maxlklhds

October 1, 2020

1 Maximum Likelihoods for Nested Adjacency Matrices in Erdős-Rényi Graphs

Let us have a sequence $(G(N,p))_{N=2}^{\infty}$ of Erdős-Rényi graphs defined in such a way as to have

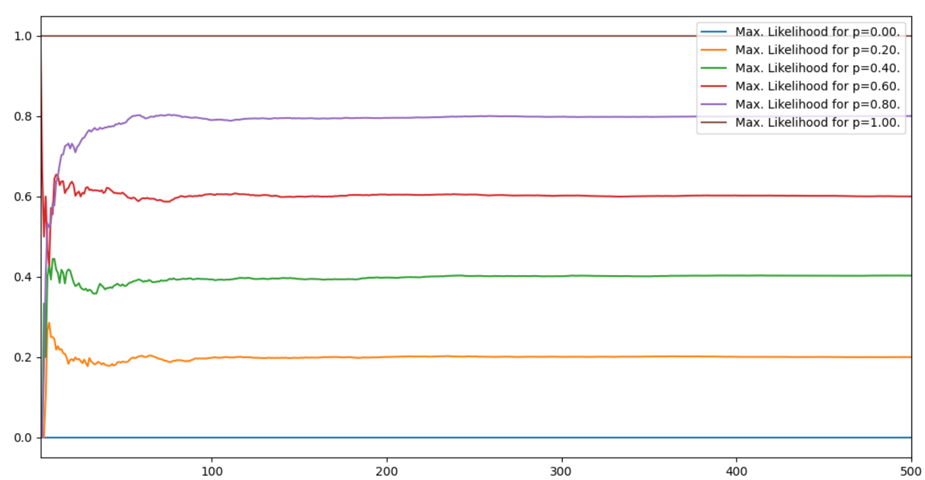
$$M_{N+1}(v,v') = M_N(v,v'),\ \forall v,v' \in [1,N]$$

That is, previous adjacency matrices are nested inside following matrices. (Perhaps it's worth mentioning that adjacency matrices herein are upper-triangular.)

Our objective is to analyze

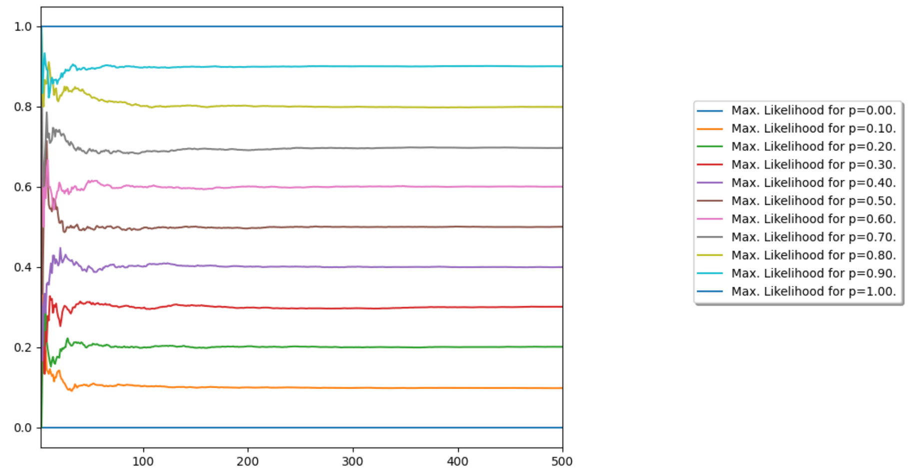
$$\lim_{N\to\infty} \hat{p}_{p,N}$$

that is, how the maximum likelihood estimator evolves as the graph increases in these circumstances. The plots below show just that.



Note that $p \in 0,1$ are trivial realizations: complete isolation and complete connectedness, respectively.

It's also interesting to note that, at the start of iterations ($N = 2$), we have that $p_{p,2} \in \{0,1\}$: both nodes are either connected (locally "equivalent" to a $p = 1$ realization) or not (likewise for $p = 0$).



Thus, it's safe to assume that

$$\lim_{N\to\infty} \hat{p}_{p,N} = p$$

a remarkable result, indeed!