BLEKINGE INSTITUTE OF TECHNOLOGY

Written test in (subject): ET2594 Capacity Analysis

Date: 2019.06.13

Name: ____

Civic number: ____________________

Number of sheets handed in:________

Mark the question(s) you have answered by putting a ring around the relevant number(s)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Instructions
A student who cannot produce valid ID will not be permitted to take the examination.
No examination scripts will be accepted by the proctor during the first hour of the examination. (Students arriving late will thus be permitted to take part in the examination).
Write your name and civic number on each sheet of paper you hand in.
Examination results are posted by e-mail no later than 10 working days after the date of the examination. Exceptions to this rule can occur. In this case, students will be informed by the teacher responsible for the course/program or by the examiner.
All blank answer sheets are to be handed in to the proctor.

(To be filled in by the proctor)

ID presented: ________
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Number of credits gained:_____ Grade:_____ ECTS:_____Examiner´s sign:_______

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I hereby sign my examination script. I am aware that by signing for my script, after correction, I waive my right to contest the examiner's comments and the credits or grade awarded.

Date______________ Signature:_____________________________________

RE-Exam In

Capacity Analysis
(2019.06.13) ET2594

Wednesday: 09:00 to 14:00

Lecturer: Siamak Khatibi

Allowed items on exam: Open book

The exam includes 5 problems (100 credit points); where for grade in ECTS you should obtain as following:

F (0-29), FX (30-49),
E (50-59), D (60-69),
C (70-79), B (80-89),
A (90-100)

Good Luck

PS: Each question is answered by a mfile using Matlab (i.e. Q1sol.m, Q2sol.m, …). In the header of each mfile you write your name and personal number. In each mfile beside your code for solution of the respective problem you should write your comments/arguments. You can write/draw on the paper as complementary material to your respective mfile (please mention in mfile you have such complementary).

You should zip all your digital materials (your mfiles related to each question, other used functions which are not standard Matlab functions, mat file, …) and upload it to the Canvas under modul "exam", in "Download your answer to REexam 2019-06-13". You deliver your complementary materials to the invigilator of exam.

Q1

Near BTH in Karlskrona there is a LIDL food shop. A group of students tried to model the queuing system of the shop. They observed that assuming a M/M/1 system model was appropriate for the purpose. They assumed also the arrival intensity and the service intensity were λ and μ respectively. Here we try to investigate the validity of their assumptions. We know that the shop is relatively small and the space for the queue length is limited. We call this limitation as L.

a) Draw a state diagram for the LIDL queuing system. (4p)

b) Calculate P_k (the probability of the having a que length of k) expressed in λ, μ and L. (4p)

c) Calculate the minimum L expressed in λ, μ and P_α where the P_α is the probability that an arriving customer is rejected; i.e. $P_{L+1} \leq P_\alpha$. (4p)

d) Calculate L for the system values according to the following table. (4p)

case	λ	μ	P_α
1	1	2	0.05
2	2	3	0.05
3	3	5	0.05
4	4	6	0.05
5	5	8	0.05
6	10	20	0.05
7	20	30	0.05
8	50	70	0.05
9	100	120	0.05

e) Plot the calculated L values from d) in relation to the used load values. (2p)

f) Comment/argue about the suggested model in relation to the 9 cases in d). (2p)

Q2

In a student project a Pizzeria restaurant (close to BTH campus) was observed and its servicing system was modelled as a M/M/1 system. The restaurant had/has a small place for serving the customers. The students noticed that each time the restaurant server (i.e. the guy behind the desk) has emptied the system (i.e. there were no other customers), he took a pause. The pause lasted an exponentially distributed time with mean of $1/\gamma$ and after that the servicing worked as usual. During the pause, all incoming customers were rejected. Assuming the arrival intensity and the service intensity are λ and μ respectively, here we try to understand the restaurant queuing system more in depth.

a) Define states and draw a state diagram. (5p)

b) Determine the state probabilities. (10p)

c) If P_{pause} (the probability of pause) is 0.05, P_0 (the probability of no customer) is 0.5, P_1 (the probability of having one costumer) is 0.75 and the service intensity of μ is 3, find γ and λ. Comment/argue your results. (5p)

Q3

Please use Q3.m in this problem. You have seen the code in assignment 9 which simulate a M/M/1 queuing system.

A) Use the code and write a Matlab function which makes it possible to simulate a M/M/1 system with respect to certain load value (ρ) and calculate the response time (the time each customer spends in the system). (5p)

B) Use your function in A) and calculate the **mean** response time when the arrival intensity λ is either 5 and 10 for ρ values of 0.25, 0,5, 0.7, 0.9, 1.0, and 1.1. Plot your results in relation to each λ in two figures. Comment your results. (8p)

C) Use your function in A) and calculate the response time when the arrival intensity λ is 5 for ρ values of 0.25, 0.9. For each ρ value, plot 1-100 of response time vector as a function of elements 2-101 by writing such as plot(resTime(2:101), resTime(1:100),'*'). From your results argue about the dependency between consecutive response times influenced by the system load. (7p)

Q4

From our observation of student's arrival to BTH, we have obtained the following model. If a student arrives at BTH at time X and a second student arrives at time Y, then

$$f_{X,Y}(x,y) = \begin{cases} \lambda^2 e^{-\lambda y} & 0 \le x < y \\ 0 & otherwise \end{cases}$$

where λ is the average number of students arrived per hour.

a) Suppose that the first student arrives 3 hours after we start watching.
i. Find the conditional PDF of the arrival time of the second student. (5p)
ii. Find the expected value of the arrival time of the second student. (5p)

b) Now suppose that the second student arrives at 5 hours.
i. Find the conditional PDF of the arrival time of the first student. (5p)
ii. Find the expected value of the arrival time of the first student. (5p)

Q5

In Karlskrona there is a cinema theater with 2 desks for serving the customers. A group of students observed the servicing system in the cinema and found out that they can model it as a M/M/2 * loss system. They observed as well that there are usually two types of customers who are attending the cinema, those who are loyal cinema visitors (VIP customers) and those who are visiting the cinema casually (New customers). One of the desks (server 1) gives services only to the New customers and if the server 1 is busy any new customers is rejected. The VIP customers are served by both desks (server 1 and server 2) and if both servers are free during an VIP arrival, server 2 is primary selected.
The New and VIP customers are arriving to the cinema with the intensities of λ_1 and λ_2. Both servers are giving services with intensity of μ.

a) Define states to describe the suggested system model by the students. (12p)

b) Assume having five cases as in following table, calculate the load on server 1, server 2 and the total load. (6p)

case	λ_1	λ_2	μ
1	1	2	1
2	2	4	10
3	5	10	15
4	10	15	20
5	20	20	30

c) In a figure using bar function in Matlab, show your results from b). (2p)

Appendix: formulas

1. Probability theory

Moments

(Discrete) (Continuous)

$$E[X] = \sum_{k=0}^{\infty} k p_k \qquad E[X] = \int_0^{\infty} x f(x) dx$$

$$E[X^2] = \sum_{k=0}^{\infty} k^2 p_k \qquad E[X^2] = \int_0^{\infty} x^2 f(x) dx$$

Variance

$$V[X] = E[X^2] - E^2[X]$$

Squared coefficient of variation

$$C^2[X] = \frac{V[X]}{E^2[X]}$$

Deterministic distribution

$$E[X] = C, \quad E[X^2] = C^2$$

Exponential distribution

$$f(t) = \mu e^{-\mu t}$$

$$E[X] = \frac{1}{\mu}, \quad Var[X] = \frac{1}{\mu^2}, \quad E[X^2] = \frac{2}{\mu^2}$$

Poisson distribution

$$P(k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

Total probability theorem

$$P(H) = \sum_i P(H \mid A_i) P(A_i)$$

General Bayes' rule

$$P(A_k \mid H) = \frac{P(A_i \cap H)}{P(H)} = \frac{P(H \mid A_k) P(A_k)}{\sum_i P(H \mid A_i) P(A_i)}$$

2. Sums

$$\sum_{k=0}^{N} \alpha^k = \frac{1-\alpha^{N-1}}{1-\alpha} \quad \alpha \neq 0$$

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha} \quad |\alpha| < 0$$

$$\sum_{k=0}^{\infty} \frac{\alpha^k}{k!} = e^a$$

$$\sum_{k=0}^{\infty} k\alpha^{k-1} = \frac{\partial}{\partial \alpha}\left(\sum_{k=1}^{\infty} e^k\right) = \frac{1}{(1-\alpha)^2}$$

$$\sum_{k=0}^{N} \binom{N}{k} a^k b^{N-k} = (a+b)^N$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad n > 0, r \geq 0$$

3 Queueing Theory

General

$T = T_Q + T_S$ total response time in the system, including waiting in queue and service time

$N = N_Q + N_S$ number of customers in the steady-state system, including the ones waiting in queue and the ones being served

Little's theorem

$E[T] = E[T_Q] + E[T_S]$ average total waiting time in the steady-state system

$E[N] = E[N_Q] + E[N_S]$ average number of customers in the steady-state system

$E[N] = \lambda_{eff} E[T]$ Little's formula

$E[N_Q] = \lambda_{eff} E[T_Q]$ Little's formula

λ_{eff} effective arrival rate

M/M/1-system

$$\rho = \frac{\lambda}{\mu}$$

$$p_k = \rho^k (1 - \rho)$$

$$E[N] = \frac{\rho}{1 - \rho}$$

$$E[T] = \frac{1/\mu}{1 - \rho}$$

$$E[T_s] = \frac{1}{\mu}$$

M/M/m-system

$$\rho = \frac{\lambda}{m\mu}$$

$$p_0 = \left[\left(\sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\right) + \left(\frac{(m\rho)^m}{m!}\right)\left(\frac{1}{1-\rho}\right)\right]^{-1}$$

$$p_k = \begin{cases} p_0\dfrac{(m\rho)^k}{m!} & k \le m \\ p_0\dfrac{(\rho)^k m^m}{m!} & k \ge m \end{cases}$$

$$E[N] = m\rho + \rho\frac{(m\rho)^m}{m!}\frac{p_0}{(1-\rho)^2}$$

M/M/m-loss system

$$p_0 = \left[\sum_{k=0}^{m}\left(\frac{\lambda}{\mu}\right)^k \frac{1}{k!}\right]^{-1}$$

$$p_k = \begin{cases} p_0\left(\dfrac{\lambda}{\mu}\right)^k \dfrac{1}{k!} & k \le m \\ 0 & k > m \end{cases}$$

$$p_m = \frac{(\lambda/\mu)^m / m!}{\sum_{k=0}^{m}(\lambda/\mu)^k / k!}$$ Erlang’s B formula

M/M/1//C finite population, single server case

$$p_0 = \left[\sum_{k=0}^{C} \left(\frac{\lambda}{\mu} \right)^k \frac{C!}{(C-k)!} \right]^{-1}$$

$$p_k = \begin{cases} p_0 \left(\frac{\lambda}{\mu} \right)^k \frac{C!}{(C-k)!} & k \le C \\ 0 & k > C \end{cases}$$

Jackson's technique

$$p(k_0, k_1, \ldots, k_n) = \prod_{i=0}^{n} p_i(k_i)$$

M/G/1-system

$E[S]$ average service time.

$$E[N] = \rho + \frac{\lambda^2 E[S^2]}{2(1-\rho)}$$ Pollaczek-Khinchines formula

For K different types of jobs

$$\rho_j = \frac{\lambda_j}{\mu_j}$$

λ_j: arrival rate of type jth jobs;

μ_j: service rate of type jth jobs

$$\lambda = \sum_{j=1}^{K} \lambda_j$$

$$\rho = \sum_{j=1}^{K} \rho_j$$

$$E[S^2] = \sum_{j=1}^{K} \frac{\lambda_j}{\lambda} E[S_j^2]$$

Priority-system: Head of line non preemptive (HOL-NP)

$E[S_j]$ average service time for class j jobs

$\overline{R}=\frac{1}{2}\sum_{j=1}^{K}\lambda_j E[S_j^2]$ expected residual service time of a job in service

K = number of priority classes

$E[T_1]=E[S_1]+\frac{\overline{R}}{1-\rho_1}$ mean time in the system for a class 1 job

$E[T_j]=E[S_j]+\frac{\overline{R}}{\left(1-\sum_{i=1}^{j-1}\rho_i\right)\left(1-\sum_{i=1}^{j}\rho_i\right)}$ mean time in the system for a class j job

$j=2,3.,..,K$

Priority-system: Preemptive resume

$E[S_j]$ average service time for class j jobs

$\overline{R}_{0j}=\frac{1}{2}\lambda_j E[S_j^2]$ expected residual service time of a class j job in service

$j=1,2,...,K,$ K = number of priority classes

$E[T_1]=E[S_1]+\frac{\overline{R}_{01}}{1-\rho_1}$ mean time in the system for a class 1 job

$$E[T_j]=\frac{1}{\left[1-\sum_{i=1}^{j-1}\rho_i\right]}\left[E[S_j]+\frac{\sum_{i=1}^{j}\overline{R}_{0i}}{\left(1-\sum_{i=1}^{j}\rho_i\right)}\right]$$ mean time in the system for a class j job

$j=2,3.,..,K$