A proof mining bibliography

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The list given here is an attempt to collect some of the many relevant references for works from and around the area of proof mining. For simplicity, the focus is (initially) on applications in analysis and on applications using functional interpretations. Through the breadth of this goal, the list nevertheless will presumably always be incomplete.

Suggestions for additions can be send to pischke@mathematik.tu-darmstadt.de.

The references are ordered alphabetically.

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