FYS2150
Lab Report: Time and Frequency

Nicholas Karlsen
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The goal for this lab was measuring time, and comparing three methods of doing so, of varying degrees of "sophistication"; an hourglass, a stopwatch and a photo-diode.

I. INTRODUCTION

The lab spanned 6 hours and consisted of measuring the period of a pendulum using three different methods of measurement; an hourglass, a stopwatch and a photo-diode connected to a computer. The experiments were performed by myself, and my lab partner Lars K. Skaarseth. Before anything else, it is worth to note that the first experiment can largely be disregarded due to an error on our part, more on this in section III F 1.

II. THEORY

The theory used in this lab report is almost entirely summed up by the following equations;

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{1}$$

Where T denotes the period a swinging pendulum, L the length of the wire by which the pendulum is suspended and g the downward acceleration on the pendulum due to gravity.

$$\vec{R} = \frac{1}{M}\sum_i m_i \vec{r}_i \tag{2}$$

Where $\vec{R}$ denotes the position of the center of mass of a body of mass M consisting of several smaller bodies of mass m_i with individual center of mass at r_i.

Further detail can be found in "Elementary Mechanics Using Python" [1], or most other books covering introductory mechanics.

III. EXPERIMENTAL PROCEDURE

A. List of equipment used

- Hourglass, generic, unknown duration
- Pendulum, Aluminum cylinder, see Fig. 1
- Stopwatch, Cielo 100MT
- Photo-diode
- Cables
- Meter-rule, Hultafors
- Power supply
- Data acquisition tool, NI USB-6211
- PC with MATLAB installed
- Umbraco Key
- Screwdrivers
- Reflective tape

B. Preparation

We connected an aluminum cylinder to a block of aluminum secured to a desk with a (i believe nylon?) string as shown in Fig. 1. For the purposes of this experiment the string is assumed to be inextensible.

The string was turned on the Outer side of the screws on the top block, and the inner at the bottom so that the string would form a "V" shape, or positive angle. This was done in an attempt to limit the sideways motion of the block as much as possible, so that more of its kinetic energy would be restricted to the forward/backward motion (from the perspective of Fig. 1), which is the motion being measured in these experiments.

The length of the string, and later, the dimensions of the pendulum (As shown in Fig. 1) were measured using a Hultafors meter-rule with an uncertainty of $\pm 1mm$ as well as a small uncertainty due to thermal expansion. Since we did not record the temperature in the room, i will assume standard temperature and pressure, thus negating the need to account for thermal expansion.

C. Hourglass

In this this initial part of the experiment, we wanted to measure the number of swings made by the pendulum for the duration of an hourglass with an unknown duration. After having released the pendulum, we let it swing for one period before starting the hourglass, then both i and my lab partner then counted the number of swings separately in an attempt to minimize the chance human error when counting. Since we only counted complete swings, the measured value has an uncertainty of $\pm 1T$. The hourglass was also kept still on a table for its complete duration.

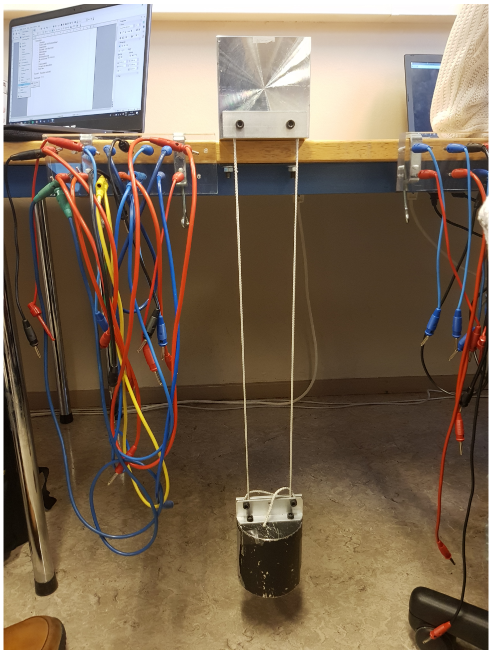

FIG. 1. A photograph showing how the pendulum was set up

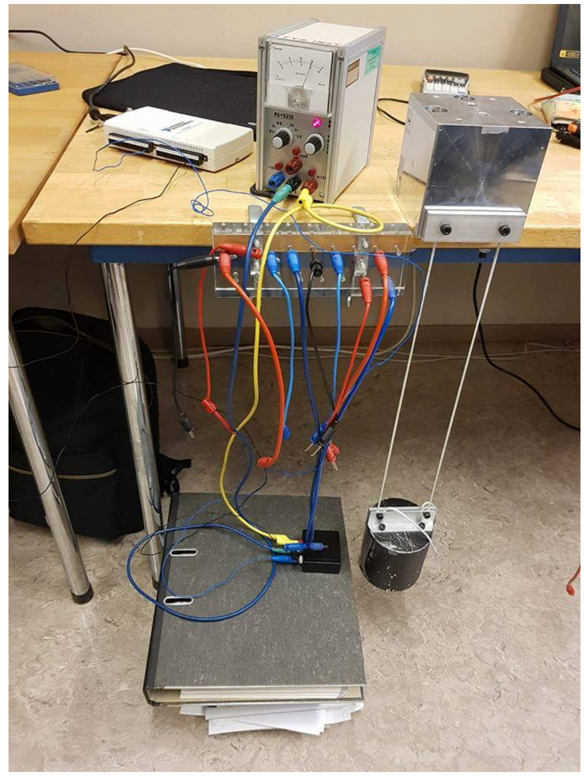

FIG. 2. A photograph showing how the photo-diode was set up and connected to the data acquisition box (NI USB-6211) for experiment no. 3 (Section III E)

D. Stopwatch

This part is mostly similar to the last one, the only difference being the method of measuring the period. This time we used a stopwatch (Cielo 100MT, uncertainty of $\pm 0.01s$) to record the time. In order to minimize human error, we opted to only measure the time every 10th swing and again, both of us counted the swings individually ensuring we would not record the time at the wrong number of swings. We used the lap function of the stop watch to save the time taken for every 10th swing as well as the total running time for the 100 swings that we recorded.

A rather significant source of error in this experiment comes from the reaction time, and to some extent the judgment of the person who is recording the time (he had to judge when the pendulum was at its apex).

E. Photo-diode

In this part, we made our measurements electronically using a photo-diode connected to a computer with MATLAB via a data acquisition box (NI USB-6211). A photo-diode sends out IR light, when this light is not reflected back to an IR sensitive receiver, the photo-diode emits a constant 5V . When the IR light is reflected back to it, it instead emits 0V. For this reason, we had to attach a reflective material to the block of aluminum. We opted for some aluminum foil. Once everything was properly connected (see Fig. 2) we ran a MATLAB script **svingeperiode.m** to record the data and started the pendulum as previously. We repeated this 4 times with different settings and conditions. Experiment 1 & 2 were identical, measured at different frequencies. For experiment 3, we positioned the photo-diode such that it would aim at the top of the cylinder when it was at rest. In experiment 4, the pendulum was given a push rather than releasing it from rest.

F. Measuring lengths

1. *Length of string*

When measuring the length of the string, we measured it between the middle of the two strings, from the bottom and top of the aluminum at the top and bottom parts of the string respectively. We took great care to ensure that the pendulum was at rest when making the measurement, and kept our heads level with the ruler, and one eye closed when taking the reading in an attempt to minimize any misreading due to parallax. We also rested the Hultafors meter-rule against a right-angle ruler in an additional attempt to keep it stable and upright. We took 3 repeated readings, one of which was performed by a different person.

After we had completed our measurements with the hourglass, we noticed a rather large initial rotational and sideways instability in the pendulum, leading us to (mistakingly, see section IV E) shorten the string by roughly 10cm. After having changed the length of the string, we repeated the exact same procedure of measuring the new length except we only did two measurements, again alternating who took the reading.

2. *Dimensions of cylinder*

The cylinder, and the T-bracket on top where the string is connected was also measured using the Hultafors meter-rule. This time, we only made one measurement per length, all done by the same person. Other than that, similar considerations were made as when measuring the string.

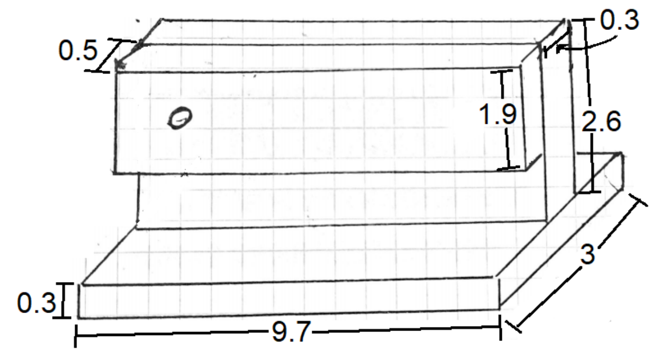

FIG. 3. A sketch showing the dimensions of the T-bracket connected to the cylinder. All numbers in cm with uncertainty $\pm 0.1 cm$ (Not to scale)

IV. RESULTS

A. Recorded lengths and dimensions

The dimensions of the pendulum, consisting of an aluminum T-bracket connected to a cylinder is given in Fig. 3 and table II respectively. The length at which it was suspended is given in table I

TABLE I. Length of string

Measurement No.	1	2	3
Length, Hourglass experiment [cm] ($\pm 0.1 cm$)	53.4	52.9	52.9
Length, Stopwatch & Photo-diode experiment [cm] ($\pm 0.1 cm$)	45.3	45.2	

TABLE II. Dimensions of Cylinder

Height [cm] (± 0.1)	10.2
Diameter [cm] (± 0.1)	9.7

B. Period recorded with hourglass

The number of oscillations during the draining of the hourglass is listed on table III

TABLE III. Measurements with Hourglass

Recording no.	Number of oscillations (± 1)
1	116
2	121
3	128
Mean	122
Std. Dev.	5

C. Period recorded with stopwatch

The period of the pendulum measured with the stopwatch is listed on table IV. We also recorded the duration of the Hourglass using the stopwatch, listed in table V

TABLE IV. Measurements with Stopwatch

Number of periods	Time per 10 oscillations [sec] (± 0.01)	Total time [min:sec] (± 0.01)
10	14.14	14.14
20	14.31	28.53
30	14.32	42.85
40	14.40	57.25
50	14.29	1:11.54
60	14.39	1:25.93
70	14.29	1:40.72
80	14.34	1:54.46
90	14.20	2:08.76
100	14.52	2:23.28
Mean period	14.33	N/A
Std. Dev	0.10	N/A

TABLE V. Duration of Hourglass measured with stopwatch

Recording no.	Time [min:sec] ($\pm 0.01 s$)
1	3:08:40
2	3:08:28

D. Period recorded with photo-diode

Fig.4, Fig.5, Fig.6 and Fig.7 shows the result from experiments 1-4 listed on table VI respectively.

TABLE VI. Measurements with photo-diode

Experiment no.	Standard deviation of mean period	Mean period	Position of diode	Total measured time [s]	Measuring frequency [KHz]
1	5.6540e-4	1.4421	Bottom	120	25
2	8.8552e-4	1.4502	Bottom	120	200
3	0.0483	1.5816	top	120	25
4	0.0026	1.4922	Bottom	120	25

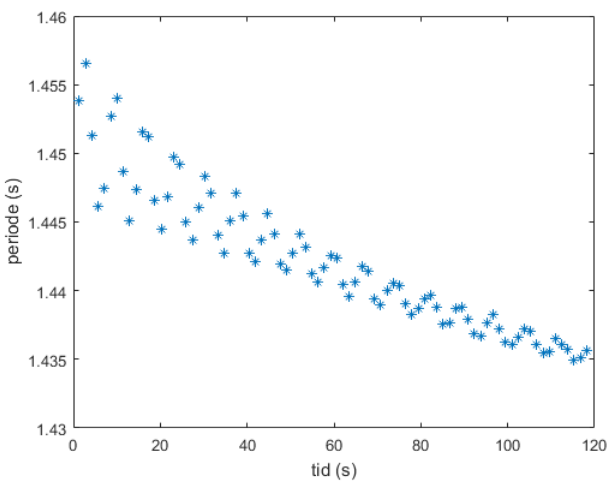

FIG. 4. Data from Experiment no. 1 using the photo-diode [a]

[a] Due to poor planning on my part, this figure, and the following all lack figure titles. I mistakingly did not keep all of the .mat files

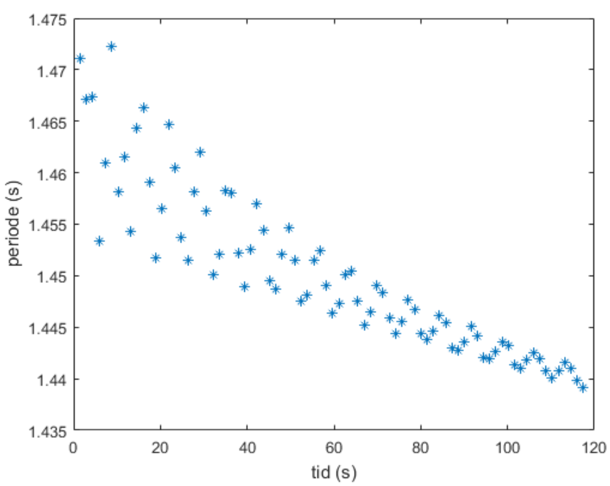

FIG. 5. Data from Experiment no. 2 using the photo-diode

E. Problems

As mentioned in the introduction, we did make one rather large experimental error, which does taint our data somewhat. After having completed our measurements with the Hourglass, we noticed a rather large initial sideways and rotational instability when attempting to start the pendulum for our measurements with the stopwatch. In order to minimize this instability, we decided to shorten the string, not considering the ramification this had on our data.

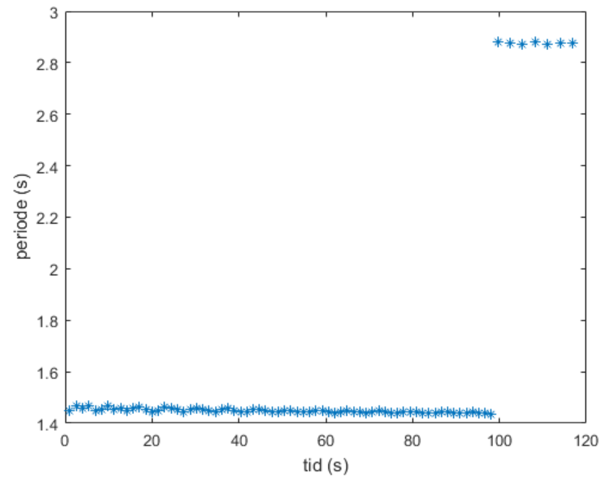

FIG. 6. Data from Experiment no. 3 using the photo-diode

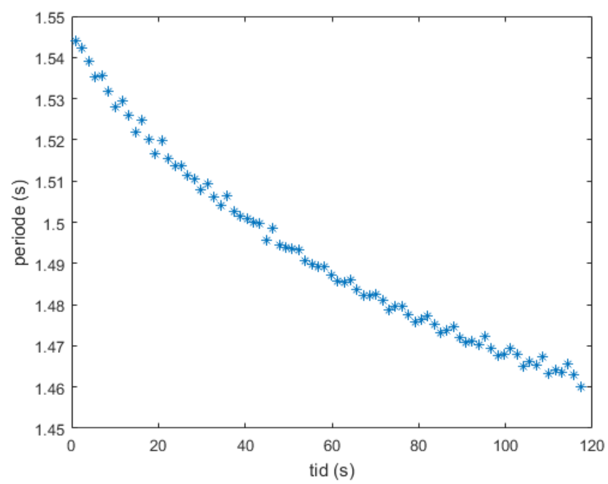

FIG. 7. Data from Experiment no. 4 using the photo-diode

V. DISCUSSION

A. Theoretical period

1. Center of mass

The first thing to consider, and compare the measured results against would be the theoretical period of the system. Since the pendulum is roughly symmetrical in certain planes, i will assume it is and only look for the vertical component of the center of mass. Using Equation 2 and my measurements in section IV A [2] i found that the center of mass of the pendulum is $\sim 7.79cm$ from the top of the pendulum. If i had disregarded the mass of the T-bracket the C.M of the cylinder would have been at $8cm\,(\pm 0.1cm)$, meaning it may be more accurate to simply neglect accounting for the T-bracket in this case. When calculating the uncertainty in the CM when accounting for the T-bracket (which is very tedious to do) i found that only one of the volumes would have an uncertainty of $\pm 2mm$. So the total uncertainty would be so large that it simply makes more sense to "ignore" the T-bracket when calculating the period of the pendulum.

2. Period

Using equation 1 along with the center of mass from the previous subsection i find that the theoretical period of the pendulum for the 52.9cm and 45.2cm strings respectively are 1.56s($\pm$0.02) and 1.46s($\pm$0.02) respectively. (Not accounting for the uncertainty that comes from my assumption that only the cylinder contributes to the C.M)

B. The hourglass

The most apparent problem with using the hourglass as a timekeeping tool is the rather large inconsistency it presents. While granted, we did only perform 3 readings of its duration using the pendulum, the spread of values we did get were rather significant with a standard deviation of 5 periods, equivalent of $\sim 8s$. The inconsistency of the hourglass was again observed when we timed it using a stopwatch (see table V), observing a $12s$ difference between our two readings. This makes the hourglass a highly inconsistent, and inaccurate tool for measuring time in general.

C. The stopwatch

In stark contrast to the hourglass, the stopwatch is a seemingly much more accurate tool for measuring time, with a standard deviation of only $0.1s$. While the stopwatch itself is, theoretically, an excellent tool for measuring the period of a pendulum in terms of its error, one does have to consider the effects of human error in this experiment, which ultimately is what makes up the inconsistency in these results. The person recording the period has to both react and judge when the pendulum is at its apex. Either way, the mean of our recordings divided by 10 results in a mean recorded period of 14.3s, while reasonably close to the theoretical period, it is outside of three standard deviations away from the mean.

D. The photo-diode

As expected, the digital, automated solution comes out as the most accurate and precise with a standard deviation in order of magnitude $10^{-4}s$ in ideal conditions (As in experiment no. 1 & 2, table VI). The "ideal conditions" being very important here. As we tested in experiment no. 3, table VI changing the position of the photo diode can have a quite significant impact on the quality of data. Note the sudden significant jump in period at $\sim 100s$ in Fig. 6, where we aimed the photo-diode much further up on the cylinder. This "error" arose because the photo-diode no longer consistently reflected its light on the aluminum foil of the pendulum. So while the photo-diode is incredibly precise and accurate, it does require careful set up to ensure that it can make its recordings properly due to how inflexible it is.

Lastly, in regards to experiment no. 4. While it does have a standard deviation that is an order of magnitude higher than for 1 & 2, this is likely due to the initial instability of the system after having received it's initial push. In retrospect, it would have been interesting to see how the period evolved in a longer timespan than just 120 seconds. The same goes for experiment 1 & 2 as well, even though they are much more stable in general, it does seem that the readings has a tendency to converge after a while.

VI. CONCLUSION

In summary, having compared the three different methods of measuring time, i think it's safe to disregard this particular hourglass as a reliable source of timekeeping. I would much rather trust my own counting than its timekeeping capabilities. As for the Stopwatch and photo-diode, i believe it's a case of what is being measured. For a uniform, controlled system such as this pendulum, the photo-diode is the most suitable and accurate of the two. Where as the Stopwatch is much more flexible in its usage, making it a much safer general choice for time keeping compared to the photo-diode, even though it is far less accurate and prone to human error.

[1] Anders Malthe-Sorenssen. *Elementary Mechanics Using Python*. Springer, 2015.

[2] The volume of the parts serve as the mass in this case, as the entire pendulum is made of the same material.