MH3510 - Assignment 2

No. :

Date :

1(i) $y_{ijk} = \theta_{ij} + \epsilon_{ijk}$, $i = 1,2,3$; $j = 1,2$; $k = 1,2,\dots,5$

Where θ_{ij} is the treatment mean of i^{th} level of factor A & j^{th} level of factor B.
ϵ_{ijk} is the error at the i^{th} level of factor A & j^{th} level of factor B at the k^{th} job.
on each make of disk drives.

Assumption : ϵ_{ijk} are iid , $E(\epsilon_{ijk}) = 0$, $Var(\epsilon_{ijk}) = \sigma^2$ and ϵ_{ijk} are normally distributed
$\Rightarrow \epsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$

1(ii) $I = 3$, $J = 2$, $r = 5$

ANOVA Table

	df	SS	MS	F	p-value
Factor A	$I-1=2$	261.267	130.6335	2.5259	0.1010
Factor B	$J-1=1$	1.633	1.633	0.031576	
Interaction	$(I-1)(J-1)=2$	768.867	384.4335	7.43345	
Error	$IJ(r-1)=24$	1241.2	51.7167		
Total	29	2272.967			

Working: $IJ(r-1) = 3(2)(5-1)$
$= 24$

$SSE = 2272.967 - 261.267 - 1.633 - 768.867$
$= 1241.2$

$MS_A = \frac{SS_A}{df_A} = \frac{261.67}{2} = 130.6335$

$MS_B = \frac{SS_B}{df_B} = \frac{1.633}{1} = 1.633$

$MS_{AB} = \frac{SS_{AB}}{df_{AB}} = \frac{768.867}{2} = 384.4335$

$MSE = SSE/df_E = \frac{1241.2}{24} = 51.7167$

$F_A = \frac{130.6335}{51.7167} = 2.5259$

$F_B = \frac{1.633}{51.7167} = 0.031576$

$F_{AB} = \frac{384.4335}{51.7167} = 7.43345$

1(iii) $H_0 : (\alpha\beta)_{ij} = 0 \quad \forall_{i,j}$ (do not exists interaction)
From F-table , $F^{0.05}_{2,24} = 3.40$
Since $F_{AB} = 7.43345 > F^{0.05}_{2,24} = 3.40 \Rightarrow$ we reject H_0.
$\Rightarrow \therefore$ There exists interaction between 2 factors.

1(iv) From 1(iii), we can see that there exists significant interaction between 2 factors (and hence, no need perform individual tests for factor A & factor B).
$\Rightarrow$ Hence, the researcher is wrong.

2(i) $y_{ij} = \theta_i + \epsilon_{ij}$ $i = 1,\dots,6$; $j = 1,\dots,5$
Where θ_i is the i^{th} treatment mean response
ϵ_{ij} is the error at i^{th} level at the j^{th} job on each make of disk drives.
Assumption: ϵ_{ij} are iid , $E(\epsilon_{ij}) = 0$, $Var(\epsilon_{ij}) = \sigma^2$ and ϵ_{ij} are normally distributed
$\Rightarrow \epsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2)$

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2(ii) ANOVA Table

Source	df	SS	MS	F
Treatment (Between groups)	5	1031.767	206.3534	3.99
Error (within groups)	24	1241.2	51.717	
Total	29	2272.967		

working: $df_T = 6-1 = 5$

$df_E = 29-5 = 24$

$SST = 261.267 + 1.633 + 768.867 = 1031.767$

$SSE = 1241.2$ (from 1(ii))

$MST = \frac{1031.767}{5} = 206.3534$

$MSE = \frac{1241.2}{24} = 51.717$

$F = \frac{206.3534}{51.717} = 3.99$

2(iii) $y_{ij} = \theta_i + \epsilon_{ij}$, $i=1,\dots,6$; $j=1,\dots,5$

Fitted treatment mean $= \hat{\theta}_i = \frac{1}{5}\sum_{j=1}^{5} y_{ij}$

Let $\hat{\theta}_a$ and $\hat{\theta}_b$ be any 2 fitted treatment means.

$\therefore \hat{\theta}_a - \hat{\theta}_b = \frac{1}{5}\sum_{j=1}^{5} y_{aj} - \frac{1}{5}\sum_{j=1}^{5} y_{bj}$

$\Rightarrow Var(\hat{\theta}_a - \hat{\theta}_b) = Var\left(\frac{1}{5}\sum_{j=1}^{5} y_{aj} - \frac{1}{5}\sum_{j=1}^{5} y_{bj}\right)$

$= \frac{1}{25}\sum_{j=1}^{5} Var(y_{aj}) + \frac{1}{25}\sum_{j=1}^{5} Var(y_{bj})$ $\Rightarrow$ (Since $y_{a1},\dots,y_{a5}, y_{b1},\dots,y_{b5}$ are independent)

$= \frac{5}{25} Var(y_{aj}) + \frac{5}{25} Var(y_{bj})$

$= \frac{1}{5}\sigma^2 + \frac{1}{5}\sigma^2$

$= \frac{2}{5}\sigma^2 = 0.4\sigma^2$ (shown)

2(iv) From ANOVA table $\Rightarrow s^2 = \frac{SSE}{df_E} = 51.717$

2(v) Let θ_{D1} and θ_{D2} be disk drive 1 & 2 respectively.

$\Rightarrow L = \theta_{D1} - \theta_{D2}$ & $\hat{L} = \hat{\theta}_{D1} - \hat{\theta}_{D2}$

$\frac{\hat{L}-L}{s.e.(\hat{L}-L)} \sim t_{n-k}$

$n-k = 29-5 = 24$

$\therefore \frac{\hat{L}-L}{s.e.(\hat{L}-L)} \sim t_{24}$

$s.e.(\hat{L}-L) = \sqrt{var(\hat{\theta}_{D1}-\hat{\theta}_{D2})\cdot s^2}$

$= \sqrt{(0.4)(51.717)}$

$= 4.5483$

$\hat{L} = 48.4 - 61.2 = -12.8$

Since expected service time is positive, we will take absolute value of L.

i.e $|\hat{L}| = |-12.8| = 12.8$

From t-table : $t_{24}^{0.025} = 2.064$

$|\hat{L}| \pm t_{24}^{0.025} \cdot s.e.(\hat{L}-L) = 12.8 \pm (2.064)(4.5483) = [3.4123, 22.188]$

$\therefore$ 95% confidence interval for the difference in expected service time is

$[3.4123, 22.188]$.