IBM Q Backend Performance Analysis

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Outline

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Quantum State Tomography

- **Fundamental Problem:** Measurement doesn't reveal the quantum state
- **Solution:** Repeated measurement in different bases → reconstructed state

1-qubit reconstruction

$$\rho = \frac{1}{2}\left(\mathbb{1} + \sum_{i=1}^{3} \alpha_i \sigma_i\right)$$

$$\rho = \frac{1}{2}\left(\mathbb{1} + \langle \hat{X} \rangle \hat{X} + \langle \hat{Y} \rangle \hat{Y} + \langle \hat{Z} \rangle \hat{Z}\right)$$

Total of 3 different sets of measurements.

2-qubit reconstruction

$$\rho = \frac{1}{4} \sum_{\vec{v}_i} \langle \sigma_{v_1} \otimes \sigma_{v_2} \rangle \, \sigma_{v_1} \otimes \sigma_{v_2}$$

$$\rho = \frac{1}{4}\left(\mathbb{1} \otimes \mathbb{1} + \langle \mathbb{1} \otimes \hat{X} \rangle \mathbb{1} \otimes \hat{X} + \ldots\right)$$

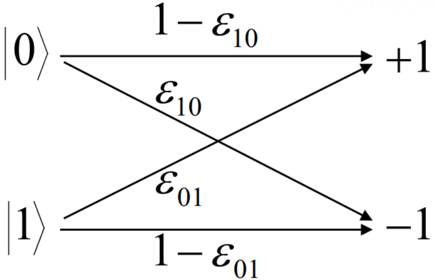
Total of 9 measurement set, for 15 non-trivial expectation values.

Calibrating for Readout - Single Qubit

- The act of measurement on superconducting qubits introduces a large source of error.
- We correct for this by initializing and measuring empty circuits.

Scheme to Calibrate

- Initialize in $|0\rangle$, $|1\rangle$.
- Measure output and derive ϵ values.



$$\overline{m} = (\epsilon_{01} - \epsilon_{10}) + (1 - \epsilon_{01} - \epsilon_{10}) \left(|\alpha|^2 - |\beta|^2\right)$$

$$\langle \hat{\sigma}_i \rangle = \frac{\overline{m}_i - \beta_0}{\beta_1} \tag{1}$$

Calibrating for Readout - 2 Qubit Case

Much less straightforward...

$$\begin{pmatrix} \overline{m}_{ij,MSQ} \\ \overline{m}_{ij,LSQ} \\ \overline{m}_{ij,corr} \end{pmatrix} = \begin{pmatrix} \beta_{0,M} \\ \beta_{0,L} \\ \beta_{0,corr} \end{pmatrix} + \begin{pmatrix} \beta_{1,M} & \beta_{2,M} & \beta_{3,M} \\ \beta_{1,L} & \beta_{2,L} & \beta_{3,L} \\ \beta_{1,corr} & \beta_{2,corr} & \beta_{3,corr} \end{pmatrix} \begin{pmatrix} \langle I\sigma_j \rangle \\ \langle \sigma_i I \rangle \\ \langle \sigma_i \sigma_j \rangle \end{pmatrix}$$

In order to find the corrected expectation values from the measured ones (the $\bar{m}$), we need to find the β values by initializing the qubits in $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ and relating the output expectation values using:

$$\begin{pmatrix} \overline{m}_{A,k} \\ \overline{m}_{B,k} \\ \overline{m}_{C,k} \\ \overline{m}_{D,k} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} \beta_{0,k} \\ \beta_{1,k} \\ \beta_{2,k} \\ \beta_{3,k} \end{pmatrix} \tag{2}$$

where the k can be MSQ, LSQ or the correlated outcomes.

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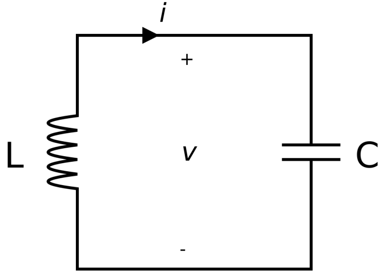
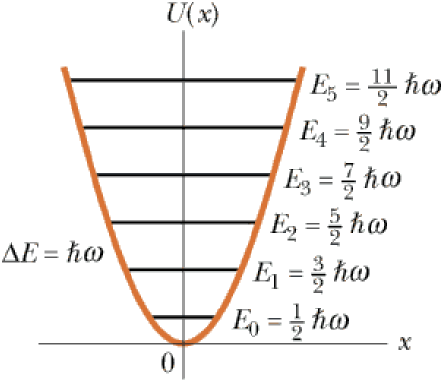
Quantum LC oscillators



Physics described by the HO Hamiltonian:

$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2$$
$$= \frac{Q^2}{2C} + \frac{\phi^2}{2L}$$

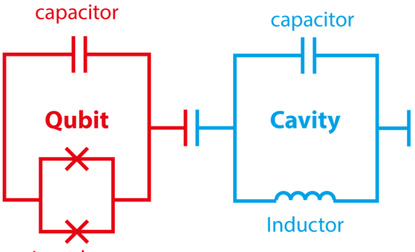
Energy Levels



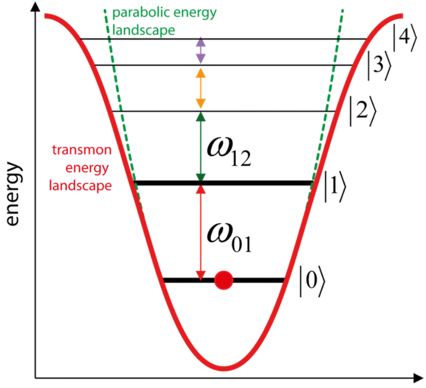
Energy spacing is **even**... not good for a qubit!

The Transmon Qubit

- **First Step:** Introduce Josephson element with non-linear inductance to shift energy levels.
- **Make it a Transmon:** Couple to regular LC circuit to reduce sensitivity to charge noise, by boosting the ratio E_J/E_C. Increases T_1 coherence times.



Unique energy spacing → confine dynamics to 2 states, $|0\rangle$ and $|1\rangle$



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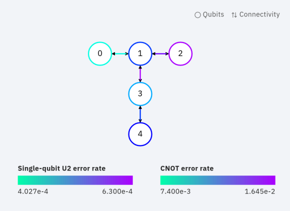
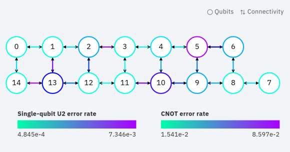
Device connections

Devices, with connections, used in our research:

Burlington

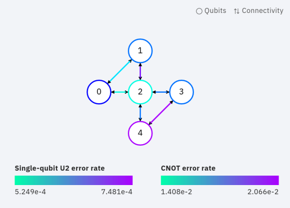


Melbourne



Only qubits 0, 1, 2 and 14 were used.

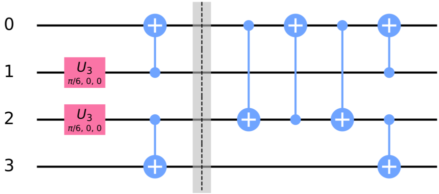
Yorktown



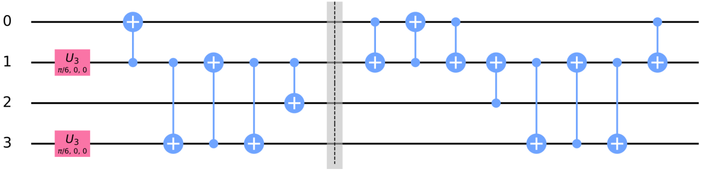
Device connections have a significant impact on the number of gates required to run a circuit.

Example: Yorktown versus Burlington

Yorktown and Burlington both 5 qubit devices, but give different number of required gates (for Purification protocol)



Yorktown (7 CNOT gates)



Burlington (13 CNOT gates)

Error percentage

Average percent error on single qubit U_2 and CNOT gates.

Backend	$U_2(\%)$	$CNOT(\%)$
Burlington	0.050	1.213
Melbourne	1.258	2.156
Yorktown	0.689	2.275

- Burlington has lowest average error, but this doesn't necessarily mean better fidelity, as the number of gates for each circuit varies per device.
- Yorktown, with highest average CNOT error, still often outperforms other backends.

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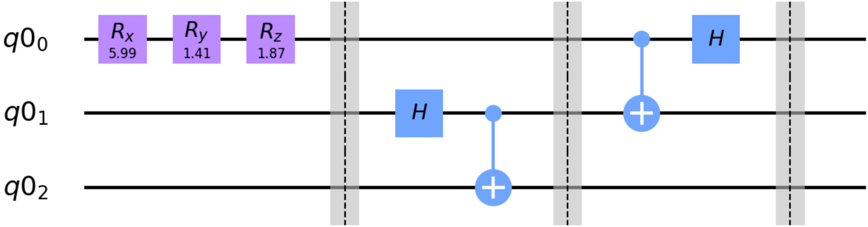
The Teleportation protocol

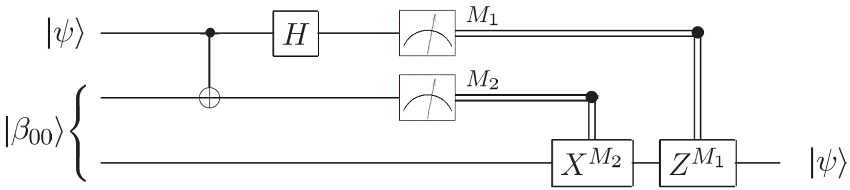
Circuit run

- Initialize random state on first qubit.
- Initialize $|\Phi^+\rangle$ Bell state on bottom qubits.
- CNOT gate on qubit 0 and 1.
- Hadamard gate on qubit 0.



The Teleportation protocol

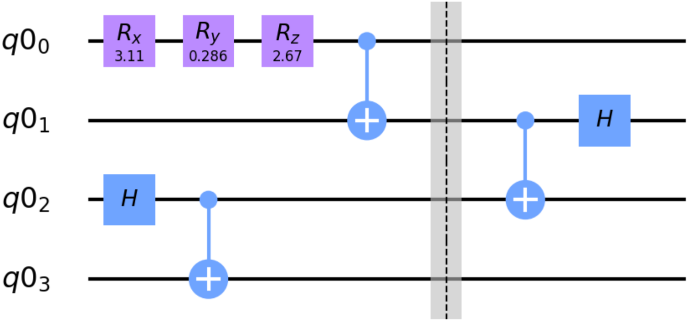
No operations possible after measurement on IBM Q devices. So, resort to post measurement techniques. If measurement of qubit 0 and 1 gives -1, Pauli-Z and Pauli-X matrix are applied, respectively.



Entanglement swap

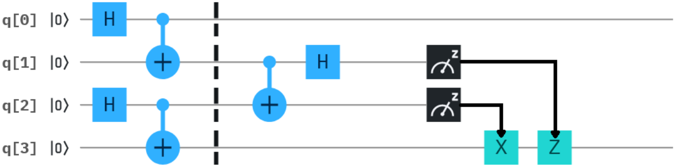
Circuit run

- Initialize random Bell-like state on top qubits.
- Initialize $|\Phi^+\rangle$ Bell state on bottom qubits.
- CNOT gate on qubit 1 and 2.
- Hadamard gate on qubit 1.



Entanglement swap

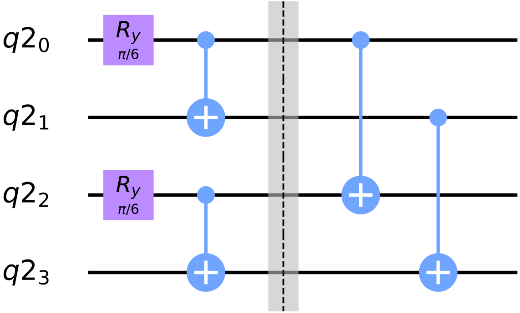
Again, resort to post measurement techniques. If measurement of qubit 1 and 2 gives -1, Pauli-Z and Pauli-X matrix are applied, respectively. This creates entanglement between qubits 0 and 3 of the random Bell-like state.



Entanglement purification

Circuit run

- Initialize two Bell-like state on top and bottom qubits by rotating $\theta = \arcsin(2F - 1)$. Where F is the input fidelity. Here $F = 0.75$.
- CNOT gate on qubit 0 and 2.
- CNOT gate on qubit 1 and 3.

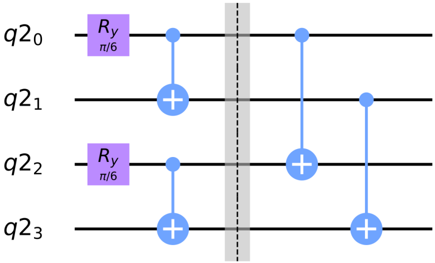


Entanglement purification

Circuit run (with states)

- $|\Psi\rangle = 0.933\,|0000\rangle + 0.067\,|1111\rangle + 0.250\,(|0011\rangle + |1100\rangle)$
- $|\Psi\rangle = 0.933\,|0000\rangle + 0.067\,|1011\rangle + 0.250\,(|0111\rangle + |1100\rangle)$
- $|\Psi\rangle = 0.933\,|0000\rangle + 0.067\,|0011\rangle + 0.250\,(|1111\rangle + |1100\rangle)$

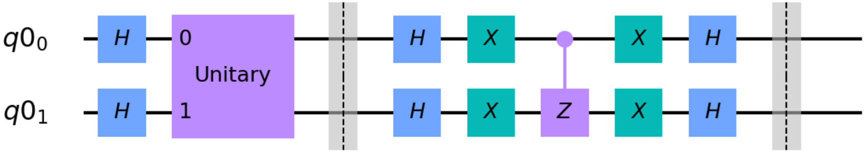
Measuring bottom two qubits in $|11\rangle$ (which occurs with a probability of 12.5%) gives the top two qubit state $|\Psi_{1,2}\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle) = |\Phi^+\rangle$.



Grover's search algorithm

Circuit run

- Apply Hadamard gate to the two qubits.
- Apply diagonal matrix with three values of 1 and one -1 (rest of matrix is 0).
- Do an inversion about the mean.



Measurement of the circuit gives position of the -1 in the diagonal matrix. For example, $|11\rangle$ measurement is the fourth position

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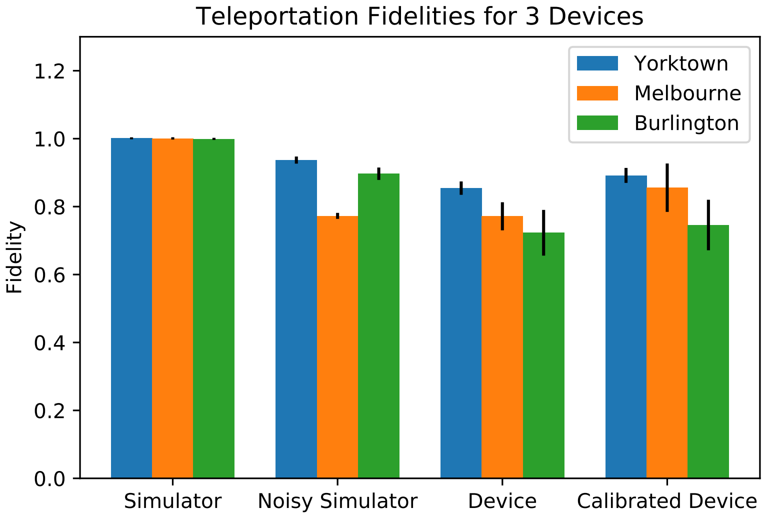
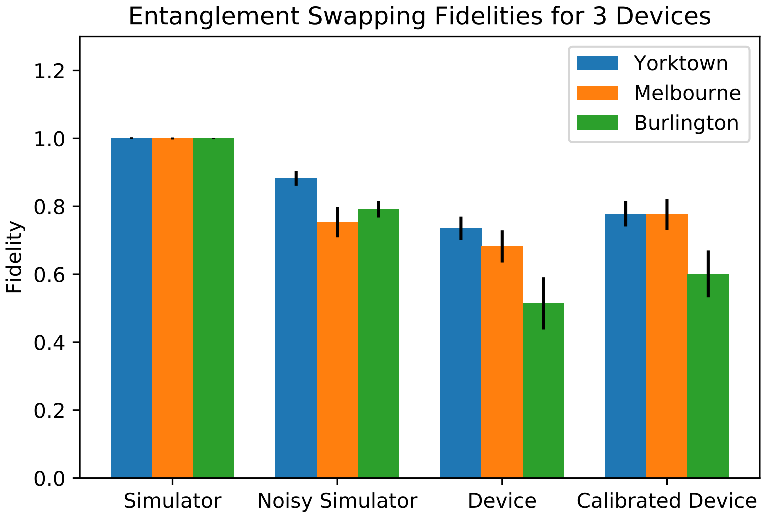
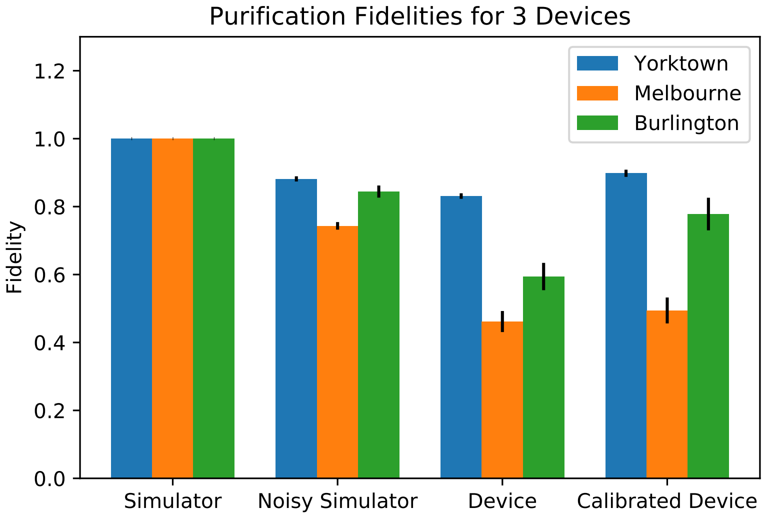
Teleportation



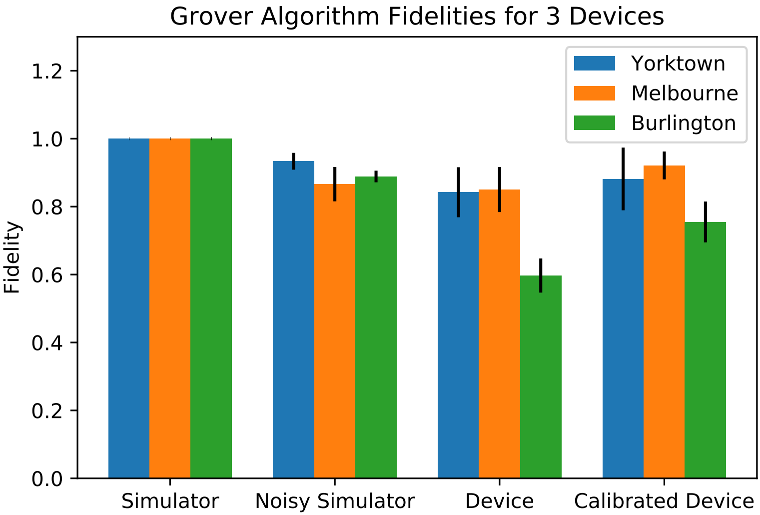
Entanglement Swapping



Entanglement Purification



Grover's Algorithm



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Conclusion/Future Outlook

- The circuit states were successfully reconstructed using one- and two-qubit state tomography.
- The optimal backend for implementation was Yorktown, since the advantages of greater inter-connectivity outweigh any disadvantage from slightly lower gate fidelities.
- Different types of circuits will favor implementation on qubits with very different physical properties.
- An interesting comparison to conduct might be the performance of different implementations with regards to a handful of well-known protocols.