# Disease Modeling - FYS3150 Computational Physics

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## INTRODUCTION

## THEORY, ALGORITHMS AND METHODS

## Runge-Kutta methods

#### **Markov Chains**

A Markov Chain aims to model the behavior of stochastic processes, that is models which time-evolution of each parameter  $x_i$  is governed by a set of transition probabilities  $P(x_i \to x_j)$ .

The k-th state of the system is given by the state vector  $\mathbf{x}_k$ , a column vector with entries  $x_i$  for each parameter of the system.

$$\mathbf{x}_{k} = \begin{pmatrix} x_{1} \\ \vdots \\ x_{i} \\ \vdots \\ x_{n} \end{pmatrix} \tag{1}$$

The transition probabilities  $P(x_i \to x_j)$  are contained within a Stochastic matrix, M, an  $n \times n$  matrix which columns are probability vectors that sum to unity. M is such that multiplying it by a state  $\mathbf{x}_k$  determines the next state,  $\mathbf{x}_{k+1}$ 

$$M\mathbf{x}_k = \mathbf{x}_{k+1} \tag{2}$$

From Theorem 18 in Lay [1, p. 277], if M is a regular stochastic matrix there exists a unique steady state vector  $\mathbf{q}$  which the system will converge towards as  $k \to \infty$  characterized by

$$\mathbf{x}_k = \mathbf{x}_{k+1} \tag{3}$$

Regardless of the initial state,  $\mathbf{x}_0$ .

## SIRS Model

The SIRS model is a type of mathematical model describing how infectious disease evolves within a population, and is a part of a family of similar models in epidemiology with various different features. In the SIRS model, the total population (N) is divided into three groups

• Susceptible (S): People who do not have the disease, and are not immune to it.

- Infected (I): People who are infected with the disease.
- Recovered (R): People who has recovered from the infection, and have developed immunity.

Where the permitted traversal from one group to another follows a cyclical pattern  $S \to I \to R \to S$ .

The rate of traversal is governed by a set of coupled differential equations,

$$S' = cR - \frac{aSI}{N}$$

$$I' = \frac{aSI}{N} - bI$$

$$R' = bI - cR$$
(4)

Where the constants a, b, c are governing the

- rate of transmission
- rate of tecovery
- rate of tmmunity loss

respectively. This system can also be modelled as a Stochastic system described as a Markov Chain, shown as a Markov diagram in Fig. 1. If we consider a small, finite timeinterval  $\Delta t$ , we can approximate the transition probabilities  $P(x_i \to x_j)$  from Eqn. 4

$$P(S \to I) = \frac{aSI}{N} \Delta t$$

$$P(I \to R) = bI \Delta t$$

$$P(R \to S) = cR \Delta t$$
(5)

It follows then, that the system will reach a steady state after a finite number of transitions for any set of initial conditions as discussed in Section

## RESULTS AND DISCUSSIONS

## CONCLUSIONS

[1] D. C. Lay, *Linear Algebra and Its Applications*, 5th ed. (Pearson, 2016).

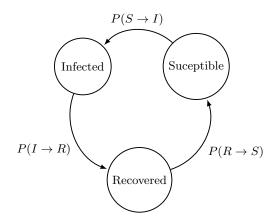


FIG. 1: The SIRS Model