Final q4 part c)

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Solve for S.S. with transport

$$\frac{dC_A}{dt} = r_A C_A - \frac{V_{max}}{K_M} C_A C_B - \frac{w \cdot D_A}{\pi R^2 L} C_A = 2 - C_A C_B - \frac{w \cdot D_A}{\pi R^2 L} C_A = 0$$

$$\frac{dC_B}{dt} = C_A C_B - C_B = 0$$

$C_A = 1$: $\quad 2 - C_B - \frac{w \cdot D_A}{\pi R^2 L} = 0 \longrightarrow C_B = 2 - \frac{w \cdot D_A}{\pi R^2 L}$

$C_B = 0$: $\quad C_A = \frac{2\pi R^2 L}{w \cdot D_A}$

Two steady state solns exist: $C_A = 1$ and $C_B = 2 - \frac{w \cdot D_A}{\pi R^2 L}$; $\quad C_A = \frac{2\pi R^2 L}{w \cdot D_A}$

Solve determinant and trace of Jacobian

$f_A = -C_B - \frac{w \cdot D_A}{\pi R^2 L} \qquad f_B = -C_A \qquad g_A = C_B \qquad g_B = C_A - 1$

General Jacobian:

$$\begin{pmatrix} -C_B - \frac{w \cdot D_A}{\pi R^2 L} & -C_A \\ C_B & C_A - 1 \end{pmatrix}$$

$\text{Trace} = (C_A - 1) + \left(-C_B - \frac{w \cdot D_A}{\pi R^2 L}\right)$

at $C_A = 1$ and $C_B = 2 - \frac{w \cdot D_A}{\pi R^2 L}$,

$\text{Trace} = -2$

$$\det(J) = (C_A - 1)\left(-C_B - \frac{w \cdot D_A}{\pi R^2 L}\right) - (-C_A \cdot C_B)$$

at $C_A = 1$ and $C_B = 2 - \frac{w \cdot D_A}{\pi R^2 L}$, $\quad \det(J) = -\left(-1\left(2 - \frac{w \cdot D_A}{\pi R^2 L}\right)\right) = 2 - \frac{w \cdot D_A}{\pi R^2 L}$

phase portrait classification

$$\frac{1}{4}(\text{trace})^2 = 1$$

for $\det(J) > 1$, $2 - \frac{w \cdot D_A}{\pi R^2 L} > 1$, the state is a spiral sink. (stable)

this occurs for $0 < \frac{w \cdot D_A}{\pi R^2 L} < 1$ which can be constructed with a system of large radius and length or small width for $0 < \det(J) < 1$, the state is a (stable) sink. this occurs when $1 < \frac{w \cdot D_A}{\pi R^2 L} < 2$

for $\det(J) < 0$, the state is an (unstable) saddle. This occurs when width of the capillary is high relative to small radius and length