End-to-end Fair Division for the CHIPS Act

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Submission Id: 530

ABSTRACT

State-of-the-art fair division tools from theoretical computer science are often insufficiently flexible to capture the complex constraints of real-world problems. Motivated by the problem of fairly dividing budget allocations to electronic design automation (EDA) companies under the CHIPS Act, we propose a multi-step fair division process that leverages tools from impartial budget division and bankruptcy problems in order to allocate funding from *projects* to *companies* in exchange for required EDA tools. Our contributions are fourfold: (1) Based on the desiderata of stakeholders of the CHIPS Act we design our multi-step fair division pipeline in a way that incentivizes strategyproof reporting for projects; (2) we prove a general impossibility theorem that establishes no mechanism can simultaneously satisfy both impartiality and the nucleolus of the corresponding coalitional problem for companies; (3) we analyze the worst-case manipulability of the pipeline from the perspective of companies; and (4) we also empirically study the average-case manipulability of the fair division pipeline over a range of synthetic scenarios. This work serves as a proof of concept of a broader goal: leveraging stakeholder feedback to design principled fair allocation mechanisms for complicated real-world decision domains.

KEYWORDS

Fair division, resource allocation, impartiality, bankruptcy problems

ACM Reference Format:
Anonymous Author(s). 2026. End-to-end Fair Division for the CHIPS Act. In *Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), Paphos, Cyprus, May 25 – 29, 2026*, IFAAMAS, 15 pages.

1 INTRODUCTION

Fair resource allocation is a foundational, ubiquitous problem. Government organizations, philanthropists, disaster relief organizations, and many other entities of various sizes continually face difficult decisions about how to effectively, efficiently, and fairly distribute a scarce resource among those in need. However, despite the potentially massive scope of these division scenarios (e.g., the Infrastructure Investment and Jobs Act (IIJA) of 2021 allocated \$1.2T for transportation and infrastructure spending [59]), they almost never use theoretically-robust tools from fair division and instead rely on ad-hoc approaches to parcel out resources. This is because fair division is not a "one-size-fits-all" approach: State-of-the-art fair division processes are insufficiently flexible to accommodate domain-specific constraints (e.g., legal safeguards, higher-order policy objectives, or particular types of strategic behavior) and potential disagreements about what type of "fairness" to prioritize.

Motivated by the compelling use case of fairly dividing budget allocations among electronic design automation (EDA) companies under the CHIPS Act, we propose a multi-step fair division process that first empowers stakeholders to define relevant fairness desiderata, after which tools from mechanism design may be deployed to design and implement a principled and transparent fair division mechanism for the relevant large-scale, real-world application. We note that this project has involved extensive discussion of a preliminary whitepaper with stakeholders in one of the CHIPS Hubs in order to establish the parameters of the fair division problem and discuss our end-to-end pipeline [32].

The CHIPS and Science Act of 2022 has allocated tens of billions of dollars toward semiconductor research and workforce training. In this context, we use *provisioning of commercial EDA tool licenses* as a context that captures aspects of the challenges in real-world fair allocation settings. Our discussion draws on the specific example of the DoD's *Microelectronics Commons* (MEC), a CHIPS Act-funded \$2B network of eight regional *Hubs* [23] [37] – as well as the challenges of mechanism design illuminated by the MEC's "Cross-Hub Enablement Solution" (CHES) which supports chip design operations and projects across six of the Hubs [39]. We emphasize that mechanism design for EDA provisioning can apply to a substantial fraction of CHIPS R&D and workforce training dollars, across programs including the DoD ME Commons, the NSTC Design Enablement Gateway [42] [53], the NAPMP [57], the NSF-DoC NNME [62], etc. Beyond the CHIPS Act, other government investments to strengthen the domestic semiconductor design and manufacturing ecosystem also require provisioning of EDA licenses; these include DARPA's Next-Generation Microelectronics Manufacturing program [21], the DoD Design to Transition Accelerator [58], and the NSF Chip Design Hub [61].

In the ME Commons, each *Hub* houses multiple *projects* that are competed Commons-wide; approximately \$270M of awards spanning 30+ projects at the eight MEC Hubs were awarded in Fall 2024 [44]. Each Hub also pursues "Hub Operations" to support its members and achieve differentiated long-term Hub sustainability; such operations include development of chip design flows, design enablements, unique IP blocks, and design prototypes that enhance competitiveness for future revenue capture. Both projects and Hub Operations require *tools* that are licensed by EDA software *companies*. As of Fall 2025, the MEC's CHES construct obtained approximately \$39M of EDA tool licenses from five suppliers (Cadence, Synopsys, Siemens EDA, Ansys, Keysight) to support one year of projects and Operations at six Hubs [39]. This leads to a key tension: Because only academic institutions, not the established companies that develop crucial tools used for research, receive funding, this creates an incentive for companies to "get their share of the pie" by drastically increasing the prices of their tools for academics. In other words, by artificially inflating prices, companies are able to, in effect, secure a portion of CHIPS funding for themselves.

This behavior by companies in turn causes tensions among projects at each Hub who compete for resources (i.e., licenses to

Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), C. Amato, L. Dennis, V. Mascardi, J. Thangarajah (eds.), May 25 – 29, 2026, Paphos, Cyprus.

tools) provided by companies. We assume throughout our paper that each Hub elicits tool needs from each of its projects, takes the union of these requests, and submits this list to companies. This avoids the strategic minefield of what happens when Hubs request too few tools for all projects to use at once, meaning projects must share tools by checking them out from a shared library. Under strategic considerations, this results in suboptimal strategies such as over- or under-reporting the number of licenses a project needs, and projects hogging tools – all of which leads to inefficiency and delayed progress on projects.

1.1 Our Contributions

Based on correspondence with an anonymous CHIPS Hub [32], we prioritize the following questions.

(1) How can we incentivize Hubs to truthfully report the tools needed for projects?
(2) How can we fairly divide resources among companies?

We address both of these problems by using tools from the field of *mechanism design*, which seeks to design rules for "games" in which individually rational and strategic agents are incentivized to behave in a way that benefits society as a whole. In particular, we propose separate interventions to address each problem. To incentivize projects to truthfully report tool needs, we propose a rental model in which projects must pay a higher cost to use tools they do not initially request. To fairly divide resources among companies, we propose a fair division scheme that draws on literature from impartial division of a dollar and bankruptcy problems in order to directly distribute each Hub's budget for tools among companies in a way that bypasses company-specific prices for tools.

1.2 Related Work

The CHIPS Act. The 2021 and 2022 CHIPS Act(s) [55, 56] (also: [54]) appropriated \$52B and authorized an additional \$200B+ in response to this urgent need. Out of the \$11B+ of R&D funding appropriated in the CHIPS Act, multiple billions are targeted to *design* and *design automation* technologies, as seen in the Design-centered R&D flagship facility of the National Semiconductor Technology Center (NSTC) [43], the NSTC's Strategic Plan [42], the National Advanced Packaging Manufacturing Program (NAPMP)'s recent \$1.6B solicitation [57], etc. These massive amounts of funding must be distributed by the government (e.g., NIST [60]) and public-private entities (e.g., Natcast [41]). Hence, fairness, transparency, and fair dealing are of paramount concern (cf. [51]).

Fair Division. There is much prior work on the problem of fair allocation, also called fair division, in the field of computational social choice. When the resource is infinitely divisible, this problem has been formalized as *cake-cutting* [52], and there have been many decades of research on how to find cake-cutting allocations that satisfy various notions of fairness such as proportionality (every agent gets their "fair share" of utility) [9, 24, 25, 27, 28, 38, 40, 48], envy-freeness (no agent envies another agent's share) [3, 6–8, 12, 13, 18, 30, 35, 45], or equitability (all agents receive the same value) [1, 46]. When the resource consists of indivisible goods, i.e., there exist atomic elements that must be allocated among agents, ensuring proportionality and envy-freeness is difficult [2]. Instead, researchers instead focus on maximin share fairness (MMS) [4, 10, 11], maximum Nash welfare [16, 20], envy-freeness up to one (or any) good [15, 49], group fairness [19], and the core (or relaxations thereof) [29]. We focus on scenarios in which agents have homogeneous valuation functions over the resource to be divided (e.g., money, space in a benchmark); the complexity in our setting comes from designing mechanisms that limit strategic manipulations of the fair division process.

Participatory Algorithm Design. We also draw on prior work in developing democratized, participatory algorithmic governance, and acknowledges that mechanism designers ought to collect and analyze data from surveys and stakeholder interviews in order to create pipelines that not only have strong theoretical guarantees, but also result in higher user satisfaction and engagement throughout the entire process [14, 34, 36, 47, 63].

2 PRELIMINARIES

Let $[n] := \{1, \ldots, n\}$ for all $n \in \mathbb{N}$. In a *coalitional game*, each coalition (set) of players has an assigned value or utility; the challenge is calculating an imputation, i.e., an assignment of values to individual members that is efficient and individually rational. The *nucleolus* of a coalitional game is a well-known solution concept (see e.g., [50]) that (roughly) minimizes the maximum gain of any coalition from deviating. It is always unique, and thus an attractive solution concept for fair division games.

2.1 The Model

We assume a collection of different *Hubs*, where each Hub consists of a collection of *projects* that each need some collection of *tools* provided by *companies*. Additionally, we assume that each Hub has a fixed total budget to spend on tools. For ease of exposition, we focus on a single Hub with total budget E; our proposed process naturally extends to multiple Hubs with separate or combined budgets.

We assume the following events occur within the Hub. A detailed description of the complete pipeline is in Appendix A. First, the Hub asks its projects to provide tool demands. These tool demands are then aggregated and made available to the companies supplying the tools. Second, the companies then collectively decide how to split the Hub's (insufficient) tool budget, E. Note that here, they directly divide the budget among themselves without explicitly considering per-tool prices. In general, an EDA company will not reveal its "price book" to a competing company. So, it is reasonable to assume that companies do not know any per-tool prices other than their own. They also would not in general know how many licenses are requested for any competitor's tool.

Our process for dividing awards among the companies is a two-step process directly applying two well-known division rules for different settings. We use the first rule to determine how much each agent is entitled towards out of a value larger than the total award amount, and the second rule determines how much each agent is actually allocated. We depict the fair division pipeline in Figure 1.

2.2 The Bankruptcy Problem with Conflicting Reports

Consider a variant of the bankruptcy problem as follows. A debtor has estate value $E > 0$ and owes a total of $D > E$ to a collection of

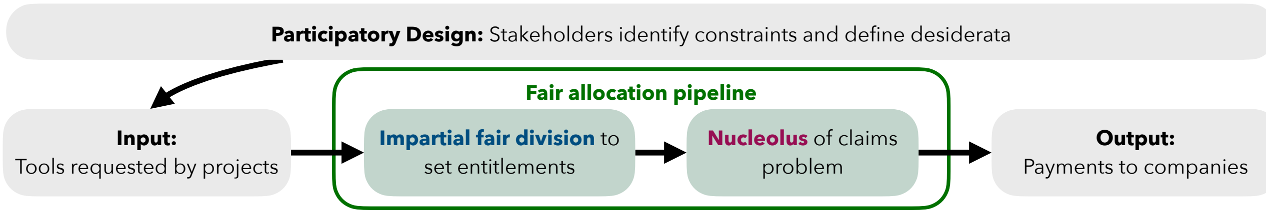

Figure 1: Illustration of the two-step fair division pipeline in the broader context of the allocation problem.

$n \geq 4$ agents. However, the agents do not necessarily agree on the split of D among themselves.

We are interested in mechanisms that first elicit from each agent i a report $r^i = (r^i_1, \ldots, r^i_n) \in \mathbb{R}^n_{\geq 0}$, where $r^i_i = 0$ and $\sum_j r^i_j = 1$, i.e., each agent reports a percentage of the (remaining) share for each other agent. An *evaluation profile* $r = \{r^1, \ldots, r^n\}$ is a collection of n reports.

2.2.1 Aggregators. In order to define the nucleolus, it is first necessary for the mechanism to first collapse a profile r into a single claims vector using an *aggregator* $A : r \mapsto A(r)$ satisfying:

- **Completeness (C):** $\sum_i A_i(r) = 1$.
- **Unanimity (U)[1]:** If all rows are consistent (i.e., up to normalization according to $D - d_i$) with some division v, then $A(r) = v$.
- **Anonymity/Neutrality (AN):** Permuting reporters leaves A unchanged; permuting columns permutes the corresponding components of $A(r)$.
- **Column-Responsiveness (CR):** Around any unanimous profile, it is possible to perturb some agent's off-diagonal reports so that the induced claims vector shifts by $\varepsilon(e_j - e_k)$ for some small $\varepsilon > 0$ and some distinct $j, k \neq i$. Here, e_ℓ is the n-dimensional basis vector with a single one in position ℓ and zeroes everywhere else. Intuitively: by altering only agent i's reports about others, one can shift a bit of claim from agent k to agent j while holding $A_i(r)$ and D fixed.

Given a profile r, aggregator A, total amount owed $D > 0$, and total amount to be distributed $E \in (0, D)$, a mechanism M outputs an allocation $M(A(r) \cdot D, E) \in \mathbb{R}^n_{\geq 0}$ with $\sum_i M_i(A(r) \cdot D, E) = E$, where $M_i(A(r) \cdot D, E)$ is agent i's allocation. We will be interested in aggregators that satisfy the following definition of *impartiality*:

Definition 2.1. Given evaluation profiles $r := \{r^i\}_{i \in [n]}$ and r' where r' differs from r only in r^i_{-i}, an assignment function $f : r \mapsto f(r)$ is *impartial* if $f_i(r') = f_i(r)$.

Note that this definition is general in that it may also apply to mechanisms.

2.2.2 Contested Garment Rule. We study the nucleolus allocation of the (standard) bankruptcy instance $(d := A(r) \cdot D, E)$ that only takes a single claims vector d and estate E as input (instead of a profile r). Note that it is generally NP-hard to compute the nucleolus of a cooperative game. [5] showed that in bankruptcy problems, the nucleolus coincides with the *contested garment rule*. Thus, the contested garment rule gives a natural solution for allocation problems where the total amount entitled by the agents exceeds the amount that can be allocated. However, [5] does not study the case of agents misreporting their debts (because in the bankruptcy situations they're interested in, the debts are already well-documented and cannot be misreported).

We now formally describe the contested garment rule $\mathrm{CG}(d, E)$. We first describe two subroutines by the contested garment rule, known as constrained equal awards (CEA) and constrained equal losses (CEL). In CEA, each agent starts with 0 and "eats" the estate E at an equal rate and stops eating once they've eaten their entitled amount, or if no more estate remains. CEL is a dual procedure, in which each agent starts with their entitled amount and "reverse-eats" until they hit 0 or the total allocation drops from D to E.

Let $d = \{d_i\}_{i \in [n]}$. We define $\mathrm{CEA}(d, E) := \{a_i\}_{i \in [n]}$, $\mathrm{CEA}(d, E)_i = a_i$, where a_i is the allocation of agent i by the CEA procedure given input d, E, and define $\mathrm{CEL}(d, E), \mathrm{CEL}_i(d, E)$ analogously. The pseudocodes for CEA and CEL are given in Algorithm 1.

Algorithm 1 Constrained equal awards and losses

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Constrained equal awards (CEA).
Agent i is active and a_i ← 0 ∀ i ∈ [n].
while E > 0 do:
    n' ← number of currently active agents
    a_i ← a_i + ϵ for all active agents i.
    E ← E − ϵn'
    Deactivate each agent j such that a_j = d_j.
Return a := {a_i}_{i∈[n]}.

Constrained equal losses (CEL).
Agent i is active and a_i ← d_i ∀ i ∈ [n].
while D > E do:
    n' ← number of currently active agents
    a_i ← a_i − ϵ for all active agents i.
    D ← D − ϵn'
    Deactivate each agent j such that a_j = 0.
Return a := {a_i}_{i∈[n]}.
```

The contested garment rule runs either CEA or CEL depending on if $E \leq D/2$ or not. If $E \leq D/2$, then we run $\mathrm{CEA}(\{d_i/2\}_{i \in [n]}, E)$; otherwise, we first allocate $d_i/2$ to agent i for all i, and then run $\mathrm{CEL}(\{d_i/2\}_{i \in [n]}, E - D/2)$ as input. We denote the output of the contested garment rule as $\mathrm{CG}(d, E), \mathrm{CG}_i(d, E)$.

Hence, returning to our original pipeline, we are given a profile r from the n agents (companies), an aggregator A, and an estate

[1] The standard bankruptcy problem studied in [5] can be defined in the same way as the bankruptcy problem with conflicting reports that we define here, but there the profile is assumed to be unanimous.

(budget) E and entitlement (total amount owed to the companies) $D > E$. We obtain a claims vector $d := A(r) \cdot D$, and we return the allocation given by $\mathrm{CG}(d, E)$.

2.3 Roadmap

In Section 3, we address the Hub reporting step of the pipeline with a simple strategyproof mechanism. In Section 4, we define our mechanism for the budget division step, proving an impossibility result and analyzing a notion of manipulability of the mechanism in Sections 4.2 and 4.3. Finally, in Section 5 we provide experiments on the manipulability of our budget division mechanism.

3 STRATEGYPROOF HUB REPORTING

Our first insight is that we can examine strategyproofness for step 1 in the pipeline independently from step 2 in order to incentivize projects to truthfully report their tool demands.

We propose the following *rental model*. An exogenous source provides a menu of estimated prices per tool. These prices are not directly what each tool costs, but could be derived from prior academic license costs or estimates from a domain expert. Let the entire estimated price vector be denoted π. Based on these prices, each project creates a list of tools they need, and they contribute the estimated cost of all of these tools to the Hub's tool rental pot. Each project is then able to use all of the tools they request for free. If they need additional tools, they pay strictly more than the estimated price for each tool they use but did not initially request, i.e., to use additional tool T, a project pays $(1 + \varepsilon)$ times the rental price of T for some $\varepsilon > 0$.

Claim 3.1. *The rental model is strategyproof for projects.*

Proof. We show that each project individually is incentivized to report truthfully. For a specific project, there are two cases for untruthful behavior: **(1)** If a project under-reports their tool needs, then they will have to rent tools at a higher rate than if they reported correctly, **(2)** if a project over-reports, then they end up paying for tools they don't need to use, which is worse than reporting correctly. Thus, reporting truthfully is best for each project. □

4 BUDGET DIVISION PIPELINE

In this section we give the details of our mechanism for the companies' side of the process via a mechanism for the bankruptcy problem with conflicting reports. Then we show that there is no mechanism obtaining impartiality and the nucleolus allocation, and thus shift our focus to bounding the amount a given agent can gain from misreporting.

4.1 The Allocation Mechanism

The high-level intuition of our approach is as follows.

(1) We first use an impartial aggregator to get companies (i.e., agents) to decide how much of the budget (i.e., estate) each of their competitors deserves given tool demands from projects.
(2) Then, we use the contested garment rule to divide up the (insufficient) budget among companies.

The first step is *impartial*: No agent can change her own claim based on the information she reports. However, an agent i *can* influence any other agent's entitlement, and this *can* influence i's resulting allocation after running the contested garment rule. In particular, since we show in the next subsection that it is impossible to achieve both the nucleolus allocation and "true" impartiality, we instead obtain impartiality in the aggregator function that defines the claims input for the mechanism. We seek to bound the amount any agent can benefit from misreporting in impartial aggregation.

As is often true in practice, we assume that $D > E$ no matter how many tools projects request, i.e., the total demand D of all companies outweighs the total tool budget E of the Hub [32]. If this is not true, then the result of the impartial division of a dollar step is exactly the final division of budget.

One may ask why we do not just run the impartial aggregator over the budget E (or equivalently scale down their claims over D down to E). However, even in the case where all the companies unanimously agree on what share of the pie each of them deserves, the solution resulting from scaling down their claims to E may not be consistent with the actual claims they deserve. For instance, consider a case with four companies, $E = 100, D = 400$ and claims vector $(50, 50, 50, 250)$. If we scale down each company's claim such that the sum of debts is E, then the resulting allocation is $(12.5, 12.5, 12.5, 62.5)$. However, the nucleolus (i.e., contested garment) solution is $(25, 25, 25, 25)$; in particular, company 4 should not be able to muscle the other three companies out of the way. This is exactly because the nucleolus and total impartiality are incompatible with each other, which we formalize next.

4.2 An Impossibility Result

In this subsection, we show that *impartial mechanisms*, however attractive they seem, are impossible to achieve while returning the nucleolus allocation of the bankruptcy problem.

Lemma 4.1. *Fix $n \geq 4$ and $E \in (0, D)$. There exists a unanimous claim vector v and distinct i, j, k with $j, k \neq i$ such that, for some $\varepsilon > 0$ that $\mathrm{CG}_i(v + \varepsilon(e_j - e_k), E) \neq \mathrm{CG}_i(v, E)$.*

Proof. For CEL: awards take the form $\mathrm{CEL}_m(d, E) = d_m - \lambda$ for non-capped agents m, where λ is the common loss level satisfying $\sum_{m \in n} \min\{d_m/2, \lambda\} = D - E$. Choose v so that agent i is not capped and exactly one of agents j and k is capped. Shifting ε units of claim from k (capped) to j (uncapped) forces λ upward. Hence $\mathrm{CEL}_i(d, E) = d_i - \lambda$ decreases. The same logic applies in the CEA regime (with roles of awards and losses swapped). □

Theorem 4.2. *Given $\mathrm{CG}(A(r) \cdot D, E)$ where A satisfies (C), (U), (AN), and (CR),* CG *cannot satisfy impartiality.*

Proof. Start from a unanimous profile r° with $A(r^\circ) = v$ as in Lemma 4.1. By (CR), there exists r' differing from r° only in agent i's off-diagonal reports such that $A(r') = v + \varepsilon(e_j - e_k)$. By Lemma 4.1, we have $\mathrm{CG}_i(r', E) \neq \mathrm{CG}_i(r^\circ, E)$, but impartiality requires equality, since r' and r° differ only in i's reports about others. This is a contradiction. □

4.3 Manipulability of Mechanisms with Impartial Aggregators

In this subsection we analyze how much an agent can change their resulting allocation if they misreport their input, subject to impartial aggregation. We first provide our definition of the *manipulability* of a mechanism M that uses aggregator A to decide claims report d.

Definition 4.3. The *additive manipulability* for agent i of a mechanism M whose claims vector is decided by aggregator A is

$$\max_{r,r'\in\mathbb{R}^{n\times n}:r'^j=r^j\,\forall j\neq i} M_i(A(r')\cdot D,E)-M_i(A(r)\cdot D,E),$$

where r' is a profile where only agent i's row is potentially manipulated (note that it doesn't have to be).

As this maximum is taken over all possible pairs r, r' that differ in only i's report, we can view r as an "initial" claims and r' as the "manipulated" claims.

4.3.1 Unrestricted Impartial Manipulation. As a baseline, we study the worst-case additive manipulability if in the aggregation step, an agent is allowed to arbitrarily impact the claims of others, but not itself (preserving impartiality).

Definition 4.4 (Impartial dictator). Given a profile $r \in \mathbb{R}^{n\times n}$ and $x \in [0,1]$, the *impartial dictator-i aggregator* $\text{dict}^i(r)$ assigns to r the vector d such that $d_i = x$ and $d_j = (1-x)r^i_j$ for every $j \neq i$.

In other words, agent i is impartially fixed to some claim x, but can arbitrarily distribute the remaining $1-x$ among the other agents. It is easy to see that the above aggregator is impartial. We analyze CEA and CEL independently on the impartial dictator aggregator.

Lemma 4.5. *The additive manipulability of* $\text{CEA}(\text{dict}^i(r)\cdot D, E)$ *is at most* $\frac{E}{2}-\frac{E}{n}$.

Proof. We want to construct claims vectors $d \neq d'$ that are equal at d_i that maximize $\text{CEA}_i(d,E) - \text{CEA}_i(d',E)$. Note we have assumed d, d' have already been normalized to sum to D.

CEA equivalently allocates $\min(d_j, s)$ to j for every j where s is the constant satisfying $\sum_{j\in[n]}\min(d_j,s) = E$. We may view s as the number of rounds of increments until all agents are deactivated. So we want to start with a claims d where agent i gets its minimum possible allocation. We do so by maximizing s, which is done by minimizing the number of agents that are incremented in each round. To do so, we assign 0 to $n-2$ other agents and $D-d_i$ to one other agent in d, reducing to a two-agent instance. If $d_i \leq D-d_i$ then agent i will be allocated $\min(d_i, E/2)$, because either both agents increment to $E/2$ or i drops out before round $E/2$. Otherwise agent i will be allocated $\max(E-(D-d_i), E/2)$. Then i's allocation can be expressed by the function:

$$\text{CEA}_i(d,E)=\begin{cases} d_i & d_i\in[0,E/2)\\ E/2 & d_i\in[E/2,D-E/2]\\ E-(D-d_i) & d_i\in(D-E/2,D]\end{cases}\tag{1}$$

On the other hand, agent i's allocation is minimized by maximizing the number of agents with entitlement E/n in d' having initially fixed d'_i, as this maximizes the number of agents incremented in each round. In this case we have:

$$\text{CEA}_i(d',E)=\begin{cases} d'_i & d'_i\in[0,E/n)\\ E/n & d'_i\in[E/n,D-(n-1)E/n]\\ E-(D-d_i) & d'_i\in(D-(n-1)E/n,D]\end{cases}\tag{2}$$

The largest gap between these two functions is when $d_i = d'_i \in [E/2, D-(n-1)E/n]$, in which the gap is exactly $E/2 - E/n$. □

Lemma 4.6. *The additive manipulability of* $\text{CEL}(\text{dict}^i(r)\cdot D, E)$ *is at most* $\frac{D-E}{2}-\frac{D-E}{n}$.

Proof. Like before we may equivalently view $\text{CEL}(d,E)$ to allocate $\max(0, d_j - s)$ to agent j for every $j \in [n]$ where s is the constant such that $\sum_{i\in[n]}\max(0,d_j-s) = E$, and we view s as the number of rounds of *decrements* (instead of increments). As we must make a total of $D-E$ decrements across the s rounds, then s is minimized when we maximize the number of decrements per round, which is at most n. Hence $\frac{D-E}{n} \leq s \leq D-E$. We consider the following cases:
Case 1: $d_i \leq \frac{D-E}{n}$: since $s \geq \frac{D-E}{n}$, agent i will always get 0.
Case 2: $\frac{D-E}{n} < d_i \leq \frac{D+(n-1)E}{n}$: then s is minimized by setting all non-i agents to at least $\frac{D-E}{n}$, obtaining $s = \frac{D-E}{n}$. It is always possible to set the other $n-1$ agents to at least $\frac{D-E}{n}$ in this case since $D - d_i \geq D - \frac{D+(n-1)E}{n} = (n-1)\frac{D-E}{n}$.
Case 3: $d_i > \frac{D+(n-1)E}{n}$: observe in this case agent i is "too rich" to set every other agent to at least $\frac{D-E}{n}$. Then s is minimized by setting all other agents to at most $d_i - E$. To prove this, let $s^* := (D-E)-(D-d_i) = d_i - E$. We show that s^* is the minimum possible number of rounds of decrements in this case. The sum of all decrements over each agent is $D-E$, and agent i needs to distribute $D-d_i$ of the estate among the other agents. To minimize s, we want to minimize the number of agents that are allocated a strictly positive amount after s rounds. Ideally, all other agents get decremented to 0 after the s rounds, so agent i still needs to be decremented at least $(D-E)-(D-d_i)$ times. Next, we show that $s = s^*$ if and only if all other agents claim at most s^*. Since $s \geq s^*$ from the previous claim, it suffices to show $s \leq s^*$. We prove the first direction by contrapositive, that $\exists\, j$ s.t. $d_j > s^* \implies s > s^*$. Then we have that after s^* rounds of decrements, the total amount decremented across all agents is

$$2s^* + (D - d_i - d_j) < 2d_i - 2E + D - d_i - d_i + E = D - E$$

where we used the assumption that $d_i \geq d_j > s^* = d_i - E$. Hence after s^* rounds, the total decremented amount has not yet reached $D-E$, so $s > s^*$. For the second direction, observe that since all other agents besides i claim at most $s^* \leq s$, they must be decremented to 0 after s rounds. So by round s we have made $D-d_i$ total decrements across all other agents. Then there are $(D-E)-(D-d_i) = s^*$ decrements that must be shouldered by agent i. This requires at least s^* rounds of decrements, and hence $s \leq s^*$. Lastly, we show that it is always possible to set all other agents to at most s^* in this case. We have

$$\begin{aligned} d_i > \frac{D+(n-1)E}{n} &\implies nd_i > D+(n-1)E\\ &\implies D-d_i < (n-1)(d_i-E)\end{aligned}$$

So the remaining $D-d_i$ can be distributed among the other $n-1$ agents such that each agent receives at most $d_i - E$.

The "worst case" of claims for agent i is when $\frac{D-E}{2} \leq d_i \leq \frac{D+(n-1)E}{n}$ and one other agent j claims $D-d_i$ and all other agents claim 0. Here agent i gets $d_i - \frac{D-E}{2}$, but can manipulate the claims to give all other agents at least $\frac{D-E}{n}$, in which case it gets $d_i - \frac{D-E}{2}$, making a gain of $(d_i - \frac{D-E}{n}) - (d_i - \frac{D-E}{2}) = \frac{D-E}{2} - \frac{D-E}{n}$. □

Then we have the following as a corollary:

Corollary 4.7. *The additive manipulability of* $\mathrm{CG}(\mathrm{dict}^i(r)\cdot D, E)$ *is at most* $\frac{E}{2} - \frac{E}{n}$.

Proof. Recall $\mathrm{CG}(d, E)$ outputs $\mathrm{CEA}(d/2, E)$ if $E \le D/2$, and $d/2 + \mathrm{CEL}(d/2, E - D/2)$ if $E > D/2$. When $E \le D/2$, manipulating $n-2$ agents to 0 yields equal manipulability for agent i whether or not we run CEA on d or $d/2$, so the manipulability of CEA upper bounds that of CG in this case. When $E > D/2$, no agent with non-zero claim can be allocated 0, so the manipulability of CEL upper bounds that of CG in this case. Lastly note that when $E \le D \le 2E$ then $\frac{D-E}{2} - \frac{D-E}{n} \le \frac{E}{2} - \frac{E}{n}$, and the corollary follows. □

4.3.2 Restricted Impartial Manipulation. In this section we analyze strategies for maximizing manipulability for a broader class of impartial aggregators beyond dictatorships. We start with a structural lemma on the worst-case manipulation strategies for CEA.

Lemma 4.8. *Fixing profile* r *and agent* i, *for an impartial aggregator* A, $\arg\max_{r':r'^j=r^j\forall j\neq i}(\mathrm{CEA}_i(A(r')D, E) - \mathrm{CEA}_i(A(r)D, E))$ *satisfies* $r'^i_j = 1$ *for some* $j \neq i$ *and* $r'^i_k = 0$ *for all other* $k \neq i, j$.

Proof. Let $j = \arg\max_{j\neq i}(A_j(r))$. If there exists a better manipulation for i that isn't of the form $(0, \dots, 0, 1)$ then there is some agent k with $A_k(r)\cdot D = x > 0$ that i can set to 0, and give x more to j. It suffices to analyze how the change in j's claim affects i's, since lowering an agent k's claim only helps i in CEA.

If $A_j(r) \ge A_i(r)$, then j was always eating with i while i was active during CEA with claims $A(r)$. Hence agent i strictly benefits from the manipulation $(0, \dots, 0, 1)$ by eating $x/(c-1)$ more, where c is the number of agents with larger claims than i in $A(r)$.

Let r' be the profile defined in the lemma statement. If $A_j(r) < A_i(r)$, then either $A_j(r') \le A_i(r')$ or $A_j(r') > A_i(r')$ after setting $r'^i_j = r^i_j + x$. For the former sub-case we have two more sub-sub-cases. First, if i, j deactivated at the same time during CEA with claims $A(r')$, then by the same reasoning as earlier i strictly benefits. Otherwise, i ate longer than j during CEA with claims $A(r)$, so during CEA with claims $A(r')$ agent j will begin to eat during time steps in which i was eating without j. Nonetheless, i's eating time is never decreased because if that were the case then the sum of resulting allocations would be less than E, since the only change was the symmetric change in r^i_j, r^i_k. Finally, when $A_j(r') > A_i(r')$, observe i's allocation is unchanged by how long j eats after i drops out. So this is identical to the case when $A_j(r') = A_i(r)$. □

Next, we show that manipulability is maximized for CEA under a certain initial structure of evaluation profiles, under a wide class of impartial aggregators.

Definition 4.9. Let r be an evaluation profile such that $A_i(r) = x$, and r' be the result of swapping only r^i in r with some r'^i such that $r'^i_j < r^i_j$. Define the function $\Delta^{(r,r',A)}(A_j(r)D) := D(A_j(r) - A_j(r'))$. An impartial aggregator A is *stable* if it satisfies $\Delta^{(r,r',A)}(r^i_j - r'^i_j) \le D(1-x)(r'^i_j - r^i_j)$ and Δ is concave.

Note that this definition is natural in that it essentially says that A guarantees that if a single agent decides that another agent's value (up to normalization) should be changed by δ, then the aggregator will not change that agent's share by δ or more, and furthermore a manipulator cannot harm an agent with a larger starting claim more than what they could harm an agent with a smaller starting claim. In particular, the impartial dictator aggregator is the "worst" stable aggregator as it satisfies the specified inequalities with equality, and its corresponding Δ function is linear.

Lemma 4.10. *Fixing an agent* i *with claim* d_i, *for a stable impartial aggregator* A, $\mathrm{CEA}_i(A(r')\cdot D, E) - \mathrm{CEA}(A(r)\cdot D, E)$ *is maximized if* r *satisfies*

$$A(r)\cdot D = \mathcal{A} := \left(E/n, \dots, d_i, E/n, \dots, D - d_i - \frac{(n-2)E}{n} \ge E/n\right)$$

and r' *is the manipulated profile defined in Lemma 4.8.*

Proof. The intuition is roughly that if there is a non-i agent with less than E/n initially, then the amount it can be lowered is at most that of if it was E/n initially, but if it was greater than E/n initially, then the amount it gets lowered from above E/n to E/n does not benefit i at all.

If $d_i \le E/n$, then agent i is allocated d_i regardless of the other shares, so we may WLOG assume $d_i > E/n$. Let $d' := A(r')D$ and j denote the agent that i assigns D to in r'. In the CEA allocation for $\mathcal{A}$, every agent gets E/n. Meanwhile, d'_i is unchanged by impartiality, $d'_j = E/n + \delta(n-2)$ for exactly one $j \neq i$ and $d_k = E/n - \delta$ for the remaining $k \neq i$ for some $\delta > 0$.

In the case that $\mathrm{CEA}_i(d', E) = d'_i = d_i$, then $\mathcal{A}$ is clearly the initial claims vector that maximizes additive manipulability, because agent i gets at least E/n and at most d_i across any claims vector. So it remains to prove the claim for the case where agent i receives less than d_i. The total sum of allocations of the lowered agents by i in $A(r')D$ is strictly less than $(n-2)\frac{E}{n}$ since i doesn't gain anything from j's change, and i, j both receive more than E/n. Hence on average, a lowered agent k is allocated $A_k(r')D < E/n$. Then

$$\sum_{k\neq i,j} \Delta^{(r,r',A)}(d'_k) \le (n-2)\cdot\Delta^{(r,r',A)}\left(\frac{d'_k}{n-2}\right)$$
$$< (n-2)\cdot\Delta^{(r,r',A)}\left(\frac{E}{n}\right) \le (n-2)\frac{E}{n},$$

where the first inequality is due to Jensen's inequality and stability, and the second is by the average $A_k(r')D$ being strictly less than E/n. This bounds the total possible gain across *all* lowered agents. At best, agent i splits this total gain equally between itself and j after the best manipulation, which is what happens in $\mathcal{A}$. Therefore i's gain is maximized if all other $n-2$ agents are set to exactly E/n in $A(r)D$, as if anyone was set to less than E/n then the total gain would be less and if anyone was set to more than E/n they can't be lowered to anything lower than they would've been lowered to if they were just at E/n due to stability. □

The previous two lemmas are analogous to Lemma 4.5, but far more general. A natural impartial aggregator to use is the following *impartial division rule* proposed in [22][2].

[2] This is known to satisfy other natural properties such as completeness, unanimity, and anonymity, and is used in real-world applications, e.g., Spliddit [31].

Definition 4.11 (Arithmetic mean). Given impartial profile r, let $\mathbf{r}$ be defined such that $\mathbf{r}^i_{jk} = r^i_j / r^i_k$ for every $j, k \neq i$ and $\mathbf{r}^i_{ij}, \mathbf{r}^i_{ji}$ for any i, j is empty. For any $\vec{x} \in \mathbb{R}^n$ let $\rho(\vec{x}) = 1/n \sum_i \vec{x}_i$, and define the *arithmetic mean aggregator* f as follows:

$$f_i(r) := f_i(\mathbf{r}) = \frac{1}{n}\left(1 - \sum_{j\neq i} \frac{1}{1 + \rho((\mathbf{r}^k_{ij})_{k\neq i,j}) + \sum_{k\in n\setminus\{i,j\}} \rho((\mathbf{r}^l_{kj})_{l\neq i,j,k})}\right) + \frac{1}{n}\left(\sum_{j\neq i} \frac{1}{1 + \rho((\mathbf{r}^k_{ji})_{k\neq i,j}) + \sum_{k\in n\setminus\{i,j\}} \rho((\mathbf{r}^l_{ki})_{l\neq i,j,k})}\right)$$

We remark that there is empirical evidence (see, e.g., Figure 5 in Appendix C.2) showing that the above rule is not only a stable aggregator, but also that no manipulation by some agent i can change another agent j's share by more than $O(n^{-1})$, which would imply an additive manipulability of $O(1)$. We implement our full pipeline using the arithmetic mean aggregator in Section 5.

4.4 Aggregation via Impartial Ranking

In the previous section, we combined impartial aggregators with the contested garment rule, and bounded an agent's gain from misreporting by changing the claims of other agents. In this section we propose an alternative impartial mechanism by using *impartial ranking* with the contested garment rule.

We first fix an claims vector $d := \{d_1, d_2, \ldots, d_n\}, D \sum_{i\in[n]} d_i = D$ where we use an impartial ranking rule to decide which agent is assigned index. Then we run the contested garment rule as before with d. Specifically, the agents still report evaluation profiles r like before, which we can directly convert into rankings (i.e., each agent submits an ordered lists of the other $n-1$ agents) as input for the impartial ranking rule. However, the values that make up d are fixed and not a function of r.

The impartial ranking rule outputs an assignment of ranks from 1 to n to each agent; we assign the agent ranked at i by the rule the claim d_i. Impartial ranking rules that satisfy other desirable properties have been studied previously, see e.g., [17, 33].

Theorem 4.12. *The additive manipulability of the contested garment rule using the impartial ranking rule to define claims is* 0.

Proof. Observe that because we fix the claims vector d and then assign claims to agents, no agent can gain from misreporting after applying the contested garment rule on d. Indeed, no matter what an agent reports, the resulting instance of the contested garment rule is identical in their perspective, since they cannot manipulate their own rank (and thus their own claim) by impartiality, and manipulating anyone else's rank doesn't change the claims vector by our fixing of d. □

In particular, this means that the mechanism $\mathrm{CG}(R(r)\cdot D, E)$ where R is some impartial ranking rule $r \mapsto \pi(d)$ (i.e., r maps to a permutation of d) is impartial and obtains the nucleolus allocation. Note that this doesn't contradict Theorem 4.2, since $R(r)$ doesn't satisfy column-responsiveness (as it is not even a function of r). We describe how to choose a specific claims vector d in Appendix B.

5 EMPIRICAL RESULTS

In previous sections, we have examined the worst-case manipulability of mechanisms with impartial aggregators. We now present experimental results that explore the average additive manipulability of the contested garment mechanism under unrestricted (i.e., $\mathrm{CG}(\mathrm{dict}^i(r)\cdot D, E)$) and restricted (i.e., $\mathrm{CG}(f(r)\cdot D, E)$) misreports from a single agent. Note that we have already proved the worst case manipulability of the unrestricted setting, but we plot it to compare the restricted manipulability with respect to this upper bound. All code used in the experiments will be released on Github upon acceptance.

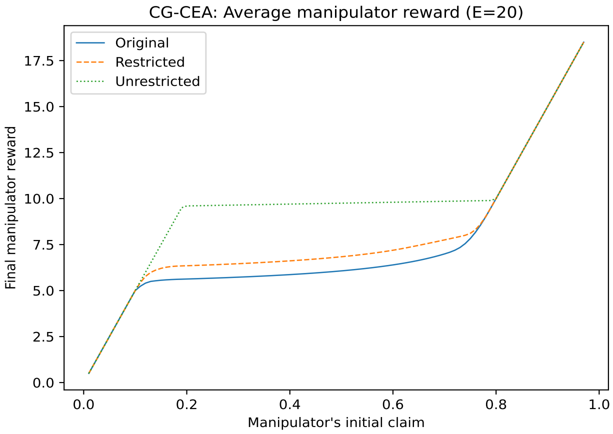
Average Additive Manipulability. We study how a potential manipulator's average (maximum) payoff varies across three treatments when starting from a unanimous outcome: (1) a *baseline* with no manipulation, (2) the agent's best *restricted* impartial manipulation in which the agent is allowed to arbitrarily change their report and an impartial aggregator with the arithmetic mean is used, and (3) the agent's best unrestricted (i.e., dictatorial) impartial manipulation in which the agent is allowed to directly modify the entries in the claims vector for all other agents while preserving their own entry and the sum of all entries in the claims vector.

Setup. For $n = 4$ agents, we consider all unanimous scenarios in which all agents agree on the division of credit among themselves and each agent deserves a multiple of 0.01 (the atomic unit of credit) out of 1 total credit to be divided. We assume the total amount owed is $D = 100$ and consider estate values $E \in \{5, 10, \ldots, 95\}$, where the contested garment rule uses CEA on the halved entitlements when $E \leq D/2$ and uses CEL after giving each agent half of their entitlement and adjusting D to $D - E/2$ when $D > E/2$.

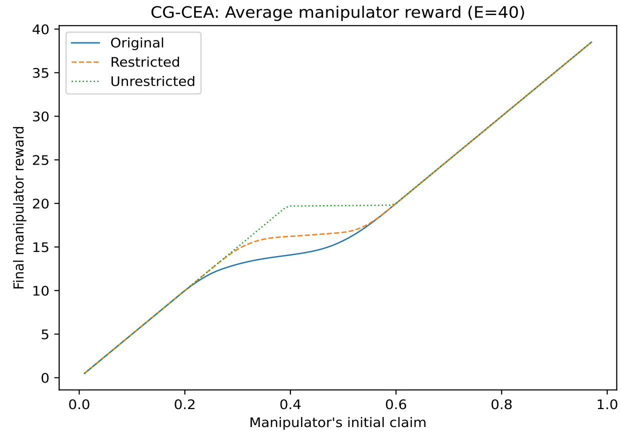
We average over unilateral (i.e., single-agent) manipulations of the following form. Consider manipulating agent i, who is assigned d_i in the unanimous claims vector. Agent i's allowable manipulations are all possible divisions of one unit of credit among the other $n-1$ agents in increments of 0.01, which are then scaled by $1 - d_i$ to preserve their unanimous share. For each estate value, each unanimous scenario, and each candidate manipulator, we search over all allowable manipulations to compute three payoffs. *Baseline*: The result of the contested garment mechanism on the truthful unanimous matrix. *Restricted*: This is the result of the single-best manipulation under the restricted scenario, i.e., the maximum possible allocation to the manipulator over all possible manipulations after running the contested garment mechanism on the result of using an impartial aggregator with the arithmetic mean. *Unrestricted*: This is the result of the single-best manipulation under the unrestricted (dictatorial) scenario, i.e., the maximum possible allocation to the manipulator when running the contested garment mechanism over all possible dictatorial manipulations.

Results. We compare the average maximum reward of both types of manipulations to the baseline value over all starting unanimous values $x_0 \in \{0.01, 0.02, \ldots, 0.97\}$ in Figures 2a and 2b for the CEA realm and Figures 2c and 2d for the CEL realm. Additional results for a wide range of E values are presented in the appendix.

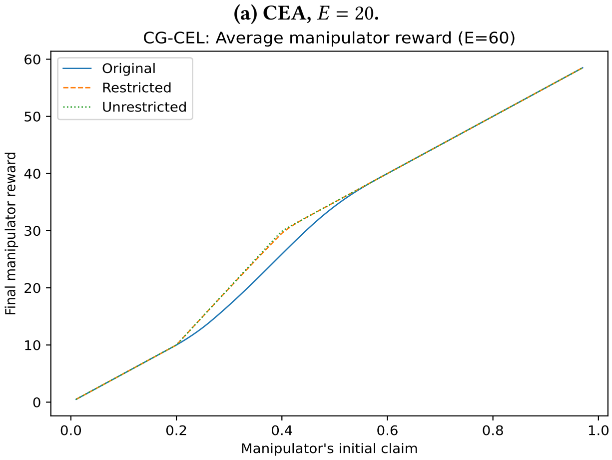
In Figures 2a and 2b, we see that for very low and very high values of the manipulator's initial value x_0, there is no benefit from either unrestricted or restricted manipulation. For moderate values of x_0, i.e., between roughly 0.2 and 0.7 in Figure 2a and 0.3 and 0.5 in Figure 2b, there is a clear separation between the benefit of unrestricted and restricted manipulations. Unrestricted



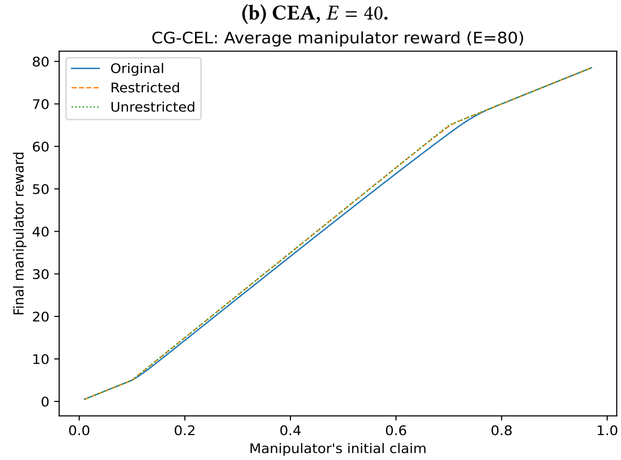
(a) CEA, $E = 20$.



(b) CEA, $E = 40$.



(c) CEL, $E = 60$.



(d) CEL, $E = 80$.

Figure 2: Average additive manipulation for CEA and CEL and various values of E. Throughout, $D = 100$.

(i.e., dictatorial) manipulations lead to large gains on average, as expected given our theoretical results, but restricted manipulations only lead to very slight gains over the baseline. This illustrates that using the impartial aggregator with the arithmetic mean severely limits the additive manipulability of the fair division pipeline. These trends are observed for all $E \leq D/2$; the only difference among results for different values of E is that as E increases, the width of the manipulation-susceptible region shrinks.

In Figures 2c and 2d, we again see that agents on the extremes never benefit: For very low and very high values of the manipulator's initial value x_0, there is no benefit from either unrestricted or restricted manipulation. For moderate values, there is a separation. However, unlike in the CEA realm where unrestricted manipulation led to much higher payoffs than restricted manipulation, in the CEL realm we observe approximately equal manipulability of unrestricted and restricted manipulations. However, the overall additive manipulability still remains relatively low, which suggests that the impartial aggregator with the arithmetic mean rule still performs well on average. These trends are observed for all $E > D/2$; the only difference among results for different values of E is that as E increases, the width of the central region where manipulation helps grows, but the maximum manipulability (i.e., difference between restricted or unrestricted and the original) shrinks.

6 DISCUSSION AND FUTURE WORK

Our work initiates the design and analysis of theoretically-grounded and stakeholder-driven fair allocation pipelines for complicated real-world problems. In particular, after proving a general impossibility theorem for the two theoretical properties we consider (i.e., impartiality and the nucleolus), we propose an explainable and implementable two-step pipeline that empirically demonstrates robustness to restricted manipulations.

A natural next step is to derive formal bounds on additive manipulability of the pipeline with various (i.e., the arithmetic and geometric mean, and the arithmetic and geometric median [22]) aggregators. Additionally, it could be instructive to study the average-case manipulability in non-unanimous settings, i.e., when companies do not agree on a division. Lastly, we plan to implement this pipeline in collaboration with one or more Hubs; this will require additional work to ensure that the participatory design process has opportunities to iterate and refine the fair division pipeline once implemented.

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A AN END-TO-END FAIR DIVISION PIPELINE FOR DOD MICROELECTRONICS COMMONS

We illustrate our ideas via a specific process for the DoD Microelectronics Commons that draws heavily on the concepts of impartial fair division and claims problems. We begin by sketching the provisioning process, along with its immutable constraints. Through this example, we highlight the kinds of nitty-gritty considerations needed for real-world relevance and impact.

A.1 Rules of the Process

A provisioning process consists of three classes of players (equivalently, agents): (1) *projects*, (2) EDA *companies*, and (3) a trusted *mediator* (i.e., the entity running the allocation process). Each project requires a particular set (or bundle) of tools to complete their work. Each company provides licenses to tools that they own. The goal of each project is to obtain a satisfactory bundle of tools; the goal of each company is to maximize the amount of money they are paid for tools that they provide. The goal of the mediator is to ensure that projects receive tool licenses and pay companies, and that companies send tool licenses and receive payment that effectively comes from projects.

Budget specification: Each collection of projects, or Hub, must provide a budget range for the amount of money they are willing to spend on tool licenses, e.g., "Out of $200M in overall project Year 2 funds, an estimated range of between $15M and $40M is anticipated to be used by the USG for EDA licenses in support of awarded projects."

Fixed prices: Each company must provide the mediator with "a la carte" book prices for each tool at the start of the fair division process. They may also provide "all-you-can-eat" (AYCE) prices for unlimited licenses to all of their tools. These prices are binding and may not be changed, and each company's prices are private to themselves, the mediator, and Hub or project leads. Prices will be kept from competitors through standard NDAs.

Substitutions: Projects must specify acceptable substitutions between tools. The mediator will provide a list of "default" substitutions among tools of the form, "1 copy of tool A is substitutable by 2 (or more) copies of tool B." Projects may override default substitution rates during the project request step.

Tool rental model: Each project will pay for the tools they request at a base rate per tool; they may also buy extra tools beyond those initially requested at a higher rate per tool. The base rate will be determined based on the fixed prices provided by the companies and the amount of money paid to each company at the end of the suggested fair division process.

No tool contention: Hubs will purchase the union of all project requests in order to prevent (potentially strategic) contention for scarce tools.

No vetoes: The results of the mechanism are binding, i.e., companies must accept their payments, and projects must accept the tools acquired on their behalf. If necessary, it may be useful to explicitly disallow side channel deals between individual projects and companies. Hubs, projects, and companies must commit to satisfying their EDA needs through the provisioning process, and it is understood that companies give "best price" deals to the mediator.

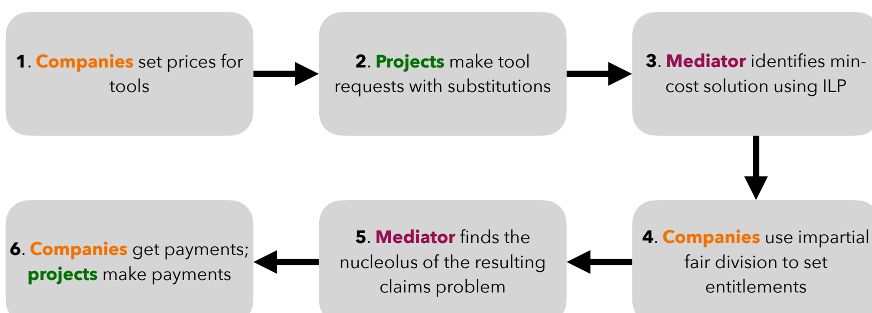


Figure 3: Illustration of the end-to-end fair division pipeline.

A.2 End-to-End Flow

We now describe the overall flow of the fair division pipeline (see Figure 3 for a visual representation), followed by additional details of several key aspects. The end-to-end flow has six steps:

1. Company pricing: EDA companies provide "a la carte" prices per tool to the moderator of the process. They may also provide "all-you-can-eat" (AYCE) pricing at the whole-Hub level.

2. Project requests: Projects provide tool demands and substitutions (if different from default rates). All requests will be anonymized for privacy reasons.

3. Bundle optimization: The mediator solves a minimum cost integer-linear program (ILP) to find the least expensive bundle of tools to acquire that satisfies the union of all project demands. Due to the (linear) structure of substitutions, this is potentially an easy problem. Here, one can view AYCE offers by companies as establishing a cap on their total *entitlement*, i.e., the amount of money they are judged to "deserve" at the end of the impartial fair division process in Step 4.

4. Impartial fair division or impartial fair ranking: Companies are provided with the minimum-cost bundle from the ILP optimization step, and each company must evaluate how much of the total tool budget their competitors deserve. This leverages industry-specific expertise (competitors should have a good sense of what each company's bundle is "worth") and allows companies to fight back against a company that price-gouges an unsubstitutable tool. In order to ameliorate strategic misreporting, we use an impartial division rule with the property that each company's report does not affect their entitlement.
5. Claims problem: The mediator then divides the (insufficient) tool budget among companies based on their entitlements at the end of Step 4 according to the contested garment rule, which recovers the nucleolus. The target division is the result of the impartial fair division step.
6. Project payments: Projects are effectively charged based on the a la carte prices of the tools that they request and the amount of money allocated to each company after solving the claims problem.

A.3 Specific Pipeline Details

Bundle Optimization Per Project (Step 3). The ILP that the mediator solves for bundle optimization for each project is as follows. Assume that there are a total of T tools collectively offered by all m companies, and there are also $0 \leq k < m$ AYCE offers from some subset of companies. The mediator would define one ILP for each project (because they may have different substitution matrices S) and return the sum of all $\vec{x}$ solutions as the overall bundle of tools. Note that if all substitutions are of a linear form (i.e., each tool is equivalent to some set of linear combinations of other tools), then this ILP becomes quite simple. For each tool T_k, there is a most cost-effective "a la carte" substitution. We may use these most cost-effective substitutions directly. In this alternative formulation, AYCE offers become upper bounds on the amount of money that will be paid to each company.

The ILP takes as input the following. Tool requests: $\vec{t}$ is a $1 \times T$ vector of the quantities of each tool requested by the project. Prices of tools: $\vec{p}$ is a $1 \times (T+k)$ vector of the price per tool (where the last k slots are prices of AYCE tools). Note that $k \leq m$ because each company can make at most one AYCE offer. Substitutions: S is a $(T+k) \times T$ matrix that encodes the accepted substitution rates between tools specified by the project. Entry S_{ij} represents the value of one copy of tool i in terms of number of copies of tool j. The last k rows represent AYCE substitutions, where entries in row S_ℓ for $\ell \in [T+1, T+k]$ are 0 for tools not offered by the company making the AYCE offer and ∞ for tools offered by the company making the AYCE offer. The ILP returns $\vec{x}$, a $1 \times (T+k)$ vector of the quantities of each tool produced by the solver.

The ILP solves for $\vec{x}$ to minimize the total price of all tools subject to (1) tool requests being met, and (2) integral numbers of each tool requested, i.e.,

$$\begin{aligned}
&\min \vec{x} \cdot \vec{p} \\
&\text{subject to} \\
&\quad \vec{x} \cdot S_{\cdot j} \geq t_j, \ \ \forall j \in [T] \\
&\quad x_i \in \mathbb{Z}_{\geq 0}, \ \ \forall i \in [T+k].
\end{aligned}$$

Democratizing Entitlements (Step 4). One downside of the impartial mechanism used in Step 4 is that it does not actively disincentivize any company from providing (ignorant or spiteful) misreports about how much their competitors deserve; impartiality only guarantees that a company's share is not affected by their judgments of others. This leaves the door open for suboptimal outcomes where, for instance, all companies spitefully determine that the most powerful company in the pool deserves no money at all. However, further democratization of Step 4 may address this issue by, for instance, taking into consideration market share and Hub consensus about how much money each company deserves, and determining entitlements based on a weighting scheme over multiple constituent elements (e.g., 30% market share, 20% Hub consensus, and 50% impartial mechanism), as has been done in, e.g., college football rankings [26]. The current setup allows for such democratization to happen essentially "for free" in the sense that the result of Step 4 would still be impartial (company reports still do not affect their entitlements) and the subsequent steps of the division process proceed unchanged.

B CLAIMS VECTORS FOR IMPARTIAL RANKING

We discuss how to design a good claims vector d here. In order to retain the impartiality that we discussed previously, the values $\{d_i\}_{i \in [n]}$ must not be functions of the actual input evaluation profiles r. Hence, the choices for claims vectors are (essentially) only limited to functions of n. Trivial options satisfying the criterion include the uniform vector $\{1/n, 1/n, \ldots, 1/n\}$ and a geometric sequence $\{1/2, 1/4, \ldots, 1/2^{n-1}, 1/2^{n-1}\}$ (note the last term is also $1/2^{n-1}$ so that the sum of all values is 1).

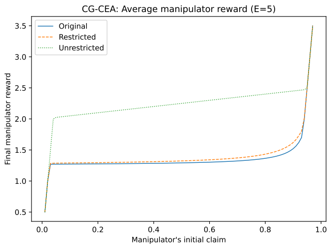
While there is no relation between these choices of d and the input evaluation profiles, we can still choose d based on a new criteria: how much an agent can (multiplicatively) manipulate the gain/loss of *another* agent. For the uniform vector this is 1, but for the geometric sequence this is at most 2^k if k is the maximum number of positions an agent can move another agent in the ranking by changing their input evaluation profile. To get polynomial manipulability on k, we propose a generalized harmonic sequence $\{\frac{1}{H_{n,p}}, \frac{1}{2^p H_{n,p}}, \ldots, \frac{1}{n^p H_{n,p}}\}$ where $H_{n,p} := \sum_{i \in [n]} \frac{1}{i^p}$ is the generalized harmonic number on n vertices and degree p. It is easily verified that the most an agent can multiplicatively manipulate the gain/loss of another agent with this claims vector is at most $(k+1)^p$. The choice of p here allows for some flexibility by the mechanism designer: for example, if they desire the maximum claim to be $1/2$, they can choose $p := p(n)$ that satisfies this condition. We remark that the main weakness of this approach is that the claims vector is chosen completely independently of the profile r, and this is exactly what allows it to achieve 0 additive manipulability.

C ADDITIONAL EXPERIMENTAL RESULTS

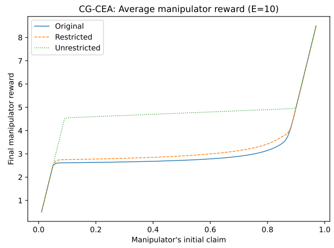
C.1 Average Manipulability

In Figure 4, we plot the average manipulability of restricted and unrestricted manipulation over a wide range of E values for the contested garment rule (which runs a combination of CEA and CEL with suitable parameters). In the CEA realm, i.e., when $E \leq D/2$, as E increases, the width of the manipulable region shrinks. In the CEL realm, i.e., when $E > D/2$, as E increases, the width of the manipulable region increases but the magnitude of manipulation decreases.

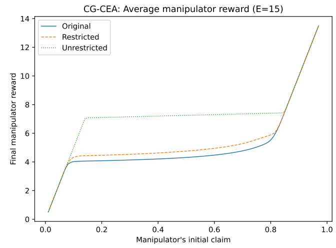
Another thing to note is that, in the CEA realm, the largest unrestricted manipulation gaps often occur when the manipulator's initial claim is E/D (e.g., when $E = 20$, the maximum difference between Unrestricted and Original occurs at $d_i = 0.2$). Intuitively, this is due to the following two phenomena: (1) This is the smallest value of d_i such that unrestricted manipulation allows the manipulator to get $E/2$; and (2) This value maximizes the chance that other agents' unanimous shares are all greater than the threshold that ensures that the manipulator is originally awarded exactly E/n (e.g., in the case of $E = 20$, this happens when all other agents have $d_j \geq 10$).



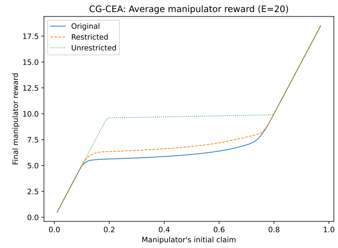
(a) CEA, $E = 5$.



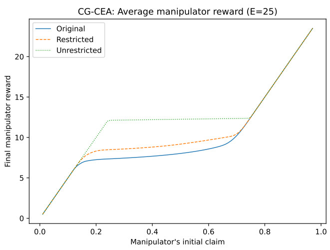
(b) CEA, $E = 10$.



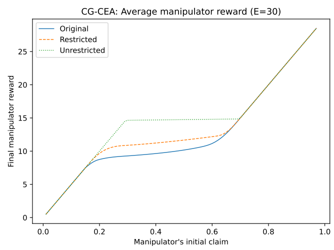
(c) CEA, $E = 15$.



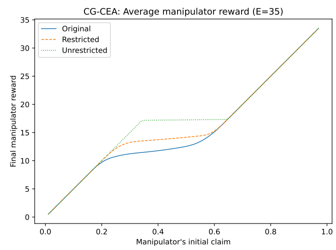
(d) CEA, $E = 20$.



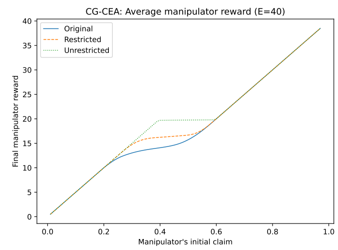
(e) CEA, $E = 25$.



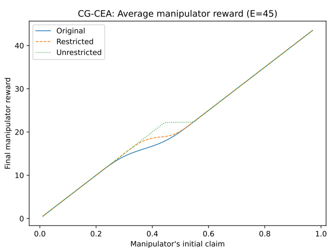
(f) CEA, $E = 30$.



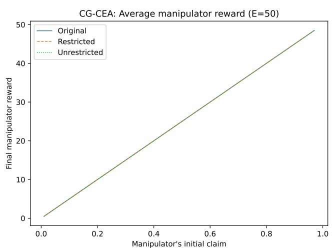
(g) CEA, $E = 35$.



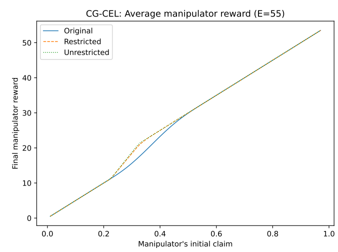
(h) CEA, $E = 40$.



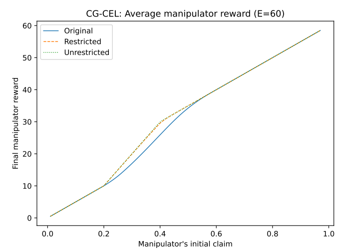
(i) CEA, $E = 45$.



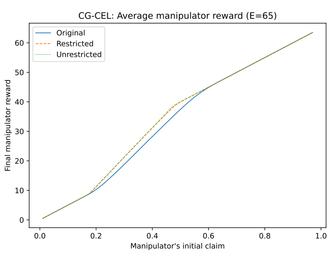
(j) CEA, $E = 50$.



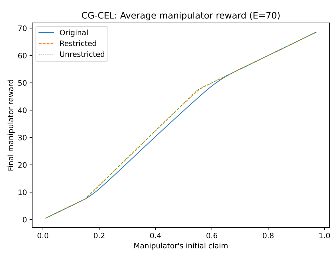
(k) CEL, $E = 55$.



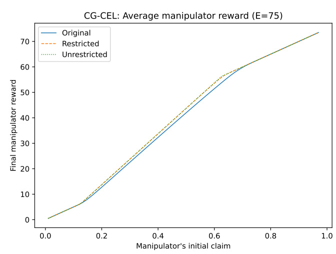
(l) CEL, $E = 60$.



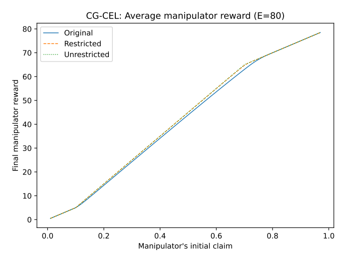
(m) CEL, $E = 65$.



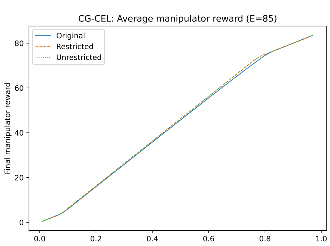
(n) CEL, $E = 70$.



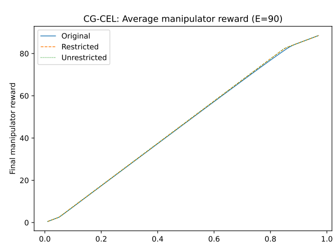
(o) CEL, $E = 75$.



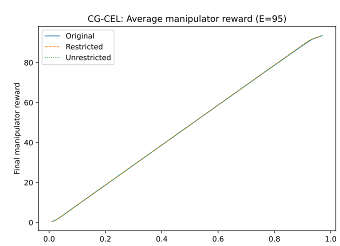
(p) CEL, $E = 80$.



(q) CEL, $E = 85$.



(r) CEL, $E = 90$.



(s) CEL, $E = 95$.

Figure 4: Average additive manipulation for CEA ($E = 5$ to 50) and CEL ($E = 55$ to 95). Throughout, $D = 100$.

C.2 Impartial Division

In Figure 5, we plot the maximum possible benefit of manipulation under the impartial rule with the arithmetic mean aggregator against the number of agents on the x axis. Notably, as the number of agents increases, the maximum possible benefit of manipulation vanishes; we hypothesize that the shape of the curve is $O(n^{-1})$.

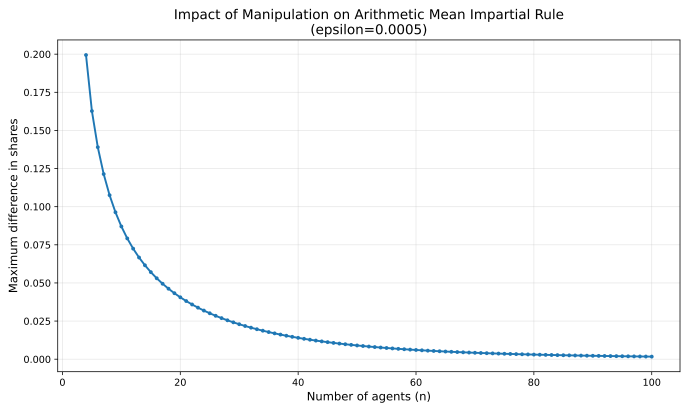


Figure 5: Maximum impact of unilateral manipulation using the impartial rule with the arithmetic mean aggregator.