Disease Modeling - FYS3150 Computational Physics

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INTRODUCTION

THEORY, ALGORITHMS AND METHODS

Runge-Kutta methods

Markov Chains

A Markov Chain aims to model the behavior of stochastic processes, that is models which time-evolution of each parameter x_i is governed by a set of transition probabilities $P(x_i \to x_j)$.

The k-th state of the system is given by the state vector $\mathbf{x}_k$, a column vector with entries x_i for each parameter of the system.

$$\mathbf{x}_k = \begin{pmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{pmatrix} \tag{1}$$

The transition probabilities $P(x_i \to x_j)$ are contained within a Stochastic matrix, M, an $n \times n$ matrix which columns are probability vectors that sum to unity. M is such that multiplying it by a state $\mathbf{x}_k$ determines the next state, $\mathbf{x}_{k+1}$

$$M\mathbf{x}_k = \mathbf{x}_{k+1} \tag{2}$$

From Theorem 18 in Lay [1, p. 277], if M is a regular stochastic matrix there exists a unique steady state vector $\mathbf{q}$ which the system will converge towards as $k \to \infty$ characterized by

$$\mathbf{x}_k = \mathbf{x}_{k+1} \tag{3}$$

Regardless of the initial state, $\mathbf{x}_0$.

SIRS Model

The SIRS model is a type of mathematical model describing how infectious disease evolves within a population, and is a part of a family of similar models in epidemiology with various different features. In the SIRS model, the total population (N) is divided into three groups

- Susceptible (S) : People who do not have the disease, and are not immune to it.
- Infected (I) : People who are infected with the disease.
- Recovered (R) : People who has recovered from the infection, and have developed immunity.

Where the permitted traversal from one group to another follows a cyclical pattern $S \to I \to R \to S$.

The rate of traversal is governed by a set of coupled differential equations,

$$\begin{aligned} S' &= cR - \frac{aSI}{N} \\ I' &= \frac{aSI}{N} - bI \\ R' &= bI - cR \end{aligned} \tag{4}$$

Where the constants a, b, c are governing the

- rate of transmission
- rate of recovery
- rate of immunity loss

respectively. This system can also be modelled as a Stochastic system described as a Markov Chain, shown as a Markov diagram in Fig. 1. If we consider a small, finite timeinterval Δt, we can approximate the transition probabilities $P(x_i \to x_j)$ from Eqn. 4

$$\begin{aligned} P(S \to I) &= \frac{aSI}{N}\Delta t \\ P(I \to R) &= bI\Delta t \\ P(R \to S) &= cR\Delta t \end{aligned} \tag{5}$$

It follows then, that the system will reach a steady state after a finite number of transitions for any set of initial conditions as discussed in Section

RESULTS AND DISCUSSIONS

CONCLUSIONS

[1] D. C. Lay, *Linear Algebra and Its Applications*, 5th ed. (Pearson, 2016).

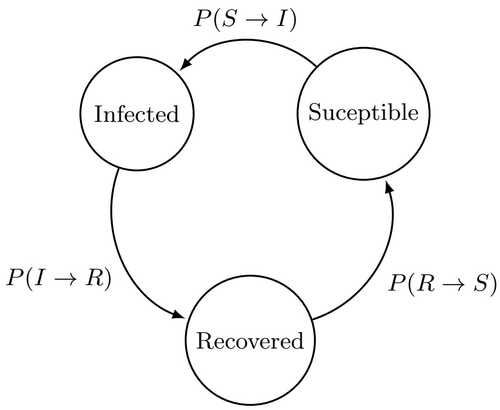

FIG. 1: The SIRS Model