

CO450 - Combinatorial Optimization

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1 Introduction

1.1 Overview of the Course

Combinatorial optimization leverages tools from: combinatorics, linear programming theory and algorithms to *efficiently* solve optimization problems on discrete structures (e.g. graphs)

The course will covering the following topics:

- Spanning trees
- Max flow, Min cut
- Matroids and matroid optimization
- Matchings and related problems
- Approximation algorithms

1.2 Review of LP theory

A linear program (LP) is an optimization problem of the form:

$$\begin{aligned} \max \quad & c^\top x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{1.1}$$

where $x \in \mathbb{R}^n$, $A \in M_{m \times n}(\mathbb{R})$, and the objective function and constraints are linear. We must also require that:

- There are a finite number of variables and constraints
- The inequalities are non-strict

Any LP has 3 possible outcomes:

1. The LP is infeasible
2. The LP is unbounded, i.e. We can achieve feasible solutions of arbitrarily “good” objective value. (For Equation (1.1), this means that $\forall v \in \mathbb{R}$ there exists a feasible solution x s.t $c^\top x > v$)
3. The LP has an optimal solution. (For Equation (1.1), this means there is a feasible solution x^* such that $c^\top x^* \geq c^\top x \forall$ feasible solutions x)

Theorem 1.1: Fundamental Theorem of Linear Programming

There are only these 3 possible outcomes

1.2.1 Duality