

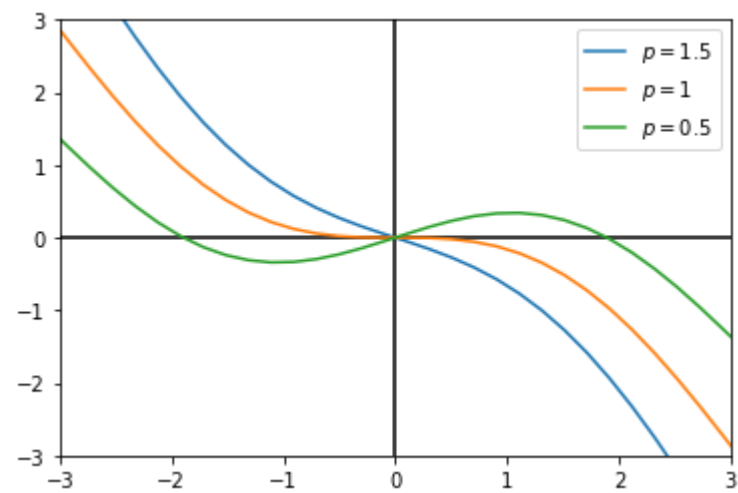
```
In [1]: import numpy as np
import matplotlib.pyplot as plt
```

Let a differential system be defined as

$$\frac{dx}{dt} = \sin(x) - px$$

with $p \in \mathbb{R}$. Thus, $\frac{dx}{dt}$ varies as the graphs below.

```
In [2]: def f(x,p):
        return np.sin(x)-p*x
x = np.linspace(-10,10,100)
figure=plt.figure()
plt.xlim(-3,3)
plt.ylim(-3,3)
plt.plot(x,0*x,'k')
plt.plot(0*x,x,'k')
plt.plot(x,f(x,1.5), label='$p=1.5$')
plt.plot(x,f(x,1), label='$p=1$')
plt.plot(x,f(x,0.5), label='$p=0.5$')
plt.legend()
plt.show()
```



Note that, for different p 's, there are different points in which the function intersects with the x -axis.

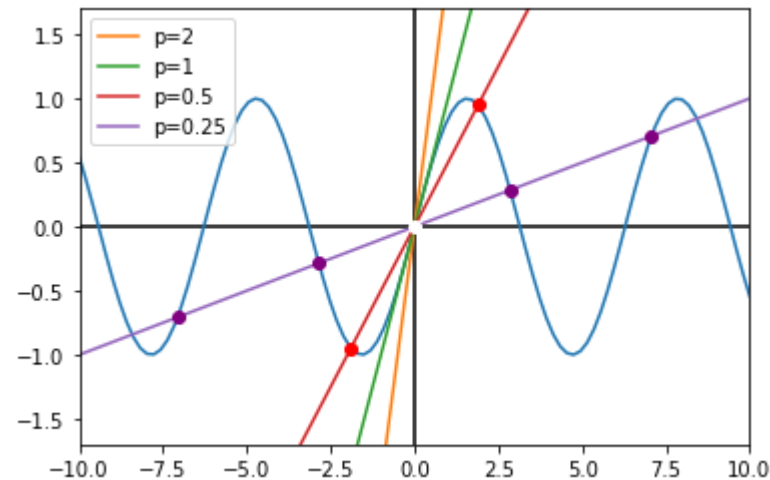
We seek to analyze the system's fixed points as p changes, i.e. its bifurcation diagram. For that, we seek x^* such that

$$\frac{dx}{dt} = 0 \iff \sin(x^*) - px^* = 0$$

It's easier to see what happens graphically:

```
In [3]: def f(x,p):
        return np.sin(x)-p*x
x = np.linspace(-10,10,100)
figure = plt.figure()
plt.xlim(-10,10)
plt.ylim(-1.7,1.7)

plt.plot(x,0*x,'k')
plt.plot(0*x,x,'k')
plt.plot(x,np.sin(x))
plt.plot(x,2*x, label='p=2')
plt.plot(x,x, label='p=1')
plt.plot(x,0.5*x, label='p=0.5')
plt.plot(x,0.1*x, label='p=0.25')
plt.legend()
plt.plot(0,0,'ow')
plt.plot(1.89549, 0.5*1.89549,'or') ## Solved sin(x) = 0.5x numerically
plt.plot(-1.89549, -1.89549*0.5,'or') ## Solved sin(x) = 0.5x numerically
plt.plot(2.85234, 0.1*2.85234,'o',color='purple') ## Idem for p = 0.1
plt.plot(-2.85234, -0.1*2.85234,'o',color='purple')
plt.plot(7.06817, 0.1*7.06817,'o',color='purple')
plt.plot(-7.06817, -0.1*7.06817,'o',color='purple')
plt.show()
```



Note that, for $p \geq 1$, only $x^* = 0$, while for $0 < p < 1$, we see that new critical points arise.

