End-to-end Fair Division for the CHIPS Act

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ABSTRACT

RH 0.1 ≪**Richard 0.1:** ANSON TODO≫ We propose a series of with provable fairness guarantees. We also provide empirical evidence of

KEYWORDS

Fair division, resource allocation, impartiality, claims problems

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1 INTRODUCTION

≪**Richard 1.1:** 8 PAGES, Abstract submission: Oct 01, 2025 Paper
RH 1.1 submission: Oct 08, 2025≫

RH 1.2 ≪**Richard 1.2:** ANSON TODO≫

≪**Richard 1.3:** ANSON GET PERSONAL COMMUNICATION REFERENCE, THEY SHOULD MOTIVATE THE PROBLEM WE'RE
RH 1.3 SOLVING≫

Fair resource allocation is a foundational, ubiquitous problem. Government organizations, philanthropists, disaster relief organizations, and many other entities of various sizes continually face difficult decisions aobut how to effectively, efficiently, and fairly distribute a scarce resource among those in need. However, despite the potentially massive scope of these division scenarios (e.g., the Infrastructure Investment and Jobs Act (IIJA) of 2021 allocated \$1.2T for transportation and infrastructure spending [6]), they almost never use theoretically-robust tools from fair division and instead rely on ad-hoc approaches to parcel out resources. This is because fair division is not a "one-size-fits-all" approach: State-of-the-art fair division processes are insufficiently flexible to accommodate domain-specific constraints (e.g., legal safeguards, higher-order policy objectives, or particular types of strategic behavior) and potential disagreements about what type of "fairness" to prioritize.

Motivated by the compelling use case of fairly dividing budget allocations among electronic design automation (EDA) companies under the CHIPS Act, we propose a multi-step fair division process that first empowers stakeholders to define relevant fairness desiderata, after which tools from mechanism design may be deployed to design and implement a principled and transparent fair division mechanism for the relevant large-scale, real-world application.

≪**Richard 1.4:** SHOULD CITE SOME MEDIA ABOUT THE
RH 1.4 CHIPS ACT HERE TOO≫ The CHIPS and Science Act of 2022 has allocated tens of billions of dollars toward semiconductor research and workforce training. A sizable portion of awarded funding thus far has been allocated to *hubs* based at universities or public-private entities, where each hub pursues many individual *projects*. Additionally, each project relies on a collection of *tools* provided by chip design software *companies* like Cadence and Synopsys. However, most notably in the DoD Microelectronics Commons (now in its second year), because only academic institutions, not the established companies that develop crucial tools used for research, receive funding, this creates an incentive for companies to "get their share of the pie" by drastically increasing the prices of their tools for academics. In other words, by artificially inflating prices, companies are able to, in effect, secure a portion of CHIPS funding for themselves.

This behavior by companies in turn causes tensions among projects at each hub who compete for resources (i.e., licenses to tools) provided by companies. Ideally, each hub would be able to truthfully elicit tool needs from each of its projects, take the union of these requests (e.g., in a certain hub H, if project P_1 needs 4 copies of tool T_1, project P_2 needs 3 copies of tool T_1, and project P_3 needs 6 copies of tool T_1, the hub H will request 13 copies of tool T_1), and submit this list to the companies. However, because tool prices are so high, the hub may not have enough budget to buy licenses for all the requested tools. This naturally would lead to a model in which hubs request too few tools for all projects to use at once, meaning projects must share tools by checking them out from a shared library. Under strategic considerations, this results in suboptimal strategies such as over- or under-reporting the number of licenses a project needs, and projects hogging tools – all of which leads to inefficiency and delayed progress on projects.

1.1 Our Contributions

We consider the following questions.

(1) How can we incentivize hubs to truthfully report the tools needed for projects? ≪**Richard 1.5:** is this question still relevant after appendicizing the hubs problem≫ RH 1.5
(2) How can we fairly divide resources among companies?

We seek to address both of these problems by using tools from the field of *mechanism design*, which seeks to design rules for "games" in which individually rational and strategic agents are incentivized to behave in a way that benefits society as a whole. In particular, we propose separate interventions to address each problem. To incentivize projects to truthfully report tool needs, we propose a rental model in which projects must pay a higher cost to use tools they do not initially request. To fairly divide resources among companies, we propose a fair division scheme that draws on literature from impartial dollar division and claims problems in economics in order to directly distribute each hub's budget for tools among companies in a way that bypasses company-specific prices for tools.

≪**Richard 1.6:** INTRODUCE THE FORMAL PROBLEM HERE≫ RH 1.6

≪**Richard 1.7:** A REITERATION OF THE PERSONAL COMMUNICATION HERE≫ RH 1.7

Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), C. Amato, L. Dennis, V. Mascardi, J. Thangarajah (eds.), May 25 – 29, 2026, Paphos, Cyprus.

1.2 Related Work

RH 1.8 ≪**Richard 1.8:** (1) CHIPS WORK (2) MECHANISMS FOR MULTIPLE PARTY PROBLEMS LIKE THIS (3) IMPARTIAL MECHANISMS AND APPLICATIONS OF TALMUD RULE≫

2 PRELIMINARIES

Let $[n] := \{1, \ldots, n\}$ for all $n \in \mathbb{N}$.

2.0.1 Hubs and Companies. We assume a collection of h different *hubs*, $H_1, \ldots, H_h$, where hub H_i consists of p_i different *projects*, $P_i^1, \ldots, P_i^{p_i}$. There are also n different *companies*, $C_1, \ldots, C_n$, and each company C_j provides t_j different *tools*, $T_j^1, \ldots, T_j^{t_j}$. Let $t_{tot} = \sum_{j=1}^n t_j$ be the total number of distinct tools offered by all companies.
RH 2.1 ≪**Richard 2.1:** are these notations necessary?≫

Each project requires some number of copies of each type of tool to be completed. Let $a(i, j) \in \mathbb{N}^{t_{tot}}$ represent the true vector of demands for project p_i^j for all $i \in [h]$, $j \in [p_i]$, i.e., $a(i, j)$ is the minimum tool demand needed to complete project p_i^j to specifications.

Additionally, we assume that each hub H_i has a budget B_i to spend on tools.

RH 2.2 *2.0.2 Pipeline.* ≪**Richard 2.2:** ANSON TODO≫ We assume the following sequence of events occurs independently for each hub H_i, for all $i \in [h]$.

(1) Each hub H_i asks its projects to provide tool demands. These tool demands are then aggregated and made available to the companies supplying the tools. This aggregation may be done at the hub level or across all hubs; our pipeline works for both cases.
(2) The companies then collectively decide how to split H_i's (insufficient) tool budget, B_i. Note that here, they directly divide the budget among themselves without explicitly considering per-tool prices. In general, an EDA company will not reveal its "price book" to a competing company. So, it is reasonable to assume that companies do not know any per-tool prices other than their own. They also would not in general know how many licenses are requested (whether per project, per Hub, or in total) for any competitor's tool.

Our process for dividing awards among the companies is a two-step process directly applying two well-known division rules for different settings. We use the first rule to determine how much each agent is entitled towards out of a value larger than the total award amount, and the second rule determines how much each agent is actually allocated. We describe them in detail below.

2.0.3 Impartial Credit Division. In this problem, we seek to divide a value of 1 among $n \geq 4$ agents[1], who each have an *evaluation profile* of how much of the item they believe *every other* agent deserves. Specifically, each agent i submits a profile $r^i \in \mathbb{R}^n_{\geq 0}$ such that $\sum_{j\in[n]} r^i_j = 1$ where $r^i_i = 0$ and $r^i_j \in [0, 1]$ for every i, j[2].

In [3], they propose a division rule that considers each agent's evaluation of the allocations that other agents should receive, and divides the entire dollar and maintains *impartiality*: no agent reports anything about their own allocations, and their report has no effect on their own allocation, i.e. their allocation is determined solely by the other agents' reports.

Definition 2.1. An impartial division rule f for a credit division problem with evaluation profiles $r := \{r^i\}_{i\in[n]}$ is a function that assigns r a vector $f(r) \in \mathbb{R}^n_{\geq 0}$ such that for any pair of inputs and all i, we have that if for all $j \neq i$, in both inputs j has the same reported value, then f assigns i the same share in both inputs.

Perhaps the simplest rule they propose that obtains impartiality as well as a number of other natural properties is the *arithmetic mean aggregator division rule*[3].

Definition 2.2 (Arithmetic mean aggregator). An arithmetic mean m-aggregator is the function $\rho(a_1, \ldots, a_m) = \frac{1}{m}\sum_i a_i$.

Definition 2.3 (Arithmetic mean division rule). Let $\rho(r)$ be the arithmetic mean of the ratios r, and define the arithmetic mean m-aggregator division rule f as follows:

$$f_i(r) = \frac{1}{n}\Big(1 - \sum_{j\neq i} \frac{1}{1+\rho(r_{ij}) + \sum_{k\in N\setminus\{i,j\}} \rho(r^{-i}_{kj})}\Big) + \frac{1}{n}\sum_{j\neq i} \frac{1}{1+\rho(r_{ji}) + \sum_{k\in N\setminus\{i,j\}} \rho(r^{-j}_{ki})}$$

We implement our full pipeline using the arithmetic mean division rule above in our empirical study in Section 5.

2.0.4 Claims Problem. In the claims problem, an estate $E \in \mathbb{Z}^+$ must be fairly allocated to a set of n agents, who each have an entitlement d_i such that $D := \sum_{i\in[n]} d_i > E$. In [1], they defined the contested garment rule and proved that it always returns the *nucleolus* of a corresponding coalitional game that is derived by the claims problem. Note that it is generally NP-hard to compute the nucleolus of a cooperative game. Thus, the contested garment rule gives a natural solution for allocation problems where the total amount entitled by the agents exceeds the amount that can be allocated. However, their study does not delve into the case of agents misreporting their debts (because in these bankruptcy situations the debts are typically already well-documented and cannot be easily misreported).

Definition 2.4. An allocation rule $g(d, E)$ for a claims problem with estate E and entitlements $\{d_i\}_{i\in[n]}$ is a function that outputs a vector $g(d, E) \in \mathbb{R}^n_{\geq 0}$ such that $\sum_{i\in[n]} g_i(d, E) = E$.

We formally describe the contested garment rule. We first describe two subroutines by the contested garment rule, known as constrained equal awards (CEA) and constrained equal losses (CEL). In CEA, each agent starts with 0 and "eats" the estate E at an equal rate and stops eating once they've eaten their entitled amount, or if no more estate remains. CEL is a dual procedure, in which each agent starts with their entitled amount and "reverse-eats" until they hit 0 or the total allocation drops from D to E.

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[1] We may assume we're dividing 1 up to normalization.

[2] In [3] they define evaluation profiles slightly differently: each agent i submits a profile of ratios $r^i \in \mathbb{R}^{(n-1)\times(n-1)}$ where $r^i_{jk} \geq 0$ and $r^i_{jk} r_{kl} r_{lj} = 1$ for all $j, k, l \in [n-1]$. This definition is *consistent* in that $r_{jk} r_{kj} = 1$, i.e. if agent i believes j deserves c times as much of the share as k, then k deserves $1/c$ times as much as j. It is easy to see that such an evaluation profile of the cardinal value form induces one of the ratio form, and vice versa up to normalization.

[3] This is known to be used in real-world applications e.g., Spliddit [4].

Let $d = \{d_i\}_{i\in[n]}$. We define $\text{CEA}(d, E) := \{a_i\}_{i\in[n]}$ and $\text{CEA}(d, E)_i = a_i$ where a_i is the allocation of agent i by the CEA procedure given input d, E, and define $\text{CEL}(d, E)$, $\text{CEL}_i(d, E)$ analogously. The pseudocodes for CEA and CEL are given below.

Algorithm 1 Constrained equal awards and losses

```
Constrained equal awards.
Agent i is active and a_i ← 0 ∀ i ∈ [n].
while E > 0 do:
    A ← number of currently active agents
    a_i ← a_i + ϵ for all active agents i.
    E ← E − ϵA
    Deactivate each agent j such that a_j = d_j.
Return a := {a_i}_{i∈[n]}.

Constrained equal losses.
Agent i is active and a_i ← d_i ∀ i ∈ [n].
while D > E do:
    A ← number of currently active agents
    a_i ← a_i − ϵ for all active agents i.
    D ← D − ϵA
    Deactivate each agent j such that a_j = 0.
Return a := {a_i}_{i∈[n]}.
```

One perspective of the contested garment rule is to run either CEA or CEL depending on if $E \le D/2$ or not. If $E \le D/2$, then we run CEA with $(\{d_i/2\}_{i\in[n]}, E)$ as input; otherwise, we first allocate $d_i/2$ to agent i for all i, and then run constrained equal losses with $(\{d_i/2\}_{i\in[n]}, E - D/2)$ as input. We denote the output of the contested garment rule as $\text{CG}(d, E)$, $\text{CG}_i(d, E)$.

Hence, returning to our original pipeline, we are given a set of evaluation profiles r from the n agents (companies), a division rule f, and an estate E and total entitlement D. We obtain an entitlement list d which is obtained by setting $d_i = f_i(r) \cdot D$, and we return the allocation given by $\text{CG}(d, E)$.

2.0.5 Roadmap. ≪**Richard 2.3:** Roadmap the paper eg In section 3 we show how to…≫ RH 2.3

3 STRATEGYPROOF HUB REPORTING

RH 3.1 ≪**Richard 3.1:** ANSON TODO; APPENDICIZE≫ Our first insight is that we can examine strategyproofness for step 1 in the pipeline independently from step 2 in order to incentivize projects to truthfully report their tool demands.

We propose the following *rental model.*

- An exogenous source provides a menu of estimated prices per tool, i.e., each tool T_j^k has an estimated price $\pi(T_j^k)$ for all $j \in [m]$ and $k \in [t_j]$. These prices are not directly what each tool costs, but could be derived from prior academic license costs or estimates from a domain expert. Let the entire estimated price vector be denoted π.
- Based on these prices, each project creates a list of tools they need, and they contribute the estimated cost of all of these tools to the hub's tool rental pot, i.e., project P_i^j contributes $a(i, j) \cdot \pi$ for all $i \in [h]$ and $j \in [p_i]$.
- Each project is then able to use all of the tools they request for free. If they need additional tools, they pay strictly more than the estimated price for each tool they use but did not initially request, i.e., to use additional tool T_j^k, a project pays $(1 + \varepsilon)\pi(T_j^k)$ for some $\varepsilon > 0$.

THEOREM 3.1. *The rental model is strategyproof on the part of projects.*

PROOF. We show that each project individually is incentivized to report truthfully. For a specific project, there are two cases for untruthful behavior.

- If a project under-reports their tool needs, then they will have to rent tools at a higher rate than if they reported correctly.
- If a project over-reports their tool needs, then they end up paying for tools they don't need to use, which is worse than reporting correctly.

Therefore, reporting truthfully is the best strategy for each project. □

One note: We assume that $D \gg B$ no matter how many tools projects request, i.e., the total demand D of all companies outweighs the total tool budget B of each hub. ≪**Richard 3.2:** insert explanation why this assumption makes sense≫ RH 3.2

4 IMPARTIAL BUDGET DIVISION

≪**Richard 4.1:** RICHARD TODO≫ RH 4.1

We will combine concepts from the impartial division of a dollar [3] and division rules for bankruptcy problems [1].

The high-level intuition of our approach is as follows.

(1) We first leverage impartial division of a dollar to get companies to decide how much of the budget each of their competitors deserves based on the tool demands from projects.
(2) Then, we use the contested garment division described above to divide up the (insufficient) money among companies. This achieves a solution concept known as the nucleolus in the economic literature.

The first step is *impartial* in that no agent can change her own entitlement based on the information she reports. However, an agent i *can* influence any other agent's entitlement, and this *can* influence i's resulting allocation after running the contested garment rule. We seek to bound the amount any agent can benefit from such misreports.

One may ask why we do not just run the impartial division of a dollar algorithm over the budget B (or equivalently scale down their claims over D down to B). Indeed, this would result in payments to companies that add up to B, and each company would not be able to influence their own share. However, even in the case where all the companies unanimously agree on what share of the pie each of them deserves, the solution resulting from scaling down their claims to B may not be consistent with the actual claims they deserve. For instance, consider a case with four companies, $B = 100$, and $D = 400$, where companies 1, 2, and 3 deserve 50 and company 4 deserves 250. If we scale down each company's debt such that the sum of debts is B, then companies 1, 2, and 3 get 12.5 and company 4 gets 62.5. However, the consistent solution requires that all four companies should be paid 25 and company 4 should not be able to muscle the other three companies out of the way.

To distinguish between the two steps of the mechanism, we will generally refer to the first step as the *division* step where we *assign* entitlements to agents, and the second step as the *allocation* step where we *allocate* money to agents based on their entitlements.

For the rest of this section we analyze how much an agent can change their resulting allocation if they misreport their input, under different impartial rules for the division step. We first provide our definition of the *manipulability* of an allocation rule g that uses a division rule f to decide the entitlement report d.

Definition 4.1. The **additive manipulability** for agent i of an allocation rule g whose entitlement is decided by an impartial division rule f is defined as

$$\max_{r,r' \in \mathbb{R}^{n\times n}} g_i(f(r') \cdot D, E) - g_i(f(r) \cdot D, E)$$

where $r'_i \neq r_i$, that is, r' is an entitlement report where agent i's row is manipulated.

4.1 Unrestricted Impartial Manipulation

As a baseline, we study the worst-case additive manipulability if an agent is allowed to arbitrarily impact the debts of others, but not itself (preserving impartiality). For simplicity, we may assume i's share is initially fixed to some "true" value, but i completely dictates the remaining shares in the credit division.

Definition 4.2 (Impartial dictator rule). Given an evaluation profile $r \in \mathbb{R}^{n\times n}$ and a fixed value $x \in [0, 1]$, the impartial dictator-i rule $f^i(r)$ assigns the vector whose i^{th} entry is x and j^{th} entry is $(D - x)r^i_j$ for every $j \neq i$ to r.

It is easy to see that the above rule is an impartial division rule. We analyze CEA and CEL independently on the impartial dictator rule.

Lemma 4.3. *The additive manipulability of* $\mathrm{CEA}(\mathrm{dict}^i(r) \cdot D, E)$ *is at most* $\frac{E}{2} - \frac{E}{n}$.

Proof. It suffices to examine the setting where we fix an agent i whose entitlement d_i is fixed but can manipulate the remaining $\{d_j\}_{j\neq i}$ such that $\sum_{j\in[n]} d_j = D$ is maintained. The constrained equal awards rule can be equivalently defined to allocate $\min(d_i, s)$ to i for every i where s is the constant satisfying $\sum_{i\in[n]} \min(d_i, s) = E$.

We may view s as the number of rounds of increments until all agents are deactivated. Then for agent i to be allocated d_i we require $d_i \leq s$ and otherwise agent i is allocated s. So we want to maximize s, which is done by minimizing the number of agents that are incremented in each round: hence the allocation for agent i is maximized by assigning 0 to $n - 2$ other agents and $k := D - d_i$ to one other agent, effectively reducing the instance to a two-agent instance. If $d_i \leq k$ then agent i will be allocated $\min(d_i, E/2)$, because either both agents increment to $E/2$ or i drops out before round $E/2$. Otherwise agent i will be allocated $\max(E - k, E/2)$ by the same reasoning. Hence, in this case, the allocation that i gets can be represented by the following function:

$$\mathrm{CEA}_i(d, E) = \begin{cases} d_i & d_i \in [0, E/2) \\ E/2 & d_i \in [E/2, D - E/2] \\ E - k & d_i \in (D - E/2, D] \end{cases} \tag{1}$$

On the other hand, agent i's allocation is minimized by maximizing the number of agents with entitlement E/n with respect to having fixed d_i, as this maximizes the number of agents incremented in each round. In this case we have:

$$\mathrm{CEA}_i(d, E) = \begin{cases} d_i & d_i \in [0, E/n) \\ E/n & d_i \in [E/n, D - (n-1)E/n] \\ E - k & d_i \in (D - (n-1)E/n, D] \end{cases} \tag{2}$$

The largest gap between these functions is when $d_i \in [E/2, D - (n-1)E/n]$, in which the gap is exactly $E/2 - E/n$. □

Lemma 4.4. *The additive manipulability of* $\mathrm{CEL}(\mathrm{dict}^i(r) \cdot D, E)$ *is at most* $\frac{D-E}{2} - \frac{D-E}{n}$.

Proof. We can analyze the setting where we fix an agent i whose entitlement d_i is fixed, but they can manipulate the remaining entitlements. Like in CEA case we may equivalently view the constrained equal losses rule $\mathrm{CEL}(d, E)$ to allocate $\max(0, d_i - s)$ to agent i for every $i \in [n]$ where s is the constant such that $\sum_{i\in[n]} \max(0, d_i - s) = E$, and we can view s as the number of rounds of *decrements*.

As we must make a total of $D-E$ decrements across the s rounds, then s is minimized when we maximize the number of decrements per round, which is at most n. Hence $\frac{D-E}{n} \leq s \leq D-E$. We consider the following cases:

(1) $d_i \leq \frac{D-E}{n}$: since $s \geq \frac{D-E}{n}$ as above, agent i will always get 0 no matter how they manipulate the entitlements.
(2) $\frac{D-E}{n} < d_i \leq \frac{D+(n-1)E}{n}$: then the maximum manipulability is achieved by agent i setting all other agents to at least $\frac{D-E}{n}$, which achieves the minimum $s = \frac{D-E}{n}$. It is always possible to set the other $n-1$ agents to at least $\frac{D-E}{n}$ in this case since $D - d_i \geq D - \frac{D+(n-1)E}{n} = (n-1)\frac{D-E}{n}$.
(3) $d_i > \frac{D+(n-1)E}{n}$: observe in this case agent i is "too rich" to set every other agent to at least $\frac{D-E}{n}$. Then the maximum manipulability is achieved by agent i setting all other agents to at most $d_i - E$. To prove this, let $s^* := (D-E) - (D-d_i) = d_i - E$. We show that s^* is the minimum possible number of rounds of decrements in this case. Indeed, the sum of all decrements for each agent is exactly $D-E$, and agent i needs to distribute $D - d_i$ of the estate among the other agents. To minimize s, we want to minimize the number of agents that are allocated a strictly positive amount after s rounds. Then ideally, all other agents get decremented to 0 after the s rounds, so agent i still needs to be decremented at least $(D-E) - (D-d_i)$ times. Next, we show that $s = s^*$ if and only if all other agents are entitled at most s^*. Since $s \geq s^*$ from the previous claim, it suffices to show $s \leq s^*$. We prove the first direction by contrapositive, specifically that if there exists a j with $d_j > s^*$, then $s > s^*$. In this case, we have that

after s^* rounds of decrements, the total amount decremented across all agents is

$$\begin{aligned} 2s^* + (D - d_i - d_j) &< 2d_i - 2E + D - d_i - d_i + E \\ &= D - E \end{aligned}$$

where we used the assumption that $d_i \geq d_j > s^* = d_i - E$. Hence after s^* rounds, the total decremented amount has not yet reached $D - E$, so $s > s^*$. To prove the second direction, observe that since all other agents besides agent i are entitled at most $s^* \leq s$, they must be decremented to 0 after s rounds. So by round s we have made $D - d_i$ total decrements across all other agents. Then there are $(D - E) - (D - d_i) = s^*$ decrements that must be shouldered by agent i. This requires at least s^* rounds of decrements, and hence $s \leq s^*$. It remains to show that it is always possible to set all other agents to at most s^* in this case. We have

$$\begin{aligned} d_i > \frac{D + (n-1)E}{n} &\implies nd_i > D + (n-1)E \\ &\implies D < nd_i - (n-1)E \\ &\implies D - d_i < (n-1)(d_i - E) \end{aligned}$$

Hence, the remaining $D - d_i$ can be distributed among the other $n - 1$ agents such that each agent receives at most $d_i - E$.

The "worst case" of entitlements for agent i is when $\frac{D-E}{2} \leq d_i \leq \frac{D+(n-1)E}{n}$ and one other agent j is entitled $D - d_i$ and all other agents are entitled 0. Here agent i gets $d_i - \frac{D-E}{2}$, but can manipulate the entitlements to give all other agents at least $\frac{D-E}{n}$, in which case it gets $d_i - \frac{D-E}{2}$, achieving the maximum manipulability of $(d_i - \frac{D-E}{n}) - (d_i - \frac{D-E}{2}) = \frac{D-E}{2} - \frac{D-E}{n}$. □

Since contested garment first allocates half of everyone's entitlements and then runs $\text{cel}(E - D/2, d/2)$, the manipulability is not affected in this instantiation of CEL in the contested garment allocation. Thus, we have the following corollary:

Corollary 4.5. *The additive manipulability of* $H(\mathit{dictator}, \text{cg}, E, R)$ *when* $E \leq D/2$ *is at most* $\frac{E}{2} - \frac{E}{n}$, *and at most* $\frac{D-E}{2} - \frac{D-E}{n}$ *when* $E \geq D/2$.

4.2 Restricted Impartial Manipulation

Next, we study how using an impartial credit division mechanism to determine "fair" entitlements before running the contested garment allocation algorithm affects the worst-case additive manipulability. Each agent i submits an entitlement report $(\cdot)$ such that $\sum_{j\neq i} d_j = D$, where they do not report their own entitlement (so the entitlement report is length $n - 1$).

We first prove two structural lemmas on the worst-case types of manipulation possible under a wide range of impartial credit division rules.

Lemma 4.6 (Best manipulation, CEA). *When running* CEA, *for any agent i, the entitlement report* $(0, 0, \ldots, 0, D)$ *maximizes i's allocation.*

Proof. Let agent j be the richest agent besides i initially. If there exists a better manipulation that is not $(0, \ldots, 0, D)$ then there exists some agent k with a non-zero reported entitlement x by i that we can set to 0, and give x to j. We only care about what happens to j, since lowering an agent k can only help i.

If j is already above i, then of course they were always eating with i for the entirety of i's eating period. Hence agent i strictly benefits by eating $x/(c-1)$ where c is the number of agents richer than i.

If j is initially below i, then i must be the richest. Either j is below or equal i after receiving x from k, or it is strictly richer. For the former case we have two more sub-cases. Firstly, if i and j initially stopped eating at the same time, then by the same reasoning as above i strictly benefits. Otherwise, i ate longer than j initially, so j will begin to eat during time steps in which i was not eating together with j. Nonetheless, i's eating time is never decreased because if that were the case then the sum of resulting allocations would be strictly less than E, since the only change was the symmetric change in j, k's entitlement. To be extra clear, note that the sum of allocations is at most i's new eating time (i.e. i's allocation) plus $d_j + x$ plus the new remaining allocation, and $d_j + x$ plus the new remaining allocation must be equal to d_j plus the initial remaining allocation; therefore in order to still sum to E it must be that i's new eating time is at least equal to its initial eating time. This holds even when $d_j + x = d_i$.

Finally, in the case that $d_j + x > d_i$, i.e. j is now above i, observe i's allocation is unchanged by how long j eats after i drops out. So this is identical to the case when $d_j + x = d_i$. □

Lemma 4.7 (Best initial structure, CEA). *When running* CEA, *fixing* E, n, d_i *(but not* D*), the entitlement* $\mathcal{A} := (E/n, E/n, \ldots, E/n, d_i)$ *maximizes the additive manipulability for i.* ≪**Richard 4.2:** formalize this statement≫ RH 4.2

Proof. Observe that if $d_i \leq E/n$, then agent i is allocated d_i regardless of the other entitlements, so we may WLOG assume $d_i > E/n$. Furthermore, any agent that wants to maximize additive manipulability will use the best manipulation as defined in Lemma 4.6. Hence it suffices to only analyze what the best manipulation does to instances where $d_i > E/n$.

For some agent i with entitlement d_i, let $\text{BM}_f(d_i)$ denote the new entitlement of agent i using division rule f after applying the best manipulation. When the rule f is obvious we may drop the subscript.

Clearly, in the initial contested garment allocation for $\mathcal{A}$, every agent gets E/n. After the best manipulation, d_i is unchanged, $d_j = E/n + \delta(n-2)$ for exactly one $j \neq i$ and $d_k = E/n - \delta$ for the remaining $k \neq i$. Let us define $\text{pg}_i^A := \sum_k d_k^A - \text{BM}(d_k^A)$ where the sum is over $k \neq i$ such that $d_k^A > \text{BM}(d_k^A)$. In the resulting allocation, agent i receives at most $\min(d_i, E/n + \text{pg}_i^{\mathcal{A}}/2)$ because by definition of $\text{BM}(d_j)$, agent j will never be deactivated while there is remaining estate to be eaten. In the case that agent i receives d_i due to the impartial rule we use, then $\mathcal{A}$ is clearly the initial entitlement that maximizes additive manipulability, because agent i gets at least E/n and at most d_i across any entitlement.

It remains to prove the claim for the case where agent i receives less than d_i after the best manipulation in $\mathcal{A}$. Observe that we may further WLOG assume that ≪**Richard 4.3:** fix this notation≫ RH 4.3 $d_{raise} > E/n$, as otherwise all other agents have entitlement at most E/n, in which we can simply apply extended monotonicity to get that agent i's gain from any of the $n-2$ lowered agents is at most that of in $\mathcal{A}$. Then observe that the total sum of entitlements of the lowered agents in $\mathcal{B}$ is strictly less than $E - \frac{E}{n} - \frac{E}{n} = (n-2)\frac{E}{n}$ since agent i does not eat from itself or *raise*, and i and *raise* both receive strictly more than E/n in the initial entitlement. Hence on average, a lowered agent j in $\mathcal{B}$ has $d_j < E/n$. Then we have

$$\sum_{j \in [n-2]} M(d_j) \leq (n-2)M(\sum_{j \in [n-2]} \frac{d_j}{n-2})$$
$$\leq (n-2)M(\frac{E}{n})$$

where the first line is due to Jensen's inequality and the concavity of rule M, and the second is by the average d_j being strictly less than E/n and the nondecreasingness of M.

This bounds the total gain across all lowered agents. At best, agent i splits this total gain equally between itself and *raise* after the best manipulation, which is what happens in $\mathcal{A}$. So since the total gain after best manipulation of $\mathcal{B}$ is strictly less than that of $\mathcal{A}$, agent i's individual gain must be less. □

Putting these two lemmas together implies that for any ≪**Richard 4.4:** concave?≫ rule, the maximum manipulability is achieved under the initial structure A with the manipulation A. ≪**Richard 4.5:** need to add something saying that the actual impartial rule we use cannot lower the $n-2$ E/n's by x much $\implies$ additive manip is at most because from E/n to $E/n + x(n-2)/2$ so ideally $x \leq O(1/n)$ to have $O(1)$ manip≫

RH 4.4

RH 4.5

We use the duality of CEA and CEL, specifically **??**.

Lemma 4.8 (Best manipulation, CEL). *When running* CEL, *for any agent i, the entitlement report* $(\frac{D}{n-1}, \frac{D}{n-1}, \dots, \frac{D}{n-1})$ *maximizes i's allocation.*

Proof. ≪**Richard 4.6:** maximize the number of people eating with you for as long as possible. basically anyone who is too high is wasting their excess?≫

RH 4.6

Without loss of generality we may assume that $d_i > \frac{D-E}{n}$. Clearly we must minimize $\mathrm{CEA}(d, \sum d - E)$. Similarly to before, this will be accomplished when maximizing the number of agents that "eat" alongside agent i. To argue this formally, ≪**Richard 4.7:** my problem with this is is setting everyone equal actually going to bring everyone closer to equal in reality or should you e.g., give all the poorer people than you as much money as possible and richer people as little as possible. doesn't this become super rule dependent?≫ □

RH 4.7

Lemma 4.9 (Best initial structure, CEL). *When running* CEL, *the entitlement* $\mathcal{A} := (0, 0, \dots, 0, d_i, D - d_i)$ *maximizes the additive manipulability for i.* ≪**Richard 4.8:** formalize this statement≫

RH 4.8

Proof. □

4.3 Impartial Ranking

In the previous sections, we combined impartial credit division rules with the contested garment rule, allowing agents to gain from misreporting by changing entitlements of other agents. In this section we propose an impartial mechanism by using impartial ranking with the contested garment rule. We first fix an entitlement list $\{x_1, x_2, \dots, x_n\}$ where we use an impartial ranking algorithm to decide which agent is assigned index. Then we run the contested garment rule as before with these entitlements. Specifically, the agents still report evaluation profiles like before, and we transform these into rankings as input for the impartial ranking rule. Impartial ranking rules that satisfy other desirable properties are known to exist and have been studied previously, see e.g. [2, 5].

Observe that because we fix the entitlement list and then assign entitlements to agents, no agent can gain from misreporting after applying the contested garment rule on the resulting entitlement list. Indeed, no matter what an agent reports, the resulting instance of the contested garment rule is identical in their perspective, since they cannot manipulate their own rank (and thus their own entitlement) by impartiality, and manipulating anyone else's rank doesn't change the entitlement list by our fixing of the entitlement list.

It remains to design a good entitlement list. However, in order to retain the impartiality that we discussed in the previous paragraph, the values $\{x_i\}_{i \in [n]}$ must not be functions of the actual input evaluation profiles. Hence the choices for entitlement lists are (essentially) only limited to functions of n. Trivial options satisfying the criterion include the uniform vector $\{1/n, 1/n, \dots, 1/n\}$ and a geometric sequence $\{1/2, 1/4, \dots, 1/2^{n-1}, 1/2^{n-1}\}$ (note the last term is also $1/2^{n-1}$ so that the sum of all values is 1).

While there is no relation between these choices of $\{x_i\}_{i \in [n]}$ and the input evaluation profiles, we can still choose $\{x_i\}_{i \in [n]}$ based on a new criteria: how much an agent can (multiplicatively) manipulate the gain/loss of *another* agent. For the uniform vector this is 1, and for the geometric sequence this is at most 2^k if k is the maximum number of positions an agent can move another agent in the ranking by changing their input evaluation profile. To get polynomial manipulability on k, we propose a generalized harmonic sequence $\{\frac{1}{H_{n,p}}, \frac{1}{2^p H_{n,p}}, \dots, \frac{1}{n^p H_{n,p}}\}$ where $H_{n,p} := \sum_{i \in [n]} \frac{1}{i^p}$ is the generalized harmonic number on n vertices and degree p. It is easily verified that the most an agent can multiplicatively manipulate the gain/loss of another agent with this entitlement list is at most $(k+1)^p$.

The choice of p here allows for some flexibility by the mechanism designer: for example, if they desire the maximum entitlement to be 1/2, they can choose $p := p(n)$ that satisfies it. Similarly, they can even choose p such that the maximum entitlement is equal to the maximum entitlement that the impartial credit division algorithm returns on the input evaluation profiles, but this turns the entitlement list into a function of the evaluation profiles.

5 EXPERIMENTAL SETUP

This section specifies the datasets, design, and evaluation protocol used to study manipulation under two bankruptcy rules, *Constrained Equal Awards* (CEA) and *Constrained Equal Losses* (CEL). All definitions are stated for a general number of agents n. Our experiments instantiate the pipeline with $n = 4$, but every step extends verbatim to $n = 5, 6, 7$ or larger.

≪**Anson 5.1:** Experiments to include: average manipulability for both CEA and CEL; it'd be nice to have one combined graphic. Extensions to more than 4 agents.≫ AK 5.1

5.1 Research Questions

We study how a potential manipulator's *average payoff* varies with the estate across three treatments: a baseline with no manipulation, a restricted manipulation channel, and an unrestricted manipulation channel. We also ask whether CEA and CEL differ in their sensitivity to manipulation when evaluated on the same scenarios, estate grids, and numerical settings; answers come from computing payoffs at each estate level, aggregating across scenarios and over the identity of the manipulator, and comparing the resulting estate–payoff curves.

5.2 Synthetic Scenarios

Each run begins with a scenario vector $v \in \mathbb{R}^n_{\geq 0}$ whose entries sum exactly to 1.0000. A diverse collection of 16 such vectors provides variation in how weight is distributed across agents. For any scenario v, the truthful report matrix is

$$R_{\text{truth}} = \mathbf{1}\, v^\top \in \mathbb{R}^{n \times n},$$

i.e., n identical rows equal to v. Estate sizes are varied on fixed grids that expose changes in the sets of binding constraints. For CEA we evaluate $D \in \{15, 20, 25, 30, 35, 40, 45\}$; for CEL we evaluate $D \in \{60, 65, 70, 75, 80, 85, 90, 95\}$. Manipulation search is performed on a discrete grid with resolution $\texttt{SCALE} = S = 100$, so one grid unit corresponds to $1/S = 0.01$. The manipulator's diagonal entry remains fixed at its truthful value $f \in [0, 1]$; the residual mass $1 - f$ can be redistributed subject to a per-recipient floor $\alpha(1 - f)$ with $\alpha = 0.01$ (implemented as *at least* one grid unit on the S=100 grid).

5.3 Manipulation Treatments

For each estate, each scenario, and each candidate manipulator $m \in \{1, \ldots, n\}$, we compute three payoffs.

Baseline (no manipulation). We use the truthful consensus matrix R_{truth}. We multiply initial claims by a fixed scale $S = 100$ to express shares in claim units. The allocation rule (CEA or CEL) is then applied, and the manipulator's payoff is her award.

Restricted manipulation. The manipulator may alter *only the off–diagonal entries of her own report row*; her self entry is held fixed at its truthful value. The remaining mass in that row is redistributed across the other $n-1$ columns on the $S = 100$ grid, and each recipient must receive at least a lower bound equal to one percent. The row must continue to sum to one. After the manipulation, the entire matrix is passed through an impartial-division routine that maps reports to a share vector with nonnegative components summing to one. These shares, scaled by $S = 100$, define the claims passed to the allocation rule,either CEA or CEL. The restricted payoff for agent m is the *maximum* award it can obtain over all feasible redistributions of her row that satisfy the lower bound and grid constraints.

Unrestricted manipulation. The manipulator may alter *any entries in the report matrix except her own self entry*; manipulation may occur in other agents' rows as well, subject to each manipulated row remaining a probability vector on the $S = 100$ grid (nonnegative and summing to one). No impartial division is applied in this treatment. The unrestricted payoff is the *maximum* award achievable over all feasible edits.

In both manipulation treatments the space of permissible edits is finite on the $S = 100$ grid, and we enumerate it deterministically; numerical tolerances and solver settings used by CEA and CEL are kept identical across treatments.

5.4 Allocation Rules

Both rules map a claims vector $c \in \mathbb{R}^n_{\geq 0}$ to awards $a \in \mathbb{R}^n_{\geq 0}$ that sum exactly to the estate D, solved with the same numerical routine. ≪**Richard 5.2:** these definitions may be a little redundant as we've already defined them earlier, also would need to state that this is equivalent to the formulation of cea/cel that we use earlier≫ RH 5.2

Constrained Equal Awards (CEA).. Awards cap each claim at a common ceiling λ:

$$a_i = \min\{c_i, \lambda\}, \qquad i = 1, \ldots, n,$$

where λ is chosen to satisfy the mass-balance equation

$$\sum_{i=1}^{n} \min\{c_i, \lambda\} = D.$$

Constrained Equal Losses (CEL).. Awards subtract a common loss λ and truncate at zero:

$$a_i = \max\{c_i - \lambda,\ 0\}, \qquad i = 1, \ldots, n,$$

with λ chosen to satisfy

$$\sum_{i=1}^{n} \max\{c_i - \lambda,\ 0\} = D.$$

Numerics. In both cases λ is found by bisection with 60 iterations and tolerance 10^{-12}. When necessary a proportional adjustment is applied so that $\sum_i a_i = D$ to machine precision. Using an identical iteration budget and tolerance for CEA and CEL ensures observed differences reflect rule structure rather than solver artifacts.

5.5 Search Procedure and Fixed Parameters

Let n be the number of agents, and let the manipulator be m. We work on a discrete grid with $\texttt{SCALE} = S$ units (in our runs $S = 100$) and a per–recipient floor equal to αS units (in our runs $\alpha = 0.01$, so the floor is 1 unit). Denote by $k = n - 1$ the number of *other* recipients in the manipulator's row. After reserving the floor of αS for each of the k recipients, the number of *free* units is

$$U = S - (\alpha S)\, k.$$

Every feasible manipulation corresponds to a weak composition of U into k parts. By the stars-and-bars formula, the number of candidate rows we must evaluate is

$$N_{\text{cand}} = \binom{U + k - 1}{k - 1}.$$

Under our default settings $S = 100$ and $\alpha = 0.01$, the floor is 1 unit and thus $U = 100 - k$. Hence

$$N_{\text{cand}} = \binom{100 - k + k - 1}{k - 1} = \binom{99}{k - 1}.$$

For example, when $n = 4$ ($k = 3$) we have $N_{\text{cand}} = \binom{99}{2} = 4{,}851$; when $n = 6$ ($k = 5$), $N_{\text{cand}} = \binom{99}{4} = 3{,}764{,}376$; when $n = 7$ ($k = 6$), $N_{\text{cand}} = \binom{99}{5} = 75{,}287{,}520$. Each candidate row is evaluated once (restricted: with impartial division; unrestricted: directly), and the

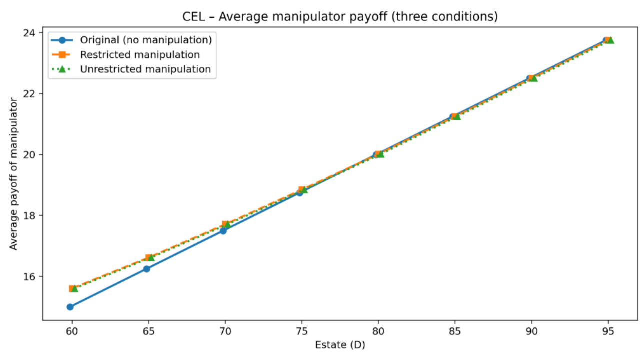

Figure 1: CEL – Average manipulator payoff

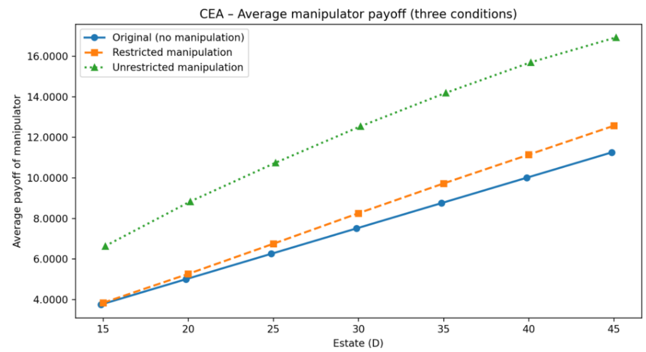

Figure 2: CEA – Average manipulator payoff

allocation rule (CEA or CEL) is solved by bisection with 60 iterations and tolerance 10^{-12}. We apply no rounding anywhere, preserving differences at the 0.001 level.

Remark (alternative parameterization). If one prefers to reason in terms of a smaller *residual* unit budget U_0 (e.g., $U_0 = 60$ units) after fixing the manipulator's diagonal entry and before floors are applied, then the free units are $U^\star = U_0 - (\alpha S)\,k$, and the count becomes

$$N_{\text{cand}} = \binom{U^\star + k - 1}{k - 1}.$$

For instance, with $U_0 = 60$ and $\alpha S = 1$, we obtain $N_{\text{cand}} = \binom{59}{2} = 1{,}711$ for $n = 4$, $\binom{59}{4} = 455{,}126$ for $n = 6$, and $\binom{59}{5} = 5{,}006{,}386$ for $n = 7$.

5.6 Result Analysis

CEL plot. The figure plots the *average payoff of the manipulator* on the vertical axis against the estate D on the horizontal axis. The horizontal axis is labeled "Estate (D)" with ticks at 60, 65, 70, 75, 80, 85, 90, 95, and the vertical axis is labeled "Average payoff of manipulator." Three series appear on the same axes: the *original* (no manipulation) treatment is a blue solid line with circle markers, the *restricted* manipulation treatment is an orange dashed line with square markers, and the *unrestricted* manipulation treatment is a green dotted line with triangle markers. The three curves rise almost perfectly linearly. At the low end there is a small manipulation premium: at $D = 60$ the original line is about 15.0 while both manipulation lines are about 15.6; at $D = 65$ the original is about 16.2 and the manipulation lines are about 16.6; and at $D = 70$ the original is about 17.5 while the manipulation lines are roughly 17.7. From $D = 75$ onward the curves are essentially coincident—around 18.8 at $D = 75$, 20.0 at $D = 80$, 21.3 at $D = 85$, 22.5 at $D = 90$, and 23.8 at $D = 95$—with differences on the order of a few hundredths at most. In short, under CEL the effect of manipulation is small at very low estates and vanishes as the estate increases. ≪**Richard 5.3:** this figure may be okay but maybe we want a result that shows more
RH 5.3 obvious separation between restricted and unrestricted?≫

CEA plot. The plotting conventions are identical, with "Estate (D)" on the horizontal axis taking values 15, 20, 25, 30, 35, 40, 45 and "Average payoff of manipulator" on the vertical axis. Here the three lines are clearly separated across the full range. At $D = 15$ the original line is about 3.7, the restricted line (after impartial division) is about 3.8, and the unrestricted line is about 6.6. At $D = 25$ the original rises to about 6.3, the restricted to about 6.8, and the unrestricted to about 10.7. At $D = 35$ the values are roughly 8.7 for the original, 9.7 for the restricted, and 14.2 for the unrestricted. At the top of the range, $D = 45$, the original, restricted, and unrestricted lines are about 11.2, 12.6, and 16.9, respectively. The gaps therefore expand with the estate: the restricted-minus-original difference grows from roughly 0.1 at $D = 15$ to about 1.4 at $D = 45$, and the unrestricted-minus-original difference grows from about 2.9 to about 5.7 over the same interval. All three CEA curves are close to linear in D, but the slopes differ: the unrestricted line increases fastest, followed by the restricted line, and then the original line.

Comparison. Looking across the two plots, the takeaway is straightforward. Under CEL, the three lines—original, restricted, and unrestricted—almost sit on top of one another. Any edge from manipulating is tiny on average and essentially disappears once the estate passes about 75. Under CEA, by contrast, the gaps never close. The unrestricted line stays highest and pulls away as the estate grows, while the restricted line still sits above the original even after impartial division. All curves appear almost straight because, over wide ranges of the estate, awards rise at a nearly constant rate; under CEA, however, this pattern produces a clear and widening gap between the manipulation curves—especially the unrestricted case—and the no-manipulation baseline, and that gap remains visible across the estate range.

6 CONCLUSION

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