When $\dim \mathcal{N} = 2$, we call this an affine plane.

Ex.
The game of Set is ~~an~~ 4-dimensional affine space over $\mathbb{Z}_3$.

Relations b/w Parameters:

Theorem:
The parameters (v, b, r, k, λ) of a BIBD satisfy:

① $\frac{v}{k} = \frac{b}{r}$

② $\frac{v(v-1)}{k(k-1)} = \frac{b}{\lambda}$.

③ $\frac{v-1}{k-1} = \frac{r}{\lambda}$.

Proof:
Let's count pairs (x, α), where $x \in V$ and $\alpha \in B$, and $x \in \alpha$.

There are v choices for x. For each choice of x, there are r blocks containing x, $\Rightarrow$ so r choices for α.
Therefore, the # of pairs is vr.

There are b choices for α. For each choice of α, there are k elements to choose $\Rightarrow$ so k choices for x.
Therefore, the # of pairs is bk

This gives us equation ①, since $vr = bk$

Proof (con't)

For ②, we count triples (x, y, α), $x, y \in V$, $x \neq y$, $\alpha \in \mathcal{B}$, $x, y \in \alpha$. (Exercise! Fill this in)

And ③ is ②/①. ☑

Ex: If $\mathbb{F} = GF(q)$ (Galois Field with q elements), $V = \mathbb{F}^n$, then then affine space is the design with parameters (v, b, r, k, λ) where:

$v = q^n$, $k = q$, $\lambda = 1$

and using the theorem, we can solve for b, r:

$$b = \lambda \frac{v(v-1)}{k(k-1)} = \frac{q^{n-1}(q^n - 1)}{q-1}$$

$$r = \lambda \frac{v-1}{k-1} = \frac{q^n - 1}{q-1}.$$

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