Final q4 part b)

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Derive the transport term with dependence on cell parameters

At steady-state, transport rate of A into the capillary = rate out

$\rightarrow \quad K_d C_A^{compartment} = \text{rate out}$

Diffusion to the end of the capillary is equal to the diffusive flux integrated over the cross-sectional area of the capillary.

$$K_d C_A^{compartment} = \frac{\partial N_A^{compartment}}{\partial t} \cdot \frac{1}{V_{compartment}} = \frac{1}{V_{compartment}} \int_{surface} D_A \frac{\partial C_A(r)}{\partial r}$$

where r = distance from end of capillary.
change from moles units to Molar units

for the cross-sectional area $W \cdot H \longrightarrow K_d C_A^{compartment} = W \cdot H \cdot D_A \frac{\partial C_A(r)}{\partial r} \cdot \frac{1}{\pi R^2 H}$

Integrate and solve: Choose bounds to be distance, concentration at compartment $\longrightarrow$ distance, concentration at end of capillary

$$\int_{r=L}^{r=0} K_d C_A^{compartment} \partial r = \int_{C_A = C_A^{compartment}}^{C_A=0} \frac{1}{\pi R^2 H} \cdot W \cdot H \cdot D_A \, \partial C_A(r) \longrightarrow K_d C_A^{compartment} = \frac{W \cdot D_A \cdot C_A^{compartment}}{\pi R^2 \cdot L}$$

Solve for $K_d \longrightarrow \quad K_d = \frac{W \cdot D_A}{\pi R^2 \cdot L}$

The full rate equation for C_A in the compartment:

$$\frac{\partial C_A}{\partial t} = r_A C_A - \frac{V_{max}}{K_M} C_A C_B - \frac{W \cdot D_A}{\pi R^2 L} C_A$$