Quantum State Tomography and Post Measurement Analysis in Qiskit

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Abstract

The IBM Quantum Experience is a public platform for executing quantum circuits on superconducting back-ends. We execute the Teleportation protocol, Grover's search algorithm, Entanglement Swapping and Entanglement Purification on three superconducting devices available from IBMQ. We analyze the results from post measurement circuits and reconstruct the final quantum state using state tomography.

I. Introduction

Quantum computers are defined by their use of quantum mechanical phenomena in order to manipulate data. Modeling the functionality of these computers is done with quantum circuits which are based on quantum bits (or qubits). In this project we simulate and measure circuits in Qiskit, which is an open-source quantum computing software development framework. In order to analyze the results we use 1- and 2 qubit Quantum state tomography. Quantum state tomography is a method for determining the final quantum state for real computation. Although a quantum state cannot be directly determined from one measurement, preparing the same state and measuring it in different basis allows us to construct an approximation of the quantum state.

The purpose of this report is to examine various circuit in order to determine if the post measurement method is valid for different circuits and if we can improve the results form various devices by implementing 1- and 2 qubit state tomography. In the case of for example quantum teleportation or entanglement swapping circuit it is impossible to run the original circuits on real devices since they require that initial measurements dictate which operators are used on the qubits. This is impossible on IBMQ backends because they do not support operations after measurements. The goal of these measurements is to determine the fidelity of a prepared state to a target pure state. To this end we analyze density matrices and Pauli set plots of the final states. Our state tomography results also allow us to correct for readout error, which is the error caused by the measurement of the qubits in different basis [1].

In this report we first discuss the theory needed to understand state tomography and readout error correction. Then we present our methods for developing Qiskit software and explain the relevant circuits. In results chapter the measurement data is shown and from this data we derive important performance metrics like the fidelity for each circuit on different backends. Finally in the last chapter we discuss the conclusion, recommendations and outlook.

II. Theory

In a classical computer its internal state is measured at different points in time in order to debug the system. However, for a quantum computer, the analogy would be the measurement of its density matrix, which is called state tomography. We first define the density matrix of a single qubit,

$$\rho = \frac{1}{2}\left(I + \sum_i \alpha_i \sigma_i\right) \tag{1}$$

where σ_i are all the Pauli-matrices and α_i are the real-valued coefficients. Using the trace orthogonality of the Pauli-matrices,

$$\mathrm{Tr}\left(\sigma_j \sigma_k\right) = 2\delta_{jk} \tag{2}$$

we can derive the real-valued coefficients by calculating the expectation values of the different Pauli-matrices.

$$\mathrm{Tr}\left(\rho\sigma_i\right) = \langle\sigma_i\rangle = \alpha_i \tag{3}$$

By measuring the single qubit in the different basis (X,Y and Z) we can derive these expectation values. This requires a repeated preparation and measuring of the final state. In reality, the measured expectation values are estimations of $\langle X\rangle, \langle Y\rangle, \langle Z\rangle$. Often in a quantum computer the measurements are only done in the Z-basis. Other operators are realized using rotation operators before the final measurement.

In order to convert the estimated- to real expectation values we correct for readout error, which will give us a better estimation of the density matrix ρ. If $\epsilon_{10}, \epsilon_{01}$ are the probabilities that a $|0\rangle$ state gives an eigenvalue back of -1 and a $|1\rangle$ state which is measured as a eigenvalue 1, and if α, β are coefficients of the final state $|\psi\rangle$, then the measured expectation value $\langle m\rangle$ in the Z-basis is

$$\langle m\rangle = \tag{4}$$

III. Results

IV. Conclusion

To conclude, we live in a society

References

[1] Michael Nielsen and Isaac Chuang. *Quantum computation and quantum information*. Cambridge New York: Cambridge University Press, 2010. ISBN: 9781107002173.