# Reviewer 1

Review Report Form

English language and style

( ) English very difficult to understand/incomprehensible  
( ) Extensive editing of English language and style required  
( ) Moderate English changes required  
(x) English language and style are fine/minor spell check required  
( ) I don't feel qualified to judge about the English language and style

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|  | Yes | Can be improved | Must be improved | Not applicable |
| Does the introduction provide sufficient background and include all relevant references? | (x) | ( ) | ( ) | ( ) |
| Are all the cited references relevant to the research? | (x) | ( ) | ( ) | ( ) |
| Is the research design appropriate? | (x) | ( ) | ( ) | ( ) |
| Are the methods adequately described? | (x) | ( ) | ( ) | ( ) |
| Are the results clearly presented? | (x) | ( ) | ( ) | ( ) |
| Are the conclusions supported by the results? | (x) | ( ) | ( ) | ( ) |

Comments and Suggestions for Authors

The authors present a new data structure called multiset-trie, which is used to store and process the set of multisets. The manuscript extends the ideas of trie and capture the requirement of the operations that need to be handled in the collection of multisets. The novel part of the manuscript is mathematical modeling, mathematical analysis, and empirical evaluation. The manuscript reads well and is of interest to the algorithms community. I recommend acceptance.

Submission Date

10 December 2022

Date of this review

10 Jan 2023 00:01:43

# Reviewer 2

Review Report Form

English language and style

( ) English very difficult to understand/incomprehensible  
( ) Extensive editing of English language and style required  
( ) Moderate English changes required  
(x) English language and style are fine/minor spell check required  
( ) I don't feel qualified to judge about the English language and style

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| Does the introduction provide sufficient background and include all relevant references? | ( ) | ( ) | (x) | ( ) |
| Are all the cited references relevant to the research? | (x) | ( ) | ( ) | ( ) |
| Is the research design appropriate? | ( ) | ( ) | (x) | ( ) |
| Are the methods adequately described? | (x) | ( ) | ( ) | ( ) |
| Are the results clearly presented? | (x) | ( ) | ( ) | ( ) |
| Are the conclusions supported by the results? | ( ) | ( ) | (x) | ( ) |

Comments and Suggestions for Authors

The paper proposes a data structure that stores multisets and can answer some natural queries on it.  
Each multiset is represented as a vector of natural numbers, and such vectors are stored in a trie. The queries are answered using naive search on the trie. The model is evaluated in a random scenario, in which each multiset is equally probable.

All in all, I think it would be difficult to improve this paper so that it would deliver results that deserve publishing.

**Reviewer:** I do not see any novelty or importance in those results. The data structure is naive and offers no improvements over a naive approach.

**Response:**

The multiset-trie is different to trie in the following features. 1) Since we are storing multisets the ordering of elements is not important and we use the ordering for the efficient search. The elements of multiset are ordered before they are entered into mset-trie. Experiments show that we achieve the best performance if the elements are ordered by the increasing/decreasing frequency of multiset elements. 2) While the insert, find and delete operations work in the same way as in the ordinary trie, the sub-multiset and super-multiset operations exploit the artificial ordering of multisets for a fast search of multisets (from mset-trie) that are either subsumed or subsume a parameter multiset, respectively.

The artificial ordering of multisets acts as a constraint that narrows down the search space for sub-multiset and super-multiset operations. Consider for instance the following example. Suppose we are searching for all sub-multisets of m={1,3,3,3,5}. In mset-trie we search (using the artificial ordering of elements) the paths from the root to some leaf. In each search step we either take the next element from m, or not! The search space is threfore constrained by the elements of the parameter multiset.

Similarly, the parameter multiset is used as a constraint also in the case of operation super-multiset. In this case, the paths from root to some leaf of mset-trie must include all elements of m={1,4,6}, plus, they can include some additional elements but only when all the preceeding elements match the existing path. For example, after m is matched in prefix {1,4} we can only accept any number of elements 5, but then we have to match an instance of 6, and after we can accept any elements higher than 6 (belonging to some multiset in mset-trie) . Again, the parameter multset m constrains the search space.

Also in the case that the above presented extension of trie can be called “naive”, by the best of our knowledge, nobody has used and explored such an extension of trie before. And, in spite of the limited empirical study, we can say that the sub-multiset and super-multiset operations are fast. For example, judged from the empirical analysis, the number of visited nodes (of mset-trie) for the existance queries is low. The same is derived also from the mathematical analysis of the mset-trie.

**Reviewer:** The evaluated model is strange: an a-priori bound on the cardinality is artificial and not justified.

Uniform distribution over all possible multisets is completely unrealistic.

No particular application is given. In a fixed application one could discuss number of attributes, the distribution of values for an item etc.

**Response:**

As with the analysis of any data-structure, it is impossible to predict characteristic properties of data as those differ over various domains. However, in the process of analyzing the data-structure, the usage of uniform distribution seems to be one of standard benchmarks.

We understand that the reviewer asks for a specific application, together with discussion on number of attributes, distribution of input data etc. However, since this is a new data structure which can be used in any domain using multiset containment operations, we felt that providing a discussion (or even fine-tuning the parameters) for any specialized domain would decrease the strength this paper.

**Reviewer:** Important implementation details are not described: for a fixed node the way the children are accessed matters. Using a table makes large overhead. Using a search tree adds time complexity. Using hash tables makes accessing the next node difficult. This is not addressed at all.

**Response:** Implementation of the mset-trie is given in the source code. The conclusion section includes the paragraph stating the inefficient implementation of the links from a node to its children. They are implemented by an array of a fixed size. In the same paragraph we continue “A custom-implemented small and extendable hash table would significantly decrease the amount of space needed to represent a multiset-trie.”

**Reviewer:** The experiments are run on randomly created data with uniform distribution. This gives little insight into real life performance; in fact probably could be calculated.

**Response:** Experiments 1-3 use artificially generated data which *allow us* to investigate the behaviour of mset-trie operations when varying: 1) the size of the alphabet, 2) the maximal degree of a node (maximal multiplicity of elements in multisets), and 3) the density of mset-trie. Further, Experiment 4 uses real-world data (English dictionary). In this experiment we study the influence of the mapping of letters from the alphabet into an interval [0,N] (N is the size of alphabet) to the performance of mset-trie.

We believe that Experiments 1-4 give a general picture of the multiset-trie behavior and performance.

We have added a new Experiment 5 which includes the comparison of the mset-trie with the main-memory implementation of the inverted index.

**Reviewer:** A natural approach to answer your queries is the range orthogonal queries on vectors of natural numbers. This problem in particular has well analyzed and implemented data structures, say k-d trees and others. They literally appear in textbooks. You should at least compare to them.

**Response:** In the case of k-d trees, the values within the query range (or variouos orthognoal ranges) translate to some (usually geometric) meaning. More formally, for range queries to even make sense, a domain of data needs to form a metric space (i.e. must be equipped with a reasonable distance function).

Indeed one can define (edit) distance on the domain of sets, and in our algorithm elements are indeed ordered initially. However this order bears absolutely no meaning and could be selected in arbitrary way with no impact to the resulting output (and maybe with a slight impact on preformance).  
This is however far from the case if one would go and build a k-d tree structure on top of this, as range queries are highly sensitive on the ordering of elements.

Submission Date

10 December 2022

Date of this review

16 Dec 2022 13:10:28

# Reviewer 3

Review Report Form

English language and style

( ) English very difficult to understand/incomprehensible  
(x) Extensive editing of English language and style required  
( ) Moderate English changes required  
( ) English language and style are fine/minor spell check required  
( ) I don't feel qualified to judge about the English language and style

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Comments and Suggestions for Authors

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|  | Yes | Can be improved | Must be improved | Not applicable |
| Does the introduction provide sufficient background and include all relevant references? | ( ) | (x) | ( ) | ( ) |
| Are all the cited references relevant to the research? | (x) | ( ) | ( ) | ( ) |
| Is the research design appropriate? | ( ) | (x) | ( ) | ( ) |
| Are the methods adequately described? | (x) | ( ) | ( ) | ( ) |
| Are the results clearly presented? | ( ) | (x) | ( ) | ( ) |
| Are the conclusions supported by the results? | (x) | ( ) | ( ) | ( ) |

**Reviewer:** Some parts of the text are difficult to read. Try to clarify your definitions.

And Appendix of definitions would be very useful.

**Response:** XXX Matjaž, je to glede matematičnega dela?

**Reviewer:** Also, in the experiments is necessary to clarify how is build the input data.

**Response:** The generation of data is described in Section 5, subsection entitled *Data generation* (page 16).

**Reviewer:** And tell us a little bit more about the C++ implementation and how the structure is coded.

**Response:** XXX Mogoče lahko vprašamo Mikito, da napiše en ali dva odstavka o implementaciji.

**Reviewer:** And finally, regarding the performance, is it possible to compare it with another data structure commonly used?

**Response:** We added a new Experiment 5 that compares mset-trie with the main-memory implementation of the inverted index.

Submission Date

10 December 2022

Date of this review

11 Jan 2023 17:14:35

# Reviewer 4

Review Report Form

English language and style

( ) English very difficult to understand/incomprehensible  
( ) Extensive editing of English language and style required  
( ) Moderate English changes required  
(x) English language and style are fine/minor spell check required  
( ) I don't feel qualified to judge about the English language and style

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|  | Yes | Can be improved | Must be improved | Not applicable |
| Does the introduction provide sufficient background and include all relevant references? | (x) | ( ) | ( ) | ( ) |
| Are all the cited references relevant to the research? | (x) | ( ) | ( ) | ( ) |
| Is the research design appropriate? | ( ) | (x) | ( ) | ( ) |
| Are the methods adequately described? | ( ) | ( ) | (x) | ( ) |
| Are the results clearly presented? | ( ) | (x) | ( ) | ( ) |
| Are the conclusions supported by the results? | ( ) | (x) | ( ) | ( ) |

Comments and Suggestions for Authors

The authors present a trie-based data structure for indexing a set of multisets, supporting, among common trie operations, containment queries to report multisets that are supersets or subsets of the queried multiset.

The manuscript is well-structured, and language mistakes are few in numbers.

The topic and the results meet the scope of the journal Algorithms.

The data structure is well-explained, and evaluated under various angles.

**Reviewer:** Nevertheless, I am not satisfied with the mathematical presentation proving the expectancy of various characteristics of the proposed data structure.

**Response:** XXX Matjaž.

**Reviewer:** Also, the experiments are lacking any comparison with existing solutions, which are only briefly sketched in one of the last sections of the manuscript.

**Response:** A new experiment that compares mset-trie with the main-memory implementation of the inverted index is added the section Experiment 5.

**Reviewer:** I would think positively about accepting the paper if

(1) the authors can make the proofs in Section 4 better understandable, and

(2) augment the experiments in Section 5 with a thorough comparison with existing data structures.

**Response:** (1) We hope that the reviewer will find the updated version of Section 4 to be more readable, with additional details facilitating understanding.

(2) A new experiment is added comparing mset-trie to inverted index in section *5.5 Experiment 5.* The results of the experiment show that mset-trie outperforms inverted file significantly.

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**Reviewer:** Detailed comments follow, where numbers at the beginning are the line numbers printed on the right side of the manuscript pages.

4: "such as sub-multiset and super-multiset": these are not operations but objects.

**Response:**

**Reviewer:** 8: What precisely are the time and space complexities of your data structure?

**Response:**

**Reviewer:** 13: "a particular" : I think any element is allowed to have duplicates in a multiset, not just a particular element.

**Response:**

**Reviewer:** 62/63: What are the merits for being height-balanced for this use case? Any trie that is "full" forms a complete n-ary tree.

**Response:**

**Reviewer:** 73: What is the "time complexity space"?

**Response:**

**Reviewer:** 74: empirical analyses, ->  empirical, analyses

**Response:**

**Reviewer:** 100: Multiset -> A multiset

**Response:**

**Reviewer:** 104: What is $n$? Is $n$ a user-defined constant?

**Response:**

**Reviewer:** Figure 1: I would write the stored sets as leaf-labels. Further, I see no advantage for writing $c\_j$ instead of just $j$.

**Response:**

**Reviewer:** 156: For what is the variable $dev$ used? Can you give an intuition behind it?

**Response:** XXX Jaz ne vem. Bi moral pogledati bolj natančno. Izgleda kot neko dodatno presenečenje. Matjaž, ti poznaš?

**Reviewer:** You described up to Section 3.3 standard trie operations, which you could put into the appendix since there is nothing new to see for those readers familiar with trie data structures.

**Response:** XXX Kaj ti meniš? Želimo imeti kompletno predstavitev z vsemi operacijami?

**Reviewer:** 190: tire -> trie

**Response:** Corrected.

**Reviewer:** I think Section 3.5 is identical to Section 3.4 due to the symmetry of the problem.

**Response:** XXX Meni se zdi, da ima prav. Kaj narediti?? Napišemo, da se strinjamo vendar za kompletnost predstavljamo oba primera?

**Reviewer:** 254: What do you mean with a "quite" precise upper bound?

**Response:** XXX

**Reviewer:** 255: "appears" : Is this not clear by symmetry?

**Response:** XXX

**Reviewer:** 269: Since you need $\Omega(n\sigma)$ time to transform a given input multiset into your type Multiset representation, I am not convinced with your O(1) time.

**Response:** XXX

**Reviewer:** 279: The selection of the same letter $M$ and $\mathcal{M}$ might be confusing. Maybe you can select a different letter for one of the two entities?

**Response:** XXX

**Reviewer:** Lemma 1 : I think you should find Lemma 1 in literature for the standard trie data structure, which you can cite here.

**Response:** XXX

**Reviewer:** Lemma 2: What is a "generating function"?

**Response:** XXX

**Reviewer:** Page 10:

I could not follow the proofs since it seems that some notations have not been introduced in a sufficiently detailed way:

- What does $\tilde$ mean?

- What is $\mathcal{B}\_0$?

- What is $Bernoulli()$?

**Response:** XXX

**Reviewer:** - Referring to the book of Gardiner [27] to possibly omit crucial definitions and keep the proof shortly is in stark contrast to the elaborative description of standard trie operations in the previous section. A more detailed description in how the proof works is more than welcome.

- What is a probability generating function?

- What is $1^-$?

- What is $G'\_X$?

**Response:** XXX

**Reviewer:** Definition 1: What are the intuitions behind $\alpha$ and $\beta$?

**Response:** XXX

**Reviewer:** 321: previous Definition 1 -> Definition 1

**Response:** XXX

**Reviewer:** 336: Here, the time bound of $O(|M|)$ is fine with me since this is also the number of elements we report, so we are output-sensitive for this query.

However, if we have a query like ${1^n, 2^n, ..., \sigma^0}$, then the time stays the same with fewer up to none elements to report, which is not optimal.

**Response:** XXX

**Reviewer:** 340: Can you give some informal conclusion on what the cumbersomely long equations for the time complexities mean?

**Response:** XXX

**Reviewer:** It would be also good to make a distinction between expected and worst case scenarios. Currently, the analysis seems to mix both worlds.

**Response:** XXX

**Reviewer:** 345: on the -> the

**Response:** XXX

**Reviewer:** 399: the relation : with respect to what?

**Response:** XXX

**Reviewer:** Figure 2: I have not understood what probability is addressed in this figure.

**Response:** XXX

**Reviewer:** Figure 3: I would rename the y-axis to the ratio you compute.

**Response:** XXX

**Reviewer:** 398: Note that analyzing -> Analyzing

**Response:** XXX

**Reviewer:** (15): What is $p$ in this equation? It seems to be only used in the limit?

**Response:** XXX

**Reviewer:** Figure 8 caption: Exsitence -> Existence

**Response:** Corrected.

**Reviewer:** 638: Why is your time complexity constant?

**Response:** XXX

**Reviewer:** 677-679: Can you explain in detail where the problem was and how it got solved?

**Response:**

**Reviewer:** 680-691: Are you aware of the Leapfrog Triejoin algorithm, which seems to be closely related to the problem addressed here.

Todd L. Veldhuizen: Incremental Maintenance for Leapfrog Triejoin. CoRR abs/1303.5313 (2013)

**Response:** XXX

**Reviewer:** 695: The performance also depends on what type of queries you want to optimize.

**Response:**

**Reviewer:** 707: How can the space complexity of your data structure be O(|M|) when storing a constant number of elements (|M| = O(1)) already costs you $\Omega(\sigma)$ nodes in the trie?

**Response:** XXX

**Reviewer:** 715-719: Specifying the implementation of your data structure as you did here would make sense to me if you would have practically evaluated the time/space of your data structure. It seems that such an experiment is missing, in particular with comparison to other approaches.

**Response:** We have added a new section Experiment 5 where we compare mset-trie to the inverted file.

**Reviewer:** Finally, I could not compile the code (https://github.com/nick-ak96/mstrie) since it seems that some header files are not included such as <memory> needed for std::shared\_ptr<>.

Maybe you can check whether you can make your code more portable?

g++ -DHAVE\_CONFIG\_H -I. -I..    -std=c++14 -Ilib -g -O2 -MT core/mstrie.o -MD -MP -MF $depbase.Tpo -c -o core/mstrie.o core/mstrie.cpp &&\

mv -f $depbase.Tpo $depbase.Po

In file included from core/mstrie.cpp:17:

core/mstrie.hpp:68:7: error: ‘shared\_ptr’ in namespace ‘std’ does not name a template type

   68 |  std::shared\_ptr<std::vector<std::shared\_ptr<MstrieNode>>> mult\_switch;

      |       ^~~~~~~~~~

core/mstrie.hpp:13:1: note: ‘std::shared\_ptr’ is defined in header ‘<memory>’; did you forget to ‘#include <memory>’?

12 | #include <queue>

  +++ |+#include <memory>

**Response:** XXX Sistem je narejen na MAC-u.Tu ne vem kaj napisati... Mogoče najboljše, če napišemo, da Mikita ni dostopen...

Submission Date

10 December 2022

Date of this review

16 Jan 2023 07:59:48