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1 Graph Theory Primer

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Let $G = (V, E)$ be a graph, where V is the vertex set and E is the edge set.

Definition 1.1. The **degree** of a vertex $v \in V$ (denoted $\deg(v)$) is the number of edges with one end in v . (i.e. The size of the set $\{va \mid a \in V, a \neq v\}$).

A **walk** is a sequence of vertices $v_1v_2 \dots v_k$ where v_iv_{i+1} is an edge. A **path** is a walk where all vertices are distinct. A **cycle** is a walk where $v_1 = v_k$ and v_1, \dots, v_{k-1} are distinct.

Finally, we say a graph is **connected** if there exists a path between any two vertices in G .

Definition 1.2. For $S \subset V$, the **cut** induced by S is the set of all edges with one end in S and one end not in S , denoted $\delta(S) = \{uv \in E \mid u \in S, v \notin S\}$. Given two vertices $s, t \in V$ with $s \in S, t \notin S$, we call $\delta(S)$ an s, t -**cut**

An s, t -**path** is a path with starting vertex s and ends on t .

Theorem 1.1. There exists an s, t -path if and only if every s, t -cut is nonempty

Definition 1.3. A **tree** is a connected graph with no cycles. A **spanning tree** is a subgraph that is a tree and has vertex set V

Note the following:

- A tree on n vertices contains $n - 1$ edges.
- If T is a tree, then adding an edge $uv \notin T$ creates exactly one cycle C . Moreover, if xy is an edge in C , then $T + uv - xy$

Let $D = (N, A)$ be a directed graph. N is a set of nodes and A is a set of ordered pairs of nodes (called arcs).

Definition 1.4. For an arc (u, v) , we call u the **tail** and v the **head**

Definition 1.5. The **out-degree** of node u (denoted $d(u)$ or $d^{\text{out}}(u)$) is the number of arcs with tail u . The **in-degree** of node u (denoted $d(\bar{u})$ or $d^{\text{in}}(u)$) is the number of arcs with head u .