Project 1

FYS3150 - Computational Physics

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Abstract

This is an abstract

1 Introduction

2 Theory

We have second order inhomogeneous differential equation, Eqn. 1

$$\frac{d^2u(x)}{dx^2} = f(x) \tag{1}$$

With boundary conditions $\ddot{u}(x) = f(x)$, $u(0) = u(1) = 0$ and $x \in (0, 1)$.

If we let $f(x) = 100e^{-10x}$, then it can be shown analytically that the differential equation has a solution Eqn. 2

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \tag{2}$$

This problem can also be solved numerically by discretization. By taylor expansion in the form Eqn. 3

$$-\frac{v_{i+1} + v_{i-1} + 2v_i}{h^2} = f_i \tag{3}$$

where $h = 1/(n+1)$ is the step size for $n+1$ points.

This problem can be written in the form

$$A\vec{v} = \vec{b} \tag{4}$$

Where A $\in R^{n\times n}$ is a tridiagonal matrix with diagonal elements 2 and -1 (Eqn. 5) and $\vec{b} = h^2(f_1, \dots, f_n)$.

$$A = \begin{bmatrix} 2 & -1 & 0 & \dots & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & -1 \\ \dots & \dots & \dots & \dots & -1 & 2 \end{bmatrix} \tag{5}$$

By matrix multiplication, if one multiplies the i-th row of A by $\vec{v}$, we get

$$\begin{aligned} -1v_{i-1} + 2v_i - 1v_{i+1} &= -\left(v_{i+1} + v_{i-1} - 2v_i\right) \\ &= h^2 f_i \end{aligned} \tag{6}$$

Which is equivalent to the discretized differential equation Eqn. 3, showing that the differential equation can be solved as a linear algebra problem rather than iteratively.

2.1 Algorithms

2.2 Tests

The algorithms which were outlined in section 2.1 were implemented in Python 2.7, and the all of the source code can be found on my github: https://github.com/nicholaskarlsen/FYS3150

2.3 Discussion