PS5 q1

Friday, April 30, 2021 9:45 AM

$$\frac{\partial N_1}{\partial t} = F(D_2) - \gamma_N N_1$$

$$\frac{\partial D_1}{\partial t} = G(N_1) - \gamma_D D_1$$

$$\frac{\partial N_2}{\partial t} = F(D_1) - \gamma_N N_2$$

$$\frac{\partial D_2}{\partial t} = G(N_2) - \gamma_D D_2 \qquad (1)$$

N_x = Active notch in cell x
D_x = Active delta in cell x

F [=] Rate of Notch activation
G [=] Rate of Delta activation

γ [=] Inactivation rate constant
eq. $p^* \xrightarrow{\gamma_D} p$

$$f(D') = \frac{F(D')}{\gamma_N} = \frac{D'^2}{0.1 + D'^2}$$

eqn. (6)

$$g(N) = \frac{G(N)}{\gamma_D} = \frac{1}{1 + 10N^2}$$

$\gamma_D / \gamma_N \ll 1 \Rightarrow \gamma_N / \gamma_D \gg 1$

let $\gamma_N / \gamma_D = v'$
let $\partial\tau = \gamma_D \partial t$

In terms v'

$$\frac{\partial N_1}{\partial \tau} = \frac{F(D_2)}{\gamma_D} - \frac{\gamma_N}{\gamma_D} N_1 = v'\left(f(D_2) - N_1\right)$$

$$\frac{\partial D_1}{\partial \tau} = \frac{G(N_1)}{\gamma_D} - D_1 = g(N_1) - D_1$$

$$\frac{\partial N_2}{\partial \tau} = \frac{F(D_1)}{\gamma_D} - \frac{\gamma_N}{\gamma_D} N_2 = v'\left(f(D_1) - N_2\right)$$

$$\frac{\partial D_2}{\partial \tau} = \frac{G(N_2)}{\gamma_D} - D_2 = g(N_2) - D_2$$

Active Notch Steady-State → solve for Notch in terms Delta

at Q.S.S.,

$$\frac{1}{v'}\frac{\partial N_1}{\partial \tau} = f(D_2) - N_1 \approx 0 \rightarrow f(D_2) \approx N_1$$

$$\frac{1}{v'}\frac{\partial N_2}{\partial \tau} = f(D_1) - N_2 \approx 0 \rightarrow f(D_1) \approx N_2$$

Sub-in Notch approximations into Delta dynamical equations

$$\frac{\partial D_1}{\partial \tau} = g(f(D_2)) - D_1$$

$$\frac{\partial D_2}{\partial \tau} = g(f(D_1)) - D_2$$

Define these functions in Python program, along w/ functional forms of $f(D)$, $g(N)$ from above. Create Streamplot in D_1, D_2 plane