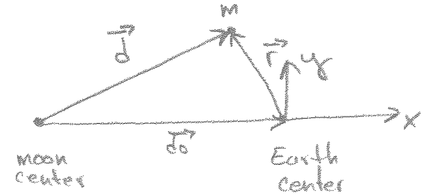
9.3

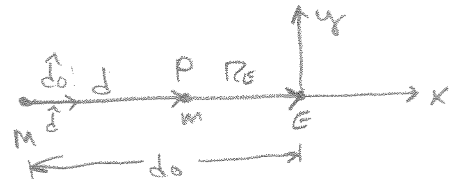
a) $\vec{F}_{tid} = -GM_m m\left(\frac{\hat{d}}{d^2} - \frac{\hat{d_o}}{d_o^2}\right)$



$d = d_o - R_e = d_o\left(1 - \frac{R_e}{d_o}\right)$

$d^{-2} = d_o^{-2}\left(1 - \frac{R_e}{d_o}\right)^{-2}$

$\quad \cong d_o^{-2}\left(1 + \frac{2R_e}{d_o}\right)$ {binomial expansion}



$\vec{F}_{tid_p} \cong -GM_m m\left(d^{-2}\hat{d} - d_o^{-2}\hat{d_o}\right)$

$\quad \cong -GM_m m\left(d_o^{-2}\left(1 + \frac{2R_e}{d_o}\right)\hat{d} - d_o^{-2}\hat{d_o}\right)$

$\quad \cong -\frac{GM_m m}{d_o^2}\left(1 + \frac{2R_e}{d_o} - 1\right)\hat{x}$ {$\hat{d}$ and $\hat{d_o}$ are in $\hat{x}$ direction}

$$\boxed{\vec{F}_{tid_p} \cong -\frac{2GM_m m R_e}{d_o^3}\hat{x}}$$

$G = 6.7\times10^{-11}\ \frac{N\cdot m^2}{kg^2}$

$M_m = 7.3\times10^{22}\ k$

$R_e = 6.4\times10^{6}\ m$

$d_o = 3.8\times10^{8}\ m$

$\frac{F_{tid}}{F_g} = \frac{2GM_m R_e}{d_o^3 g}$

$\quad = \frac{2(6.7\times10^{-11})(7.3\times10^{22})(6.4\times10^{6})}{(3.8\times10^{8})^3(9.81)}$

$$\boxed{\frac{F_{tid}}{F_g} = 1.2\times10^{-7}}$$

b) the force will be the same magnitude but opposite direction.

$d = d_o + R_E$

$d^{-2} = d_o^{-2}\left(1 + \frac{R_e}{d_o}\right)^{-2}$

$d^{-2} \cong d_o^{-2}\left(1 - \frac{2R_e}{d_o}\right)$

$F_{tid_R} = -\frac{GM_m m}{d_o^2}\left(1 - \frac{2R_e}{d_o} + 1\right)\hat{x}$

$$\boxed{\vec{F}_{tid_R} = \frac{2GM_m m R_e}{d_o^3}\hat{x}}$$

9.5

$$\vec{F}_{td} = -GM_m m\left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_o}{d_o^2}\right)$$

$$U_{td} = -\int_{\infty\,(\hat{d})}^{d} \vec{F}\cdot d\vec{r} - \int_{\infty\,(\hat{d}_o)} \vec{F}\cdot d\vec{r}$$

$$= -\int_{\infty}^{d} -GM_m m \frac{dd'}{d'^2} - \int_{x}^{0} -GM_m m \frac{dx'}{d_o^2}$$

$$= GM_m m\left[\int_{\infty}^{d} \frac{dd'}{d'^2} + \int_{x}^{0} \frac{dx'}{d_o^2}\right]$$

$$= GM_m m\left[-\frac{1}{d'}\Big|_{\infty}^{d} + \frac{x'}{d_o^2}\Big|_{x}^{0}\right]$$

$$= GM_m m\left[-\frac{1}{d} - \frac{x}{d_o^2}\right]$$

$$U_{td} = -GM_m m\left[\frac{1}{d} + \frac{x}{d_o^2}\right]$$

At point P $d = d_o - R_e$, $x = -R_e$

$$U_{td}(P) = -GM_m m\left[\frac{1}{d_o - R_e} - \frac{R_e}{d_o^2}\right]$$

$$= -\frac{GM_m m}{d_o}\left[\frac{1}{1-\frac{R_e}{d_o}} - \frac{R_e}{d_o}\right]$$

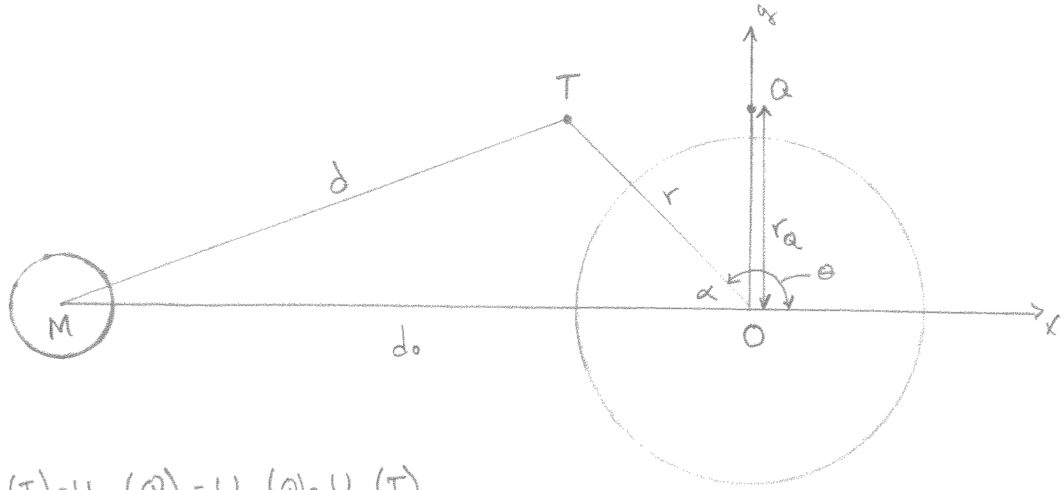
$$= -\frac{GM_m m}{d_o}\left[\frac{d_o + \frac{R_e^2}{d_o} - R_e}{d_o - R_e}\right]$$

$$= -\frac{GM_m m}{d_o}\left[\frac{d_o - R_e}{d_o - R_e} + \frac{\frac{R_e^2}{d_o}}{d_o - R_e}\right]$$

$$\cong -\frac{GM_m m}{d_o}\left[1 + \frac{\frac{R_e^2}{d_o}}{d_o}\right] \quad \{d_o - R_e \approx d_o\}$$

$$U_{td}(P) \cong -\frac{GM_m m}{d_o}\left[1 + \frac{R_e^2}{d_o^2}\right]$$

9.6



$$U_{eg}(T) - U_{eg}(Q) = U_{tid}(Q) - U_{tid}(T)$$

$$U_{eg}(T) - U_{eg}(Q) = mgh \quad \{h \text{ is difference between OT and OQ}\}$$

$$U_{tid} = -GM_m m\left(\frac{1}{d} + \frac{x}{d_0^2}\right)$$

$$U_{tid}(Q) = -GM_m m\left(\frac{1}{\sqrt{d_0^2 + r_Q^2}} + \frac{0}{d_0^2}\right)$$

$$= -GM_m m\left(d_0^2 + r_Q^2\right)^{-1/2} \quad \{r_Q \approx R_E\}$$

$$\cong -GM_m m\left(d_0^2 + R_E^2\right)^{-1/2}$$

$$\cong -GM_m m\, d_0^{-1}\left(1 + \frac{R_E^2}{d_0^2}\right)^{-1/2}$$

$$U_{tid}(Q) \cong -\frac{GM_m m}{d_0}\left(1 - \frac{R_E^2}{2d_0^2}\right) \quad \{(1+\varepsilon)^{-1/2} \cong 1 - \tfrac{1}{2}\varepsilon\}$$

$$U_{tid}(T) = -GM_m m\left(\frac{1}{d} + \frac{x}{d_0^2}\right)$$

$$d^2 = d_0^2 + r^2 - 2d_0 r\cos\alpha$$

$$= d_0^2 + r^2 - 2d_0 r\cos(\pi - \theta)$$

$$d^2 = d_0^2 + r^2 + 2d_0 r\cos\theta \quad \{\cos(\pi-\theta) = \cos\pi\cos\theta + \sin\pi\sin\theta\}$$

$$d^{-1} = \left(d_0^2 + r^2 + 2d_0 r\cos\theta\right)^{-1/2}$$

$$= d_0^{-1}\left(1 + \frac{r^2}{d_0^2} + 2\frac{r}{d_0}\cos\theta\right)^{-1/2}$$

Let $\varepsilon = \frac{r^2}{d_0^2} + \frac{2r}{d_0}\cos\theta$

$$(1+\varepsilon)^{-1/2} \cong 1 - \frac{1}{2}\varepsilon + \frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{2}\varepsilon^2 \cong 1 - \frac{1}{2}\varepsilon + \frac{3}{8}\varepsilon^2$$

$$d^{-1} \cong d_0^{-1}\left(1 - \frac{1}{2}\frac{r^2}{d_0^2} - \frac{r}{d_0}\cos\theta + \frac{3}{8}\left(\frac{r^2}{d_0^2} + \frac{2r}{d_0}\cos\theta\right)\right)$$

$$d^{-1} \cong d_0^{-1}\left(1 - \frac{1}{2}\frac{r^2}{d_0^2} - \frac{r}{d_0}\cos\theta + \frac{3}{8}\left(\left(\frac{r^2}{d_0^2}\right)^2 + \frac{4r^3}{d_0^3}\cos\theta + \frac{4r^2}{d_0^2}\cos^2\theta\right)\right)$$

$$\cong d_0^{-1}\left(1 - \frac{r^2}{2d_0^2} - \frac{r}{d_0}\cos\theta + \frac{3}{8}\left(\frac{4r^2}{d_0^2}\cos^2\theta\right)\right)$$

$$d^{-1} \cong d_0^{-1}\left(1 - \frac{r^2}{2d_0^2} - \frac{r}{d_0}\cos\theta + \frac{3}{2}\frac{r^2}{d_0^2}\cos^2\theta\right)$$

$$x = -r\cos\alpha$$

$$= -r\cos(\pi-\theta)$$

$$x = r\cos\theta$$

$$U_{tid}(T) \cong -\frac{GM_m m}{d_0}\left(1 - \frac{r^2}{2d_0^2} - \frac{r}{d_0}\cos\theta + \frac{3}{2}\frac{r^2}{d_0^2}\cos^2\theta + \frac{r\cos\theta}{d_0}\right)$$

$$U_{tid}(t) \cong -\frac{GM_m m}{d_0}\left(1 - \frac{r^2}{2d_0^2} + \frac{3}{2}\frac{r^2}{d_0^2}\cos^2\theta\right)$$

$$U_{tid}(Q) - U_{tid}(T) = -\frac{GM_m m}{d_0}\left[\left(1 - \frac{R_e^2}{2d_0^2}\right) - \left(1 - \frac{r^2}{2d_0^2} + \frac{3}{2}\frac{r^2}{d_0^2}\cos^2\theta\right)\right] \quad \{r \cong R_e\}$$

$$= \frac{GM_m m}{d_0}\left[\frac{3}{2}\frac{R_e^2}{d_0^2}\cos^2\theta\right]$$

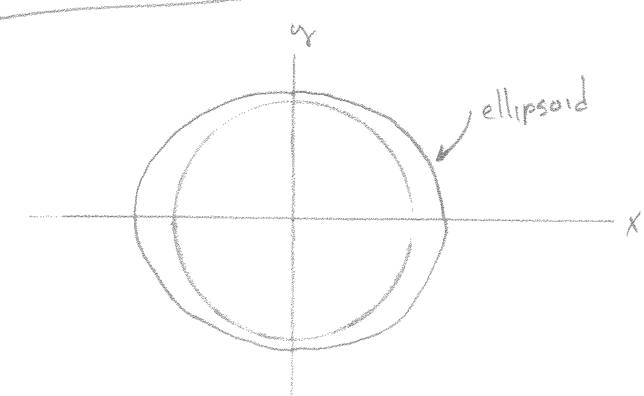
$$mgh = \frac{GM_m m}{d_0}\frac{3}{2}\frac{R_e^2}{d_0^2}\cos^2\theta$$

$$g = \frac{GM_e}{R_e^2}$$

$$\frac{GM_e}{R_e^2}h = \frac{GM_m}{d_0^3}\frac{3}{2}R_e^2\cos^2\theta$$

$$h = \frac{3}{2}\frac{M_m}{M_e}\frac{R_e^4}{d_0^3}\cos^2\theta$$

$$\boxed{h = h_0\cos^2\theta, \quad h_0 = \frac{3}{2}\frac{M_m}{M_e}\frac{R_e^4}{d_0^3}}$$



9.10

$$\left(\frac{d\vec{Q}}{dt}\right)_{s_0} = \left(\frac{d\vec{Q}}{dt}\right)_s + \vec{\Omega}\times\vec{Q}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{s_0} = \left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega}\times\vec{r}$$

$$\left(\frac{d^2\vec{r}}{dt^2}\right)_{s_0} = \left(\frac{d}{dt}\right)_{s_0}\left[\left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega}\times\vec{r}\right]$$

$$\left(\frac{d}{dt}\right)_{s_0} = \left(\frac{d}{dt}\right)_s + \vec{\Omega}\times \_$$

$$\left(\frac{d^2\vec{r}}{dt^2}\right)_{s_0} = \left(\frac{d}{dt}\right)_s\left[\left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega}\times\vec{r}\right] + \vec{\Omega}\times\left[\left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega}\times\vec{r}\right]$$

$$= \left(\frac{d^2\vec{r}}{dt^2}\right)_s + \left(\frac{d}{dt}\right)_s\left(\vec{\Omega}\times\vec{r}\right)\Big] + \vec{\Omega}\times\left[\left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega}\times\vec{r}\right]$$

$$= \ddot{\vec{r}}_s + \left(\frac{d\vec{\Omega}}{dt}\right)_s\times\vec{r}_s + \vec{\Omega}\times\dot{\vec{r}}_s + \vec{\Omega}\times\dot{\vec{r}}_s + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}_s)$$

$$\left(\frac{d^2\vec{r}}{dt^2}\right)_{s_0} = \ddot{\vec{r}}_s + 2\vec{\Omega}\times\dot{\vec{r}}_s + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}_s) + \dot{\vec{\Omega}}\times\vec{r}_s$$

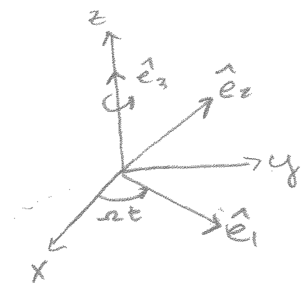
$$\ddot{\vec{r}}_s = \ddot{\vec{r}}_{s_0} - 2\vec{\Omega}\times\dot{\vec{r}}_s - \vec{\Omega}\times(\vec{\Omega}\times\vec{r}_s) - \dot{\vec{\Omega}}\times\vec{r}_s$$

$$\vec{F}_s = F_{s_0} - 2m\vec{\Omega}\times\dot{\vec{r}}_s - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}_s) - m\dot{\vec{\Omega}}\times\vec{r}_s$$

$$F_{az} = -m\dot{\vec{\Omega}}\times r_s$$

$$\boxed{F_{az} = m\vec{r}_s\times\dot{\vec{\Omega}}}$$

9.11



$\vec{\Omega} = \Omega\hat{z} = \Omega\hat{e}_3$

x-y-z is the inertial frame
$\hat{e}_1$-$\hat{e}_2$-$\hat{e}_3$ is the rotating frame

$$\hat{e}_1 = \cos\Omega t\,\hat{x} + \sin\Omega t\,\hat{y}$$
$$\hat{e}_2 = -\sin\Omega t\,\hat{x} + \cos\Omega t\,\hat{y}$$
$$\hat{e}_3 = \hat{z}$$
$$\dot{\hat{e}}_1 = -\Omega\sin\Omega t\,\hat{x} + \Omega\cos\Omega t\,\hat{y} = \Omega\hat{e}_2$$
$$\dot{\hat{e}}_2 = -\Omega\cos\Omega t\,\hat{x} - \Omega\sin\Omega t\,\hat{y} = -\Omega\hat{e}_1$$
$$\dot{\hat{e}}_3 = 0$$

$$\vec{r} = x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3$$
$$\dot{\vec{r}} = \dot{x}\hat{e}_1 + x\dot{\hat{e}}_1 + \dot{y}\hat{e}_2 + y\dot{\hat{e}}_2 + \dot{z}\hat{e}_3 + z\dot{\hat{e}}_3$$
$$= \dot{x}\hat{e}_1 + x\Omega\hat{e}_2 + \dot{y}\hat{e}_2 - y\Omega\hat{e}_1 + \dot{z}\hat{e}_3$$
$$\dot{\vec{r}} = (\dot{x} - \Omega y)\hat{e}_1 + (\dot{y} + \Omega x)\hat{e}_2 + \dot{z}\hat{e}_3$$
$$\dot{r}^2 = (\dot{x} - \Omega y)^2 + (\dot{y} + \Omega x)^2 + \dot{z}^2$$
$$= \dot{x}^2 - 2\Omega\dot{x}y + \Omega^2 y^2 + \dot{y}^2 + 2\Omega\dot{y}x + \Omega^2 x^2 + \dot{z}^2$$
$$\dot{r}^2 = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Omega^2(x^2 + y^2) + 2\Omega(x\dot{y} - \dot{x}y)$$

$$L = \frac{1}{2}m\left[(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Omega^2(x^2 + y^2) + 2\Omega(x\dot{y} - \dot{x}y)\right] - U(x, y, z)$$

$$\frac{\partial L}{\partial x} = m(\Omega^2 x + \Omega\dot{y}) - \frac{\partial U}{\partial x}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{d}{dt}\left(m(\dot{x} - \Omega y)\right)$$
$$= m(\ddot{x} - \Omega\dot{y})$$

$$\frac{\partial L}{\partial y} = m(\Omega^2 y - \Omega\dot{x} - \frac{\partial U}{\partial y})$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = \frac{d}{dt}\left(m(\dot{y} + \Omega x)\right)$$

$$= m(\ddot{y} + \Omega\dot{x})$$

$$\frac{\partial L}{\partial z} = -\frac{\partial U}{\partial z}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{z}} = \frac{d}{dt} m\dot{z}$$

$$= m\ddot{z}$$

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$$

1) $m\ddot{x} = m(\Omega^2 x + 2\Omega\dot{y}) - \frac{\partial U}{\partial x}$

2) $m\ddot{y} = m(\Omega^2 y - 2\Omega\dot{x}) - \frac{\partial U}{\partial y}$

3) $m\ddot{z} = -\frac{\partial U}{\partial z}$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})$$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = \vec{\Omega}(\vec{\Omega}\cdot\vec{r}) - \vec{r}(\vec{\Omega}\cdot\vec{\Omega})$$

$$= \Omega^2\hat{e}_3(\hat{e}_3\cdot\vec{r}) - \vec{r}\Omega^2$$

$$= \Omega^2\hat{e}_3(\hat{e}_3\cdot(x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3)) - \vec{r}\Omega^2$$

$$= \Omega^2\hat{e}_3(z) - \vec{r}\Omega^2$$

$$= \Omega^2 z\hat{e}_3 - \Omega^2 x\hat{e}_1 - \Omega^2 y\hat{e}_2 - \Omega^2 z\hat{e}_3$$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = -\Omega^2 x\hat{e}_1 - \Omega^2 y\hat{e}_2$$

$$\vec{\Omega}\times\dot{\vec{r}} = \Omega\hat{e}_3\times(\dot{x}\hat{e}_1 + \dot{y}\hat{e}_2 + \dot{z}\hat{e}_3)$$

$$\vec{\Omega}\times\dot{\vec{r}} = \Omega\dot{x}\hat{e}_2 - \Omega\dot{y}\hat{e}_1$$

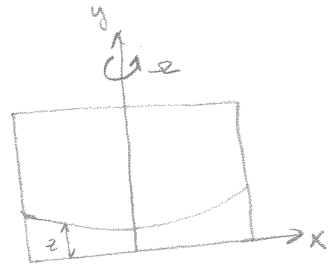
$$m\ddot{\vec{r}} = m[\Omega^2 x\hat{e}_1 + 2\Omega\dot{y}\hat{e}_1 + \Omega^2 y\hat{e}_2 - 2\Omega\dot{x}\hat{e}_2] - \vec{\nabla}\cdot U(\vec{r})$$

$$= m[2\Omega\dot{y}\hat{e}_1 - 2\Omega\dot{x}\hat{e}_2 + \Omega^2 x\hat{e}_1 + \Omega^2 y\hat{e}_2] - \vec{\nabla}\cdot U(\vec{r})$$

$$= m[-2(\vec{\Omega}\times\dot{\vec{r}}) - \vec{\Omega}\times(\vec{\Omega}\times\vec{r})] - \vec{\nabla}\cdot U(\vec{r})$$

$$m\ddot{\vec{r}} = \underset{F_{cor.}}{2m\dot{\vec{r}}\times\vec{\Omega}} + \underset{F_{centr.}}{m(\vec{\Omega}\times\vec{r})\times\Omega} - \underset{\text{Force due to Potential}}{\vec{\nabla}\cdot U(\vec{r})} + \underset{\text{External Net Force}}{\vec{F}_{ext}}$$

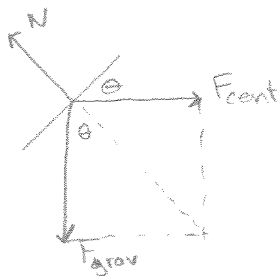
9.14



$$\vec{F}_{cent} = \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$
$$= \Omega^2 r \hat{r}$$

$$\vec{F}_{grav} = -mg\hat{z}$$

Forces on surface



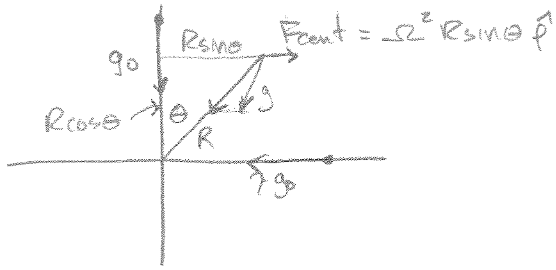
$$\tan\theta = \frac{F_{cent}}{F_{grav}}$$
$$= \frac{\Omega^2 r}{mg}$$
$$= \frac{\Omega^2 x}{mg} \quad \{r = x\}$$

$$\frac{dy}{dx} = \frac{\Omega^2 x}{mg} \qquad \left\{\tan\theta = \frac{dy}{dx}\right\}$$

$$dy = \frac{\Omega^2 x}{mg} dx$$

$$\boxed{y = \frac{\Omega^2}{2mg} x^2 + C}$$

9.15



$\vec{g} = \vec{g}_0 + \Omega^2 R\sin\theta\,\hat{\rho}$

$\vec{g}(0) = \vec{g}_0$

$\vec{g}(\pi/2) = \vec{g}_0 + \Omega^2 R\,\hat{\rho}$

$g(\pi/2) = g_0 - \Omega^2 R$

$\lambda g_0 = g_0 - \Omega^2 R$

$g_0(\lambda - 1) = -\Omega^2 R$

$\Omega^2 R = g_0(1-\lambda)$

$\vec{g}(\theta) = -g_0\hat{r} + \Omega^2 R\sin\theta\,\hat{\rho}$

$= -g_0\hat{r} + g_0(1-\lambda)\sin\theta\,\hat{\rho}$

$= -g_0(\cos\theta\,\hat{z} + \sin\theta\,\hat{\rho}) + g_0\sin\theta\,\hat{\rho} - g_0\lambda\sin\theta\,\hat{\rho}$

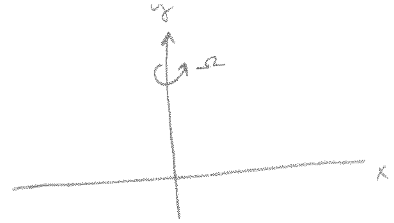
$= -g_0\cos\theta\,\hat{z} - g_0\sin\theta\,\hat{\rho} + g_0\sin\theta\,\hat{\rho} - g_0\lambda\sin\theta\,\hat{\rho}$

$= -g_0\cos\theta\,\hat{z} - g_0\lambda\sin\theta\,\hat{\rho}$

$g(\theta) = \left((-g_0\cos\theta)^2 + (-\lambda g_0\sin\theta)^2\right)^{1/2}$

$$\boxed{g(\theta) = g_0\left(\cos^2\theta + \lambda^2\sin^2\theta\right)^{1/2}}$$

9.18



$$\vec{F} = F_{cor} + F_{cent} + F_{ext}$$
$$= 2m\vec{v}\times\vec{\Omega} + m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) - mg\hat{y}$$
$$= 2mv\Omega\hat{z} + \Omega^2 x\hat{x} - mg\hat{y}$$
$$\vec{F} = m\Omega^2 x\hat{x} - mg\hat{y} \quad \{2mv\Omega\hat{z} \text{ is not in the x-y plane}\}$$

$m\ddot{y} = -mg$

$\ddot{y} = -g$

$\dot{y} = -gt + \dot{y}_0$

$y = -\frac{1}{2}gt^2 + \dot{y}_0 t + y_0$

$m\ddot{x} = m\Omega^2 x$

$x = Ae^{\lambda t}$

$\lambda^2 = \Omega^2$

$\lambda = \pm\Omega$

$x = Ae^{\pm\Omega t}$

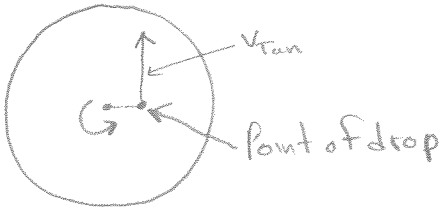
$x = A_1e^{\Omega t} + A_2e^{-\Omega t}$

In the y direction the particle behaves as if in an inertial system.

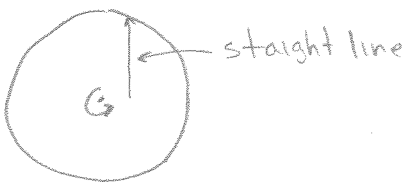
For x motion as $t\to\infty$ $x\to\infty$. If $A_2=0$ then as $t\to\infty$ $x\to 0$. If $A_1=0$ then as $t\to\infty$ $x\to\infty$.

9.19

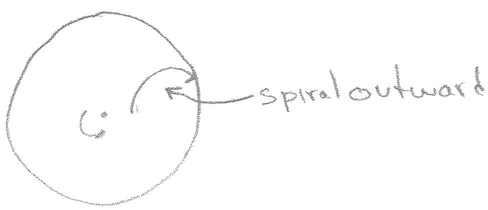
a)



Inertial observer



Observer on merry-go-round

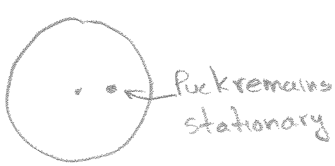


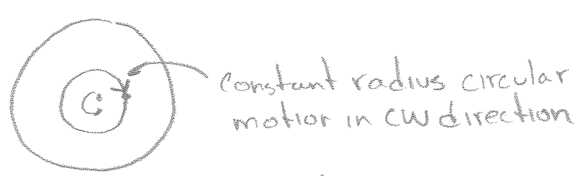
The inertial observer sees the puck travel in a straight line. the merry-go-round observer see the puck spiral outward to the right. This spiral motion is due to both a centrifugal and Coriolis force.

b)

Inertial observer

observer on merry-go-round





The inertial observer sees a stationary puck. The merry-go-round observe sees the puck move in a circle centered on the axis of rotation. No Coriolis force exists $\{\vec{V}_{rel}=0\}$ hence no spiral path. Only a centrifugal force exists with a magnitude of $\Omega^2 r$.