1 General definition of matched filter

For real signals $x_i[n]$ and $r[n]$, where $x_i[n] \neq 0 \forall n \in \Gamma$, and $r[n] \neq 0 \forall n \in (-\infty, \infty)$ the matched filter output is given by:

$$MF_i = \frac{\sum_{n\in\Gamma} x_i[n] r[n]}{\sqrt{\sum_{n\in\Gamma}(x_i[n])^2}\sqrt{\sum_{n\in\Gamma}(r[n])^2}}, \tag{1}$$

For complex-valued signals $x_i[n]$ and $r[n]$, where $x_i[n] \neq 0 \forall n \in \Gamma$, and $r[n] \neq 0 \forall n \in (-\infty, \infty)$ the matched filter output is given by:

$$MF_i = \frac{\sum_{n\in\Gamma} x_i[n] r^c[n]}{\sqrt{\sum_{n\in\Gamma}|x_i[n]|^2}\sqrt{\sum_{n\in\Gamma}|r[n]|^2}}, \tag{2}$$

where $(\cdot)^c$ denotes complex conjugation.

2 Doing a running matched filter

When you are a signal like $r[n]$ that is going on through the whole experiment, then the matched filter may be computed in real-time across a sliding window as follows. Let $N = |\Gamma|$, i.e., the total number of samples over which $x_i[n]$ is non-zero. Let $\vec{r}[n] = r[n]r[n-1]\ldots r[n-(N-1)]$ denote the vector of samples with time cursor at n, and looking back at the last N samples, starting at n. Let $\vec{x}_i[n] = x_i[0]x_i[1]\ldots x_i[N-1]$, assuming $x_i[n]$ is indeed non-zero from $n = 0$ till $n = N-1$. Then the running matched filter output $MF_i[n]$ is given as:

$$MF_i[n] = \frac{\vec{r}[n]^T \vec{x}_i[n]}{\sqrt{\vec{x}_i[n]^T \vec{x}_i[n]}\sqrt{\vec{r}[n]^T \vec{r}[n]}}, \tag{3}$$

where $(\cdot)^T$ denotes the transpose of the vector. Equation (4) is given for real-valued signals. For complex-valued signals the formulation changes to:

$$MF_i[n] = \frac{\vec{r}[n]^H \vec{x}_i[n]}{\sqrt{\vec{x}_i[n]^H \vec{x}_i[n]}\sqrt{\vec{r}[n]^H \vec{r}[n]}}, \tag{4}$$

where $(\cdot)^T$ denotes the conjugate transpose of the vector.