Teleportation

April 14, 2020

1 Quantum Teleportation

This script teleports an arbitrary state from one qbit to another using the well-known teleportation protocol. Single qbit tomography and readout calibration are used to reconstruct the output state, in order to compare with the expected result.

[1]:
```
#importing necessary modules
import numpy as np
import pandas as pd
import random
%config InlineBackend.figure_format = 'svg' # Makes the images look nice
%matplotlib inline

# qiskit modules
import qiskit
from qiskit import ClassicalRegister, QuantumRegister, QuantumCircuit, Aer
from qiskit.tools.visualization import plot_state_city, plot_bloch_multivector,␣
↪plot_histogram, plot_state_paulivec
from qiskit.tools.monitor import job_monitor
from qiskit.quantum_info import state_fidelity, DensityMatrix

# tomography functions
from qiskit.ignis.verification.tomography import state_tomography_circuits,␣
↪StateTomographyFitter

# readout calibration
from qiskit.ignis.mitigation.measurement import complete_meas_cal,␣
↪CompleteMeasFitter
```

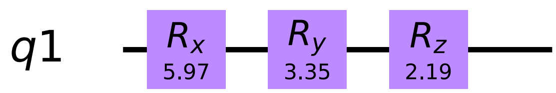
[2]:
```
# importing IBMQ backends
from qiskit import IBMQ
ibmq_provider = IBMQ.load_account() # credentials stored on disk
device = ibmq_provider.get_backend('ibmq_16_melbourne')

# necessary imports for noise modelling
from qiskit.providers.aer.noise import NoiseModel
noise_model = NoiseModel.from_backend(device)
coupling_map = device.configuration().coupling_map
```

```
basis_gates = noise_model.basis_gates
```

1.1 Defining the expected state

[3]:
```
R=[]
for i in range(3):
    R.append(10*random.random())

qreg = QuantumRegister(3)
qreg_exp = QuantumRegister(1)
qc_expected = QuantumCircuit(qreg_exp)
qc_expected.rx(np.pi/5*R[0],0)
qc_expected.ry(np.pi/5*R[1],0)
qc_expected.rz(np.pi/5*R[2],0)
qc_expected.draw(output='mpl')
```

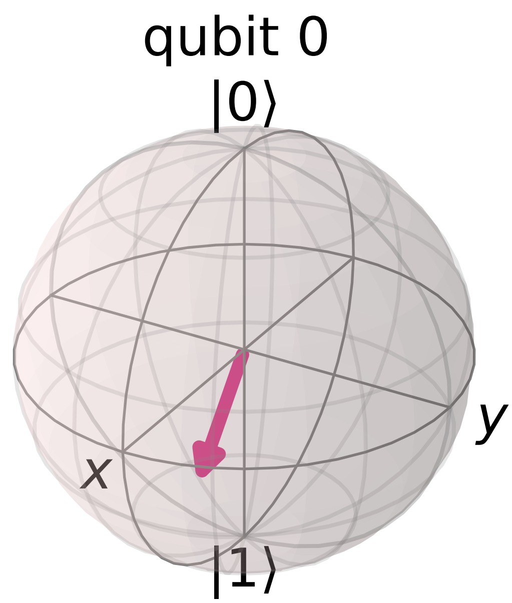
[3]:



[4]:
```
# visualizing the expected state on the Bloch sphere
job = qiskit.execute(qc_expected, Aer.get_backend('statevector_simulator'))
psi_expected = job.result().get_statevector(qc_expected)
print('The expected state is:', psi_expected)
plot_bloch_multivector(psi_expected)
```

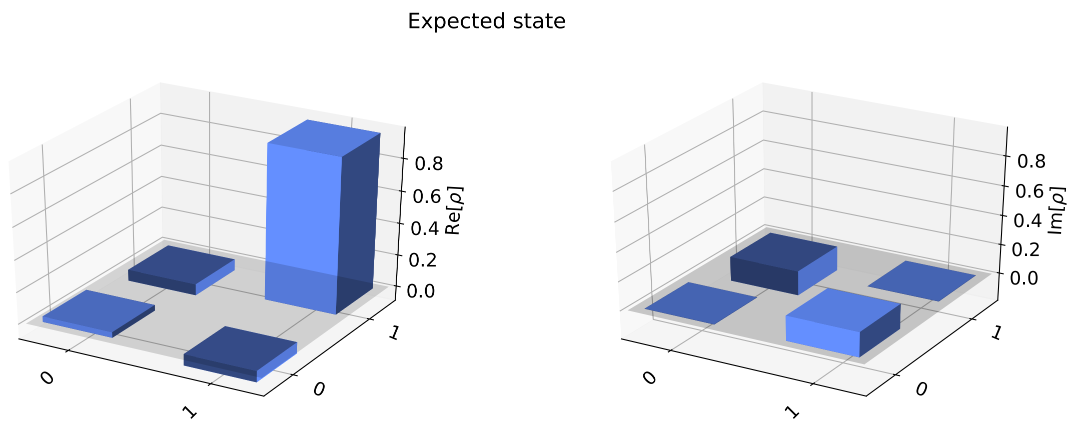
```
The expected state is: [ 0.18761614+0.j         -0.37928017-0.90606111j]
```

[4]:



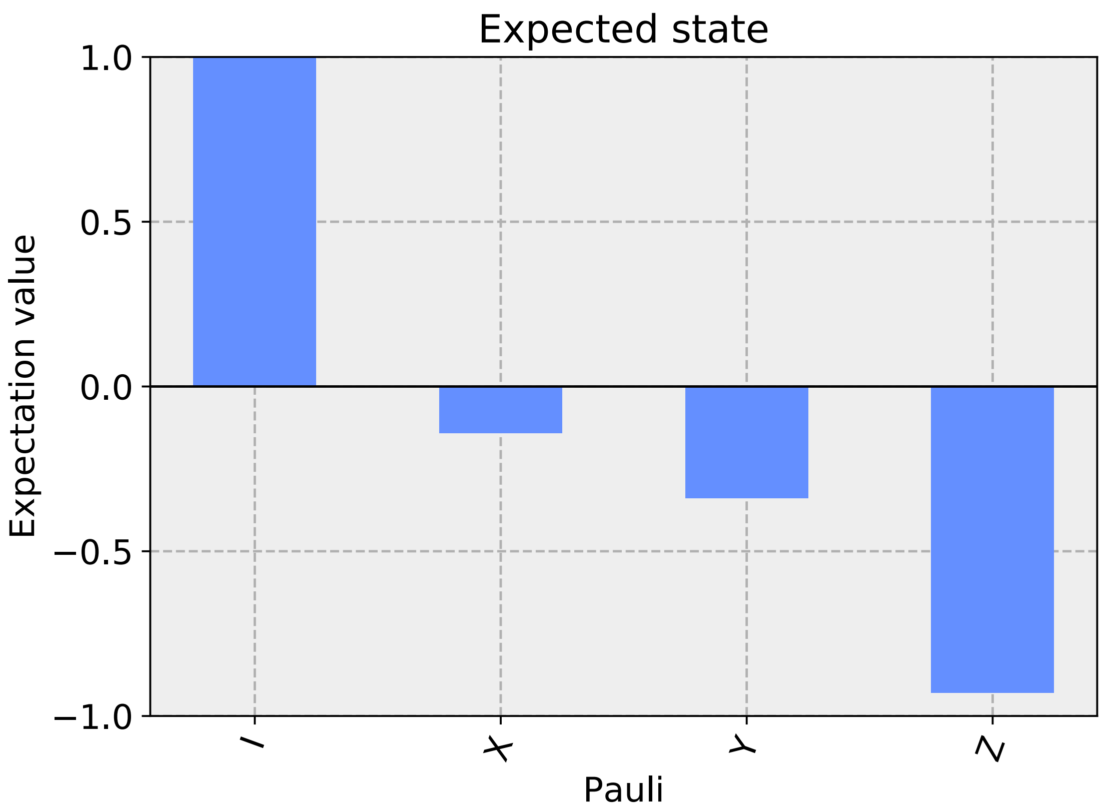
[5]:
```
# plotting the density matrix
plot_state_city(psi_expected, title='Expected state')
```

[5]:



[24]:
```
plot_state_paulivec(psi_expected, title='Expected state')
```

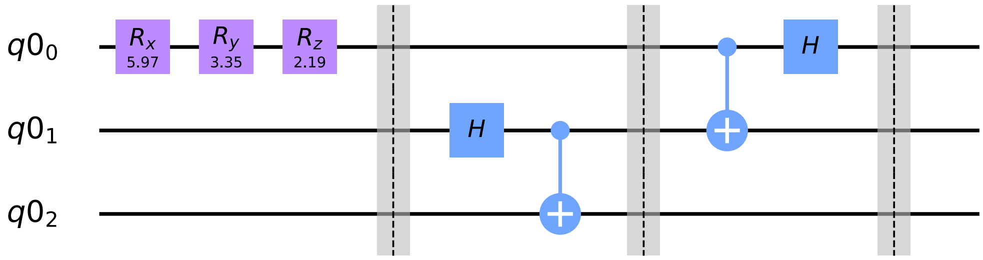
[24]:



1.1.1 Defining the circuit

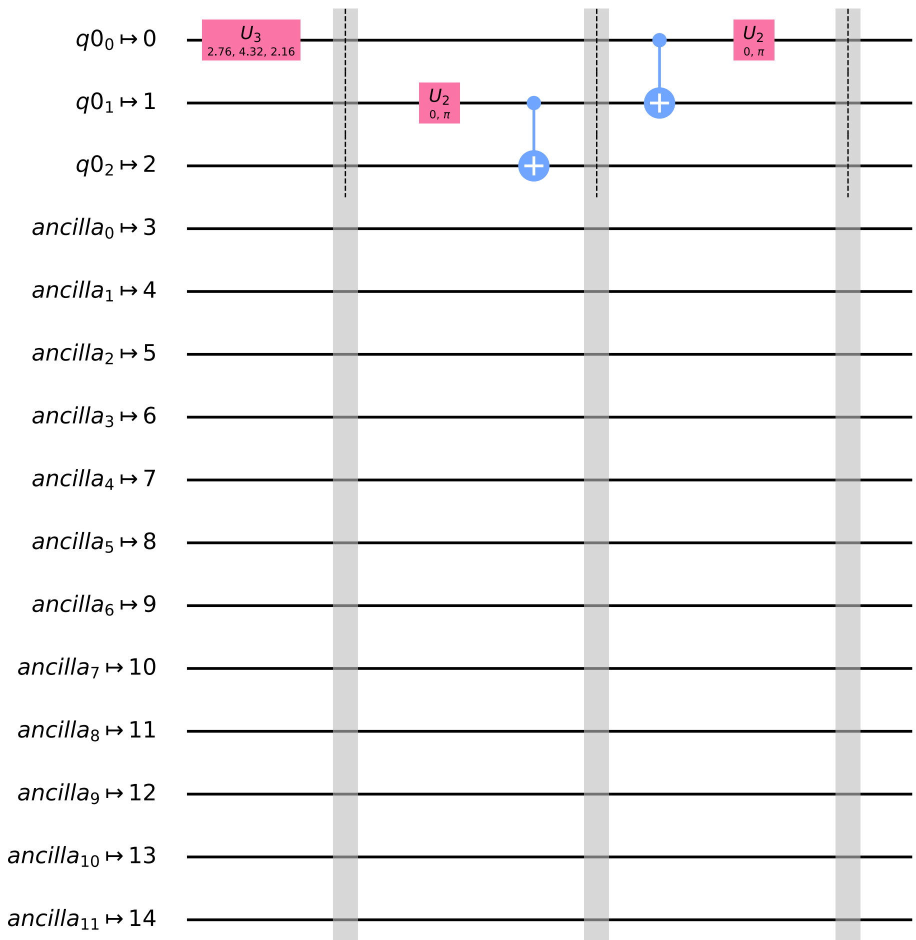
[6]:
```
qc = QuantumCircuit(qreg)
qc.rx(np.pi/5*R[0],0)
qc.ry(np.pi/5*R[1],0)
qc.rz(np.pi/5*R[2],0)
qc.barrier()
qc.h(1)
qc.cx(1,2)
qc.barrier()
qc.cx(0,1)
qc.h(0)
qc.barrier()
qc.draw(output='mpl')
```

[6]:



[7]:
```
# what does this circuit look like on the device?
transpile_qc = qiskit.compiler.transpile(qc,device)
transpile_qc.draw(output='mpl')
```

[7]:

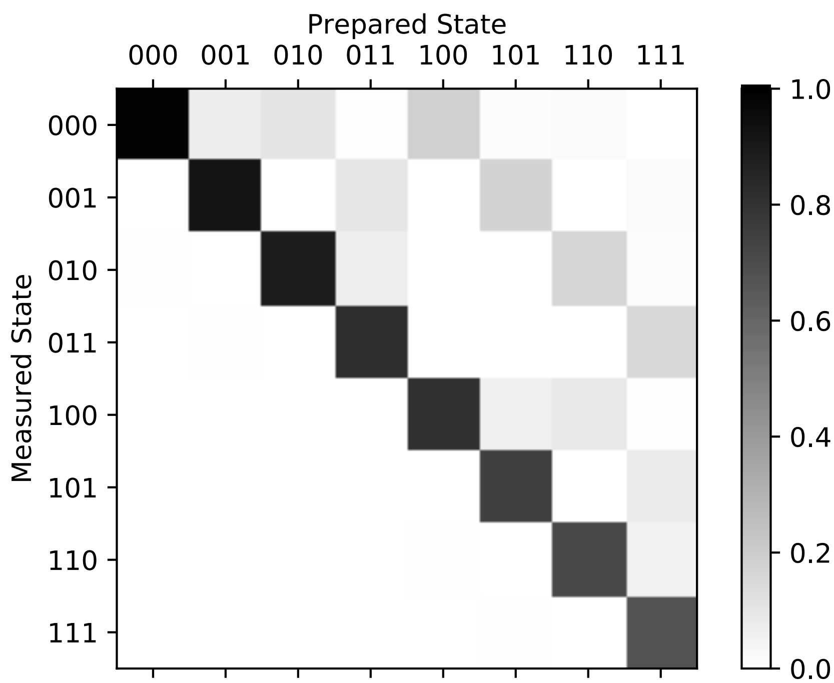


1.2 Readout correction

This is done by initializing the three qbits in all eight combinations of 000, 001, etc and measuring the output to estimate the likelihood of a bit flip.

[8]:
```
#readout calibration measurements
cal_circuits, state_labels = complete_meas_cal(qr = qc.qregs[0], circlabel =␣
↪'measerrormitcal')
cal_job = qiskit.execute(cal_circuits,backend = device,shots = 8192,␣
↪optimization_level = 0)
job_monitor(cal_job)
calres = cal_job.result()
meas_fitter = CompleteMeasFitter(calres, state_labels)
meas_fitter.plot_calibration()
```

```
Job Status: job has successfully run
```



[9]:
```
# post-processing for finding the readout error correction
counts = [0,0,0,0,0,0,0,0]
bits = ['000', '100', '001', '101', '010', '110', '011', '111']
for i in range(8):
    counts[i] = calres.get_counts(cal_circuits[i])
    for bit in bits:
        if bit not in counts[i]:
            counts[i][bit] = 0
```

```python
m0 = 0
m1 = 0
for i in range(8):
    for num, bit in enumerate(bits):
        if num%2 == 0 and i < 4:
            m0 += float(counts[i][bit]) / (8192*4)
        if num%2 == 1 and i < 4:
            m0 -= float(counts[i][bit]) / (8192*4)
        if num%2 == 1 and i > 3:
            m1 -= float(counts[i][bit]) / (8192*4)
        if num%2 == 0 and i > 3:
            m1 += float(counts[i][bit]) / (8192*4)

beta0 = float(m0 + m1) / 2
beta1 = float(m0 - m1) / 2
print('The value of beta 0:', beta0)
print('The value of beta 1:', beta1)

cons_F = beta1
F_aa = 1/2 + beta1 /2
print('The conservative fidelity is:', cons_F)
print('The average assignment fidelity is:', F_aa)
```

```
The value of beta 0: 0.184539794921875
The value of beta 1: 0.811309814453125
The conservative fidelity is: 0.811309814453125
The average assignment fidelity is: 0.9056549072265625
```

1.3 State Tomography

In order to reconstruct the output state of the circuit, it is necessary to perform single qbit tomography to build up the state we have from a large number of measurements in the 3 cardinal bases.

[10]:
```python
# defining the tomography circuits
qst_full_circuits = state_tomography_circuits(qc,qreg)
# the three circuits are in the order [ZZX, ZZY, ZZZ]
qst_circuit = [qst_full_circuits[24], qst_full_circuits[25],␣
↪qst_full_circuits[26]]
```

[11]:
```python
paulis = [('Z','Z','X'), ('Z','Z','Y'), ('Z','Z','Z')]
basis = ['X', 'Y', 'Z']
backends = ['simulator', 'simulator_noise', device]

for num, backend in enumerate(backends):
    state = {'X': {'0': 0, '1': 0}, 'Y': {'0': 0, '1': 0}, 'Z': {'0': 0, '1':␣
↪0}}
```

```
    state_df = pd.DataFrame(data=[state,state,state,state])
    if num == 0:
        job_sim = qiskit.execute(qst_circuit, Aer.
↪get_backend('qasm_simulator'), shots=8192)
        tom = StateTomographyFitter(job_sim.result(), qst_circuit)
    if num == 1:
        job_sim = qiskit.execute(qst_circuit, Aer.
↪get_backend('qasm_simulator'), shots=8192,
                                 noise_model=noise_model,
                                 coupling_map=coupling_map,
                                 basis_gates=basis_gates)
        tom = StateTomographyFitter(job_sim.result(), qst_circuit)
    if num == 2:
        job_dev = qiskit.execute(qst_circuit, backend, shots=8192)
        job_monitor(job_dev)
        tom = StateTomographyFitter(job_dev.result(), qst_circuit)
    for pauli in paulis:
        for bit in bits:
            if bit not in tom.data[pauli]:
                tom.data[pauli][bit]=0

    # 4 possible outcomes of teleport protocol, where the final state is either
    # psi, X @ psi, Z @ psi, X @ Z @ psi. We store all 4 seperately, to then␣
↪transform
    # the resulting density matrices appropriately before finding fidelity
    for i in range(4):
        for j in range(3):
            state_df.loc[i][basis[j]] = {'0': tom.data[(paulis[j])][bits[2*i]],
                                          '1': tom.
↪data[(paulis[j])][bits[2*i+1]]}

    if num == 0:
        state_sim = state_df
        print('The full state for the simulator is', state_sim)
    if num == 1:
        state_sim_noise = state_df
        print('The full state for the noisy simulator is', state_sim_noise)
    if num == 2:
        state_dev = state_df
        print('The full state for the device is', state_dev)
```

```
The full state for the simulator is                        X
Y                      Z
0  {'0': 846, '1': 1190}  {'0': 695, '1': 1321}  {'0': 74, '1': 1932}
1  {'0': 1177, '1': 913}  {'0': 1401, '1': 674}  {'0': 70, '1': 2007}
2  {'0': 856, '1': 1165}  {'0': 1405, '1': 658}  {'0': 1990, '1': 76}
3  {'0': 1169, '1': 876}  {'0': 686, '1': 1352}  {'0': 1957, '1': 86}
```

```
The full state for the noisy simulator is                          X
Y                      Z
0  {'0': 1155, '1': 1017}  {'0': 1056, '1': 1101}  {'0': 583, '1': 1588}
1   {'0': 1330, '1': 749}   {'0': 1364, '1': 685}  {'0': 513, '1': 1474}
2   {'0': 1088, '1': 953}   {'0': 1416, '1': 604}  {'0': 1957, '1': 124}
3   {'0': 1182, '1': 718}   {'0': 916, '1': 1050}  {'0': 1841, '1': 112}
Job Status: job has successfully run
The full state for the device is                          X
Y                      Z
0  {'0': 1302, '1': 1095}  {'0': 1161, '1': 1211}  {'0': 781, '1': 1677}
1   {'0': 1289, '1': 781}   {'0': 1491, '1': 660}  {'0': 699, '1': 1355}
2    {'0': 971, '1': 968}   {'0': 1404, '1': 613}  {'0': 1841, '1': 139}
3   {'0': 1121, '1': 665}    {'0': 740, '1': 912}  {'0': 1576, '1': 124}
```

[12]:

```
# defining the matrices to reconstruct the density matrices
Iden = np.matrix('1, 0; 0, 1')
XPaul = np.matrix('0, 1; 1, 0')
YPaul = np.matrix('0, 0-1j; 0+1j, 0')
ZPaul = np.matrix('1, 0; 0, -1')

state_tot=[state_sim,state_sim_noise,state_dev]
rho = [0,0,0,0]
r_tot = np.zeros((4,3))
for num, states in enumerate(state_tot):
    rho_tot=[0,0,0,0]
    for i in range(4):
        for j in range(3):
            r_tot[i,j] = (states[basis[j]][i]['0']*1 +␣
↪states[basis[j]][i]['1']*(-1)) \
                        /(states[basis[j]][i]['0'] + states[basis[j]][i]['1'])
    for i in range(4):
        rho_tot[i] = (1/
↪2)*(Iden+r_tot[i,0]*XPaul+r_tot[i,1]*YPaul+r_tot[i,2]*ZPaul)
    rho[num] = rho_tot

r_tot[:,:] = (r_tot[:,:] - beta0) / beta1
rho_corr = [0,0,0,0]
for i in range(4):
        rho_corr[i] = (1/
↪2)*(Iden+r_tot[i,0]*XPaul+r_tot[i,1]*YPaul+r_tot[i,2]*ZPaul)
rho[3] = rho_corr

for ind in range(4):
    rho[ind][1] = ZPaul @ rho[ind][1] @ ZPaul
    rho[ind][2] = XPaul @ rho[ind][2] @ XPaul
    rho[ind][3] = ZPaul @ XPaul @ rho[ind][3] @ XPaul @ ZPaul
```

```
# # finding the average density matrix from the four possible outcomes
# rho_avg = [0,0,0,0]
# for i in range(4):
#     rho_avg[i] = np.mean(np.array([rho[i][0], rho[i][1], rho[i][2],␣
↪rho[i][3]]), axis=0)
```

1.4 Calculating Fidelities

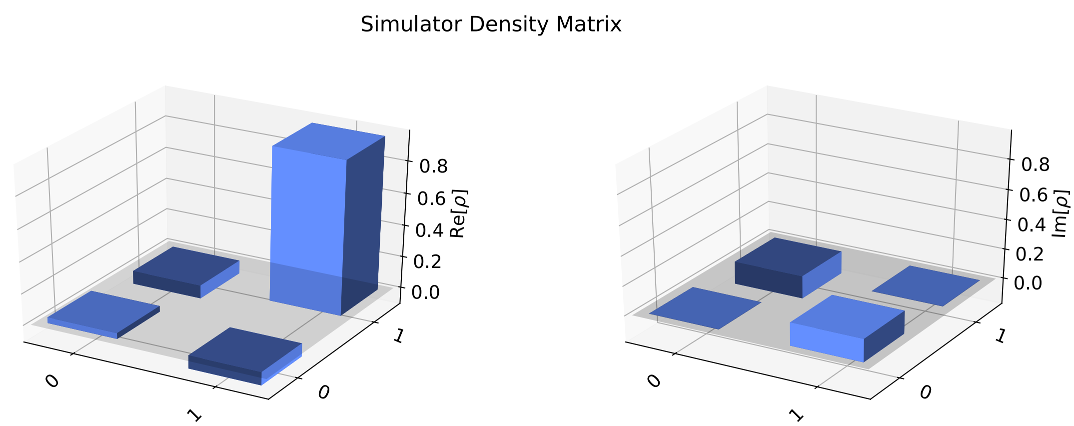
[13]:
```
Fidelity = np.zeros((4,4))
for i in range(4):
    for j in range(4):
        Fidelity[i,j] = state_fidelity(psi_expected, rho[i][j], validate=False)

F = np.mean(Fidelity,axis=1)
print('Simulator Fidelity:', F[0])
print('Noisy Simulator Fidelity:', F[1])
print('Device Fidelity:', F[2])
print('Corrected Device Fidelity:', F[3])
```

```
Simulator Fidelity: 0.9979464626289786
Noisy Simulator Fidelity: 0.857173871328877
Device Fidelity: 0.824352080542855
Corrected Device Fidelity: 0.8997881878964933
```

1.5 Visualizing Output States

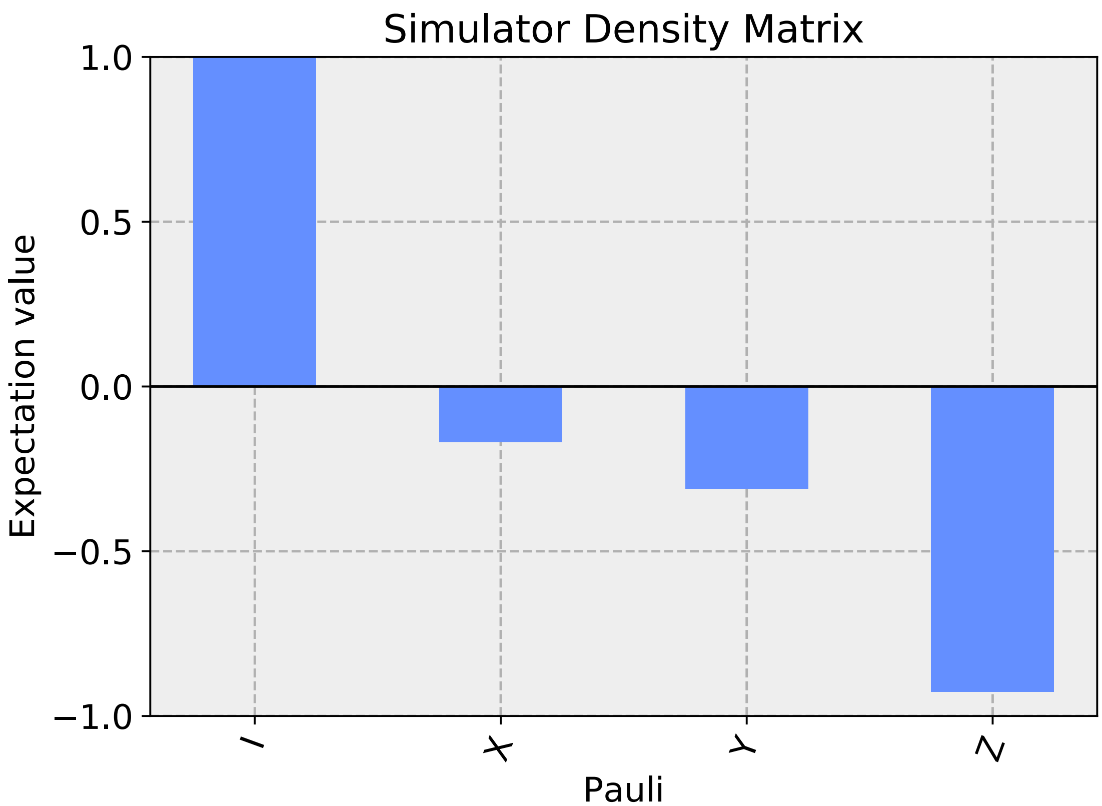
[14]:
```
plot_state_city(rho[0][0], title='Simulator Density Matrix')
```

[14]:



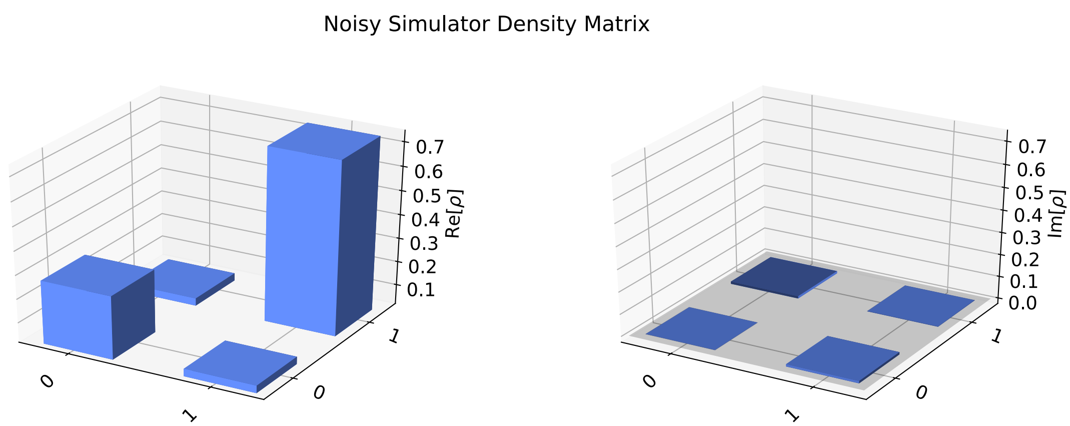
[15]:
```
plot_state_paulivec(rho[0][0], title='Simulator Density Matrix')
```

[15]:



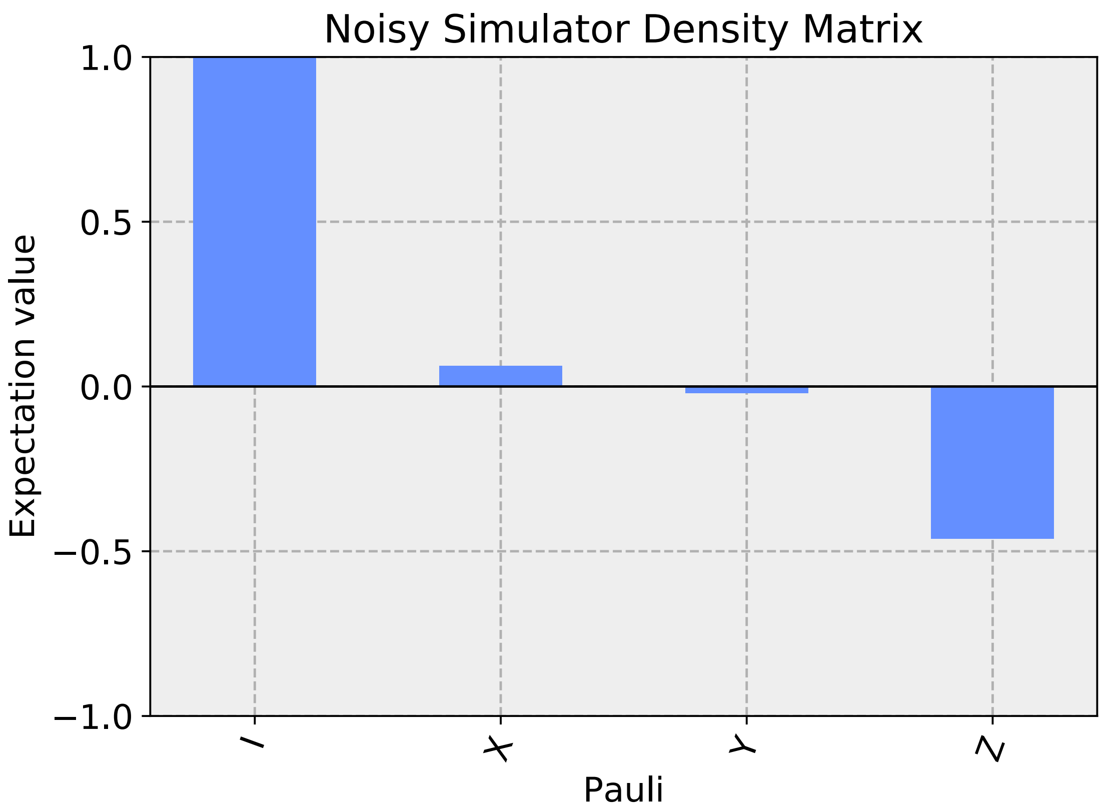
[16]:
```
plot_state_city(rho[1][0], title='Noisy Simulator Density Matrix')
```

[16]:



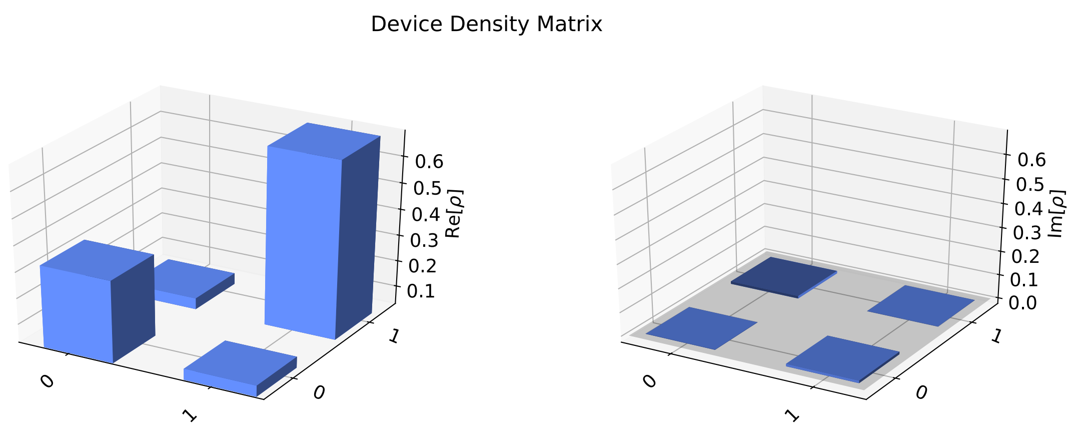
[17]:
```
plot_state_paulivec(rho[1][0], title='Noisy Simulator Density Matrix')
```

[17]:



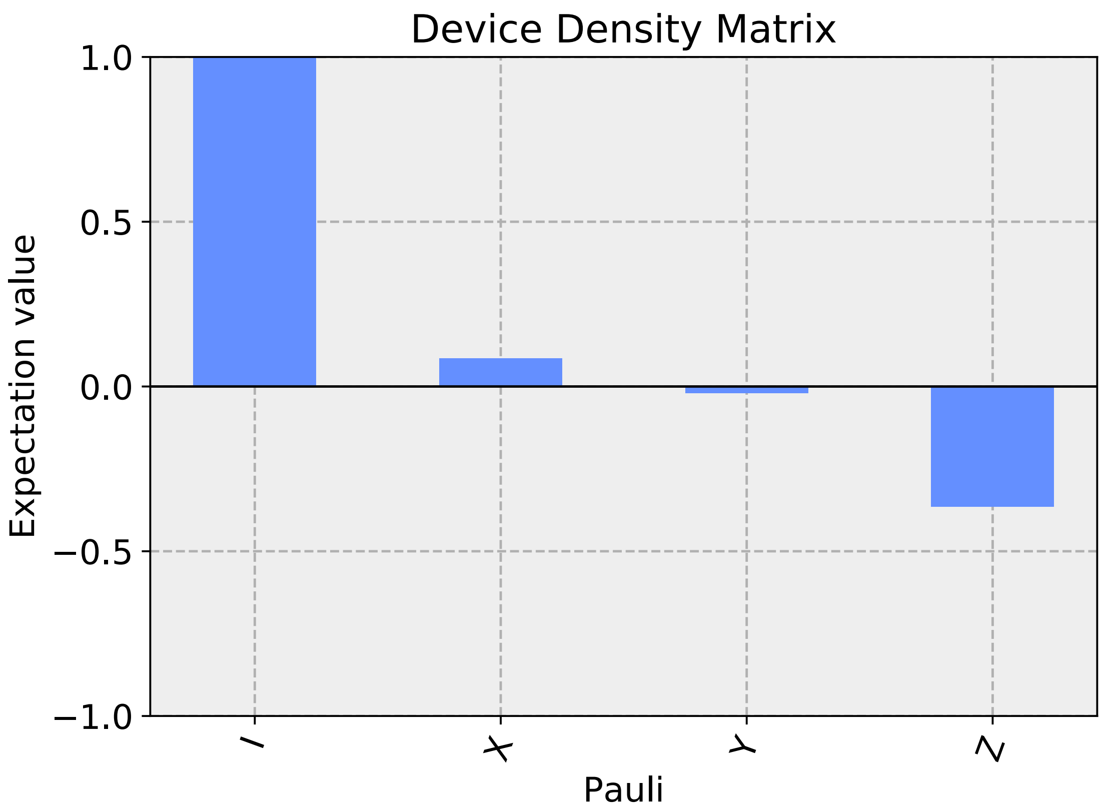
[18]:
```
plot_state_city(rho[2][0], title='Device Density Matrix')
```

[18]:



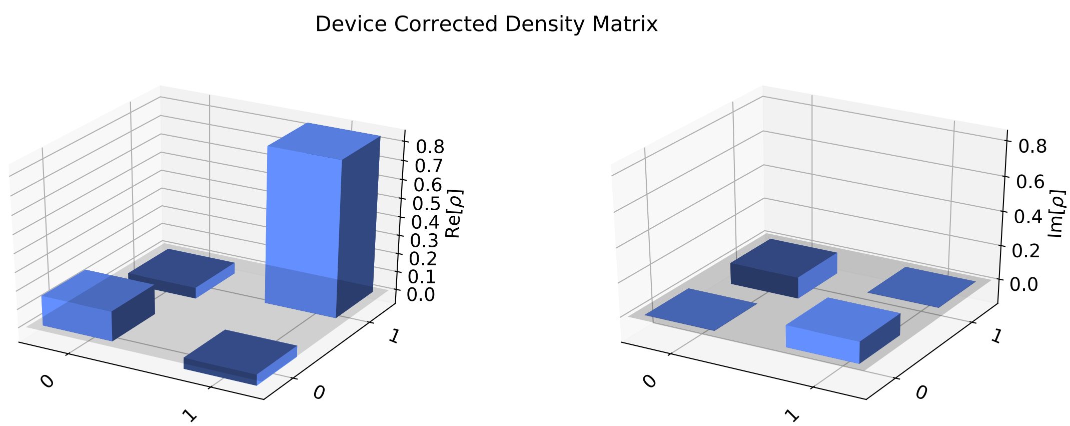
[19]:
```
plot_state_paulivec(rho[2][0], title='Device Density Matrix')
```

[19]:



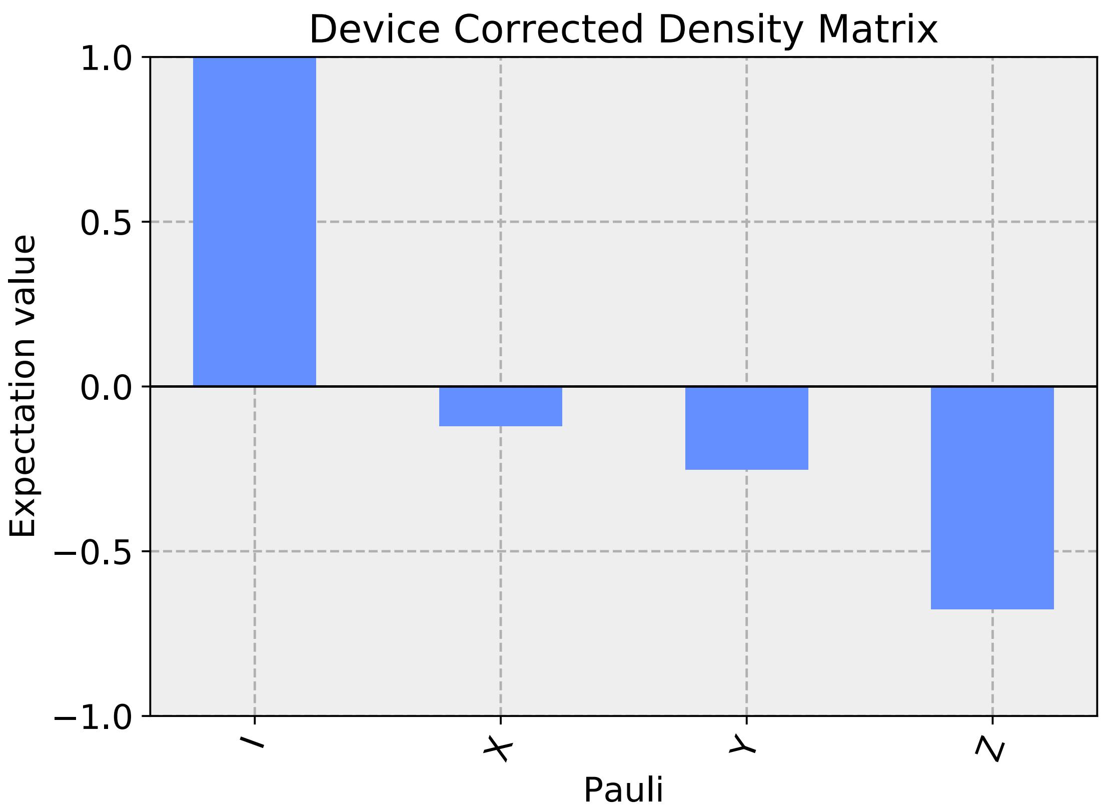
[20]:
```
plot_state_city(rho[3][0], title='Device Corrected Density Matrix')
```

[20]:



[21]:
```
plot_state_paulivec(rho[3][0], title='Device Corrected Density Matrix')
```

[21]:



[22]:
```
# checking validity of the density matrices
print('Is the simulator density matrix valid?', DensityMatrix(rho[0][0]).
 ↪is_valid())
print('Is the device density matrix valid?', DensityMatrix(rho[2][0]).
 ↪is_valid())
```

```
Is the simulator density matrix valid? True
Is the device density matrix valid? True
```

[23]:
```
import qiskit.tools.jupyter
%qiskit_version_table
```

```
<IPython.core.display.HTML object>
```