alpha + \beta_1\boldsymbol{x}_{1t} + \beta_2\boldsymbol{x}_{2t} + \cdots + \beta_j\boldsymbol{x}_{jt}
\end{aligned}
$$

Where:

- $\boldsymbol{X}_t$ is the matrix of predictor values at time $t$
- $\alpha$ is an intercept coefficient
- $\beta$ is a vector of regression coefficients
- $\mathbb{E}(\boldsymbol{Y}_t|\boldsymbol{X}_t) = logit^{-1}(\alpha + \boldsymbol{X}_t\beta)$

# Some other relevant distributions

Many other useful GLM probability distributions exist. Some of these include:

***Negative Binomial*** – overdispersed integers in $(0, 1, 2, \dots)$

***Bernoulli*** – presence-absence data in $\{0, 1\}$

***Student's T*** – heavy-tailed (skewed) real values in $(-\infty, \infty)$

***Lognormal*** – heavy-tailed (right skewed) real values in $(0, \infty)$

***Gamma*** – lighter-tailed (less skewed) real values in $(0, \infty)$

***Multinomial*** – integers representing $K$ unordered categories in $(0, 1, \dots, K)$

***Ordinal*** – integers representing $K$ ordered categories in $(0, 1, \dots, K)$

**GLMs allow us to build models that respect the bounds and distributions of our observed data**

**They traditionally assume the appropriately transformed mean response depends *linearly* on the predictors**

**But there are many other properties we'd like to model**

# Remember these?

Temporal autocorrelation

Lagged effects

***Non-Gaussian data and missing observations***

***Measurement error***

***Time-varying effects***

***Nonlinearities***

***Multi-series clustering***

# Remember these?

Temporal autocorrelation

Lagged effects

Non-Gaussian data and missing observations

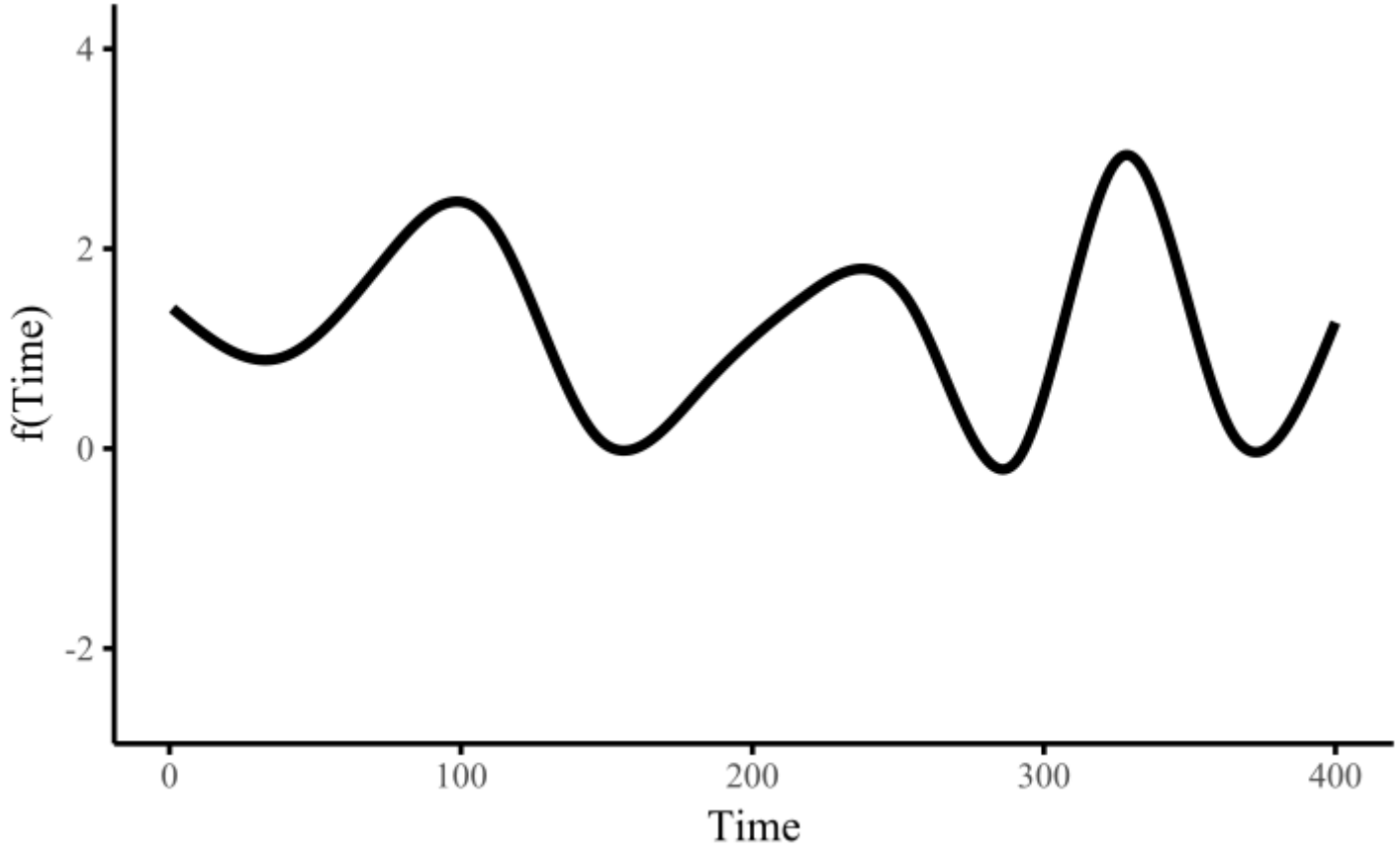
Measurement error

Time-varying effects

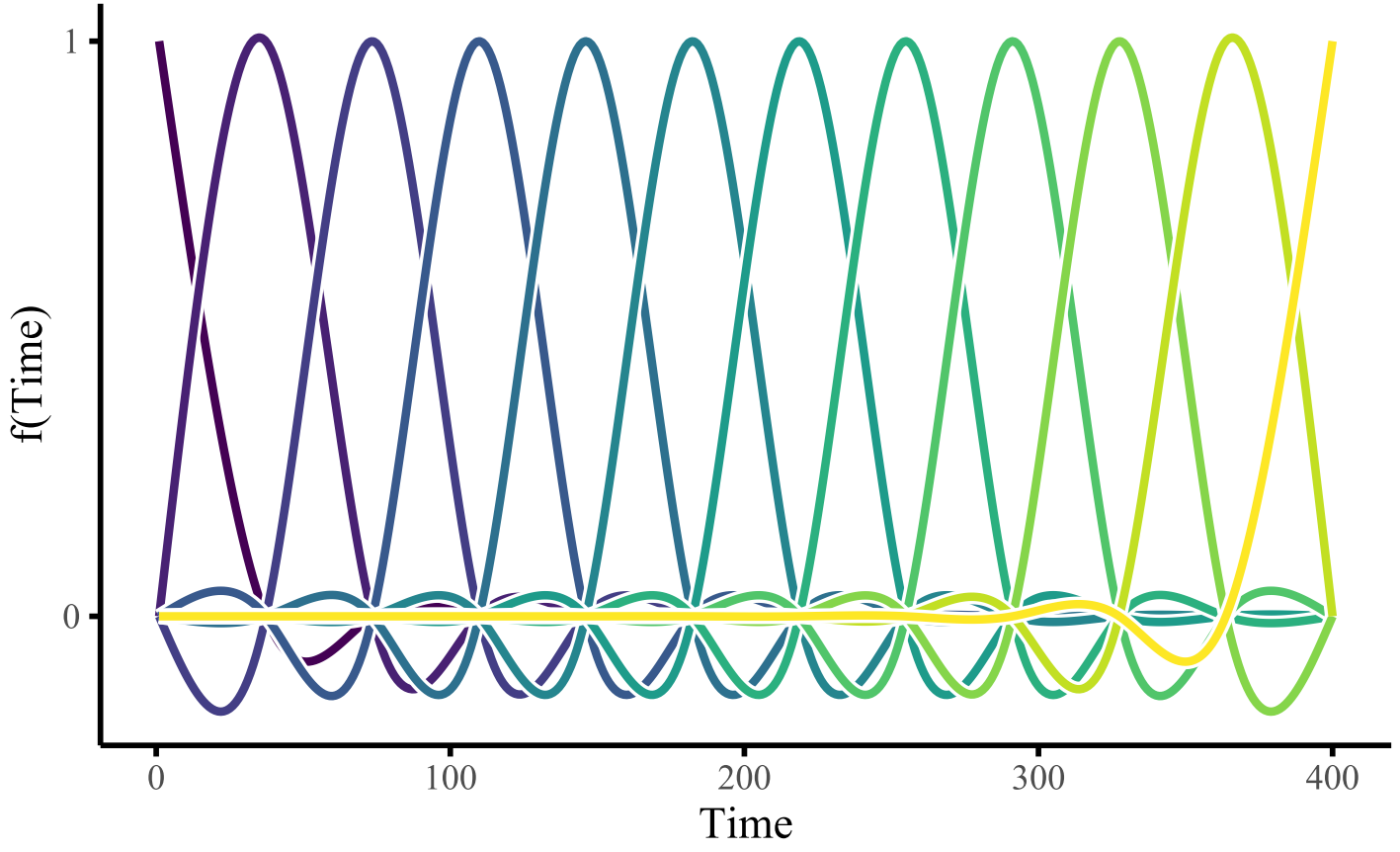
***Nonlinearities***

Multi-series clustering

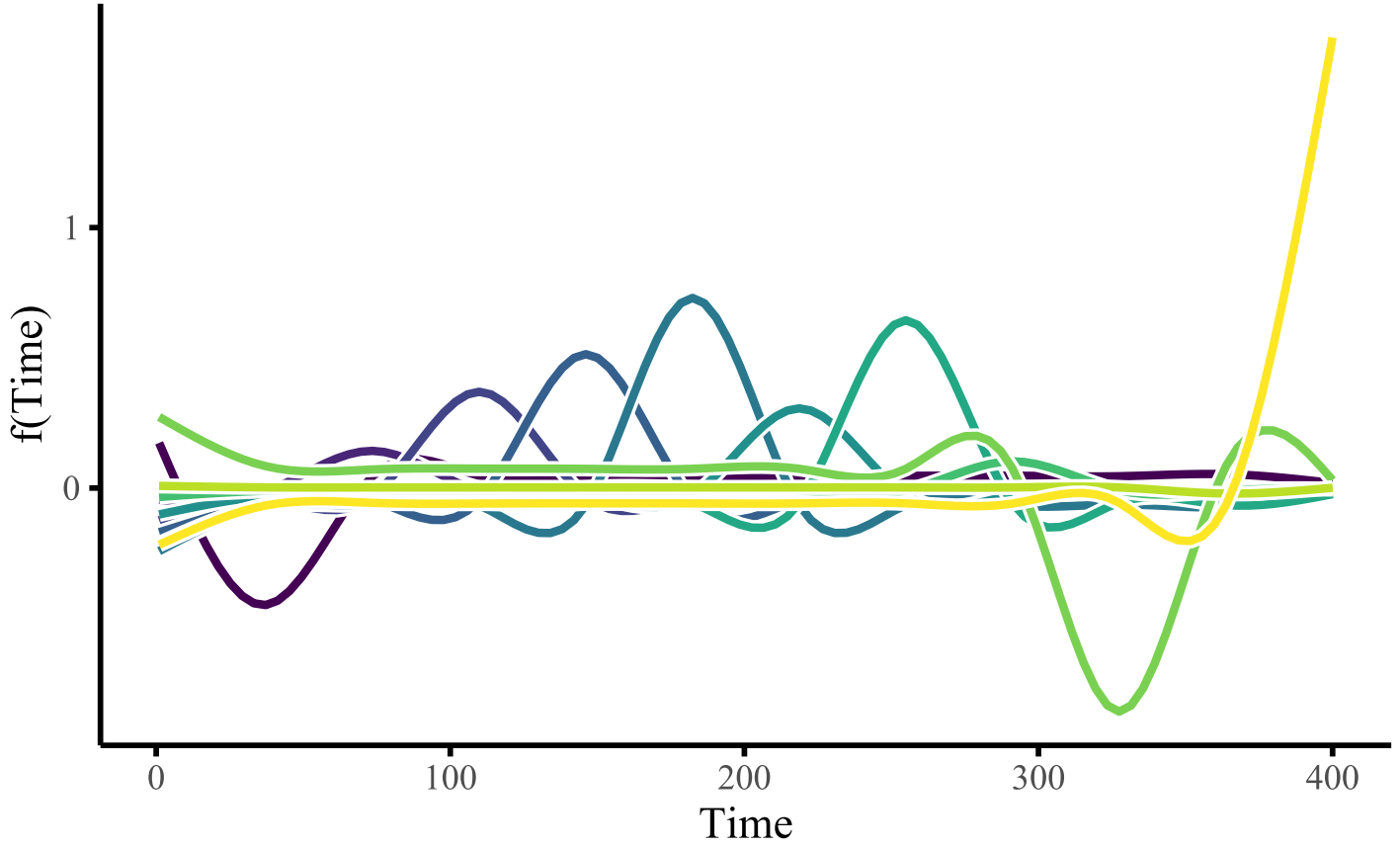
# GAMs use splines ...



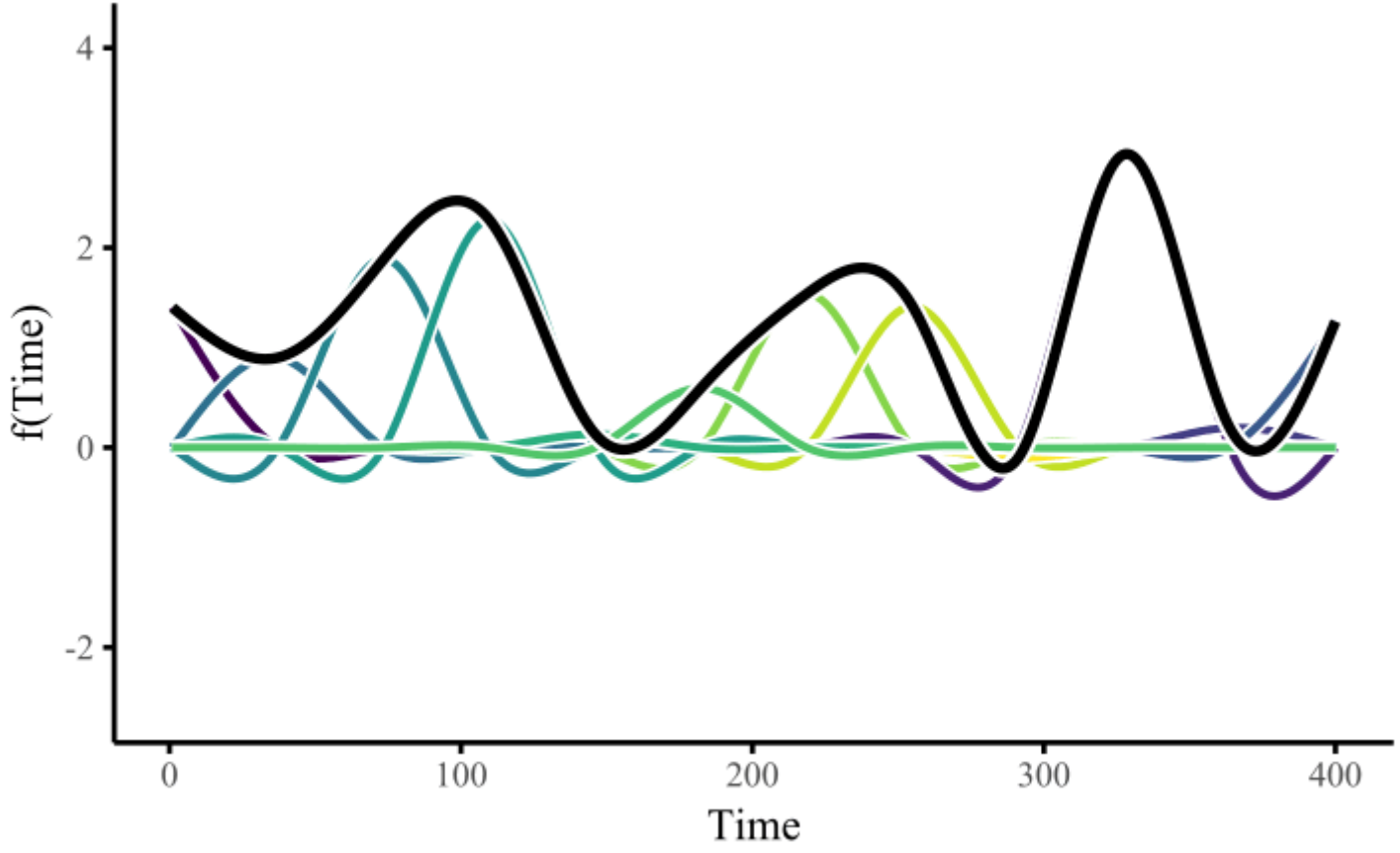
# ... made of basis functions



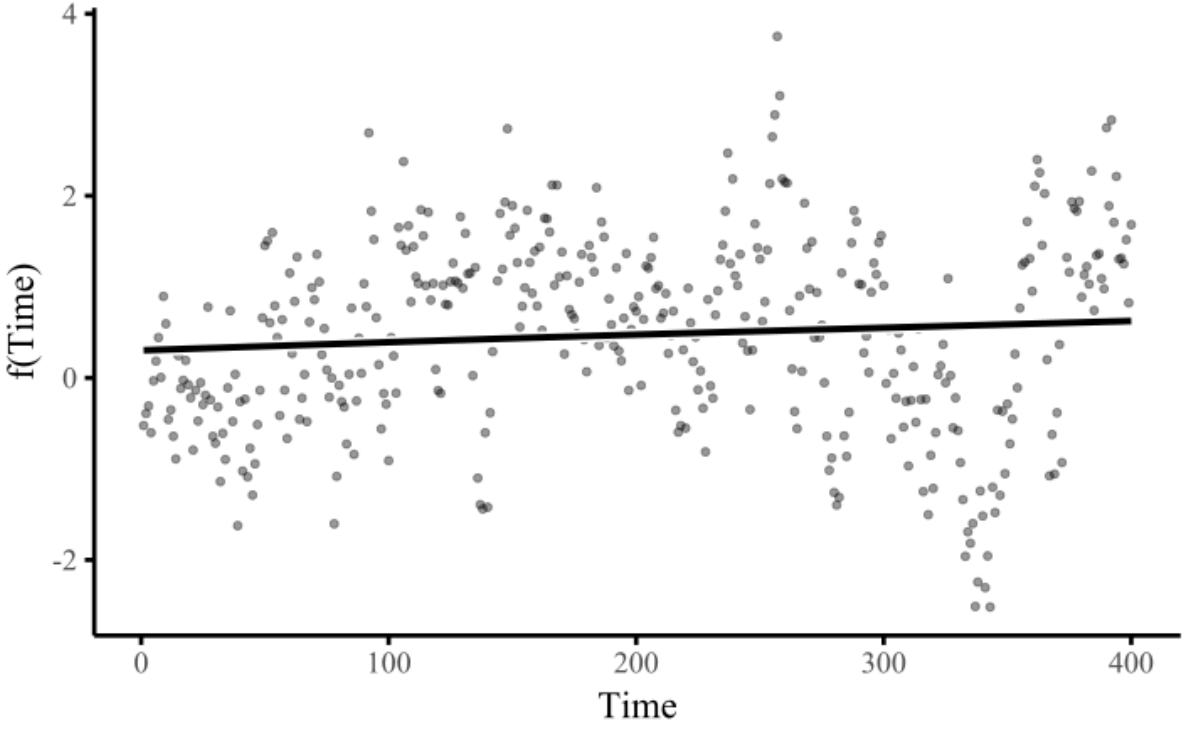
# Weighting basis functions ...

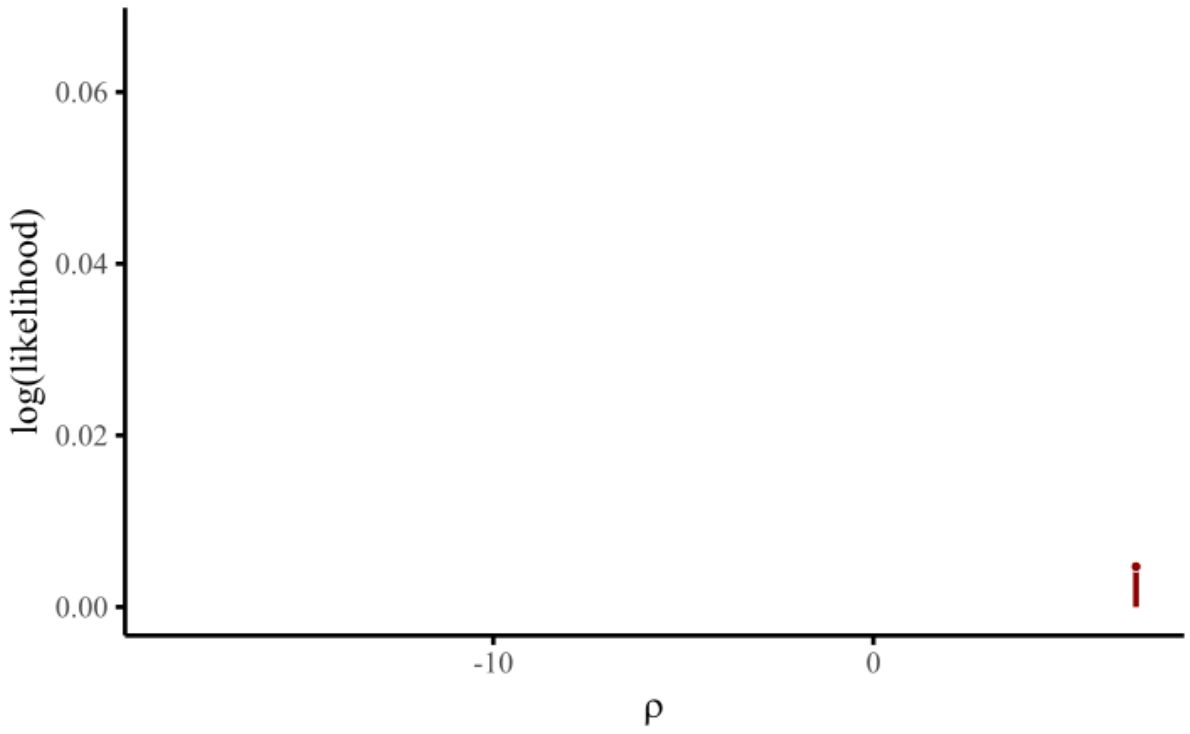


# ... gives a spline $(f(x))$

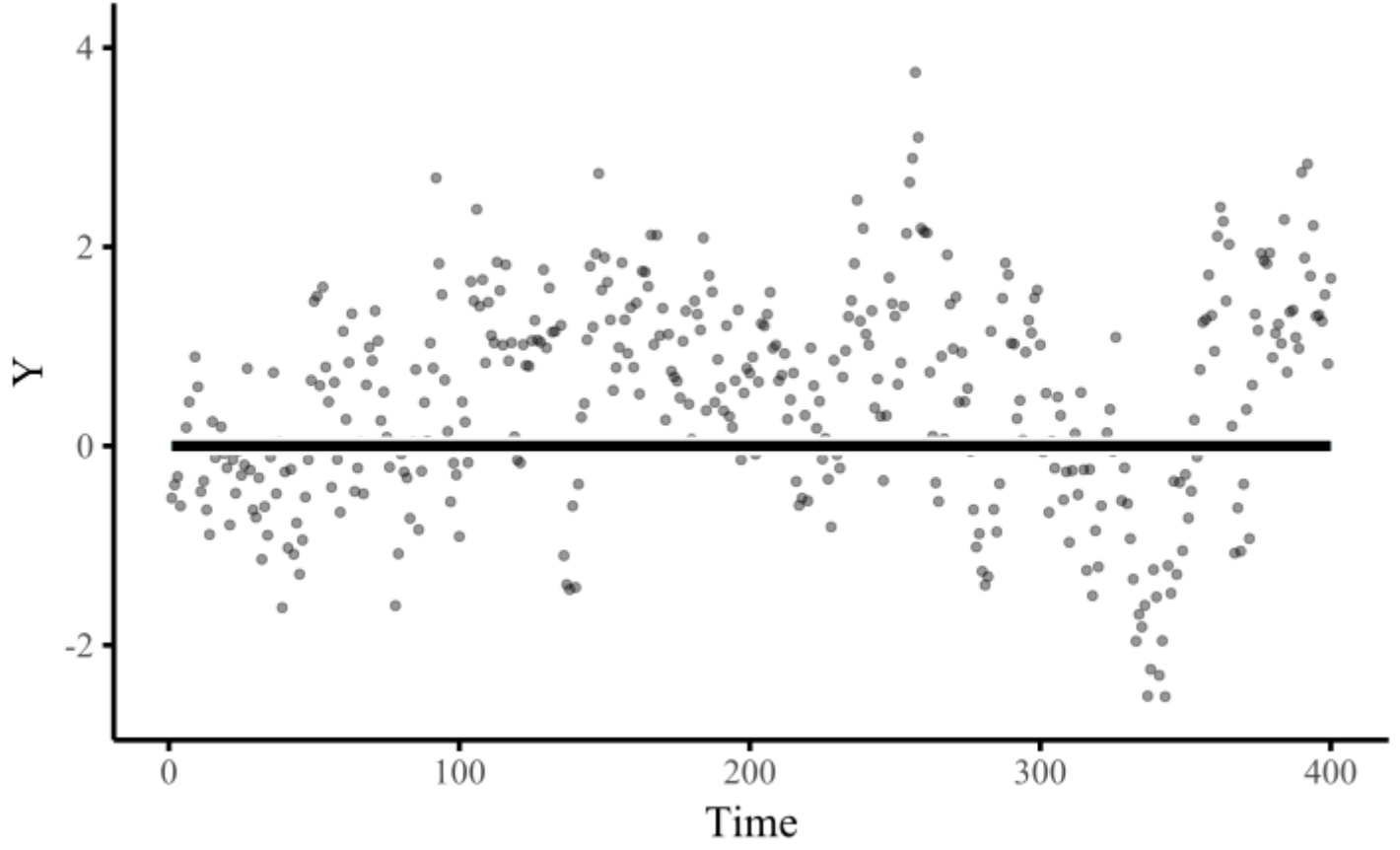


# Penalize $f''(x)$ to learn weights



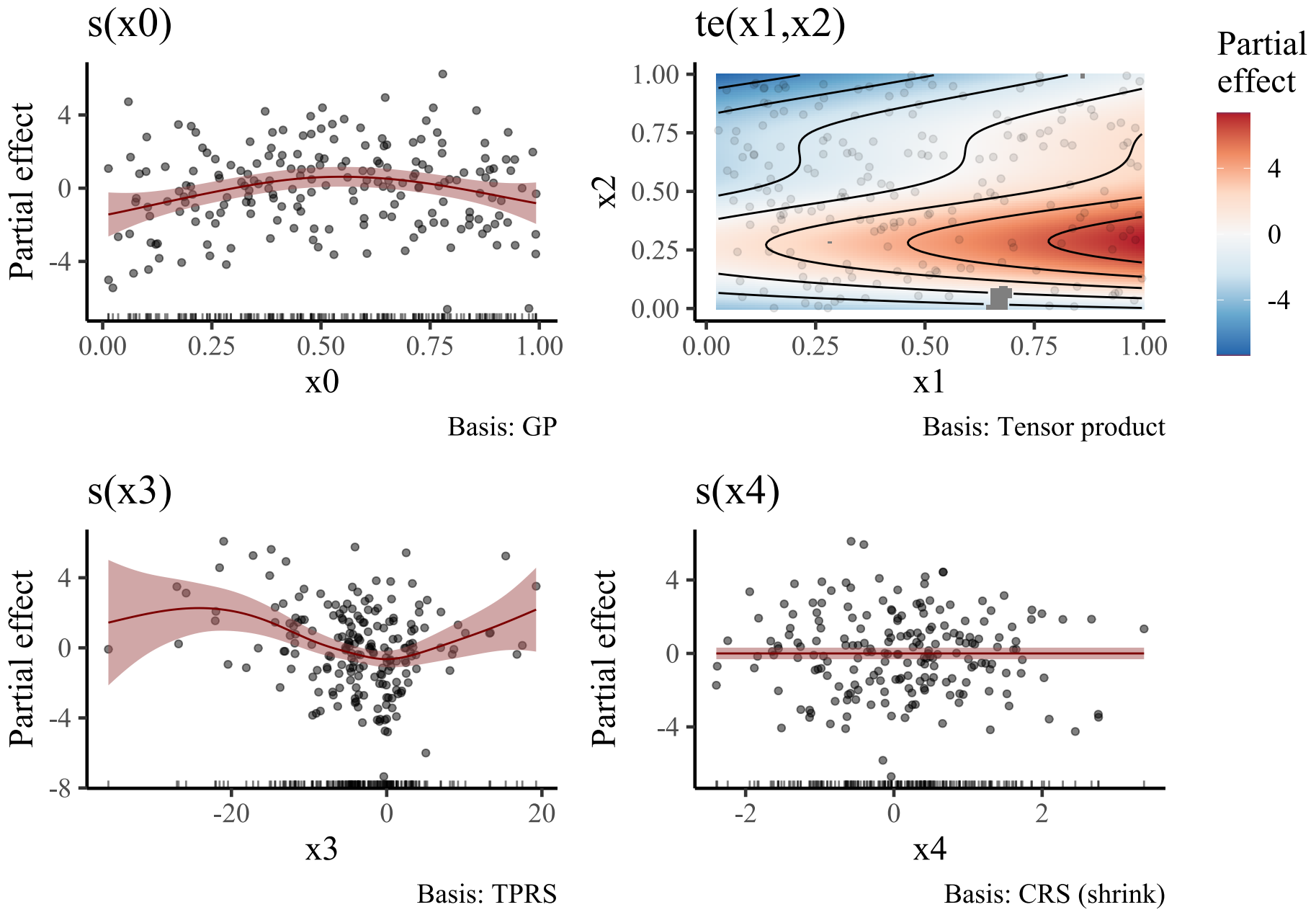



# Penalize $f''(x)$ to learn weights



**GAMs are just fancy GLMs, where some (or all) of the predictor effects are estimated as (possibly nonlinear) smooth functions**

**But the complexity they can handle is *enormous***

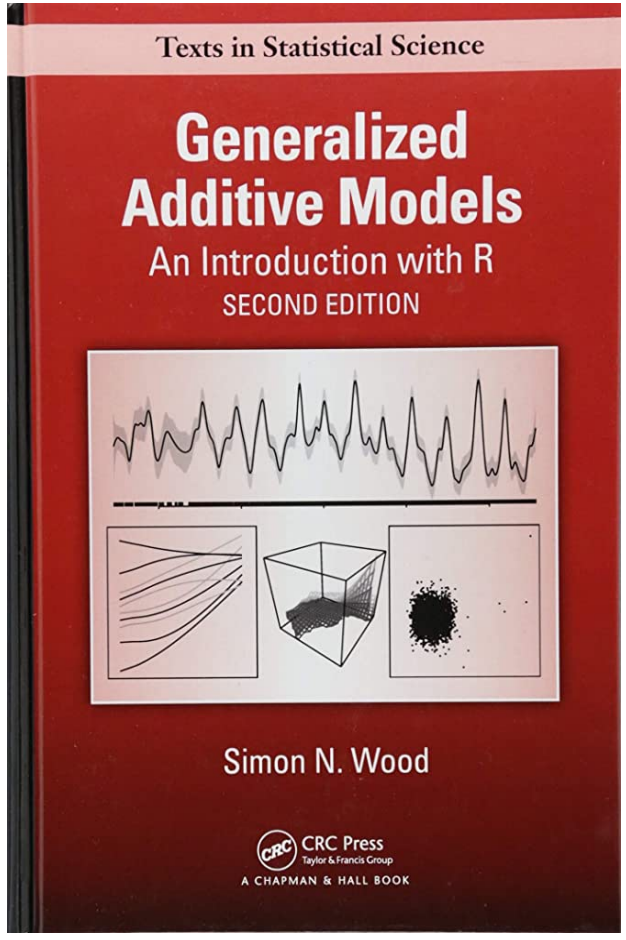

s(x0)
Partial effect
4
0
-4
0.00
0.25
0.50
0.75
1.00
x0
Basis: GP
te(x1,x2)
x2
1.00
0.75
0.50
0.25
0.00
x1
Basis: Tensor product
Partial effect
4
0
-4
s(x3)
Partial effect
-8
-20
20
x3
Basis: TPRS
s(x4)
Partial effect
-2
2
x4
Basis: CRS (shrink)

# GAMs easy to fit in R

$$\mathbb{E}(\boldsymbol{Y_t}|\boldsymbol{X_t}) = g^{-1}(\alpha + \sum_{j=1}^{J} f(x_{jt}))$$

Where:

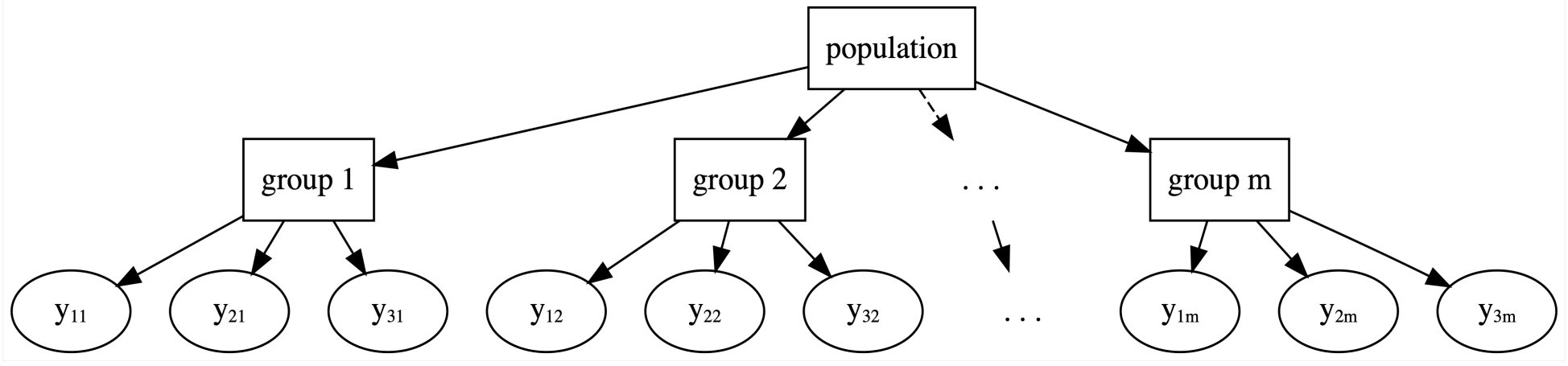
- $g^{-1}$ is the *inverse* of the link function
- $\alpha$ is the intercept
- $f(x)$ are potentially nonlinear functions of the $J$ predictors



# But how can GAMs and GLMs be useful for modelling ecological time series?

# Temporal random effects

# Random effects are *hierarchical*



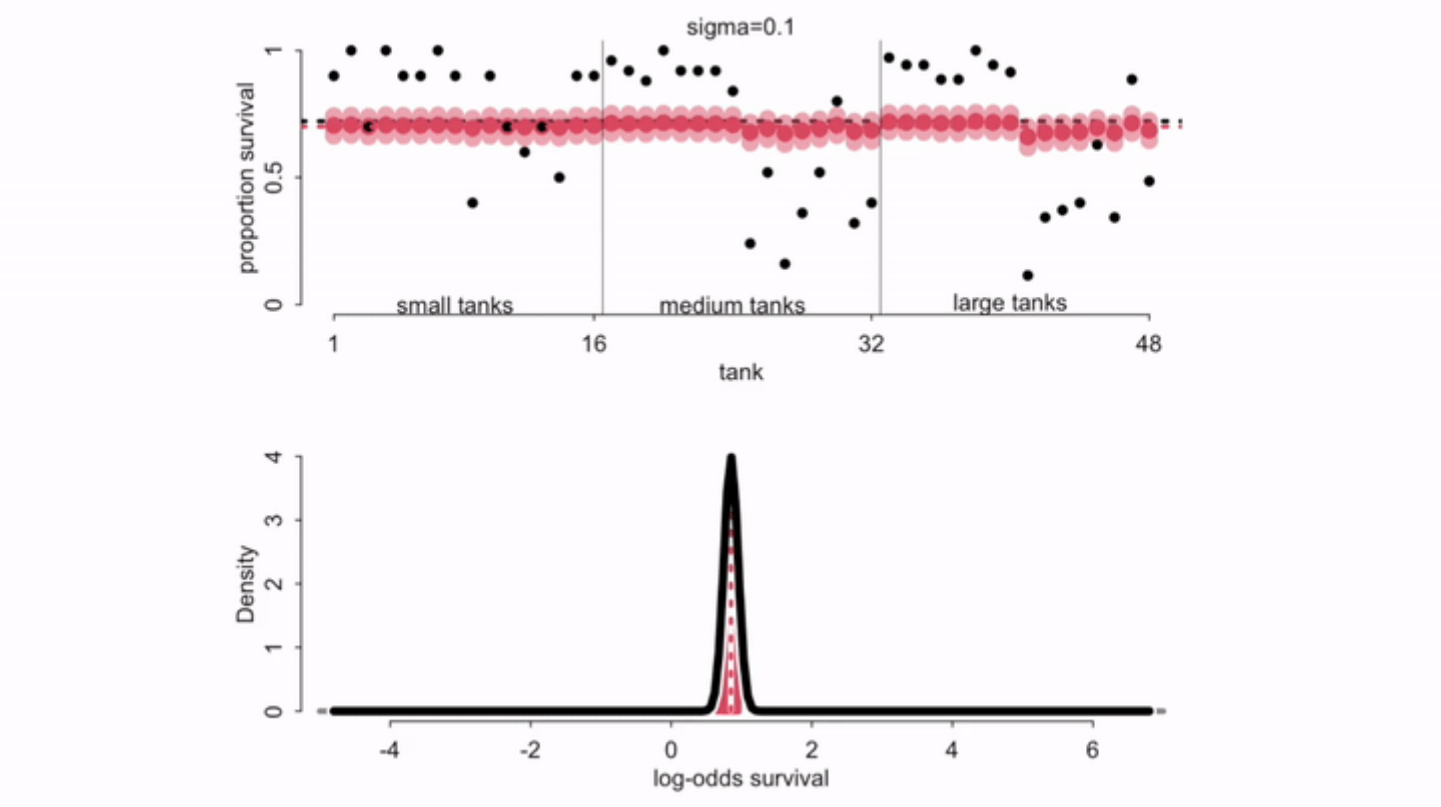

Johnson *et al* 2021

**Hierarchical models *learn from all groups at once* to inform group-level estimates**

**This induces *regularization*, where noisy estimates are pulled towards the overall mean**

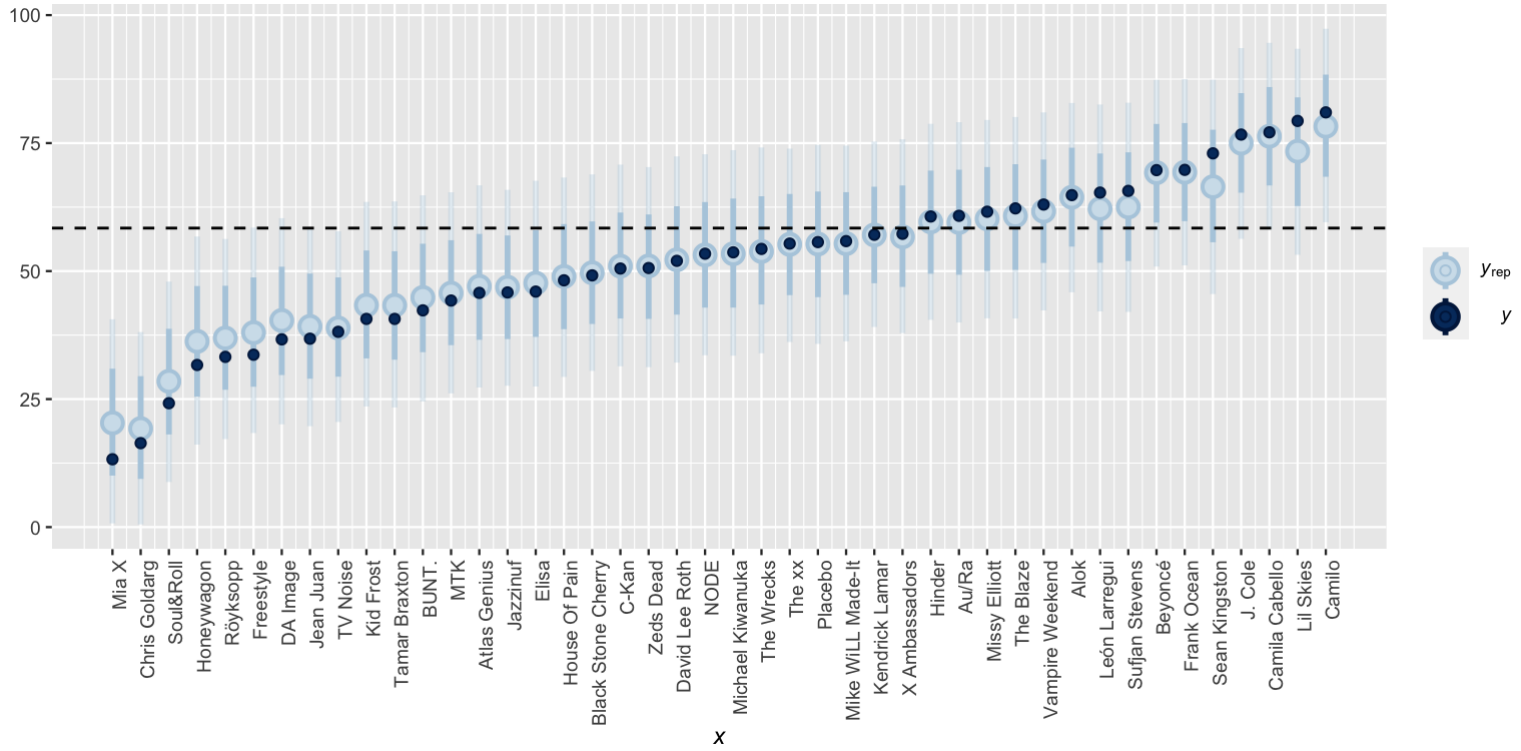
**The regularization is known as partial pooling**

# Partial pooling in action




McElreath 2023

# Noisy estimates *pulled* to the mean




Johnson *et al* 2021

# How can they be modelled?

$$
\begin{aligned}
\boldsymbol{Y}_t &\sim \text{Poisson}(\lambda_t) \\
log(\lambda_t) &= \beta_{year[year_t]} \\
\beta_{year} &\sim \text{Normal}(\mu_{year}, \sigma_{year}) \\
\mu_{year} &\sim \text{Normal}(0, 1) \\
\sigma_{year} &\sim \text{Exponential}(2)
\end{aligned}
$$

Where we have multiple time points per year, and:

- $\beta_{year}$ are yearly intercepts (*one effect per year*)
- $\mu_{year}$ estimates *mean effect among all years*
- $\sigma_{year}$ estimates *how much effects vary across years*

# Modelling with the `mvgam` 📦

Bayesian framework to fit Dynamic GLMs and Dynamic GAMs

- Hierarchical intercepts, slopes *and smooths*
- Latent dynamic processes
- State Space models with measurement error

Built off the `mgcv` 📦 to construct penalized smoothing splines

Convenient and familiar R formula interface

Uni- or multivariate series from a range of response distributions

Uses Stan for efficient Hamiltonian Monte Carlo sampling

# Example of the interface

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

Where `y` = response, `x`'s = covariates, and `series` = a grouping term

# Typical formula syntax

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# A random intercept effect

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# A random slope effect

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# A linear parametric effect

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# A one-dimensional smooth

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# A two-dimensional smooth

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# Data and response distribution

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# latent dynamics

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# Sampler parameters

```
model ← mvgam(
  formula = y ~
    s(series, bs = 're') +
    s(x0, series, bs = 're') +
    x1 +
    s(x2, bs = 'tp', k = 5) +
    te(x3, x4, bs = c('cr', 'tp')),
  data = data,
  family = poisson(),
  trend_model = 'AR1',
  burnin = 500,
  samples = 500,
  chains = 4,
  parallel = TRUE
  )
```

# Example data (long format)

| y | series | time |
|---|---|---|
| 1 | species_1 | 1 |
| 1 | species_2 | 1 |
| NA | species_3 | 1 |
| NA | species_4 | 1 |
| 1 | species_1 | 2 |
| 0 | species_2 | 2 |
| 3 | species_3 | 2 |
| 4 | species_4 | 2 |

# Response (NAs allowed)

| y | series | time |
|---|---|---|
| 1 | species_1 | 1 |
| 1 | species_2 | 1 |
| NA | species_3 | 1 |
| NA | species_4 | 1 |
| 1 | species_1 | 2 |
| 0 | species_2 | 2 |
| 3 | species_3 | 2 |
| 4 | species_4 | 2 |

# Series indicator (as `factor`)

| y | series | time |
|---|---|---|
| 1 | species_1 | 1 |
| 1 | species_2 | 1 |
| NA | species_3 | 1 |
| NA | species_4 | 1 |
| 1 | species_1 | 2 |
| 0 | species_2 | 2 |
| 3 | species_3 | 2 |
| 4 | species_4 | 2 |

# Time indicator

| y | series | time |
|---|---|---|
| 1 | species_1 | 1 |
| 1 | species_2 | 1 |
| NA | species_3 | 1 |
| NA | species_4 | 1 |
| 1 | species_1 | 2 |
| 0 | species_2 | 2 |
| 3 | species_3 | 2 |
| 4 | species_4 | 2 |

# Any other predictors

| y | series | time | x0 | x1 | x2 | x3 | x4 |
|---|---|---|---|---|---|---|---|
| 1 | species_1 | 1 | -0.38 | A | 0.20 | 1.18 | -0.72 |
| 1 | species_2 | 1 | -0.71 | A | -2.67 | 1.02 | 0.67 |
| NA | species_3 | 1 | 0.05 | B | -0.33 | 0.12 | 1.50 |
| NA | species_4 | 1 | 0.77 | B | 0.65 | 0.86 | -0.49 |
| 1 | species_1 | 2 | 0.29 | A | -0.25 | 1.18 | -0.82 |
| 0 | species_2 | 2 | 0.34 | A | -0.15 | 2.12 | 0.20 |
| 3 | species_3 | 2 | -0.38 | B | -0.81 | 1.33 | -1.15 |
| 4 | species_4 | 2 | 1.32 | B | 0.22 | -0.72 | 1.36 |

# Examples

# The data structure

```
dplyr::glimpse(model_data)
```

```
## Rows: 199
## Columns: 6
## $ series  <fct> PP, PP, PP, PP, PP, PP, PP, PP, PP, PP, PP, PP, PP, PP, PP,
PP…
## $ year    <fct> 2004, 2004, 2004, 2004, 2004, 2004, 2004, 2004, 2004, 2004,
20…
## $ time    <dbl> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
18,…
## $ count   <int> 0, 1, 2, NA, 10, NA, NA, 16, 18, 12, NA, 3, 2, NA, NA, 13,
NA,…
## $ mintemp <dbl> -9.710, -5.924, -0.220, 1.931, 6.568, 11.590, 14.370,
16.520, …
## $ ndvi    <dbl> 1.4658889, 1.5585069, 1.3378172, 1.6589129, 1.8536561,
```

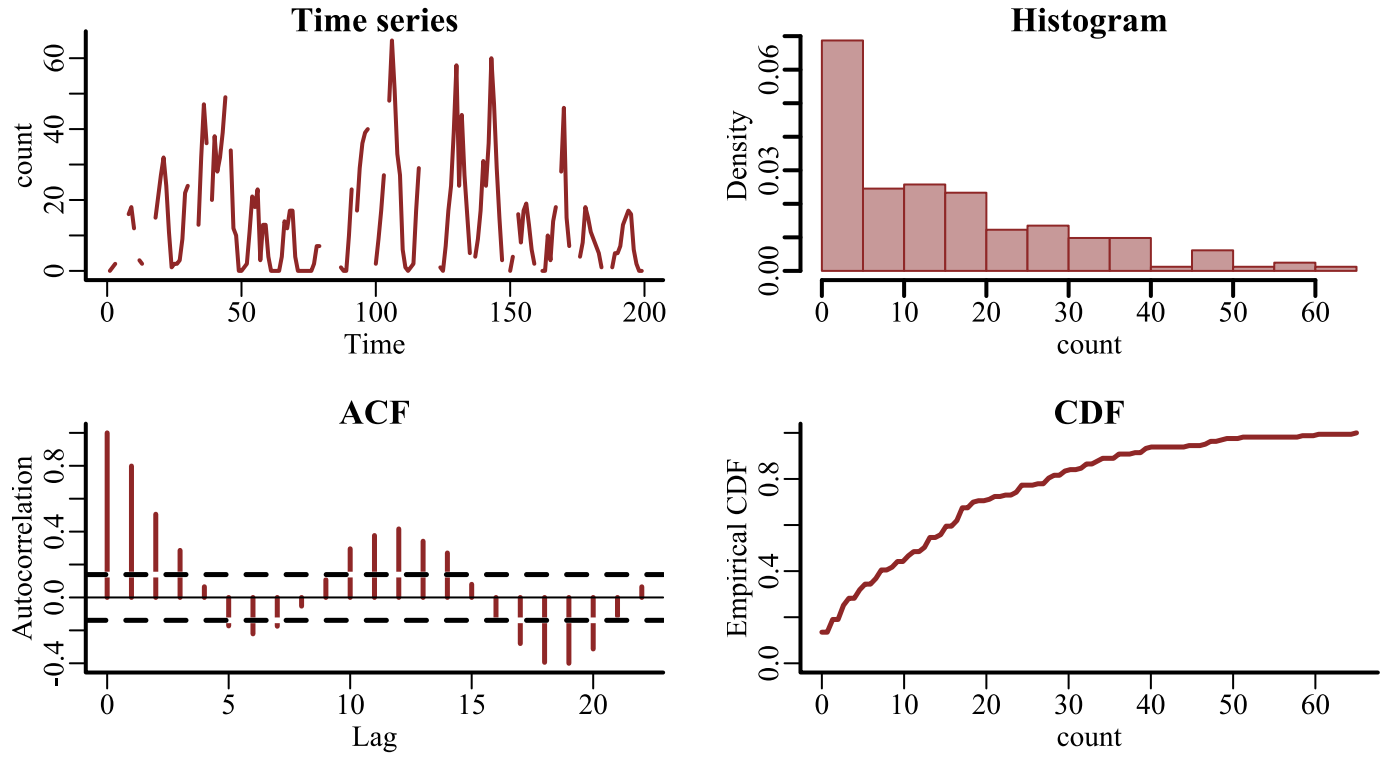
# The observations

Code Plot

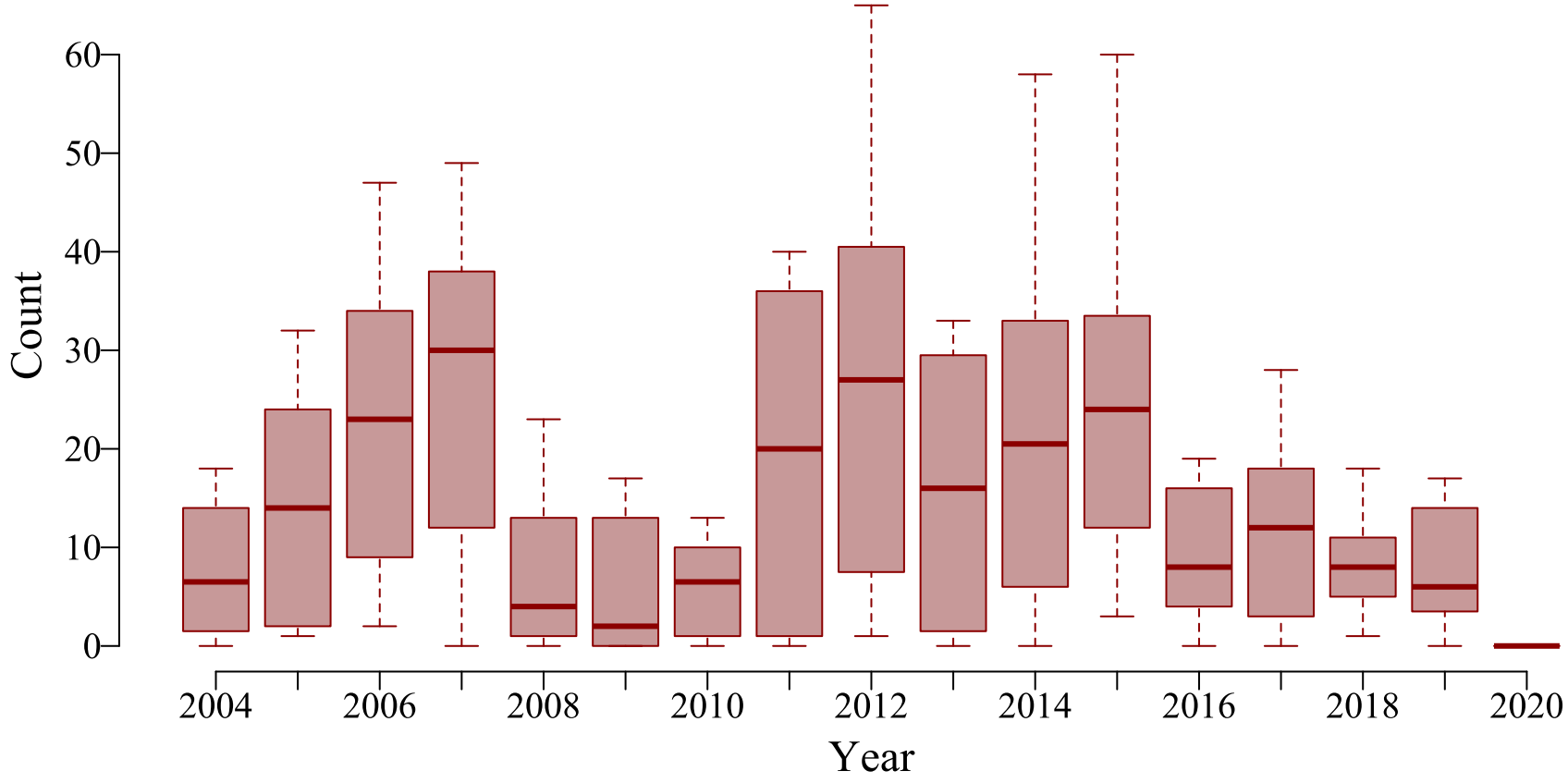
```
# use mvgam's plot utility to view properties of the observations
plot_mvgam_series(data = model_data, y = 'count')
```

# The observations

Code Plot



# Yearly heterogeneity



# Yearly random intercepts

```
year_random ← mvgam(count ~
                      s(year, bs = 're') - 1,
                    family = poisson(),
                    data = model_data,
                    trend_model = 'None',
                    burnin = 500,
                    samples = 500,
                    chains = 4)
```

Random effect basis in `mgcv` language

Global intercept suppressed

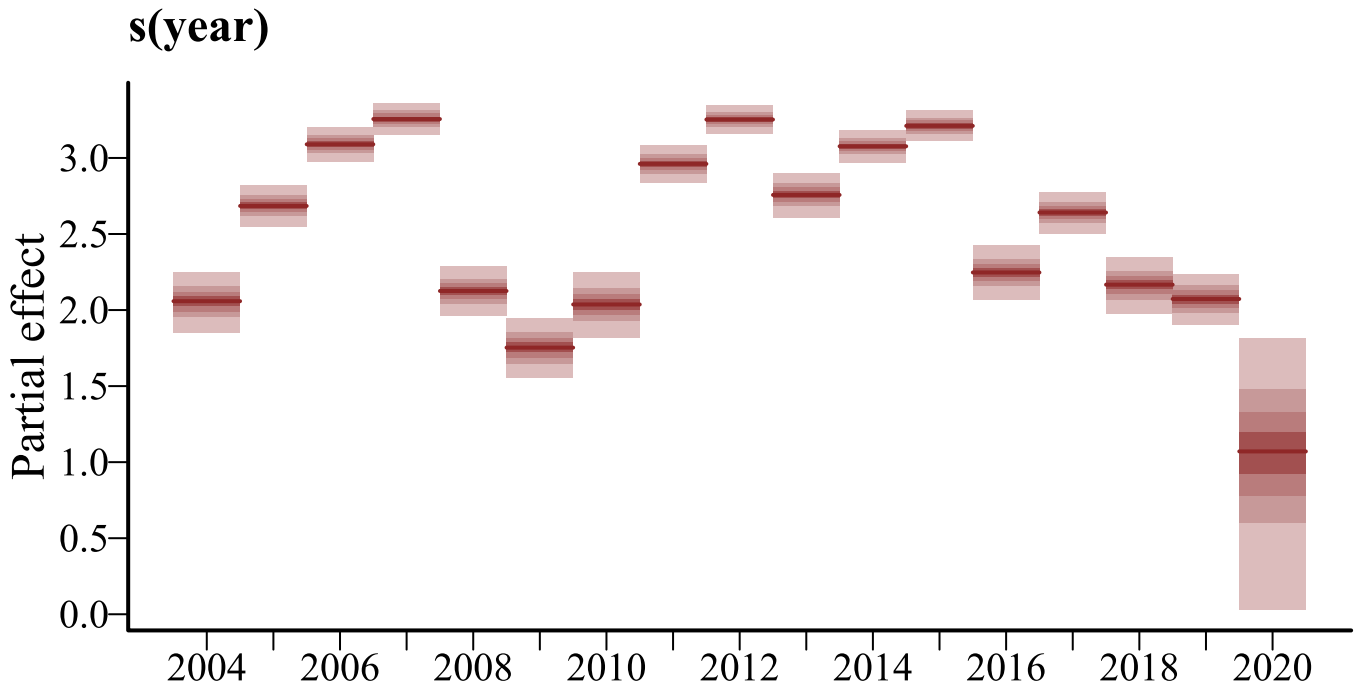
# Estimated yearly intercepts

Code Plot

```
# plot the random effect posterior estimates
plot(year_random, type = 're')
```

# Estimated yearly intercepts

Code Plot



# Population parameters

Code Means SDs

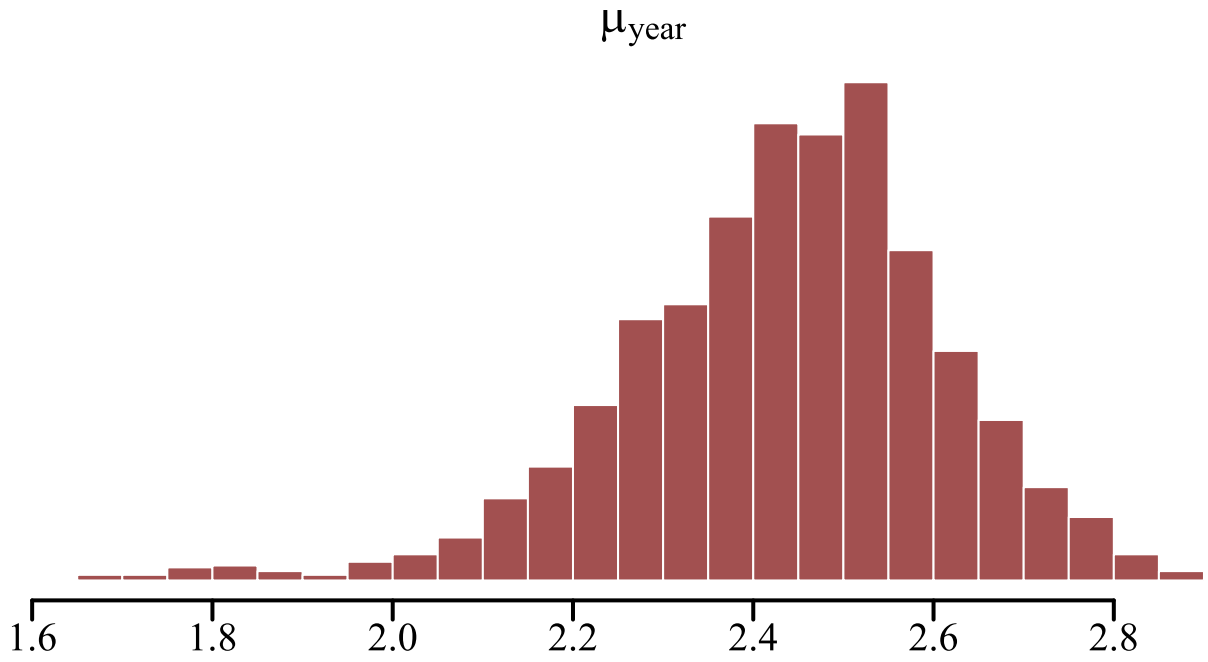
```
# extract population estimates
pop_params ← as.data.frame(year_random,
                           variable = c('mean(year)',
                                        'sd(year)'))

# plot as histograms
hist(pop_params$`mean(year)`, main = expression(mu[year]))

hist(pop_params$`sd(year)`, main = expression(sigma[year]))
```

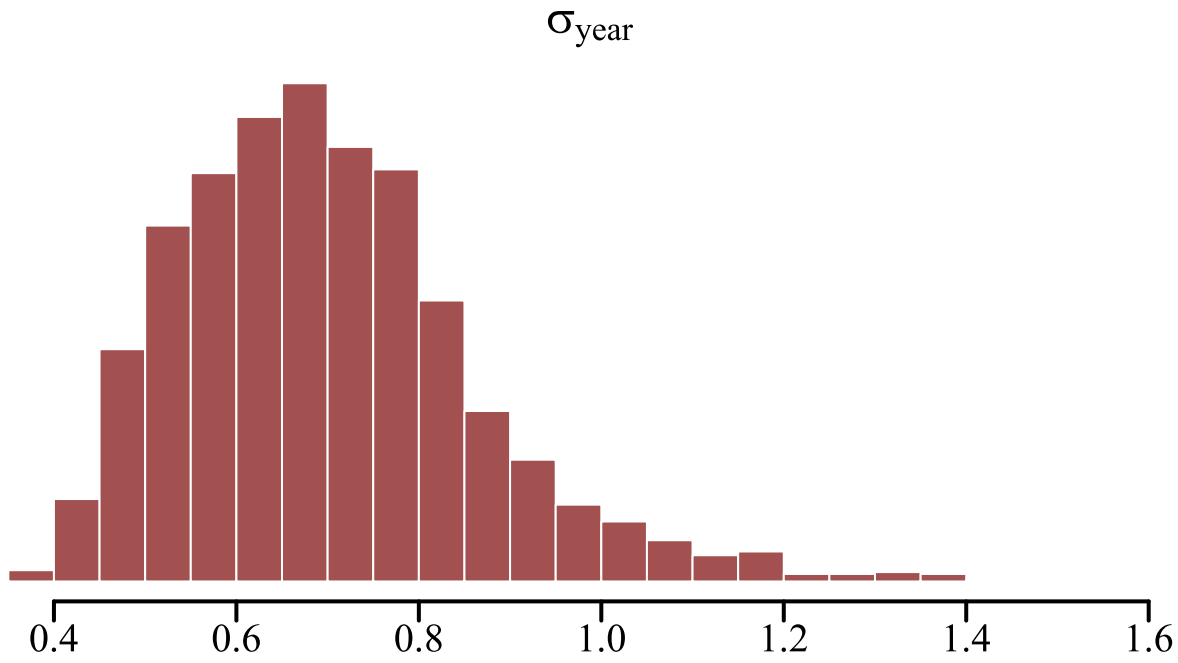
# Population parameters

Code | Means | SDs



# Population parameters

Code Means SDs



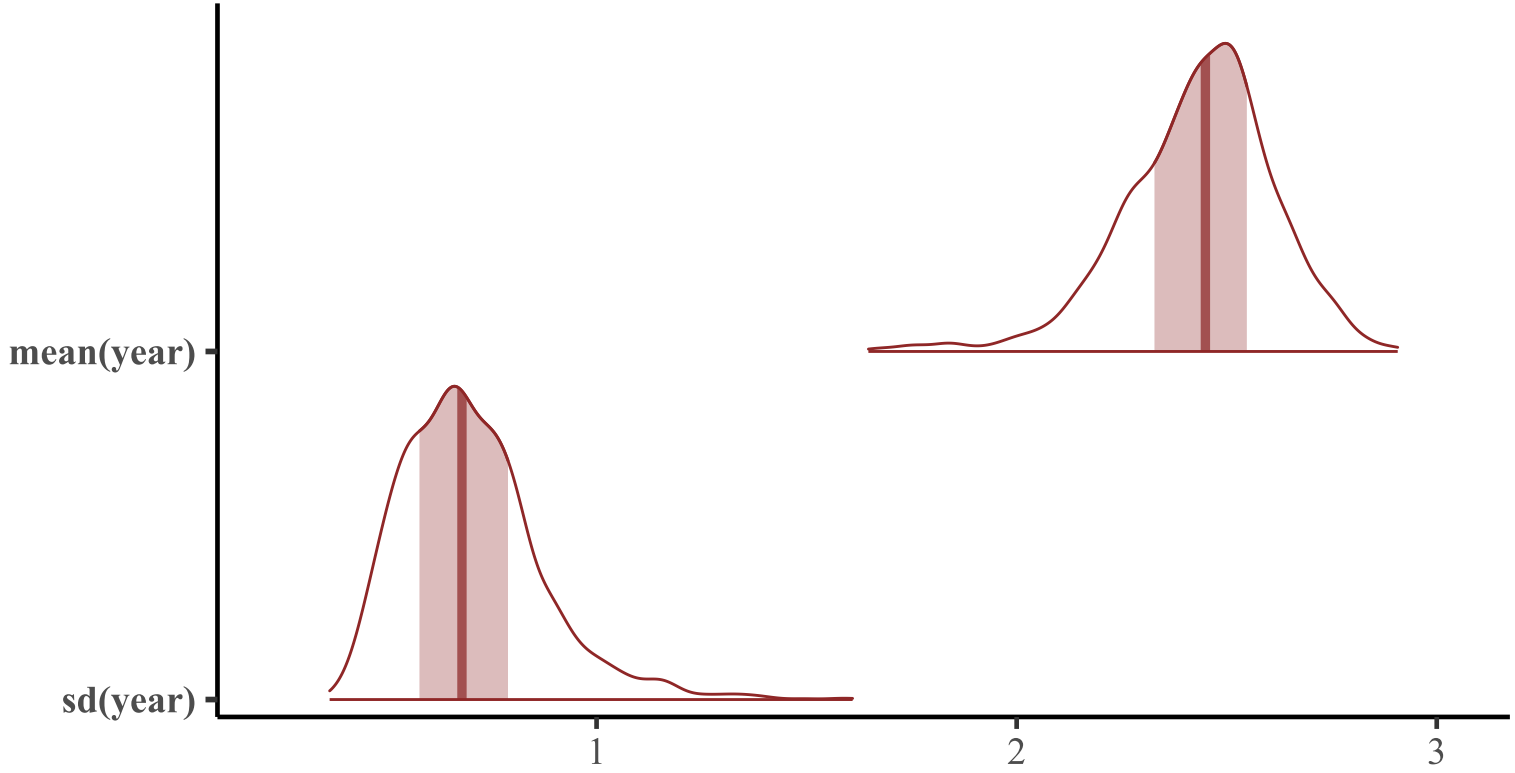
# Or using bayesplot

Code Plot

```
# use bayesplot utilities to plot parameter estimates
mcmc_plot(year_random, type = 'areas',
          variable = c('mean(year)', 'sd(year)'))
```

# Or using bayesplot

Code | Plot



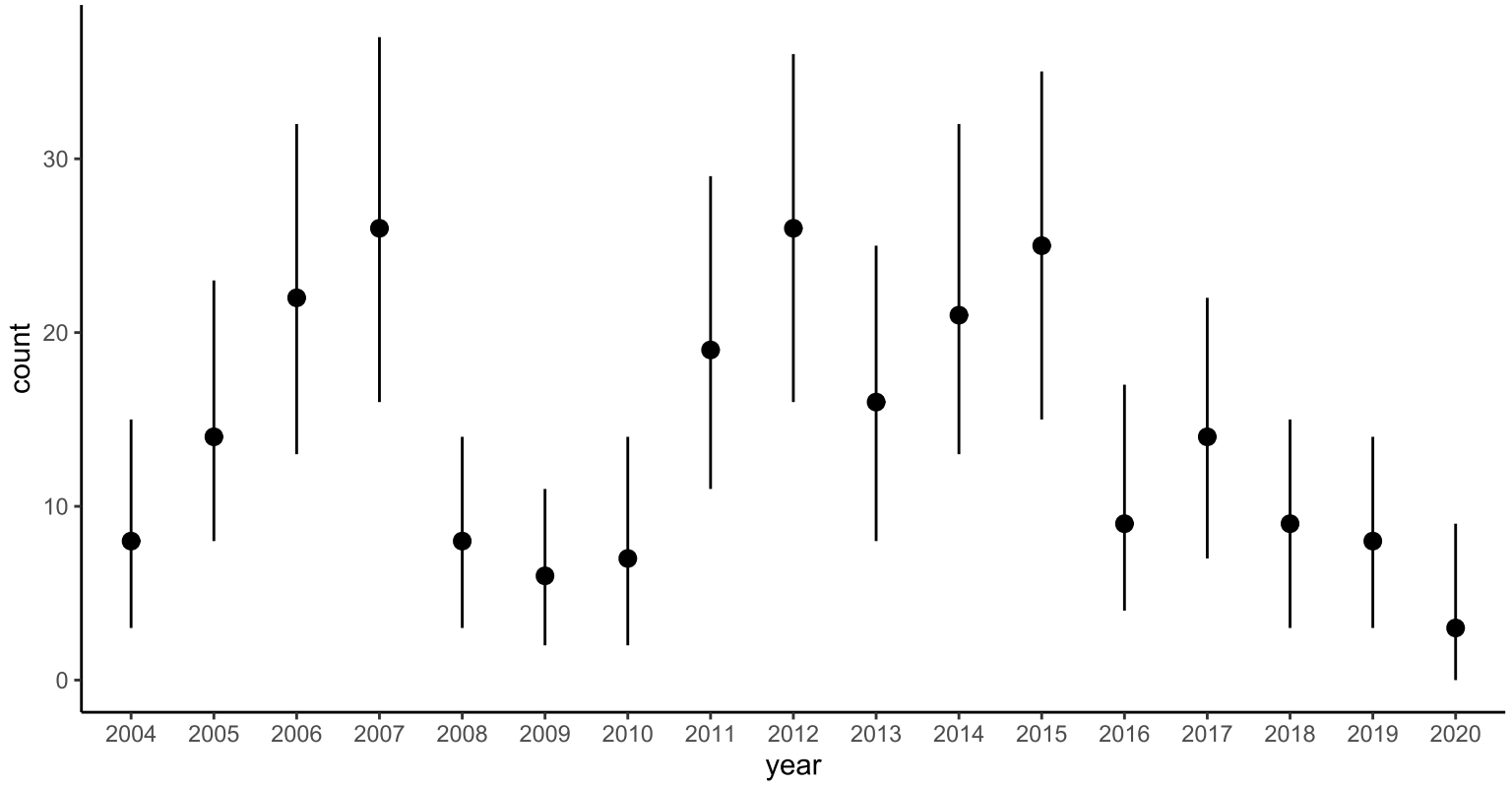
# Conditional predictions

Code Plot

```
# use marginaleffects utilities to plot conditional predictions
library(ggplot2)
plot_predictions(year_random,
                 condition = 'year',
                 n_cores = 2) +
  theme_classic()
```

# Conditional predictions

Code Plot



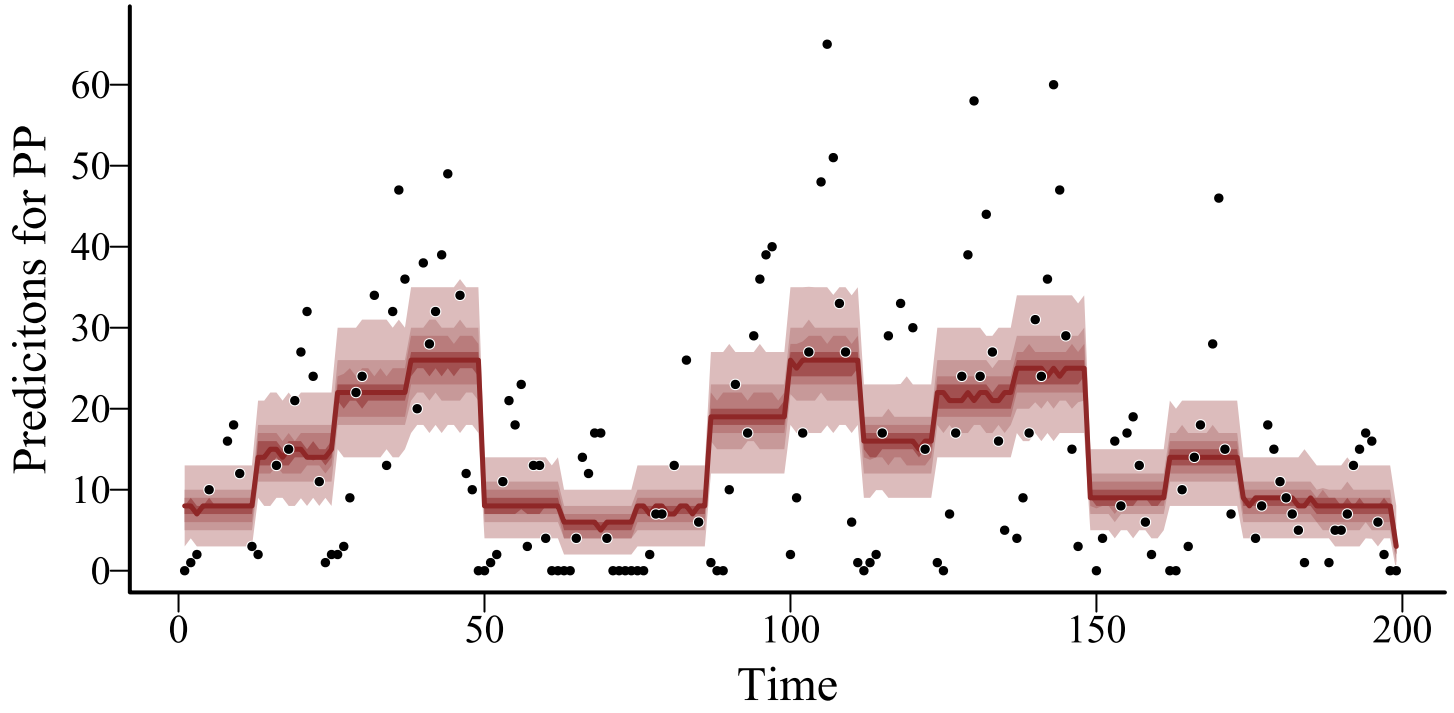
# Hindcast predictions

Code Plot

```
# use mvgam's plot to view hindcast predictions
plot(year_random, type = 'forecast')
```

# Hindcast predictions

Code Plot



# mvgam with yearly smooth

```
model_data %>%
  dplyr::mutate(year = as.numeric(as.character(year))) → model_data

year_smooth ← mvgam(count ~
                      s(year, bs = 'tp', k = 15),
                    family = poisson(),
                    data = model_data,
                    trend_model = 'None',
                    burnin = 500,
                    samples = 500,
                    chains = 4)
```

A thin plate regression spline of the numeric `year` variable

Retain intercept because smooths are zero-centered

# Coefficients uninterpretable

```
rownames(coef(year_smooth))
```

```
##  [1] "(Intercept)" "s(year).1"   "s(year).2"   "s(year).3"   "s(year).4"
##  [6] "s(year).5"   "s(year).6"   "s(year).7"   "s(year).8"   "s(year).9"
## [11] "s(year).10"  "s(year).11"  "s(year).12"  "s(year).13"  "s(year).14"
```

We ***must*** use predictions and plots to understand the model

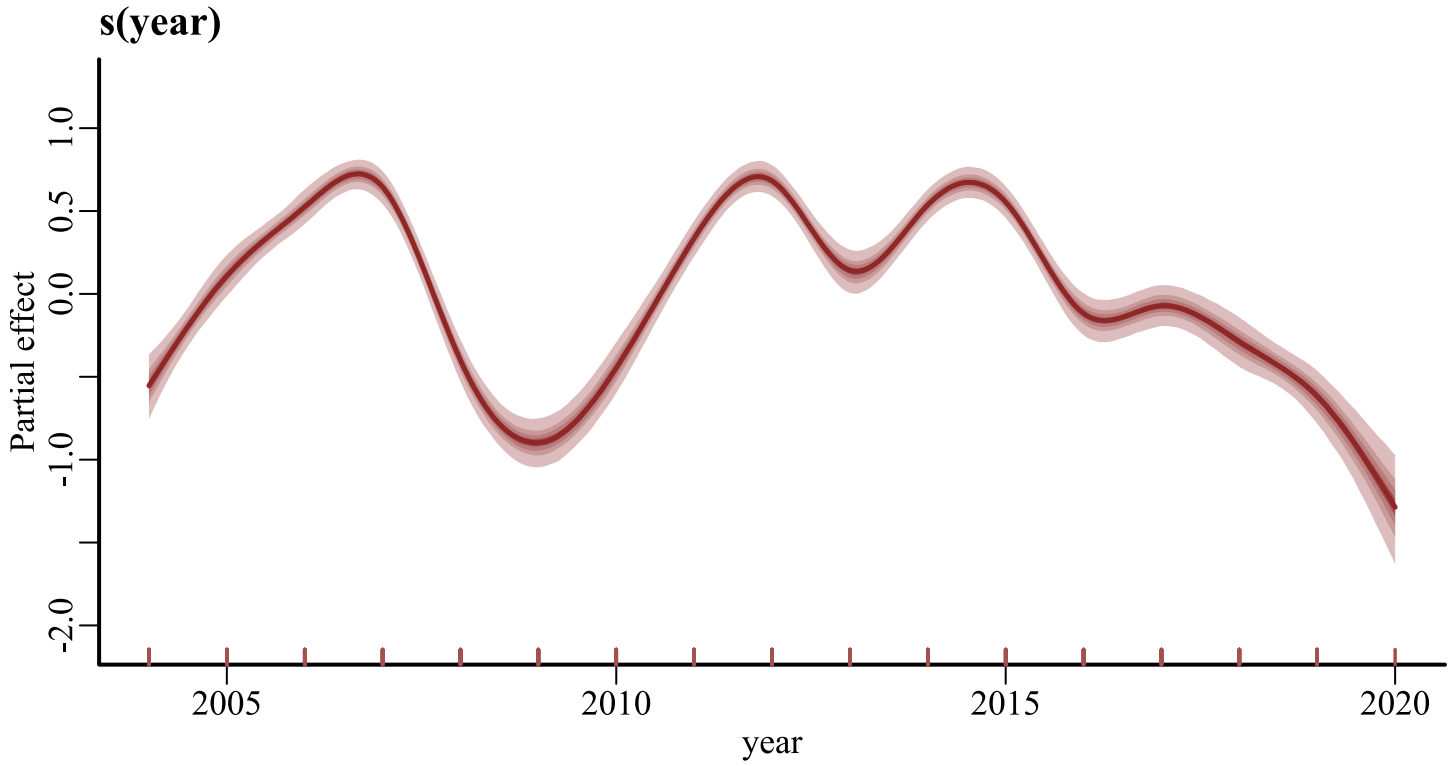
# Estimated yearly smooth

Code Plot

```
# plot the smooth effect posterior estimates
plot(year_smooth, type = 'smooth')
```

# Estimated yearly smooth

Code Plot



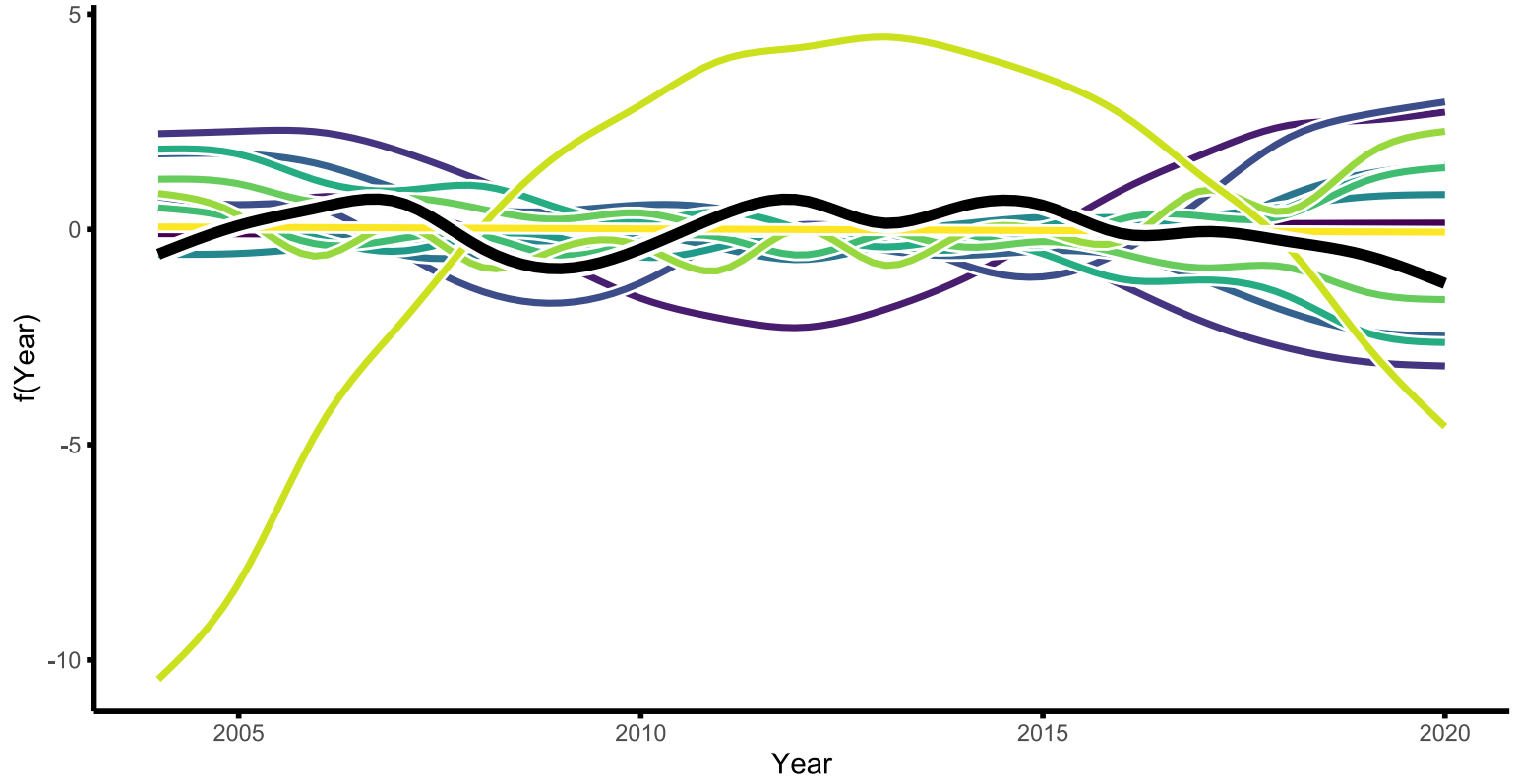
# Plotting basis functions

Code Plot

```
# plot the basis functions with gratia
library(ggplot2); library(viridis); library(gratia)
theme_set(theme_classic())
ggplot(basis(year_smooth$mgcv_model),
       aes(x = year, y = value, color = bf)) +
  geom_borderline(linewidth = 1.25, bordercolour = "white") +
  geom_borderline(data = smooth_estimates(year_smooth$mgcv_model),
                  inherit.aes = FALSE,
                  mapping = aes(x = year, y = est), linewidth = 2) +
  scale_color_viridis(discrete = TRUE) +
  theme(legend.position = 'none', axis.line = element_line(size = 1),
        axis.ticks = element_line(colour = "black", size = 1)) +
  ylab('f(Year)') + xlab('Year')
```

# Plotting basis functions

Code Plot



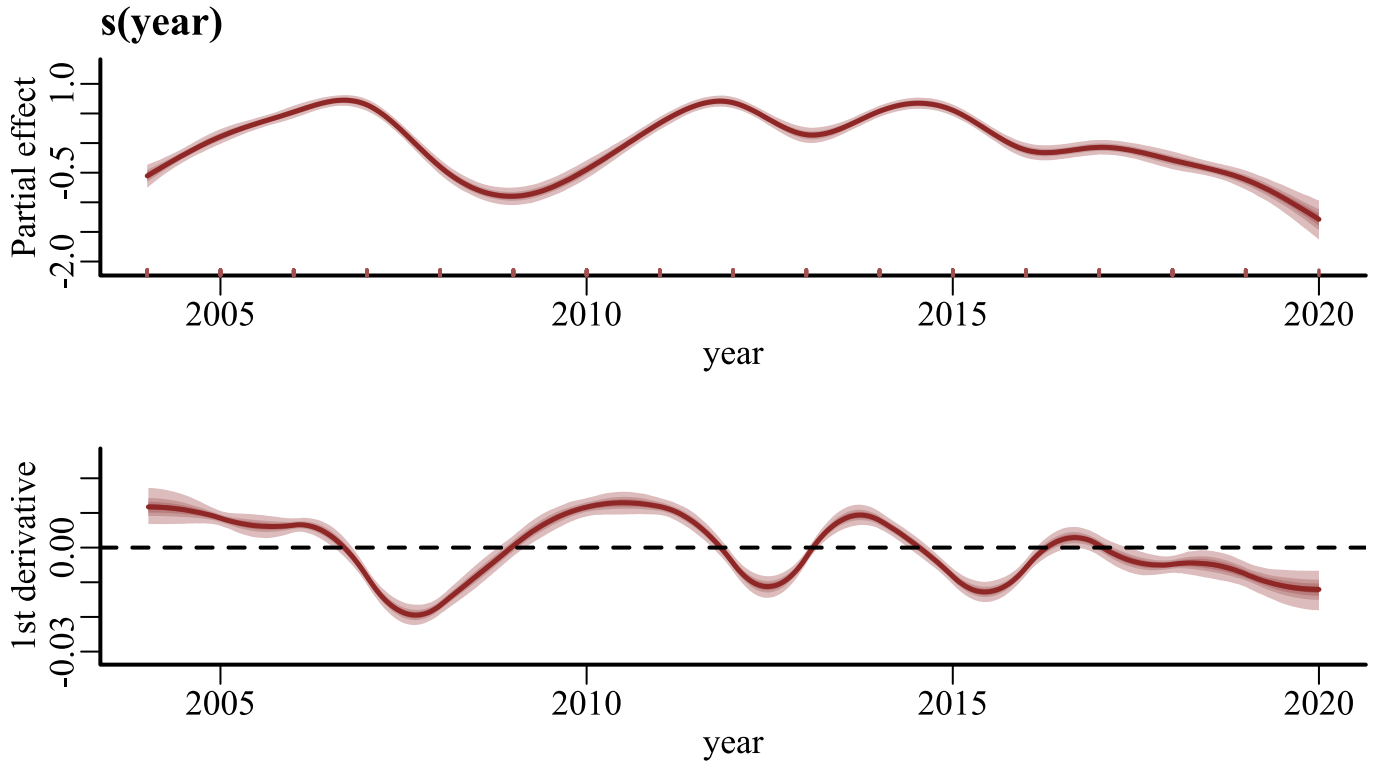
# Rates of change

Code Plot

```
# plot the smooth effect posterior estimates
plot(year_smooth, type = 'smooth', derivatives = TRUE)
```

# Rates of change

Code Plot



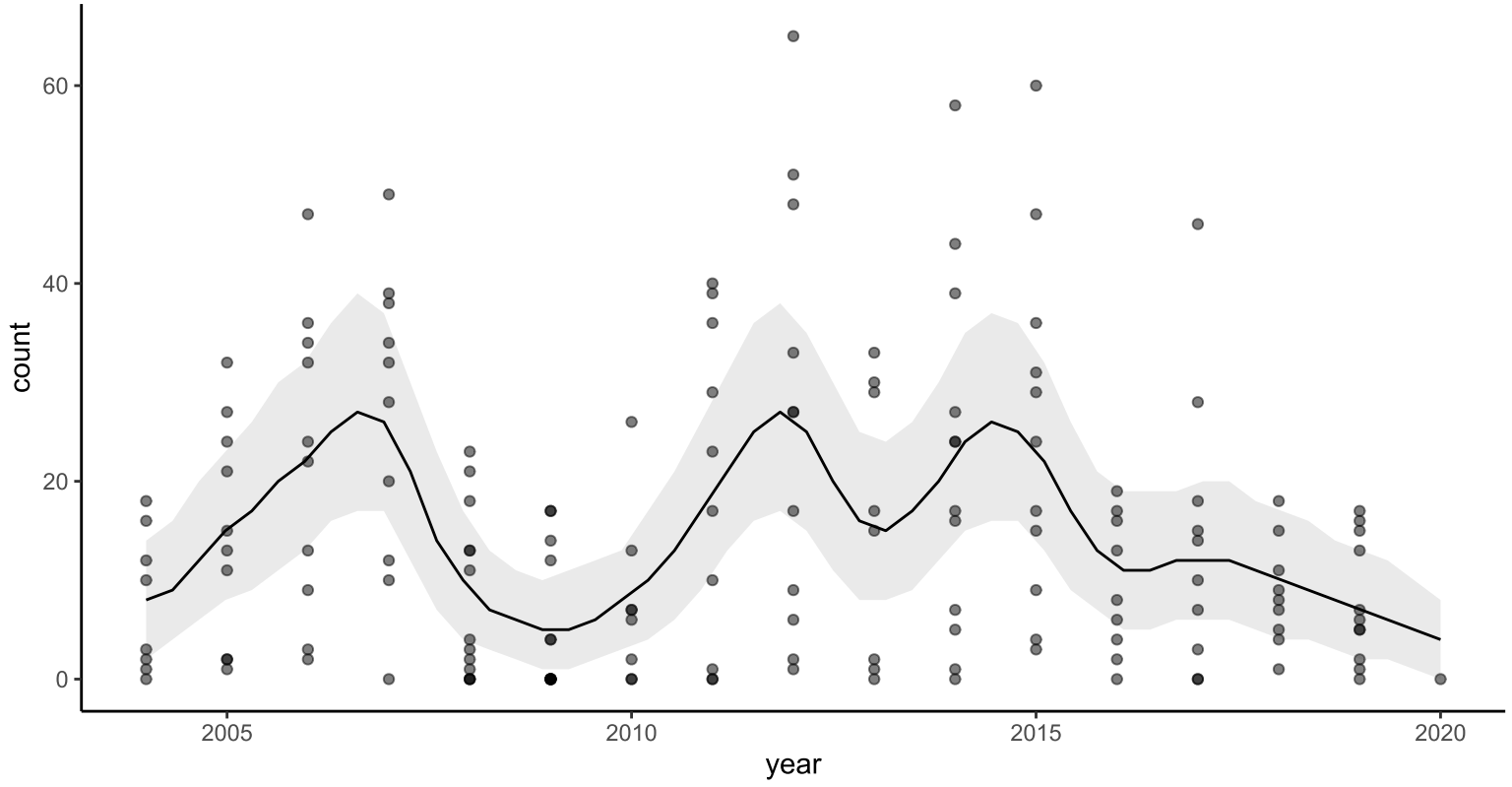
# Conditional predictions

Code Plot

```
# use marginaleffects utilities to plot conditional predictions
library(ggplot2)
plot_predictions(year_smooth,
                 condition = 'year',
                 points = 0.5,
                 n_cores = 2) +
  theme_classic()
```

# Conditional predictions

Code Plot



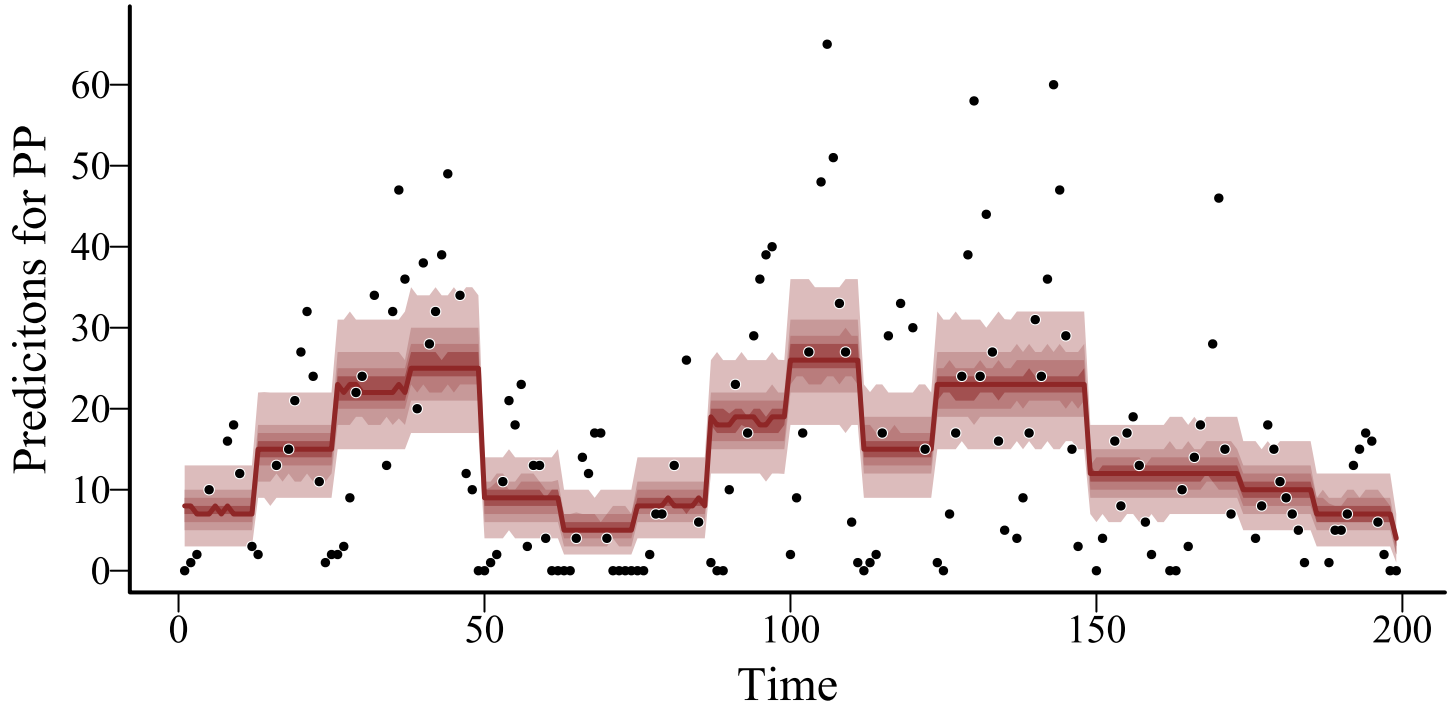
# Hindcast predictions

Code Plot

```
# use mvgam's plot to view hindcast predictions
plot(year_smooth, type = 'forecast')
```

# Hindcast predictions

Code Plot

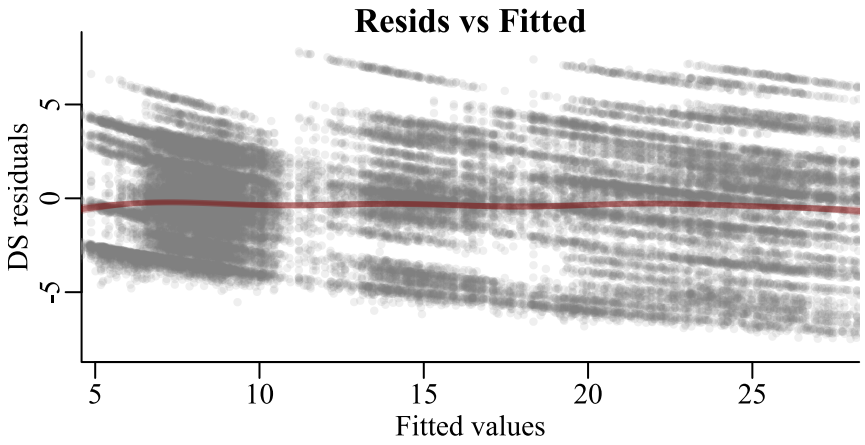


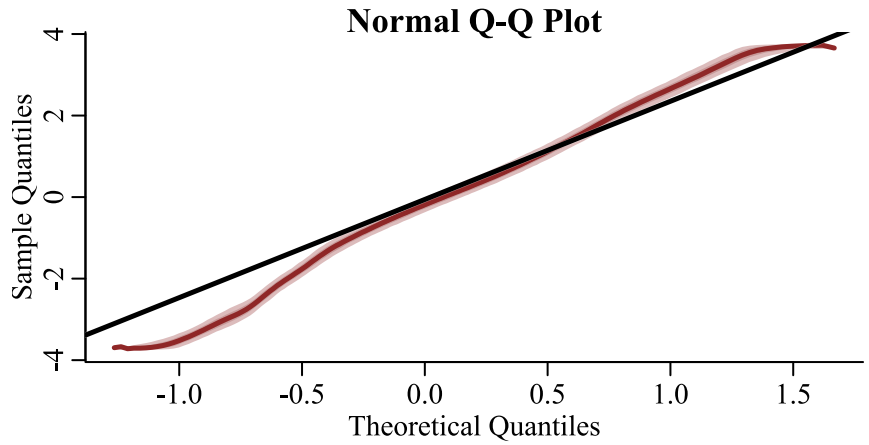
**Forecasts will differ. Why?**

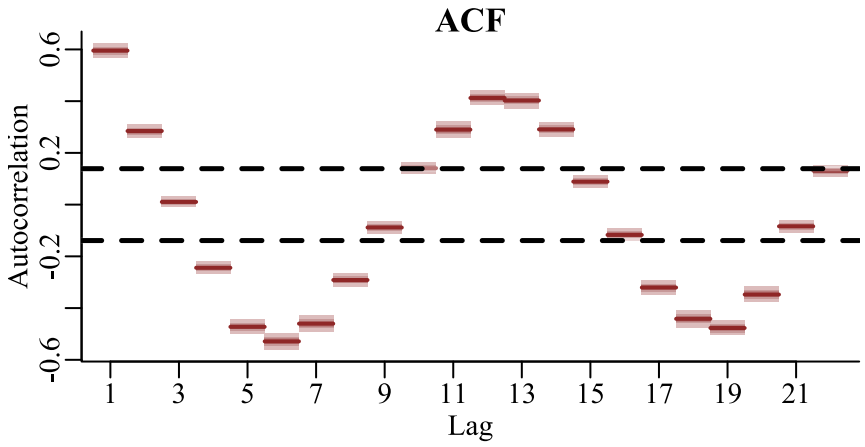
**We will explore this further in the tutorial and in the next lecture**

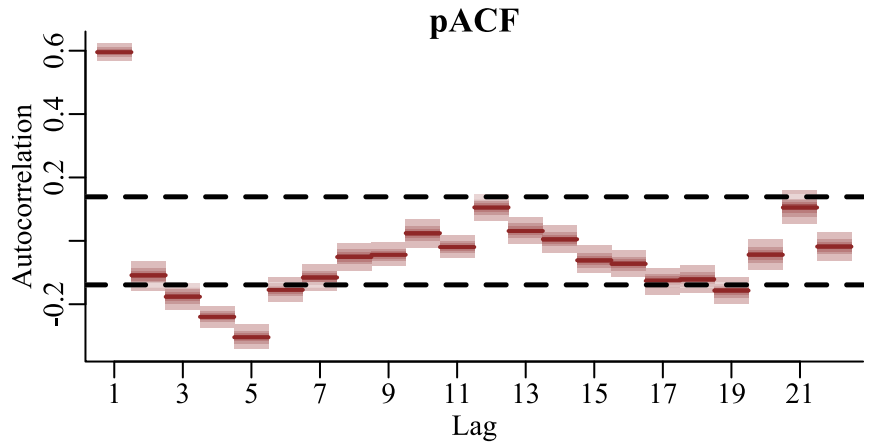
**But how do model diagnostics look?**

# Random year diagnostics

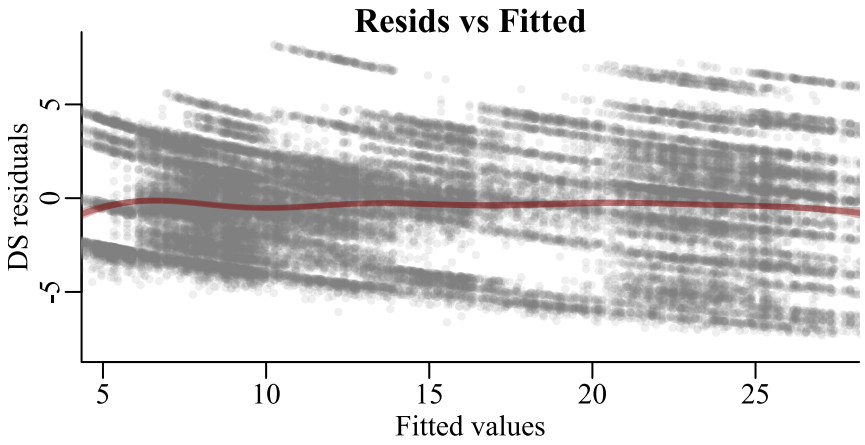


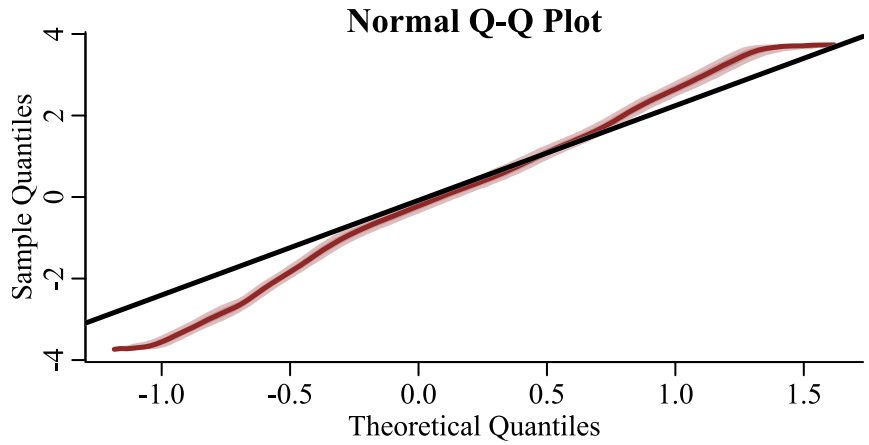



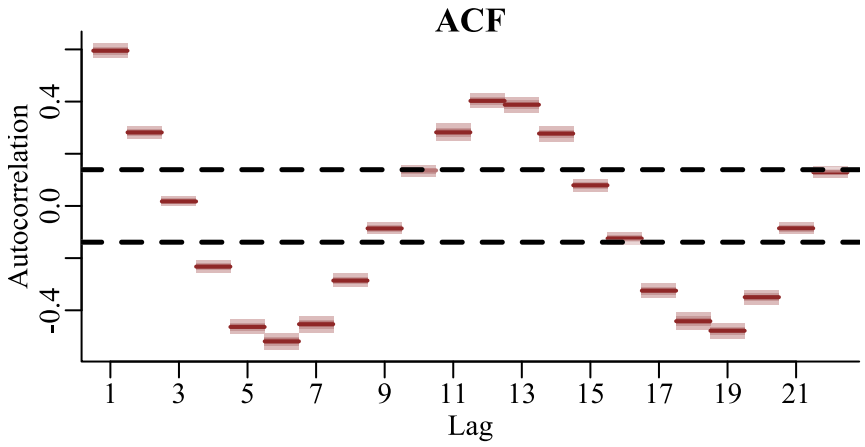



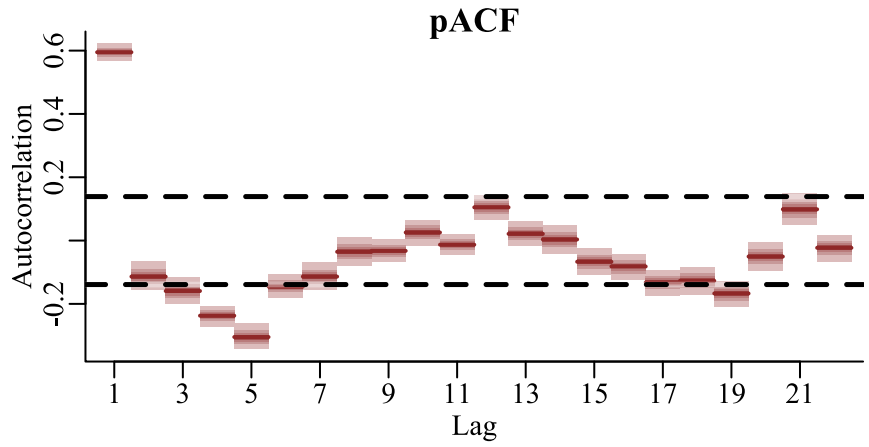



# Smooth year diagnostics



**Randomized quantile residuals show evidence of unmodelled autocorrelation and seasonality**

**How can we deal with the seasonality?**

# Adding a smooth of mintemp

```
year_temp_smooth ← mvgam(count ~
                           s(year, bs = 'tp', k = 15) +
                           s(mintemp, bs = 'tp', k = 8),
                         family = poisson(),
                         data = model_data,
                         trend_model = 'None',
                         burnin = 500,
                         samples = 500,
                         chains = 4)
```

A thin plate regression spline of `mintemp`

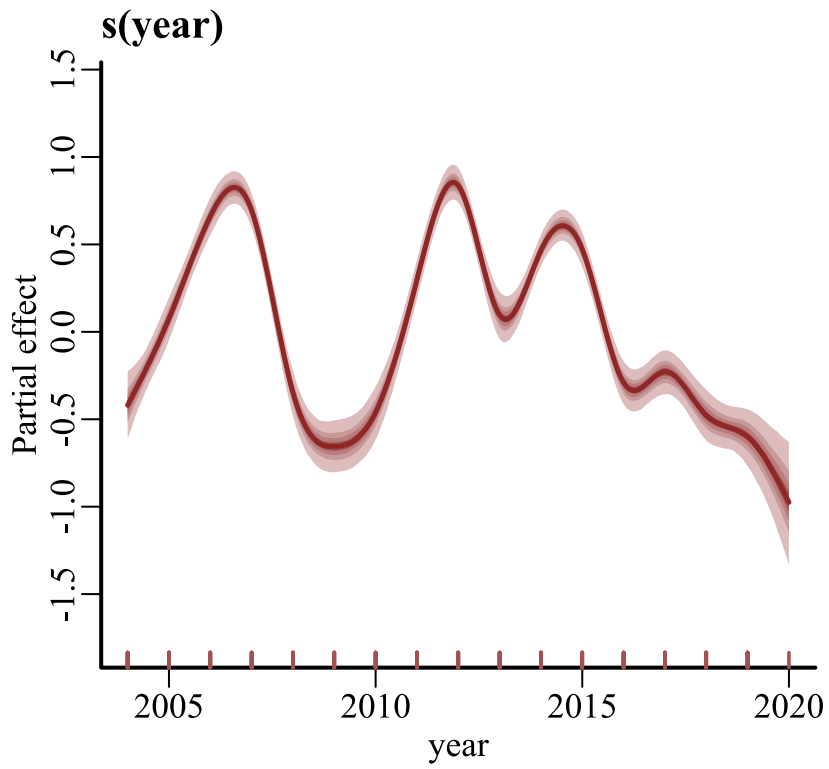
# Estimated smooths

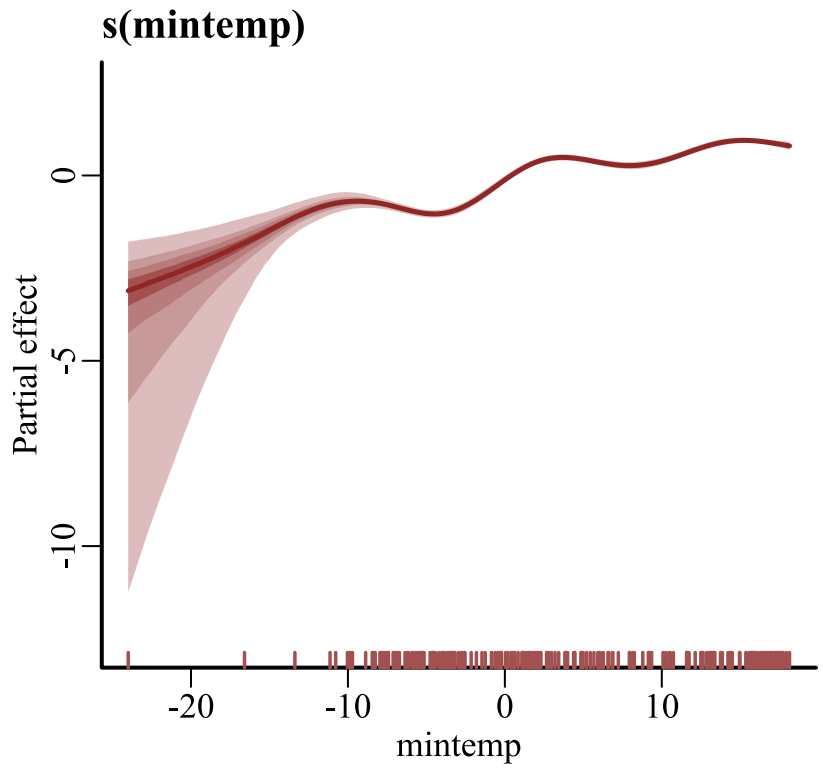
Code Plot

```
# use mvgam's plot to view both smooth functions
plot(year_temp_smooth, type = 'smooth')
```

# Estimated smooths

Code Plot



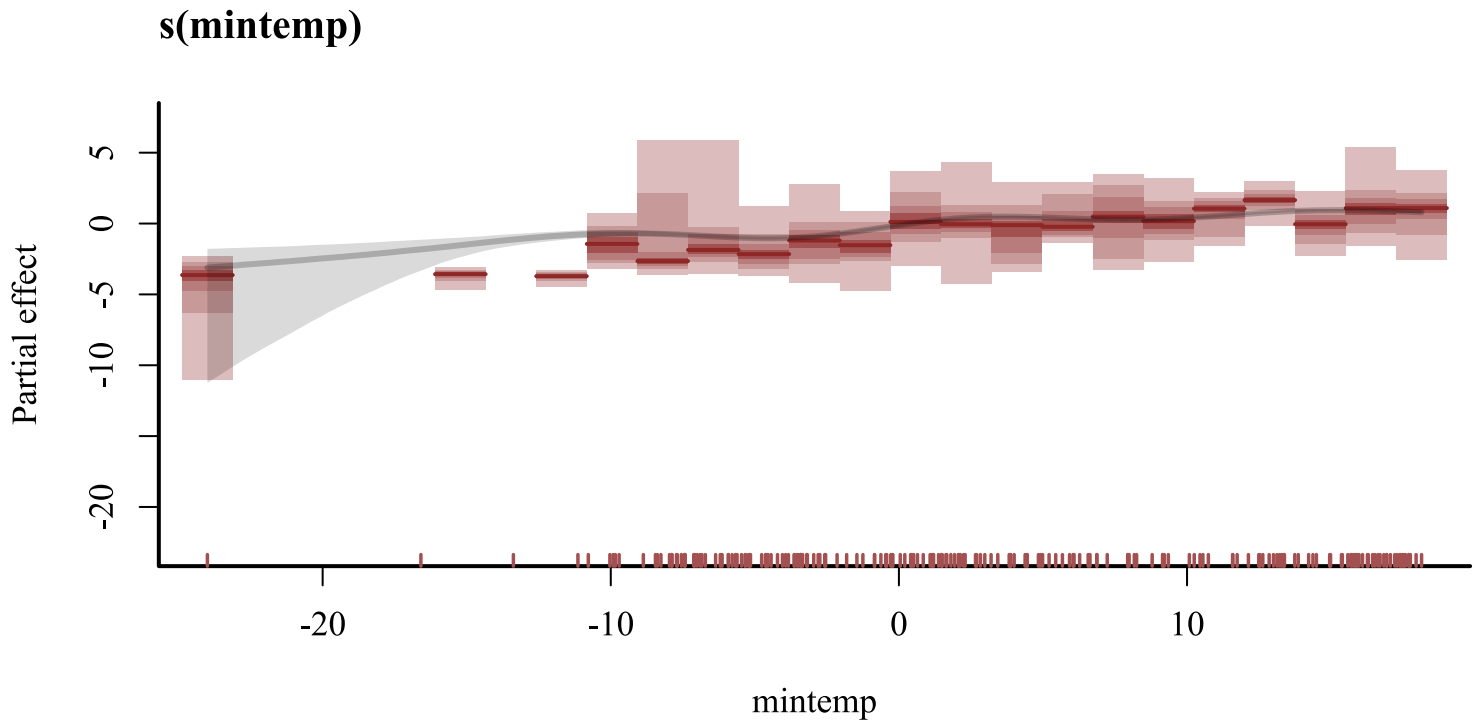
# Partial residuals

Code Plot

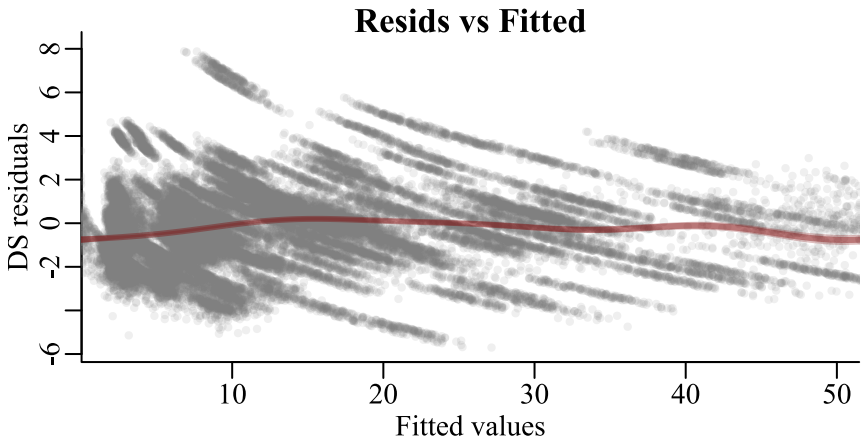
```
# use mvgam's plot_mgvam_smooth to view partial residuals
plot_mvgam_smooth(year_temp_smooth, smooth = 2, residuals = TRUE)
```

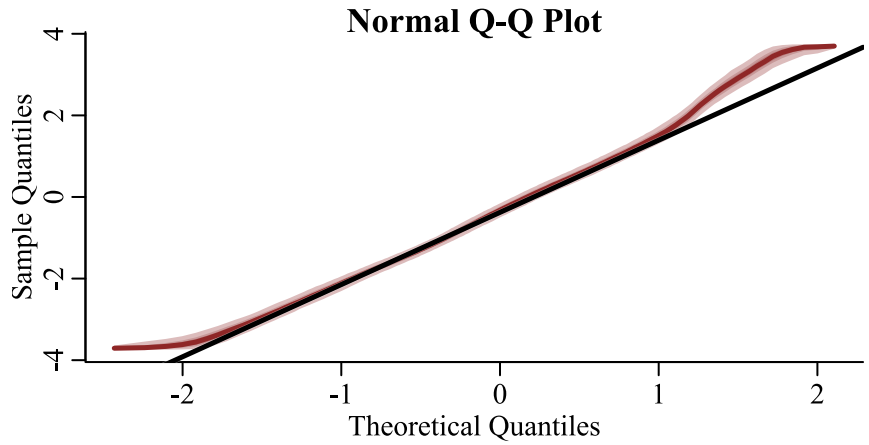
# Partial residuals

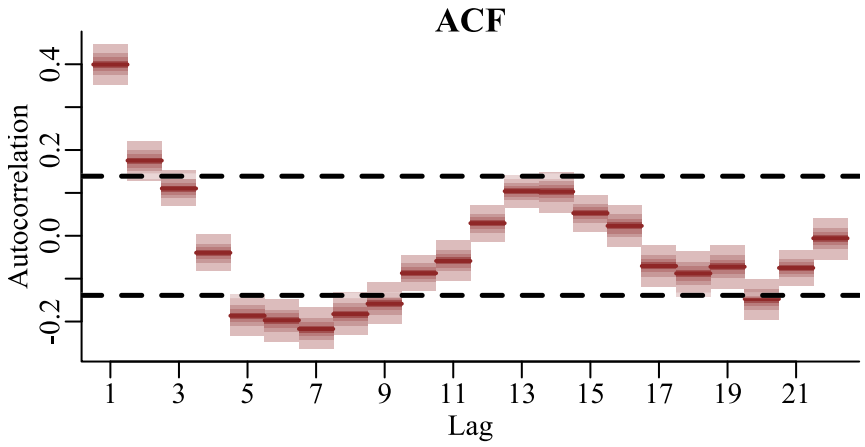
Code Plot

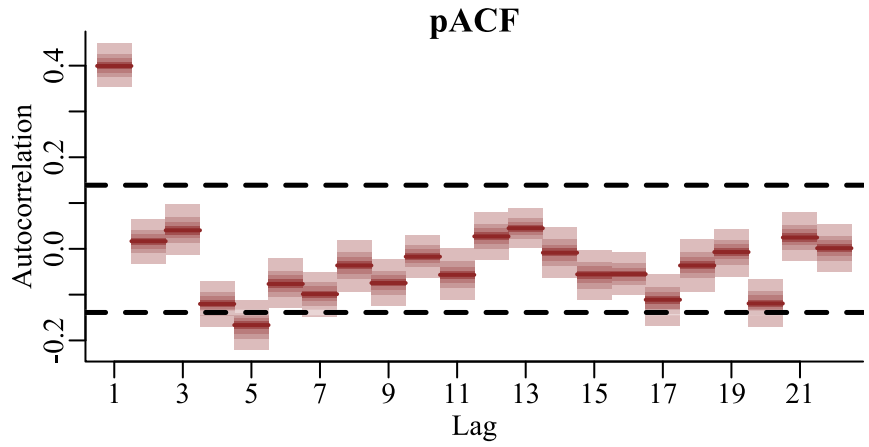


# Diagnostics



**Randomized quantile residuals still show evidence of unmodelled autocorrelation**

**How can we deal with this?**

# Adding a latent AR term

```
year_temp_smooth_ar ← mvgam(count ~
                              s(year, bs = 'tp', k = 15) +
                              s(mintemp, bs = 'tp', k = 8),
                            family = poisson(),
                            trend_model = 'AR1',
                            data = model_data,
                            burnin = 500,
                            samples = 500,
                            chains = 4)
```

A latent (unobserved) dynamic model for residuals

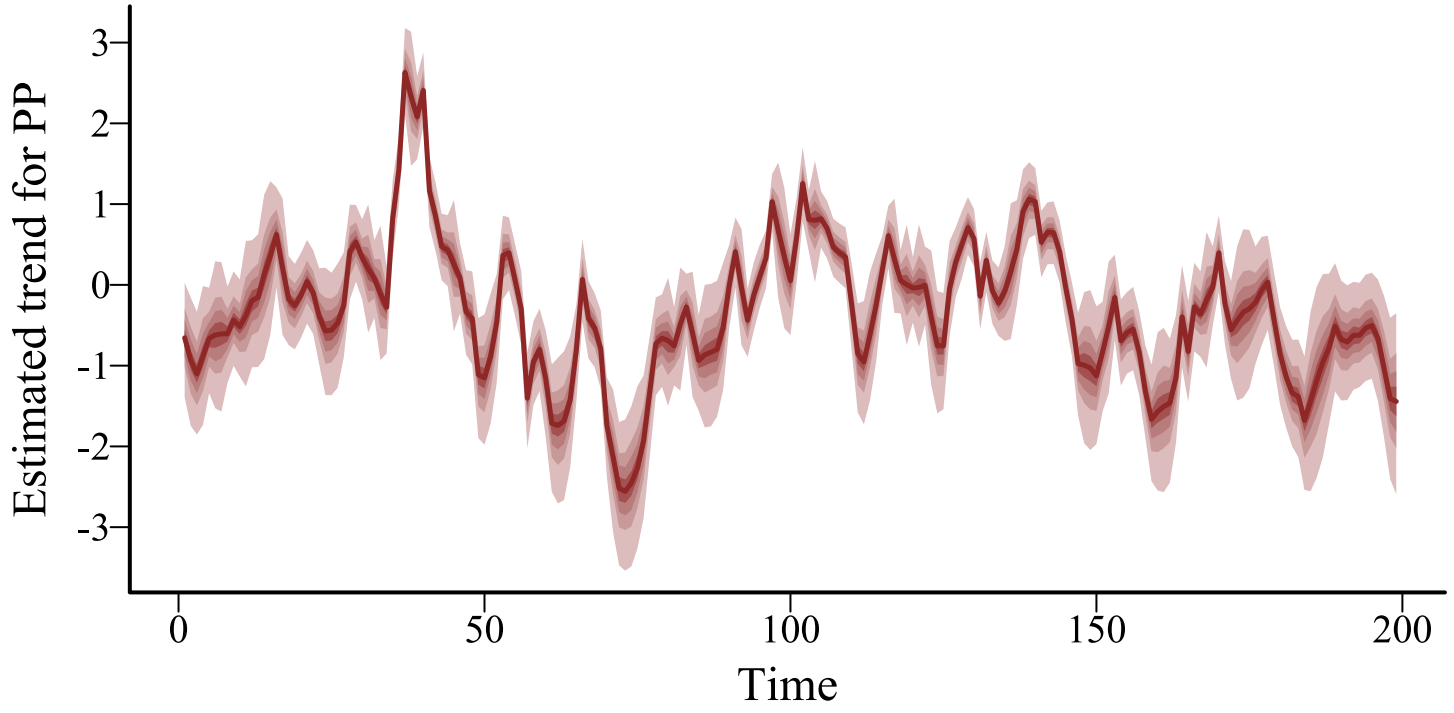
# The latent trend

Code Plot

```
# use mvgam's plot to view the trend
plot(year_temp_smooth_ar, type = 'trend')
```

# The latent trend

Code Plot



# AR parameters

Code AR1s SDs

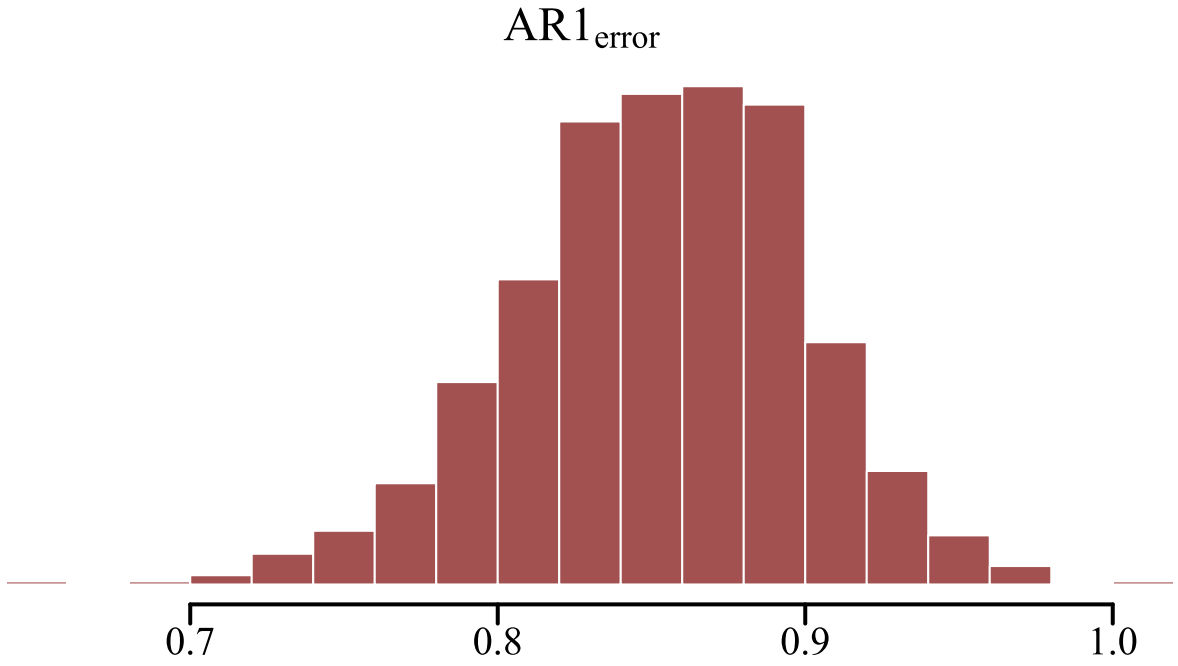
```
# extract AR estimates
ar_params ← as.data.frame(year_temp_smooth_ar,
                          variable = c('ar1[1]',
                                       'sigma[1]'))

# plot as histograms
hist(ar_params$`ar1[1]`, main = expression(AR1[error]))

hist(ar_params$`sigma[1]`, main = expression(sigma[error]))
```

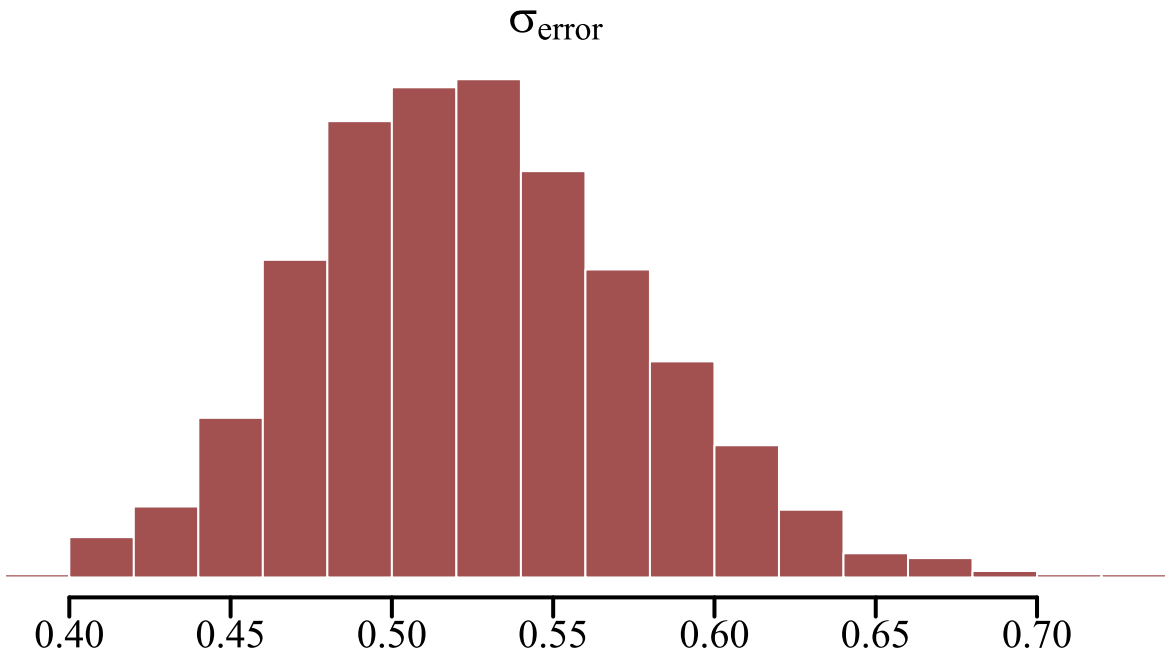
# AR parameters

Code AR1s SDs

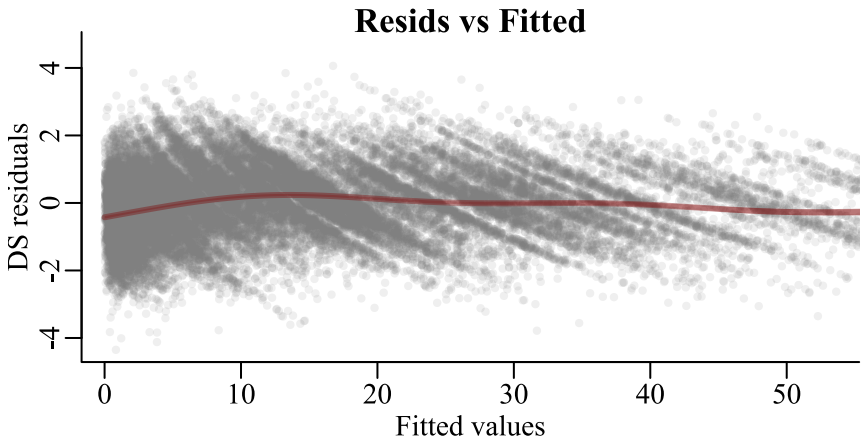


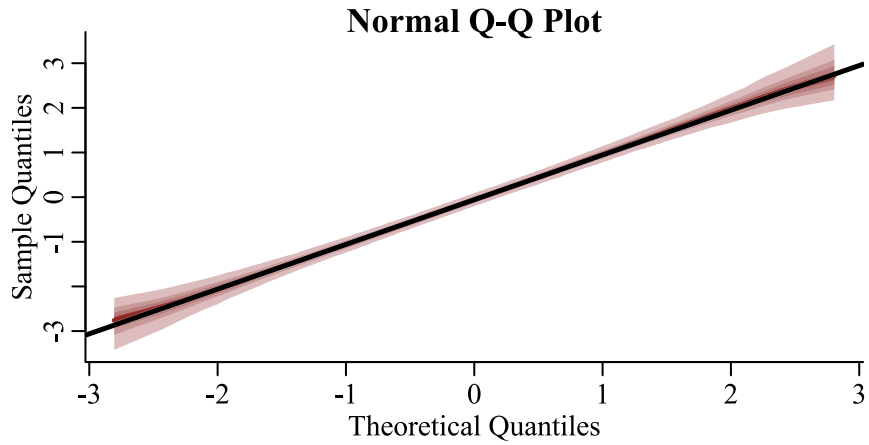
# AR parameters

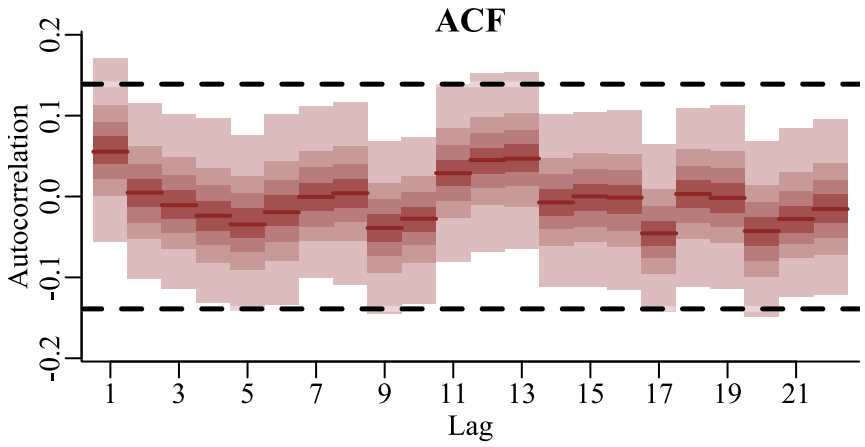
Code AR1s SDs

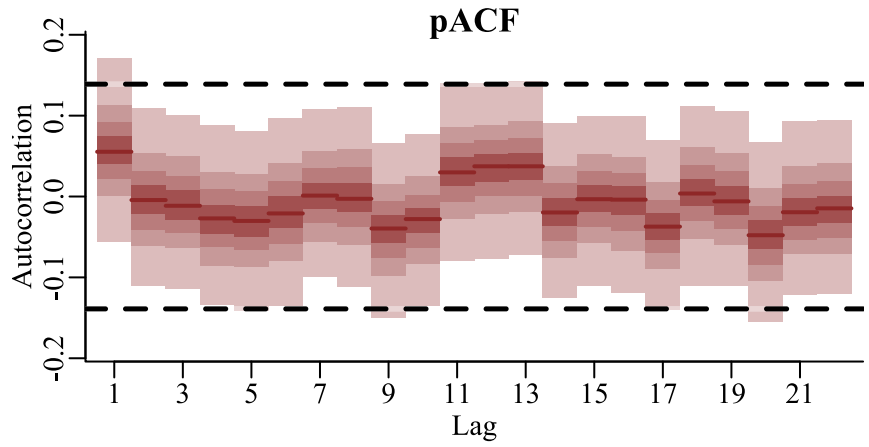


# Diagnostics

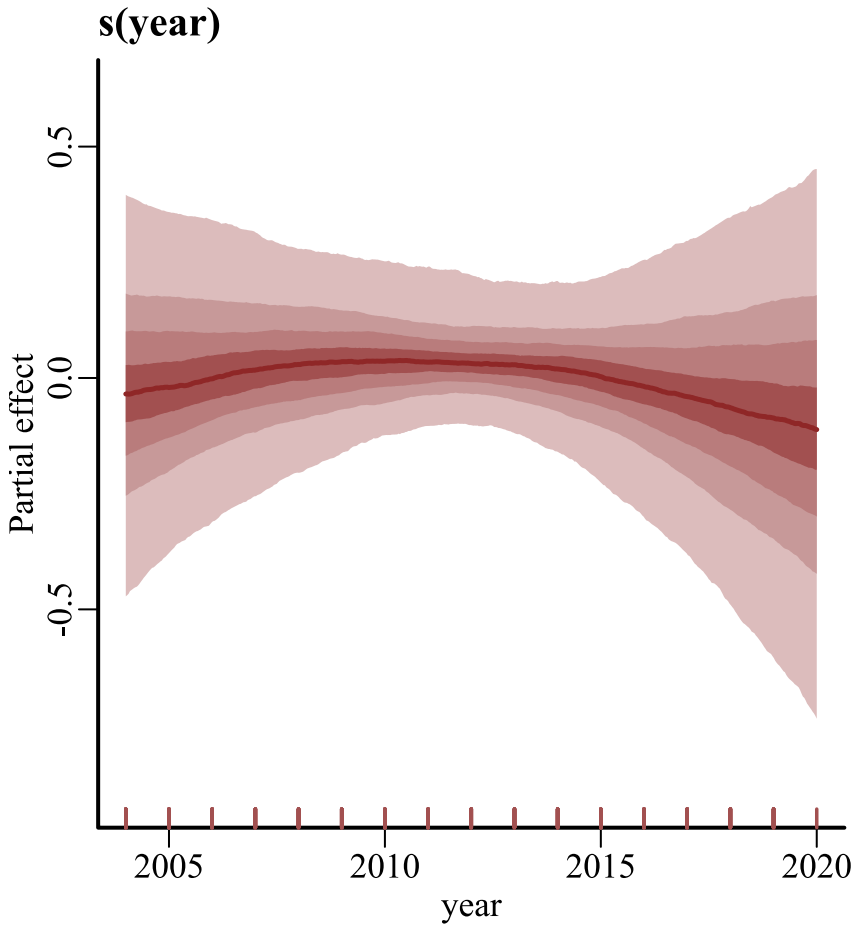


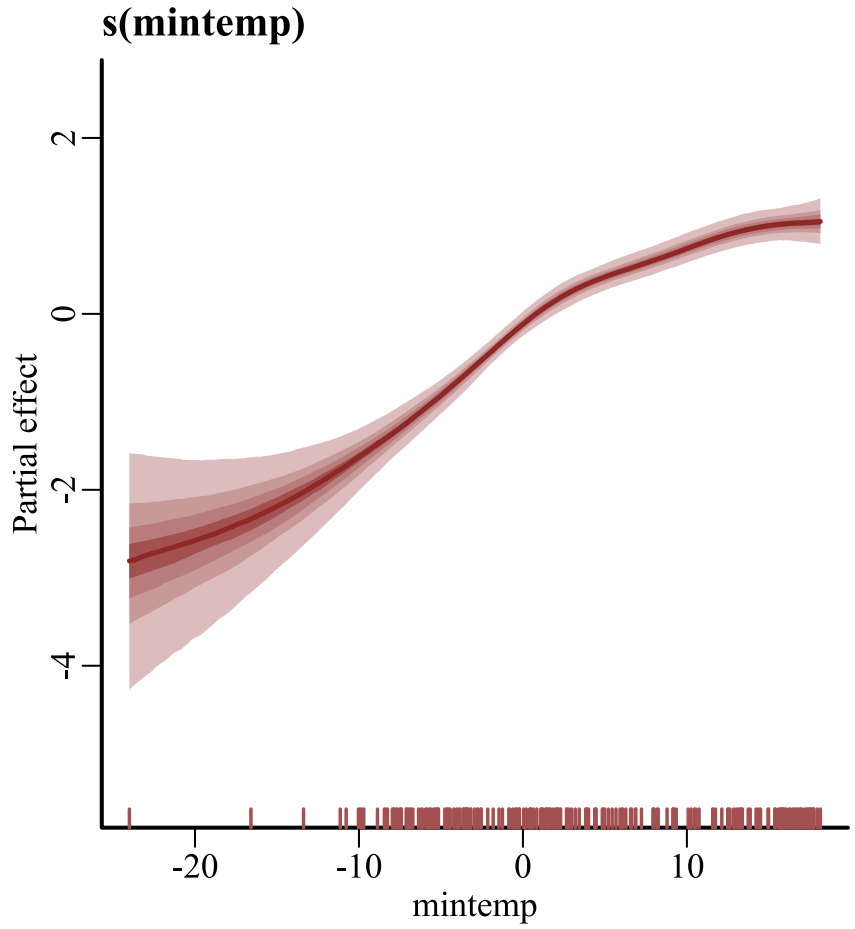



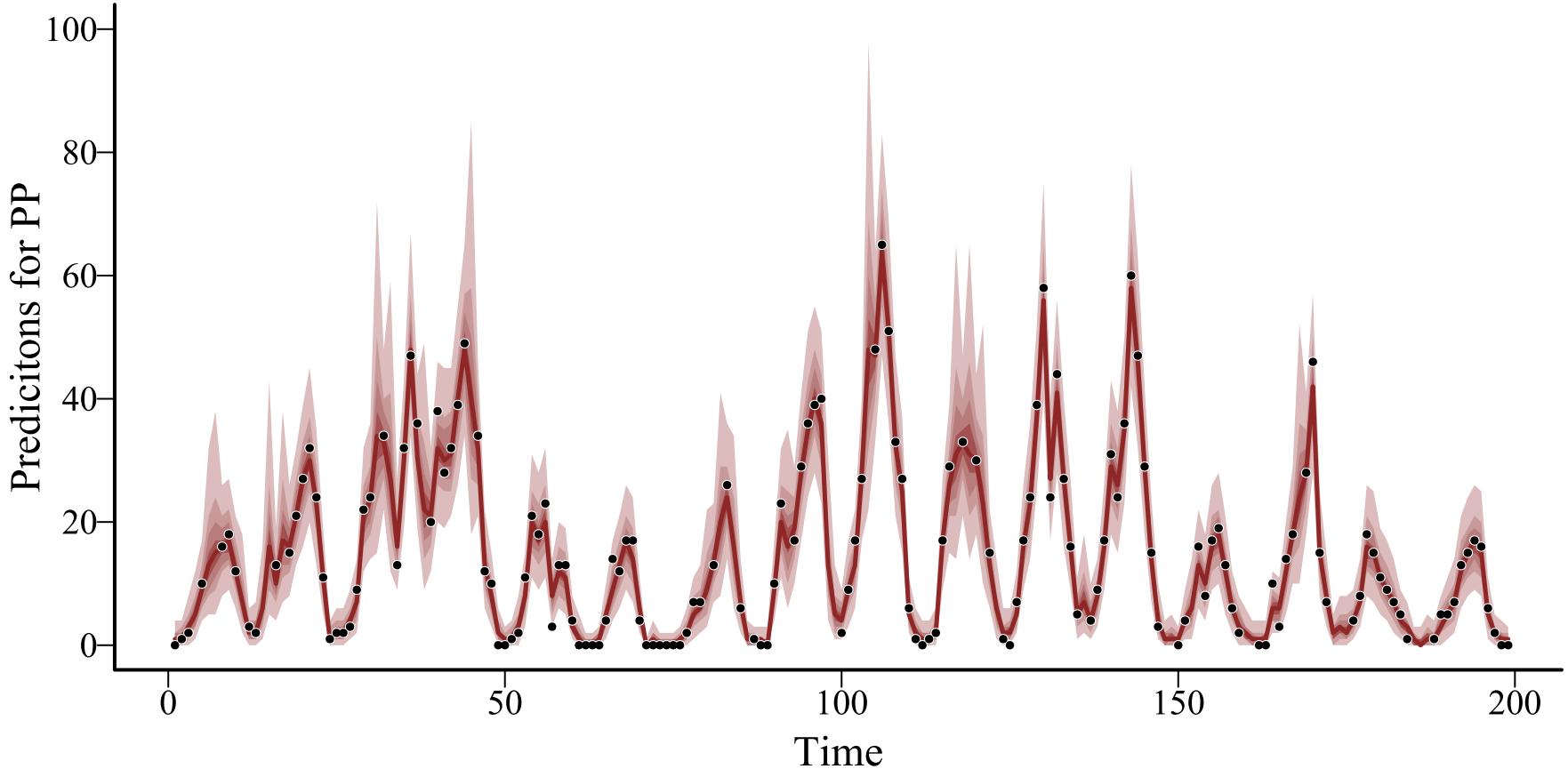



# Updated smooths



# Hindcast predictions



# In the next lecture, we will cover

Latent autoregressive processes

Latent Gaussian Processes

Dynamic coefficient models