FYS2150
Lab Report: Elasticity

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A study on two different methods to determine the Young's modulus of a brass rod.

I. INTRODUCTION

II. THEORY

A. Euler-Bernoulli beam theory

$$h(m) = \frac{mgl^3}{48EI} \tag{1}$$

$$E = \frac{4l^3 g}{3\pi|A|d^4} \tag{2}$$

B. Errors

When performing arithmetic operations on recorded data, the uncertainty in the data must also carry over to the derived results. How these uncertainties carry over in different operations can be found in Practical Physics [?].

III. EXPERIMENTAL PROCEDURE

A. Three-point flexural test

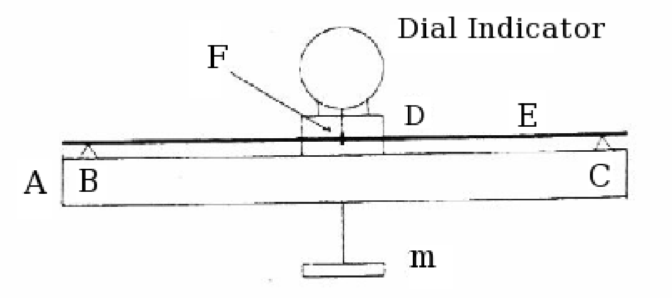


FIG. 1: Apparatus for measuring the deflection of a rod

Using 1 as a reference; The brass rod, A, was laid on the "knives" B and C. In the middle of the rod, there was a ring as shown in Fig. 2. The flat surface of the ring was in contact with the needle of the dial indicator at G. In order to ensure that the flat surface of the ring was at right angle with the needle, we turned the rod such that

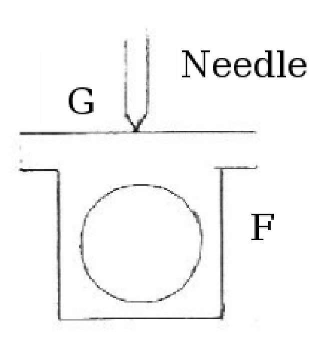


FIG. 2: Cross-section of apparatus where the dial indicator meets the ring in Fig. 1

the reading of the dial indicator would be at a minimum, as the skewer the surface, the greater the reading. This process was repeated at the start of every attempt of the experiment.

B. Measuring the speed of sound in the rod

The brass rod, with a ring attached to it (same as before), was laid to rest on the flat side of the ring on a solid surface such that the rod is held up by the ring. We also made sure that the rod was not to be disturbed in any way while it was vibrating. When hit with a hammer, it will emit a sound consisting of different frequencies. Following are the two different methods we used for determining the root frequency of the rod. During both experiments, we ensured there were no significant noise pollution during our recording (By which i mean people performing the same experiment as us).

1. *By hearing for beats*

A speaker was connected to a signal generator. We started the signal generator at 1200Hz and hit the brass rod with a plastic hammer on the the flat surface on one end of the rod. By ear, there was an audible beat due to the superposition of the two signals. We adjusted the signal generator such that the the frequency of the beat was minimized, and there was essentially no audible difference between the two signals. We did this by trying above and below where we thought the root frequency was, eventually zeroing in on a value.

TABLE I: Flex of beam, h(m), with rough m.

Attempt no.	h(0kg) [mm]	h(0.5kg) [mm]	h(1kg) [mm]	h(1.5kg) [mm]	h(2.0kg) [mm]	h(2.5kg) [mm]	h(3.0kg) [mm]	h(3.5kg) [mm]
1	9.44	8.72	8.00	7.28	6.58	5.84	5.15	4.43
2	9.42	8.70	7.98	7.26	6.53	5.80	5.09	4.39
3	9.42	8.71	7.98	7.26	6.53	5.80	5.09	4.37
4	9.41	8.69	7.97	7.25	6.52	5.79	5.08	4.36
5	9.42	8.70	7.98	7.26	6.70	5.87	5.19	4.51

2. By Fourier transform

A USB microphone was placed close to the rod, and faced towards it. The microphone was connected to a computer running matlab, with a script that collects audio data from it and Fourier transforms it using FFT. The recordings made were made with a sampling frequency of 8×1024 Hz and varying durations. As before, we hit the rod using a plastic hammer and recorded the data.

IV. RESULTS

A. Results from Three-point flexural test

Table I contains the data recorded with the dial indicator

B. Results from measuring the speed of sound in the rod

When hearing for beats, me and my labpartner decided that the root frequency was ≈ 1240 Hz.

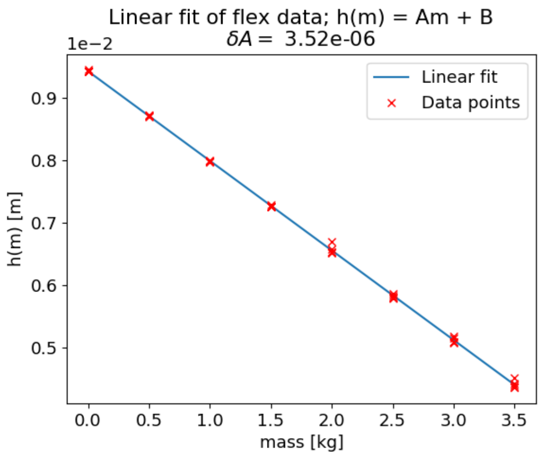
Fig. 4 contains the data and derived results from our fourth attempt of the experiment. We performed a total of 7 attempts, which all yielded in similar results to attempt no. 4. The data yielded from all of the attempts is summarized in Fig. 5 which shows the peaks in the frequency domain in one plot. Table II contains all of the relevant numbers related to each attempt.

TABLE II: FFT data

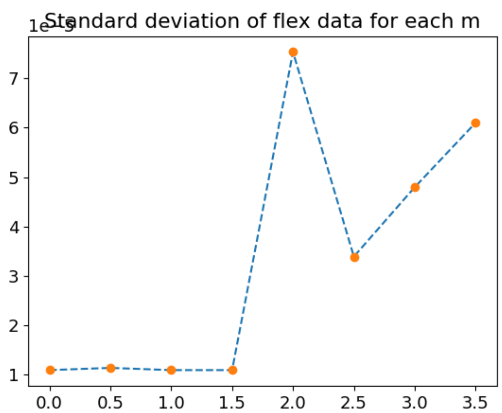
Attempt no.	f [Hz]	Δf [Hz]	t [s]	f_s [Hz]
1	1213.60	0.10	10	8192
2	1213.60	0.10	10	8192
3	1213.65	0.05	20	8192
4	1213.72	0.04	25	8192
5	1213.72	0.04	25	8192
6	1213.72	0.07	15	8192
7	1213.73	0.07	15	8192

V. DISCUSSION

VI. CONCLUSION



(a) Flex of rod



(b) Standard deviation of data

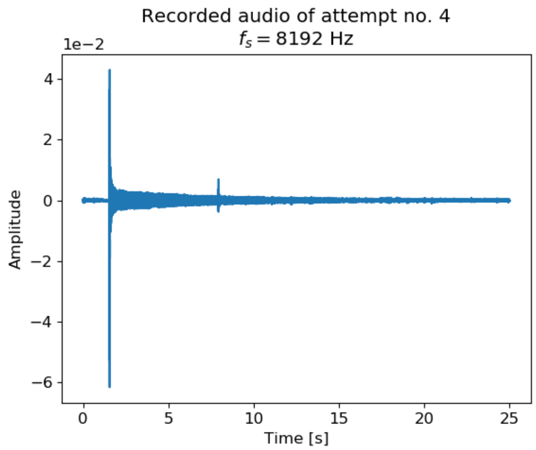
FIG. 3: (a) Shows the flex of the brass beam measured by the dial indicatior. (b) Shows the standard deviation of the data points in (a) at their respective masses

Appendix: Code

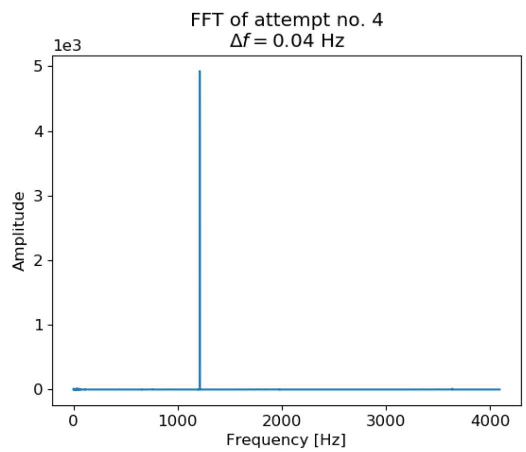
All of the code used to produce this report. Anything noteworthy should already be mentioned in the main body of the report.

scripts/FFTlyd.py

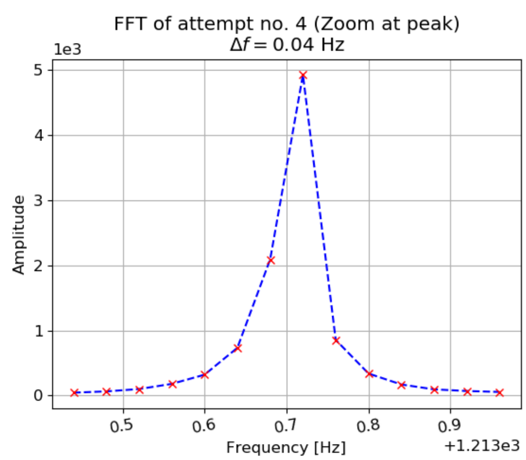
```
#!/usr/bin/env python
# -*- coding: utf-8 -*-
"""
Generates the same figures as FFTlyd.m
author: Nicholas Karlsen
"""
import scipy.io as sio
import matplotlib.pyplot as plt
import numpy as np


# Sets font size of matplot
plt.rcParams.update({'font.size': 12})


```



(a) Time domain



(b) Frequency domain



(c) Zoomed frequency domain

FIG. 4: All of the plots generated for attempt no. 4

```
def import_matlab(filename):
    # Opens .mat file
    mfile = sio.loadmat(filename)
    # Fetches data
    data = mfile.get("data")
```

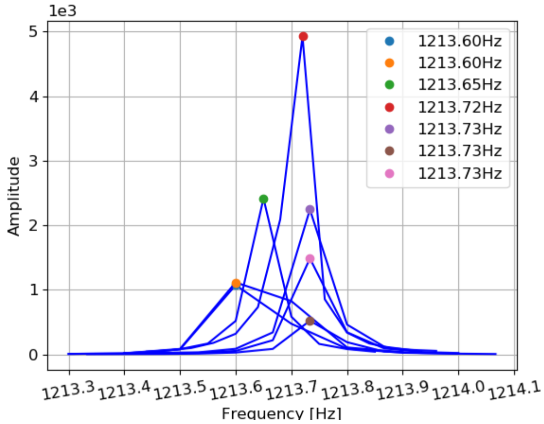


FIG. 5: Zoomed frequency plot for all 7 attempts.

```
    energi = mfile.get("energi")
    fut = mfile.get("fut")
    L = mfile.get("L")
    t = mfile.get("t")

    return data, energi, fut, L, t


rel_path = "data/"
n = 1
mat_file = "forsok%i.mat" % n


def raw_fig(filename):
    data, energi, fut, L, t = import_matlab(filename)
    plt.plot(t, data)
    plt.xlabel("Time [s]")
    plt.ylabel("Amplitude")
    plt.ticklabel_format(style='sci', axis='y', scilimits=(0,0))


raw_fig(rel_path + "forsok1.mat")
plt.title("Recorded audio of attempt no. 1\n$f_s = 8192$ Hz")
plt.savefig("raw_exp2_1.png")
plt.close()

raw_fig(rel_path + "forsok4.mat")
plt.title("Recorded audio of attempt no. 4\n$f_s = 8192$ Hz")
plt.savefig("raw_exp2_4.png")
plt.close()


def figure1(filename):
    data, energi, fut, L, t = import_matlab(filename)
    fut = np.transpose(fut)
    fh = int(len(energi) / 2.0)    # half lenght of data
    # Only plot first half of data, as FF mirrors in half-way point.
    plt.plot(fut[:fh], energi[:fh])
    plt.xlabel("Frequency [Hz]")
    plt.ylabel("Amplitude")
    plt.ticklabel_format(style='sci', axis='y', scilimits=(0,0))


figure1(rel_path + "forsok1.mat")
plt.title("FFT of attempt no. 1\n$\Delta f=0.10$ Hz")
plt.savefig("energy_exp2_1.png")
plt.close()

```

```python
figure1(rel_path + "forsok4.mat")
plt.title("FFT of attempt no. 4\n$\Delta f=0.04$ Hz")
plt.savefig("energy_exp2_4.png")
plt.close()

eigenfreqs = []


def figure2(filename, style="-", cross=0):
    data, energi, fut, L, t = import_matlab(filename)
    fut = np.transpose(fut)

    fh = int(len(energi) / 2.0)   # half lenght of data
    ipeak = np.argmax(energi[:fh])

    eigenfreqs.append(fut[ipeak])

    i = ipeak
    while energi[i] > np.amax(energi[:fh]) * 0.01:
        i -= 1

    j = ipeak
    while energi[j] > np.amax(energi[:fh]) * 0.01:
        j += 1

    plt.plot(fut[i:j], energi[i:j], color="blue", linestyle=style)
    if cross == 1:
        plt.plot(fut[i:j], energi[i:j], "rx")
    else:
        plt.plot(fut[ipeak], energi[ipeak], "o", label="%.2fHz" % fut[ipeak])

    plt.grid("on")

figure2(rel_path + "forsok1.mat", style="--", cross=1)
plt.xlabel("Frequency [Hz]")
plt.ylabel("Amplitude")
plt.ticklabel_format(style='sci', axis='y', scilimits=(0,0))
plt.xticks(rotation=10)
plt.title("FFT of attempt no. 1 (Zoom at peak)\n$\Delta f=0.10$ Hz")
plt.savefig("freq_exp2_1.png")
plt.close()


figure2(rel_path + "forsok4.mat", style="--", cross=1)
plt.xlabel("Frequency [Hz]")
plt.ylabel("Amplitude")
plt.ticklabel_format(style='sci', axis='y', scilimits=(0,0))
plt.xticks(rotation=10)
plt.title("FFT of attempt no. 4 (Zoom at peak)\n$\Delta f=0.04$ Hz")
plt.savefig("freq_exp2_4.png")
plt.close()


for i in range(1, 8):
    figure2(rel_path + "forsok%i.mat" % i)

plt.xlabel("Frequency [Hz]")
plt.ylabel("Amplitude")
plt.legend()
plt.ticklabel_format(style='sci', axis='y', scilimits=(0,0))
plt.xticks(rotation=10)
plt.savefig("freq_exp2_all.png")
plt.close()
```

scripts/lab_data.py

```python
#!/usr/bin/env python
# -*- coding: utf-8 -*-
"""
Contains all of the data collected in the
```

```
Elacticity lab, module 2 of FYS2150
author: Nicholas Karlsen
"""

from pylab import *
import scipy.constants as const
import FYS2150lib as fys


rcParams.update({'font.size': 13})  # Sets font size of plots


def E_sound(f, L, d, M):
    '''
    Returns youngs modulus given
    f = root frequency
    L = lenght of rod
    d = diameter of rod
    M = mass of rod
    '''
    return (16.0 * M * L * f**2) / (np.pi * d**2)


def E_sound_error(E, sd, sf, sL, sM, d, f, L, M):
    return E * np.sqrt((2 * sd / d)**2 + (2 * sf / f)**2 +
                       (2 * sL / L)**2 + (2 * sM / M)**2)


def weight_data(set=1):
    "set decides which data set the function returns."
    set = set.lower()   # Forces lowercase
    sets = ["masses", "rod"]
    # Mass of weights measured with balance
    m_a_balance = 500.1e-3
    m_b_balance = 1000.3e-3
    m_c_balance = 2000.5e-3

    # Mass of reference weights
    m_reference = array([0.5, 1.0, 2.0])
    m_reference_balance = array([500.0e-3, 999.9e-3, 2000.1e-3])  # Weighed

    # Using linear fit to correct for error in balance
    a, b, da, db = fys.linfit(m_reference, m_reference_balance)
    # Corrected masses
    m_a = (m_a_balance - b) / a     # approx 500g
    m_b = (m_b_balance - b) / a     # approx 1000g
    m_c = (m_c_balance - b) / a     # approx 2000g

    if set == sets[0]:  # Return corrected masses
        return m_a, m_b, m_c

    if set == sets[1]:
        return

    if set not in sets:
        print "Invalid set"
        print "List of valid sets:", sets
        print "exiting..."
        exit()


def experiment1_data():
    m_a, m_b, m_c = weight_data("masses")
    mass_dat = array(
        [0, m_a, m_b, m_a + m_b, m_c, m_a + m_c,
         m_b + m_c, m_a + m_b + m_c])               # [Kg]

    # Round 1: (in order)
    h_1 = array([9.44, 8.72, 8.00, 7.28, 6.58, 5.84, 5.15, 4.43]) * 1e-3  # [m]
    # Round 2: (in order)
```

```
h_2 = array([9.42, 8.70, 7.98, 7.26, 6.53, 5.80, 5.09, 4.39]) * 1e-3  # [m]
# Round 3: (in order)
h_3 = array([9.42, 8.71, 7.98, 7.26, 6.53, 5.80, 5.09, 4.37]) * 1e-3  # [m]
# Round 4: (in order)
h_4 = array([9.41, 8.69, 7.97, 7.25, 6.52, 5.79, 5.08, 4.36]) * 1e-3  # [m]
# Round 5: (in order)
h_5 = array([9.42, 8.70, 7.98, 7.26, 6.70, 5.87, 5.19, 4.51]) * 1e-3  # [m]

h_mean = (h_1 + h_2 + h_3 + h_4 + h_5) / 5.0

m, c, dm, dc = fys.linfit(mass_dat, h_mean)

mass = linspace(0, 3.5, 8)
h_mass = m * mass + c  # h(m)


def plotdata():
    h_sets = [h_1, h_2, h_3, h_4, h_5]
    plot(mass, h_mass, label="Linear fit")
    # errorbar(mass, m * mass + c, yerr=dm, color='blue', fmt='o', label='Error Range')

    for dat in h_sets:
        plot(mass_dat, dat, "x", color="r")
    plot(NaN, NaN, "xr", label="Data points")
    xlabel("mass [kg]")
    ylabel("h(m) [m]")
    ticklabel_format(style='sci', axis='y', scilimits=(0,0))
    legend()
    title("Linear fit of flex data; h(m) = Am + B\n$\delta A =$ %.2e" % dm)
    savefig("figs/h_m_fig.png")
    close()
plotdata()

def plot_stddev():
    """Plots the standard deviation of h(m)
    as m is increased"""
    deviation = np.zeros(len(h_1))
    for i in xrange(len(h_1)):
        deviation[i] = fys.stddev(array([h_1[i],
                                         h_2[i],
                                         h_3[i],
                                         h_4[i],
                                         h_5[i]]))[0]
        print i
    print deviation
    print len(mass_dat), len(deviation)
    plot(mass_dat, deviation, linestyle="--")
    plot(mass_dat, deviation, "o")
    ticklabel_format(style='sci', axis='y', scilimits=(0,0))
    plt.title("Standard deviation of flex data for each m")
    savefig("figs/h_m_deviation.png")
    close()
plot_stddev()

# lengde mellom yttersidene til festepunktene til knivene
# PEE WEE 2m Y612CM LUFKIN +- 0.01cm
l_AB = 133.9 * 1e-2  # [m]
# diameter til festepunkter
# Moore & Wright 1965 MI +- 0.01mm
l_AB_diameter = 4.09 * 1e-3  # [mm]
# anta festepunktet er p  midtden s  trekk fra diameter totalt sett
l = l_AB - l_AB_diameter

#M linger av stangens diameter d p  forskjellige punkter
# Moore & Wright 1965 MI +- 0.01mm
d = array([15.98, 15.99, 15.99, 16.00, 15.99, 15.99, 15.98, 15.99, 15.99, 15.99]) * 1e-3  # [m]
d_m = mean(d); #m

A = abs((h_mass - c) / mass)

```

```
    E = mean(4.0 * l**3 * const.g / (3 * pi * A * d_m**4)[1:-1])
    print E

def experiment_2():
    ''' Data pertaining to the audio exp.'''


if __name__ == "__main__":
    experiment1_data()
```

scripts/FYS2150lib.py

```
#!/usr/bin/env python
# -*- coding: utf-8 -*-
"""
A collection of commonly used functions in FYS2150.
author: Nicholas Karlsen
"""
import numpy as np


def stddev(x):
    """
    Finds the standard deviation, and standard deviation of
    a 1D array of data x.
    See. Eqn D. Page 24 squires
    """
    n = len(x)
    sigma = np.sqrt((np.sum(x**2) - 1.0 / n * np.sum(x)**2) / (n - 1))
    sigma_m = np.sqrt((np.sum(x**2) - 1.0 / n * np.sum(x)**2) / (n * (n - 1)))

    return sigma, sigma_m


def linfit(x, y):
    """
    Finds the line of best-fit in the form y=mx+c given two
    1D arrays x and y.
    """
    n = np.size(y)
    D = np.sum(x**2) - (1.0 / n) * np.sum(x)**2
    E = np.sum(x * y) - (1.0 / n) * np.sum(x) * np.sum(y)
    F = np.sum(y**2) - (1.0 / n) * np.sum(y)**2

    dm = np.sqrt(1.0 / (n - 2) * (D * F - E**2) / D**2)
    dc = np.sqrt(1.0 / (n - 2) * (float(D) / n + np.mean(x)) *
                 ((D * F - E**2) / (D**2)))
    m = float(E) / D
    c = np.mean(y) - m * np.mean(x)

    return m, c, dm, dc
```