ORF435 Final Project

Nicholas Kim

Section 1: Ms. Kim's Individual Retirement

The Goal:
The goal we will look at is relatively straightforward. Given a certain post-retirement annual spending rate, the expected remaining lifetime at the retirement age, and the discount rate, the desired goal will be the total amount of cash required at the moment of retirement to sustain all the discounted outflows from annual spending. Here, this will be in terms of a % of the final salary as the initial annual expense, which will then increase at the inflation rate until death.

We note that for this report, the expected remaining lifetime for ages 67 and 70 are 17 years and 15 years respectively. [(*as interpolated from HW8's specification)]

Defining the "Best" Strategy:
In determining the optimal retirement strategy, we take a holistic approach centered primarily around the log-utility of money and risk analysis. The basic framework of our approach is as such.

Factor 1) - Summation of yearly spending utilities (or $\sum_i ln(s_i)$, where s_i = money spent during year i))
-Each year, an individual receives an amount of utility proportional to the log-utility of spending (eg, they make 150,000 and save 12%, so that year they spent 132,000 to add ln(132,000) to their "lifetime utility".)
-This utility can be summed across years
-Therefore, an optimal retirement strategy is the one that offers a high lifetime utility, defined as the summation of annual utilities

There are two core variables to consider for this model, where we analyze the later as a function of the former.

1) **Annual Savings Amount** - The percentage of annual salary to be put into savings
2) **Post-Retirement Spending Amount** - The average annual spending post-retirement (as a proportion of the final pre-retirement salary)

The amount that one can expect to spend post-retirement is a distribution with directive positive correlation to the annual savings amount. Then, the basic question/interplay of the two variables can be stated as such. A higher savings amount corresponds to a lower utility/standard-of-living pre-retirement, and a higher-post retirement spending amount corresponds to a higher utility/standard-of-living post-retirement. The higher one's saving amount is, the more one can spend more post-retirement, but at a certain point, excessive saving may impact quality of life pre-retirement enough to not justify the utility increase post-retirement.

Caveats to utility: There are some caveats to the usage of these utilities, which is why we do not base our analysis solely around it. For one, it assumes that one gets an equal utility from spending x amount of money at any point in their lives, which is not necessarily true as ones preferences, needs, and desires fluctuate/move around certain intermediate goals not specified in this planning problem. In addition, for most normal consumers, log utilities can over-value extreme upsides when it comes to wealth. For instance, if at retirement, one had more than enough to comfortably sustain life, and then one had the option of gaining 2x their savings at 50% chance, and losing 50% at the same probability, most, barring some rare external motivations, would find this an overly risky prospect with more to lose than gain (eg, the upside may be gaining a bigger house, and the downside may entail losing the ability to travel and pay rent).

As such, we introduce a "utility-cap" for post-retirement spendings at 80% of the final salary annually. This avoids optimizing for extraneously aggressive savings that may arise in the utility representation.

Factor 2) - Downside Risk Index As per the above concerns with pure utility and the nature of "quantized expenses" that may make savings below certain thresholds much worse than the utility would imply, we add a weighting based on the logarithms of the CGaR and downside risk L1 metrics. The risk metrics are defined as such.

CGaR/Conditional GaR) - Average deficit to goal of all outcomes at/below the 5th percentile
Downside Risk L1) - Average of all outcomes short of the goal

(Note that in our analysis, we also compute three other risk metric for qualitative understanding purposes. These are the variance of average savings, GaR (similar to CGaR, but instead of the average deficit, simply the outcome at the 5th percentile), and probability of goal)

The final proposed goal measure is Factor 1) - Factor 2), which translates to
Best Strategy = $\boxed{max[(\sum_i ln(yearly\ spending)) - (ln(CGaR) + ln(DownsideL1))]}$
where we exclude excess utility in factor 1) coming from above 80% final-salary post retirement spendings.

In short, we seek to find the optimal trade off between the negative utility of saving more, and the positive utility of having more money for retirement, and then account for the specific intertwining of downside risk metrics to supplement the outcomes of the initial utility-based analysis.

Modeling Assumption - We **do not** take it that there is some **fixed post-retirement spending amount**. Instead, **we assume that one will want to spend all accumulated savings over the course of retirement**, with the aim being to remove a portion of arbitrariness in the optimization process. We represent the spending of all accumulated savings in terms of a percentage-of-final-salary annual spending.

(Note: while we do make an explicit recommendation of our analyzed "best strategy" in this report, we also do our best to provide sufficient data to allow Ms. Kim to make her own call on what is best should she please, as we acknowledge the possibility of incomplete information wrt her utility drivers)

Testing Process/Raw Data:

We first generated simulated data based on monthly asset/factor returns and regime switching. In addition, we seek to make this more robust than the standard pure markov approach with the following modification:

Trend Representation: If a market emerges out of a certain regime, it would seem that, at least for a short while, its probability of reverting back into its previous regime should be lower. As such, we include a decay factor, where the relative probability of regime switching back to the previous regime after emerging out of it is made less likely, with this correction linearly decaying over a period of 4 months (with its initial strength being by a factor of 1.5, so say the standard markov regime change percentages are 98% stay the same, 2% switch. The switching probability x, fulfilling x*49 = (1-x), would be changed to fulfill x*49*1.5 = (1-x))

While we lack the data in our given spreadsheet to do so (only a single average treasury return vs. discrete maturities), another hypothetical idea (perhaps for the future) could be to incorporate yield curve data, where an inversion would increase the probability of switching to a crash regime from a normal regime.

Then, we established three asset allocation strategies for savings to test against each other. The allocations are as follows.

Allocation A) Sharpe Optimal (see appendix for calculation process):

0.1% US Equities
29.1% US Treasuries
14.4% High Yield Bonds
23.7% Corp. Bonds
0.0% Real Estate
0.1% Commodities
31.5% Cash

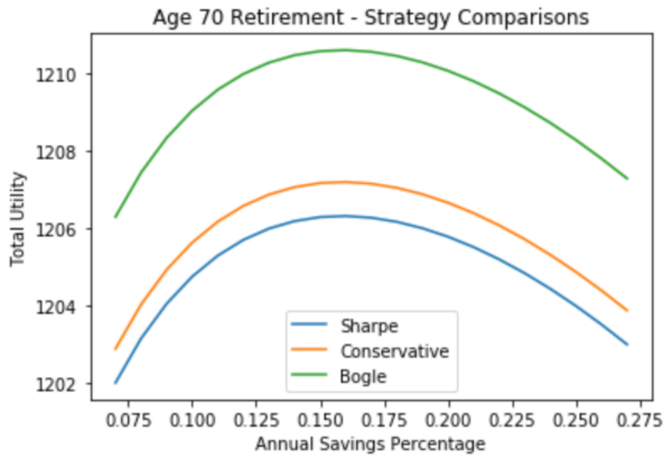
Allocation B) Conservative (bonds/treasuries/cash) -
0.0% US Equities
40.0% US Treasuries
20.0% High Yield Bonds
20.0% Corp. Bonds
0.0% Real Estate
0.0% Commodities
20.0% Cash

Allocation C) Bogle Rule -
(100-age)% US Equities
(age-10)% US Treasuries
0.0% High Yield Bonds
0.0% Corp. Bonds
0.0% Real Estate
0.0% Commodities
10.0% Cash

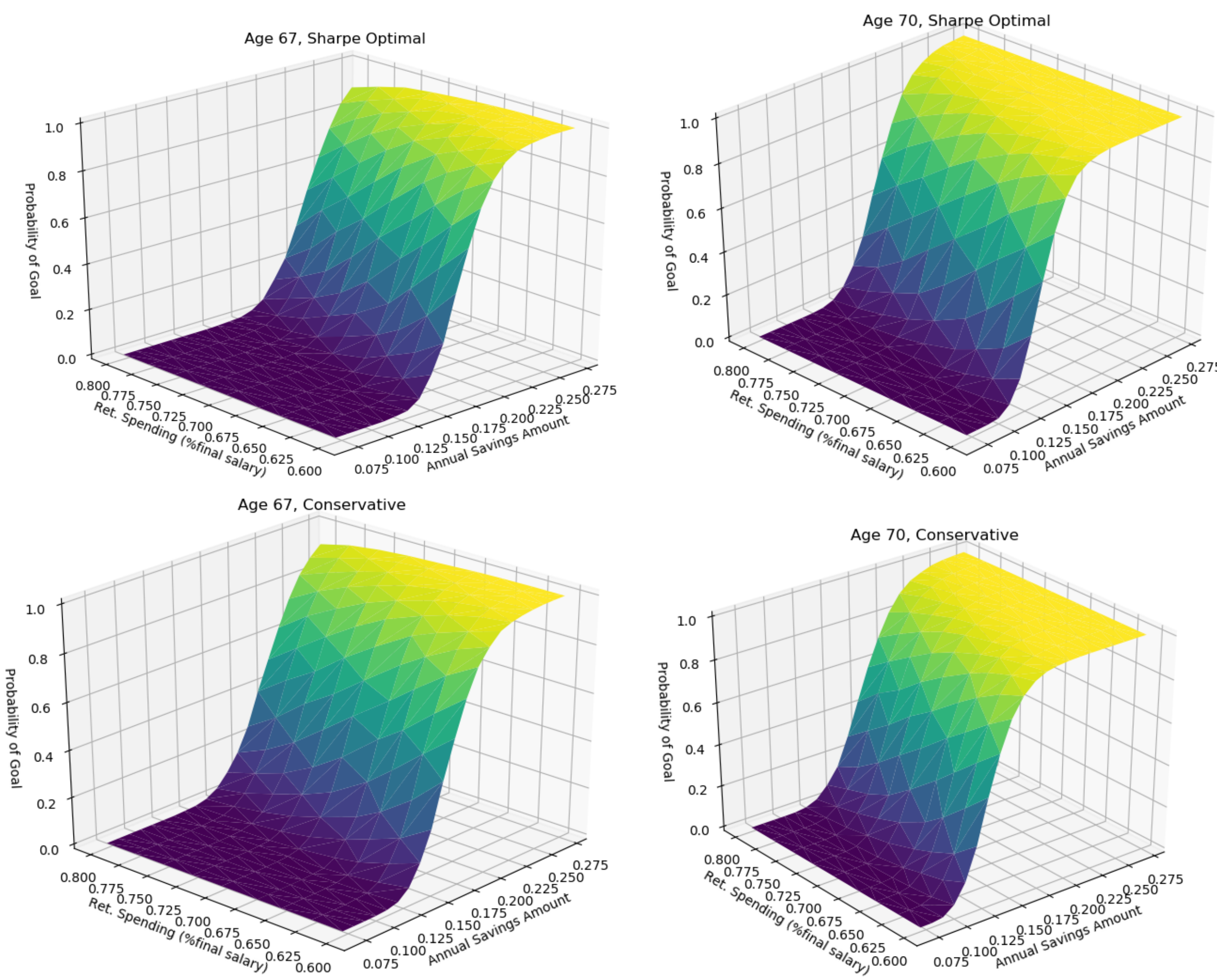
Now, we test the effectiveness of all three strategies against annual saving percentages ranging from **7%-28%**.

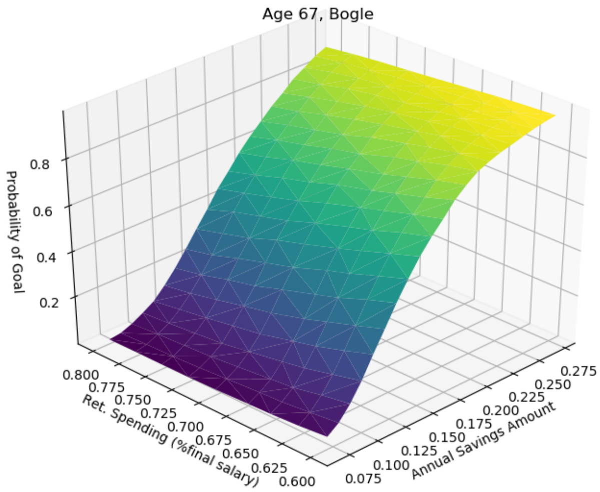
After running through 10,000 scenario simulations, where each scenario runs through a simulated wealth path until 75 years, we observe the following relationship between the total-lifetime-utility and savings percentage for each investment strategy at retirement (67 years and 70 years).

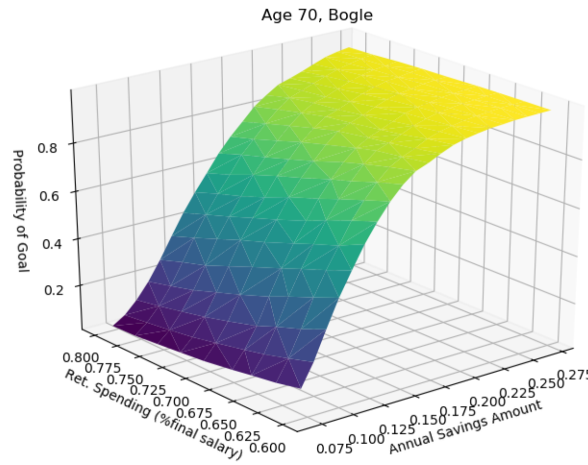
We produce the following plots for the utilities.
NOTE: these are unadjusted for excess utilities above the 80% retirement spending cap. The corrections will be factored in in the following analysis section.)



In addition, we also created 3d contour plots for each strategy, detailing their probabilities of reaching a spending goal (defined as % of final salary per year) for a given savings percentage.







Data Analysis:

Optimal Savings Percentage - As one can see from the initial 2d graphs, there is a concave relationship (and thus a maximum) between savings and utility for all strategies.

For retirement at **67 years**, save **19%** salary per year
For retirement at **70 years**, save **16%** salary per year

Below, we have the risk profiles for each strategy at the optimal savings percentage.
Note - the "probability to hit goal" in the tables is with reference to a 60%-of-final-salary post-retirement annual spending goal. This is constant value is useful for comparison/analysis, but does not relate to the calculation of utility, which again assumes that one expects to spend all their accumulated savings over their expected remaining lifetime as the "maximum retirement spending % of salary", which is simply the percentage-of-final-salary annual spending possible given that one expects to liquidate all their savings (where "max ret spend %salary" refers not the maximum saving percentage ever encountered, but rather the average of the maximum-feasible percentage spending for each trial).

Sharpe Optimal

	Data Type	67 years	70 years
0	Probability to hit Goal	0.77911	0.865067
1	Mean	5.66838e+06	5.68223e+06
2	Variance	5.16932e+11	5.70776e+11
3	GaR	533026	344859
4	CGaR	787644	585717
5	Downside Risk L1	358925	309223
6	Savings%	0.19	0.16
7	Total Utility	1150.03	1206.3
8	Pre-Ret Util	929.169	1009.99
9	Post-Ret Util	220.857	196.307
10	Max Ret. Spend %Salary	0.66454	0.70058

Conservative Allocation

	Data Type	67 years	70 years
0	Probability to hit Goal	0.850075	0.906047
1	Mean	5.99724e+06	6.04317e+06
2	Variance	7.82955e+11	8.73778e+11
3	GaR	438759	240798
4	CGaR	736934	521498
5	Downside Risk L1	384103	327750
6	Savings%	0.19	0.16
7	Total Utility	1150.94	1207.18
8	Pre-Ret Util	929.169	1009.99
9	Post-Ret Util	221.769	197.185
10	Max Ret. Spend %Salary	0.703095	0.745081

Bogle Rule

	Data Type	67 years	70 years
0	Probability to hit Goal	0.851074	0.889055
1	Mean	7.85954e+06	8.00085e+06
2	Variance	8.69432e+12	9.17405e+12
3	GaR	1.06376e+06	700871
4	CGaR	1.55331e+06	1.20285e+06
5	Downside Risk L1	829135	761816
6	Savings%	0.19	0.16
7	Total Utility	1154.62	1210.59
8	Pre-Ret Util	929.169	1009.99
9	Post-Ret Util	225.453	200.593
10	Max Ret. Spend %Salary	0.921424	0.98645

Further Investigation of Utility

The initial utility curves, unadjusted for extraneous retirement savings would seem to suggest that the bogle rule is the best choice for utility maximization. **However**, we must remember that utility functions are heuristic/imperfect, and it would seem there's a relevant investigation. We find that while the bogle rule generates high averaged utility, **most of this upside comes from excess in post-retirement spending**, which we have described counterarguments for (to counting as strongly as they do in utility) in the initial section.

In short, the **Bogle Rule utilities are overestimates** practically speaking. Adjusting for the 80% spending utility cap, the Bogle Rule loses the following utility.
Year 67: -2.4
Year 70: -3.5

Despite this penalization, one can see the Bogle Rule still marginally offers the highest utilities. Analysis of the risk measures however shows additional undesirable properties of the Bogle allocation.

<u>Risk Measure Analysis</u>
First, we note that the Bogle Rule has by far the highest variance, being 10x greater than that of the other two allocations. Additionally, it also has GaR, CGaR, and downside risk that are roughly double that of the other two. In addition to its worse risk profile, its probability of goal is only equal to that of the conservative allocation for 67 years and actually less than it for 70 years.

Thus, the Bogle Rule has a close effective utility to that of the Conservative Allocation, but it has a significantly worse risk profile. **As such, we would prefer the Conservative Allocation to the Bogle Rule**.

Comparing the Conservative Allocation to the Sharpe Optimal one, it would seem that the Conservative Allocation is the clear winner. It offers higher utility, a significantly higher probability of goal (85 vs 78 and 91 vs 87 percent respectively), and roughly equivalent GaR, CGaR, and downside risk.

Explicitly calculating based upon our proposed best-strategy formula, we have

Sharpe 67: 1150.03 - (ln(787644) + ln(358925)) = <u>1123.66</u>
Sharpe 70: 1206.30 - (ln(585717) + ln(3309223)) = <u>1180.38</u>
Conservative 67: 1150.94 - (ln(736934) + ln(384103)) = <u>1124.57</u>
Conservative 70: 1207.18 - (ln(521498) + ln(327750)) = <u>1181.32</u>
Bogle 67: 1152.22 - (ln(1553310) + ln(829135)) = <u>1124.34</u>
Bogle 70: 1207.09 - (ln(1202850) + ln(761816)) = <u>1179.55</u>

The order preference of allocations for these optimal savings percentages is then $\boxed{\text{Conservative} > \text{Bogle} > \text{Sharpe}}$ for year 67, and $\boxed{\text{Conservative} > \text{Sharpe} > \text{Bogle}}$ for year 70

Final Recommendation and Remarks:

Based on the above analysis, we recommend the following.

Final Recommendation

For Retirement at Age 67: - $\boxed{\text{Conservative Allocation, 19\% Savings Percentage}}$
For Retirement at Age 70: - $\boxed{\text{Conservative Allocation, 16\% Savings Percentage}}$

Where the "Conservative Allocation" consists of
0.0% US Equities
40.0% US Treasuries
20.0% High Yield Bonds
20.0% Corp. Bonds
0.0% Real Estate

0.0% Commodities
20.0% Cash

Bogle Rule Remarks/Potential Benefits -
Cons) While this offers high average utility, its skew towards excessive retirement savings, variance, and overall risk profile render it less desirable.
Pros) As one can see from the 3d contour plots, the Bogle Rule offers a **much higher return for the low savings percentages** than the other strategies. Whereas conservative/sharpe allocation have flat progressions and then a steep increase in goal feasibility to around the 15% mark, the Bogle rule is more linear in nature. Even saving only 12.5% annually will offer around a 70% chance of being able to spend 60% of final-salary per year post-retirement!.

Potential Use Case) If Ms. Kim has immediate expenses to tend to and would prefer low savings, she may consider opting for the Bogle Rule.

Sharpe Optimal remarks/Potential Benefits -
Cons) Overall, the sharpe optimal seems to fall short. Its main characteristic, as observed from the contour plot, is that it requires significant annual savings percentages to actualize its growth potential. While the growth has desirable variance/downside risk properties, it simply does not grow fast enough to reach retirement goals with the reasonable range of saving percentages. In particular, sharpe optimal struggles greatly with the 67 year retirement.
Pros) On the other hand, if one has the ability to comfortably save large amounts of annual salary each year, and will likely retire at 70 (or perhaps potentially even later) then the Sharpe Optimal strategy has a strong probability of goal and risk profile. At the upper ends of savings percentages, the Sharpe Optimal offers a higher probability of goal than the Bogle allocation while also maintaining a slightly superior risk profile to the Conservative Allocation.

Potential Use Case) If Ms. Kim has the freedom to save her salary aggressively and will retire relatively late at 70 or older, she may consider opting for the Sharpe optimal allocation

This concludes our recommendation for Ms. Kim's retirement planning. To see specific details of the calculations/simulations, please see Appendix 1).

Section 2: Mr./Ms. Kim - Condo and Retirement Planning

The Goals:
Here, we consider two goals in tandem with the combined income of Mr./Ms. Kim.

Goal 1) The retirement goal, defined similarly as in part 1)
Goal 2) The condo goal. A $275000 downpayment is desired for a condo in 10 years, when Mr./Ms. Kim will both be age 35.

Defining the "Best" Strategy:
We seek to use an approach that synthesizes the combined importance of both goals. In this case, we will still make use of the holistic utility measure, where we consider the fulfillment of the downpayment as a utility-increasing expenditure. We will also look at maximizing the combined probability of reaching both the downpayment and retirement goal, and we will look at minimizing the relevant risk metrics such as

downside risk/CGaR.

The core variables to consider for this model are

1) **Annual Savings Amount - Pre Downpayment** - Percentage of Mr./Ms. Kim's annual salary to be put into savings before condo purchase (that is, for some percentage x, save x% of Mr. Kim's salary, and x% of Ms. Kim's salary)
2) **Annual Savings Amount - Post Downpayment** - Percentage of Mr./Ms. Kim's annual salary to be put into savings after condo purchase
3) **Post-Retirement Spending Amount** - Average annual spending post-retirement

Not explicitly in the model but relevantly, we also look to analyze the **Probability of Downpayment fulfillment** - Likelihood of being able to fulfill a $275000 downpayment after 10 years. However, while we consider the potential negative utilities one could factor into the model from late downpayments, a preliminary mathematical analysis would suggest that a late downpayment is exceedingly unlikely even with single digit savings amounts. As an example, if they both save only 8% of income before age 35, and we assume a near-worst case scenario where Ms. Kim's salary does not increase at all for 5 years (a 7σ event happening 5 times in a row), they will still have enough to lay down the 275,000 down payment at age 35. As it would turn out from simulations on data (in the following section), this expectation is verified. Therefore, we will seek to primarily analyze the retirement goal in the context of a practically guaranteed subtraction of savings at age 35.

We use the same best strategy metric as in the previous section, with a slight modification to lower the utility cap from 80 to 60 percent for post-retirement expenditures, owing to the significantly higher income of dual earning families that outpaces the utility in standard of living. In other words, a two people can expect to make around twice the money of one, but the living expenditures of two people together is sizeably lower than that of 2x the living expenditures of one lone individual.

Best Strategy = $\boxed{max[(\sum_i ln(yearly\ spending)) - (ln(CGaR) + ln(DownsideL1))]}$
where we exclude excess utility in factor 1) coming from above 60% final-salary post retirement spendings.

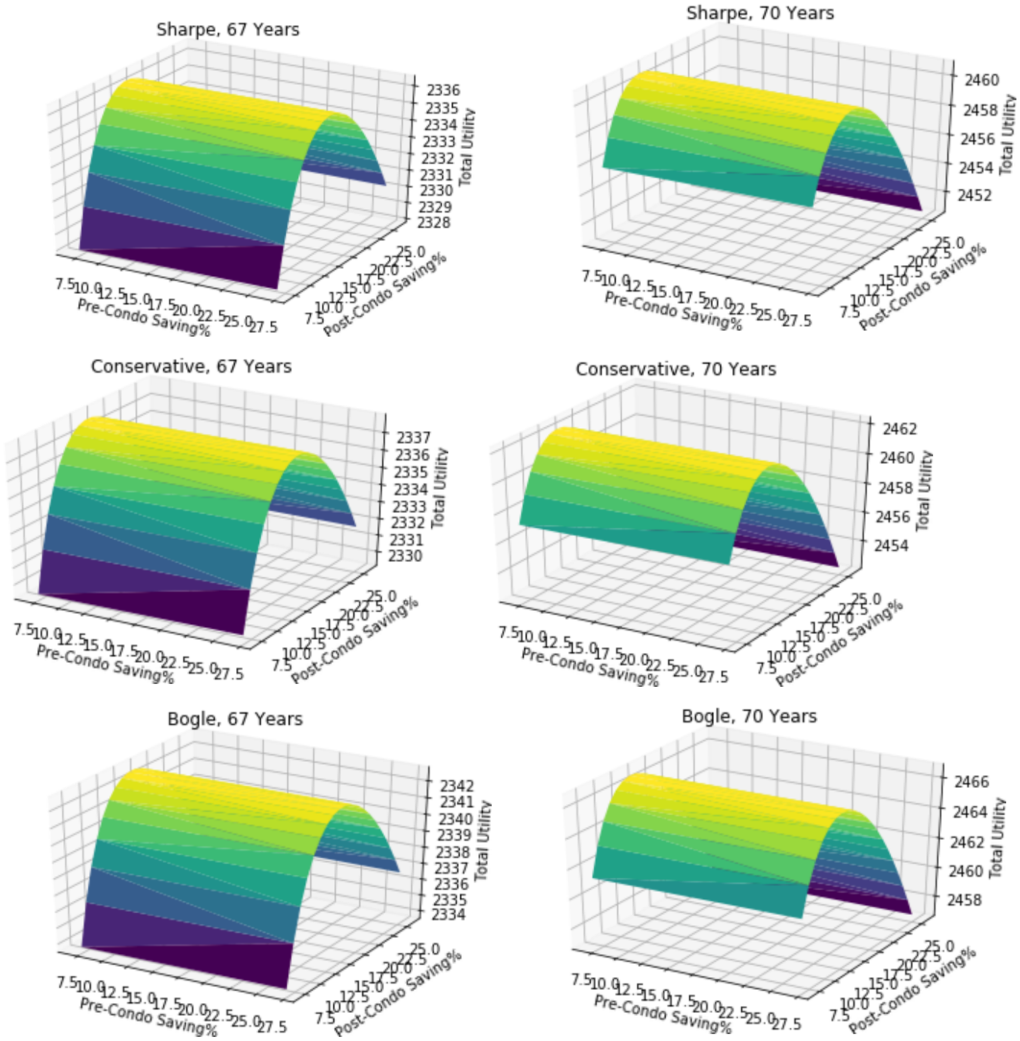
Testing Process/Raw Data:

We use the same simulated data as in part 1), and also test with the same three possible allocations, Allocation A) Sharpe Optimal, Allocation B) Conservative (bonds/treasuries/cash), and Allocation C) Bogle Rule.

Now, we simulate wealth paths for 1,000 scenario simulations given all possible percent saving combinations for the pre-post condo payment in the range 7% to 26%. That is, we test all 400 such possible combinations of the pre-post condo savings possibilities in the range, and measure the total utility of each. After running the simulations, we obtain the following contour plots, and also record the utility maximizing amount/savings combination for each strategy and retirement horizon (67 or 70 years).

Note: as in the previous section, the plotted utilities are unadjusted for the utility cap on excess retirement spendings. These are again adjusted/acounted for in the following analysis.

(additional note: while the pre-condo savings impact percentage is visually underrepresented given the graph scale v. overall utility differences (and the much shorter timeframe of impact vs. post-condo savings), its relevant impact on the maximization problem is far from the flat one implied, as captured by our recording of the various utility-optimal savings combinations.)



We note the max utility savings combinations for each retirement timeline and allocation strategy.

```
[2337.01434229624, (0.23, 0.15)] sharpe 67
[2460.38825534025, (0.21, 0.13)] sharpe 70
[2338.35620583088, (0.24, 0.15)] cons 67
[2461.66460176387, (0.22, 0.13)] cons 70
[2343.36994592758, (0.26, 0.15)] bogle 67
[2466.28611952009, (0.26, 0.13)] bogle 70
```

Data Analysis:

First, we briefly note that the pre-condo savings are all higher than the post-condo savings in the optimal utility models. This is what would be expected going in intuitively (even with an overwhelmingly high probability to hit the goal), as the purchase of the condo offsets the compounding trajectory of early savings growth and thus should be compensated for in the initial stages to reestablish a reliable trajectory to retirement savings.

Here, we list out the risk profiles for each of the above savings combinations/strategies. Note that the "goal" in question here is with respect to a 50% final-salary annual spending post-retirement. (note again: "max ret spend %salary" refers not the maximum saving percentage ever encountered, but rather the average of the maximum-feasible percentage spending for each trial)

Sharpe 67

	Data Type	67 years
0	Probability to hit Goal	0.53465347
1	Mean	9253212.84
2	Variance	1.5175E+12
3	GaR	1752059.01
4	CGaR	2234592.85
5	Downside Risk L1	962827.807
6	Savings%	(0.23, 0.15)
7	Total Utility	2337.01434
8	Pre-Ret Util	1993.25386
9	Post-Ret Util	343.760484
10	Max Ret. Spend %Salary	0.50237589
11	35yo savings	632939.121
12	Condo Date	35
13	Condo Prob	1

Sharpe 70

	Data Type	70 years
0	Probability to hit Goal	0.712871
1	Mean	9422276
2	Variance	1.57E+12
3	GaR	1232903
4	CGaR	1717192
5	Downside Risk L1	824394.4
6	Savings%	(0.21, 0.13)
7	Total Utility	2460.388
8	Pre-Ret Util	2154.552
9	Post-Ret Util	305.8358
10	Max Ret. Spend %Salary	0.537887
11	35yo savings	548771.3
12	Condo Date	35
13	Condo Prob	1

Conservative 67

	Data Type	67 years
0	Probability to hit Goal	0.658416
1	Mean	9784719
2	Variance	2.31E+12
3	GaR	1620197
4	CGaR	2204478
5	Downside Risk L1	1003007
6	Savings%	(0.24, 0.15
7	Total Utility	2338.356
8	Pre-Ret Util	1993.252
9	Post-Ret Util	345.1043
10	Max Ret. Spend %Salary	0.531232
11	35yo savings	639250.6
12	Condo Date	35
13	Condo Prob	1

Conservative 70

	Data Type	70 years
0	Probability to hit Goal	0.762376
1	Mean	10005466
2	Variance	2.42E+12
3	GaR	1135941
4	CGaR	1643667
5	Downside Risk L1	748744.8
6	Savings%	(0.22, 0.13)
7	Total Utility	2461.665
8	Pre-Ret Util	2154.551
9	Post-Ret Util	307.114
10	Max Ret. Spend %Salary	0.571179
11	35yo savings	554271.4
12	Condo Date	35
13	Condo Prob	1

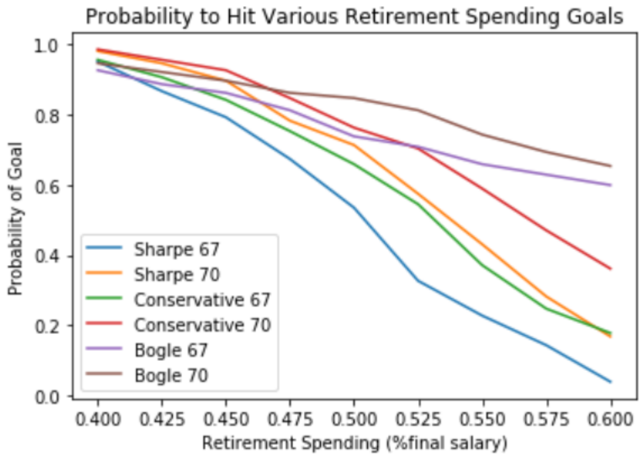
Bogle 67

	Data Type	67 years
0	Probability to hit Goal	0.737623762
1	Mean	12518662.52
2	Variance	2.03775E+13
3	GaR	2385831.254
4	CGaR	3531023.719
5	Downside Risk L1	1476009.945
6	Savings%	(0.26, 0.15)
7	Total Utility	2343.369946
8	Pre-Ret Util	1993.247991
9	Post-Ret Util	350.121955
10	Max Ret. Spend %Salary	0.679663845
11	35yo savings	683929.8037
12	Condo Date	35
13	Condo Prob	1

Bogle 70

	Data Type	70 years
0	Probability to hit Goal	0.846535
1	Mean	12892567
2	Variance	2.12E+13
3	GaR	1576296
4	CGaR	2673072
5	Downside Risk L1	1571269
6	Savings%	(0.26, 0.13)
7	Total Utility	2466.286
8	Pre-Ret Util	2154.543
9	Post-Ret Util	311.7433
10	Max Ret. Spend %Salary	0.735994
11	35yo savings	593365.4
12	Condo Date	35
13	Condo Prob	1

Now, for these various optimal savings amounts, we plot the percentage likelihoods of hitting various post-retirement spending goals.



First, we quickly note that the probability of achieving the condo goal was a strict 1 in all the optimal allocation cases, and indeed some quick math would verify that the condo goal is not at all difficult to achieve, even with a savings rates in the single digits. This allows us simplify our analysis away from juggling the optimum of two goals to focusing on the retirement goal in the context of an assured down payment setback at 35 years

We first analyze the 70 year retirement options.

Now, despite the high utility of the bogle strategy, we encounter a similar phenomenon as with the Ms. Kim's solo retirement planning, where **a large part of this "utility edge" comes from extraneous savings in retirement** that can be accrued from highly aggressive early savings amounts and having large downside risks. As per the reasoning in the "defining the best strategy" section, we establish a utility cutoff at 60% per-year spending of the final salary at retirement, which gives us the following utility decreases for the bogle rule.

At 70 years, the bogle rule offers on average, the ability to spend 73.66% of the final combined income of Mr./Ms. Kim. On average, this would amount to 1291415.6745 * 0.7855 $\approx$ 1,000,000 of annual spending post retirement. Applying our previously suggested utility cap for amounts $>$ 60%, the bogle rule experiences a utility decline of approximately -6.5

```
possible_spending(expectedSavings(1291415.674521951, 15, 0.80), 1291415.674521951, 15)
```

```
(0.8, 315.00982097722846)
```

```
possible_spending(expectedSavings(1291415.674521951, 15, 0.60), 1291415.674521951, 15)
```

```
(0.5999999999999999, 308.53697434706345)
```

```
315.00982097722846 - 308.53697434706345
```

```
6.472846630165009
```

The Sharpe and Conservative asset allocations are both below the utility cap threshold, so the adjusted total utilities from the three allocations for the 70 year period are now as follows

Conservative (2461.66) $>$ Sharpe (2460.39) $>$ Bogle (2459.79))

We also note the downside risk metrics of the Bogle strategy, with a variance approximately 10x and CGaR approximately 2x that of the other two strategies, which would overall render it undesirable.

Considering the conservative allocation vs the sharpe allocation, the conservative allocation once again seems to be the clear winner. It offers higher probability of goal and expected utilities, all while maintaining more desirable profiles in most risk metrics (GaR, CGaR, downside risk). While there is a higher variance in the conservative allocation, we believe this to be offset by the probability of goal and expected spending % possible post retirement (with this edge being represented in-part by the utility as well).

Explicitly calculating out the values of our best-strategy metric, we have

Sharpe 70: 2460.39 - (ln(1717192) + ln(824394)) = 2432.41
Conservative 70: 2461.665 - (ln(1643667) + ln(748744.8)) = 2433.83
Bogle 70: 2459.78 - (ln(2673072) + ln(1571269)) = 2430.71

As such, for the 70 year retirement, we recommend the Conservative Allocation, saving 22% pre-condo purchase and 13% post-condo purchase.

Now, we analyze the year 67 retirement.

Much of what was said before applies similarly to the general profiles of each of the investment strategies. However, something to consider is that the bogle rule in this case does not seem to overshoot into extraneous-savings territory so much, offering a mean post retirement spending capacity of 67.97%. The extraneous utility offered above 60% spending here is equivalent to only 3.29. With that being said, there is the still the large downside risk present to perhaps be wary of, being on average 2x worse in GaR/CGaR and 10x in variance.

The adjusted total utilities from the three allocations for the 67 year period are as follows

Bogle (2339.89) > Conservative (2338.36) > Sharpe (2337.01))

Calculating out the explicit goal function values, we have

Explicitly calculating out the values of our best-strategy metric, we have

Sharpe 67: 2337.01 - (ln(2234592.85) + ln(962827.8)) = 2308.61
Conservative 67: 2338.36 - (ln(2204478) + ln(1003007)) = 2309.93
Bogle 67: 2339.89 - (ln(353102.719) + ln(1476009.945)) = 2313.19

So it would seem that the downside risks for the bogle rule are sufficiently compensated by the practical utilities of the upside. As such, we recommend the bogle rule for the year 67 retirement goal with the caveat that if one feels a log utility underestimates their own aversion to large downside risk, one should choose the conservative allocation.

Final Recommendation and Remarks:

Based on the above analysis, we recommend the following.

Final Recommendation

For Retirement at Age 67:

Bogle Allocation, 24% Savings Percentage pre-condo, 15% Savings Percentage post-condo

For Retirement at Age 70:

$\boxed{\text{Conservative Allocation, 22\% Savings Percentage pre-condo, 13\% Savings Percentage post-condo}}$

Where the "Conservative Allocation" consists of
0.0% US Equities
40.0% US Treasuries
20.0% High Yield Bonds
20.0% Corp. Bonds
0.0% Real Estate
0.0% Commodities
20.0% Cash

Our analysis of the alternative options for each retirement age are largely similar as per the last section, with some minor additional analysis.

Sharpe: For the Sharpe allocation, we cannot recommend as a choice for either retirement timeline, even in the part 1 potential use case of having many assets to spare. It would seem that the impact of the -275000 down payment is large wrt the "growth optimal" progression of the Sharpe, and as such even relatively large savings contributions fail to see any reasonable benefit to payoffs in the form of utility/goal probabilities.

Conservative: For the conservative allocation at 67 years, in addition to(/perhaps in the same vein as) the utility-question case we already went over, we might recommend it over the bogle allocation if one is satisfied with a lower level of retirement spending. At the %40 final salary retirement spending range (which we note is still expected to be >500,000 per year with the combined salary), the conservative allocation, in addition to having better risk metrics, also has a higher probability of goal, which gives consideration for use for a small marginal tradeoff of our utility heuristic.

This concludes our recommendation for Mr./Ms. Kim's combined financial planning. To see specific details of the calculations/simulations, please see Appendix 2)