Project 1

FYS3150 - Computational Physics

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Abstract

This is an abstract

1 Introduction

2 Method

We have second order inhomogeneous differential equation, Eqn. 1

$$\frac{d^2u(x)}{dx^2} = f(x) \tag{1}$$

With boundary conditions $\ddot{u}(x) = f(x)$, $u(0) = u(1) = 0$ and $x \in (0, 1)$.

If we let $f(x) = 100e^{-10x}$, then it can be shown analytically that the differential equation has a solution Eqn. 2

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \tag{2}$$

This problem can also be solved numerically by discretization. By taylor expansion in the form Eqn. 3

$$-\frac{v_{i+1} + v_{i-1} + 2v_i}{h^2} = f_i \tag{3}$$

where $h = 1/(n+1)$ is the step size for $n+1$ points.

This problem can be written in the form

$$A\vec{v} = \vec{b} \tag{4}$$

Where A $\in R^{n \times n}$ is a tridiagonal matrix with diagonal elements 2 and -1 (Eqn. 5) and $\vec{b} = h^2(f_1, \ldots, f_n)$.

$$A = \begin{bmatrix} 2 & -1 & 0 & \ldots & \ldots & 0 \\ -1 & 2 & -1 & 0 & \ldots & 0 \\ 0 & -1 & 2 & -1 & \ldots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & -1 \\ \ldots & \ldots & \ldots & \ldots & -1 & 2 \end{bmatrix} \tag{5}$$

By matrix multiplication, when we multiply the i-th row of A by $\vec{v}$, we get

$$\begin{aligned} -1v_{i-1} + 2v_i - 1v_{i+1} &= -\left(v_{i+1} + v_{i-1} - 2v_i\right) \\ &= h^2 f_i \end{aligned} \tag{6}$$

Which is equivalent to the discretized differential equation Eqn. 3, showing that the differential equation can be solved as a linear algebra problem rather than iteratively.

2.1 Analysis

The algorithms which were outlined in section 2 were implemented in Python 2.7, and the all of the source code can be found on my github: https://github.com/nicholaskarlsen/FYS3150

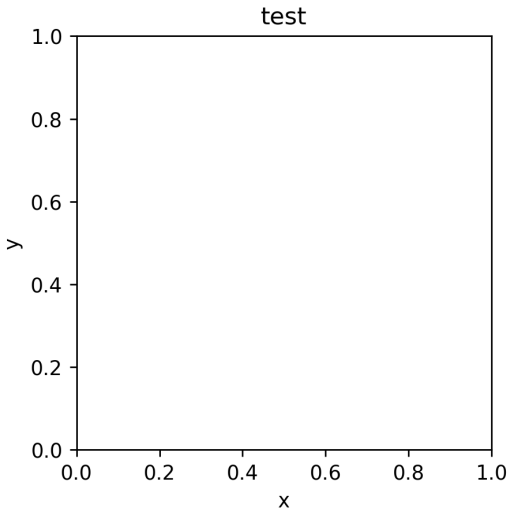

Figure 1: This is a caption

2.2 Discussion