End-to-end Fair Division for the CHIPS Act

Anonymous Author(s)
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ABSTRACT

State-of-the-art fair division tools from theoretical computer science are often insufficiently flexible to capture the complex constraints of real-world problems. Motivated by the problem of fairly dividing budget allocations to electronic design automation (EDA) companies under the CHIPS Act, we propose a multi-step fair division process that leverages tools from impartial budget division and bankruptcy problems in order to allocate funding from *projects* to *companies* in exchange for required EDA tools. Our contributions are fourfold: (1) Based on the desiderata of stakeholders of the CHIPS Act we design our multi-step fair division pipeline in a way that incentivizes strategyproof reporting for projects; (2) we prove a general impossibility theorem that establishes no mechanism can simultaneously satisfy both impartiality and the nucleolus for companies; (3) we analyze the worst-case manipulability of the pipeline from the perspective of companies; and (4) we also empirically study both average and worst-case manipulability of the fair division pipeline over a range of synthetic scenarios. This work serves as a proof of concept of a broader goal: leveraging stakeholder feedback to design principled fair allocation mechanisms for complicated real-world decision domains.

KEYWORDS

Fair division, resource allocation, impartiality, bankruptcy problems

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1 INTRODUCTION

≪**Richard 1.1:** ANSON GET PERSONAL COMMUNICATION REFERENCE, THEY SHOULD MOTIVATE THE PROBLEM WE'RE SOLVING≫ RH 1.1

Fair resource allocation is a foundational, ubiquitous problem. Government organizations, philanthropists, disaster relief organizations, and many other entities of various sizes continually face difficult decisions aobut how to effectively, efficiently, and fairly distribute a scarce resource among those in need. However, despite the potentially massive scope of these division scenarios (e.g., the Infrastructure Investment and Jobs Act (IIJA) of 2021 allocated \$1.2T for transportation and infrastructure spending [8]), they almost never use theoretically-robust tools from fair division and instead rely on ad-hoc approaches to parcel out resources. This is because fair division is not a "one-size-fits-all" approach: State-of-the-art fair division processes are insufficiently flexible to accommodate domain-specific constraints (e.g., legal safeguards, higher-order policy objectives, or particular types of strategic behavior) and potential disagreements about what type of "fairness" to prioritize.

Motivated by the compelling use case of fairly dividing budget allocations among electronic design automation (EDA) companies under the CHIPS Act, we propose a multi-step fair division process that first empowers stakeholders to define relevant fairness desiderata, after which tools from mechanism design may be deployed to design and implement a principled and transparent fair division mechanism for the relevant large-scale, real-world application. We note that this project has involved extensive discussion of a preliminary whitepaper with stakeholders in one of the CHIPS hubs in order to establish the parameters of the fair division problem and discuss our end-to-end pipeline [1].

≪**Richard 1.2:** SHOULD CITE SOME MEDIA ABOUT THE CHIPS ACT HERE TOO≫ The CHIPS and Science Act of 2022 has RH 1.2
allocated tens of billions of dollars toward semiconductor research and workforce training. A sizable portion of awarded funding thus far has been allocated to *hubs* based at universities or public-private entities, where each hub pursues many individual *projects*. Additionally, each project relies on a collection of *tools* provided by chip design software *companies* like Cadence and Synopsys. However, most notably in the DoD Microelectronics Commons (now in its second year), because only academic institutions, not the established companies that develop crucial tools used for research, receive funding, this creates an incentive for companies to "get their share of the pie" by drastically increasing the prices of their tools for academics. In other words, by artificially inflating prices, companies are able to, in effect, secure a portion of CHIPS funding for themselves.

This behavior by companies in turn causes tensions among projects at each hub who compete for resources (i.e., licenses to tools) provided by companies. Ideally, each hub would be able to truthfully elicit tool needs from each of its projects, take the union of these requests (e.g., in a certain hub H, if project P_1 needs 4 copies of tool T_1, project P_2 needs 3 copies of tool T_1, and project P_3 needs 6 copies of tool T_1, the hub H will request 13 copies of tool T_1), and submit this list to the companies. However, because tool prices are so high, the hub may not have enough budget to buy licenses for all the requested tools. This naturally would lead to a model in which hubs request too few tools for all projects to use at once, meaning projects must share tools by checking them out from a shared library. Under strategic considerations, this results in suboptimal strategies such as over- or under-reporting the number of licenses a project needs, and projects hogging tools – all of which leads to inefficiency and delayed progress on projects.

≪**Anson 1.3:** Connect our model to the CHIPS Act – introduce the provisioning problem and all its constraints.≫ AK 1.3

1.1 Our Contributions

We consider the following questions.

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(1) How can we incentivize hubs to truthfully report the tools needed for projects?
(2) How can we fairly divide resources among companies?

We seek to address both of these problems by using tools from the field of *mechanism design*, which seeks to design rules for "games" in which individually rational and strategic agents are incentivized to behave in a way that benefits society as a whole. In particular, we propose separate interventions to address each problem. To incentivize projects to truthfully report tool needs, we propose a rental model in which projects must pay a higher cost to use tools they do not initially request. To fairly divide resources among companies, we propose a fair division scheme that draws on literature from impartial division of a dollar and bankruptcy problems in economics in order to directly distribute each hub's budget for tools among companies in a way that bypasses company-specific prices for tools.

RH 1.4 ≪**Richard 1.4:** INTRODUCE THE FORMAL PROBLEM HERE≫

≪**Richard 1.5:** A REITERATION OF THE PERSONAL COMMUNICATION HERE≫ RH 1.5

1.2 Related Work

≪**Richard 1.6:** (1) CHIPS WORK (2) MECHANISMS FOR MULTIPLE PARTY PROBLEMS LIKE THIS (3) IMPARTIAL MECHANISMS AND APPLICATIONS OF TALMUD RULE≫ RH 1.6

2 PRELIMINARIES

Let $[n] := \{1, \ldots, n\}$ for all $n \in \mathbb{N}$. The *nucleolus* of a coalitional game is a well-known solution concept (see e.g., [7]) that (roughly) minimizes the maximum gain of any coalition from deviating. It is also always unique, hence it is an attractive solution for fair division games.

2.1 The Model

We assume a collection of different *hubs*, where each hub consists of a collection of *projects* that each need some collection of *tools* provided by *companies*. Additionally, we assume that each hub has a fixed total budget to spend on tools. For ease of exposition, we focus on a single hub with total budget E; our proposed process naturally extends to multiple hubs with separate or combined budgets.

We assume the following events occur within the hub.

(1) The hub asks its projects to provide tool demands. These tool demands are then aggregated and made available to the companies supplying the tools.
(2) The companies then collectively decide how to split the hub's (insufficient) tool budget, E. Note that here, they directly divide the budget among themselves without explicitly considering per-tool prices. In general, an EDA company will not reveal its "price book" to a competing company. So, it is reasonable to assume that companies do not know any per-tool prices other than their own. They also would not in general know how many licenses are requested for any competitor's tool.

Our process for dividing awards among the companies is a two-step process directly applying two well-known division rules for different settings. We use the first rule to determine how much each agent is entitled towards out of a value larger than the total award amount, and the second rule determines how much each agent is actually allocated. ≪**Anson 2.1:** Include figure of pipeline.≫ AK 2.1

2.2 The Bankruptcy Problem with Conflicting Reports

Consider a variant of the bankruptcy problem as follows. A debtor has estate value $E > 0$ and owes a total of $D > E$ to a collection of $n \geq 4$ agents. However, the agents do not necessarily agree on the split of D among themselves.

We are interested in mechanisms that first elicit from each agent i a report $r^i = (r^i_1, \ldots, r^i_n) \in \mathbb{R}^n_{\geq 0}$, where $\sum_j r^i_j = 1$ and $r^i_i = 0$, i.e., each agent reports a percentage of the share among all agents excluding themselves[1]. Given an *evaluation profile* $r = (r^1, \ldots, r^n)$ where each r^i is a report, a mechanism M outputs an allocation $M(r, E) \in \mathbb{R}^n_{\geq 0}$ with $\sum_i M_i(r, E) = E$, where $M_i(r, E)$ is agent i's allocation given profile r and estate E.

2.2.1 Aggregators. In order to define the nucleolus, it is first necessary for the mechanism to first collapse a profile r into a single claims vector using an *aggregator* $A : r \mapsto A(r)$ satisfying:

- **Completeness (C):** $D \sum_i A_i(r) = D$.
- **Unanimity (U):** If all rows equal v, then $A(r) = v$.
- **Anonymity/Neutrality (AN):** Permuting reporters leaves A unchanged; permuting columns permutes the corresponding components of $A(r)$.
- **Column-Responsiveness (CR):** Around any unanimous profile, it is possible to perturb some agent's off-diagonal reports so that the induced claims vector shifts by $\varepsilon(e_j - e_k)$ for some small $\varepsilon > 0$ and some distinct $j, k \neq i$. Here, e_ℓ is the n-dimensional basis vector with a single one in position ℓ and zeroes everywhere else. Intuitively: by altering only agent i's reports about others, one can shift a bit of claim from agent k to agent j while holding $A_i(r)$ and D fixed.

We will be interested in aggregators that satisfy the following:

Definition 2.1. Given evaluation profiles $r := \{r^i\}_{i \in [n]}$ and r' where r' differs from r only in r^i_{-i}, an assignment function $f : r \mapsto f(r)$ is **impartial** if $f_i(r') = f_i(r)$.

Note that this definition is general in that it may also apply to mechanisms.

2.2.2 Contested Garment Rule. We study $M(r, E) := \mathrm{Nuc}(A(r) \cdot D, E)$, i.e. the nucleolus allocation of the (standard) bankruptcy instance ($d := A(r) \cdot D, E$) that only takes a single claims vector d and estate E as input (instead of a profile r). Note that it is generally NP-hard to compute the nucleolus of a cooperative game. [2] showed that in bankruptcy problems, the nucleolus coincides with the *contested garment rule*. Thus, the contested garment rule gives a natural solution for allocation problems where the total amount entitled by the agents exceeds the amount that can be allocated. However, [2] does not delve into the case of agents misreporting their debts (because in the bankruptcy situations they're interested in, the debts are already well-documented and cannot be misreported).

[1]We can define the standard bankruptcy problem studied in [2] in the same way, but with all r^i's equal.

We now formally describe the contested garment rule $\mathrm{CG}(d, E)$. We first describe two subroutines by the contested garment rule, known as constrained equal awards (CEA) and constrained equal losses (CEL). In CEA, each agent starts with 0 and "eats" the estate E at an equal rate and stops eating once they've eaten their entitled amount, or if no more estate remains. CEL is a dual procedure, in which each agent starts with their entitled amount and "reverse-eats" until they hit 0 or the total allocation drops from D to E.

Let $d = \{d_i\}_{i \in [n]}$. We define $\mathrm{CEA}(d, E) := \{a_i\}_{i \in [n]}, \mathrm{CEA}(d, E)_i = a_i$, where a_i is the allocation of agent i by the CEA procedure given input d, E, and define $\mathrm{CEL}(d, E), \mathrm{CEL}_i(d, E)$ analogously. The pseudocodes for CEA and CEL are given below.

Algorithm 1 Constrained equal awards and losses

```
Constrained equal awards.
Agent i is active and a_i ← 0 ∀ i ∈ [n].
while E > 0 do:
    n' ← number of currently active agents
    a_i ← a_i + ϵ for all active agents i.
    E ← E − ϵn'
    Deactivate each agent j such that a_j = d_j.
Return a := {a_i}_{i∈[n]}.

Constrained equal losses.
Agent i is active and a_i ← d_i ∀ i ∈ [n].
while D > E do:
    n' ← number of currently active agents
    a_i ← a_i − ϵ for all active agents i.
    D ← D − ϵn'
    Deactivate each agent j such that a_j = 0.
Return a := {a_i}_{i∈[n]}.
```

One perspective of the contested garment rule is to run either CEA or CEL depending on if $E \leq D/2$ or not. If $E \leq D/2$, then we run CEA with $(\{d_i/2\}_{i \in [n]}, E)$ as input; otherwise, we first allocate $d_i/2$ to agent i for all i, and then run constrained equal losses with $(\{d_i/2\}_{i \in [n]}, E - D/2)$ as input. We denote the output of the contested garment rule as $\mathrm{CG}(d, E), \mathrm{CG}_i(d, E)$.

Hence, returning to our original pipeline, we are given a set of evaluation profiles r from the n agents (companies), an aggregator A, and an estate (budget) E and entitlement (total amount owed to the companies) $D > E$. We obtain a claims vector $d := A(r) \cdot D$, and we return the allocation given by $\mathrm{CG}(d, E)$.

2.3 Roadmap

In Section 3, we address the hub reporting step of the pipeline with a simple strategyproof mechanism. In Section 4, we define our mechanism for the budget division step, proving an impossibility result and analyzing a notion of manipulability of the mechanism in Sections 4.2 and 4.3. Finally, in Section 5 we provide experiments on the manipulability of our budget division mechanism.

3 STRATEGYPROOF HUB REPORTING

≪**Richard 3.1:** ANSON TODO; APPENDICIZE? I feel like for completeness it still helps to have the full pipeline if we end up having space for it in the end. just left to verify the language/notations are still consistent here≫ Our first insight is that we can examine RH 3.1 strategyproofness for step 1 in the pipeline independently from step 2 in order to incentivize projects to truthfully report their tool demands.

We propose the following *rental model.*

- An exogenous source provides a menu of estimated prices per tool, i.e., each tool T_j^k has an estimated price $\pi(T_j^k)$ for all $j \in [m]$ and $k \in [t_j]$. These prices are not directly what each tool costs, but could be derived from prior academic license costs or estimates from a domain expert. Let the entire estimated price vector be denoted π.
- Based on these prices, each project creates a list of tools they need, and they contribute the estimated cost of all of these tools to the hub's tool rental pot, i.e., project P_i^j contributes $a(i, j) \cdot \pi$ for all $i \in [h]$ and $j \in [p_i]$.
- Each project is then able to use all of the tools they request for free. If they need additional tools, they pay strictly more than the estimated price for each tool they use but did not initially request, i.e., to use additional tool T_j^k, a project pays $(1 + \varepsilon)\pi(T_j^k)$ for some $\varepsilon > 0$.

Theorem 3.1. *The rental model is strategyproof on the part of projects.*

Proof. We show that each project individually is incentivized to report truthfully. For a specific project, there are two cases for untruthful behavior: **(1)** If a project under-reports their tool needs, then they will have to rent tools at a higher rate than if they reported correctly, **(2)** if a project over-reports their tool needs, then they end up paying for tools they don't need to use, which is worse than reporting correctly. Thus, reporting truthfully is the best strategy for each project. ◻

One note: As is often true in practice ≪**Anson 3.2:** cite??≫, we AK 3.2 assume that $D > E$ no matter how many tools projects request, i.e., the total demand D of all companies outweighs the total tool budget E of the hub. If this is not true, then the result of the impartial division of a dollar step is exactly the final division of budget.

4 BUDGET DIVISION PIPELINE

In this section we give the details of our mechanism for the companies' side of the process via a mechanism for the bankruptcy problem with conflicting reports. Then we show that there is no mechanism obtaining impartiality and the nucleolus allocation, and thus shift our focus to bounding the amount a given agent can gain from misreporting.

4.1 The Allocation Mechanism

The high-level intuition of our approach is as follows.

(1) We first use an impartial aggregator to get companies (i.e. agents) to decide how much of the budget (i.e. estate) each of their competitors deserves based on the tool demands from projects.
(2) Then, we use the contested garment rule to divide up the (insufficient) budget among companies. This achieves a solution concept known as the nucleolus in the economic literature.

The first step is *impartial* in that no agent can change her own claim based on the information she reports. However, an agent i *can* influence any other agent's entitlement, and this *can* influence i's resulting allocation after running the contested garment rule. In particular, since we show in the next subsection that it is impossible to achieve both the nucleolus allocation and "true" impartiality, we instead obtain impartiality in the aggregator function that defines the claims input for the mechanism. We seek to bound the amount any agent can benefit from such misreports resulting from impartial aggregations.

One may ask why we do not just run the impartial aggregator over the budget E (or equivalently scale down their claims over D down to E). However, even in the case where all the companies unanimously agree on what share of the pie each of them deserves, the solution resulting from scaling down their claims to E may not be consistent with the actual claims they deserve. For instance, consider a case with four companies, $E = 100, D = 400$ and claims vector $(50, 50, 50, 250)$. If we scale down each company's claim such that the sum of debts is E, then the resulting allocation is $(12.5, 12.5, 12.5, 62.5)$. However, the nucleolus (i.e. contested garment) solution is $(25, 25, 25, 25)$; in particular, company 4 should not be able to muscle the other three companies out of the way. This is exactly because the nucleolus and total impartiality are incompatible with each other, which we formalize next.

4.2 An Impossibility

In this subsection, we show that *impartial mechanisms*, however attractive they seem, are impossible to achieve while returning the nucleolus allocation of the bankruptcy problem.

Lemma 4.1. *Fix $n \geq 4$ and $E \in (0, D)$. There exists a unanimous claim vector v and distinct i, j, k with $j, k \neq i$ such that, for some $\varepsilon > 0$ that $\mathrm{CG}_i(v + \varepsilon(e_j - e_k), E) \neq \mathrm{CG}_i(v, E)$.*

Proof sketch. For CEL: awards take the form $\mathrm{CEL}_m(d, E) = d_m - \lambda$ for non-capped agents m, where λ is the common loss level satisfying $\sum_{m \in n} \min\{d_m/2, \lambda\} = D - E$. Choose v so that agent i is not capped and exactly one of j, k is capped. Shifting ε units of claim from k (capped) to j (uncapped) forces λ upward. Hence $\mathrm{CEL}_i(d, E) = d_i - \lambda$ decreases. The same logic applies in the CEA regime (with roles of awards and losses swapped). ◻

Theorem 4.2 (Incompatibility). *Suppose $M(r, E) = \mathrm{Nuc}(A(r), E)$ where A satisfies (C), (U), (AN), and (CR). Then M cannot satisfy impartiality.*

Proof. Start from a unanimous profile r° with $A(r^\circ) = v$ as in Lemma 4.1. By (CR), there exists r' differing from r° only in agent i's off-diagonal reports such that $A(r') = v + \varepsilon(e_j - e_k)$. By Lemma 4.1, we have $M_i(r', E) \neq M_i(r^\circ, E)$, but impartiality requires equality, since r' and r° differ only in i's reports about others. This is a contradiction. ◻

4.3 Manipulability of Mechanisms with Impartial Aggregators

In this subsection we analyze how much an agent can change their resulting allocation if they misreport their input, subject to impartial aggregation. We first provide our definition of the *manipulability* of a mechanism M that uses aggregator A to decide the claims report d.

Definition 4.3. The *additive manipulability* for agent i of a mechanism M whose claims vector is decided by aggregator A is

$$\max_{r, r' \in \mathbb{R}^{n \times n} : r'^j = r^j \forall j \neq i} M_i(A(r') \cdot D, E) - M_i(A(r) \cdot D, E)$$

that is, r' is a profile where only agent i's row is manipulated (but it also may not be).

As this maximum is taken over all possible pairs r, r' that differ in only i's report, we can view r as an "initial" claims and r' as the "manipulated" claims.

4.3.1 *Unrestricted Impartial Manipulation.*

As a baseline, we study the worst-case additive manipulability if in the aggregation step, an agent is allowed to arbitrarily impact the claims of others, but not itself (preserving impartiality).

Definition 4.4 (Impartial dictator). Given a profile $r \in \mathbb{R}^{n \times n}$ and $x \in [0, 1]$, the *impartial dictator-i aggregator* $\mathrm{dict}^i(r)$ assigns to r the vector d such that $d_i = x$ and $d_j = (1 - x) r^i_j$ for every $j \neq i$.

It is easy to see that the above aggregator is impartial. We analyze CEA and CEL independently on the impartial dictator aggregator.

Lemma 4.5. *The additive manipulability of $\mathrm{CEA}(\mathrm{dict}^i(r) \cdot D, E)$ is at most $\frac{E}{2} - \frac{E}{n}$.*

Proof. We want to construct claims vectors $d \neq d'$ that are equal at d_i that maximize $\mathrm{CEA}_i(d, E) - \mathrm{CEA}_i(d', E)$. Note we have assumed d, d' have already been normalized to sum to D.

CEA equivalently allocates $\min(d_j, s)$ to j for every j where s is the constant satisfying $\sum_{j \in [n]} \min(d_j, s) = E$. We may view s as the number of rounds of increments until all agents are deactivated. So we want to start with a claims d where agent i gets its minimum possible allocation. We do so by maximizing s, which is done by minimizing the number of agents that are incremented in each round. To do so, we assign 0 to $n - 2$ other agents and $D - d_i$ to one other agent in d, reducing to a two-agent instance. If $d_i \leq D - d_i$ then agent i will be allocated $\min(d_i, E/2)$, because either both agents increment to $E/2$ or i drops out before round $E/2$. Otherwise agent i will be allocated $\max(E - (D - d_i), E/2)$. Then i's allocation can be expressed by the function:

$$\mathrm{CEA}_i(d, E) = \begin{cases} d_i & d_i \in [0, E/2) \\ E/2 & d_i \in [E/2, D - E/2] \\ E - (D - d_i) & d_i \in (D - E/2, D] \end{cases} \tag{1}$$

On the other hand, agent i's allocation is minimized by maximizing the number of agents with entitlement E/n in d' having initially fixed d'_i, as this maximizes the number of agents incremented in each round. In this case we have:

$$\mathrm{CEA}_i(d', E) = \begin{cases} d'_i & d'_i \in [0, E/n) \\ E/n & d'_i \in [E/n, D - (n-1)E/n] \\ E - (D - d_i) & d'_i \in (D - (n-1)E/n, D] \end{cases} \tag{2}$$

The largest gap between these two functions is when $d_i = d'_i \in [E/2, D - (n-1)E/n]$, in which the gap is exactly $E/2 - E/n$. ◻

Lemma 4.6. *The additive manipulability of* $\mathrm{CEL}(\mathrm{dict}^i(r)\cdot D, E)$ *is at most* $\frac{D-E}{2}-\frac{D-E}{n}$.

Proof. Like before we may equivalently view $\mathrm{CEL}(d,E)$ to allocate $\max(0,d_j-s)$ to agent j for every $j\in[n]$ where s is the constant such that $\sum_{i\in[n]}\max(0,d_j-s)=E$, and we view s as the number of rounds of *decrements* (instead of increments). As we must make a total of $D-E$ decrements across the s rounds, then s is minimized when we maximize the number of decrements per round, which is at most n. Hence $\frac{D-E}{n}\le s\le D-E$. We consider the following cases:

(1) $d_i\le\frac{D-E}{n}$: since $s\ge\frac{D-E}{n}$, agent i will always get 0.
(2) $\frac{D-E}{n}<d_i\le\frac{D+(n-1)E}{n}$: then s is minimized by setting all non-i agents to at least $\frac{D-E}{n}$, obtaining $s=\frac{D-E}{n}$. It is always possible to set the other $n-1$ agents to at least $\frac{D-E}{n}$ in this case since $D-d_i\ge D-\frac{D+(n-1)E}{n}=(n-1)\frac{D-E}{n}$.
(3) $d_i>\frac{D+(n-1)E}{n}$: observe in this case agent i is "too rich" to set every other agent to at least $\frac{D-E}{n}$. Then s is minimized by setting all other agents to at most d_i-E. To prove this, let $s^*:=(D-E)-(D-d_i)=d_i-E$. We show that s^* is the minimum possible number of rounds of decrements in this case. The sum of all decrements over each agent is $D-E$, and agent i needs to distribute $D-d_i$ of the estate among the other agents. To minimize s, we want to minimize the number of agents that are allocated a strictly positive amount after s rounds. Ideally, all other agents get decremented to 0 after the s rounds, so agent i still needs to be decremented at least $(D-E)-(D-d_i)$ times. Next, we show that $s=s^*$ if and only if all other agents claim at most s^*. Since $s\ge s^*$ from the previous claim, it suffices to show $s\le s^*$. We prove the first direction by contrapositive, that $\exists\, j$ s.t. $d_j>s^*\implies s>s^*$. Then we have that after s^* rounds of decrements, the total amount decremented across all agents is

$$2s^*+(D-d_i-d_j)<2d_i-2E+D-d_i-d_i+E=D-E$$

where we used the assumption that $d_i\ge d_j>s^*=d_i-E$. Hence after s^* rounds, the total decremented amount has not yet reached $D-E$, so $s>s^*$. For the second direction, observe that since all other agents besides i claim at most $s^*\le s$, they must be decremented to 0 after s rounds. So by round s we have made $D-d_i$ total decrements across all other agents. Then there are $(D-E)-(D-d_i)=s^*$ decrements that must be shouldered by agent i. This requires at least s^* rounds of decrements, and hence $s\le s^*$. Lastly, we show that it is always possible to set all other agents to at most s^* in this case. We have

$$\begin{aligned} d_i>\frac{D+(n-1)E}{n} &\implies nd_i>D+(n-1)E\\ &\implies D<nd_i-(n-1)E\\ &\implies D-d_i<(n-1)(d_i-E)\end{aligned}$$

So the remaining $D-d_i$ can be distributed among the other $n-1$ agents such that each agent receives at most d_i-E.

The "worst case" of claims for agent i is when $\frac{D-E}{2}\le d_i\le\frac{D+(n-1)E}{n}$ and one other agent j claims $D-d_i$ and all other agents claim 0. Here agent i gets $d_i-\frac{D-E}{2}$, but can manipulate the claims to give all other agents at least $\frac{D-E}{n}$, in which case it gets $d_i-\frac{D-E}{2}$, making a gain of $(d_i-\frac{D-E}{n})-(d_i-\frac{D-E}{2})=\frac{D-E}{2}-\frac{D-E}{n}$. □

Then we have the following as a corollary:

Corollary 4.7. *The additive manipulability of* $\mathrm{CG}(\mathrm{dict}^i(r)\cdot D,E)$ *is at most* $\frac{E}{2}-\frac{E}{n}$.

Proof. Recall $\mathrm{CG}(d,E)$ outputs $\mathrm{CEA}(d/2,E)$ if $E\le D/2$, and $d/2+\mathrm{CEL}(d/2,E-D/2)$ if $E>D/2$. When $E\le D/2$, manipulating $n-2$ agents to 0 yields equal manipulability for agent i whether or not we run CEA on d or $d/2$, so the manipulability of CEA upper bounds that of CG in this case. When $E>D/2$, no agent with nonzero claim can be allocated 0, so the manipulability of CEL upper bounds that of CG in this case. Lastly note that when $E\le D\le 2E$ then $\frac{D-E}{2}-\frac{D-E}{n}\le\frac{E}{2}-\frac{E}{n}$, and the corollary follows. □

4.3.2 Restricted Impartial Manipulation. In this section we analyze strategies for maximizing manipulability for a broader class of impartial aggregators beyond dictatorships. We start with a structural lemma on the worst-case manipulation strategies for CEA.

Lemma 4.8. *Fixing profile r and agent i, for an impartial aggregator A,* $\arg\max_{r':r'^j=r^j\forall j\neq i}(\mathrm{CEA}_i(A(r')D,E)-\mathrm{CEA}_i(A(r)D,E))$ *satisfies* $r'^i_j=1$ *for some* $j\neq i$ *and* $r'^i_k=0$ *for all other* $k\neq i,j$.

Proof. Let $j=\arg\max_{j\neq i}(A_j(r))$. If there exists a better manipulation for i that isn't of the form $(0,\dots,0,1)$ then there is some agent k with $A_k(r)\cdot D=x>0$ that i can set to 0, and give x more to j. It suffices to analyze how the change in j's claim affects i's, since lowering an agent k's claim only helps i in CEA.

If $A_j(r)\ge A_i(r)$, then j was always eating with i while i was active during CEA with claims $A(r)$. Hence agent i strictly benefits from the manipulation $(0,\dots,0,1)$ by eating $x/(c-1)$ more, where c is the number of agents with larger claims than i in $A(r)$.

Let r' be the profile defined in the lemma statement. If $A_j(r)<A_i(r)$, then either $A_j(r')\le A_i(r')$ or $A_j(r')>A_i(r')$ after setting $r'^i_j=r^i_j+x$. For the former sub-case we have two more sub-subcases. First, if i,j deactivated at the same time during CEA with claims $A(r')$, then by the same reasoning as earlier i strictly benefits. Otherwise, i ate longer than j during CEA with claims $A(r)$, so during CEA with claims $A(r')$ agent j will begin to eat during time steps in which i was eating without j. Nonetheless, i's eating time is never decreased because if that were the case then the sum of resulting allocations would be less than E, since the only change was the symmetric change in r^i_j,r^i_k. Finally, when $A_j(r')>A_i(r')$, observe i's allocation is unchanged by how long j eats after i drops out. So this is identical to the case when $A_j(r')=A_i(r)$. □

Next, we show that manipulability is maximized for CEA under a certain initial structure of evaluation profiles, under a wide class of impartial aggregators.

Definition 4.9. Let r be an evaluation profile such that $A_i(r)=x$, and r' be the result of swapping only r^i in r with some r'^i such that $r'^i_j<r^i_j$. Define the function $\Delta^{(i,j)}(r^i_j-r'^i_j):=D(A_j(r)-A_j(r'))$. An impartial aggregator A is *stable* if it is nondecreasing, concave, and $\Delta^{(i,j)}(r^i_j-r'^i_j)\le D(1-x)(r'^i_j-r^i_j)$.

Note that this definition is natural in that it essentially says that A guarantees that **(1)** for any $a > b > c > d$, a manipulator cannot lower an agent's claim from b down to c yet l and **(2)** if a single agent decides that another agent's value (up to normalization) should be changed by δ, then the aggregator will not change that agent's share by δ or more. In particular, the impartial dictator aggregator is the "worst" stable aggregator as it satisfies the specified inequalities with equality.

LEMMA 4.10. *Fixing an agent i with claim d_i, for a stable impartial aggregator A, $\mathrm{CEA}_i(A(r')\cdot D, E) - \mathrm{CEA}(A(r)\cdot D, E)$ is maximized if r satisfies*

$$A(r)\cdot D = \mathcal{A} := \left(E/n, \ldots, d_i, E/n, \ldots, D - d_i - \frac{(n-2)E}{n} \geq E/n\right)$$

and r' is the manipulated profile defined in Lemma 4.8.

PROOF. The intuition is that if there is a non-i agent with less than E/n initially, then the amount it can be lowered is at most that of if it was E/n initially, but if it was greater than E/n initially, then the amount it gets lowered from above E/n to E/n does not benefit i at all.

If $d_i \leq E/n$, then agent i is allocated d_i regardless of the other shares, so we may WLOG assume $d_i > E/n$. Let $d' := A(r')D$ and j denote the agent that i assigns D to in r'. In the CEA allocation for $\mathcal{A}$, every agent gets E/n. Meanwhile, d'_i is unchanged by impartiality, $d'_j = E/n + \delta(n-2)$ for exactly one $j \neq i$ and $d_k = E/n - \delta$ for the remaining $k \neq i$ for some $\delta > 0$.

In the case that $\mathrm{CEA}_i(d', E) = d'_i = d_i$, then $\mathcal{A}$ is clearly the initial claims vector that maximizes additive manipulability, because agent i gets at least E/n and at most d_i across any claims vector. So it remains to prove the claim for the case where agent i receives less than d_i. The total sum of allocations of the lowered agents by i in $A(r')D$ is strictly less than $(n-2)\frac{E}{n}$ since i doesn't gain anything from j's change, and i, j both receive more than E/n. Hence on average, a lowered agent k is allocated $A_k(r')D < E/n$. Let $g_A : \mathbb{R} \mapsto \mathbb{R}$ be the function assigning to a claim d_j the (absolute value of the) amount it can be changed by from any manipulation with aggregator A. In particular, g is a concave function by the stability of A. Then

$$\begin{aligned}\sum_{k\neq i,j} g() &\leq (n-2)\cdot g(\frac{}{n-2}) \\ &\leq (n-2)\cdot g(E/n) \\ &\leq (n-2)\frac{E}{n}\end{aligned}$$

where the first line is due to Jensen's inequality and stability, and the second is by the average $A_k(r')D$ being strictly less than E/n. Therefore i's gain is maximized if all other $n-2$ agents are set to E/n in $A(r)D$. This bounds the total possible gain across *all* lowered agents. At best, agent i splits this total gain equally between itself and j after the best manipulation, which is what happens in $\mathcal{A}$. ◻

The previous two lemmas are analogous to Lemma 4.5, but far more general. A natural impartial aggregator to use is the following *impartial division rule* proposed in [4][2].

[2]This is known to satisfy other natural properties such as completeness, unanimity, and anonymity, and is used in real-world applications e.g. Spliddit [5].

Definition 4.11 (Arithmetic mean). Given evaluation profiles r, let $\mathbf{r}$ be defined such that $\mathbf{r}^i_{jk} = r^i_j/r^i_k$ for every $j, k \neq i$ and $\mathbf{r}^i_{ij}, \mathbf{r}^i_{ji}$ for any i, j is empty. For any $\vec{x} \in \mathbb{R}^n$ let $\rho(\vec{x}) = 1/n \sum_i \vec{x}_i$, and define the **arithmetic mean aggregator** f as follows:

$$\begin{aligned} f_i(r) := f_i(\mathbf{r}) = \frac{1}{n}&\left(1 - \sum_{j\neq i} \frac{1}{1 + \rho((\mathbf{r}^k_{ij})_{k\neq i,j}) + \sum_{k\in n\setminus\{i,j\}} \rho((\mathbf{r}^l_{kj})_{l\neq i,j,k})}\right) \\ &+ \frac{1}{n}\left(\sum_{j\neq i} \frac{1}{1 + \rho((\mathbf{r}^k_{ji})_{k\neq i,j}) + \sum_{k\in n\setminus\{i,j\}} \rho((\mathbf{r}^l_{ki})_{l\neq i,j,k})}\right)\end{aligned}$$

We remark that there is empirical evidence (we refer the reader to the appendix for details) showing that the above rule is not only a stable aggregator, but also that no manipulation by some agent i can change another agent j's share by more than $O(n^{-\epsilon})$ for some constant $\epsilon \in (0,1)$, which would imply an additive manipulability of $O(n^{\epsilon'})$ for some other constant $\epsilon' \in (0,1)$. We implement our full pipeline using the arithmetic mean aggregator above in our empirical study in Section 5.

4.4 Aggregation via Impartial Ranking

In the previous section, we combined impartial aggregators with the contested garment rule, and bounded an agent's gain from misreporting by changing the claims of other agents. In this section we propose an impartial mechanism by using *impartial ranking* with the contested garment rule.

We first fix an claims vector $d := \{d_1, d_2, \ldots, d_n\}, D \sum_{i\in[n]} d_i = D$ where we use an impartial ranking rule to decide which agent is assigned index. Then we run the contested garment rule as before with d. Specifically, the agents still report evaluation profiles r like before, which we can directly convert into rankings (i.e. each agent submits an ordered lists of the other $n-1$ agents) as input for the impartial ranking rule. However, the values that make up d are fixed and not a function of r.

The impartial ranking rule outputs an assignment of unique ranks from 1 to n to each agent, and then we assign the agent ranked at i by the rule the claim d_i. Impartial ranking rules that satisfy other desirable properties are known to exist and have been studied previously, see e.g. [3, 6].

THEOREM 4.12. *The additive manipulability of the contested garment rule using the impartial ranking rule above to define the claims is* 0.

PROOF. Observe that because we fix the claims vector d and then assign claims to agents, no agent can gain from misreporting after applying the contested garment rule on d. Indeed, no matter what an agent reports, the resulting instance of the contested garment rule is identical in their perspective, since they cannot manipulate their own rank (and thus their own claim) by impartiality, and manipulating anyone else's rank doesn't change the claims vector by our fixing of d. ◻

In particular, this means that the mechanism $\mathrm{CG}(R(r)\cdot D, E)$ where R is some impartial ranking rule $r \mapsto \pi(d)$ (i.e. r maps to a permutation of d) is impartial and obtains the nucleolus allocation. Note that this doesn't contradict Theorem 4.2, since $R(r)$ doesn't satisfy column-responsiveness.

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It remains to design a good claims vector d. However, in order to retain the impartiality that we discussed previous, the values $\{d_i\}_{i\in[n]}$ must not be functions of the actual input evaluation profiles r. Hence the choices for claims vectors are (essentially) only limited to functions of n. Trivial options satisfying the criterion include the uniform vector $\{1/n, 1/n, \ldots, 1/n\}$ and a geometric sequence $\{1/2, 1/4, \ldots, 1/2^{n-1}, 1/2^{n-1}\}$ (note the last term is also $1/2^{n-1}$ so that the sum of all values is 1).

While there is no relation between these choices of d and the input evaluation profiles, we can still choose d based on a new criteria: how much an agent can (multiplicatively) manipulate the gain/loss of *another* agent. For the uniform vector this is 1, but for the geometric sequence this is at most 2^k if k is the maximum number of positions an agent can move another agent in the ranking by changing their input evaluation profile. To get polynomial manipulability on k, we propose a generalized harmonic sequence $\{\frac{1}{H_{n,p}}, \frac{1}{2^p H_{n,p}}, \ldots, \frac{1}{n^p H_{n,p}}\}$ where $H_{n,p} := \sum_{i\in[n]} \frac{1}{i^p}$ is the generalized harmonic number on n vertices and degree p. It is easily verified that the most an agent can multiplicatively manipulate the gain/loss of another agent with this claims vector is at most $(k+1)^p$. The choice of p here allows for some flexibility by the mechanism designer: for example, if they desire the maximum claim to be $1/2$, they can choose $p := p(n)$ that satisfies it.

5 EMPIRICAL RESULTS

In previous sections, we have examined the worst-case manipulability of mechanisms with impartial aggregators. We now present experimental results that study the average manipulability of the contested garment mechanism with an impartial aggregator using the arithmetic mean rule. We run two experiments. The first explores the average additive manipulability of the contested garment mechanism under unrestricted (i.e., $\mathrm{CG}(\mathrm{dict}^i(r)\cdot D, E)$) and restricted (i.e., $\mathrm{CG}(f(r)\cdot D, E)$) misreports from a single agent. Note that we have already proved the worst case manipulability of the unrestricted (i.e., impartial dictatorial) setting, but we plot it to compare the restricted manipulability with respect to this upper bound. The second illustrates how payoffs for a given manipulator change as we increase the number of agents.

5.1 Average Additive Manipulability

We begin by studying how a potential manipulator's average (maximum) payoff varies across three treatments when starting from a unanimous outcome: (1) a *baseline* with no manipulation, (2) the agent's best *restricted* impartial manipulation in which the agent is allowed to arbitrarily change their report and an impartial aggregator with the arithmetic mean is used, and (3) the agent's best unrestricted (i.e., dictatorial) impartial manipulation in which the agent is allowed to directly modify the entries in the claims vector for all other agents while preserving their own entry and the sum of all entries in the claims vector.

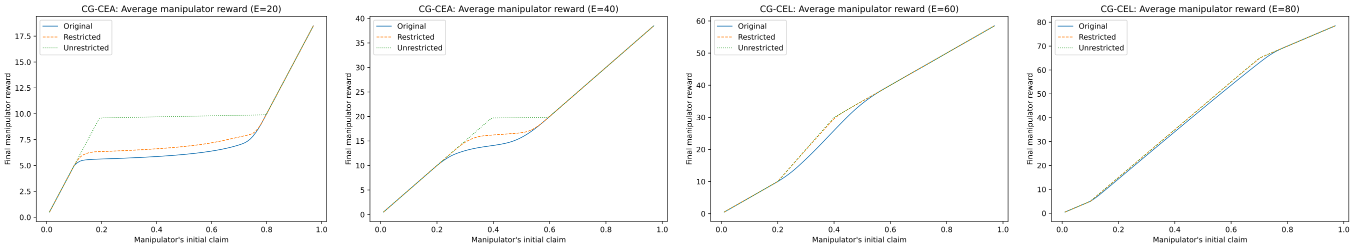
For $n = 4$ agents, we consider all unanimous scenarios in which all agents agree on the division of credit among themselves and each agent deserves a multiple of 0.01 (the atomic unit of credit) out of 1 total credit to be divided. We assume the total amount owed is $D = 100$ and consider estate values $E \in \{5, 10, \ldots, 95\}$, where the contested garment rule uses CEA on the halved entitlements when $E \leq D/2$ and uses CEL after giving each agent half of their entitlement and adjusting D to $D - E/2$ when $D > E/2$.

We average over unilateral (i.e., single-agent) manipulations of the following form. Consider manipulating agent i, who is assigned d_i in the unanimous claims vector. Agent i's allowable manipulations are all possible divisions of one unit of credit among the other $n-1$ agents in increments of 0.01, which are then scaled by $1-d_i$ to preserve their unanimous share. For each estate value, each unanimous scenario, and each candidate manipulator, we search over all allowable manipulations to compute three payoffs. *Baseline*: The result of the contested garment mechanism on the truthful unanimous matrix. *Restricted*: This is the result of the single-best manipulation under the restricted scenario, i.e., the maximum possible allocation to the manipulator over all possible manipulations after running the contested garment mechanism on the result of using an impartial aggregator with the arithmetic mean. *Unrestricted*: This is the result of the single-best manipulation under the unrestricted (dictatorial) scenario, i.e., the maximum possible allocation to the manipulator when running the contested garment mechanism over all possible dictatorial manipulations.

We compare the average maximum reward of both types of manipulations to the baseline value over all starting unanimous values $x_0 \in \{0.01, 0.02, \ldots, 0.97\}$ in Figures 1a and 1b for the CEA realm and Figures 1c and 1d for the CEL realm. Additional results for a wide range of E values are presented in the appendix.

In Figures 1a and 1b, we see that for very low and very high values of the manipulator's initial value x_0, there is no benefit from either unrestricted or restricted manipulation. For moderate values of x_0, i.e., between roughly 0.2 and 0.7 in Figure 1a and 0.3 and 0.5 in Figure 1b, there is a clear separation between the benefit of unrestricted and restricted manipulations. Unrestricted (i.e., dictatorial) manipulations lead to large gains on average, as expected given our theoretical results, but restricted manipulations only lead to very slight gains over the baseline. This illustrates that using the impartial aggregator with the arithmetic mean severely limits the additive manipulability of the fair division pipeline. These trends are observed for all $E \leq D/2$; the only difference is that as E increases, the width of the central region where manipulation helps shrinks.

In Figures 1c and 1d, we again see that for very low and very high values of the manipulator's initial value x_0, there is no benefit from either unrestricted or restricted manipulation. For moderate values, there is a separation. However, unlike in the CEA realm where unrestricted manipulation led to much higher payoffs than restricted manipulation, in the CEL realm we observe approximately equal manipulability of unrestricted and restricted manipulations. However, the overall additive manipulability still remains relatively low, which suggests that the impartial aggregator with the arithmetic mean rule still performs well on average. These trends are observed for all $E > D/2$; the only difference is that as E increases, the width of the central region where manipulation helps grows, but the maximum manipulability (i.e., difference between restricted or unrestricted and the original) shrinks.



(a) CEA, $E = 20$. **(b) CEA, $E = 40$.** **(c) CEL, $E = 60$.** **(d) CEL, $E = 80$.**

Figure 1: Average additive manipulation for CEA and CEL and various values of E.

5.2 Manipulability as n Increases

In our second experiment, we test the vulnerability of the contested garment rule to a specific claim type as the number of agents increases; throughout, due to the exponential blowup in the number of potential manipulations as n increases, we test $n \in \{4, 5, 6, 7\}$. We note that the trends we observe in both settings hold across all values of E; additional results are in the appendix. ≪**Anson 5.1:** Remake these plots, trim whitespace, etc.≫

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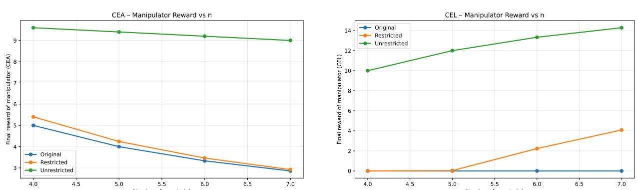

(a) CEA, $E = 40$. **(b) CEL, $E = 60$.**

Figure 2: Manipulator reward as n increases.

In the CEA setting, i.e., when $E \le D/2$, we start from a unanimous division in which $n - 1$ agents each deserve E/n and the manipulating agent deserves $D - \frac{n-1}{n}E$. This unanimous division results in a final allocation in which all agents get E/n after running the contested garment mechanism, but because $n - 1$ agents are exactly at that key threshold, this intuitively gives the manipulating agent the greatest power to affect how much other agents are allocated. As shown in Figure 2a, as n increases, even though the additive manipulability of unrestricted manipulations remains large, the additive manipulability of restricted manipulations decreases, which is an encouraging signal that CEA remains robust to restricted manipulation as n increases.

In the CEL setting, i.e., when $E > D/2$, we start from a unanimous division in which the manipulator deserves $(D - E)/2$, one other agent deserves (essentially) the remainder $(D + E)/2 - (n - 2) \cdot 0.01$, and all other agents deserve the minimum possible value of 0.01. As noted in the proof of Lemma 4.6, this unanimous division intuitively maximizes the manipulator's ability to increase their own share by increasing agents from 0 to a positive value so that they can simultaneously eat away at the remaining resource after giving each agent half of their share. As shown in Figure 2b, as n increases, the additive manipulability of both the unrestricted and restricted settings increases. This demonstrates one weakness of using an impartial arithmetic mean aggregator: The pipeline is not sybil-proof in domains where $E > D/2$, i.e., an adversary can bribe additional companies that do not deserve any payments to join the division instance in order to increase their own final reward.

6 CONCLUSION

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A MISSING PROOFS

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During submission, supplementary material can be included as an appendix in the pdf but like the other supplementary material, it is not required to be read by the reviewers. ≪**Richard A.1:** does this mean we can put the appendix in the same main.tex submission≫

RH A.1

IFAAMAS will not publish your supplementary material. Nevertheless, if your paper gets accepted, then the readers of your published paper should have access to the same information as the reviewers of your submission. This entails that you should make your (suitably revised) supplementary material openly available in archival form at the time of publication of your paper, and that you should include a reference to the supplementary material in the camera-ready version of your paper. For code or data you may wish to use a service such as Zenodo or github. For a technical appendix with additional proof details or similar, you may wish to use an archival preprint server such as arXiv.

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