MATH 4620 Exercise Set

For the 7th Edition

****

HW 1
IV.18 # 12, 20, 38, 41, 46, 50, 55, 56
IV.19 # 23, 26, 29
****

HW 2
IV.20 # 10, 14, 28, 29
IV.21 # 5
IV.22 # 14, 16, 24, 27, 31
IV.23 # 34, 35
****

HW 3
Compute the product

$$(0\iota + 1(123) + 0(132) + 0(12) + 1(13) + 1(23))(1\iota + 1(123) + 0(132) + 1(12) + 0(13) + 1(23))$$

in the group ring $\mathbb{Z}_2 S_3$.
IV.24 # 16, 17
V.26 # 17, 18, 22
(Second isomorphism theorem for rings) Let M and N be ideals of a ring R and let

$$M + N = \{m + n \mid m \in M, n \in N\}.$$

Show that $M+N$ is an ideal of R and that $(M+N)/N$ is naturally isomorphis to $M/(M \cap N)$. (Use Theorem 34.5 to help you.)

Third isomorphism theorem for rings) Let M and N be ideals of a ring R such that $M \leq N$. Show that there is a natural isomorphism mapping R/N onto $(R/M)/(N/M)$. (Use Theorem 34.7 to help you.)
V.27 # 35, 36, 37, 38

****

HW 4
IV.25 # 6, 12, 16
Let R be a ring with unit element. Using its elements we define a ring $\widetilde{R}$ by defining $a \oplus b = a + b + 1$, and $a \cdot b = ab + a + b$, where $a, b \in R$ and where the addition and multiplication on the right-hand side of these relations are those of R.
(a) Prove that $\widetilde{R}$ is a ring under operations $\oplus$ and $\cdot$.
(b) What acts as the zero-element of $\widetilde{R}$?
(c) What acts as the unit-element of $\widetilde{R}$?
(d) Prove that R is isomorphic to $\widetilde{R}$.

VI.29 #5, 24, 31, 32, 34

****
HW 5
VI.30 #21, 23, 24, 25
VI.31 #10, 23, 25, 27

****
HW 6
VI.30 #26, 27
VI.31 #13
VI.32 #1, 5, 6

Dr. Frey's Bonus (but required) Fun Problems (These problems go together.)

1. Show that the polynomial $x^3 + x^2 - 2x - 1$ is irreducible over $\mathbb{Q}$.

2. Show that $2\cos(2\pi/7)$ satisfies $x^3 + x^2 - 2x - 1 = 0$. (Hint: Use the fact that $2\cos(2\pi/7) = e^{2\pi i/7} + e^{-2\pi i/7}$.)

3. Show that it is impossible to construct a regular septagon by straightedge and compass.

****
HW 7
VI.33 #9,10

Dr. Frey's Bonus (but required) Fun Problems (These problems go together.)

1. Construct a field with 64 elements as we did in class (with the conversion table between additive and multiplicative elements).

2. Construct a field with 49 elements as we did in class (with the conversion table between additive and multiplicative elements).

3. Suppose that F is a field of order 1024 and $F^\times = \langle \alpha \rangle$. List the elements of each subfield of F.

****
HW 8
X.48 #16, 18, 20, 34, 36
X.50 #20

****
HW 9
X.50 #22 X.54 #4

****

HW 10

Dr. Frey's Fun (but required) Bonus Questions:

1. Find $G(K, F)$ where $K = \mathbb{Q}\left(\sqrt{2}, \sqrt[3]{2}, e^{\frac{2\pi i}{3}}\right)$ and $F = \mathbb{Q}$.
2. Calculate the splitting field and the Galois group of the polynomial $f(x) = x^4 + x^2 + 1$ over $\mathbb{Q}$. To what familiar group is the Galois group isomorphic? Show the correspondence between subgroups and intermediate fields noting which subfields are normal and which are not.
3. Calculate the splitting field and the Galois group of the polynomial $f(x) = x^3 - 2$ over $\mathbb{Q}$. To what familiar group is the Galois group isomorphic? Show the correspondence between subgroups and intermediate fields noting which subfields are normal and which are not.
4. Show that the Galois group of the polynomial $f(x) = x^5 - 10x^4 + 2x^3 - 24x^2 + 2$ over $\mathbb{Q}$ is S_5. Is the equation $f(x) = 0$ solvable by radicals?

****

HW 11
IX.45 #10, 25, 26, 30
IX.46 #2, 4, 12, 17
IX.47 #6
IX.49 #6

****