MH4518 Group 7 Presentation

By: Hrishi, Nicholas and Ziheng

Agenda

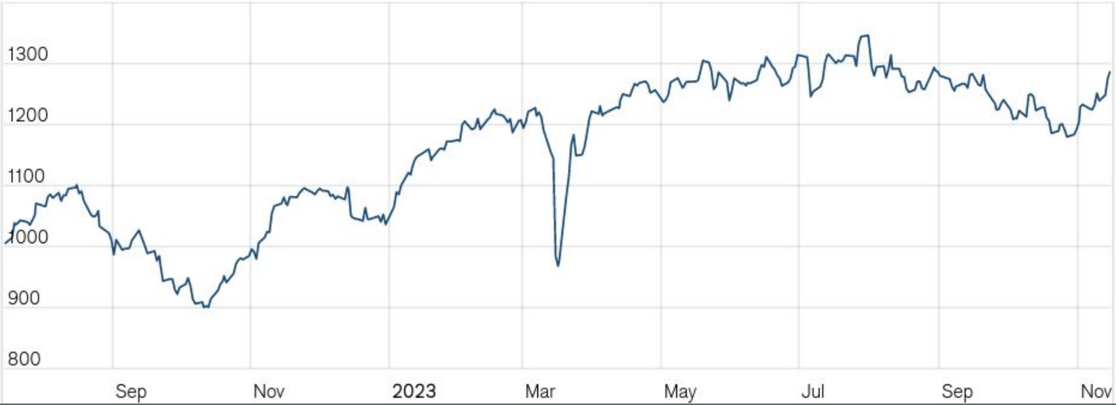
1. Product
2. Methodology
   a. Sourcing our Data
   b. Simulation Techniques
   c. Models
3. Models and Results
   a. Geometric BM
   b. Heston (Stochastic Volatility)

Product

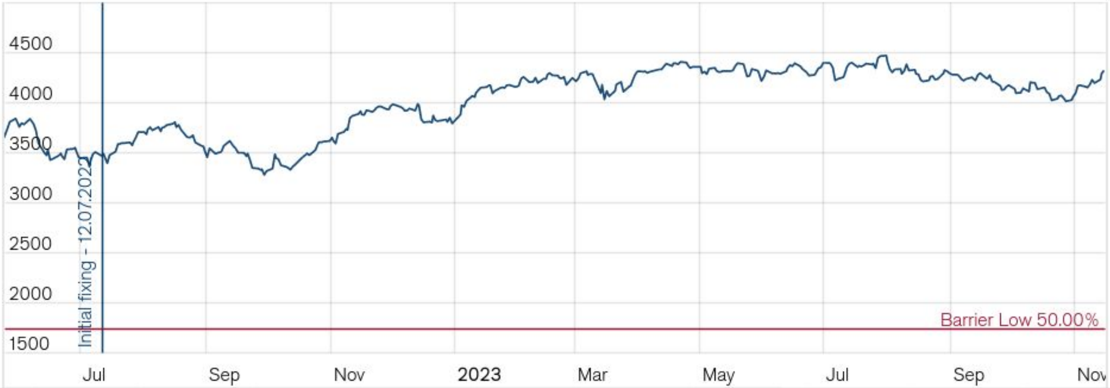
Our Product: Outperformance Bonus Certificate on EURO STOXX 50

- Underlying Asset: EURO STOXX 50 Index (SX5E on Bloomberg)
- Feature: Barrier Down-and-Out Call Option
- Dates
    - Initial Fixing Date: 12 July 2022
    - Final Fixing Date: 14 July 2025
- Prices
    - SX5E Initial Level: EUR 3,487.05
    - Barrier: EUR 1,743.525 (50% of initial level)
    - Initial Offering: EUR 1000

Product Chart in EUR



Underlying Chart



Product - Payoff

Payoff Scenarios:

a) $if\ \min(S_t) > V,$

$$payoff = max\left(1000\left(1 + 1.5\left(\frac{S_T - S_0}{S_0}\right)\right), 1000\right)$$

a) The Reference Index never reached its Barrier during the lifetime of the Certificate (best case)

- You will receive EUR 1,000 per Certificate plus 150% of the positive performance of the Reference Index, calculated from its Initial Level, or
- You will receive a Minimum Repayment of EUR 1,000 per Certificate even if the performance of the Reference Index is negative.

Product - Payoff

Payoff Scenarios:

a) $$if \ \min(S_t) > V,$$
$$payoff = max\left(1000\left(1 + 1.5\left(\frac{S_T - S_0}{S_0}\right)\right), 1000\right)$$

b) $$if \ \min(S_t) \leq V \, and \, S_T > S_0,$$
$$payoff = 1000\left(1 + 1\left(\frac{T - S_0}{S_0}\right)\right)$$
$$= 1000\left(\frac{S_T}{S_0}\right)$$

b) The Reference Index has reached its Barrier during the lifetime of the Certificate and closes at or above its Initial Level on the Final Fixing Date

- You will receive EUR 1,000 per Certificate plus 100% of the positive performance of the Reference Index, calculated from its Initial Level.

Product - Payoff

Payoff Scenarios:

a) $if\ \min(S_t) > V,$

$$payoff = max\left(1000\left(1 + 1.5\left(\frac{S_T - S_0}{S_0}\right)\right), 1000\right)$$

b) $if\ \min(S_t) \leq V\ and\ S_T > S_0,$

$$payoff = 1000\left(1 + 1\left(\frac{T - S_0}{S_0}\right)\right)$$

$$= 1000\left(\frac{S_T}{S_0}\right)$$

c) $if\ \min(S_t) \leq V\ and\ S_t < S_0,$

$$payoff = 1000\left(\frac{S_T}{S_0}\right)$$

c) The Reference Index closes below its Initial Level on the Final Fixing Date and its Barrier has been reached during the lifetime of the Certificate

- Your redemption amount will be reduced by 1% for each percentage point the Reference Index closes below its Initial Level.

Product - Payoff

Payoff Scenarios:

a) $if\ \min(S_t) > V,$

$$payoff = max\left(1000\left(1 + 1.5\left(\frac{S_T - S_0}{S_0}\right)\right), 1000\right)$$

b) $if\ \min(S_t) \leq V\ and\ S_T > S_0,$

$$payoff = 1000\left(1 + 1\left(\frac{T - S_0}{S_0}\right)\right)$$

$$= 1000\left(\frac{S_T}{S_0}\right)$$

c) $if\ \min(S_t) \leq V\ and\ S_t < S_0,$

$$payoff = 1000\left(\frac{S_T}{S_0}\right)$$

Payoff Function:

$$\chi(S_T) = \begin{cases} \max\left(1000\left(1 + 1.5\left(\frac{S_T - S_0}{S_0}\right)\right), 1000\right), & \min(S_t) > V \\ 1000\left(\frac{S_t}{S_0}\right) & \min(S_t) \leq V \end{cases}$$

Product - Timeline

Methodology - Sourcing our Data

Data Sources

- Stock Data S_t: ^STOXX50E from Yahoo Finance
  - Dates: 2 Aug 2022 - 13 Nov 2023 (daily)
- Interest Rate r: Euro Area Yield Curves with 1Y Maturity from ECB
  - Dates: 6 Sep 2004 - 13 Nov 2023 (daily)
- Actual Derivative Price: Web-scraped from Credit Suisse
  - Dates: 15 Jul 2022 - 10 Nov 2023 (daily)
- Options Quote Prices for Underlying Asset: Taken from Bloomberg Terminal
  - Dates: 11 Aug 2023 - 15 Mar 2024 (option prices for 11 different expiration dates)

Methodology - Simulation Techniques

Techniques Used

1. Monte Carlo
2. Antithetic Variates
3. Control Variates
4. 1-shoot Stratified Sampling (not applicable to path-dependent derivatives)
5. Empirical Martingale Correction

Models - Geometric Brownian Motion (GBM)

General Process

1. Obtain / calculate model parameters from historical data
    a. Risk-free interest rate r (taken from ECB)
    b. Initial asset price for each day in window S_0
    c. Total simulation duration T
    d. Volatility σ
2. Plug model parameters into our simulation functions (MC, AV, CV, SS, EMS) to simulate N price paths of the SX5E stock
3. Calculate payoff of each price path for N simulations
4. Estimate option price by “pulling back” from payoff and taking mean

Models - Geometric Brownian Motion (GBM)

Simulation Setup

- Historical Data
  - For each day in sliding window (e.g. 9 Aug 2023),
    - r: Taken from previous day (e.g. 8 Aug 2023)
    - S_0: Taken from current day (e.g. 8 Aug 2023)
    - σ: Calculated from previous M timesteps (e.g. 252 working days)
    - T: Calculated from current day (e.g. 485 working days from final fixing date)

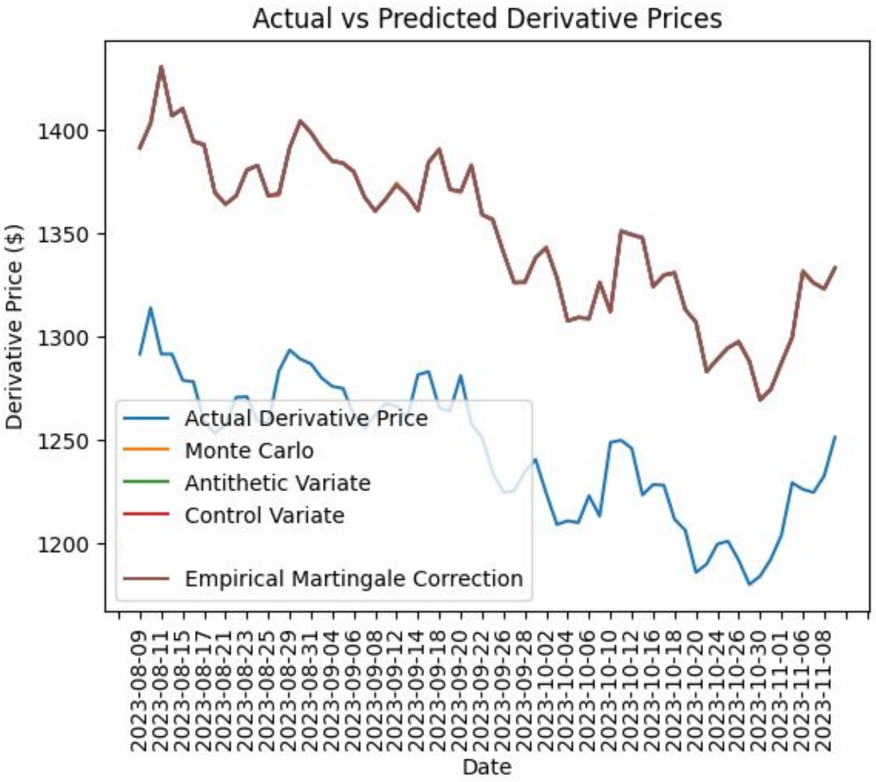
Models - Geometric Brownian Motion (GBM)

Simulation Setup

- No. of Simulations N: 1,000,000
- Sliding Window
    - Start Date: 9 Aug 2023
    - End Date: 9 Nov 2023
    - Total Working Days: 66
- Lookback Periods
    - 3 months (63 working days)
    - 1 year (252 working days)

Models - Geometric Brownian Motion (GBM)

How much is the product worth today?



Models - Geometric Brownian Motion (GBM)

```
Mean Absolute Error from Actual Price for 1000000 simulations (Lookback 252 days):
- Monte Carlo: 111.67965241821518 EUR
- Antithetic Variates: 111.76689883672678 EUR
- Control Variates: 111.75460378848314 EUR
- Empirical Martingale Correction: 111.51634024679343 EUR
```

```
Mean Estimated Variance for 1000000 simulations (Lookback 252 days):
- Monte Carlo: 0.135644774298475
- Antithetic Variates: 0.13569600160397383
- Control Variates: 0.003957229202394658
- Empirical Martingale Correction: 0.13558974773461657
```

```
Variance Reduction % for 1000000 simulations (Lookback 252 days):
- Antithetic Variates: -0.04%
- Control Variates: 97.08%
- Empirical Martingale Correction: 0.04%
```

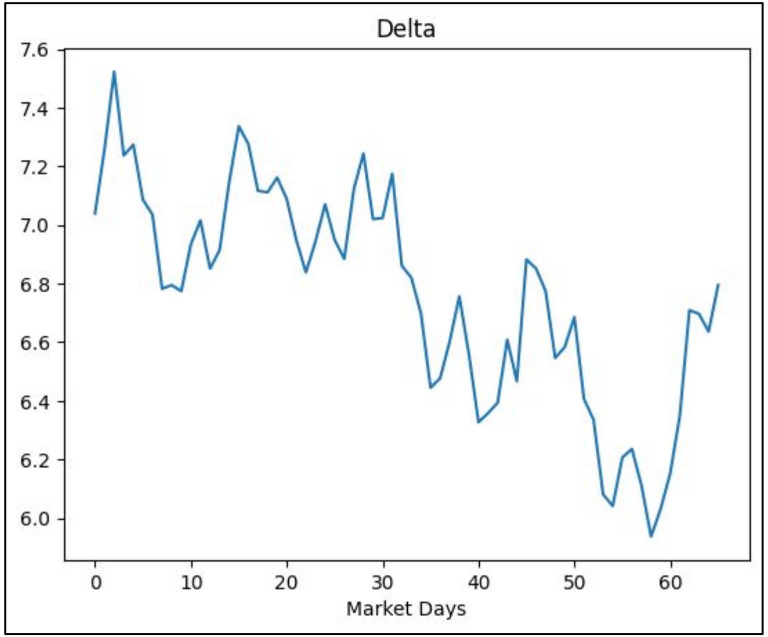
Models - Geometric Brownian Motion (GBM)

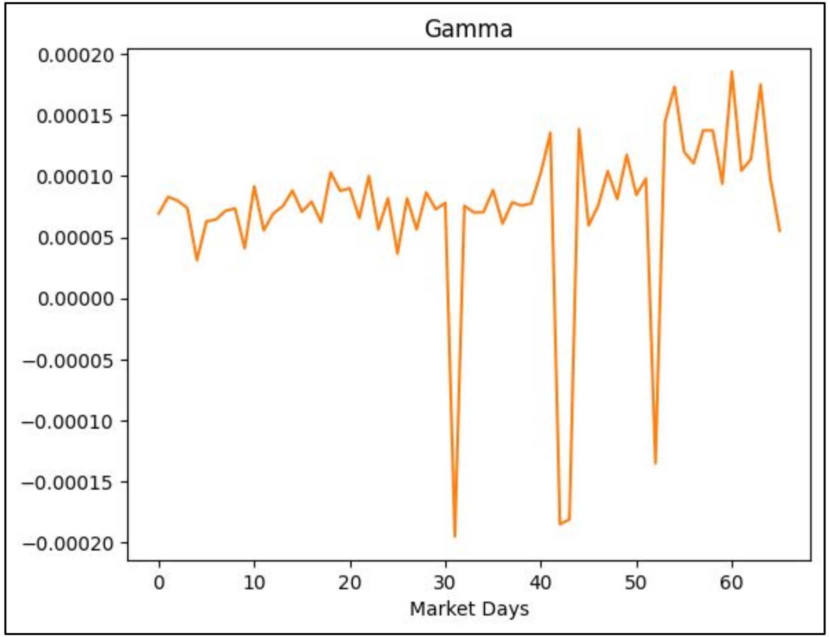
```
Mean Absolute Error from Actual Price for 1000000 simulations (Lookback 63 days):
- Monte Carlo: 104.7021450502505 EUR
- Antithetic Variates: 104.72002892291869 EUR
- Control Variates: 104.72701499035304 EUR
- Empirical Martingale Correction: 104.50549300979961 EUR
```

```
Mean Estimated Variance for 1000000 simulations (Lookback 63 days):
- Monte Carlo: 0.11138777898728294
- Antithetic Variates: 0.11140412994080132
- Control Variates: 0.0025065628219082844
- Empirical Martingale Correction: 0.11138612972251363
```

```
Variance Reduction % for 1000000 simulations (Lookback 63 days):
- Antithetic Variates: -0.01%
- Control Variates: 97.75%
- Empirical Martingale Correction: 0.00%
```

Sensitivities for Monte Carlo Simulation





Models - Geometric Brownian Motion (GBM)

Simulation Enhancement: Just-in-time compilation (Numba)

- Rewrote simulation functions to comply with Numba's
- Adding a @jit in front of the simulation function compiles the Python code to machine code, which runs hundreds of times faster

Simulation Enhancement: Parallel processing (concurrent.futures)

- Used multiple processes to parallelize different simulation days
- Divides runtime by no. of CPUs available

Improvement:

- Cut projected simulation time of 1 million simulations over 66 days using 5 simulation techniques from 30h to 1h

Models - Heston SV

Heston Pricing Formula:

$$C = S_t.\left[\frac{1}{2} + \frac{1}{\pi}\int_0^{\inf} Re\left[\frac{e^{-is\ln K} f_1(s,\nu,x)}{is}\right]ds\right] - Ke^{-rt}\left[\frac{1}{2} + \frac{1}{\pi}\int_0^{\inf} Re\left[\frac{e^{-is\ln K} f_2(s,\nu,x)}{is}\right]ds\right]$$

Simplified (Single Integral) Formula for Heston Pricing:

$$C = \frac{1}{2}(S_t - Ke^{-r(T-t)}) + \frac{1}{\pi}\int_0^{\infty} Re\left[e^{r(T-t)}\frac{f(s-i)}{is.K^{is}} - K\frac{f(s)}{is.K^{is}}\right]ds$$

$$where:$$

$$f(x) = e^{ixrT} S_t^{ix}\left(\frac{1-g.e^{d.\tau}}{1-g}\right)^{\frac{-2\kappa\theta}{\sigma^2}} \times exp\left[\frac{\tau\kappa\theta}{\sigma^2}(\kappa - \sigma\rho.i.x - d) + \frac{\nu}{\sigma^2}(\kappa - \sigma\rho.i.x - d)\frac{1-e^{d_j.\tau}}{1-g_j.e^{d_j.\tau}}\right]$$

$$and$$

$$d = \sqrt{(\rho.\sigma.i.x)^2 - \sigma^2.(2.i.x - x^2)}$$

$$g = \frac{\kappa - \rho.\sigma.i.x + d}{\kappa - \rho.\sigma.i.x - d}$$

Models - Heston SV

Dataset:

- ❖ Options Quote Prices for Underlying Asset: Taken from Bloomberg Terminal
  - ➢ Expiration Dates: 11 Aug 2023 - 15 Mar 2024 (11 different dates)
- ❖ Dataset example:

Strike	Ticker	ExDt	Bid	Ask	IVM	Mid	DM	DB	DA	sDM	Volm
08/11/23											
3275	WSX5EB 8/	08/11/23	1107.699951	1110.100098	0	1108.900024	1	1	0.993345141	1	0
3300	WSX5EB 8/	08/11/23	1082.699951	1085.100098	0	1083.900024	1	1	0.99320507	1	0
3325	WSX5EB 8/	08/11/23	1057.699951	1060.100098	0	1058.900024	1	1	0.993060529	1	0
3350	WSX5EB 8/	08/11/23	1032.699951	1035.199951	125.6420975	1033.949951	0.999665141	1	0.99243933	0.999665141	0
3375	WSX5EB 8/	08/11/23	1007.699951	1010.199951	121.7026291	1008.949951	0.999679089	1	0.992270708	0.999679089	0
3400	WSX5EB 8/	08/11/23	982.6999512	985.1999512	117.7696152	983.9499512	0.999694169	1	0.992095292	0.999694169	0
3425	WSX5EB 8/	08/11/23	957.6999512	960.1999512	113.8392639	958.9499512	0.999710321	1	0.991912067	0.999710321	0
3450	WSX5EB 8/	08/11/23	932.6999512	935.1999512	109.90522	933.9499512	0.999727726	1	0.991720974	0.999727726	0
3475	WSX5EB 8/	08/11/23	907.6999512	910.1999512	105.9605713	908.9499512	0.999746442	1	0.991519511	0.999746442	0
3500	WSX5EB 8/	08/11/23	882.6999512	885.1999512	101.9891052	883.9499512	0.999766886	1	0.991309166	0.999766886	0
3525	WSX5EB 8/	08/11/23	857.6999512	860.1999512	97.98062897	858.9499512	0.999789	1	0.991088569	0.999789	0
3550	WSX5EB 8/	08/11/23	832.6999512	835.1999512	93.91284943	833.9499512	0.99981308	1	0.990856826	0.99981308	0
3575	WSX5EB 8/	08/11/23	807.8000488	810.1999512	102.1004105	809	0.999236882	1	0.990612507	0.999236882	0
3600	WSX5EB 8/	08/11/23	782.8000488	785.1999512	98.55029297	784	0.999239087	1	0.990355253	0.999239087	0
3625	WSX5EB 8/	08/11/23	757.8000488	760.1999512	95.02385712	759	0.999241769	1	0.990083396	0.999241769	0
3650	WSX5EB 8/	08/11/23	732.8000488	735.1999512	91.52035522	734	0.999244988	1	0.989795983	0.999244988	0
3675	WSX5EB 8/	08/11/23	707.8000488	710.3000488	93.23561096	709.0500488	0.998652339	1	0.988842905	0.998652339	0
3700	WSX5EB 8/	08/11/23	682.8000488	685.3000488	89.75177765	684.0500488	0.998632789	1	0.988498569	0.998632789	0
3725	WSX5EB 8/	08/11/23	657.8000488	660.3000488	86.28860474	659.0500488	0.998612285	1	0.988133371	0.998612285	0
3750	WSX5EB 8/	08/11/23	632.8000488	635.3000488	82.85087585	634.0500488	0.998589993	1	0.987742484	0.998589993	0
3775	WSX5EB 8/	08/11/23	607.8000488	610.3000488	79.43598175	609.0500488	0.998566031	1	0.987324119	0.998566031	0
3800	WSX5EB 8/	08/11/23	582.8000488	585.3000488	76.04009247	584.0500488	0.998540759	1	0.986876905	0.998540759	0
3825	WSX5EB 8/	08/11/23	557.8000488	560.3000488	72.66053009	559.0500488	0.998514533	1	0.986395001	0.998514533	0
3850	WSX5EB 8/	08/11/23	532.8000488	535.3000488	69.30786896	534.0500488	0.99848491	1	0.985874474	0.99848491	0
3875	WSX5EB 8/	08/11/23	507.8000488	510.3000488	65.97357178	509.0500488	0.99845314	1	0.985312045	0.99845314	0
3900	WSX5EB 8/	08/11/23	482.8000488	485.3999023	65.53482819	484.0999756	0.997626543	1	0.983832836	0.997626543	0
3925	WSX5EB 8/	08/11/23	457.8999023	460.3999023	64.22728729	459.1499023	0.99677676	1	0.983128428	0.99677676	0

Models - Heston SV

- Initializing parameters and defining objective function
  - $\Theta = (\kappa, \theta, \xi, \rho, V_0) = (0.5, 0.5, 0.5, -0.5, 0.17)$
  - r, S_t
  - Objective Function: $\min_{\Theta} \sum_{i=1}^{N} (P_i(\Theta) - M_i)^2$
  - Upper and Lower Bounds: (3, 3, 5, 0, 1), (10^{-2}, 10^{-2}, 10^{-2}, -1, 10^{-2})
  - Optimization Function: Levenberg Marquardt algorithm

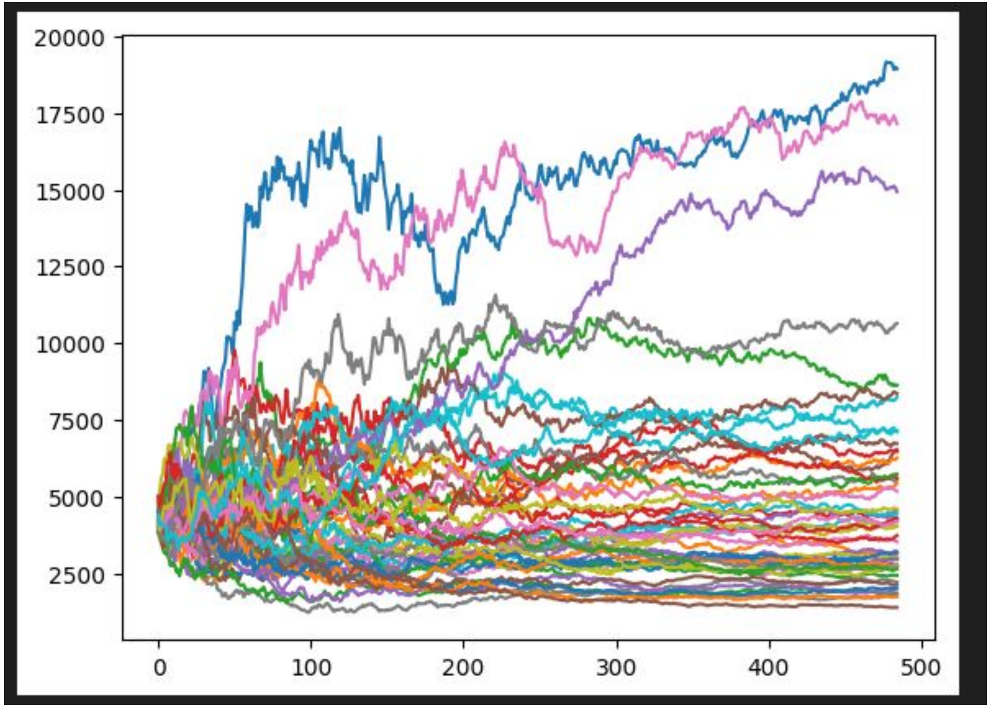
- Calibration Results:
  - ➢ $\Theta = (\kappa, \theta, \xi, \rho, V_0) = (9.82380774, 0.05478766, 0.02353233, -0.95117468, 2.83318275)$

Models - Heston SV

Simulation Setup

- No. of Simulations: 1,000,000
- Simulation Dates
    - 9 Aug 2023 :(
- $\Theta = (\kappa, \theta, \xi, \rho, V_0) = (9.82380774, 0.05478766, 0.02353233, -0.95117468, 2.83318275)$
- S_t, r

Example of Simulated Underlying Paths (Monte Carlo)



Models - Heston SV Results

After applying payoff function + discounting factor:

Predicted Value for 9 Aug 2023: 1504.36 EUR

Estimated Variance: 1.238661441

95% Confidence Interval:
[1502.18, 1506.54]

Absolute Error: 212.90

Thank You!

Q & A