Numbers in the Fibonacci Sequence Circuit
PC/CP220 Project Phase II

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Equations

Truth table of the Numbers in the Fibonacci Sequence Circuit

Number (n)	$a_2a_1a_0$	n^{th} term in the Fibonacci Sequence	$b_3b_2b_1b_0$
0	000	0	0000
1	001	1	0001
2	010	1	0001
3	011	2	0010
4	100	3	0011
5	101	5	0101
6	110	8	1000
7	111	13	1101

Truth tables for each bit

a_2	a_1	a_0	b_3	b_2	b_1	b_0
0	0	0	0	0	0	0
0	0	1	0	0	0	1
0	1	0	0	0	0	1
0	1	1	0	0	1	0
1	0	0	0	0	1	1
1	0	1	0	1	0	1
1	1	0	1	0	0	0
1	1	1	1	1	0	1

A K-Map can be used to determine the simplest equation for each bit.

b_3

b_3		a_0	
		0	1
a_2a_1	00	0	0
	01	0	0
	11	1	1
	10	0	0

- a_2a_1 (a_0 is irrelevant)

$b_3 = a_2a_1$

b_2

b_2		a_0	
		0	1
a_2a_1	00	0	0
	01	0	0
	11	0	1
	10	0	1

- a_2a_0 (a_1 is irrelevant)

$b_2 = a_2a_0$

b_1

b_1		a_0	
		0	1
a_2a_1	00	0	0
	01	0	1
	11	0	0
	10	1	0

- $\bar{a}_2a_1a_0$
- $a_2\bar{a}_1\bar{a}_0$

$b_2 = \bar{a}_2a_1a_0 + a_2\bar{a}_1\bar{a}_0$

b_0

b_0		a_0	
		0	1
a_2a_1	00	0	1
	01	1	0
	11	0	1
	10	1	1

- $\bar{a}_1a_0$ (a_2 is irrelevant)
- $a_2\bar{a}_1$ (a_0 is irrelevant)
- $\bar{a}_1a_0$ (a_2 is irrelevant)
- $\bar{a}_2a_1\bar{a}_0$

$b_0 = \bar{a}_1a_0 + a_2\bar{a}_1 + \bar{a}_1a_0 + \bar{a}_2a_1\bar{a}_0$

Testing

b_3

```
(%i1) b3:a2 and a1;
(%o1)     a2∧a1

(%i2) b3,a2=false,a1=false,a0=false;
(%o2)     false

(%i3) b3,a2=false,a1=false,a0=true;
(%o3)     false

(%i4) b3,a2=false,a1=true,a0=false;
(%o4)     false

(%i5) b3,a2=false,a1=true,a0=true;
(%o5)     false

(%i6) b3,a2=true,a1=false,a0=false;
(%o6)     false

(%i7) b3,a2=true,a1=false,a0=true;
(%o7)     false

(%i8) b3,a2=true,a1=true,a0=false;
(%o8)     true

(%i9) b3,a2=true,a1=true,a0=true;
(%o9)     true
```

Matches the original truth table, therefore the equation is correct.

b_2

```
(%i1) b2:a2 and a0;
(%o1)   a2∧a0

(%i2) b2,a2=false,a1=false,a0=false;
(%o2)   false

(%i3) b2,a2=false,a1=false,a0=true;
(%o3)   false

(%i4) b2,a2=false,a1=true,a0=false;
(%o4)   false

(%i5) b2,a2=false,a1=true,a0=true;
(%o5)   false

(%i6) b2,a2=true,a1=false,a0=false;
(%o6)   false

(%i7) b2,a2=true,a1=false,a0=true;
(%o7)   true

(%i8) b2,a2=true,a1=true,a0=false;
(%o8)   false

(%i9) b2,a2=true,a1=true,a0=true;
(%o9)   true
```

Matches the original truth table, therefore the equation is correct.

b_1

```
(%i1) b1:((not a2) and a1 and a0) or (a2 and (not a1) and (not a0));
(%o1)   ¬a2∧a1∧a0∨a2∧¬a1∧¬a0

(%i2) b1,a2=false,a1=false,a0=false;
(%o2)   false

(%i3) b1,a2=false,a1=false,a0=true;
(%o3)   false

(%i4) b1,a2=false,a1=true,a0=false;
(%o4)   false

(%i5) b1,a2=false,a1=true,a0=true;
(%o5)   true

(%i6) b1,a2=true,a1=false,a0=false;
(%o6)   true

(%i7) b1,a2=true,a1=false,a0=true;
(%o7)   false

(%i8) b1,a2=true,a1=true,a0=false;
(%o8)   false

(%i9) b1,a2=true,a1=true,a0=true;
(%o9)   false
```

Matches the original truth table, therefore the equation is correct.

b_0

```
(%i1) b0:((not a2) and a1 and (not a0)) or (a2 and (not a1)) or ((not a1) and a0) or (a2 and a0);
(%o1)   ¬a2∧a1∧¬a0∨a2∧¬a1∨¬a1∧a0∨a2∧a0

(%i2) b0,a2=false,a1=false,a0=false;
(%o2)   false

(%i3) b0,a2=false,a1=false,a0=true;
(%o3)   true

(%i4) b0,a2=false,a1=true,a0=false;
(%o4)   true

(%i5) b0,a2=false,a1=true,a0=true;
(%o5)   false

(%i6) b0,a2=true,a1=false,a0=false;
(%o6)   true

(%i7) b0,a2=true,a1=false,a0=true;
(%o7)   true

(%i8) b0,a2=true,a1=true,a0=false;
(%o8)   false

(%i9) b0,a2=true,a1=true,a0=true;
(%o9)   true
```

Matches the original truth table, therefore the equation is correct.

Summary

The equations of the output bits are:

- $b_3 = a_2a_1$
- $b_2 = a_2a_0$
- $b_1 = \bar{a}_2a_1a_0 + a_2\bar{a}_1\bar{a}_0$
- $b_0 = \bar{a}_2a_1\bar{a}_0 + a_2\bar{a}_1 + \bar{a}_1a_0 + a_2a_0$

The equations were tested in Maxima, and were proven to be correct.