

Project 1

FYS3150 - Computational Physics

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Abstract

This is an abstract

1 Introduction

2 Method

We have second order inhomogeneous differential equation, Eqn. 1

$$\frac{d^2 u(x)}{dx^2} = f(x) \quad (1)$$

With boundary conditions $\ddot{u}(x) = f(x)$, $u(0) = u(1) = 0$ and $x \in (0, 1)$.

If we let $f(x) = 100e^{-10x}$, then it can be shown analytically that the differential equation has a solution Eqn. 2

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \quad (2)$$

This problem can also be solved numerically by discretization. By Taylor expansion in the form Eqn. 3

$$-\frac{v_{i+1} + v_{i-1} + 2v_i}{h^2} = f_i \quad (3)$$

where $h = 1/(n+1)$ is the step size for $n+1$ points.

This problem can be written in the form

$$A\vec{v} = \vec{b} \quad (4)$$

Where $A \in R^{n \times n}$ is a tridiagonal matrix with diagonal elements 2 and -1 (Eqn. 5) and $\vec{b} = h^2(f_1, \dots, f_n)$.

$$A = \begin{bmatrix} 2 & -1 & 0 & \dots & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & -1 \\ \dots & \dots & \dots & \dots & -1 & 2 \end{bmatrix} \quad (5)$$

By matrix multiplication, when we multiply the i -th row of A by \vec{v} , we get

$$\begin{aligned} -1v_{i-1} + 2v_i - 1v_{i+1} &= -(v_{i+1} + v_{i-1} - 2v_i) \\ &= h^2 f_i \end{aligned} \quad (6)$$

Which is equivalent to the discretized differential equation Eqn. 3, showing that the differential equation can be solved as a linear algebra problem rather than iteratively.

2.1 Analysis

The algorithms which were outlined in section 2 were implemented in Python 2.7, and the all of the source code can be found on my github: <https://github.com/nicholaskarlsen/FYS3150>

2.2 Discussion