Nick Kerner
Homework 8
Chapter 6: 9
Dr Frey's Bonus but required fun question
Chapter 7: K2, K3, K5

Geometry

Chapter 6: problem 9

Given $X \neq P$ on l, by proposition 6.6 we know there are two limiting parallel rays to l' emanating from P (on opposite sides of $\overleftrightarrow{PQ}$. Choose the one on the same side of $\overleftrightarrow{PQ}$ as X and drop a perpendicular from X to our limiting parallel ray and call the foot Y. Know we know that $\overrightarrow{PY}$ is interior to $\angle QPX$ ($\overrightarrow{PY} \neq \overrightarrow{PX}$ as limiting parallel rays cannot have a common perpendicular to their respective line by Hilbert's Hyperbolic axiom of parallels, so XY is a segment not a point).

Case 1: $\overrightarrow{PY}$ is interior to $\angle XPX'$

Then by crossbar theorem we know that $\overrightarrow{PY}$ must intersect XX'.

Case 2: $\overrightarrow{PY} = \overrightarrow{PX'}$

Our limiting parallel cannot intersect l'. Contradiction.

Case 3: $\overrightarrow{PY}$ is interior to $\angle QPX'$.

By crossbar theorem, $\overrightarrow{PY}$ intersects QX', but our limiting parallel cannot intersect l'. Contradiction.

Therefore we know that $\overrightarrow{PY}$ intersects XX' (not at an endpoint as it would be the parallel with common perpendicular to l' or it would intersect l') call this point Z.

Case 1: Y = Z.

$XY = XZ$, clearly.

Case 2: $Y \neq Z$.

Consider $\triangle XYZ$. We know that $\angle XYZ$ is a right angle, so its measure is 90°, so since we are in hyperbolic a triangle's angle sum must be less than 180, so both other angles have measures less than 90°. So we know by proposition 4.5 that since $\angle XYZ > \angle XZY$ then $XZ > XY$.

Hence $XZ \geq XY$.

To use Aristotle's axiom, we must show that $\angle XPY$ is acute. Now we know that $\angle QPY$ is acute (proposition 6.6) and we know that it is interior to $\angle QPX$ which is right, so by angle subtraction we know that $\angle XPY$ is acute.

By Aristotle's axiom, given any segment AB (A on l, B on $\overrightarrow{PY}$), there is some X with foot Y such that $XY > AB$. So we know by segment addition that $XX' > XZ \geq XY > AB$.

QED

Dr Frey's Bonus but required fun question
Proof:

Assume to the contrary that $\delta(AMN) = \delta(ABC)$. By proposition 6.1 we know that $\delta(ABC) = \delta(MCB) + \delta(MCA)$. Additionally we know that $\delta(MCA) = \delta(MNC) + \delta(AMN)$, so we know that $\delta(ABC) = \delta(MCB) + \delta(MCA) = \delta(MCB) + (\delta(MNC) + \delta(AMN)) = \delta(MCB) + \delta(MNC) + \delta(ABC)$. Therefore $\delta(MCB) + \delta(MNC) = 0$ so $\delta(MCB) = 0$ and $\delta(MNC) = 0$. Contradiction. Hence $\triangle AMN$ and $\triangle ABC$ cannot be similar.

QED

Proof:

Assume to the contrary that $MN \cong BL$. Choose D such that $M * N * D$ and $ND \cong MN$. So $\angle CND$ and $\angle MNA$ are vertical angles, $MN \cong ND$ and $AN \cong NC$ so by SAS we know that $\triangle ANM \cong \triangle CND$.

So we know that $BM \cong MA \cong DC$.

Additionally we know that

$$\begin{aligned} MD &= MN + ND \\ &\cong MN + MN \\ &\cong BL + BL \\ &\cong BL + LC \\ &= BC \end{aligned}$$

and $MC \cong MC$ so by SSS we know that $\triangle MBC \cong \triangle CDM$.

Finally we know that $\angle BMC^\circ + \angle CMD^\circ + \angle DMA^\circ = 180$ as they make a straight line. So (substitutions are either renaming angles or derived from congruent triangles).

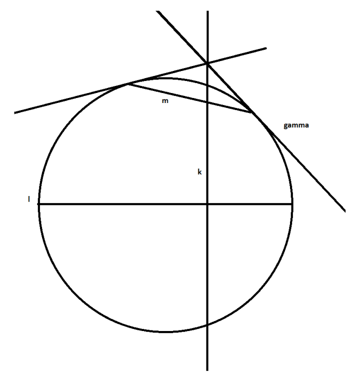
$$
\begin{aligned}
180 &= \angle BMC^\circ + \angle CMD^\circ + \angle DMA^\circ \\
&= \angle MCD^\circ + \angle CMD^\circ + \angle DMA^\circ \\
&= (\angle MCN^\circ + \angle NCD^\circ) + \angle CMD^\circ + \angle DMA^\circ \\
&= \angle MCN^\circ + \angle NAM^\circ + \angle CMD^\circ + \angle DMA^\circ \\
&= \angle NAM^\circ + \angle MCN^\circ + \angle CMD^\circ + \angle DMA^\circ \\
&= \angle NAM^\circ + \angle MCN^\circ + \angle MCB^\circ + \angle DMA^\circ \\
&= \angle NAM^\circ + (\angle MCN^\circ + \angle MCB^\circ) + \angle DMA^\circ \\
&= \angle NAM^\circ + \angle NCB^\circ + \angle DMA^\circ \\
&= \angle CAB^\circ + \angle ACB^\circ + \angle DMA^\circ \\
&= \angle CAB^\circ + \angle ACB^\circ + \angle MDC^\circ \\
&= \angle CAB^\circ + \angle ACB^\circ + \angle CBM^\circ \\
&= \angle CAB^\circ + \angle ACB^\circ + \angle CBA^\circ
\end{aligned}
$$

So $\delta(ABC) = 0$. Contradiction.

QED

Chapter 7: K2

a.



b.

c.

Chapter 7: K3

Chapter 7: K5