CEL cases.

Endowment = 70. claim = 100. consensus = (0.4, 0.3, 0.2, 0.1)

$\therefore$ claim = $100 \times [0.4, 0.3, 0.2, 0.1] = [40, 30, 20, 10]$ C stands for claim.

$\sum_{j=1}^{n} \max(C_j - X, 0) = E$ where C_j is the claim of agent

$\therefore X = \frac{100-70}{4} = 7.5$ $\therefore y_0 = 32.5$. $y = [32.5, 22.5, 12.5, 2.5]$

$\boxed{\text{Restricted:}}$ $[0.4, a, b, c]$

1-3 column $S_1 = a + 3\times0.3$ $S_2 = b + 3\times0.2$ $S_3 = c + 3\times0.1$

$\because a+b+c = 0.6$ $S_1+S_2+S_3 = 1.6+1.8 = 3.4$ (So column 0 cannot change)

$c_0 = 100\times\frac{1.6}{4} = 40$ $y_0^{remain} = 40 - 7.5 = 32.5$

$\because$ the summation $C_1+C_2+C_3 = 60$ the same as original case. no change $[32.5, 22.5, 12.5, 2.5]$

$\boxed{\text{unbounded}}$ (A)

$$\begin{cases} S_0 = 0.4\times4 = 1.6 \quad S_1+S_2+S_3 = 4\times0.6 = 2.4 \quad C_0 = 100\times\frac{1.6}{4} = 40. \quad C_1+C_2+C_3 = 60 \\ T = C_1+C_2+C_3 = 60 \quad \triangleq k = |\{j: C_j > X\}| \in \{1,2,3,4\} \end{cases}$$

$\sum_{j: C_j > \lambda} (C_j - \lambda) = 70$ $k=4$

$k=4$ ① $40 + 60 - 4X = 70$ $X = 7.5$

$k=3$ ② $X = \frac{100 - C_{least} - 70}{3} \le \frac{30}{3} = 10$. ~~$E$ C_{least}~~ $\because X = 10$ but I want $X < 7.5$

$\therefore$ $C_{least} > 7.5$ $\therefore$ manipulator will be worse of

$\therefore$ so if more ~~to~~ agent is kicked off, No improvement will occur. (A)

$\therefore$ original = ~~32.5~~ restrict = 32.5 unbounded = 32.5.

now change agent. $y = \{32.5, 22.5, 12.5, 2.5\}$

skew agent=1 22.5

skew agent=2 12.5

skew agent=3 2.5

$\boxed{\text{unbounded}}$ $\sum_{j\in S}(C_j - X) = E = 70.$

~~$k\ k \le |S|$~~: $C - kX = 70$ $X = \frac{C-70}{k}$

so for $k \in (1,2,3,4)$

$X = \frac{100-70}{4} = 7.5$. However, we have shown above (A). If we ~~k~~ kick one agent out, then X always ↑ manipulator ↓. regardless consensus matrix

$\boxed{\text{restricted}}$

similarly $[0.4, a, b, c]$ where $a+b+c = 6$.

$$S_m = \begin{cases} 1.2 & m=1 \\ 0.8 & m=2 \\ 0.4 & m=3 \end{cases} \qquad \because X = \frac{100-70}{4} = 7.5$$

$$y^{remain} = C_m - \lambda = \begin{cases} 30 - 7.5 = 22.5 & m=1 \\ 20 - 7.5 = 12.5 & m=2 \\ 10 - 7.5 = 2.5 & m=3 \end{cases}$$

$S_0 = 1.6$

CEL case

For concensus matrix $M(0) = [0.7, 0.1, 0.1, 0.1]$

$S_0 = 0.7 \times 4 = 2.8$ $S_1 = S_2 = S_3 = 0.4$. claim $= (70, 100, 10, 10)$

$\boxed{\text{original}}$ $100 - 4X = 70$ $X = \frac{100-70}{4} = 7.5 < 10$ ✓

$y = (62.5, 2.5, 2.5, 2.5)$ $y_0 = 62.5$.

$\boxed{\text{restricted}}$ $(0.7, a, b, c)$

$S_0 = 2.8$. $S_1 + S_2 + S_3 = 1.2$ $C_1 + C_2 + C_3 = 30$

$X = \frac{C_0 + C_1 + C_2 + C_3 - E}{K}$ $y_0 = E - X = 62.5$.

since E, $C_1 + C_2 + C_3$ fix, $4K \downarrow$ No way X will $\downarrow$ so manipulator $\downarrow$.

$\boxed{\text{unbounded}}$ As shown above No way we can manipulate others to increase y_0.

However, For agent = 1 could. $\boxed{62.5 \to 62.5 \to 62.5}$

$y = \{62.5, 2.5, 2.5, 2.5\}$ $y_1 = 2.5 \to \boxed{\text{original}}$.

$\boxed{\text{restricted}}$ $[0.7, 0.3, 0, 0]$. $S_1 = 0.3 + 3 \times 0.1 = 0.6$ $C_1 = 100 \times \frac{0.6}{4} = 15$.

$C_2 = C_3 = 100 \times \frac{0.3}{4} = 7.5$.

claim $[70, 15, 7.5, 7.5]$ $X_1 = C_1 - X = 15 - 7.5 = 7.5$

$\boxed{\text{unbounded}}$: $S_1 = 0.3 \times 4 = 1.2$ $C_1 = 100 \times \frac{1.2}{4} = 30$ $C_2 = C_3 = 0$. $\therefore X = (70 - x) + (30 - x) = 7$

$x = 15$

$X_1 = 30 - 15 = 15$.

$\therefore$ $\boxed{2.5 \to 7.5 \to 15 \quad \text{if improves.}}$

CEL. [0.2, 0.2, 0.3, 0.2] agent = 0

$30+30+20+20-4x=70 \quad x=7.5$.

$\boxed{\text{original}}$ $x_0 = c_0 - x = 20-7.5=12.5$

$\boxed{\text{restricted}}$

$$\boxed{\begin{array}{ll} S_0 = 0.2\times4=0.8 & c_0 = 25\times0.8 \\ S_1 = 0.3\times3+a & c_1 = 25\cdot(a+0.9) \\ S_2 = 0.3\times3+b & c_2 = 25\times(b+0.9) \\ S_4 = 0.2\times3+c & c_3 = 25\times(c+0.6) \end{array}}$$

$(20-x)+(25(a+0.9)-x)+25(b+0.9)-x+25[(c+0.6)-x]=70$

$80+25(a+b+c)-4x=70$

$y_0 = c_0 - x = 20-7.5=12.5$

$\boxed{\text{unbounded}}$

manipulation space $4\times0.8=3.2$ since $c_1+c_2+c_3=80$ so $\boxed{12.5, 12.5, 12.5}$ No improvent.

$\therefore$ $20-x+80-3x=70 \quad x=7.5 \quad y_0=c_0-x=20-7.5=12.5$

$\boxed{\text{original}}$ agent = 1.

$x_1 = 30-7.5=22.5$.

$\boxed{\text{restricted}}$ ~~$S_0 = 0.8-a-c$~~ $S_1 = 0.3\times4=1.2$ $S_0+S_2+S_3=2.8$ same above

~~S_2~~

~~S_3~~

x still $=7.5$ $\quad x_0 = 20-7.5=12.5$

$\boxed{\text{unbounded}}$

$a+b+c=3.2$

~~$S_0=0.8 \quad c_1 = 25(a+1.2)$~~

~~$S_1 = a+1.2 \quad c_2 = 25(b+1.2)$~~

~~$S_3 = c+0.8 \quad c_3 = 25$~~

$$\left\{\begin{array}{ll} S_0 = 0.8 & c_0 = 20 \\ S_1 = a+1.2 & c_1 = 25(a+1.2) \\ S_2 = b+1.2 & c_2 = 25(b+1.2) \\ S_3 = c+0.8 & c_3 = 25(c+0.8) \end{array}\right.$$

$20+25(a+b+c) = 175 \quad 175-4x=70 \quad x=20-7.5=12.5$.

So 12.5, 12.5, 12.5 no improvement.

CEL. extreme skew. $E=70$. $C=100$ $[0.7.\ 0.2.\ 0.05.\ 0.05]$

(agent = 0)

$\boxed{\text{original}}$ $70+20+5+5-4\lambda=70$ $\lambda=7.5$ $C_2=C_3=5<7.5$ X

so only agent 0 and 1. $(70-\lambda)+(20-\lambda)=70$.

$\lambda=10$ $x^{original}=(60,10,0,0)$

$\boxed{\text{Restricted}}$

$a_1+a_2+a_3=0.3$

$$\begin{cases} S_1 = a_1+0.6 \\ S_2 = a_2+0.15 \\ S_3 = a_3+0.15 \end{cases} \quad \begin{cases} C_1 = 25(a_1+0.6) \\ C_2 = 25(a_2+0.15) \\ C_3 = 25(a_3+0.15) \end{cases}$$

$\because \max(70-\lambda)$ $a_2=0.3$ $a_1=a_3=0$

$C_1=15$ $C_2=11.25$ $C_3=3.75$

$(70-\lambda)+(15-\lambda)+(11.25-\lambda)=70 \Rightarrow \lambda=\frac{96.25-70}{3}=8.75.$

$x_0=70-\cancel{8.75}\ 8.75=61.25$ $x_0=61.25>60$. ↑

$\boxed{\text{unbounded}}$ $\sum_{j=0}^{3}(C_j-\lambda)=70 \Rightarrow \lambda=7.5$

$[0.7,a_1,a_2,a_3]$ $S_1=S_2=S_3=0.4$ $x_0=C_0-\lambda=70-7.5=62.5$ ✓

$\boxed{60 \to 61.25 \to 62.25}$

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agent = 1 $x=10$ $y=1$ the as previous.

$\boxed{\text{restricted}}$ $C_1=22.5$ $C_2=C_3=3.75$. only 0, 1 positive $(70-\lambda)+(22.5-\lambda)=70$ $\lambda=11.25$ $x_1=22.5-11.25=\boxed{11.25}$

$\boxed{\text{unbounded}}$ $\cancel{(70-\lambda)+(30-\lambda)}$ $\lambda=\frac{100-70}{4}=7.5$

$x_1=C_1-\lambda=20-\lambda=\boxed{12.5}$ If $\lambda=\frac{70+C_1+C_j-70}{3}=\frac{20+C_j}{3}<7.5$.

$\therefore C_j<2.5$ However $C_j>\lambda$ No improvement. $\boxed{10 \to 11.25 \to 12.5}$

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agent = 2 all 0. $7.5>5$, so only agent 0 and 1 are positive

while $70-x+20-x=70$ $x=10>\lambda\ 5$. Not work

$\boxed{\text{Restricted}}$ $x_{min}=\frac{C_0+C_1+C_3-70}{3}\geq\frac{70+20-70}{3}$ No improvement.

$\boxed{\text{unbounded}}$ $\cancel{x\leq 70}$ manipulation spare $0.95\times 4=3.8$

$C_{1max}=20+25\times 3.8=115$ $70-x+115-x=70$ $x=57.5$ $x_2=5-57.5\leq 0$

so no reward.