Quantum State Tomography and Post Measurement Analysis in Qiskit

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Abstract

The IBM Quantum Experience is a public platform for executing quantum circuits on superconducting back-ends. We execute the Teleportation protocol, Grover's search algorithm, Entanglement Swapping and Entanglement Purification on three superconducting devices available from IBMQ. We analyze the results from post measurement circuits and reconstruct the final quantum state using state tomography.

I. Introduction

Quantum computers exploit quantum mechanical phenomena in order to perform calculations and manipulate data in ways that would be impossible on classical computers. Quantum algorithms have been developed to teleport data from one place to another, search databases efficiently, and factor large numbers quickly. These algorithms will comprise the building blocks of the quantum computers of the future, and the current efforts towards realizing a truly universal quantum computer centre around improving the manipulation of quantum bits (or qubits), the basic units of computation in these algorithms.

There are many approaches to creating these qubits. Useful qubits instantiate a set of properties that can be at odds with each other at times. For example, a desirable trait like the ability to accurately control the state of the qubit often conflicts with the desire for the qubit to be long-lived (i.e. to have a long coherence time where its quantum state is safe from environmental degradation).

The various implementations make the trade-off between desirable traits in different ways, and a leading approach for creating qubits uses LC circuits in superconductors [1]. Already in 2014, the first demonstrated universal gate set on superconducting qubits with an average gate fidelity over 99 per cent for all gates was realized [2]. As the field has progressed since then, superconducting qubits have only become more attractive as the building blocks for quantum computers. Now, multi-qubit devices that use superconductors are available publicly, and capable of performing quantum computations that implement small (less than 10 qubits, with limited depth) circuits.

We have simulated and executed a handful of foundational circuits on superconducting devices provided publicly by the IBM Quantum Experience. We use Qiskit, an open-source quantum computing software development platform which has quickly become the most popular means with which to program circuits on publicly available quantum computers. Using 1- and 2-qubit state tomography and post-measurement selection schemes, we reconstruct the average output to characterize the fidelity with which three different superconductor devices implement our chosen circuits.

The purpose of this report is to examine various circuit in order to determine if the post measurement method is valid for different circuits and if we can improve the results form various devices by implementing 1- and 2 qubit state tomography. In the case of for example quantum teleportation or entanglement swapping circuit it is impossible to run the original circuits on real devices since they require that initial measurements dictate which operators are used on the qubits. This is impossible on IBMQ backends because they do not support operations after measurements. The goal of these measurements is to determine the fidelity of a prepared state to a target pure state. To this end we analyze density matrices and Pauli set plots of the final states. Our state tomography results also allow us to correct for readout error, which is the error caused by the measurement of the qubits in different basis [3].

In this report we first discuss the theory needed to understand state tomography and readout error correction. Then we present our methods for developing Qiskit software and explain the relevant circuits. In results chapter the measurement data is shown and from this data we derive important performance metrics like the fidelity for each circuit on different backends. Finally in the last chapter we discuss the conclusion, recommendations and outlook.

II. Theory

i. One-Qubit State Tomography

In a classical computer its internal state is measured at different points in time in order to debug the system. However, for a quantum computer, the analogy would be the measurement of its density matrix, which is called state tomography. Multiple copies of the state are required to distinguish between non-orthogonal states, which are generated by repeating the experiment. We first define the density matrix of a single qubit,

$$\rho = \frac{1}{2}\left(I + \sum_i \alpha_i \sigma_i\right) \tag{1}$$

where σ_i are all the Pauli-matrices and α_i are the real-valued coefficients. Using the trace orthogonality of the Pauli-matrices (because the trace of a operator gives the average value of the observable),

$$\mathrm{Tr}\left(\sigma_j \sigma_k\right) = 2\delta_{jk} \tag{2}$$

we can derive the real-valued coefficients by calculating the expectation values of the different Pauli-matrices.

$$\mathrm{Tr}\left(\rho\sigma_i\right) = \langle\sigma_i\rangle = \alpha_i \tag{3}$$

By measuring the single qubit in the different basis (X,Y and Z) we can derive these expectation values. This requires a repeated preparation and measuring of the final state, and in the limit of a large sample size this will give a good estimation of ρ. In reality, the measured expectation values are estimations of $\langle\hat{X}\rangle$, $\langle\hat{Y}\rangle$, $\langle\hat{Z}\rangle$. Often in a quantum computer the measurements are only done in the Z-basis. Other operators are realized using rotation operators before the final measurement.

In order to convert the estimated- to real expectation values we correct for readout error, which will give us a better estimation of the density matrix ρ. If $\epsilon_{10}, \epsilon_{01}$ are the probabilities that a $|0\rangle$ state gives an eigenvalue back of -1 and a $|1\rangle$ state which is measured as a eigenvalue 1 (note that this is not a quantum effect but classical error), and if α, β are coefficients of the final state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, then the measured expectation value $\overline{m}$ in the Z-basis is

$$\begin{aligned} \overline{m} &= (1-\epsilon_{10})|\alpha|^2 + \epsilon_{01}|\beta|^2 - (1-\epsilon_{01})|\beta|^2 \\ &\qquad -\epsilon_{10}|\alpha|^2 \\ &= (\epsilon_{01}-\epsilon_{10}) + (1-\epsilon_{01}-\epsilon_{10})\left(|\alpha|^2 - |\beta|^2\right) \\ &= \beta_0 + \beta_1\langle\hat{Z}\rangle \end{aligned} \tag{4}$$

From error measurements β_0 and β_1 are derived by measuring different states on all relevant qubits. The assumptions we make here are the following: initialization of the initial me create for the readout error measurements are almost perfect (99% accurate) and the gates used to create the final states also have a high fidelity (exceeding 99.9%). From this expression $\langle\hat{Z}\rangle$ and the other expectation values $\langle\hat{\sigma}_i\rangle$ are derived from all the different measurements $\overline{m}_i$.

$$\langle\hat{\sigma}_i\rangle = \frac{\overline{m}_i - \beta_0}{\beta_1} \tag{5}$$

So the final density matrix ρ_{fin} is defined as:

$$\rho_{fin} = \frac{1}{2}\left(I + \sum_i \frac{\overline{m}_i - \beta_0}{\beta_1}\sigma_i\right) \tag{6}$$

ii. Two-Qubit State Tomography

This procedure can be generalized to more qubits, so similar to the one-qubit case the density matrix on n number of qubits is expressed as

$$\rho = \sum_{\vec{v}_i} \frac{\mathrm{Tr}\left(\sigma_{v_1} \otimes \cdots \otimes \sigma_{v_n}\rho\right)\sigma_{v_1} \otimes \cdots \otimes \sigma_{v_n}}{2^n} \tag{7}$$

where we sum over vectors $\vec{v}_i = (v_1, ..., v_n)$. The entries can be all the different combinations of basis. What these products of Pauli matrices mean is that we measure different bits in various basis. Rewriting this general expression for a two qubit system gives

$$\begin{aligned}\rho &= \frac{1}{4}\sum_{\vec{v}_i} \mathrm{Tr}\left(\sigma_{v_1} \otimes \sigma_{v_2}\rho\right)\sigma_{v_1} \otimes \sigma_{v_2} \\ &= \frac{1}{4}\sum_{\vec{v}_i} \langle \sigma_{v_1} \otimes \sigma_{v_2} \rangle \sigma_{v_1} \otimes \sigma_{v_2}\end{aligned} \tag{8}$$

where we sum over all the combinations of basis (I,X,Y and Z). Note that for a two qubit system we can choose to only measure one bit, that why the I matrix is included here it means you do not measure that particular qubit. So in total we have fifteen coefficient which can be extracted from 9 different circuits for the basis combinations. The nine measurement basis are the two-qubit correlations and then we have 6 basis containing the identity (not counting II). These six are the Bloch vectors of the most- and least significant bits, and those are derived from the original measurements by counting up all counts of the basis where there is now an identity matrix. For example if we want to know the expectation value of $\langle XI \rangle$ we can use three different measurement results ($\langle XX \rangle$, $\langle XY \rangle$ and $\langle XZ \rangle$), and in this case combine the counts from the least significant qubit.

Again we can correct for readout error, but for two-qubit systems this is a bit more complicated. Ideally we want the measured expectation value in a particular basis for two different qubits to coincide with the actual expectation values of the most-, least significant qubit and the two-qubit correlation. But the readout is compromised by readout error and crosstalk between the qubits. As a result of these errors the MSQ, LSQ and correlations results are changed. We define the measured result in a specific basis $\overline{m}_{ij}$ using a matrix notation:

$$\begin{pmatrix}\overline{m}_{ij,MSQ}\\ \overline{m}_{ij,LSQ}\\ \overline{m}_{ij,corr}\end{pmatrix} = \begin{pmatrix}\beta_{0,M}\\ \beta_{0,L}\\ \beta_{0,corr}\end{pmatrix} + \begin{pmatrix}\beta_{1,M} & \beta_{2,M} & \beta_{3,M}\\ \beta_{1,L} & \beta_{2,L} & \beta_{3,L}\\ \beta_{1,corr} & \beta_{2,corr} & \beta_{3,corr}\end{pmatrix}\begin{pmatrix}\langle I\sigma_j\rangle\\ \langle \sigma_i I\rangle\\ \langle \sigma_i\sigma_j\rangle\end{pmatrix} \tag{9}$$

Note that instead of 2 beta's we now have 4 for the offset β_0 and for the MSQ, LSQ and correlations correction for each type of measurement (in the following order: β_1, β_2 and β_3). If we define B as the matrix containing beta's (except for β_0) then we can rewrite the expectation values in terms of the measured results as follows:

$$\begin{pmatrix}\langle I\sigma_j\rangle_{est}\\ \langle \sigma_i I\rangle_{est}\\ \langle \sigma_i\sigma_j\rangle_{est}\end{pmatrix} = B^{-1}\left(\begin{pmatrix}\overline{m}_{ij,MSQ}\\ \overline{m}_{ij,LSQ}\\ \overline{m}_{ij,corr}\end{pmatrix} - \begin{pmatrix}\beta_{0,M}\\ \beta_{0,L}\\ \beta_{0,corr}\end{pmatrix}\right) \tag{10}$$

We need four calibration circuits to calibrate all the β coefficients: one circuit that has no operators, a circuit with an X-gate for the MSQ, one for the LSQ and one circuit where both qubits have a X-gate. From these configurations we calculate the desired parameters, where the circuits have letters A,B,C and D in that specific order.

$$\begin{pmatrix}\overline{m}_{A,k}\\ \overline{m}_{B,k}\\ \overline{m}_{C,k}\\ \overline{m}_{D,k}\end{pmatrix} = \begin{pmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}\begin{pmatrix}\beta_{0,k}\\ \beta_{1,k}\\ \beta_{2,k}\\ \beta_{3,k}\end{pmatrix} \tag{11}$$

where k can be MSQ, LSQ or correlated and the matrix is called M_k. These relation are derived from formula 9, where the expectation values are either 1 or -1 depending on the type of circuit. Finally we express the beta's in terms of these calibration results:

$$\begin{pmatrix} \beta_{0,k} \\ \beta_{1,k} \\ \beta_{2,k} \\ \beta_{3,k} \end{pmatrix} = M_k^{-1} \begin{pmatrix} \overline{m}_{A,k} \\ \overline{m}_{B,k} \\ \overline{m}_{C,k} \\ \overline{m}_{D,k} \end{pmatrix} \tag{12}$$

In conclusion, it is also possible to correct for readout error in a two-qubit system using calibration circuits and then transforming the results with matrices consisting of the derived beta's.

III. Methods

IV. Results

V. Conclusion

To conclude, we live in a society

References

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