Final q4 part a)

Wednesday, May 19, 2021 10:16 AM

a) Solve for steady state C_A, C_B values w/o transport:

$$\frac{dC_A}{dt} = 2 - C_A C_B = 0 \quad @ \text{ S.S.}$$

$$\frac{dC_B}{dt} = C_A C_B - C_B = 0 \quad @ \text{ S.S.}$$

$$\rightarrow (C_A - 1) C_B = 0 \qquad \text{for } C_A = 1, \; 2 - (1) C_B = 0 \rightarrow C_B = 2$$

Solve for eigenvalues at $C_A = 1$, $C_B = 2$ to determine stability:

$$f_A = \frac{\partial}{\partial_A}\left(\frac{dC_A}{dt}\right) = -C_B \qquad f_B = \frac{\partial}{\partial_B}\left(\frac{dC_A}{dt}\right) = -C_A$$

$$g_A = \frac{\partial}{\partial_A}\left(\frac{dC_B}{dt}\right) = C_B \qquad g_B = \frac{\partial}{\partial_B}\left(\frac{dC_B}{dt}\right) = C_A - 1$$

eigen values at S.S. values:

$$\left|\begin{pmatrix} -2-\lambda & -1 \\ 2 & -\lambda \end{pmatrix}\right| = (-2-\lambda)(-\lambda) + 2 = 0 \rightarrow 2\lambda + \lambda^2 + 2 = 0$$

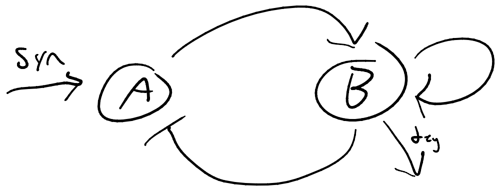
$$\lambda = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i$$

calculate trace $= f_A + g_B = -2 + 0 = -2$

calc determinant $= f_A g_B - f_B g_A = 0 + 2 = 2$

Conclusion:

The no-transport system reaches steady state at $C_A = 1$, $C_B = 2$.
The eigenvalues have real parts < 0, so the state is stable. Trace is < 0 and determinant is > 0, so the state is a sink


Syn
A
B