CO450 - Combinatorial Optimization (Fall 2019)

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0 Introduction

0.1 Overview of the Course

Combinatorial optimization leverages tools from: combinatorics, linear programming theory and algorithms to *efficiently* solve optimization problems on discrete structures (e.g. graphs)

The course will covering the following topics:

- Spanning frees - *Given connected, undirected graphs with edge costs, find the minimum spanning tree*
- Max flow, Min cut
- Matroids and matroid optimization
- Matchings and related problems
- Approximation algorithms

0.2 Review of LP theory

A linear program (LP) is an optimization problem of the form:

$$\begin{aligned} \max \quad & c^\intercal x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{0.1}$$

where $x \in \mathbb{R}^n$, $A \in M_{m\times n}(\mathbb{R})$, and the objective function and constraints are linear. We must also require that:

- There are a finite number of variables and constraints
- The inequalities are non-strict

Any LP has 3 possible outcomes:

1. The LP is infeasible

2. The LP is unbounded, i.e. We can achieve feasible solutions of arbitrarily "good" objective value. (For (0.1), this means that $\forall v \in \mathbb{R}$ there exists a feasible solution x s.t $c^\intercal x > v$)

3. The LP has an optimal solution. (For (0.1), this means there is a feasible solution x^* such that $c^\intercal x^* \geq c^\intercal x$ $\forall$ feasible solutions x)