From Drip to Dispossession: International Technology Diffusion and Technology Stealing*

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October 17, 2025

Abstract

Production technologies differ enormously across countries. Given that wages are lower in poor countries, it is puzzling that international technology diffusion occurs so slowly. I develop a model of the speed of technology diffusion consistent with this phenomenon: although firms want to transfer production technology abroad to enjoy lower wages, doing so exposes them to the risk of foreign competitors imitating or stealing their technology. Critically, foreign governments cannot credibly commit ex ante to prevent technology stealing. In an optimal-contracting framework, slow technology transfers incentivize foreign governments to limit the rate of stealing by back-loading promises of future transfers. However, in the long run, once the firm has no technology left to transfer, its foreign competitors steal its technology, abetted by their government. Firms prefer higher short-run profits from producing abroad at lower wages over maintaining their long-run technological lead. Quantitatively, the model generates slow international technology diffusion but eventual catch-up over multiple decades.

*I would like to thank Patrick Kehoe, Elena Pastorino, Luigi Bocola, Adrien Auclert, Pete Klenow, Yuliy Sannikov, Kyle Bagwell, Aniket Baksy, Rafael Berriel, Ege Ercan, Zunda Winston Xu, and seminar participants at Stanford University for their helpful comments.

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Productivity differs persistently and enormously across countries, and is primarily driven by differences in the technology in use (Jones, 2016). The natural question that arises is, to paraphrase Lucas (1990), why does technology not flow from rich to poor countries? As wages are lower in countries behind the technological frontier, firms can transfer production technology, broadly defined here as methods to transform economic inputs into outputs, to those countries, employ their cheaper labour, and earn higher profits. Doing so does not reduce the stock of technology in the origin country: it has been understood since at least Romer (1990) that technology is non-rival. With such private benefits to closing international technology differences, that these gaps are not closed means that there must be a countervailing cost to technology transfer.

I argue that transferring technology abroad is *risky* for the advanced country ("home") firm. The precise way in which this risk materialized has varied across historical episodes. Some firms have been subjected to the outright nationalization of their foreign operations, which has been especially prevalent in capital-intensive industries like oil extraction. Other firms have been pressured into entering joint ventures with local firms, with the explicit aim of technology transfer. Yet more firms have seen returns underwhelm due to regulatory burdens, increases in taxes, and the imposition of capital controls. But even in the absence of such explicit coercion, once a firm has built production facilities abroad and trained a local workforce, its foreign competitors can more easily imitate, or even steal, the technology than it could a technology that is kept at home. By doing so, those foreign competitors become more productive, and compete harder against the home firm, eroding its profits.

The key insight is that the foreign government, which is sovereign under international law, cannot commit to *not* doing this at some point in the future. In particular, the government can neither credibly promise that a future government will stop the firm's competitors imitating its technology, nor that it will not engage in outright technology *expropriation*, where it both allows imitation and harasses the home firm until its foreign operations are unprofitable. Therefore, the home firm must give the foreign government an incentive to disallow technological expropriation.

I build a model where firms own production technologies, meaning that the knowledge of how to use it resides within the firm. Home-country firms, who have the technological lead, choose whether and how to transfer it to foreign countries, which they must do if they wish to produce abroad; transfers are non-reversible. Only after the technology has been transferred can it be imitated by foreign competitors.

In my model, the home firm faces an optimal contracting problem: it offers an implicit contract of technology transfers to the foreign government that maximizes the present value of its profits, subject to the government's lack of commitment. The foreign government's objective at all times is to maximize the profits of foreign firms. Technology diffusion is initially slow in equilibrium because home firms hold back technology to incentivize the foreign government to limit the rate of imitation by foreign competitors, with the promise of future technology transfers. Were home firms to transfer the frontier technology abroad without delay and close the technology gaps, as they would under full commitment, they would leave themselves with no way to incentivize the foreign government to limit imitation by foreign firms. As more technology is transferred, the gains from stealing the technology become larger, so the rate of transfer must increase to disincentivize stealing.

The long-run outcome is quite favourable to foreign firms, who eventually get the frontier technology. This is because the home firm maximizes the timespan it enjoys profitable foreign operations by promising to eventually transfer the frontier technology. Once it has done so, it has nothing left to promise, and so the foreign government allows the technology to be imitated. Despite this, I show in a quantitative example that it is entirely possible that the home firm prefers the large short-run profits it receives from transferring technology and producing abroad using cheaper labour over maintaining its technological lead in the long run. This suggests that a laissez-faire approach by the home government to home firms moving production abroad may not be optimal in the long run.

Literature Mansfield (1961), Nelson (1968), and Findlay (1978) are early contributions to the literature on technology diffusion, and model diffusion as occurring at an exogenous rate, or as slowed by a physical cost of adopting technology. A similar approach is taken in Grossman and Helpman (1991), Helpman (1993), Parente and Prescott (1994), and Sala-i-Martin and Barro (1997). Wang (1990) constructs a model where technology is embodied in international capital flows. More recently, Holmes, McGrattan and Prescott (2015) model Chinese government policy where automobile manufacturers must form joint ventures with Chinese firms to enter the Chinese market, explicitly enabling technology transfer. Branstetter, Glennon and Jensen (2018) explain the mechanics of how firms transfer technology abroad. Relative to this literature, I provide a theory that endogenizes the speed of technology diffusion.

There is a large literature discussing strategic incentives for countries to perform research and development (R&D) in the presence of technology spillovers and technology competition. Earlier contributions include Muniagurria and Singh (1997) and Lai (1998), while Benavente, Bravo-Ortega, Egaña-delSol and Hall (2024) is more recent. Instead of studying how the technology frontier moves outwards, this paper studies how below-frontier countries are brought up to it. This paper complements a large literature that has studied technology spillovers from trade and foreign direct investment (see e.g. the references in Keller, 2004) by offering a theory that links the size of spillovers back to firms' propensity for foreign direct investment.

This paper also complements other main explanations for slow technology diffusion across countries. One is that countries are using the technology level most appropriate for their capital intensity (Basu and Weil, 1998). However, the local capital scarcity in poor countries is mostly due to their low level of technology (as opposed to a low savings rate, cf. Gourinchas and Jeanne, 2013), which suggests there exist deeper barriers to technology flows. One barrier may be inefficient financial markets in poor countries that prevent their firms from upgrading their technology (Cole, Greenwood and Sanchez, 2016). This paper argues that the risk of technology stealing is another such barrier.

There is a long tradition in international macroeconomics of modelling governments as agents with limited commitment, most often in the context of sovereign defaults. Dovis (2018) considers sovereign default as part of an implementation of implicit contract between governments and bondholders under limited commitment and asymmetric information. The problem of international investors and firms contracting with a government with limited commitment is also studied in Aguiar and Amador (2011) and Opp (2012). In their models, the concern lies in physical expropriation of sunk investments, and not expropriation of production technologies.

Finally, technology stealing and international technology diffusion is a key driver of contemporary geopolitical rivalries and tensions. In this way, this paper contributes to a new literature on geoeconomics, which Mohr and Trebesch (2025) survey.

1 Model

There are two countries, home and foreign. Foreign variables are denoted with stars (*). Time is continuous. There are a continuum of varieties of goods, indexed in $[0, 1]$. Each variety $i \in [0, 1]$ has two sub-varieties, one of which is produced by one home firm and the other by one foreign firm.

Firms use a single factor of production, labour l. Production functions are linear: the home producer of variety i produces quantity $y_{it} = q_{it} l_{it}$, where l_{it} denotes efficiency units of labour, and q_{it} denotes the *technology level* used by the home firm at time t. This will be the main object of interest. The foreign producer of variety i produces quantity $y^*_{it} = q^*_{it} l^*_{it}$. For simplicity, I assume that there is frictionless free trade between the two countries, so consumers and firms face the same prices across the two countries. The home firm has discount rate r, and the foreign firm has discount rate r^*, which I take to be exogenous.

There is a unit measure of consumers in each country. Each consumer inelastically supplies a unit of labour. Consumers are hand-to-mouth and have identical preferences across countries

$$u\Big(\{x_{it}\}_{i=0}^{1}, \{x_{it}^{*}\}_{i=0}^{1}\Big),$$

where x_{it} denotes the household's consumption of the home sub-variety of i, and x_{it}^{*} denotes the household's consumption of the foreign sub-variety of i. Consumers are subject to the budget constraint

$$\int_0^1 [p_{it}x_{it} + p_{it}^{*}x_{it}^{*}]di \leq \begin{cases} w_t + \int_0^1 \pi_{it}di, & \text{home} \\ w_t^{*} + \int_0^1 \pi_{it}^{*}di, & \text{foreign,} \end{cases}$$

where p_{it} and p_{it}^{*} denote the prices of the home and foreign sub-variety, respectively, w_t and w_t^{*} denote the home and foreign wage rate, and π_{it} denotes the flow profits of home firms producing variety i.

The consumers maximize their utility subject to their budget constraint, so they have demand function $x_{it}(p_{it}, p_{it}^{*})$ for the home firm and $x_{it}^{*}(p_{it}^{*}, p_{it})$ for the foreign firm, where p_{it} is the home firm's price, p_{it}^{*} is the foreign firm's price. The resource constraints are $\int_0^1 l_{it}di = 1$ and $\int_0^1 l_{it}^{*}di = 1$.

Pricing The flow profits of the firms are

$$\pi_{it} = x_{it}(p_{it}, p_{it}^{*})\,(p_{it} - c_{it}) \tag{1}$$

$$\pi_{it}^{*} = x_{it}^{*}(p_{it}^{*}, p_{it})\,(p_{it}^{*} - c_{it}^{*}) \tag{2}$$

where c_{it} and c_{it}^{*} are the marginal cost of the home and foreign firm, respectively. If the firms produce domestically, $c_{it} = w_t/q_{it}$ and $c_{it}^{*} = w_t^{*}/q_{it}^{*}$. Pricing strategies follow the Nash equilibrium of the stage game, assuming no collusion,[1] so

$$p_{it}(c_{it}, c_{it}^{*}) = \arg\max_{p_{it}}\Big\{x_{it}(p_{it}, p_{it}^{*}(c_{it}^{*}, c_{it}))\,(p_{it} - c_{it})\Big\} \tag{3}$$

$$p_{it}^{*}(c_{it}^{*}, c_{it}) = \arg\max_{p_{it}^{*}}\Big\{x_{it}^{*}(p_{it}^{*}, p_{it}(c_{it}, c_{it}^{*}))\,(p_{it}^{*} - c_{it}^{*})\Big\} \tag{4}$$

The pricing strategies are thus functions of marginal cost $p_{it}(c_{it}, c_{it}^{*})$ and $p_{it}^{*}(c_{it}^{*}, c_{it})$. Thus, substituting in these pricing strategies into the expression for flow profits, I can deduce that flow profits are also functions of marginal costs: $\pi_{it}(c_{it}, c_{it}^{*})$ and $\pi_{it}^{*}(c_{it}^{*}, c_{it})$.

I assume that preferences are such that demand for variety i is sufficiently elastic for profits to decline after cost increases:

$$\pi_{it}(\alpha c_{it}, \alpha c_{it}^{*}) + \pi_{it}^{*}(\alpha c_{it}^{*}, \alpha c_{it}) \leq \pi_{it}(c_{it}, c_{it}^{*}) + \pi_{it}^{*}(c_{it}^{*}, c_{it}) \tag{5}$$

for $\alpha > 1$.

Partial equilibrium I solve the model for the home and foreign firm i in partial equilibrium, where (i) the demand functions x_{it} and x_{it}^{*} solve the consumer's problem, given wages and profits; (ii) prices p_{it} and p_{it}^{*} are best responses for one another, as defined in equations (3) and (4); and (iii) firms take the wages w_t and w_t^{*} as given.

Balanced growth path Assume that the home firm has access to technology level $\bar{q}_{it}$ and the foreign firm has access to technology level $\bar{q}_{it}^{*}$. On the balanced growth path, firms are capable of improving at rate g any technology they have access to, so that $\bar{q}_{it} = e^{gt}\bar{q}_{i0}$ and $\bar{q}_{it}^{*} = e^{gt}\bar{q}_{i0}^{*}$. Similarly, wages w_t and w_t^{*} grow at rate g. I let tildes ($\tilde{}$) denote detrended variables, so that $w_t = \tilde{w}_t e^{gt}$, and so on.

[1] The assumption of no collusion follows from the structure of the technology transfer game outlined below: the home firm is playing a repeated game with the foreign government, whereas the identity of the foreign firm is immaterial, and indeed the identity of the foreign firm may change from time to time.

I will build the result in three steps. First, I consider the baseline case where firms must produce domestically, which I call technological autarky. Then I open the economy up to technology flows. I consider the case where the foreign government has full commitment and show it features neither slow technology diffusion nor technology stealing. Finally, I consider the case where the foreign government has limited commitment.

1.1 Baseline: Technological Autarky

First, consider the case with no technology transfer, and firms must produce domestically. The home firm i operates its best technology $\bar{q}_{it}$ and the foreign firm i operates its best technology $\bar{q}^*_{it}$. Without loss of generality, I assume the home firm is the one with the higher technology level: $\bar{q}_{it} > \bar{q}^*_{it}$ (otherwise relabel the countries). On the balanced growth path, marginal cost is constant over time: the home firm has cost $\bar{c}_i = w_t/\bar{q}_{it}$ and the foreign firm $\bar{c}^*_i = w^*_t/\bar{q}^*_{it}$. For ease of exposition, I will drop the i subscripts going forward. In this baseline case, the stage game determining prices is identical at all points in time, so the total discounted profits of each firm are simply the flow profits, as stated in Proposition 1.

Proposition 1: *A firm's value function is a scaled version of the present value of its flow profits. In technological autarky, its value function is equal to flow profits:*

$$\begin{aligned}\Pi^a &\equiv (r-g)\int_0^\infty e^{-rt}\pi_t(\bar{c},\bar{c}^*)dt \\ &= (r-g)\int_0^\infty e^{-rt}e^{gt}\tilde{\pi}(\bar{c},\bar{c}^*)dt \\ &= (r-g)\int_0^\infty e^{-(r-g)t}dt \times \tilde{\pi}(\bar{c},\bar{c}^*) \\ &= \tilde{\pi}(\bar{c},\bar{c}^*).\end{aligned}$$

Similarly, the foreign firm's value function is $\Pi^{*a} = \tilde{\pi}^*(\bar{c}^*,\bar{c})$.

The second line follows the first because firm profits grow at rate g, as the demand for their products grow at rate g on the balanced growth path. This case is rather sedate, so I next introduce technology diffusion.

1.2 Technology Diffusion

Say if home firm i can profitably outsource production abroad. Clearly, a precondition is that foreign wages need to be lower: $w^*_t < w_t$. To outsource, the home firm exports the production technology $q_t < \bar{q}_t$ abroad at time t. Technology exports are irreversible: $\dot{q}_t \geq 0$ for all t. This captures the notion that technology export requires training foreign workers to use a technology; once trained, they cannot be untrained, and can bring their knowledge to the foreign firms. If the home firm chooses to transfer the technology level $q_t \leq \bar{q}_t$ abroad and use it in foreign production, its flow profit is $\pi(c_t, c^*_t)$, where its marginal cost is $c_t = w^*/q_t$, and its competitor's marginal cost is $c^*_t = w^*/q^*_t$. The foreign firm can costlessly improve its technology q^*_t up to the level of the home firm's transferred technology q_t through technology imitation.

The foreign government is mercantilist: it aims to maximize the present value of profits earned by foreign firms from production, discounted at rate r^*. It is sufficient here to note that it is an empirical regularity that governments routinely adopt import tariffs and other policies that benefit domestic producers, at the expense of other economic agents (Baldwin, 1989); a full theory expositing the foundations of such governmental preferences is beyond the scope of this paper.

The foreign government has two policy tools at its disposal: first, it is able to limit the rate at which the foreign firm imitates the home firm, i.e. the foreign government controls q^*_t:

it may choose to allow no imitation, so that $q^*_t = \bar{q}^*_t$, or it may choose to allow any amount of imitation up to full imitation $q^*_t = q_t$, where q_t is the transferred technology. Second, it is able to impose additional costs on the home producer's outsourced production, which I term *protectionism*, as it is somewhat reminiscent of trade protectionism in its rationale of protecting the competitiveness of domestic firms. I model costs as a labour wedge $\tau_w \geq 0$ on the home producer, standing in for red tape, who faces the wage schedule ϕw^*, where $\phi = 1 + \tau_w \geq 1$.[2] In keeping in line with the foreign government's mercantilism, I assume that this wedge obtains no value for the government aside from furthering the profits of its producers.

Technology transfer under full commitment The home firm offers an implicit contract specifying a path of technology transfer q_t, a path of permissible imitation q^*_t, and a path of permissible protectionism ϕ_t. When facing a foreign government with full commitment, its optimal contract problem is stated in Definition 1, where I use the notation $\tilde{r} = r - g$ and $\tilde{r}^* = r^* - g$.

Definition 1: *The home firm's optimal contract problem under full commitment is to choose the technology transfer path c_t, the level of foreign imitation c^*_t, and its labour wedge ϕ_t to maximize its value*

$$\Pi_0 = \max_{\{c_t, c^*_t, \phi_t\}} \left\{ \tilde{r} \int_0^\infty e^{-rt} \pi_t(\phi_t c_t, c^*_t) dt \right\}$$

subject to the definitions of π and π^* in (1) and (2); the irreversibility of transfers $\dot{q}_t \geq 0$ so $\dot{c}_t / c_t \leq g$; that only transferred technology can be imitated $c^*_t \leq c_t \leq \bar{c}^*$; the subsidy constraint $\phi_t \geq 1$; and the foreign government's participation constraint that foreign profits must weakly increase relative to technological autarky:

$$\Pi^*_0 = \tilde{r}^* \int_0^\infty e^{-r^* t} \pi^*_t(c^*_t, \phi_t c_t) dt \geq \Pi^{*a} = \tilde{\pi}^*(\bar{c}^*, \bar{c}).$$

On the extensive margin, the home firm decides whether to enter into the contract or not. It enters if the value of the optimal contract Π_0 exceeds its value producing at home $\Pi^a = \tilde{\pi}(\bar{c}, \bar{c}^*)$. The key result about the full commitment case is stated in Proposition 2.

Proposition 2: *Under full commitment, and under the assumption that total profits of the home and foreign firm decline in their costs (eqn. 5), the home firm's optimal contract transfers its frontier technology $\bar{q}_t$ at all times t, so $c_t \equiv w^*_t / \bar{q}_t$ for all t.*

Proof. Consider a contract that sets $c^*_t \geq c_t = \alpha w^*_t / \bar{q}_t$ for $\alpha > 1$ and some t. Then $\pi_t(w^*_t/\bar{q}_t, c^*_t/\alpha) + \pi^*_t(c^*_t/\alpha, w^*_t/\bar{q}_t) \geq \pi_t(c_t, c^*_t) + \pi_t(c^*_t, c_t)$ follows from equation (5). Thus, the contract is dominated by setting $c_t = w^*_t/\bar{q}_t$: the home firm earns higher flow profits, and the foreign firm also earns higher flow profits, which relaxes the foreign government's participation constraint. □

Intuitively, Proposition 2 holds because maximal technology transfer allows the home firm to minimize its production costs, which increases its profits, and there are no downsides to doing so. The contract also usually prescribes no imitation $c^*_t = \bar{c}^*$ and no protectionism $\phi_t = 1$, although to satisfy the foreign government's participation constraint, the contract may allow for some periods of time where foreign firms imitate the technology and/or the government implements protectionism. Thus, under full commitment, technology diffusion is not slow and technology stealing does not occur.

[2]The assumption that $\tau_w \geq 0$ is tantamount to an assumption that there is an upper bound to the subsidies that the foreign government can provide subsidies to the home firm.

1.3 Limited Commitment

Now consider a foreign government that lacks the ability to commit. In particular, its best deviation, given its objective of maximizing the profit of foreign producers, is to allow full technology imitation $q_t^* = q_t$ and $\phi_t = +\infty$, which I collectively term *technological expropriation*. The home producer faces such a high wedge on its labour input that it returns to producing at home, where its cost is $\bar{c}$.

Profit maximization by the home producer again entails offering an implicit contract with the foreign government, specifying a path of technology export q_t, a path of permissible imitation q_t^*, a path of permissible protectionism ϕ_t, and the punishment in case the foreign government deviates from permissible imitation or protectionism. The home firm continues to have full commitment regarding its technology exports, so it can credibly promise the maximal punishment in response to such a deviation. As technology exports are irreversible, the maximal punishment is merely to stop all further technology export. The implicit contract between the home producer and the foreign government must always make the foreign government prefer continuing with the contract than taking its best deviation of complete imitation. The form of this constraint, termed a *sustainability constraint* by Chari and Kehoe (1990), is derived in Lemma 1.

Lemma 1: *The optimal contract must satisfy the sustainability constraint given by*

$$\tilde{r}^*[\tilde{\pi}^*(c_t, \bar{c}) - \tilde{\pi}^*(c_t^*, \phi_t c_t)] = \partial_1 \tilde{\pi}^*(c_t, \bar{c})\, \dot{c}_t \tag{ST}$$

for all $t \geq 0$ in which the home producer outsources production abroad.

Proof. The constraint that the foreign government prefers to adhere to the contract is given by

$$\Pi_t^* \equiv \tilde{r}^* \int_t^\infty e^{-r^*(s-t)} \pi_s^*(c_s^*, \phi_s c_s) ds \geq \tilde{r}^* \int_t^\infty e^{-r^*(s-t)} \pi_s^*(c_t, \bar{c}) ds.$$

Using the assumption of balanced growth and $\tilde{r}^* = r^* - g$, this is

$$\tilde{\Pi}_t^* \equiv \tilde{r}^* \int_t^\infty e^{-\tilde{r}^*(s-t)} \tilde{\pi}^*(c_s^*, \phi_s c_s) ds \geq \tilde{r}^* \int_t^\infty e^{-\tilde{r}^*(s-t)} \tilde{\pi}^*(c_t, \bar{c}) ds,$$

where the left-hand side of the inequality is the present value of foreign producer profits from adhering to the contract, and the right-hand side of the inequality is the present value of foreign producer profits under the best deviation. The subgame that starts after a deviation is essentially static, like the case of technological autarky considered previously. Hence, the right-hand side of the constraint can be simplified to $\tilde{\pi}^*(c_t, \bar{c})$. The home producer prefers as little imitation by the foreign producer as possible, which is equivalent to giving the foreign producer as little profit as possible. This means that the constraint must bind for all t. Combining these facts, the constraint may be written as

$$\tilde{\Pi}_t^* \equiv \tilde{r}^* \int_t^\infty e^{-\tilde{r}^*(s-t)} \tilde{\pi}^*(c_s^*, \phi_s c_s) ds = \tilde{\pi}^*(c_t, \bar{c})$$

Differentiating the left-hand side of the constraint with respect to t and using the fact that the constraint binds, I obtain

$$\begin{aligned} &- \tilde{r}^* \tilde{\pi}^*(c_t^*, \phi_t c_t) + (\tilde{r}^*)^2 \int_t^\infty e^{-\tilde{r}^*(s-t)} \tilde{\pi}^*(c_s^*, \phi_s c_s) ds \\ &= \tilde{r}^*[\tilde{\Pi}_t^* - \tilde{\pi}^*(c_t^*, \phi_t c_t)] \\ &= \tilde{r}^*[\tilde{\pi}^*(c_t, \bar{c}) - \tilde{\pi}^*(c_t^*, \phi_t c_t)]. \end{aligned}$$

Differentiating the right-hand side of the constraint with respect to t obtains

$$\partial_1 \tilde{\pi}^*(c_t, \bar{c})\, \dot{c}_t,$$

which proves the lemma. □

Lemma 1 has the following interpretation: if the foreign firm's flow profits are below the flow profits it would make under the best deviation of full technological expropriation, it must be compensated by an increase in the promise value delivered by the contract. We can rearrange constraint (ST) to show the following:

$$\dot{c}_t = \frac{\tilde{r}^*[\tilde{\pi}^*(c_t, \bar{c}) - \tilde{\pi}^*(c_t^*, \phi_t c_t)]}{\partial_1 \tilde{\pi}^*(c_t, \bar{c})}. \tag{6}$$

The implication of this is that the path of technology exports q_t is fully pinned down once the path of imitation q_t^* has been chosen. The sustainability constraint reduces the number of controls for the home producer from two to one. I am now ready to state the home firm's optimal contract problem.

Definition 2: *The home producer's optimal contract problem under limited commitment is to choose the technology transfer path $\{c_t\}$, the level of foreign imitation $\{c_t^*\}$, and its labour wedge $\{\phi_t\}$ to maximize*

$$\Pi_0 = \max_{\{c_t, c_t^*, \phi_t\}} \left\{ \tilde{r} \int_0^\infty e^{-rt} \pi_t(\phi_t c_t, c_t^*) dt \right\}$$

subject to the definitions of π and π^* in (1) and (2), the sustainability constraint that $\dot{c}$ follows the law of motion in (6), that only transferred technology can be imitated $c_t^* \leq c_t \leq \bar{c}^*$; the subsidy constraint $\phi_t \geq 1$; and the foreign government's participation constraint that foreign profits must weakly increase relative to technological autarky: $\Pi_0^* \geq \Pi^{*a}$.

The firm's value function can be written in detrended form:

$$\tilde{\Pi}_t = \max_{\{c_s, c_s^*, \phi_s\}} \left\{ \tilde{r} \int_t^\infty e^{-\tilde{r}s} \tilde{\pi}(\phi_s c_s, c_s^*) ds \right\}.$$

For $t > 0$, the sustainability constraint (ST) takes one control away from the home producer. In formulating the Hamilton-Jacobi-Bellman (HJB) equation, I have the home firm choose c_t^* and ϕ_t, from which c_t follows from (ST). The HJB is

$$\tilde{r}\tilde{\Pi}(c) = \max_{c^* \leq w^*/\bar{q}^*, \phi \geq 1} \left\{ \tilde{r}\tilde{\pi}(\phi c, c^*) + \tilde{\Pi}'(c) \frac{\tilde{r}^*[\tilde{\pi}^*(c, \bar{c}) - \tilde{\pi}^*(c^*, \phi c)]}{\partial_1 \tilde{\pi}^*(c, \bar{c})} \right\}.$$

In the following proposition, the home firm's HJB is divided through by the home firm's discount rate r.

Proposition 3: *The HJB equation for the value of the home producer is*

$$\tilde{\Pi}(c) = \max_{c^* \leq w^*/\bar{q}^*, \phi \geq 1} \left\{ \tilde{\pi}(\phi c, c^*) + \tilde{\Pi}'(c) \frac{\tilde{r}^*}{\tilde{r}} \frac{\tilde{\pi}^*(c, \bar{c}) - \tilde{\pi}^*(c^*, \phi c)}{\partial_1 \tilde{\pi}^*(c, \bar{c})} \right\}. \tag{7}$$

At an interior solution for the maximization problem on the right-hand side of (7), the first-order conditions are

$$\begin{aligned}
[c^*]: \quad & \partial_2 \tilde{\pi}(\phi c, c^*) = \tilde{\Pi}'(c) \frac{\tilde{r}^*}{\tilde{r}} \frac{\partial_1 \tilde{\pi}^*(c^*, \phi c)}{\partial_1 \tilde{\pi}^*(c, \bar{c})} \\
[\phi]: \quad & \partial_1 \tilde{\pi}(\phi c, c^*) = \tilde{\Pi}'(c) \frac{\tilde{r}^*}{\tilde{r}} \frac{\partial_2 \tilde{\pi}^*(c^*, \phi c)}{\partial_1 \tilde{\pi}^*(c, \bar{c})}.
\end{aligned}$$

I can immediately deduce some observations about the solution to the home producer's contracting problem.

Proposition 4: *The home producer's optimal contract, should it decide to produce abroad, has the following properties:*

(i) There is a stopping time $\tau < \infty$ at which technology transfers stop, so long as the foreign firm's net increase in flow profits after technology expropriation $\tilde{\pi}^*(c_t, \bar{c}) - \tilde{\pi}^*(c_t^*, \phi_t c_t) > \kappa > 0$ is bounded away from zero.

(ii) At time τ, the home firm's technology is stolen: $\tilde{\Pi}(c_\tau) = \tilde{\pi}(\bar{c}, c_\tau)$.

(iii) The initial technology transfer q_0 is such that $c_0 = w^*/q_0$ is the global maximum of $\tilde{\Pi}$. Technology diffusion only occurs if $\tilde{\Pi}(c_0) \geq \pi(\bar{c}, \bar{c}^*)$.

(iv) q_0 must satisfy $w^*/q_0 \equiv c_0 \leq \bar{c} = w/\bar{q}$.

(i) follows from (6) that $\dot{c}_t$ is bounded away from zero: the home firm transfers a non-vanishing flow of technology at every point in time in order to fend off the foreign government's temptation to permit expropriation. Hence, in finite time, the home firm transfers the frontier technology $\bar{q}$, at which point it has no more technology to transfer and must stop. Depending on the structure of the profit functions, the home firm may stop transfers before full transfer.

(ii) states that in the long run, the home firm's technology is stolen in equilibrium. Once technology transfer stops, the foreign government no longer has any incentive to disallow the foreign firm to steal the technology, so the foreign firm does so and produces at marginal cost $c_\tau = w^*/q_\tau$. The foreign government can maximize the profits of the foreign firm by setting the home firm's wedge ϕ to raise its marginal cost ϕc_t to be $\bar{c}$, so that it is indifferent between producing in the two countries. Hence, the home firm's value once the stopping time is reached comprises the profits it obtains from producing at home. From (ii) and (iii), it is clear the home firm's value function $\tilde{\Pi}$ exists in the domain $[c_\tau, c_0]$.

To see that (iv) holds, note that if the initial technology transfer q_0 did not satisfy $w^*/q_0 \equiv c_0 \leq \bar{c} = w/\bar{q}$, then the home producer would earn lower flow profits until c_t reaches $\bar{c}$ than if it had merely kept production at home. Then the home firm could strictly increase its value by keeping production at home until time t and following the technology transfer path from t onwards.

2 The case of Bertrand competition

In this section, I present the special case where the home and foreign firms engage in Bertrand competition to sell a homogeneous good. In this setting, I can derive a closed-form solution to the problem. The demand function is $x_t(p, p^*) = x_t(p)\mathbf{1}_{[p \leq p^*]}$, where $x_t(p)$ is declining in p. I make the following assumption.

Assumption 1: *$\bar{c}^* = w_t^*/\bar{q}_t^* \leq p^m(w_t^*/\bar{q}_t)$, where $p^m(w_t^*/\bar{q}_t)$ is the price charged by a monopolist with cost $w_t^*/\bar{q}_t$. As the monopolist's price is increasing in its own cost, $\bar{c}^* < p^m(c)$ for all $c > w_t^*/\bar{q}_t$. In addition, $x_t(\bar{c}^*) > 0$.*

Under Assumption 1, for any feasible cost of production that the home firm might enjoy in the foreign country, and any feasible cost of the foreign producer, the foreign producer's cost is below the home producer's monopoly price, and hence the home producer never charges the monopoly price.

Consider any subgame where the home firm has transferred technology to produce abroad. From Proposition 4(d), recall that the home firm must enjoy lower costs abroad to outsource: $c < \bar{c} = w_t/\bar{q}_t$. I will carry this assumption going forward. Further, the home firm's optimal contract prescribes $\phi_t = 1$ for all $t < \tau$, where τ is the time at which all technology has been transferred: $q_\tau = \bar{q}_t$. The proof is in Lemma 2.

Lemma 2: *Let τ be the smallest t for which $q_t = \bar{q}$, and hence $c_\tau = w^*/\bar{q}$. Then the home firm's optimal contract permits no protectionism $\phi_t = 1$ and no imitation $c_t^* = \bar{c}^* = w^*/\bar{q}^*$ for all $t < \tau$. Full technology transfer occurs.*

Proof. From Proposition 4(iv), the home firm requires lower production costs abroad to transfer technology: $c_t \leq \bar{c}$ for all t. Hence, under Bertrand competition, $\pi(\bar{c}, c_t) = 0$, so the home firm makes no profits forever once it is expropriated. The home firm thus wants to delay expropriation as late as possible, which is accomplished by transferring technology until the frontier is reached. Allowing a wedge $\phi_t > 1$ would simply reduce the home firm's profits, while the foreign firm continues to earn no profit, so does nothing to relax the sustainability constraint (ST). Similarly, allowing for $c_t^* < \bar{c}^*$ reduces the price that the home firm can charge, which reduces its profits (under Assumption 1). The foreign firm continues to earn no profit, so this does nothing to relax the sustainability constraint (ST). Hence, at any $t < \tau$, neither allowing $\phi_t > 1$ nor allowing $c_t^* < \bar{c}^*$ provides any benefit to the home firm. $\square$

A corollary of Lemma 2 is that home's profit in $t < \tau$ is $\pi_t(c_t, \bar{c}_t^*) = x_t(\bar{c}_t^*)(\bar{c}_t^* - c_t)$, and foreign's profit in $t < \tau$ is $\pi^*(c_t^*, c_t) = 0$. This is because $c_t^* > c_t$, and the non-selling firm has zero flow profit in Bertrand equilibrium. As $c_t < \bar{c}$, the profit that the foreign firm makes after technological expropriation is $\pi^*(c_t, \bar{c}) = x(\bar{c})(\bar{c} - c_t)$. The following proposition solves the dynamics of the home firm's optimal contract in closed form.

Proposition 5: *The home firm's optimal contract, should it decide to produce abroad, produces the following dynamics:*

(i) The expropriation date is at

$$\tau = t + \frac{1}{r^*} \log \frac{\bar{c} - c_\tau}{\bar{c} - c_t}. \tag{8}$$

(ii) The home firm's technology transfers reduce its marginal cost of production abroad at rate

$$\dot{c}_t = -\tilde{r}^*(\bar{c} - c_t). \tag{9}$$

(iii) The home firm's marginal cost of production abroad is

$$\bar{c} - c_s = (\bar{c} - c_t)e^{\tilde{r}^*(s-t)}. \tag{10}$$

(iv) The home firm's value function $\tilde{\Pi}$ is, for $r^* \neq r$,

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*)\left[(\bar{c}^* - \bar{c})\left[1 - \left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{\tilde{r}/\tilde{r}^*}\right] + \frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}(\bar{c} - c)\left[\left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - 1\right]\right]. \tag{11}$$

For $r = r^*$, it is

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*)\left[(\bar{c}^* - \bar{c})\frac{c - c_\tau}{\bar{c} - c_\tau} - (\bar{c} - c)\log\frac{\bar{c} - c}{\bar{c} - c_\tau}\right]. \tag{12}$$

(v) The home firm's initial cost of production abroad is characterized by, for $r^* \neq r$,

$$\bar{c} - c_0 = (\bar{c} - c_\tau)\left[1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right]^{\frac{\tilde{r}^*}{\tilde{r}^* - \tilde{r}}}. \tag{13}$$

For $r = r^*$, it is characterized by

$$\bar{c} - c_0 = (\bar{c} - c_\tau)\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right). \tag{14}$$

(vi) The net gain to the home firm of moving production abroad relative to autarky profits Π^a is, for $r^* \neq r$,

$$\begin{aligned}\tilde{\Pi}(c_0) - \tilde{\Pi}^a = x(\bar{c}^*)\Bigg[&-(\bar{c}^* - \bar{c})\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}} \\ &+ r\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}^*}{\tilde{r}^* - \tilde{r}}}\frac{(\bar{c} - c_\tau)(\bar{c}^* - c_\tau)}{\tilde{r}^*(\bar{c} - c_\tau) - (\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)}\Bigg].\end{aligned} \tag{15}$$

For $r = r^*$, it is

$$\tilde{\Pi}(c_0) - \tilde{\Pi}^a = \tilde{x}(\bar{c}^*)(\bar{c} - c_\tau)\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right). \tag{16}$$

The proof is in Appendix A.1. From (8) and (9), one can see that when facing a foreign country that is *less* patient, which is to say that r^* is higher, the pace of technology transfers is *faster* ($\dot{c}_t$ is larger in magnitude), and the date at which the foreign country reaches the frontier τ is brought forward. The faster pace of transfers is needed to temper the impatient foreign country's desire to expropriate technology. Does this mean that a more impatient foreign country necessarily gets more technology at any given point in time? No: there is also an extensive margin. The faster pace of technology transfer reduces the profitability of technology transfer to the home firm; if the required pace is too fast, then introducing the technology abroad may become unprofitable for the home firm, who instead prefers to keep production at home. This is discussed in Proposition 6.

Proposition 6: *If $r = r^*$, the home firm always gains from technology transfer abroad: $\Pi(c_0) - \Pi^a > 0$. Say if $\bar{c} \leq \bar{c}^*$, so that the home firm is the selling firm in technology autarky. Then if r^* is sufficiently large relative to r, the net gains of technology transfer to the home firm $\Pi(c_0) - \Pi^a$ become negative, and the home firm does not produce abroad.*

The proof is in Appendix A.2.

3 Quantitative Example

To illustrate the dynamics of this setting, I solve the Bertrand competition model under CES demand function $x(p) = ap^{-\sigma}$. The parameters are in Table 1. The setting features equal discount rates across countries ($r = r^*$) and $\bar{c} = w/\bar{q} = 3 = w^*/\bar{q}^* = \bar{c}^*$; the home firm is three times more productive than the foreign firm, but home wages are three times as high as foreign wages, so each firm faces the same marginal cost.[3] Hence, neither firm makes profits in technology autarky under Bertrand competition.

Table 1: Parameter Values

Parameter		Value
a	Market size	1
σ	Demand elasticity	1.5
$\tilde{w}$	Domestic wage	3
$\tilde{w}^*$	Foreign wage	1
$\tilde{r}$	Domestic discount rate less growth ($r - g$)	5%
$\tilde{r}^*$	Foreign discount rate less growth ($r^* - g$)	5%
$\tilde{\bar{q}}$	Domestic technology level	1
$\tilde{\bar{q}}^*$	Foreign technology level	1/3

Given that $\bar{c} = \bar{c}^*$ under my parameterization, I can deduce from (14) that $\bar{c} - c_0 = (\bar{c} - c_\tau)\exp(-1)$. Then inserting this result into (8), I deduce that $\tau = 1/\tilde{r}^*$. Hence, under an annual discount rate of $\tilde{r}^* = r^* - g = 5\%$, it takes $\tau = 20$ years for the foreign firm to reach the technology frontier. Hence, the model quantitatively produces slow catch-up to the technological frontier.

The dynamics of the optimal contract are illustrated in Figure 1. Initially, technology transfers are limited, but over time, as more technology is transferred, transfers also ac-

[3] Jones (2016, Table 6) finds that US total factor productivity is approximately 3 times higher than an average of other countries, after adjusting for differences in human capital.

celerate to offset the increased incentive for technology stealing. At $t = \tau$, full technology transfer is reached, and the foreign firm steals the technology and produces at the frontier.

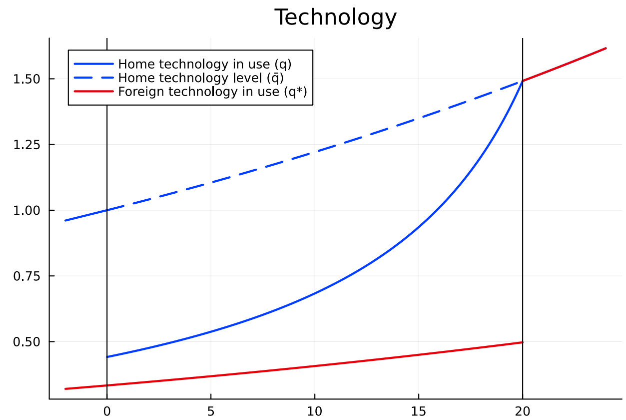

Figure 1: Home and Foreign Technology in Bertrand Competition
Parameter values in Table 1. The line at $t = 20$ denotes τ, at which full technology transfer occurs. This plot assumes balanced growth of $g = 2\%$.

I contrast the full-commitment and no-commitment cases in Figure 2. The left panel plots the dynamics of the optimal contract when the foreign government has full commitment. As shown in Proposition 2, there is full technology transfer at $t = 0$. The optimal contract simply permits no imitation by the foreign firm and no protectionism by the foreign government. This satisfies the participation constraint because the foreign firm makes no profits in the absence of technology transfer, and also makes none with it.

The right panel plots the dynamics of the optimal contract when the foreign government lacks commitment. As initial technology transfers are limited, the home firm's production costs are relatively high, and flow profits are relatively small. Over time, as more technology is transferred, the home firm's production costs decline exponentially, and flow profits increase. After full technology transfer at $t = \tau$, the foreign firm starts producing at the frontier technology, while the foreign government imposes a wedge ϕ on the home firm to raise its marginal cost to $\bar{c}$, which is its cost of producing at home. In this particular example, firm revenues, prices, quantities sold, and consumer surplus are constant over time, owing to Assumption 1. In more general settings of competition between firms, consumer surplus may increase over time as technology diffuses.

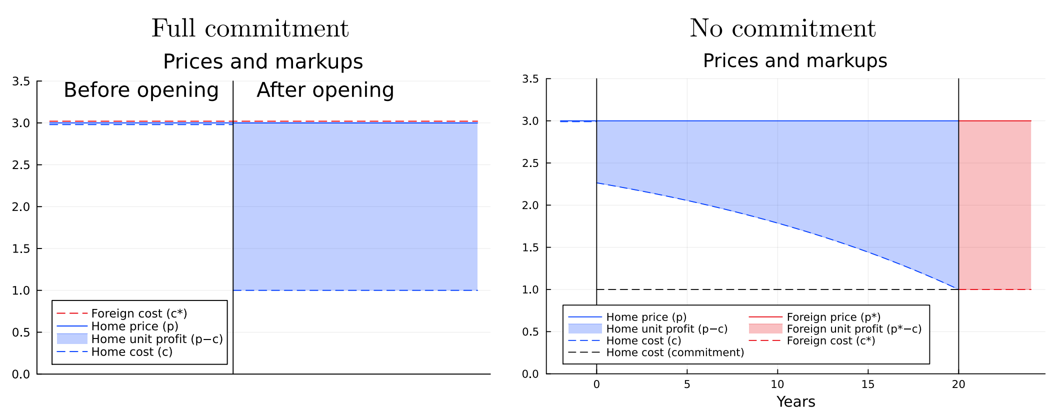

Figure 2: Revenues and Profits in Bertrand Competition
Parameter values in Table 1. The line at $t = 20$ denotes τ, the date at which full technology transfer occurs.

The evolution of each firm's value over time is shown in Figure 3. The home firm gains value from entering into foreign production, but the declines over time as t approaches τ, and becomes zero forevermore after τ. Note that the home firm prefers to make profits from transferring its technology abroad and paying lower wages, even though it knows it will have its technology stolen in the long run! In this setting, the home firm does not profit from maintaining its long-run technological lead by keeping production at home. The foreign firm's value prior to τ is the discounted present value of the profits it will make after τ, and hence increases as τ nears. Overall, even without policy commitment, technology transfers increase the combined value of the firms substantially (purple dashed line), although not as much as it would under full commitment (black dashed line).

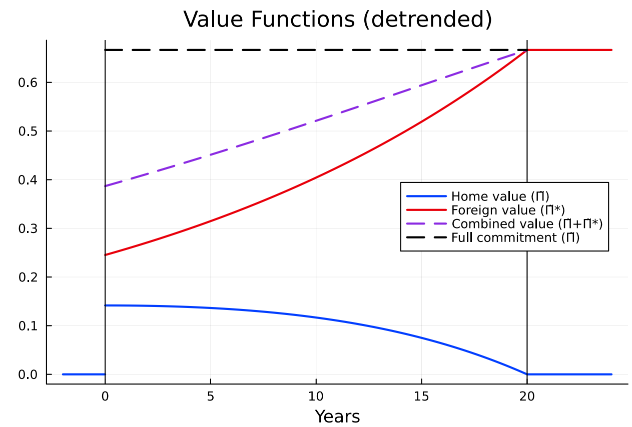

Figure 3: Home and Foreign Technology in Bertrand Competition
Parameter values in Table 1. The line at $t = 20$ denotes τ, at which full technology transfer occurs.

4 Conclusion

This paper develops a model that endogenizes the observed slow rate of technology diffusion across countries in a two-firm partial equilibrium setting along a balanced growth path. Firms transfer technology slowly to incentivize foreign governments to prevent their technology from being imitated or stolen. Hence, technology stealing occurs in the long run, only after all technology has been transferred.

Two natural directions for extensions emerge: first, it is not one firm that is offered the opportunity to produce abroad, and when many firms have the opportunity, the relative factor prices between countries will evolve over time as their relative technology levels evolve. Second, a simple assumption was made that the foreign government maximizes foreign firm profits, whereas the foreign government may also weigh consumer surplus. In an extension, there could be uncertainty about the foreign government's type, in a Fudenberg and Levine (1992) or Faingold and Sannikov (2011) setting.

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A Proofs

A.1 Proposition 5

Path of technology transfers To start, I derive the path of c. Recall that the foreign firm enjoys flow profits of $\tilde{\pi}^*(c_t, \bar{c}) = \tilde{x}(\bar{c})(\bar{c} - c_t)$ after expropriation. This has partial derivative $\partial_1 \tilde{\pi}^*(c_t, \bar{c}) = -\tilde{x}(\bar{c})$. Thus, substituting the result of Lemma 2 into the evolution of $\dot{c}_t$ given by (6), I can show that the pace of technology transfer is

$$\dot{c}_t = \frac{\tilde{r}^*[\tilde{x}(\bar{c})(\bar{c} - c_t) - 0]}{-\tilde{x}(\bar{c})} = -\tilde{r}^*(\bar{c} - c_t).$$

Rearranging and taking the integral:

$$\int_t^s \frac{\dot{c}_u}{\bar{c} - c_u} du = -\tilde{r}^* \int_t^s du,$$

and evaluating the integrals obtains

$$-\Big[\log(\bar{c} - c_u)\Big]_t^s = -\tilde{r}^* \Big[u\Big]_t^s \implies -\log(\bar{c} - c_s) + \log(\bar{c} - c_t) = -\tilde{r}^*(s - t).$$

This obtains (9) and (10).

Next, I calculate τ. It is optimal for the home firm to promise to eventually transfer all technology, so that $q_\tau = \bar{q}$, at which home's cost is $c_\tau = w^*/\bar{q}$. Substituting in $s = \tau$ into (10), we have $\bar{c} - c_\tau = (\bar{c} - c_t)e^{\tilde{r}^*(\tau - t)}$. Rearranging obtains

$$\tau = t + \frac{1}{\tilde{r}^*} \log \frac{\bar{c} - c_\tau}{\bar{c} - c_t}.$$

Value function Next, I solve for the form of the home firm's value function. Recall that under Assumption 1, home's flow profits at time s are $\tilde{x}(\bar{c}^*)(\bar{c}^* - c_s)$, and foreign's flow profits after expropriation are $\tilde{x}(\bar{c})(\bar{c} - c)$. Putting these functions and their derivatives into the HJB equation (7), I obtain

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*)(\bar{c}^* - c) + \tilde{\Pi}'(c) \frac{\tilde{r}^*}{\tilde{r}} \frac{\tilde{x}(\bar{c})(\bar{c} - c)}{-\tilde{x}(\bar{c})} = \tilde{x}(\bar{c}^*)(\bar{c}^* - c) - \tilde{\Pi}'(c) \frac{\tilde{r}^*}{\tilde{r}} (\bar{c} - c).$$

Per the usual method of solving first-order linear ODEs, let $\tilde{\Pi}(c) = v_1(c)v_2(c)$, which implies $\tilde{\Pi}'(c) = v_1'(c)v_2(c) + v_1(c)v_2'(c)$. Substituting this into the HJB and re-arranging obtains

$$v_1(c)v_2(c) + [v_1'(c)v_2(c) + v_1(c)v_2'(c)] \tfrac{\tilde{r}^*}{\tilde{r}} (\bar{c} - c) = \tilde{x}(\bar{c}^*)(\bar{c}^* - c).$$

Grouping $v_2(c)$ terms,

$$v_1(c)v_2'(c) \tfrac{r^*}{r} (\bar{c} - c) + v_2(c)[v_1(c) + v_1'(c) \tfrac{\tilde{r}^*}{\tilde{r}} (\bar{c} - c)] = \tilde{x}(\bar{c}^*)(\bar{c}^* - c). \tag{17}$$

Set the coefficient on v_2 to zero: $v_1(c) + v_1'(c) \frac{\tilde{r}^*}{\tilde{r}} (\bar{c} - c) = 0$. Then

$$\frac{v_1'(c)}{v_1(c)} = -\frac{\tilde{r}}{\tilde{r}^*} \frac{1}{\bar{c} - c} \implies \log v_1(c) = \frac{\tilde{r}}{\tilde{r}^*} \log(\bar{c} - c) + \log \kappa_1$$

where I integrate the first equation to obtain the second; $\log \kappa_1$ is the constant of integration. Exponentiating both sides obtains

$$v_1(c) = \kappa_1 (\bar{c} - c)^{\tilde{r}/\tilde{r}^*} \quad \text{and} \quad v_1'(c) = -\tfrac{\tilde{r}}{\tilde{r}^*} \kappa_1 (\bar{c} - c)^{[\tilde{r}/\tilde{r}^*] - 1}. \tag{18}$$

Substituting this back into the HJB in (17) obtains

$$\kappa_1 v_2'(c) \tfrac{\tilde{r}^*}{\tilde{r}} (\bar{c} - c)^{1 + [\tilde{r}/\tilde{r}^*]} = \tilde{x}(\bar{c}^*)(\bar{c}^* - c) \implies v_2'(c) = \frac{\tilde{x}(\bar{c}^*)}{\kappa_1} \frac{\tilde{r}}{\tilde{r}^*} \frac{\bar{c}^* - c}{(\bar{c} - c)^{1 + [\tilde{r}/\tilde{r}^*]}}.$$

It can be directly verified that for $r \neq r^*$,

$$v_2(c) = \frac{\tilde{x}(\bar{c}^*)}{\kappa_1} \frac{\bar{c}^*(1 - \frac{\tilde{r}}{\tilde{r}^*}) - \bar{c} + \frac{\tilde{r}}{\tilde{r}^*}c}{(1 - \frac{\tilde{r}}{\tilde{r}^*})(\bar{c} - c)^{\tilde{r}/\tilde{r}^*}} + \kappa_2,$$

where κ_2 is a constant of integration. Thus, multiplying $v_1(c)$ with $v_2(c)$, I obtain

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*) \frac{\bar{c}^*(1 - \frac{\tilde{r}}{\tilde{r}^*}) - \bar{c} + \frac{\tilde{r}}{\tilde{r}^*}c}{1 - \frac{\tilde{r}}{\tilde{r}^*}} + \kappa_1\kappa_2(\bar{c} - c)^{\tilde{r}/\tilde{r}^*}.$$

The boundary condition is $0 = \Pi(c_\tau)$. Hence,

$$\kappa_1\kappa_2 = -\tilde{x}(\bar{c}^*) \frac{\bar{c}^*(1 - \frac{\tilde{r}}{\tilde{r}^*}) - \bar{c} + \frac{\tilde{r}}{\tilde{r}^*}c_\tau}{(1 - \frac{\tilde{r}}{\tilde{r}^*})(\bar{c} - c_\tau)^{\tilde{r}/\tilde{r}^*}}.$$

Substituting this back into the preceding equation:

$$\begin{aligned}
\tilde{\Pi}(c) &= \frac{\tilde{x}(\bar{c}^*)}{1 - \frac{\tilde{r}}{\tilde{r}^*}} \left[\bar{c}^*(1 - \tfrac{\tilde{r}}{\tilde{r}^*}) - \bar{c} + \tfrac{\tilde{r}}{\tilde{r}^*}c - [\bar{c}^*(1 - \tfrac{\tilde{r}}{\tilde{r}^*}) - \bar{c} + \tfrac{\tilde{r}}{\tilde{r}^*}c_\tau] \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] \\
&= \tilde{x}(\bar{c}^*) \left[\Big(\bar{c}^* - \bar{c}\tfrac{\tilde{r}^*}{\tilde{r}^* - \tilde{r}} \Big) \left[1 - \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] + \tfrac{\tilde{r}}{\tilde{r}^* - \tilde{r}} c - \tfrac{\tilde{r}}{\tilde{r}^* - \tilde{r}} c_\tau \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] \\
&= \tilde{x}(\bar{c}^*) \left[\Big(\bar{c}^* - \bar{c} - \bar{c}\tfrac{\tilde{r}}{\tilde{r}^* - \tilde{r}} \Big) \left[1 - \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] + \tfrac{\tilde{r}}{\tilde{r}^* - \tilde{r}} c - \tfrac{\tilde{r}}{\tilde{r}^* - \tilde{r}} c_\tau \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] \\
&= \tilde{x}(\bar{c}^*) \left[\Big(\bar{c}^* - \bar{c} \Big) \left[1 - \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] + \tfrac{\tilde{r}}{\tilde{r}^* - \tilde{r}} \left[-\bar{c} \left[1 - \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] + c - c_\tau \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] \right].
\end{aligned}$$

Now

$$\begin{aligned}
-\bar{c} \left[1 - \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] + c - c_\tau \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} &= -(\bar{c} - c) + (\bar{c} - c_\tau) \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \\
&= -(\bar{c} - c) + (\bar{c} - c) \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{[\tilde{r}/\tilde{r}^*] - 1} \\
&= (\bar{c} - c) \left[\Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{[\tilde{r}/\tilde{r}^*] - 1} - 1 \right].
\end{aligned}$$

Hence, the home firm's value function is

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*) \left[\Big(\bar{c}^* - \bar{c} \Big) \left[1 - \Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{\tilde{r}/\tilde{r}^*} \right] + \frac{\tilde{r}}{\tilde{r}^* - \tilde{r}} (\bar{c} - c) \left[\Big(\frac{\bar{c} - c}{\bar{c} - c_\tau} \Big)^{[\tilde{r}/\tilde{r}^*] - 1} - 1 \right] \right].$$

This obtains (11).

Meanwhile, for $r = r^*$, it can be directly verified that

$$v_2(c) = \frac{\tilde{x}(\bar{c}^*)}{\kappa_1} \left[\frac{\bar{c}^* - \bar{c}}{\bar{c} - c} - \log(\bar{c} - c) \right] + \kappa_2$$

where κ_2 is a constant of integration. Thus, multiplying $v_1(c)$ with $v_2(c)$, I obtain

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*)(\bar{c} - c) \left[\frac{\bar{c}^* - \bar{c}}{\bar{c} - c} - \log(\bar{c} - c) \right] + \kappa_1\kappa_2(\bar{c} - c).$$

The boundary condition is $0 = \Pi(c_\tau)$. Hence,

$$\kappa_1\kappa_2 = -\tilde{x}(\bar{c}^*) \left[\frac{\bar{c}^* - \bar{c}}{\bar{c} - c_\tau} - \log(\bar{c} - c_\tau) \right].$$

Substituting this back into the preceding equation:

$$\begin{aligned}
\tilde{\Pi}(c) &= \tilde{x}(\bar{c}^*)(\bar{c}-c)\left[\frac{\bar{c}^*-\bar{c}}{\bar{c}-c} - \log(\bar{c}-c) - \frac{\bar{c}^*-\bar{c}}{\bar{c}-c_\tau} + \log(\bar{c}-c_\tau)\right] \\
&= \tilde{x}(\bar{c}^*)(\bar{c}-c)\left[(\bar{c}^*-\bar{c})\left(\frac{1}{\bar{c}-c} - \frac{1}{\bar{c}-c_\tau}\right) - \log\frac{\bar{c}-c}{\bar{c}-c_\tau}\right] \\
&= \tilde{x}(\bar{c}^*)(\bar{c}-c)\left[(\bar{c}^*-\bar{c})\left(\frac{c-c_\tau}{(\bar{c}-c)(\bar{c}-c_\tau)}\right) - \log\frac{\bar{c}-c}{\bar{c}-c_\tau}\right]
\end{aligned}$$

and hence the home firm's value function is

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*)\left[(\bar{c}^*-\bar{c})\frac{c-c_\tau}{\bar{c}-c_\tau} - (\bar{c}-c)\log\frac{\bar{c}-c}{\bar{c}-c_\tau}\right].$$

This obtains the value function (12).

Initial technology transfer At $t=0$, the home firm selects c_0 at the global maximum of Π. For $r \neq r^*$, the derivative of $\tilde{\Pi}$ with respect to c is

$$\begin{aligned}
\tilde{\Pi}'(c) = \tilde{x}(\bar{c}^*)\Bigg[&\frac{\bar{c}^*-\bar{c}}{\bar{c}-c_\tau}\frac{\tilde{r}}{\tilde{r}^*}\left(\frac{\bar{c}-c}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - \frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}\frac{\tilde{r}-\tilde{r}^*}{\tilde{r}^*}\frac{\bar{c}-c}{\bar{c}-c}\left(\frac{\bar{c}-c}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} \\
&- \frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}\left[\left(\frac{\bar{c}-c}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - 1\right]\Bigg],
\end{aligned}$$

which simplifies to

$$\tilde{\Pi}'(c) = \tilde{x}(\bar{c}^*)\left[\frac{\bar{c}^*-\bar{c}}{\bar{c}-c_\tau}\frac{\tilde{r}}{\tilde{r}^*}\left(\frac{\bar{c}-c}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} + \frac{\tilde{r}}{\tilde{r}^*}\left(\frac{\bar{c}-c}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - \frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}\left[\left(\frac{\bar{c}-c}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - 1\right]\right],$$

and further simplifies to

$$\tilde{\Pi}'(c) = \tilde{x}(\bar{c}^*)\left[\left(\frac{\bar{c}^*-\bar{c}}{\bar{c}-c_\tau}\frac{\tilde{r}}{\tilde{r}^*} + \frac{\tilde{r}}{\tilde{r}^*} - \frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}\right)\left(\frac{\bar{c}-c}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} + \frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}\right],$$

For $r \neq r^*$, the first-order condition of Π with respect to c is $0 = \Pi'(c_0)$, which can be rearranged to show

$$\left(\frac{\bar{c}^*-\bar{c}}{\bar{c}-c_\tau}\frac{\tilde{r}}{\tilde{r}^*} + \frac{\tilde{r}}{\tilde{r}^*} - \frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}\right)\left(\frac{\bar{c}-c_0}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} = -\frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}$$

and hence

$$\begin{aligned}
\left(\frac{\bar{c}-c_0}{\bar{c}-c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} &= -\frac{\frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}}{\frac{\bar{c}^*-\bar{c}}{\bar{c}-c_\tau}\frac{\tilde{r}}{\tilde{r}^*} + \frac{\tilde{r}}{\tilde{r}^*} - \frac{\tilde{r}}{\tilde{r}^*-\tilde{r}}} \\
&= \frac{1}{1 - \frac{\tilde{r}^*-\tilde{r}}{\tilde{r}^*}[\frac{\bar{c}^*-\bar{c}}{\bar{c}-c_\tau} + 1]} \\
&= \frac{1}{1 - \frac{\tilde{r}^*-\tilde{r}}{\tilde{r}^*}\frac{\bar{c}^*-c_\tau}{\bar{c}-c_\tau}},
\end{aligned}$$

where the second lines from dividing the numerator and denominator through by $\tilde{r}/(\tilde{r}^*-\tilde{r})$. Hence,

$$\bar{c} - c_0 = (\bar{c}-c_\tau)\left[1 - \frac{\tilde{r}^*-\tilde{r}}{\tilde{r}^*}\frac{\bar{c}^*-c_\tau}{\bar{c}-c_\tau}\right]^{\tilde{r}^*/(\tilde{r}^*-\tilde{r})}.$$

This obtains the expression for c_0 in (13).

For $r = r^*$, the first-order condition of Π with respect to c is

$$0 = \tilde{\Pi}'(c_0) = \tilde{x}(\bar{c}^*)\left[(\bar{c}^* - \bar{c})\frac{1}{\bar{c} - c_\tau} + \log\frac{\bar{c} - c_0}{\bar{c} - c_\tau} + 1\right].$$

Rearranging,

$$\log\frac{\bar{c} - c_0}{\bar{c} - c_\tau} = -\left[1 + \frac{\bar{c}^* - \bar{c}}{\bar{c} - c_\tau}\right].$$

Hence,

$$\log\frac{\bar{c} - c_0}{\bar{c} - c_\tau} = -\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau},$$

and hence

$$\bar{c} - c_0 = (\bar{c} - c_\tau)\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right).$$

This obtains the expression for c_0 in (14).

Initial value Next, I solve for $\Pi(c_0)$. For $r \neq r^*$, to start, rearrange (13) to

$$\frac{\bar{c} - c_0}{\bar{c} - c_\tau} = \left[1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right]^{\frac{\tilde{r}^*}{\tilde{r}^* - \tilde{r}}}.$$

Substituting this into (11), I obtain

$$\begin{aligned}\Pi(c_0) = \tilde{x}(\bar{c}^*)\Bigg[&(\bar{c}^* - \bar{c})\left[1 - \left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}}\right] \\ &+ \frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}(\bar{c} - c_\tau)\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}^*}{\tilde{r}^* - \tilde{r}}}\left[\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{-1} - 1\right]\Bigg].\end{aligned}$$

Now

$$\begin{aligned}&\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{-1} - 1 \\ &= \frac{\tilde{r}^*(\bar{c} - c_\tau)}{\tilde{r}^*(\bar{c} - c_\tau) - (\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)} - \frac{\tilde{r}^*(\bar{c} - c_\tau) - (\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)}{\tilde{r}^*(\bar{c} - c_\tau) - (\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)} \\ &= \frac{(\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)}{\tilde{r}^*(\bar{c} - c_\tau) - (\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)}\end{aligned}$$

and hence

$$\begin{aligned}\tilde{\Pi}(c_0) = \tilde{x}(\bar{c}^*)\Bigg[&(\bar{c}^* - \bar{c})\left[1 - \left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}}\right] \\ &+ \tilde{r}\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}^*}{\tilde{r}^* - \tilde{r}}}\frac{(\bar{c} - c_\tau)(\bar{c}^* - c_\tau)}{\tilde{r}^*(\bar{c} - c_\tau) - (\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)}\Bigg].\end{aligned}$$

Under Bertrand competition, in technological autarky, if $\bar{c} \leq \bar{c}^*$, the home firm's profits are $\tilde{\Pi}^a = \tilde{x}(\bar{c}^*)(\bar{c}^* - \bar{c})$. Hence, the net gain to the home firm of producing abroad is

$$\begin{aligned}\tilde{\Pi}(c_0) - \tilde{\Pi}^a = \tilde{x}(\bar{c}^*)\Bigg[&-(\bar{c}^* - \bar{c})\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}} \\ &+ \tilde{r}\left(1 - \frac{\tilde{r}^* - \tilde{r}}{\tilde{r}^*}\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)^{\frac{\tilde{r}^*}{\tilde{r}^* - \tilde{r}}}\frac{(\bar{c} - c_\tau)(\bar{c}^* - c_\tau)}{\tilde{r}^*(\bar{c} - c_\tau) - (\tilde{r}^* - \tilde{r})(\bar{c}^* - c_\tau)}\Bigg].\end{aligned}$$

This obtains the expression in (15).

For $r = r^*$, note from (14) that

$$c_0 - c_\tau = (\bar{c} - c_\tau) - (\bar{c} - c_0) = (\bar{c} - c_\tau)\left[1 - \exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)\right].$$

Inserting this result and (14) into (12) obtains

$$\begin{aligned}
\tilde{\Pi}(c_0) &= \tilde{x}(\bar{c}^*)\left[(\bar{c}^* - \bar{c})\left[1 - \exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)\right] - (\bar{c} - c_\tau)\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)\log\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)\right] \\
&= \tilde{x}(\bar{c}^*)\left[(\bar{c}^* - \bar{c})\left[1 - \exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)\right] + (\bar{c}^* - c_\tau)\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)\right] \\
&= \tilde{x}(\bar{c}^*)\left[(\bar{c}^* - \bar{c}) + (\bar{c} - c_\tau)\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right)\right].
\end{aligned}$$

The net gain to the home firm of producing abroad is

$$\tilde{\Pi}(c_0) - \tilde{\Pi}^a = \tilde{x}(\bar{c}^*)(\bar{c} - c_\tau)\exp\left(-\frac{\bar{c}^* - c_\tau}{\bar{c} - c_\tau}\right).$$

This obtains the expression in (16).

A.2 Proposition 6

For the $r = r^*$ case, it suffices to examine (16). I established in (14) that $c_0 < \bar{c}$. By the irreversibility of technology transfers, $c_\tau \le c_0 < \bar{c}$. Substituting this result into (16), I can deduce that $\tilde{\Pi}(c_0) - \tilde{\Pi}^a > 0$.

For the $r^* > r$ case, start by examining equation (11), which is reproduced here for convenience:

$$\tilde{\Pi}(c) = \tilde{x}(\bar{c}^*)\left[(\bar{c}^* - \bar{c})\left[1 - \left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{\tilde{r}/\tilde{r}^*}\right] + \frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}(\bar{c} - c)\left[\left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - 1\right]\right].$$

Hold r constant and take $r^* \to \infty$. Thus, $\tilde{r}/\tilde{r}^* \to 0$. For the first time inside the brackets on the RHS,

$$\left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{\tilde{r}/\tilde{r}^*} \to 1 \implies 1 - \left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{\tilde{r}/\tilde{r}^*} \to 0.$$

Also,

$$\left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - 1 \to \frac{\bar{c} - c_\tau}{\bar{c} - c} - 1 < \infty$$

and hence

$$\frac{\tilde{r}}{\tilde{r}^* - \tilde{r}}(\bar{c} - c)\left[\left(\frac{\bar{c} - c}{\bar{c} - c_\tau}\right)^{[\tilde{r}/\tilde{r}^*]-1} - 1\right] \to 0,$$

so $\tilde{\Pi}(c) \to 0$ for all $c < \bar{c}$, and hence $\tilde{\Pi}(c_0) \to 0$. Under Bertrand competition, in technological autarky, if $\bar{c}^* > \bar{c}$ as assumed in the statement of the proposition, the home firm's profits are $\tilde{\Pi}^a = \tilde{x}(\bar{c}^*)(\bar{c}^* - \bar{c}) > 0$. Hence, $\tilde{\Pi}^a > 0$, and hence $\tilde{\Pi}(c_0) - \tilde{\Pi}^a < 0$ for sufficiently large r^*.