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In [1]: import numpy as np
import matplotlib.pyplot as plt
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Let a differential system be defined as

$$x_{n+1} = \frac{\sin(x_n)}{p}$$

with  $p \in \mathbb{R}$ .

## Algorithm for finding fixed points

To create our bifurcation diagram, we simply iterate

$$x_{n+1} = f(x_n) = \frac{\sin(x_n)}{p}$$

for some  $x_0$  and for different  $p$ 's. Note that  $x^* = 0$  is an attracting fixed point for  $p > 1$ , since

$$\left| \frac{df}{dx} \right|_{x^*=0} = \left| \frac{\cos(0)}{p} \right| = \left| \frac{1}{p} \right| < 1$$

and becomes a repelling fixed point for  $p < 1$ , since

$$\left| \frac{df}{dx} \right|_{x^*=0} = \left| \frac{\cos(0)}{p} \right| = \left| \frac{1}{p} \right| > 1$$

The code is in `strogatz-cont.py`.

Although initially there are simple bifurcations (going from right to left), there quickly appears some chaotic phases, followed by simple bifurcations "breaks" inbetween.

Just a side note: for each  $(p, x^*)$ , I plotted its corresponding *pixel* in the plane. So when it's all blue, (basically) *all* single pixels are blue/are a fixed point.

Below is the Lyapunov exponents' plot, against  $p$ .

