

Analysis of Steady-State Behavior in Server Queues using Markov Chains and Eigenvalues in the M/M/1 Model

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Abstract—This document is a model and instructions for L^AT_EX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.

Index Terms—component, formatting, style, styling, insert.

I. INTRODUCTION

Queueing behavior arises naturally in nearly every computing environment where resources are shared among multiple tasks. Whenever incoming work arrives faster than it can be immediately processed, the excess work must wait, forming a queue. This phenomenon appears in a broad range of systems, including CPU scheduling, packet forwarding in routers, job dispatching in cloud infrastructures, disk I/O scheduling, and networked application servers. The performance of these systems is heavily influenced by their queueing characteristics, making analytical models essential for understanding and improving real-world performance [1].

Modern computing workloads are highly variable and unpredictable. Task arrivals do not occur at fixed intervals, and service times fluctuate due to user behavior, network delays, resource contention, and software-level scheduling. These uncertainties make deterministic approaches nonoptimal. However, probabilistic theory enables engineers to determine whether a server will remain stable under a particular load, estimate average waiting times, and understand how performance degrades as traffic increases.

To formally analyze such behavior, queueing systems are commonly modeled as stochastic processes, with one of the simplest and most fundamental being the M/M/1 queue. In this model, arrivals follow a Poisson process with service times that follow an exponential distribution, all handled by a single server. Despite its simplicity, the model can capture trends and predict behaviors such as stability, queue buildup, and performance spikes. [2].

The M/M/1 queue can be naturally represented as a Markov chain, where each state corresponds to the current number of jobs in the system. Transitions between states reflect arrivals and completions of jobs, producing a stochastic matrix representing the dynamic behavior of the queue. The steady-state

distribution corresponds to the eigenvector associated with eigenvalue 1, consistent with standard results in Markov chain theory [3]. This connection between queueing systems and linear algebra provides a powerful framework for analyzing server load and predicting performance under varying traffic conditions.

In this paper, we examine the steady-state behavior of the M/M/1 queue using Markov chains and eigenvalue analysis. We first derive the transition matrix of the system and compute its stationary distribution using the eigenvector method. We then compare this theoretical distribution with the known closed-form solution of the M/M/1 queue. To validate the results, a queue simulation is implemented in C++, allowing us to observe empirical queue behavior under different arrival and service rates. By comparing theoretical predictions with simulation outcomes, we illustrate how mathematical modeling can determine whether a server can withstand a particular load and how queue lengths evolve over time.

II. THEORETICAL FOUNDATION

A. Matrices

1) *Definition:* A matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. The individual items in a matrix are called its elements or entries. The size of a matrix is described in terms of the number of rows and columns it contains. Generally, a general $m \times n$ matrix may be denoted as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

with A denoting the matrix, m the number of rows, n the number of columns, and a_{ij} the element in row i and column j . A matrix with size $n \times n$ is called a square matrix of order n and the elements a_{ii} (where the row and column indices are equal) form the main diagonal of matrix A .

2) *Row and Column Vectors*: A matrix with only one row or one column is called a row matrix (or a row vector) or column matrix (or a column vector), respectively. A general $m \times 1$ column matrix and a $1 \times n$ row matrix \mathbf{b} may be denoted as:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \text{ and } \mathbf{b} = [b_1 \quad b_2 \quad \cdots \quad b_n]$$

3) *Matrix Addition and Subtraction*: If A and B are matrices of the same size, their sum ($A + B$) and difference ($A - B$) are obtained by adding or subtracting their corresponding entries. Matrices of different sizes cannot be added or subtracted. In matrix notation, if $A = [a_{ij}]$ and $B = [b_{ij}]$ have the same size, then $(A \pm B)_{ij} = a_{ij} \pm b_{ij}$.

4) *Scalar Multiplication*: If A is any matrix and k is any scalar, then the scalar multiple kA is the matrix obtained by multiplying every entry of A by k . In matrix notation, if $A = [a_{ij}]$, then $(kA)_{ij} = ka_{ij}$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \rightarrow 2A = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$$

5) *Matrix Multiplication*: If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is defined to be the $m \times p$ matrix C whose entries are given by:

$$c_{ij} = \sum_{k=1}^n a_{ik}b_{kj}$$

In simpler terms, to find entry c_{ij} of the product matrix C , multiply the corresponding entries of the i^{th} row of matrix A with the j^{th} column of matrix B and add the results. Given the example below:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 5 & 0 \end{bmatrix}$$

$$C = AB = \begin{bmatrix} 25 & 7 \\ 22 & 15 \end{bmatrix}$$

Since A is a 2×3 matrix and B is a 3×2 matrix, the resulting product AB is a 2×2 matrix. For example, to determine entry c_{11} of the product matrix AB , we multiply the corresponding entries of the first row of matrix A with the first column of matrix B and add the results.

6) *Transpose of a Matrix*: Given a matrix A of size $m \times n$, the transpose of A , denoted by A^T , is the $n \times m$ matrix obtained by interchanging the rows and columns of A . In matrix notation, if $B = A^T$, then the entries of B are defined as $b_{ij} = a_{ji}$, $1 \leq i \leq n$, $1 \leq j \leq m$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

B. Eigenvalues and Eigenvectors

If A is an $n \times n$ matrix, then a nonzero vector \mathbf{v} in R^n is called an eigenvector of A if there exists a scalar λ such that:

$$A\mathbf{v} = \lambda\mathbf{v}$$

The scalar λ is called the eigenvalue of A corresponding to the eigenvector \mathbf{v} . In other words, multiplying the matrix A by the vector \mathbf{v} results in a new vector that is a scalar multiple of the original vector \mathbf{v} .

Given a matrix A with size $n \times n$, the eigenvalues and eigenvectors are found by solving the following characteristic equation:

$$\begin{aligned} Ax &= \lambda x \\ IAx &= \lambda Ix \\ Ax &= \lambda Ix \\ (A - \lambda I)x &= 0 \end{aligned}$$

Since $x = 0$ is the only trivial solution, for $(A - \lambda I)x = 0$ to have non-trivial solutions, the matrix $(A - \lambda I)$ must be singular, and therefore $\det(A - \lambda I)$ must be zero. The polynomial given by $\det(A - \lambda I) = 0$ is called the characteristic equation of A , and the solutions to such equation are the eigenvalues of A , otherwise denoted as the characteristic roots.

C. Markov Chains

A markov chain is a mathematical system used to model systems that transition between different states over time. The defining characteristic of a markov chain is that the probability of transitioning to the next state depends only on the current state, and not on the sequence of states that preceded it. This property is known as the markov property.

A markov chain is commonly represented using a matrix formed by probability vectors which represent the likelihood of the system transitioning from one state to another. This matrix is called the transition matrix and the state vectors at successive time intervals can be defined by $x(n+1) = Px(n)$, where P is the transition matrix and P_{ij} is the probability that the system will be in state i at time $n+1$ given that it was in state j at time n .

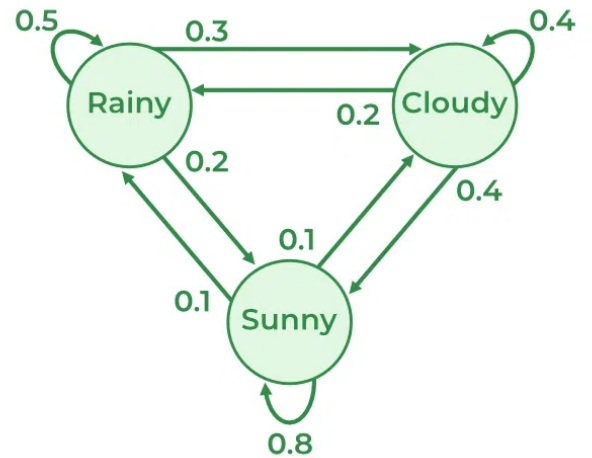


Fig. 1. Example of a Markov Chain representing weather states [6].

D. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \quad (1)$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use “(1)”, not “Eq. (1)” or “equation (1)”, except at the beginning of a sentence: “Equation (1) is . . .”

E. L^AT_EX-Specific Advice

Please use “soft” (e.g., `\eqref{Eq}`) cross references instead of “hard” references (e.g., (1)). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don’t use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in L^AT_EX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you’ve discovered a new method of counting.

BIB_TE_X does not work by magic. It doesn’t get the bibliographic data from thin air but from .bib files. If you use BIB_TE_X to produce a bibliography you must send the .bib files.

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Do not use `\nonumber` inside the `{array}` environment. It will not stop equation numbers inside `{array}` (there won’t be any anyway) and it might stop a wanted equation number in the surrounding equation.

F. Some Common Mistakes

- The word “data” is plural, not singular.
- The subscript for the permeability of vacuum μ_0 , and other common scientific constants, is zero with subscript formatting, not a lowercase letter “o”.
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited,

such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)

- A graph within a graph is an “inset”, not an “insert”. The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
- Do not use the word “essentially” to mean “approximately” or “effectively”.
- In your paper title, if the words “that uses” can accurately replace the word “using”, capitalize the “u”; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones “affect” and “effect”, “complement” and “compliment”, “discreet” and “discrete”, “principal” and “principle”.
- Do not confuse “imply” and “infer”.
- The prefix “non” is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the “et” in the Latin abbreviation “et al.”.
- The abbreviation “i.e.” means “that is”, and the abbreviation “e.g.” means “for example”.

An excellent style manual for science writers is [?].

G. Authors and Affiliations

The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

H. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is “Heading 5”. Use “figure caption” for your Figure captions, and “table head” for your table title. Run-in heads, such as “Abstract”, will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and,

conversely, if there are not at least two sub-topics, then no subheads should be introduced.

I. Figures and Tables

a) *Positioning Figures and Tables:* Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation “Fig. 2”, even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

Table Head	Table Column Head		
	Table column subhead	Subhead	Subhead
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^aSample of a Table footnote.

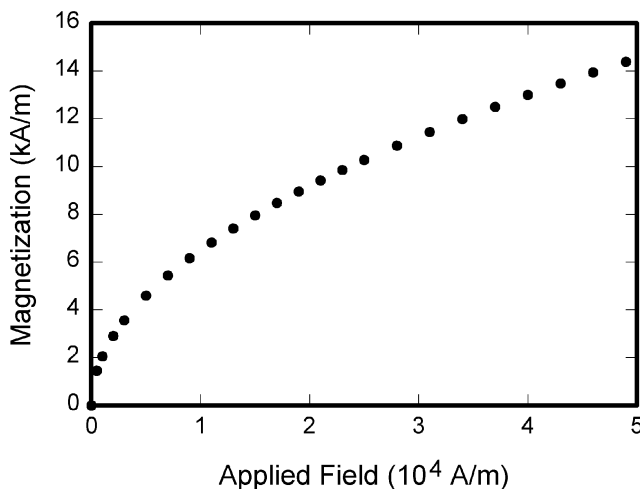


Fig. 2. Example of a figure caption.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity “Magnetization”, or “Magnetization, M”, not just “M”. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write “Magnetization (A/m)” or “Magnetization {A[m(1)]}”, not just “A/m”. Do not label axes with a ratio of quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

ACKNOWLEDGMENT

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks ...”. Instead, try “R. B. G. thanks...”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

REFERENCES

Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use “Ref. [3]” or “reference [3]” except at the beginning of a sentence: “Reference [3] was the first ...”

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors’ names; do not use “et al.”. Papers that have not been published, even if they have been submitted for publication, should be cited as “unpublished” [4]. Papers that have been accepted for publication should be cited as “in press” [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

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