Strategic Incentives in Large Markets

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Outline

1. Incentive Compatibility of Large Centralized Matching Markets - Lee
   - Motivation
   - Results
   - Discussion
2. Optimal Truncation in Matching Margets (Coles and Shorrer GEB)
   - Matching markets
   - Current Work and Previous Work
   - Model
   - The Individual Agent Problem
   - Optimal Behavior - Single Agent
   - The Preference List Submission Game
3. Comparative Statics
   - Safety of Truncation
   - Risk Aversion
   - Correlated Preferences
   - Equilibrium and Welfare

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The Small Core Literature

- Roth Peranson (AER 1999)
  - Deals with the redesign of the medical match
- Motivation: crisis of confidence concerning whether the matching algorithm was unreasonably favorable to employers at the expense of applicants

Short Lists

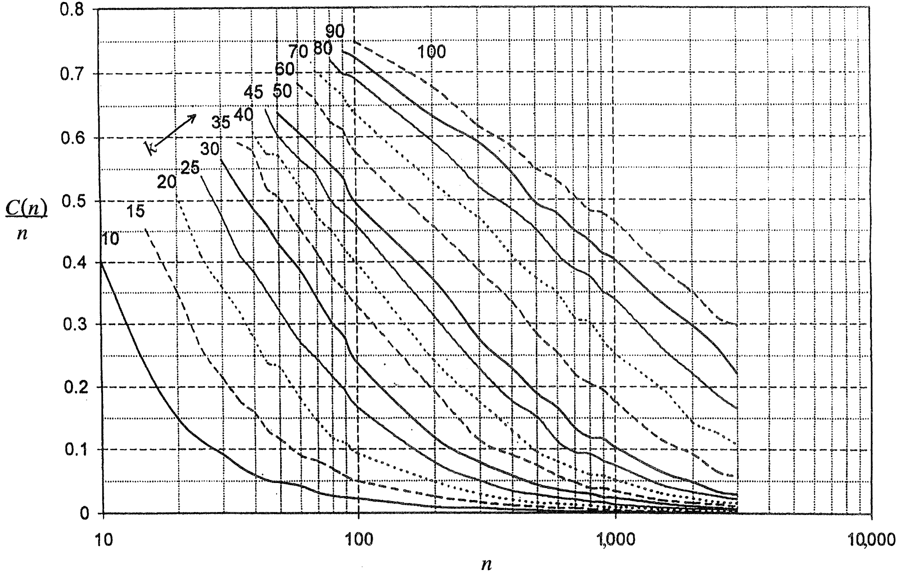

FIGURE 2. SIZE OF THE SET OF STABLE MATCHINGS AS A FRACTION OF n FOR DIFFERENT VALUES OF k (UNCORRELATED PREFERENCES)

Notes: $C(n)$ is the number of applicants who get different stable matches, when the market size is n; k is the number of

Long Lists

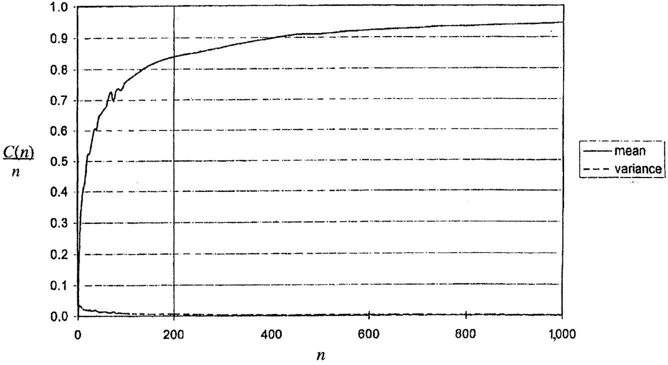

FIGURE 1. SIZE OF THE SET OF STABLE MATCHINGS AS A FRACTION OF n, WHEN $k = n$ (UNCORRELATED PREFERENCES)

Note: $C(n)$ is the number of applicants who get different stable matches, when the market size is n.

The Small Core Literature (contd.)

- Immorlica Mahdian (2005)
  - Roth and Peranson's computational experiment is correct (when one side has short lists)
- Kojima, Pathak (AER 2009)
  - Also in the many to one setting
- Pittel, Knuth
  - with long lists there are many matches (expected number super-linear in n)
  - men and women ranks are "far" under DAA (sum of ranks $n \log n$ vs. $\frac{n^2}{\log n}$ - with $p \xrightarrow[n\to\infty]{} 1$)

The Principle of Deferred decisions

- We can generate preferences "on the run" only when needed
  - That is, generate the next woman on a man's list only when he is rejected
  - When multiple offers are at hand, break the tie.
- This makes the "random markets" simpler
  - So we can use the probabilistic method

Example

- Based on Pittel 89', which is used in the proof of the main result.
- In the uniform private value setting, prove that the sum of ranks of men under the men proposing algorithm, r_n, is at most $(1+\varepsilon)n\log n$ with high probability for large n.
- At each round, pick a single man, choose at random (uniformly) a woman which he has not yet proposed to.
  - She is his next "pick." Make him propose to her
- The woman accepts the man with probability $\frac{1}{m}$, where m is the number of men who proposed to her (including).
- Slight adaptation: allow men to also draw previously visited women - they are then rejected with probability 1.
  - This only increases the number of offers men make

Example (contd.)

- The sum of men's ranks are less then the number of offers they make.
- This is a coupon collector problem, known to have expectation nH_n, $H_n = 1 + \frac{1}{2} + ... + \frac{1}{n}$
- Let A be the event that a certain woman has not been proposed to in the first $(1+\varepsilon)\, n \log n$ steps
  - This is the only way that the algorithm runs longer than $M = (1+\varepsilon)\, n \log n$
- $Pr\left\{r_n < (1+\varepsilon)\, n \log n\right\} \geq 1 - nPr\left\{A\right\} = 1 - n\left(1 - \frac{1}{n}\right)^M \geq 1 - n\exp\left(\frac{-M}{n}\right) = 1 - \frac{1}{n^{\varepsilon}} \to 1$

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Setting

- 1-1 matching markets with n men and n women
- Object of interest - stable matching
  - We know that if all men prefer stable matching μ to μ', women have the reverse preferences
  - There exists a best and worst stable matching for each gender, μ_W and μ_M
- Assumptions on utility: (the strong set of assumptions in this paper)

$$U_{f,w} = U\left(C_w, \zeta_{f,w}\right)$$

$$V_{f,w} = V\left(C_f, \eta_{f,w}\right)$$

- Same utility functions, continuous and strictly increasing (or just common/private value)
  - common and private values for partners.
  - bounded support
  - Implicit assumption U is bounded on the domain

Setting

- Positive density for common values (no tiers)
- bounded private values, i.i.d (no dependence on common part, no "geography")
- Same distribution for every n
- All matches are preferred to staying unmatched

Main Results

- Observation: Fix others' submitted lists. Under complete information, no woman can get a better match than μ_W and no man can get better than μ_M under any stable matching mechanism.
- This means that the maximal gains from manipulation for an agent can be bounded by the utility difference between the best and worst matches.

Definition

$A_F(\varepsilon; U, V)$ is the set of firms for which the firm optimal match, and the worker optimal match differ by less than ε.

Theorem

For every $\varepsilon > 0$, $\mathbb{E}\left[\frac{|A_F(\varepsilon;U,V)|}{n}\right] \underset{n\to\infty}{\longrightarrow} 1$.

Main Results (contd.)

- The proof of the pure common values case is easy (only one stable matching)
- Let's look at the pure private values case:
  - Provides good intuition to what happens within a "tier"
- First, it is enough to show that the fraction of firms with "bad" worst match rank converges to 0.
- Pittel showed that the sum of worst ranks: $\left(\frac{n^2}{\log n}\right)^{-1} \sum R^F_{\mu W} \to 1$
- Denote by $\bar{B}(\gamma_n; U, V)$ the set of firms with worst stable partner ranked worst than $\gamma_n n$
- Fix a market instance. $\frac{\gamma_n}{n}\left|\bar{B}\left(\gamma_n, u, v\right)\right| \leq \frac{1}{n}\sum_{f\in\bar{B}}\frac{R^F_{\mu W}}{n} \leq \frac{1}{n}\sum_{f}\frac{R^F_{\mu W}}{n}$
- $\frac{\left|\bar{B}(\gamma_n,u,v)\right|}{n} \leq \sum_f \frac{R^F_{\mu W}}{n^2} \times \frac{1}{\gamma_n} = \sum_f \frac{R^F_{\mu W}}{n^2(\log n)^{-1}} \times \frac{1}{\gamma_n \log n} \underset{p}{\to} 0$
  - when we choose $\gamma_n = \frac{1}{\log\log n}$.

Intuition

- Pittel's well known results: there are many matches and the sum of best and worst ranks are $n \log n$ vs. $\frac{n^2}{\log n}$
- So, even in the worst matching the average rank is $\frac{n}{\log n}$.
- Bounding utilities (+continuity) means that this rank is in a percentile that goes to 0
  - However slowly.
- Then, show that this applies to most individuals (not just expectation)
- What would happen if we have two tiers of schools and two tiers of students with private values assuring that there are no inter-tier matches?

The General Case

- WLOG - uniform iid random variables.
- Lower bound:

Proposition

Fix $\varepsilon > 0$. *For every* $\bar{c} \in (0,1]$,
$\frac{1}{n}\left|\left\{f \in F \mid C_f \geq \bar{c} \text{ and } U_f^{\mu^W} \leq U(\bar{c},1) - \varepsilon\right\}\right| \xrightarrow{p} 0$ *as* $n \to \infty$

- Intuition: the matching is almost assortative in common values, so in large markets there is a large "competitive fringe" below every common value level (one use of the full support assumption).
- iid of private values implies that some partner from the fringe will be "perfect" in the private value component (and will consider the high partner attractive)

The General Case

- Step 1: Take $\hat{c} < \bar{c}$ and $\hat{\varepsilon} > 0$ such that

$$U(\hat{c}, 1-\hat{\varepsilon}) \geq U(\bar{c}, 1) - \varepsilon \ AND$$
$$V(\hat{c}, 1) < V(\bar{c}, 1-\hat{\varepsilon})$$

  - This means that $\bar{c}$ and $\hat{c}$ are close and $\hat{\varepsilon}$ is small
    - the second part is implicit in the paper, but is necessary for the next stage
- Take $\tilde{c} \in (\hat{c}, \bar{c})$ with $V(\bar{c}, 1-\hat{\varepsilon}) \geq V(\tilde{c}, 1)$
- This induces a partition into tiers: $(0, \hat{c})$ $(\hat{c}, \tilde{c})$ $(\tilde{c}, \bar{c})$ $(\bar{c}, 1)$

The General Case

- Step 2: inspect tier 1 of F, and tiers 1-3 of W
  - with the uniform distribution, and large n, this implies way more w's than f's.
  - Given a realization of common and private values

$$\bar{F}(c_F) = \{f \in F | c_f \geq \bar{c}\}, \;\; \bar{W}(c_W) = \{w \in W | c_w \geq \hat{c}\}$$

  - Also,

$$B_{\bar{F}}(c_F, c_W, \zeta, \eta) = \left\{ f \in \bar{F}(c_F) | \; u_f^{\mu^W} \leq u(\bar{c}, 1) - \varepsilon \right\}$$

$$B_{\bar{W}}(c_F, c_W, \zeta, \eta) = \left\{ w \in \bar{W}(c_W) | \; u_w^{\mu^W} \leq V(\tilde{c}, 1) \right\}.$$

- This is a set of firms in $\bar{F}$ that do badly at some stable matching, and the set of workers in $\bar{W}$ that do badly in all stable matching.

The General Case

- Step 3: construct a bipartite graph.
- $\bar{F}$ and $\bar{W}$ are the set of nodes.
- $f \in \bar{F}$ and $w \in \bar{W}$ are connected iff

$$\zeta_{f,w} \leq 1 - \hat{\varepsilon} \text{ or } \eta_{f,w} \leq 1 - \hat{\varepsilon}$$

- From independence we get that the probability of a potential edge is $1 - \hat{\varepsilon}^2$ and their occurrence is independent.

The General Case

- Observation: $B_{\bar{F}}$ and $B_{\bar{W}}$ are a bi-clique
- Proof: if $f \in B_{\bar{F}}$ and $w \in B_{\bar{W}}$ were not connected:

$$u_{f,w} = U(c_w, \zeta_{f,w}) > U(\hat{c}, 1-\hat{\varepsilon}) \geq U(\bar{c}, 1) - \varepsilon$$

$$v_{f,w} = V(c_f, \eta_{f,w}) > V(\bar{c}, 1-\hat{\varepsilon}) \geq V(\tilde{c}, 1)$$

- The last part is by choice of $\hat{c}, \hat{\varepsilon}, \tilde{c}$.
- This implies that the pair (f, w) blocks the matching μ^W - a contradiction.

Theorem

(Dawande, Keskinocak, Swaminathan, and Tayur (2001)). Consider a random bipartite graph $G(V_1 \cup V_2, p)$, where $0 < p < 1$ is a constant, $|V_1| = |V_2| = n$, and $\beta(n) = log(n)/log(\frac{1}{p})$. If a maximal balanced bi-clique of this graph has size $B \times B$, then $P(\beta(n) \leq B \leq 2\beta(n)) \to 1$, as $n \to \infty$.

- Conclusion: $\min\left\{\frac{1}{n}|B_{\bar{F}}|, \frac{1}{n}|B_{\bar{W}}|\right\} \xrightarrow{p} 0$ as $n \to \infty$
- But as we have more tiers of W, $B_{\bar{W}}$ must be large:

$$\frac{|B_{\bar{W}}|}{n} \geq \frac{|\{w \in W | C_w > \hat{c}\}|}{n} - \frac{|\{f \in F | C_f > \tilde{c}\}|}{n} \xrightarrow{p} \tilde{c} - \hat{c} > 0$$

- Choose $\varepsilon' < \frac{\tilde{c}-\hat{c}}{2}$

$$P\left(\frac{|B_{\bar{F}}|}{n} > \varepsilon'\right) \leq P\left(\min\left\{\frac{1}{n}|B_{\bar{F}}|, \frac{1}{n}|B_{\bar{W}}|\right\} > \varepsilon'\right) + P\left(\frac{1}{n}|B_{\bar{W}}| \leq \varepsilon'\right)$$

The General Case

Upper bound

Proposition

Fix $\varepsilon > 0$. For every $\bar{c} \in [0,1]$,
$$\frac{1}{n}\left|\left\{f \in F \middle| \ C_f \leq \bar{c} \text{ and } U_f^{\mu^F} \geq U(\bar{c},1)+\varepsilon\right\}\right| \xrightarrow{p} 0 \text{ as } n \to \infty$$

- The proof is just "accounting" given the lower bound
- Pick $\bar{\bar{c}} \in (\bar{c},1)$ with $U(\bar{\bar{c}},1) \leq U(\bar{c},1)+\varepsilon$
- $\left|\left\{f \in F \middle| \ C_f \leq \bar{c} \text{ and } U_f^{\mu^F} \geq U(\bar{c},1)+\varepsilon\right\}\right| \leq$
  $\left|\left\{w \in W \middle| \ C_w \geq \bar{\bar{c}} \text{ and } V_w^{\mu^F} \leq V(\bar{c},1)\right\}\right|$
- For every successful firm, there is a miserable worker.
- Use the lower bound

The General Case

Theorem

For any $\varepsilon' > 0 \quad P\left(\frac{1}{n}\left|\{f \in F \mid \Delta(f;U,W) > \varepsilon\}\right| > \varepsilon'\right) \to 0$

- Choose a large K such that $\frac{1}{K} < \varepsilon'$ and

$$\left|c - c'\right| \le \frac{1}{K} \Rightarrow \left|U(c,1) - U(c',1)\right| \le \frac{\varepsilon}{2}$$

- Claim:

$$\begin{aligned} &\left|\{f \in F \mid \Delta(f;U,W) > \varepsilon\}\right| \le \\ &\sum_{k \ge 1}\left|f \in F \mid C_f \ge \frac{k}{K} \text{ and } U_f^{\mu^W} \le U\left(\frac{k}{K},1\right) - \frac{\varepsilon}{4}\right| \\ &+\sum_{k \ge 1}\left|f \in F \mid C_f \le \frac{k}{K} \text{ and } U_f^{\mu^F} \ge U\left(\frac{k}{K},1\right) + \frac{\varepsilon}{4}\right| \\ &+\left|f \in F \mid C_f \le \frac{1}{K}\right| \end{aligned}$$

- 3 components: unlucky, lucky, and low common value.

The General Case

- Consider f with $c_f \in (\frac{k}{K}, \frac{k+1}{K})$
- $\Delta(f; u, w) > \varepsilon \Rightarrow u_f^{\mu^F} - u_f^{\mu^W} > \varepsilon \geq \left(U\left(\frac{k+1}{K}, 1\right) + \frac{\varepsilon}{4}\right) - \left(U\left(\frac{k}{K}, 1\right) - \frac{\varepsilon}{4}\right)$
- So f is in one of the sets corresponding to RHS. Finish by using the bounds.

Equilibrium

Theorem

For any $\varepsilon, \delta, \theta > 0$, there exists N such that with probability at least $(1-\delta)$ a market of size $n > N$ has an $\varepsilon-$Nash equilibrium in which $(1-\theta)$ proportion of agents reveal their true preferences.

- This is due to the positive spillover from truncation strategies
  - Truncation is crucial

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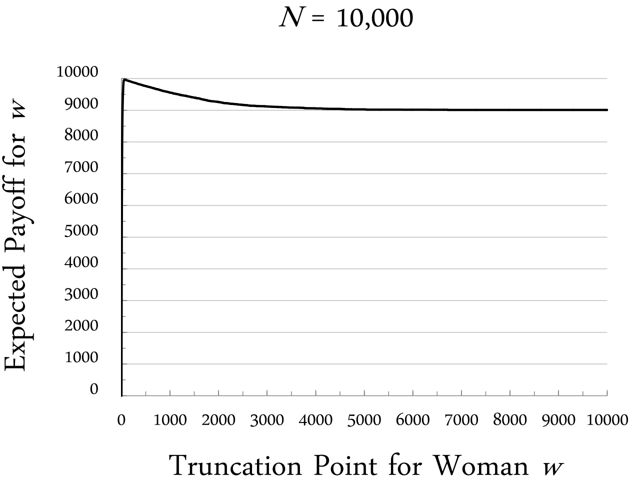
Discussion of Assumptions

- Bounded (and independent of n) utilities.
  - Both from below and from above
- Rate of convergence



Discussion of Assumptions

- Many **iid** random draws, including bilateral independence

Example

Consider the case of Harvard and Stanford, both preferring to hire students from the other institution, while students all prefer to stay in the same area.

Example

Consider a hospital with a generous support for families (or health plan) and interns with children (or sick interns).

- Dependence in private values

Discussion of Assumptions

Example

Schools are indifferent (or have tiers), everyone wants the same schools.

Discussion of Assumptions

- Timing - when do interviews happen?
- What is the real important friction?
  - we know that in many settings submitted lists are shorter than required

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Centralized Matching Markets

- National Resident Matching Program (NRMP).
- Public School System.
- Kidney Transplants.
- Do we observe strategic behavior?
  - Yes.

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Strategic Behavior

- Can't we design a mechanism that eliminates the need for strategizing?
- No! (Roth, 1982)
- But, we can make sure that for one side of the market, truth telling will be a dominant strategy.
  - This can be achieved using the Deferred Acceptance Algorithm (Gale and Shapley, 1962).

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Previous Work

- Roth, 1982. Dubins and Freedman 1981.
- Roth and Peranson 1999, Immorlica and Mahdian 2005, Kojima and Pathak 2009. Lee 2014.
- Pittel 1989,1992. Knuth 1976. McVitie and Wilson 1970.
    - Ashlagi Knoria Leshno 2013.
- Roth and Rothblum 1999. Ehlers and Masso 2007.

This Paper

Strategic behavior in the setting of the 1-1 market with incomplete information under the Gale-Shapley Algorithm.

1. **Characterization.**
   1. Optimal strategic misrepresentation for a participant in a benchmark, low information case.
   2. Equilibrium.
   3. Safety of truncation.
2. **Comparative statics.** How does optimal behavior change when the environment changes?
   1. Unbalanced markets.
   2. Risk aversion.
   3. Correlated preferences.
   4. Preferences between equilibria.

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Marriage Markets and Matching

- A *marriage market* consists of a triplet $(\mathscr{M}, \mathscr{W}, u)$.
  - When $|\mathscr{M}| = |\mathscr{W}| = N$ we say that it is a balanced marriage market of size N.
- *Preferences* for man $m \in \mathscr{M}$ are given by a *(1-1) von Neumann-Morgenstern* utility $u_m : \mathscr{W} \cup \{m\} \to \mathbb{R}$.
  - Similarly for women.
- u is the profile of preferences for men and women.
- A (deterministic) matching μ is a mapping from $\mathscr{M} \cup \mathscr{W}$ to $\mathscr{M} \cup \mathscr{W}$ such that:
  - For every $m \in \mathscr{M}, \mu(m) \in \mathscr{W} \cup \{m\}$.
  - For every $w \in \mathscr{W}, \mu(w) \in \mathscr{M} \cup \{w\}$.
  - For every $m, w \in \mathscr{M} \cup \mathscr{W}, \mu(m) = w$ if and only if $\mu(w) = m$.
- u_m induces the preference list $P_m := (w_1, w_4, m, w_2 \ldots)$ iff $u_m(w_1) > u_m(w_4) > u_m(m) > u_m(w_2) > \ldots$
  - We sometimes write $P_m = (w_1, w_4)$ and ignore the *unacceptable* women.
  - Similarly for women.

Stable Matching

Given a profile of preferences:

- Pair (m, w) <u>blocks</u> matching μ if m prefers w to $\mu(m)$ and w prefers m to $\mu(w)$.
- Matching μ is individually rational if $(\forall x)$ x weakly prefers $\mu(x)$ to x.
- Matching μ is stable if it is individually rational and has no blocking pair.

The Men-Proposing Deferred Acceptance Algorithm

- *MP-DA* takes as its input a preference list profile P for agents $\mathscr{M} \cup \mathscr{W}$, and the output is a matching $\mu^M[P]$.
  - When P is clear from the context, we write μ^M to denote $\mu^M[P]$.
- The algorithm works iteratively as follows:

Step 1. Each man proposes to his favorite woman.

Step 2. Women place best acceptable man "on a string," reject the others.

Step 3. Men not on a string propose to their second favorite woman.

...

Termination when all men are either on a string, or have exhausted their list of women. Tentative matches become final.

Properties of MP-DA

1. The MP-DA Algorithm yields a matching that is stable (with respect to reported preferences).
2. This matching is the optimal stable matching for men, in the sense that compared to any other stable matching, each man is at least as well off.
3. This matching is the worst stable matching for women. Each woman weakly prefers every other stable matching.

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The Preference List Submission Problem for Men

- Each $i \in \mathscr{M} \cup \mathscr{W}$ must submit a preference list $\hat{P}_i$ to *MP-DA*.
  - $\hat{P}_i$ is chosen from the set of i's possible preference lists $\mathscr{P}_i$.
- The agent's beliefs about what preference lists others will report are represented by the random variable $\tilde{P}_{-i}$.
- Agent i solves the *Preference List Submission Problem*:

$$\max_{\hat{P}_i \in \mathscr{P}_i} \mathbb{E}[u_i(\mu^M[\hat{P}_i, \tilde{P}_{-i}](i))].$$

Theorem

(Dubins and Freedman; Roth) In the Preference List Submission Problem, truthful reporting is an optimal strategy for men.

The Preference List Submission Problem for Women

Example

m_1	m_2	m_3	w_1	w_2	w_3
$\mathbf{w_3}$	$\mathbf{w_2}$	$\mathbf{w_1}$	m_1	m_1	$\mathbf{m_1}$
w_1	w_1	w_2	m_2	m_3	m_2
w_2	w_3	w_3	$\mathbf{m_3}$	$\mathbf{m_2}$	m_3

The Preference List Submission Problem for Women

Example

m_1	m_2	m_3	w_1	w_2	w_3
$\mathbf{w_3}$	$\mathbf{w_2}$	$\mathbf{w_1}$	m_1	m_1	$\mathbf{m_1}$
w_1	w_1	w_2	m_2	m_3	m_2
w_2	w_3	w_3	$\mathbf{m_3}$	$\mathbf{m_2}$	m_3

Example

m_1	m_2	m_3	w_1	w_2	w_3
$\mathbf{w_3}$	w_2	w_1	m_1	m_1	$\mathbf{m_1}$
w_1	$\mathbf{w_1}$	$\mathbf{w_2}$	$\mathbf{m_2}$	$\mathbf{m_3}$	m_2
w_2	w_3	w_3		m_2	m_3

Example

m_1	m_2	m_3		w_1	w_2	w_3
$\mathbf{w_3}$	$\mathbf{w_2}$	$\mathbf{w_1}$		m_1	m_1	$\mathbf{m_1}$
w_1	w_1	w_2		m_2	m_3	m_2
w_2	w_3	w_3		$\mathbf{m_3}$	$\mathbf{m_2}$	m_3

Example

m_1	m_2	m_3		w_1	w_2	w_3
$\mathbf{w_3}$	w_2	w_1		m_1	m_1	$\mathbf{m_1}$
w_1	$\mathbf{w_1}$	$\mathbf{w_2}$		$\mathbf{m_2}$	$\mathbf{m_3}$	m_2
w_2	w_3	w_3			m_2	m_3

Example

m_1	m_2	m_3		w_1	w_2	w_3
$\mathbf{w_3}$	w_2	w_1		m_1	m_1	$\mathbf{m_1}$
w_1	w_1	$\mathbf{w_2}$			$\mathbf{m_3}$	m_2
w_2	w_3	w_3			m_2	m_3

The Truncation Problem

- Let P_w be the preference list for woman w.
- Let $k \in \{0, 1, 2..., N\}$. Define P_w^k to be the k-truncation of w's list.
- Woman w solves:

$$\max_{k \in \{0,...,N\}} \mathbb{E}[u_w(\mu^M[P_w^k, \tilde{P}_{-w}](w))].$$

- Shorthand:

$$\max_{k \in \{0,...,N\}} \mathbb{E}[v(k, \tilde{P}_{-w})].$$

The Truncation Problem

- In some cases, all we need to consider are truncation strategies.
  - Under 'symmetry' truncation is optimal (Roth and Rothblum '99).
- In general, detailed information is necessary to benefit from non-truncation strategies.
  - Truncation strategies, on the other hand, generate a simple trade-off.
- Computationally easy.
- Natural metric for the extent of manipulation.

Truncation and Achievable Mates

- Given a preference list profile P, man m is *achievable* for w if there is some stable matching μ in which m is matched to w.

Proposition

Let P be the preference list profile of all agents in $\mathscr{M} \cup \mathscr{W}$. Then $\mu^M[P_w^k, P_{-w}](w)$ is w's least preferred achievable mate under P with rank $\leq k$. Should no such mate exist, $\mu^M[P_w^k, P_{-w}](w) = w$.

Interval Decomposition

- If m_i is the last man on w's truncated preferences list:

$$\underbrace{m_1 \succ m_5 \succ \dots \quad \succ m_h}_{\text{unmatched}} \succ \underbrace{m_{12} \succ \dots \quad \dots \quad m_9 \succ m_l}_{\text{improvement}} \succ \underbrace{m_4 \succ m_{10} \succ \dots}_{\text{no effect}}$$

$\uparrow$ Best stable mate

$\uparrow$ Worst stable mate

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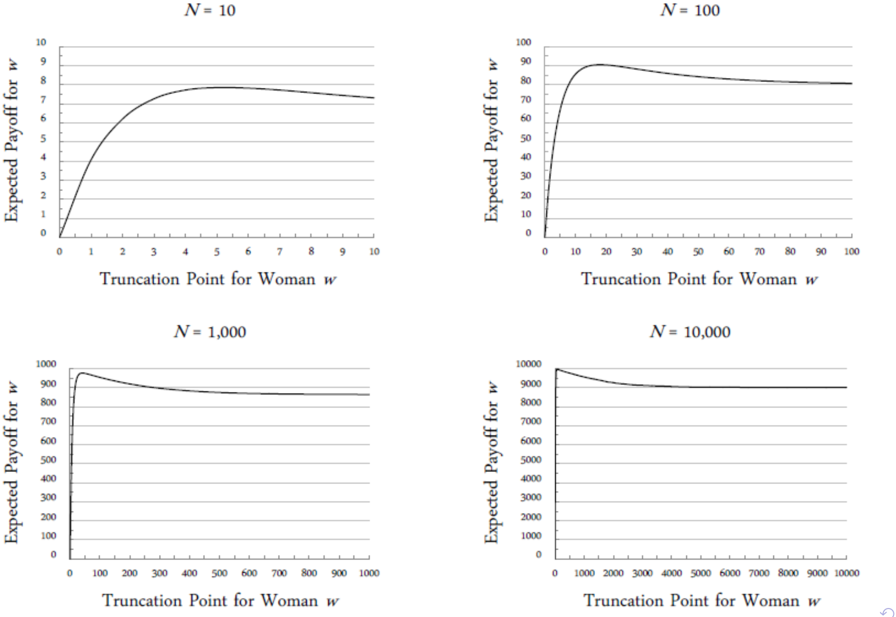
Uniform Market

- *In a uniform market* each player is equally likely to have any full preference list.
  - Being unmatched is ranked at the bottom.
- Players identically value a match with their rth ranked choice $\forall\ r \in \{1, \ldots, N\}$ and have identical value to being unmatched.
- Suppose that woman w has preferences $u_w(\cdot)$ linear in the rank of her match (where being unmatched is treated as rank $N+1$).

Simulation

Payoffs Linear in Rank



Large Balanced Market

Theorem

Let woman w have uniform beliefs and preferences linear in rank (or any strictly increasing, convex transformation of such preferences). Then $\lim_{N \to \infty} \frac{k^*(N)}{N} = 0$.

Proof.

Intuition:

1. Under *WP-DA* the expected partner rank asymptotes to $\log N$, while under *MP-DA* it asymptotes to $\frac{N}{\log N}$.

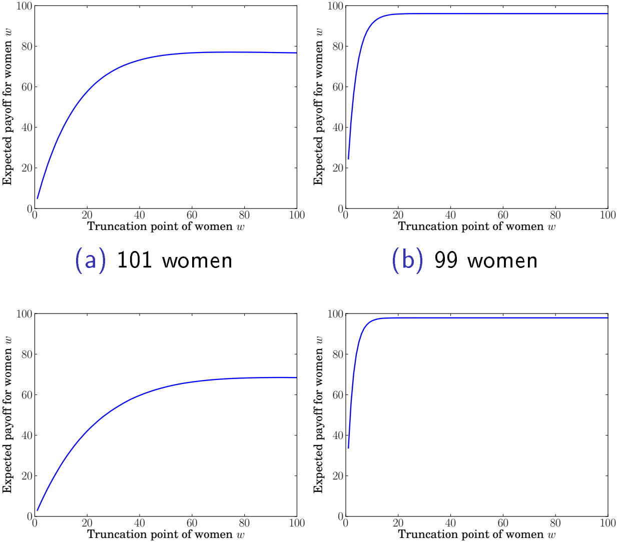
   * As a fraction of N both approach 0.

2. By keeping the top $7 \log^2 N$ men, a woman does better than she does by truth telling.

3. By keeping a constant fraction of her list, a women does not do much better than by truth telling (low likelihood of *any* stable mate so low down the list). □

Simulation

Payoffs Linear in Rank



(a) 101 women

(b) 99 women

(c) 105 women

(d) 95 women

Large Balanced Market

Theorem

Given $L > 0$, consider a market with N men and $N + L$ women. Let woman w have uniform beliefs and preferences linear in rank (or any strictly increasing, concave transformation of such preferences). Then $\frac{k_*(N,L)}{N} \geq \frac{L}{L+1}$ *so* $\lim_{N\to\infty} \frac{k_*(N,L)}{N} \geq \frac{L}{L+1}$. *In particular* $\frac{k_*(N,L)}{N} \geq \frac{1}{2}$ *and* $\lim_{N\to\infty} \frac{k_*(N,L)}{N} \geq \frac{1}{2}$.

- Not a "large market" result!

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The Preference List Submission Game

- The *Preference List Submission Game* is the Bayesian game described by
$$\left\langle I, \mathscr{P}, \mu^M[\cdot], U, \phi(\cdot) \right\rangle$$
- A pure strategy for agent i is a mapping $s_i : U_i \to \mathscr{P}_i$, and a mixed strategy for i is a mapping $\sigma_i : U_i \to \Delta(\mathscr{P}_i)$.
- Objective: a Bayesian Nash equilibrium *in truncation strategies* $\sigma = (\sigma_{m_1}, \ldots, \sigma_{m_N}, \sigma_{w_1}, \ldots, \sigma_{w_{N+L}})$ in which men report truthfully and women mix over truncation strategies.
  - Does it exist? (when strategies are not restricted to truncation strategies)
  - Properties.

Equilibrium

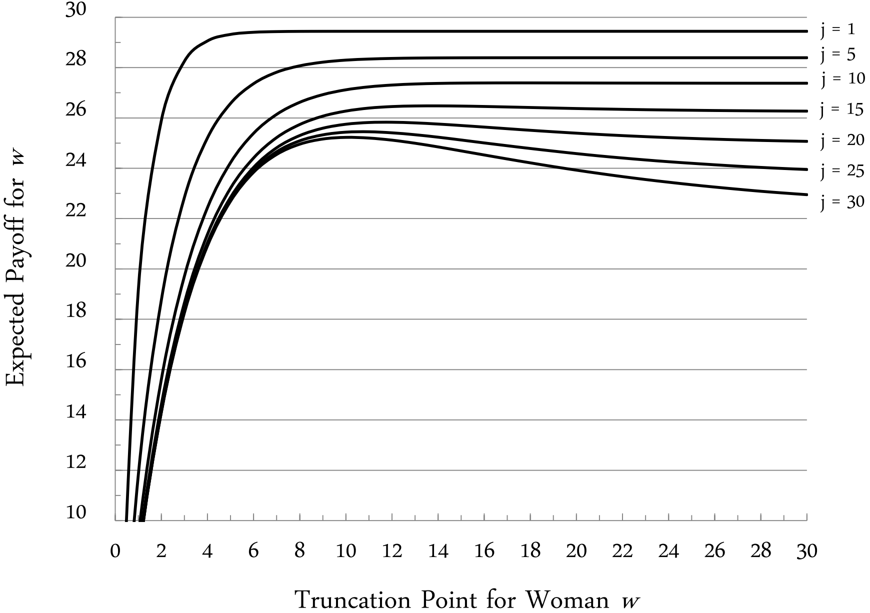
Existence

Theorem

In uniform markets, there exists a symmetric equilibrium $((\sigma_m), (\sigma_w))$ where men each use the strategy σ_m of truthful reporting and women each use the strategy σ_w, which is a mixture over truncation strategies.

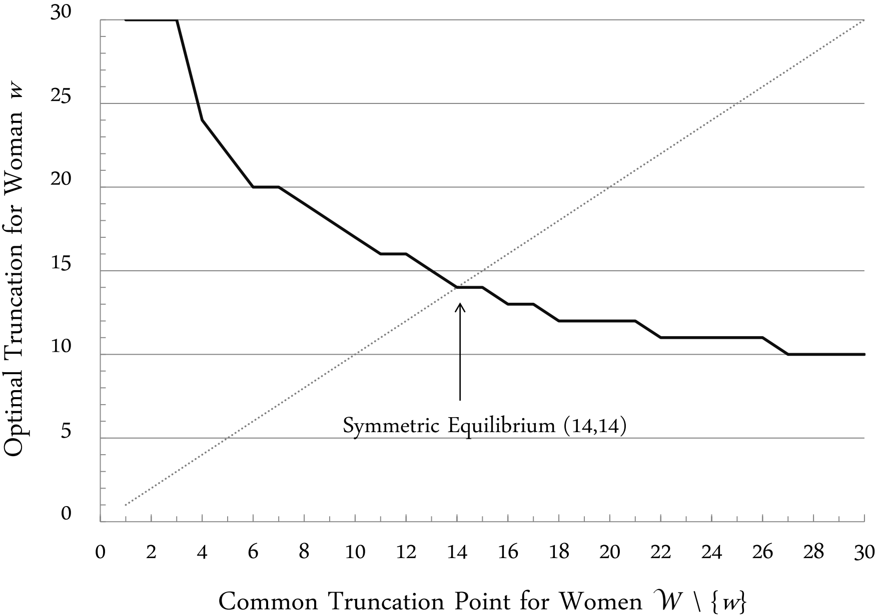
Equilibrium

Simulation



Equilibrium

Simulation



Unbalanced Markets

Conjecture

Fix L>3. In a uniform markets with N men and $N+L$ women, where MP-DA is used, there exists a symmetric equilibrium where all women submit truncated lists longer than $\frac{N}{2}$.

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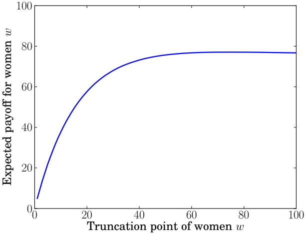
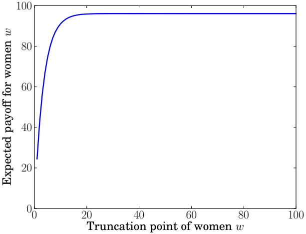
Simulation

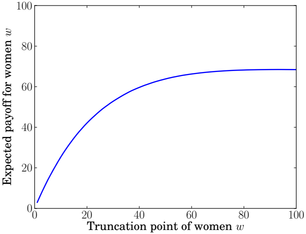
Payoffs Linear in Rank



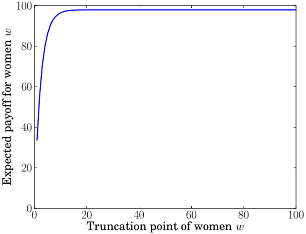
(e) 101 women



(f) 99 women



(g) 105 women



(h) 95 women

Safety

Theorem

Fix $L>0$ and $\delta \in (0,1)$. For N large enough, in a uniform market with N men and $N+L$ women, if all other agents report truthfully and woman w submits a truncation list of length less than δN, she will be unmatched with probability at least $\frac{.49+L}{N+L}$.

Theorem

Fix $L \geq 0$. For a uniform market with $N+L$ men and N women, a woman that submits a truncation containing more than $L+(2+a)\log^2 N$ men will be matched with probability at least $1-O\left(N^{-c(a)}\right)$, where $c(a)=2a\left[3+(4a+9)^{\frac{1}{2}}\right]^{-1}$. In particular a women that submits a truncated list of more than $L+10\log^2 N$ men will be unmatched with probability at most $O\left(\frac{1}{N^2}\right)$.

ε-Equilibrium

- What is the important friction?

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Truncation and Risk Aversion

Setting

- Since we are not dealing with utility of money, there is no notion of risk neutrality.
    - However, we can compare levels of risk aversion.
- Let $\psi(\cdot)$ be any strictly increasing, concave transformation.
- Recall:
$$v(k, P_{-w}) \equiv u_w(\mu^M[P_w^k, P_{-w}](w))$$
- Define:
$$v_\psi(k, P_{-w}) \equiv \psi(u_w(\mu^M[P_w^k, P_{-w}](w))$$

Truncation and Risk Aversion

Results

Theorem

Let $\tilde{P}_{-w}$ be any random variable distributed over $\mathscr{P}_{-w}$. Then $\forall k \in \{1, \ldots, N-1\}, \forall t \in \{1, \ldots, N-k\}$ *we have*

$$\begin{array}{rcl} \mathbb{E}\left[v(k, \tilde{P}_{-w})\right] & \leq & \mathbb{E}\left[v(k+t, \tilde{P}_{-w})\right] \quad \Rightarrow \\ \mathbb{E}\left[v_\psi(k, \tilde{P}_{-w})\right] & \leq & \mathbb{E}\left[v_\psi(k+t, \tilde{P}_{-w})\right] \end{array} .$$

Furthermore, if i) $\psi(\cdot)$ is strictly concave, and ii) under $\tilde{P}_{-w}$, each man is achievable for w with positive probability, then the second inequality is strict.

Truncation and Risk Aversion

Rank of w's Partner	1	2	…	k	$k+1$	…	w
Probability under k+1 Truncation	q_1^{k+1}	q_2^{k+1}	…	q_k^{k+1}	q_{k+1}^{k+1}	-	q_w^{k+1}
Probability under k Truncation	q_1^{k}	q_2^{k}	…	q_k^{k}	-	-	q_w^{k}

Sorting of Truncation Points

Corollary

Let k_i^l be the minimum optimal truncation point (by rank) and let k_i^h be the maximum optimal truncation point for woman $i \in \{w, w_\psi\}$. Then $k_w^l \leq k_{w_\psi}^l$ and $k_w^h \leq k_{w_\psi}^h$. Furthermore, if conditions i) and ii) from the theorem hold, then $k_w^h \leq k_{w_\psi}^l$.

i) $\psi(\cdot)$ is strictly concave.
ii) under $\tilde{P}_{-w}$, each man is achievable for w with positive probability.

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Truncation and Correlation

- Correlation in preferences is a natural phenomenon.
- We consider two models of correlation:
  - Introducing a probability that other women share w's beliefs exactly.
  - Introducing "noisy" preferences.

Truncation and Correlation

- $p(P_{\mathscr{M}}, P_{\mathscr{W}\setminus\{w\}})$
- $p^C(P_{\mathscr{M}}, P_{\mathscr{W}\setminus\{w\}}) \equiv \begin{cases} p^M(P_{\mathscr{M}}) & \text{if } P_{\hat{w}} = P_w \quad \forall \hat{w} \in \mathscr{W}\setminus\{w\} \\ 0 & \text{otherwise} \end{cases}$
- $p^\alpha(P_{-w}) \equiv (1-\alpha)p(P_{-w}) + \alpha p^C(P_{-w})$

- $k^h(\alpha, p, u_w)$ and $k^l(\alpha, p, u_w)$ are the optimal choices involving the least and most truncation respectively.

Truncation and Correlation

Proposition

Let $\alpha, \alpha' \in [0,1]$ with $\alpha' > \alpha$. Then $k^l(\alpha', p, u_w) \geq k^l(\alpha, p, u_w)$ and $k^h(\alpha', p, u_w) \geq k^h(\alpha, p, u_w)$.

- Intuition: when there is a unique stable matching, it can never hurt to submit a full list.

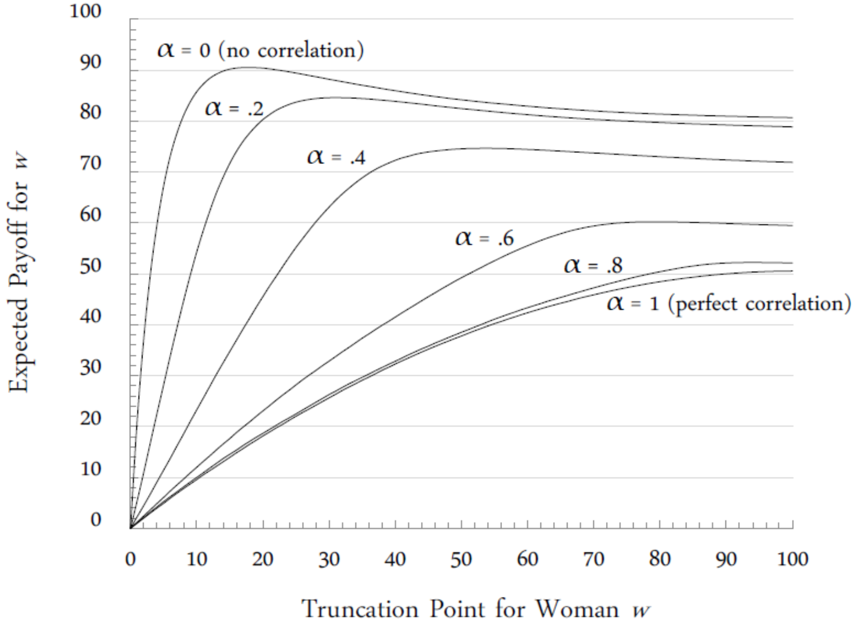
Truncation and Correlation

“Noisy” Preferences

- Woman w_j's rankings over men are then determined by the sum $\alpha \cdot r_i + (1-\alpha)q_{ij}$. (Caldarelli and Capocci)
    - $r_i \sim U[0,1]$, $q_{ij} \sim U[0,1]$.
    - $\alpha \in [0,1]$ is a parameter that we will vary.

Truncation and Correlation

"Noisy" Preferences



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Equilibrium

Welfare

Theorem

Let σ and σ' be equilibria in truncation strategies in which each woman truncates more under σ than under σ' (in the sense of first order stochastic dominance). Then compared to the outcomes in σ', under σ,

Theorem

i) welfare for women is weakly greater.

ii) welfare for men is weakly lower.

iii) the expected number of matches is weakly lower.

Furthermore, under both σ and σ', i), ii) and iii) hold in comparison to the outcomes from truthful reporting of preferences to MP-DA.

Equilibrium

Welfare

Let w_1, w_2 be two symmetric women.

Theorem

Consider any asymmetric equilibrium where w_1 truncates more then w_2 (in the sense of first order stochastic dominance). Then i) if w_1 and w_2 swap strategies, the resulting profile will also be an equilibrium and ii) w_2 prefers the original equilibrium, in which she truncates less.

Thank You