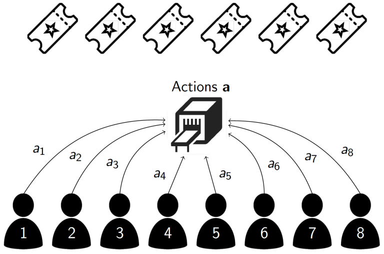
Lotteries for shared experiences

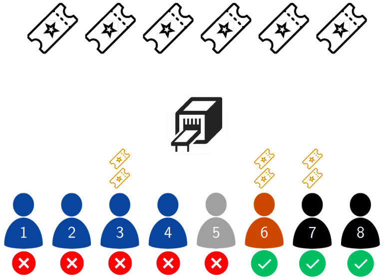
Columbia Business School
AT THE VERY CENTER OF BUSINESS™

Nick Arnosti
Carlos Bonet

PROBLEM

- We must allocate k identical tickets to a set of agents N.
- N is partitioned into groups: each agent i belongs to a group G_i.
- Agent i is happy $(u_i = 1)$ if collectively G_i receives $|G_i|$ or more tickets.





PERFORMANCE METRICS

- **Efficiency:** Random allocation p is β-efficient if

$$U(p) = \frac{1}{k} \sum_i u_i(p) \geq \beta.$$

- **Fairness:** Random allocation p is β-fair if for every i and j,

$$\frac{u_i(p)}{u_j(p)} \geq \beta.$$

We seek worst-case guarantees in the family of instances

$$I(\alpha, \kappa) = \left\{ \frac{\#\ tickets}{\#\ agents} \leq \alpha, \quad \frac{\max\ group\ size\ - 1}{\#\ tickets} \leq \kappa \right\}$$

0 α 1
supply ≪ demand supply = demand

0 κ 1
large groups small groups

MECHANISMS

- **Individual Lottery (IL)**
  1. Each agent i requests a number of tickets $a_i \in \{1, \ldots, k\}$.
  2. Process agents in uniformly random order.
     Give agents their request, while tickets remain.
- **Group Lottery (GL)**
  1. Each agent declares a subset of agents $a_i \subseteq N$.
     A set of agents S is a valid group if for every $i \in S, a_i = S$.
  2. Process valid groups in uniformly random order.
     Give valid groups their request, while tickets remain.
- **Weighted Individual Lottery (IW)**
  Individual lottery where agents are sampled without replacement with probability inversely proportional to their request.

MAIN RESULTS

Mechanism	Action set	Efficiency	Fairnes
Individual Lottery	$\{1,2,\ldots,k\}$	0	0
Group Lottery	$\{0,1\}^N$	$1 - \kappa$	$1 - 2\kappa$
Weighted IL	$\{1,2,\ldots,k\}$	$1 - \kappa - \alpha/2$	$1 - 2\kappa - \alpha/2$

CONCLUSIONS

- The Individual Lottery is inefficient and unfair.
- If possible, use Group Lottery. Otherwise, a small change improves performance: use Weighted Individual Lottery.