Exchange Rates and International Risk Sharing under Home Portfolio Bias*

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Abstract

The standard view in international macroeconomics is that exchange rate dynamics are inconsistent with the notion that international financial markets enable countries to share risk effectively. I develop a model of international financial frictions that is consistent with two facts about international asset holdings: (i) home portfolio bias and (ii) the elasticity of substitution in international portfolio choice, and I show that these portfolio facts characterize the extent to which countries share risk in equilibrium. When matched to observed portfolio allocations and elasticities, the model implies extensive international risk sharing, yet it solves the key Backus-Smith exchange rate puzzle, which is that a country's consumption increases when its consumption bundle becomes more expensive (a real exchange rate appreciation). In the model, a shock that increases the relative demand for a country's goods raises their price and increases their firms' profits; under home portfolio bias, it also raises the relative income of domestic households, who own most of the country's firms, so they consume more. More generally, this mechanism delivers the procyclical, volatile, and persistent exchange rates seen in the data, whereas other popular shocks in the literature cannot do so when matched to observed portfolios.

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A central question in international macroeconomics is the extent to which international financial markets allow countries to mitigate aggregate shocks. Despite substantial cross-border asset holdings, the standard view is not much: if investors were able to trade financial assets to fully share risk amongst themselves, then a country's consumption would decline relative to other countries when its exchange rate *appreciates*. This is inconsistent with the data, which show that a country's consumption declines when its exchange rate *depreciates*: this is the Backus and Smith (1993) puzzle. Full risk sharing therefore predicts the wrong correlation between exchange rates and the state of the economy. In response to this fundamental challenge for models of open economies, researchers have shown that models that shut down most cross-border asset trade, so as to dramatically weaken international risk sharing, do make progress on matching exchange rate dynamics. However, the deeper puzzle remains: how is it that in the world, cross-border asset trade is substantial, but exchange rate dynamics seem to require that the same asset trade is severely limited?

In this paper, I show that facts about foreign asset holdings discipline the amount of international risk sharing. To do so, I build a simple model of international business cycles with financial frictions, where households must hold money to buy goods, with the key friction that repatriating foreign income to obtain money requires costly financial intermediation. My model is consistent with two key facts about foreign asset holdings: (i) home portfolio bias, which is the fraction of wealth that investors hold in domestic assets rather than foreign assets, and (ii) the elasticity of substitution of portfolio holdings across countries in response to expected returns, which I refer to as the *portfolio elasticity*. In the model, these two portfolio facts characterize how much international risk sharing occurs in equilibrium.

I then show that a model with financial frictions of the size needed to match the portfolio facts suffices to quantitatively resolve the Backus-Smith puzzle, and is consistent with two other major exchange rate patterns: first, exchange rates are much more volatile than most macroeconomic aggregates; and second, exchange rate fluctuations are persistent, with a half life of around 5 years (the *purchasing power parity* puzzle of Rogoff, 1996). In an extension, I show that departing from standard preferences also allows the model to match key correlations between exchange rates and international asset prices.

Why does the Backus-Smith puzzle emerge with full risk sharing? In the international setting, different countries consume different goods, so full risk sharing equalizes not marginal utility across countries, but rather the marginal utility *per additional dollar*. Thus, if a country's consumption declines, so the marginal utility of its households increases, it can only be that the value of an additional dollar has declined for them. This means that its domestic price level, expressed in dollars, must have increased relative to other countries, which is a real exchange rate appreciation. But in the data, a decrease in a country's relative consumption is correlated with exchange rate depreciation.

In my model, a combination of home portfolio bias and shocks to the relative demand for country-specific output goods resolves the Backus-Smith puzzle. When the relative demand for American goods increases, they become more expensive relative to foreign goods. American households, who mostly consume American goods, face a higher price level relative to foreign households, which is a US real exchange rate appreciation. American firms, which produce American goods, become more profitable and pay out more dividends to investors. With imperfect financial markets, an increase in American dividends generically increases the return on American-originated securities relative to their foreign counterparts to maintain securities market clearing. Under home portfolio bias, American households own mostly American securities, so they become relatively wealthier, and they consume more. Hence, American households increase their relative consumption following a US real exchange rate appreciation, as in the data, resolving the Backus-Smith puzzle.

The key difference in my setting is that marginal utility per dollar is no longer equalized across countries: instead, the amount of international risk sharing, which I define as the cross-country co-movement of the marginal utility per dollar, is determined by the amount of home portfolio bias and the portfolio elasticity. Nevertheless, a quantification of the model suggests that marginal utility per dollar does still co-move strongly: the Backus-Smith puzzle is neither evidence that international risk sharing is especially weak, nor evidence that international financial markets fail to allow countries to mitigate aggregate shocks.

In Section 1, I build on the international business cycle model of Backus, Kehoe and Kydland (1993). Households trade a complete set of contingent claims to dividend income (as in Arrow, 1964 and Debreu, 1959), capturing the wide variety of assets traded across countries. Following the international monetary model of Lucas (1982), I introduce money as a payment technology: consumers must buy goods with money. The model introduces a financial friction: consumers must procure costly financial services to turn dividend income into money. The need for financial services varies by household, who are therefore heterogeneous, and by the country the income originates from.

The average household needs to use more financial services to repatriate foreign dividend income relative to domestic income, making foreign asset holdings less attractive. Section 2 shows that these financial frictions introduce home bias in aggregate country-level portfolios. Section 3 shows how aggregate portfolios characterize international risk sharing, and breaks down the model's mechanism for how home portfolio bias and relative demand shocks resolve the Backus-Smith puzzle. Conversely, productivity shocks raise the supply of a country's good and lower its price while increasing its consumption; hence, they continue to drive the opposite correlation between relative consumption and real exchange rate.

In Section 4, I quantify the model to match the portfolio facts, and show that home portfolio bias and relative demand shocks are key to matching the exchange rate facts. The model also matches classical business cycle moments. Then I apply a Kalman filter to reconstruct the time series of US productivity and relative demand shocks using business cycle data. In line with the business cycle literature, it shows that productivity shocks drive most GDP and consumption fluctuations, but relative demand shocks drive most exchange rate fluctuations. However, a counterfactual decomposition indicates that both productivity and relative demand shocks are needed to match the Backus-Smith correlation seen in the data.

Section 5 considers two other popular explanations for the Backus-Smith puzzle. I add intermediation shocks as in Itskhoki and Mukhin (2021) and discount factor shocks as in Kekre and Lenel (2024a) to the model, and re-run the Kalman filter to back out the time series of these shocks. The results suggest that there is little room for these shocks to explain exchange rate movements. This is because in my environment where foreign asset holdings are as in the data, there is substantially more international risk sharing than in their environments, so these shocks do not generate large exchange rate volatility.

Section 6 extends the model with habit-formation preferences, which generate quantitatively-plausible risk premia. I show that when currencies are correlated with asset prices as they are in the data, the model predicts that *safe* currencies—those that tend to appreciate during global financial crises—pay investors lower returns on average, instead of the same average returns predicted by the doctrine of *uncovered interest parity.* In other words, investors demand less compensation for exchange rate risk on safe currencies. In equilibrium, these lower returns are delivered in the form of lower interest rates, which makes progress on the currency premium puzzle of Hassan, Mertens and

Wang (2024).

Literature French and Poterba (1991) document that home portfolio bias persisted despite moves toward open financial markets in the 1980s, and consider various international financial frictions that might inhibit portfolio diversification. The large following literature is surveyed by Lewis (1999) and Coeurdacier and Rey (2013). One popular approach posits that holding home assets helps households hedge against various kinds of risk. For example, Pavlova and Rigobon (2007) show that home assets help households hedge against exchange rate risk caused by discount rate shocks: households have a high discount rate precisely when domestic goods are expensive.[1] The main difficulty faced by this set of models is that asset trade allows households to implement perfect international risk sharing, which is inconsistent with exchange rate facts.[2] Hence, returning to the approach of French and Poterba (1991), I let international financial frictions determine portfolio allocations.

This paper also develops the general equilibrium macroeconomic implications of the new literature on inelastic international financial markets following Koijen and Yogo (2020), Camanho, Hau and Rey (2022), and Jiang, Richmond and Zhang (2024b, 2025), who provide estimates of the portfolio elasticity. In this sense, it complements the work of Kleinman, Liu, Redding and Yogo (2023), who discuss its implications for convergence to steady state in the neoclassical growth model.

My treatment of the exchange rate puzzles has been influenced by a literature that documents mechanisms that unsatisfactory to resolve these puzzles. Chari, Kehoe and McGrattan (2002) show that monetary shocks and nominal price rigidities replicate neither the cyclicality nor the persistence of exchange rate fluctuations. They and Lustig and Verdelhan (2019) show that restricting international asset trade to only nominal risk-free bonds also does not help explain the cyclicality of exchange rate fluctuations.[3] This complements earlier work by Baxter and Crucini (1995), Kollmann (1996), Heathcote and Perri (2002), and Kehoe and Perri (2002), who show that such restrictions do not

[1] Another approach posits that holding home equities helps households insure against non-insurable labor income risk. While Baxter and Jermann (1997) show that this approach fails in the simplest one-good setting, Heathcote and Perri (2013) show that this mechanism helps in economies with multiple goods and investment. Coeurdacier and Gourinchas (2016) show that adding international trade in bonds also overturns Baxter and Jermann's result.

[2] Say if there are N countries, and each country has just two assets that can be traded (a stock and a bond), and no asset is redundant. To avoid perfect international risk sharing, there must be risks that are not spanned by the asset payoffs, so the model needs at least $2N + 1$ shock processes. As the number of tradable assets increases, the required number of shocks becomes more demanding.

[3] Some recent work finds that a combination of domestic and international market incompleteness may resolve the puzzle, under certain assumptions about the correlation between idiosyncratic uninsurable household-level risk and aggregate consumption (Marin and Singh, 2025; Acharya, Challe and Coulibaly, 2025).

improve on the fit of the international business cycle model of Backus, Kehoe and Kydland (1992). It also complements Cole and Obstfeld (1991), who show that asset trade is redundant with sufficient goods trade. Hence, my mechanism relies neither on nominal rigidities, nor on restrictions on the span of traded assets.

Models of international risk sharing and exchange rates can be roughly divided into three categories, depending upon their degree of international asset market segmentation. The first category of papers allow investors to trade a complete set of financial securities across countries. This approach is taken by many models which link exchange rates with a broader class of risky assets, such as Verdelhan (2010), Colacito and Croce (2011, 2013), and Colacito, Croce, Ho and Howard (2018), as the First Welfare Theorem makes computing allocations and asset prices particularly tractable. As investors implement full risk sharing when allowed to trade a complete set of securities, the positive correlation between relative consumption and exchange rate appreciations is the key puzzle for these papers. I show how to resolve the puzzle while retaining the tractability afforded by complete securities markets.

The second category features some degree of international financial frictions. Existing approaches include limited financial participation (Alvarez, Atkeson and Kehoe, 2002, 2009), portfolio adjustment costs (Fukui, Nakamura and Steinsson, 2023; Guo, Ottonello and Perez, 2023), and convenience yields (Jiang, Krishnamurthy and Lustig, 2023; Jiang, Krishnamurthy, Lustig and Sun, 2024a; Kekre and Lenel, 2024b). I show that these mechanisms can be disciplined by international portfolio facts.

A third category features a strong degree of segmentation between asset markets between countries. Corsetti, Dedola and Leduc (2008) make progress on the cyclicality puzzle with a low trade elasticity and restricting international asset trade to one-period bonds denominated in a global numéraire. A popular recent strand of literature follows Gabaix and Maggiori (2015): in these models, households and firms are limited to trading assets domestically; international asset trade is restricted to risk-averse financial intermediaries. In particular, Itskhoki and Mukhin (2021) and papers that borrow its mechanism, such as Kekre and Lenel (2024a), do resolve many of the exchange rate puzzles, but they ignore that foreign asset holdings allow countries to share risk: instead, their foreign portfolio shares are essentially zero.[4] I show that the exchange rate puzzles can be resolved in a model that is consistent with international portfolio facts and amenable to standard approaches to asset pricing.

[4]In Itskhoki and Mukhin (2021), financial intermediaries sell short-term bonds in one currency and buy short-term bonds in the other currency. There are no international equity holdings.

1 Model

The world economy has countries $i = 1, \dots, N$. Within each country lives a large number of infinitely-lived households and firms. In each country, the identical home firms own physical capital and produce a traded country-specific intermediate good by hiring labor from home households, as in the real business cycle model of Backus, Kehoe and Kydland (1992). Final goods producers combine intermediate goods from different countries to produce non-traded final goods that households consume and non-traded capital goods that firms invest in. The home intermediate good makes up a large share of home final goods and a small share of foreign final goods, reflecting home bias in consumption.

Each country has its own currency, in which local prices are denominated. In particular, country i's final good costs P_i^F units of local currency. The nominal exchange rate $\mathcal{E}_{ij}$ is the price of a unit of country j's currency in country i's currency. More simply, it is the units of i's currency per unit of j's currency; a good that costs P units of j's currency equivalently costs $\mathcal{E}_{ij}P$ units of i's currency. A higher $\mathcal{E}_{ij}$ means that j's currency appreciates, and i's currency depreciates. The real exchange rate $\mathcal{Q}_{ij}$ is the relative price of each country's (non-traded) final good:

$$\mathcal{Q}_{ij} \equiv \frac{P_j^F \mathcal{E}_{ij}}{P_i^F}.$$

In the data, real and nominal exchange rates move close to one-for-one (see Figure 9), so here I assume they move one-for-one.

1.1 Timing

The timing within each period t is a slight modification of the international monetary model of Lucas (1982). Before any actions are taken, the state of the world s^t is revealed. I use $s^t = (s_0, \dots, s_t)$ to denote the history of events from the beginning of time through to t. The probability at time 0 of a particular history s^t is $\pi(s^t)$. The timing of a household's actions is illustrated in Figure 1.

At the beginning of the period, intermediate goods firms hire workers to produce output, which they sell to final goods producers on credit, and credit wages to workers and dividend payouts to investors. Next, an asset market opens in each country, where households trade the securities of that country and may take out dividend income from their investment position. Simultaneously, a foreign exchange market opens, where households and final goods producers arrange to exchange currencies. Settlement in these markets is delayed until the end of the period. After asset trading

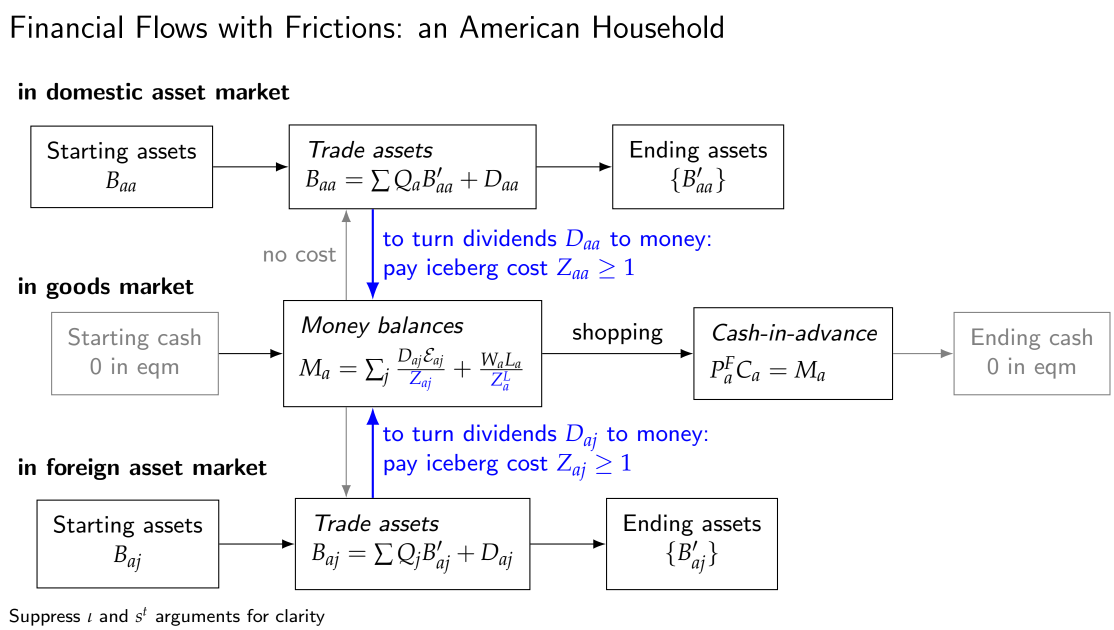

Figure 1: Household financial flows and timing

ends, households convert their promised wages and dividends into money, which is the payment technology used in the goods market.[5] The friction is that households need to use financial services to obtain money. After receiving money, households trade it for goods with final goods producers.

At the end of the period, settlement occurs in the intermediate goods market, each country's asset market, and foreign exchange market: final goods producers use their accumulated money to settle their transactions with intermediate firms, who in turn use the money to settle their wage and dividend obligations, which was used to back the money that households received. With the obligations fulfilled, money is withdrawn from the economy.

In this setup, money is used as a unit of value and as a medium of exchange, but not as a store of value: no money is held across periods, and therefore money imposes no distortions on the real economy other than the financial services consumed in its creation.

1.2 Households

Let ι denote a household in country i.

In this economy, each country has an asset market in which households may trade a complete set of

[5] Lucas (1980) models why money may be an efficient payments technology for goods markets.

state-contingent one-period nominal bonds, denominated in the local currency. Let $B_{ij}(\iota, s^{t+1})$ denote household ι's holdings of bonds in country j's asset markets, which pay out $B_{ij}(\iota, s^{t+1})$ units of j's currency if the particular state s^{t+1} occurs, and 0 otherwise. Let $Q_j(s_{t+1} \mid s^t)$ denote the price of this bond in units of currency j at time t. The price of this bond in time 0 is $Q_j(s^{t+1}) = \prod_{\tau=0}^{t} Q_j(s_{\tau+1} \mid s^\tau)$.

In each country j's asset market, household ι faces a sequence of budget constraints

$$\sum_{s_{t+1}} Q_j(s_{t+1} \mid s^t) B_{ij}(\iota, s^{t+1}) + D_{ij}(\iota, s^t) \leq B_{ij}(\iota, s^t), \tag{1}$$

where the household takes $D_{ij}(\iota, s^t)$ units of j's currency out for consumption as dividends, and reinvests the rest. At time 0, the household faces a budget constraint

$$\sum_j Q_j(s^0)\mathcal{E}_{ij}(s^0) B_{ij}(\iota, s^0) \leq \bar{B}_i(\iota) \tag{2}$$

that limits the value of its initial portfolio holdings by its endowment $\bar{B}_i(\iota)$, which is specified in country i's currency.

Households must hold money $M_i(\iota, s^t)$ in order to buy goods, imposing a *cash-in-advance* constraint:

$$P_i^F(s^t) C_i^g(\iota, s^t) \leq M_i(\iota, s^t), \tag{3}$$

where $P_i^F(s^t)$ is the price of the final good consumed by country i's households, and $C_i^g(\iota, s^t)$ is household ι's consumption of goods. I distinguish consumption of goods, which directly contribute to household utility, from household consumption of financial services, which do not. Instead, financial services comprise all of the services that sit between a household's receipt of income and a household's receipt of goods: maintenance of a bank account, interest rate spreads on loans and mortgages, payments (e.g. credit cards), insurance, etc.

I model financial services as a heterogeneous iceberg cost that households pay to obtain money from its dividend and wage income. For dividend income originating in country j, the household ι must expend a fraction $1 - 1/Z_{ij}(\iota)$ of the income on financial services, and receives the remaining fraction $1/Z_{ij}(\iota)$ as money. Hence, in state s^t, given a household who takes out $D_{ij}(\iota, s^t)$ in dividend income from each country j, the household realizes the amount of money

$$\sum_j \frac{D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{Z_{ij}(\iota)}.$$

Households are heterogeneous in their $Z_{ij}(\iota)$, but these costs are otherwise non-stochastic. For the average household ι, $Z_{ij}(\iota)$ will be larger than $Z_{ii}(\iota)$, so that the household will optimally choose to hold a home-biased portfolio.

Household ι supplies $L_i(\iota, s^t)$ units of labor, and under the prevailing wage of $W_i(s^t)$, it receives labor income $W_i(s^t)L_i(\iota, s^t)$. Country i households similarly expend a fraction $1 - 1/Z_i^L$ of their labor income on financial services in order to convert it into money. Household ι's money balances comprise its dividend and labor income net of financial services costs, and unspent balance from the previous period:

$$M_i(\iota, s^t) = \sum_j \frac{D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{Z_{ij}(\iota)} + \frac{W_i(s^t)L_i(\iota, s^t)}{Z_i^L} + \big[M_i(\iota, s^{t-1}) - P_i^F(s^{t-1})C_i(\iota, s^{t-1})\big]. \tag{4}$$

Proceeding under the usual assumption that monetary policy keeps the nominal interest rate positive, households do not carry money balances across periods. Thus, using the law of motion of money balances (4), the cash-in-advance constraint (3) can be written independently of money balances as

$$P_i^F(s^t)C_i^g(\iota, s^t) = \sum_j \frac{D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{Z_{ij}(\iota)} + \frac{W_i(s^t)L_i(\iota, s^t)}{Z_i^L}. \tag{5}$$

Household ι's problem is to choose paths of consumption of goods $C_i^g(\iota, s^t)$, labor $L_i(\iota, s^t)$, and asset portfolio $B_{ij}(\iota, s^t)$ to maximize its expected utility

$$\sum_t \sum_{s^t} \beta^t \pi(s^t) u\big(C_i^g(\iota, s^t) - v(L_i(\iota, s^t))\big) \tag{6}$$

subject to budget constraints (1) and (2), cash-in-advance constraint (5), and no-short-sale constraint $D_{ij}(\iota, s^t) \geq 0$ for all j and s^t. Flow utility u is power utility (e.g. CRRA or Campbell and Cochrane habits), and the disutility of labor $v(L)$ follows Greenwood, Hercowitz and Huffman (1988, hereafter GHH) as being stated in consumption units, which will be convenient for proving an aggregation result later. GHH preferences increase the household's marginal utility of consumption when they supply more labor.

Household ι's first-order condition for labor equates the disutility of labor with the real wage:

$$v'(L_i(\iota, s^t)) = \frac{W_i(s^t)}{P_i^F(s^t)Z_i^L}. \tag{7}$$

Household ι's portfolio composition is determined by its idiosyncratic draw of $Z_{ij}(\iota)$ for $j = 1, \ldots, N$: as the cash-in-advance constraint (5) is linear in dividend income $D_{ij}(\iota, s^t)$, in each state s^t household ι chooses to take income only from the country with the highest return after adjusting for Z:

$$j(\iota, s^t) = \arg\max_j \frac{\mathcal{E}_{ij}(s^t)}{Q_j(s^t)\mathcal{E}_{ij}(s^0)Z_{ij}(\iota)}. \tag{8}$$

1.3 Firms

Each country's firms produce a country-specific traded intermediate good. A representative firm in country i enters the state s^t with capital $K_i(s^{t-1})$, whereupon it is subjected to a productivity shock to labor-augmenting technology $A_i(s^t)$. Technology has a global component a_{Gt} and a country-specific idiosyncratic component a_{Iit}:

$$\log A_i(s^t) \equiv a_{it} = \Gamma_i^a a_{Gt} + a_{Iit},$$

where Γ_i^a denotes the loading of country i on the global component a_{Gt}. The two processes a_{Gt} and a_{Iit} are AR(1) with persistence ρ_a:

$$\begin{aligned} a_{Gt+1} &= \rho_a a_{Gt} + \varepsilon_{Gt+1} \\ a_{Iit+1} &= (1-\rho_a)\log \bar{A}_i + \rho_a a_{Iit} + \varepsilon^a_{Iit+1}, \end{aligned}$$

where $\bar{A}_i$ is country i's steady state productivity, and the innovations ε have mean zero.

The firm hires labor $L_i(s^t)$ from i's households to operate its capital, thus producing a quantity

$$Y_i(s^t) = A_i(s^t) K_i(s^{t-1})^{\alpha} L_i(s^t)^{1-\alpha} \tag{9}$$

of the country i-specific intermediate good, which it sells at the competitive price $P_i^X(s^t)$. The firm chooses its labor demand to maximize its operating income

$$R_i^K(s^t)K_i(s^{t-1}) \equiv \max_{L_i(s^t)} \Big\{ P_i^X(s^t) A_i(s^t) K_i(s^{t-1})^{\alpha} L_i(s^t)^{1-\alpha} - W_i(s^t) L_i(s^t) \Big\}.$$

The firm's first-order condition for labor is

$$W_i(s^t) = (1-\alpha) P_i^X(s^t) A_i(s^t) \left(\frac{K_i(s^{t-1})}{L_i(s^t)} \right)^{\alpha}. \tag{10}$$

The firm's labor demand and operating income are, respectively,

$$L_i(s^t) = \left[(1-\alpha) A_i(s^t) \frac{P_i^X(s^t)}{W_i(s^t)} \right]^{\frac{1}{\alpha}} K_i(s^{t-1}). \tag{11}$$

$$R_i^K(s^t) K_i(s^{t-1}) = \frac{\alpha}{1-\alpha} W_i(s^t) L_i(s^t). \tag{12}$$

The firm chooses to invest an amount $I_i^K(s^t)$ in new capital, with each unit of investment costing $P_i^K(s^t)$. The firm issues its remaining free cash flow $R_i^K(s^t)K_i(s^{t-1}) - P_i^K(s^t) I_i^K(s^t)$ as dividend income to households who hold state-contingent securities.

The firm chooses a path of investment $I_i^K(s^t)$ to maximize the present value of its dividend payouts

$$\sum_{t=0}^{\infty}\sum_{s^t} Q_i(s^t)\big[R_i^K(s^t)K_i(s^{t-1}) - P_i^K(s^t)I_i^K(s^t)\big] \tag{13}$$

where the second expression follows from the household's first-order condition for claims (82), subject to the law of motion of capital

$$K_i(s^t) = (1-\delta)K_i(s^{t-1}) + I_i^K(s^t).$$

The firm's Euler equation for capital is

$$Q_i(s^t)P_i^K(s^t) = \sum_{s_{t+1}} Q_i(s^{t+1})\big[R_i^K(s^{t+1}) + (1-\delta)P_i^K(s^{t+1})\big]. \tag{14}$$

1.4 Final Goods

Competitive final goods producers combine intermediates goods from various countries to produce non-traded final goods, as in Armington (1969). Engel (1999) and Chari, Kehoe and McGrattan (2002) provide the motivating evidence for an Armington setup instead of explicitly modelling tradable and non-tradable goods: the relative price of tradable and non-tradable goods are stable across time and do not fluctuate with the real exchange rate.[6]

For final consumption goods, the final goods producer has a production function with constant returns to scale

$$C_i(s^t) = \left[\sum_j \Omega_{ij}^F(s^t)^{\frac{1}{\theta}} X_{ij}^F(s^t)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{15}$$

where $\Omega_{ij}^F(s^t)$ is the weight of j's intermediate good in i's final good and θ is the elasticity of substitution between each country's intermediate good, or the *trade elasticity* for short. The production function for capital goods has a higher import composition

$$I_i^K(s^t) = \left[\sum_j \Omega_{ij}^K(s^t)^{\frac{1}{\theta}} X_{ij}^K(s^t)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{16}$$

with $\Omega_{ij}^K > \Omega_{ij}^F$ for $j \neq i$ (Oviedo and Singh, 2013). The economy features home bias in consumption and capital investment, so the steady state weights satisfy $\bar{\Omega}_{ii}^F > \bar{\Omega}_{ij}^F$ and $\bar{\Omega}_{ii}^K > \bar{\Omega}_{ij}^K$ for $j \neq i$, which Obstfeld and Rogoff (2000) argue to be an important element in fitting international economics facts.

[6]This is different to the cross-country cross-section, where Balassa (1964) and Samuelson (1964) show that relative prices of tradable and non-tradable goods vary widely across countries.

At state s^t, the weight is

$$\Omega_{ij}^F(s^t) = \frac{\bar{\Omega}_{ij}^F \exp(\omega_{jt})}{\sum_{\hat{j}} \bar{\Omega}_{i\hat{j}}^F \exp(\omega_{\hat{j}t})}. \tag{17}$$

I introduce shocks to international relative demand ω_{jt} for country j's intermediate good, which follow an AR(1) process

$$\omega_{jt+1} = \rho_\omega \omega_{jt} + \varepsilon_{jt}^\omega,$$

where the innovations ε_{jt}^ω have mean zero. Expenditure weights for capital goods have a similar form.

The final goods producer's problem in the final consumption goods market is to choose intermediates demand $X_{ij}^F(s^t)$ to maximize profits

$$P_i^F(s^t)\left[\sum_j \Omega_{ij}^F(s^t)^{\frac{1}{\theta}} X_{ij}^F(s^t)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} - \sum_j P_j^X(s^t)\mathcal{E}_{ij}(s^t)X_{ij}^F(s^t) \tag{18}$$

Intermediate goods follows the law of one price: in each country i, j's good is priced at $P_j^X(s^t)\mathcal{E}_{ij}(s^t)$ in units of i's currency.[7] The problem in the capital goods market is similar.

1.5 Equilibrium

Equilibrium requires various market clearing conditions, each of which hold in every period t and every state of the world s^t. C_i^g units of the final consumption good are consumed directly, and the remainder is consumed as financial services, so total household consumption is

$$C_i(s^t) \equiv \int C_i(\iota, s^t)d\iota = \int C_i^g(\iota, s^t) + \left[\frac{\sum_j D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t) + W_i(s^t)L_i(\iota, s^t)}{P_i^F(s^t)} - C_i^g(\iota, s^t)\right]d\iota, \tag{19}$$

where the bracketed term is total financial services consumption. Substituting this expression into the left-hand side of the final goods production function (eqn. 15) gets the market clearing condition for each country i's final consumption goods. Equation (16) describes market clearing for capital goods. Market clearing for intermediate goods is

$$\sum_{i=1}^N \left[X_{ij}^F(s^t) + X_{ij}^K(s^t)\right] = A_j(s^t)K_j(s^t)^\alpha L_j(s^t)^{1-\alpha} \quad \text{for all } j = 1, \ldots, N, \tag{20}$$

where labor market clearing equates labor demand with labor supply:

$$L_j(s^t) = \int L_j(\iota, s^t)d\iota \quad \text{for all } j = 1, \ldots, N.$$

[7] Under Armington trade, shipping costs, tariffs, and other importer-specific costs are absorbed into the input weight matrix Ω^F. For more details, see Section A.2.

Asset market clearing requires that the total country j-originated dividend income received by investors be supplied by the total dividend payouts of country j's firms:

$$\sum_{i=1}^{N} \int D_{ij}(\iota, s^t) d\iota = R_j^K(s^t) K_j(s^{t-1}) - P_j^K(s^t) I_j^K(s^t) \quad \text{for all } j = 1, \ldots, N. \tag{21}$$

Foreign exchange market clearing (i.e. the international balance of payments) equates the total demand for i's currency with the total supply of i's currency

$$\begin{aligned} & P_i^X(s^t) \sum_{j \neq i} \big[X_{ji}^F(s^t) + X_{ji}^K(s^t)\big] + \sum_j \int D_{ij}(\iota, s^t) \mathcal{E}_{ij}(s^t) d\iota \\ & = \sum_{j \neq i} P_j^X(s^t) \mathcal{E}_{ij}(s^t) \big[X_{ij}^F(s^t) + X_{ij}^K(s^t)\big] + \sum_j \int D_{ji}(\iota, s^t) d\iota. \end{aligned} \tag{22}$$

Finally, the model maps real and nominal variables so that consumer price inflation $P_i^F(s^{t+1})/P_i^F(s^t)$ is constant across countries, so that real and nominal exchange rates co-move one-for-one.

An equilibrium has allocations for households $B_{ij}(s^t)$, $D_{ij}(s^t)$, $L_i(s^t)$, $C_i(s^t)$, $C_i^F(s^t)$; allocations for firms $L_i(s^t)$, $Y_i^X(s^t)$, $I_i^K(s^t)$; allocations for final goods producers $X_{ij}^F(s^t)$, $X_{ij}^K(s^t)$, $C_i^F(s^t)$, $I_i^K(s^t)$; goods prices $P_i^X(s^t)$, $P_i^F(s^t)$, $P_i^K(s^t)$; wages $W_i(s^t)$; and bond prices $Q_i(0, s^t)$ that satisfy: (i) household allocations solve their problem, (ii) firm allocations solve their problem, (iii) final goods producer allocations solve their problem, and (iv) the market-clearing conditions hold.

2 Aggregate Portfolios

In this section, I will prove that the households can be aggregated to a representative household. I assume that the iceberg costs $Z_{ij}(\iota)$ are distributed according to a Fréchet (Type II extreme value) distribution.

Assumption 1. *Assume that*

$$\frac{1}{Z_{ij}(\iota)} \sim \text{Fréchet}\left(\frac{\kappa_0}{Z_{ij}}, \zeta - 1\right), \tag{23}$$

where the constant $\kappa_0 = \left(\frac{\Gamma(\zeta/(\zeta-1))}{\Gamma((\zeta-1/\gamma)/(\zeta-1))}\right)^{\gamma}$.

The distribution of $Z_{ij}(\iota)$ across households is controlled by the scale parameter Z_{ij} and shape parameter ζ. Z_{ij} measures the cost faced by an average investor from country i when realizing dividend income from country j (although the marginal investor in country j will tend to have a lower idiosyncratic draw of $Z_{ij}(\iota)$). Z_{ij} determines the weight of country j's securities in country i's aggregate portfolio. For home portfolio bias, the friction of investing at home for the average household is relatively lower than investing abroad: $Z_{ii} < Z_{ij}$ for $j \neq i$.[8]

ζ measures the dispersion in the cost of realizing capital income across households, with larger ζ indicating smaller dispersion. I will show that ζ is the aggregate country-level portfolio elasticity of substitution with respect to returns, or the *portfolio elasticity* for short. This portfolio demand elasticity is a key object for equilibrium international risk sharing in the global economy: a high elasticity implies that households are readily willing to shift their portfolio holdings to countries offering higher expected returns.

Proposition 1 shows that the heterogeneous household economy has a representative household representation.

Proposition 1 (Aggregation)**.** *Assume that $Z_{ij}(\iota)$ is distributed according to Assumption 1 and that initial household wealth is distributed such that all households in country i have the same marginal utility of wealth μ_i. Then country i's aggregate consumption $C_i(s^t)$, labor $L_i(s^t)$, and portfolios $B_{ij}(s^t)$ are identical to those of a representative household who chooses consumption of goods $C_i^g(s^t)$, labor $L_i(s^t)$, and portfolio $B_{ij}(s^t)$ to maximize*

$$\sum_t \sum_{s^t} \beta^t \pi(s^t) u\big(C_i^g(s^t) - v(L_i(s^t))\big) \tag{24}$$

subject to cash-in-advance constraint

$$P_i^F(s^t) C_i^g(s^t) = \kappa_1 D_i^{agg}(s^t) + \frac{W_i(s^t) L_i(s^t)}{Z_i^L}, \tag{25}$$

where $\kappa_1 = \kappa_0 / \Gamma(\frac{\zeta}{\zeta-1}) \approx 1$ and the dividend income aggregator is defined as

$$D_i^{agg}(s^t) \equiv \left[\sum_j \left(\frac{D_{ij}(s^t) \mathcal{E}_{ij}(s^t)}{Z_{ij}} \right)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}, \tag{26}$$

subject to a sequence of budget constraints

$$\sum_{s_{t+1}} Q_j(s_{t+1} \mid s^t) B_{ij}(s^{t+1}) + D_{ij}(s^t) \leq B_{ij}(s^t), \tag{27}$$

[8] Under the Fréchet distribution, a small number of households will draw $Z < 1$. For those households, I will impose a sufficiently large cost of moving money out of the goods market into the securities market to foreclose the possibility of a "money pump."

and time-0 budget constraint

$$\sum_j Q_j(s^0)\mathcal{E}_{ij}(s^0)B_{ij}(s^0) \leq \bar{B}_i, \tag{28}$$

where the initial endowment is $\bar{B}_i = \int \bar{B}_i(\iota)d\iota$.

The proof appears in Appendix A.1. I show that the representative household of country i's first-order condition for labor is the same as that of the heterogeneous households:

$$v'(L_i(s^t)) = \frac{1}{Z_i^L}\frac{W_i(s^t)}{P_i^F(s^t)}. \tag{29}$$

The assumption of GHH preferences is key to obtaining this result. I then show the representative household's portfolio is the same as the aggregate country-level portfolios of the heterogeneous households.[9] The representative household i's first-order condition for country j's dividend income equates the expected marginal utility of the consumption funded by receiving an additional unit of j's dividends to the marginal utility of the wealth needed to buy the claim:

$$\beta^t\pi(s^t)u'\big(C_i(s^t) - v(L_i(s^t))\big)\Big[d_{ij}^{sh}(s^t)\Big]^{-\frac{1}{\zeta}}\frac{\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)Z_{ij}} = \mu_i Q_j(s^t)\mathcal{E}_{ij}(s^0), \tag{30}$$

where country i's portfolio share in j's dividends $d_{ij}^{sh}(s^t)$ is defined as

$$d_{ij}^{sh}(s^t) = \frac{D_{ij}(s^t)\mathcal{E}_{ij}(s^t)/Z_{ij}}{D_i^{agg}(s^t)}, \tag{31}$$

and μ_i denotes the Lagrange multiplier on the time-0 budget constraint (28). I drop the constant κ_1 because it is very close to 1 for all values of ζ that I will consider. Define country i's exchange rate–adjusted expected return on country j's state-contingent securities R_{ij}^B as

$$R_{ij}^B(s^t) = \pi(s^t)\frac{\mathcal{E}_{ij}(s^t)}{Q_j(s^t)\mathcal{E}_{ij}(s^0)}. \tag{32}$$

and country i's exchange rate and financial friction–adjusted aggregate household portfolio return R_i^H as satisfying

$$R_i^H(s^t)^{-1} = \sum_j\Big[d_{ij}^{sh}(s^t) \times \big[R_{ij}^B(s^t)/Z_{ij}\big]^{-1}\Big] \tag{33}$$

Proposition 2 shows how the portfolio shares of country i shift in response to changes in relative returns.

[9] The representative household spends a different fraction of its income on financial services than the aggregate of heterogeneous households, but this affects neither prices nor allocations in the model.

Proposition 2 (Portfolio demand)**.** *The representative household i's portfolio share that is allocated to capital income from country j is*

$$d_{ij}^{sh}(s^t) = \left(\frac{R_{ij}^B(s^t)/Z_{ij}}{R_i^H(s^t)}\right)^{\zeta}, \tag{34}$$

where i's aggregate portfolio return is

$$R_i^H(s^t) = \left(\sum_j \left[R_{ij}^B(s^t)/Z_{ij}\right]^{\zeta-1}\right)^{\frac{1}{\zeta-1}}. \tag{35}$$

The proof appears in Step 7 of Appendix A.1. Country i holds more of country j's claims when their exchange rate–adjusted return is higher and when the cost of realizing the claims Z_{ij} is lower. As a corollary to Proposition 2, substituting the portfolio share (34) back into first-order condition (30) obtains

$$\beta^t \pi(s^t) \frac{u'\big(C_i(s^t) - v(L_i(s^t))\big)}{P_i^F(s^t)} R_i^H(s^t) = \mu_i. \tag{36}$$

Hence, states of the world with high portfolio returns for i's households are states with low marginal utility for i's households.

3 Mechanism: International Risk Sharing and the Exchange Rate Puzzles

Before quantifying the model, I describe how its mechanism relates to the various exchange rate puzzles identified in the literature.

3.1 International Risk Sharing

In an environment where households in different countries consume different goods, the relevant measure of amount of risk sharing is the co-movement in the marginal utility *per dollar*

$$\text{corr}\left(\Delta \log\left(\frac{u_j'(s^t)}{P_j^F(s^t)\mathcal{E}_{ij}(s^t)}\right),\ \Delta \log\left(\frac{u_i'(s^t)}{P_i^F(s^t)}\right)\right).$$

When a household's consumption bundle is expensive, its marginal utility of having an additional dollar is low, so the additional dollar's marginal utility may be *higher* to a household with a cheap consumption bundle, even if that household has a *lower* marginal utility of consumption.

The international risk sharing condition follows from dividing the portfolio first-order condition

(30) by the analogous condition with $j = i$, to obtain

$$\frac{\frac{u_j'(s^t)}{P_j^F(s^t)\mathcal{E}_{ij}(s^t)}}{\frac{u_i'(s^t)}{P_i^F(s^t)}} = \underbrace{\frac{\mu_j Z_{jj}}{\mu_i \mathcal{E}_{ij}(s^0) Z_{ij}}}_{\text{constant}} \times \left[\frac{d_{jj}^{sh}(s^t)}{d_{ij}^{sh}(s^t)}\right]^{\frac{1}{\zeta}}. \tag{37}$$

From this equation, Proposition 3 immediately follows.

Proposition 3 (Sufficient statistics)**.** *The amount of international risk sharing between country i and j, which measures co-movements in their relative marginal utility per dollar, is characterized by two sufficient statistics:*

(i) portfolio shares $d_{ij}^{sh}(s^t)$ and $d_{jj}^{sh}(s^t)$, and

(ii) the portfolio elasticity ζ.

This result, which is analogous to the celebrated result of Arkolakis, Costinot and Rodríguez-Clare (2012) in international trade, shows that household portfolios are key to international risk sharing. By re-arranging the marginal utility per dollar, the risk sharing condition (37) links real exchange rates with relative marginal utility across countries.

$$\frac{u_j'(s^t)}{u_i'(s^t)} = \underbrace{\frac{\mu_j Z_{jj}}{\mu_i \mathcal{E}_{ij}(s^0) Z_{ij}}}_{\text{constant}} \times \left[\frac{d_{jj}^{sh}(s^t)}{d_{ij}^{sh}(s^t)}\right]^{\frac{1}{\zeta}} \times \underbrace{\frac{P_j^F(s^t)\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)}}_{\text{real exch. rate}}. \tag{38}$$

Equation (38) shows that international risk sharing is key to exchange rate dynamics.

3.2 The Standard Model: Full Risk Sharing and the Backus-Smith Puzzle

In the standard model of international risk sharing, there are no international financial frictions, and dividend income from different countries are perfect substitutes. My model collapses down to the standard model when the cost of turning dividends to money is zero and portfolio holdings are perfectly elastic: $Z_{ij} = 1$ for all i, j and $\zeta \to \infty$. Substituting this into equation (37) shows that marginal utility per dollar co-moves perfectly. Substituting this into equation (38) obtains the international risk sharing condition of Backus and Smith (1993), which I state in the following proposition; I reproduce the original by turning off labor supply effects.

Proposition 4 (Backus and Smith, 1993)**.** *With $\zeta \to \infty$ and $Z_{ij} = 1$ for all i, j, households have perfect risk sharing: when country i's real exchange rate appreciates (the RHS of equation (39)* *declines), the marginal utility of i's households is high:*

$$\frac{u'(C_j(s^t))}{u'(C_i(s^t))} = \frac{\mu_j}{\mu_i \mathcal{E}_{ij}(s^0)} \underbrace{\frac{P_j^F(s^t)\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)}}_{\text{real exch. rate}} \tag{39}$$

Hence, the consumption of i's households declines relative to j's households.

The Backus-Smith puzzle is that the full risk sharing condition (39) is violated in the data: when a country's real exchange rate appreciates, it is moderately correlated with *higher* relative consumption (with a magnitude of 0.3–0.4). Proposition 4 shows that when households have the opportunity to trade securities costlessly, they implement full risk sharing: they fully smooth the marginal utility per dollar across countries; they only fail to completely smooth marginal utility because of fluctuations in the value of a dollar: final consumption goods are relatively more expensive in some states than others. i's households substitute consumption out of expensive states with high $P_i^F(s^t)$ relative to $P_j^F(s^t)\mathcal{E}_{ij}(s^t)$ toward states where consumption is cheap.

I emphasize three points about the risk sharing condition (39): first, it follows from the household block of the model alone; the precise nature of production is irrelevant. Second, the equation holds for *any* shock, so long as financial contracts can be written against it. Third, the assumption of trade in a complete set of contingent securities is not necessary for the qualitative direction of the correlation between exchange rates and marginal utility: it is sufficient for households to trade only one-period nominal risk-free bonds denominated in the various currencies, a very minimal market structure (Chari, Kehoe and McGrattan, 2002; Lustig and Verdelhan, 2019). For these reasons, the Backus-Smith puzzle has proven quite intractable.

With GHH preferences, Proposition 4 is less stark: movements in marginal utility are partly driven by fluctuations in labor supply. However, I will show quantitatively that GHH preferences alone do not resolve the puzzle.

3.3 Resolving the Backus-Smith Puzzle via Home Portfolio Bias

With home portfolio bias, following a real appreciation of currency i, the substitution effect towards states with cheap consumption outlined above is offset by a wealth effect. Following an increase in relative demand ω_{it} for country i's intermediate goods, they become more expensive and i's firms become more profitable; under home portfolio bias, claims to the profits of i's firms are predominantly held by i's households, who become wealthier and consume more. This intuition underlies why i's households consume more when i's real exchange rate appreciates, which resolves the Backus-Smith puzzle.

Specifically, an increase in relative demand for country i's goods increases its price P_i^X relative to that of foreign goods $\mathcal{E}_{ij} P_j^X$. This increases the relative price of country i's final goods

$$P_i^F = \left(\sum_j \Omega_{ij}^F \left(P_j^X \mathcal{E}_{ij} \right)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

relative to that of foreign final goods, $\mathcal{E}_{ij} P_j^F$, as $\Omega_{ii}^F > \Omega_{ji}^F$ under home consumption bias. This is an appreciation in country i's real exchange rate.

The shock raises the prices received by i's firms, and they become more profitable, so $R_i^K K_i$ increases, and their total dividend payouts $R_i^K K_i - P_i^K I_i^K$ also increase (investment expenditure increases but not enough to fully offset the increase in flow profits). To clear the asset market (eqn. 21), households in each country j receive more dividend income D_{ji} from i's firms, and so their share of dividend income originated in country i d_{ji}^{sh} must also increase. As dividend income from different countries are imperfect substitutes to the representative households, to satisfy the portfolio demand equation (34), country i's state-contingent securities must promise higher expected returns R_{ji}^B:

$$d_{ji}^{sh}(s^t) = \left(\frac{R_{ji}^B(s^t)/Z_{ji}}{R_j^H(s^t)} \right)^{\zeta}.$$

The portfolio elasticity ζ determines how much R_{ji}^B must increase for a given increase in d_{ji}^{sh}.

Under home portfolio bias, i's aggregate household portfolio puts a higher weight on i's securities relative to foreign portfolios, and i's households receive a greater share of their dividend income from i's firms. Hence, an increase in expected returns on i's securities increases the expected return on i's aggregate household portfolio R_i^H relative to the corresponding foreign return $R_j^H \mathcal{E}_{ij}(s^t)/\mathcal{E}_{ij}(s^0)$. This can be seen by noting that $d_{ii}^{sh} > d_{ji}^{sh}$ in equation (33), which characterizes R_j^H as a portfolio-weighted average of each country's return R_{ji}^B. An increase in i's portfolio returns makes i's households wealthier relative to foreign households.

Proposition 5 shows the precise relation between the real exchange rate, marginal utility, and portfolio returns by substituting the portfolio demand equation (34) into the risk-sharing condition (eqn. 38).

Proposition 5 (Portfolio return effect)**.** *With financial frictions ($\zeta < \infty$ and $Z_{ij} \geq 1$), the international risk-sharing condition takes into account portfolio returns:*

$$\frac{u_j'(s^t)}{u_i'(s^t)} = \frac{\mu_j}{\mu_i \mathcal{E}_{ij}(s^0)} \times \frac{R_i^H(s^t)}{R_j^H(s^t)\frac{\mathcal{E}_{ij}(s^t)}{\mathcal{E}_{ij}(s^0)}} \times \underbrace{\frac{P_j^F(s^t)\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)}}_{\text{real exch. rate}}. \tag{40}$$

Hence, the effect of i's real exchange rate appreciation on i's marginal utility u_i' is mostly offset by a relative increase in i's household portfolio returns R_i^H. The change in marginal utility of i's households can be decomposed to first order as

$$du_i'(s^t) = \underbrace{\bar{u}_i''}_{<0} \times \left(\frac{\bar{C}_i}{\bar{C}_i - v(\bar{L}_i)} dC_i(s^t) - \frac{v(\bar{L}_i)}{\bar{C}_i - v(\bar{L}_i)} v'(\bar{L}_i) dL_i \right), \tag{41}$$

where bars denote the steady state of the variables. Following an increase in relative demand for i's goods, there is simultaneously an increase in i's labor supply L_i, which increases marginal utility u_i', and an increase in i's consumption C_i, which decreases u_i'. Hence, i's representative household consumes relatively more than foreign households at the same time that i's real exchange rate appreciates, which resolves the Backus-Smith puzzle.

What fails without financial frictions? In that setting, increases in i's firm profits do not differentially increase the wealth of i's households relative to foreign households, so there is no difference in portfolio returns across households. Hence, i's households do not become wealthier, and do not consume more.

3.4 Exchange Rate Volatility: Amplification of Demand Shocks

Home portfolio bias also amplifies the effect of demand shocks, which is necessary to match the volatility of real exchange rates. Recall that an increase in relative demand ω_{it} for i's goods increases the income and consumption of i's households. Under home consumption bias, an increase in the relative consumption of i's households increases the relative demand for i's goods. This amplifies the effect of the initial shock on total demand for i's goods, which further raises the relative profits of i's firms, which further increases the relative income of i's households. Hence, small demand shocks are amplified into large movements in relative prices and real exchange rates.

3.5 Exchange Rate Persistence: the Purchasing Power Parity Puzzle

The purchasing power parity puzzle is that fluctuations in real exchange rates are long-lived (Rogoff, 1996). With a shock that generates large fluctuations in real exchange rates, the puzzle that real exchange rate deviations are long-lived is solved by having this shock be persistent. This follows the argument in Engel and West (2005). In my model, for demand shocks to be persistent, ρ_ω must be large.

With volatile shocks to the real exchange rate and slow mean reversion in these shocks, real exchange rates are difficult to forecast in the short-run. In this model, nominal exchange rate movements are perfectly correlated with real exchange rate movements, so they are also difficult to forecast, in line with the findings of Meese and Rogoff (1983).

3.6 Productivity vs Relative Demand Shocks

I introduce demand shocks because shocks to productivity A_i do not resolve the Backus-Smith puzzle, even with home portfolio bias and GHH preferences. This is most easily seen by looking at the demand and supply of i's intermediate goods: from the final goods producer's problem, demand is

$$\sum_j \left[X^F_{ji} + X^K_{ji}\right] = \sum_j \left[\Omega^F_{ji}\left(\frac{P^X_i \mathcal{E}_{ji}}{P^F_j}\right)^{-\theta} C_j + \Omega^K_{ji}\left(\frac{P^X_i \mathcal{E}_{ji}}{P^K_j}\right)^{-\theta} I^K_j\right]$$

and from the firm's problem, supply is

$$Y_i = A_i\left[(1-\alpha)A_i\frac{P^X_i}{W_i}\right]^{\frac{1-\alpha}{\alpha}} K_i.$$

In Figure 2, I plot the supply and demand curves for i's intermediates by varying $P^X_i/P^X_j\mathcal{E}_{ij}$ while holding other prices fixed. Then I plot the new supply and demand curves following shocks to productivity and relative demand, which I draw by holding other quantities and prices fixed at their post-shock values.

An increase in i's productivity (left panel) increases the supply of country i's intermediate goods, while demand is nearly unchanged, and hence its price P^X_i declines relative to foreign intermediates $P^X_j\mathcal{E}_{ij}$. Under home consumption bias, the price of country i's consumption bundle P^F_i declines relative to foreign bundle $P^F_j\mathcal{E}_{ij}$, so i's households choose to consume more. Hence, in response to a positive productivity shock, country i has a real depreciation at the same time its households increase

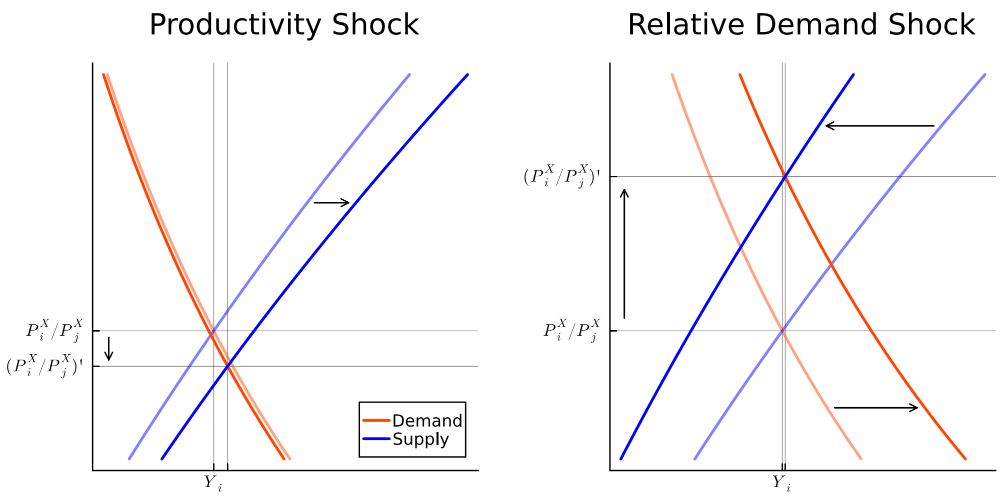

Figure 2: Supply and Demand: Productivity vs Relative Demand Shocks

their relative consumption.[10] In contrast, an increase in relative demand for i's intermediate goods (right panel) shifts its demand curve outwards. At the same time, an increase in i's wages pushes the supply curve in. Hence, i's intermediates become relatively more expensive, and i's final good also becomes relatively more expensive, so i has a real appreciation.

Equilibrium in financial markets continues to be characterized by the risk-sharing condition (40). In response to a positive productivity shock to i's firms, competition pushes down the price of i's intermediate goods, resulting in minimal increases in the profits of i's firms. Hence, there is little relative movement in the dividends paid out by i's firms, so $R_i^H/(R_j^H \mathcal{E}_{ij})$ remains steady. The decline in the real exchange rate is realized in the international risk sharing equation (40) via a large increase in consumption overwhelming a smaller increase in labor supply.

4 Quantitative Business Cycle Model

In this section, I quantify a symmetric two-country business cycle model. The two countries are the US and a composite of advanced economies that use the G10 currencies, which have freely floated against the US dollar since the end of the Bretton Woods system in 1973, and comprise most of the highly traded currencies in foreign exchange markets.[11]

[10] This result is reminiscent of Cole and Obstfeld (1991), who show that in endowment economies, international trade in assets is redundant because movements in goods prices implement international risk sharing. In a production economy, productivity shocks are the closest analog to endowment shocks.

[11] The G10 currencies are the Euro post-1999, Japanese yen, British pound, Canadian dollar, Australian dollar, Swiss franc, Swedish krona, Norwegian krone, NZ dollar; pre-1999, I represent the Euro area by the Deutschmark (using

Table 1: Quantification of Business Cycle Model

Parameter		Value	Source/Target
		Externally assigned	
$\overline{A}$	steady state TFP	$(1,1)$	normalization
Γ^a	TFP weight on global shock	$(1,1)$	normalization
μ	MU of wealth	$(1,1)$	normalization
γ	utility function curvature	2	standard
θ	trade elasticity	1.5	Feenstra et al., 2018
ζ	portfolio elasticity	21	Koijen and Yogo, 2020
$\bar{\Omega}^F_{ij}$	import share in final cons.	10%	Oviedo and Singh, 2013
$\bar{\Omega}^K_{ij}$	import share in investment	18%	Oviedo and Singh, 2013
		Endogenously chosen	
Production			
α	labor share	1/3	data
δ	depreciation rate of capital	10%	ss $I/Y \approx 23\%$
ρ_a	persistence of TFP shock	0.47	US Solow residual
σ_G	volatility of global shock	0.7%	int'l cor of Y
σ_{Ia}	vol of idiosync. TFP shock	0.4%	vol of US Solow residual
Utility			
β	time preference	0.96	ss $K/Y \approx 2$ years
χ_0	weight of labor in utility	5.81	ss $L \approx 20\%$
χ_1	1/elasticity of labor supply	0.7	vol of L
International trade			
ρ_ω	persistence of ω shock	0.88	persistence of $\mathcal{Q}$
$\sigma_{I\omega}$	vol of idiosync. ω shock	5%	vol of US Y
International finance			
Z_{ii}	cost of realizing home claims	1.07	ss $(C - C^g)/C \approx 7\%$
Z_{ij}	cost of realizing foreign claims	1.18	ss equity home bias $\approx 76\%$

4.1 Quantification

Parameter values are summarized in Table 1. The key new parameters that I introduce are the portfolio elasticity ζ and international investment frictions Z_{ij}. Koijen and Yogo (2020) estimate an asset demand system covering short-term bonds, long-term bonds, and equities across countries. To get around endogeneity problems, they use a gravity equation to predict asset demand, and a regression of asset issuance on GDP and population to predict asset supply. Using their predicted asset demand and supply, they construct an instrumental variable for expected returns, and estimate that portfolio holdings in a particular asset class in a country respond to changes in expected returns with an elasticity of 15–25. I set the portfolio elasticity ζ to 21, in line with their estimates.

To interpret the size of the international financial frictions Z, which are a one-off cost paid upon

West German data for the corresponding economic series), French franc, Italian lira, Spanish peseta, Dutch guilder, Belgian franc, Austrian schilling, Finnish markka. I construct the G10 sample by weighting each country/currency zone according to their nominal GDP in US dollars, converted at market rates. For data construction, see Appendix B.1.

converting dividends into money, it is helpful to express them as a per-period wedge on gross returns, as in Jiang, Krishnamurthy and Lustig (2023). This calls for dividing $Z - 1$ by the duration of household portfolios. In the steady state, household portfolios are equivalent a perpetual bond, for which the duration is simply

$$Duration = \frac{1}{1-\beta}.$$

Under the estimated value for $\beta = 0.96$, the duration of household portfolios is 25 years.

To quantify international financial frictions, a commonly-used measure of home portfolio bias (e.g. Coeurdacier and Rey, 2013) is

$$HB_{it} = 1 - \frac{\text{Share of Foreign Assets in } i\text{'s Portfolio}}{\text{Share of Foreign Assets in World Market Portfolio}}. \tag{42}$$

This measure is 0 when country i's portfolio treats domestic and foreign assets the same by holding them in the same ratio as their total supply, and is 1 when i's portfolio is entirely home-biased toward domestic assets. I plot these shares for the US equity portfolio in Figure 3; they imply that the average home equity bias $\overline{HB}_i$ has been 76% in the post–Bretton Woods era. Coeurdacier and Rey (2013) and Coppola, Maggiori, Neiman and Schreger (2021) document similar home portfolio bias patterns across asset classes, including bonds, equities, asset-backed securities, and banking assets, and similar patterns across countries.[12]

I choose $Z_i^L = Z_{ii}$ and Z_{ij} so that $(Z_{ii} - 1)/Duration = 28$bps and $(Z_{ij} - Z_{ii})/Duration = 44$bps to match the average 7% US household consumption share on financial services, and the average US home equity bias of 76% in the post–Bretton Woods era. Hence, to rationalize the observed pattern of home portfolio bias, it is as if the average investor faces an annual cost of 44bps of returns in holding foreign assets rather than domestic assets. This number is similar to the size of financial frictions suggested by Jiang, Krishnamurthy and Lustig (2023).

My benchmark von Neumann–Morgernstern utility function has constant relative risk aversion (Pratt, 1964):

$$u(x) = \frac{x^{1-\gamma} - 1}{1-\gamma}.$$

[12]Hence, I take the level of home equity bias as representative of the ownership of claims to home dividend income. There may be discrepancies between the two concepts due multinational corporations, where firms listed on one stock exchange own capital in other countries, and from privately-held companies, whose equity are not generally available for portfolio investment and whose ownership is presumably more home-biased.

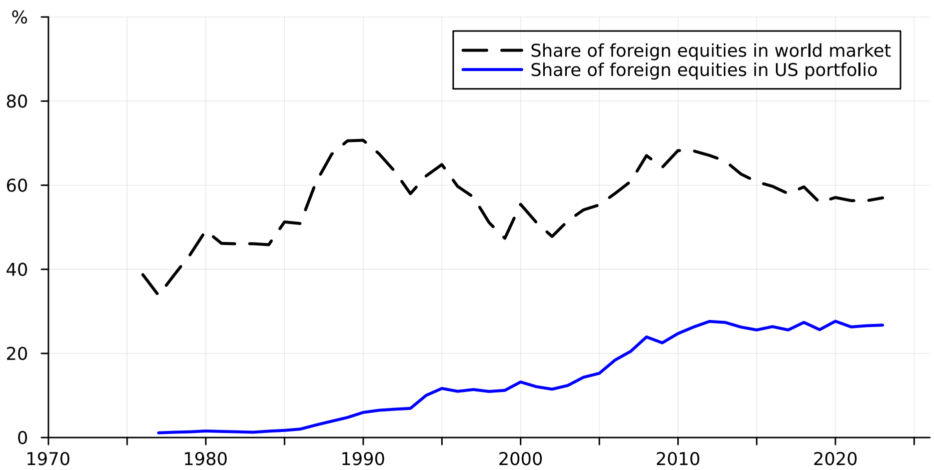

Figure 3: US Aggregate Equity Portfolio

Notes: the measure is defined in equation (42). Data from International Financial Statistics (IMF, 2025) and World Federation of Exchanges (2025).

I choose $\gamma = 2$, a standard value in the literature. The disutility of labor is also constant elasticity:

$$v(L) = \chi_0 \frac{L^{1+\chi_1}}{1+\chi_1} \tag{43}$$

I calibrate a Frisch elasticity of $1/\chi_1 = 1.4$ to match the volatility of hours worked to output, and calibrate χ_0 so that steady-state working hours are roughly 20% of total available time.[13] I normalize the initial value of each country's portfolio $\bar{B}_i$ so that both countries have equal Pareto weights in the risk-sharing condition: $\mu_i \mathcal{E}_{ij}(s^0) = \mu_j$ in equation (38). I set the elasticity of trade θ to 1.5, in line with evidence in Feenstra, Luck, Obstfeld and Russ (2018).[14] I set steady state input matrices $\bar{\Omega}^F$ and $\bar{\Omega}^K$ to match import shares in the US, which are 10% for consumption and 18% for investment (Oviedo and Singh, 2013).

I calibrate the productivity process according to the US Solow residual. I choose the persistence of relative demand ρ_ω to match the persistence of real exchange rate fluctuations. The volatility of shocks to ω is σ_ω, which I set in conjunction with the volatility of productivity shocks to target the volatility and international correlation of output. I solve the model by linearizing around the steady state.

[13]Although this value is somewhat larger than microeconometric estimates of around 1.0, it is conservative: with a smaller Frisch elasticity (and a mechanism that amplifies fluctuations in hours worked to reach its empirical level), pro-cyclical fluctuations in hours worked would more strongly dampen the cyclicality of marginal utility in equation (41), which would make the Backus-Smith puzzle easier to resolve.

[14]This is the conventional parameter used in international macroeconomics, and reflects short-run adjustment in trade balances in response to changes in relative prices.

4.2 Findings

In Table 2, I compare the model's exchange rate moments (column 2) to the data (column 1). For each puzzle, I focus on the median statistic from 100 simulations of the model of the same length as the data. For the Backus-Smith puzzle, my model has a modest positive correlation between j's relative consumption and j's real exchange rate appreciations: $\text{corr}\big[\Delta \log(C_j/C_i),\ \Delta \log(\mathcal{Q}_{ij})\big]$ is 0.15, close to its correlation in the data of 0.19.[15] I find a volatility of exchange rates 3.7 that of output, close to the 4.0 value in the data. For the purchasing power parity puzzle, the half-life of the impulse response to a shock to ω_i is 13 quarters, which is close to the 17 quarters in the data. Hence, this calibration of my model successfully reproduces the major exchange rate puzzles that do not involve risk premia.

Conversely, the standard international real business cycle model, which has neither financial frictions nor relative demand shocks (column 3), produces the Backus-Smith puzzle with a near-perfect correlation between relative consumption and exchange rate depreciation. It also produces almost no volatility in exchange rates, as productivity shocks have a relatively muted impact on international prices. Adding relative demand shocks without financial frictions (column 4) fits the volatility of exchange rates, but is unable to resolve the Backus-Smith puzzle. At the other extreme, a model with full financial autarky, so that $Z_{ij} = \infty$ for any $j \neq i$, makes the correlation between relative consumption and the real exchange rate much too strong.

In Table 3, I report business cycle correlations. Domestic business cycle moments between output, consumption, employment, and investment match the data fairly closely. There are two reasons for this: Backus, Kehoe and Kydland (1992) show that the international real business cycle model driven by productivity shocks fits the business cycle moments well. The addition of the relative demand shock does not impair its performance because relative demand shocks are qualitatively similar to productivity shocks: a positive shock $\omega_{it} > 0$ increases i's output, consumption (only with international financial frictions), labor, and investment. The crucial difference is that it causes i's exchange rate to appreciate, instead of depreciate.[16]

Finally, in Table 4, the model suggests that international risk sharing remains robust despite

[15] This value is similar to Corsetti, Dedola and Leduc (2008), who calculate a correlation of 0.25.

[16] The international correlations inherit the puzzles of the standard international real business cycle model described in Backus, Kehoe and Kydland (1993): consumption is more internationally correlated than output, employment is negatively correlated across countries, and net exports are not countercyclical. A large literature explores mechanisms to resolve each of these puzzles. As an example for net exports, see Drozd and Nosal (2012).

Table 2: US Real Exchange Rate Puzzles

	Data	Model	Variants of the Model: Standard	No fin. fric.	Fin. autarky
	(1)	(2)	(3)	(4)	(5)
Cyclicality: Backus-Smith puzzle					
$\mathrm{corr}\big[\Delta \log(C_j/C_i),\ \Delta \log(\mathcal{Q}_{ij})\big]$	0.19	0.15	-0.87	-0.23	0.88
		[0.05, 0.29]	[-0.85, -0.90]	[-0.33, -0.12]	[0.86, 0.91]
Volatility of exchange rate to GDP					
$\sigma[\log \mathcal{Q}_{ij}]/\sigma[\log Y_i]$	4.0	3.7	0.3	3.4	3.4
		[2.2, 5.3]	[0.1, 0.4]	[2.0, 5.4]	[2.2, 5.1]
Persistence of RER fluctuations *PPP puzzle (Rogoff, 1996)*					
half life of RER fluctuations	17 qtrs	13 qtrs	–	14 qtrs	13 qtrs
		[5, 27]		[6, 30]	[6, 23]
ζ		21	∞	∞	–
$Z_{ij} - Z_{ii},\ j \neq i$		0.11	0	0	∞
σ_ω (%)		4.8	0	4.8	4.8

Notes: data 1973–2019, exchange rate volatility is HP-filtered with $\lambda = 1600$, half life is the first t such that $\mathrm{corr}(y_t, y_{t-1})^t \leq 0.5$, main figures are the 50th percentile from 100 simulations of same length as data, brackets indicate 5th and 95th percentiles.

Table 3: US Business Cycle Moments

		Volatility relative to GDP		Correlation with GDP		International correlation	
Variable		Data	Model	Data	Model	Data	Model
Y	GDP	3.0%	2.8%	1.00	1.00	0.68	0.54
			[2.0, 3.7]				[0.14, 0.76]
C	Consumption	0.81	0.56	0.87	0.96	0.53	0.79
			[0.62, 0.72]		[0.91, 0.99]		[0.55, 0.90]
L	Employment	0.78	0.72	0.82	0.94	0.73	0.01
			[0.62, 0.85]		[0.89, 0.98]		[-0.45, 0.40]
I^K	Investment	2.71	2.07	0.94	0.96	0.59	0.63
			[1.81, 2.30]		[0.92, 0.98]		[0.39, 0.77]
NX	Net exports	3.48	1.37	-0.37	0.42		
			[0.76, 2.14]		[0.01, 0.72]		

Notes: data 1973–2019, HP filtered with $\lambda = 1600$, see footnote 11 for construction of the foreign data, main figures are the 50th percentile from 100 simulations of same length as data, brackets indicate 5th and 95th percentiles.

Table 4: International Risk Sharing

	Model	Variants of the Model		
		Standard	No fin. fric.	Fin. autarky
	(1)	(2)	(3)	(4)
Marginal utility per dollar				
$\text{corr}\big(\Delta\log\big(\frac{u'_j}{P^F_j \mathcal{E}_{ij}}\big),\ \Delta\log\big(\frac{u'_i}{P^F_i}\big)\big)$	0.99	1.00	1.00	0.36
	[0.98, 0.99]			[0.24, 0.50]
Marginal utility				
$\text{corr}\big(\Delta\log u'_j,\ \Delta\log u'_i\big)$	−0.68	0.93	−0.90	0.08
	[−0.73, −0.60]	[0.91, 0.95]	[−0.92, −0.87]	[−0.20, 0.08]
ζ	21	∞	∞	–
$Z_{ij} - Z_{ii},\ j \neq i$	0.11	0	0	∞
σ_ω (%)	4.8	0	4.8	4.8

Notes: main figures are the 50th percentile from 100 simulations of same length as data, brackets indicate 5th and 95th percentiles.

international financial frictions: marginal utility *per dollar* is highly correlated across countries. However, this does not extend to marginal utility itself, which is negatively correlated due to fluctuations in the real exchange rate (see eqn. 38). These results suggest that small deviations from full international risk sharing are sufficient to resolve the Backus-Smith puzzle. Under financial autarky (column 4), the co-movement in the marginal utility per dollar across countries is low. This finding is the opposite to Cole and Obstfeld (1991), who show that with only productivity shocks and no relative demand shocks, movements in the real exchange rate effectively implement international risk sharing even in financial autarky.

4.3 Reconstructing Shocks with a Kalman Filter

In this section, I reconstruct the forces that drive the data. The Kalman filter uses the decision rules of the model to back out the most likely shock that drives the data. I apply the Kalman filter to data on three time series: $\left[c_{at}, c_{jt}, q_{jt}\right]$, which are US log consumption, foreign (G10 excluding the US) log consumption, and the US real exchange rate, and use it to back out three time series of shocks $\left[\varepsilon^a_{at}, \varepsilon^a_{jt}, \varepsilon^\omega_t\right]$, which are US productivity shocks, foreign productivity shocks, and US relative demand shocks.[17]

I start by verifying that given the shocks backed out via the Kalman filter, the model matches the data well (see Appendix Figure 10). Then Figure 4 decomposes time series into their underlying

[17] As the final goods input weights Ω^F_{ij} and Ω^K_{ij} are normalized so that only the ratios of ω_j matter (and not their levels), a separate time series for foreign relative demand cannot be identified.

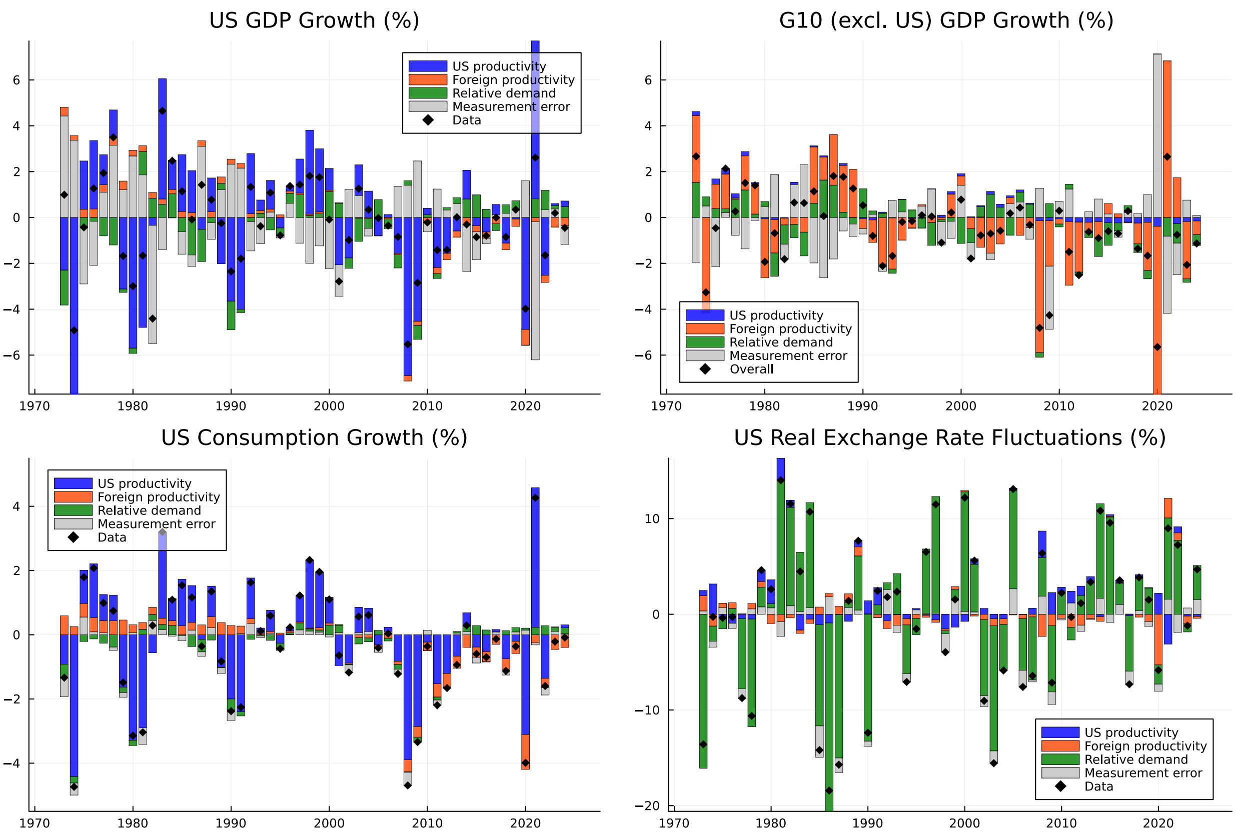

Figure 4: Kalman Filter: Shock Decomposition

Decomposition of shocks driving (a) US GDP growth, (b) foreign GDP growth, (c) US consumption growth, and (d) US real exchange rate fluctuations.

drivers. Consistent with the international real business cycle literature, US productivity shocks (in blue) drive most fluctuations in US GDP (top left) and US consumption (bottom left), while foreign productivity shocks (in orange) drive most fluctuations in foreign GDP (top right). However, domestic productivity shocks drive little of real exchange rate movements, which the Kalman filter overwhelmingly attributes to relative demand shocks (in green, bottom right panel).

Next, I decompose the contribution of the two types of shock to illustrate how the mechanism works. In Figure 5, the top left panel plots the US real exchange rate $1/q_{aj}$ in blue (higher indicates US appreciation) against US consumption relative to foreign consumption in pink. The top right panel shows that the model with all shocks reproduces the data well. The bottom left panel plots a counterfactual series with only productivity shocks: it greatly reduces the volatility of the exchange rate, and it produces a very negative -0.99 correlation between relative consumption and exchange rate appreciations, following the logic that a productivity shock increases the supply of

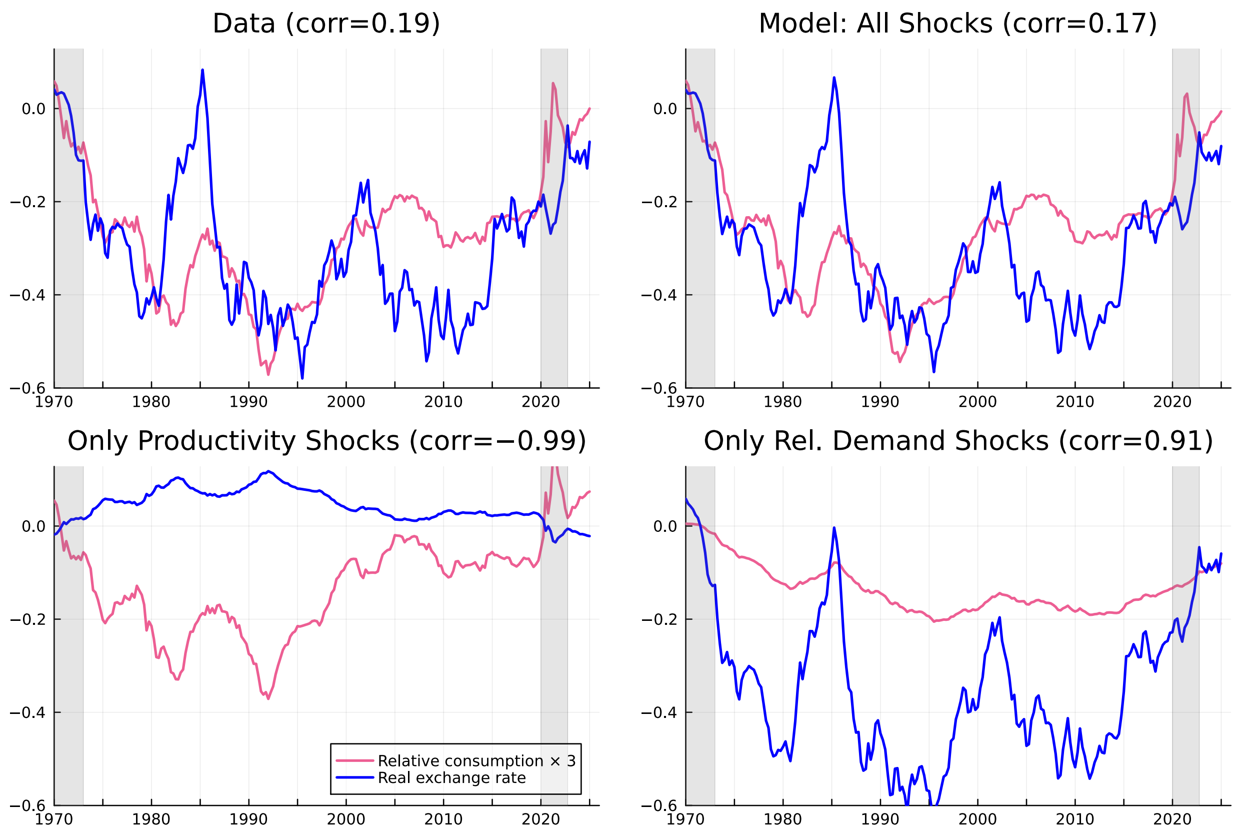

Figure 5: Kalman Filter: Decomposition of Relative Consumption and the US Real Exchange Rate

a country's intermediate goods and lowers their price, as explained in Section 3.6. This is in line with the international business cycle literature, which struggles to match exchange rate patterns with productivity shocks. The bottom right panel plots a counterfactual series with only relative demand shocks: now the correlation between relative consumption and exchange rate appreciations is too strong, and the fit of the relative consumption series is poor. With both shocks, the model produces the right correlation between relative consumption and exchange rates, which is in between these two extremes.

4.4 Subsample Analysis

The key testable prediction of the model is that the extent of international risk sharing determines the strength of the Backus-Smith correlation. Figure 3 shows that US holdings of foreign assets have increased over time, especially through the 1990s and 2000s, so the US aggregate portfolio return has more exposure to the returns on foreign assets. In the model, this diminishes the portfolio return effect in the international risk sharing equation (40), so the correlation between movements in a

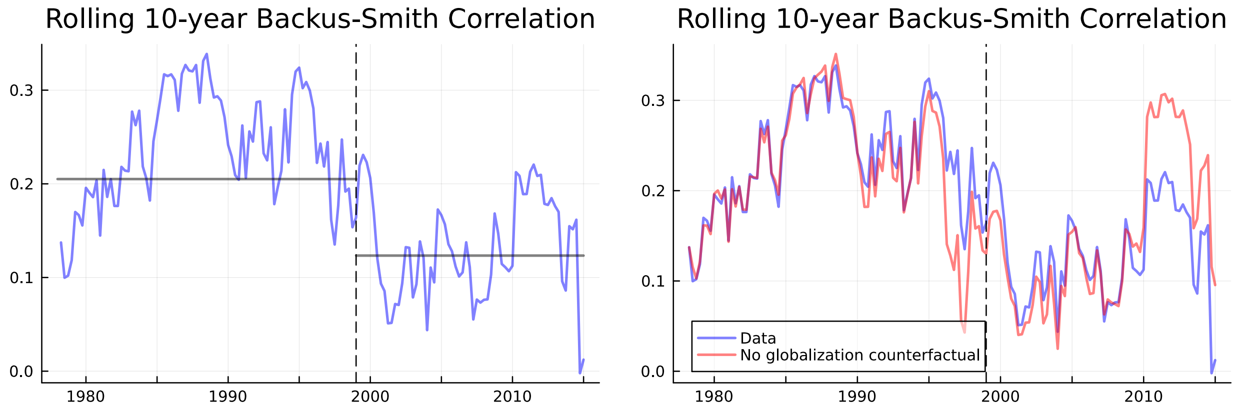

Figure 6: Backus-Smith Correlation

US vs G10 sample. The x-axis plots the centre of the 10-year rolling window.

Table 5: Subsample Analysis Quantification

Parameter		Value	Source/Target
	Early period (1973–1998)		
Z_{ij}	cost of realizing foreign claims	1.25	home equity bias $\approx$ 91%
$\bar{\Omega}^F_{ij}$	import share in final cons.	7.5%	calculations
$\bar{\Omega}^K_{ij}$	import share in final cons.	14%	calculations
	Recent period (1999–2019)		
Z_{ij}	cost of realizing foreign claims	1.25	home equity bias $\approx$ 63%
$\bar{\Omega}^F_{ij}$	import share in final cons.	11%	calculations
$\bar{\Omega}^K_{ij}$	import share in final cons.	20%	calculations

Notes: I keep the ratio of $\bar{\Omega}^F_{ij}/\bar{\Omega}^K_{ij}$ in the same ratio as calculated by Oviedo and Singh (2013). The change in the level of $\bar{\Omega}^F_{ij}$ and $\bar{\Omega}^K_{ij}$ reflect the data, where the US trade/GDP ratio is 50% higher in the late period compared with the earlier period.

country's relative consumption and appreciation in its exchange rate should decline.

The left panel of Figure 6 shows that the prediction of a decline in the Backus-Smith correlation is borne out in the data. The decline is most significant through the 1990s and 2000s, during which most movement in the US foreign portfolio share occurs.

I decompose the change in this correlation into the component driven by globalization and the component driven by a change in the underlying shock process. To do so, I re-quantify the model for the 1973–1998 period and the 1999–2019 period, focusing on the change in trade and financial openness. The changed parameters are in Table 5. To construct a counterfactual series for the Backus-Smith correlation in the absence of changes in openness, I first construct a time series of actual shocks by combining the 1973–1998 time series of shocks recovered by applying the Kalman filter with the early period parameters with the 1999–2019 time series of shocks recovered by applying the Kalman filter with the recent period parameters; I then apply the Kalman filter with the early period parameters over the entire time series to obtain the no-openness counterfactual.

The right panel of Figure 6 shows the counterfactual Backus-Smith correlation in the absence of financial globalization in red. In the recent period, it is notably higher than the data in blue, although still somewhat lower than in the early period (the Kalman filter attributes the remainder of the decline to changes in the shocks).

5 Alternative Mechanisms

Itskhoki and Mukhin (2021) and Kekre and Lenel (2024a) construct models where international asset markets are closed to investors, so that US investors cannot buy foreign assets, and vice versa. Instead, the only asset available to households are domestic nominal risk-free bonds. International asset trade is limited to specialized intermediaries, who trade the nominal risk-free bonds. They show that in such settings, shocks to Z_{ij} (which may be interpreted as foreign intermediation costs or as shocks to the behavior of noise traders) and to discount rates β can produce exchange rate dynamics seen in the data.

I put these shocks into my model by allowing $Z_{ij}(s^t)$ ($j \neq i$) and $\beta_i(s^t)$ follow an AR(1) process in the representative household's problem. I calibrate their persistence to match the annualized persistence of relative demand shocks ρ_ω of 0.88, which is representative of the values they choose. I calibrate the size of their shocks as follows: for foreign intermediation shocks, I follow Itskhoki and Mukhin (2021) by targeting a Backus-Smith correlation of -0.40 in a model that consists only of productivity and foreign intermediation shocks, which obtains a 5% annualized volatility of shocks to $Z_{ij}(s^t)$. I set the annualized volatility of discount factor shocks to 0.4%, in line with Kekre and Lenel (2024a).

I re-run the Kalman filter to back out these new shocks, adding US and foreign GDP Y and capital investment I^K as observable series. The shock decomposition for US GDP and real exchange rate fluctuations are shown in Figure 7. The results look very similar to the previous shock decomposition in Figure 4: productivity shocks drive most fluctuations in GDP growth, while relative demand shocks drive most fluctuations in the real exchange rate.

Hence, these empirical results suggest that in a model with relative demand shocks, there is little room for foreign intermediation cost/noise trader shocks and discount factor shocks to explain

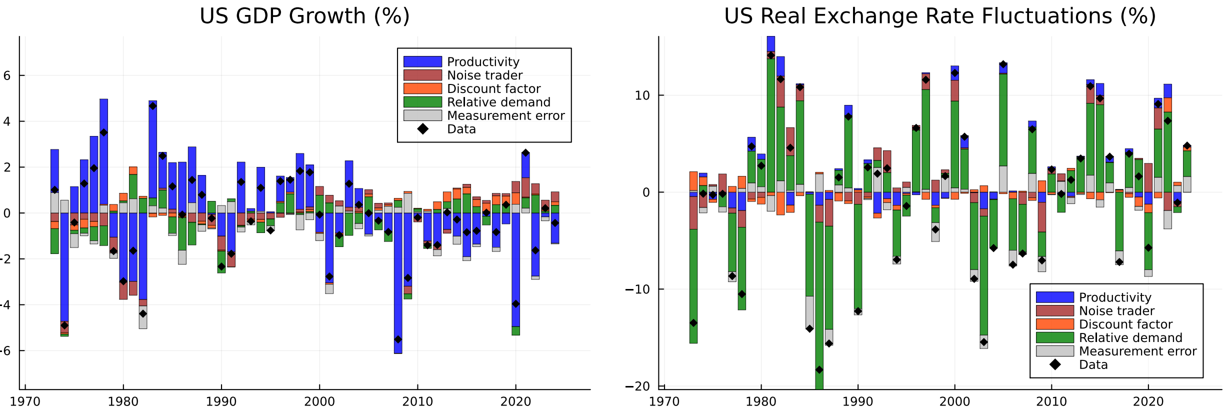

Figure 7: Kalman Filter with Additional Shocks: Shock Decomposition

Decomposition of shocks driving (a) US GDP growth and (b) US real exchange rate fluctuations.

exchange rate movements.[18] The reason for this is because when investors can trade more than just one-period nominal risk-free bonds, as they must to satisfy the documented portfolio facts, and under the empirically relevant level of trade openness, these alternative shocks are incapable of generating large exchange rate movements without generating similarly-large movements in consumption.

I plot the initial $t = 0$ impulse responses of the exchange rate and relative consumption to each of these shocks in Figure 8. Noise trader (row 1) and discount factor shocks (row 2) are only capable of generating large exchange rate movements compared to consumption when trade openness (left column, import composition of consumption $1 - \Omega_{ii}^F$) is far below its value in the data, e.g. in the *autarky limit*. In contrast, relative demand shocks (row 3) can generate large exchange rate movements compared to consumption. Variation in the level of home portfolio bias (right column) has little effect on the relative size of exchange rate movements and consumption, and instead scales up both impulse responses roughly in proportion.

6 Extension: Quantitative Risk Premium Model

A growing body of literature documents correlations between exchange rates and asset prices. Lustig and Verdelhan (2007) document that safe currencies – those with low interest rates and pay relatively low returns to investors (e.g. the US dollar and the Japanese yen) – tend to appreciate during global recessions. This supports a view that safe currencies have a lower risk premia.

[18]The results also show that productivity shocks cannot drive most movements in the real exchange rate, contrary to Corsetti, Dedola and Leduc (2008).

In this section, I show that my model is consistent with these patterns, once modified on account of the finding of Mehra and Prescott (1985) that standard CRRA preferences generate small risk premia.

6.1 Habit Formation Preferences

I re-specify household preferences with a version of the preferences of Campbell and Cochrane (1999) with an exogenously-driven habit $\tilde{H}_i(\iota, s^t)$ over the household's consumption-leisure bundle:[19]

$$u\big(C_i(\iota, s^t) - v(L_i(\iota, s^t)); s^t\big) = \frac{\big(C_i(\iota, s^t) - v(L_i(\iota, s^t)) - \tilde{H}_i(\iota, s^t)\big)^{1-\gamma}}{1-\gamma}. \tag{44}$$

Define the (scaled) surplus consumption ratio $\tilde{S}_i(\iota, s^t)$ to satisfy:

$$\big(\tilde{S}_i(\iota, s^t)\big)^{\frac{\gamma}{\gamma-1}} = \frac{C_i(\iota, s^t) - v(L_i(\iota, s^t)) - \tilde{H}_i(\iota, s^t)}{C_i(\iota, s^t) - v(L_i(\iota, s^t))}$$

so that the utility function becomes

$$u\big(C_i(\iota, s^t) - v(L_i(\iota, s^t)); s^t\big) = \big(\tilde{S}_i(\iota, s^t)\big)^{-\gamma} \frac{\big(C_i(\iota, s^t) - v(L_i(\iota, s^t))\big)^{1-\gamma}}{1-\gamma}. \tag{45}$$

The dynamics of the habit $\tilde{H}_i(s^t)$ are governed by the exogenous dynamics of the *log surplus consumption ratio* $\tilde{s}_t \equiv \log \tilde{S}_i(\iota, s^t)$ around its steady state $\bar{s} \equiv \log \bar{S}$:

$$\tilde{s}_{t+1} = (1-\rho_s)\bar{s} + \rho_s \tilde{s}_t + \lambda(\tilde{s}_t)\varepsilon_{Gt}, \tag{46}$$

where the sensitivity function λ introduces stochastic volatility:

$$\lambda(\tilde{s}_t) = \max\left\{\frac{1}{\bar{S}}\sqrt{1 - 2(\tilde{s}_t - \bar{s})} - 1, 0\right\}.$$

Observe that equation (45) shows that habit formation essentially introduces a discount factor shock $\tilde{S}_i(\iota, s^t)^{-\gamma}$. In contrast to a large literature following Backus, Foresi and Telmer (2001) that drives exchange rates dynamics with country-specific discount factor shocks, I assume that $\tilde{s}_t$ is identical across households and across countries. There are two motivations for this. With variation in discount factors across households within a country, the aggregation result in Proposition 1 would fail, as the households no longer have the same marginal utility of wealth. With variation in discount factors across countries, the finding of large fluctuations in consumption differentials across countries from

[19] Ljungqvist and Uhlig (2015) show that the external habit in the original specification of Campbell and Cochrane (1999) generates a consumption externality. An exogenous habit eliminates this externality while preserving the model's asset pricing implications (see Kehoe et al., 2022).

Section 5 would continue to hold under habit-formation preferences.[20] Instead, I will continue to limit all cross-country variation to productivity and relative demand.

In this setting, households have a habit over the consumption-leisure bundle $C_i(s^t) - v(L_i(s^t))$, rather than over consumption alone, for two reasons: first, the form of utility in equation (45) continues to be of the power utility form needed for Proposition 1 to hold. Second, a habit in consumption alone implies separable preferences between consumption and labor, which in a setting with endogenous labor supply would place a large lower bound for the volatility of the marginal disutility of labor, similar to Hansen and Jagannathan (1991), which is inconsistent with data (see Appendix A.3).

To replicate asset pricing facts, the model must balance the two forces of risk aversion and intertemporal substitution. Campbell-Cochrane preferences deliver time-varying risk aversion through the stochastic volatility term $\lambda(\tilde{s}_t)$, which determines the conditional volatility of changes in marginal utility u'_i, which is of form

$$u'_i(\iota, s^t) = \left[\tilde{S}_i(\iota, s^t)\big(C_i(\iota, s^t) - v(L_i(\iota, s^t))\big)\right]^{-\gamma}. \tag{47}$$

When surplus consumption-leisure $\tilde{s}_t$ is low, conditional volatility $\lambda(\tilde{s}_t)$ is high, so households become more risk averse: they attempt to save more in risk-free assets, pushing risk-free rates of return down, while demanding larger risk premia on risky assets, which have low returns when marginal utility is high. These preferences also deliver time-varying intertemporal substitution through mean reversion in $\tilde{s}_t$: when surplus consumption $\tilde{s}_t$ is low, households believe that their marginal utility will be higher in the future, and attempt to intertemporally substitute consumption toward the present, which pushes up risk-free rates of return. These two forces combine to keep risk-free returns stable, while allowing variation in risk premia on risky assets.[21]

6.2 Defining the Currency Risk Premium

Say if i's representative household compares holding a bond that is *risk-free* in the sense that it carries no default risk, paying out in j's currency against holding a risk-free bond paying out in i's own

[20] A tight connection between marginal utilities across countries is also consistent with Brandt, Cochrane and Santa-Clara (2006), who show that it is necessary to ensure that exchange rate volatility is much lower than the volatility of stock returns. This approach is similar in spirit to Colacito and Croce (2011), whose baseline model has Epstein-Zin preferences and identical long-run growth shocks across countries (in their setting, long-run growth shocks drive asset price volatility). I leave the question of how discount factor shocks can be reconciled with the findings of Section 5 to future work.

[21] See equation (12) of Campbell and Cochrane (1999) for a derivation.

currency. Denote the *risk-free rates*, which are the interest rates on these two bonds, as r^f_{jt} and r^f_{it}, respectively. Despite the absence of default risk, the returns of these bonds are exposed to exchange rate risk. Here, both bonds are bundles of claims to j-originated capital income. The excess return on holding the j-denominated bond, rx_{ijt}, is defined as the return on the j-denominated bond over the return on i-denominated bond:

$$rx_{ijt+1} = \left(r^f_{jt} - r^f_{it}\right) + \Delta e_{ijt+1}, \tag{48}$$

It comprises the interest rate differential $r^f_{jt} - r^f_{it}$ and the exchange rate movement Δe_{ijt+1}. Keeping with the literature, the expected excess return on the j-denominated bond $\mathrm{E}_t[rx_{ijt+1}]$ is simply j's currency risk premium.

Say if currency i is a safe currency that tends to appreciate after a negative global shock, and j is a risky currency that tends to depreciate. Currency j risk-free bonds have lower returns after negative shocks, which is when households are more risk averse. Hence, just as a risky stock—a stock that declines in price when the broader market declines—pays a larger risk premium, bonds that pay a fixed amount of a risky currency pay a risk premium relative to bonds paying in safe currencies. Investors demand compensation for holding the riskier currency in the form of higher expected returns. Proposition 6 formalizes this intuition.

Proposition 6 (Currency risk premium)**.** *The risk premium on currency j is approximately*

$$\mathrm{E}_t[rx_{ijt+1}] \approx \gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\left[\Delta e_{ijt+1}, \varepsilon_{Gt+1}\right]. \tag{49}$$

The proof appears in Appendix A.4.[22] Under balanced growth in this model, the real exchange rate is stationary in the long run (i.e. prices adhere to purchasing power parity). Under the assumption of equal inflation between countries, nominal exchange rates are also stationary in the long run:

$$\mathrm{E}\left[\Delta e_{ijt+1}\right] = \mathrm{E}\left[\Delta e_{ijt+1} + \Delta p^F_{jt+1} - \Delta p^F_{it+1}\right] = 0$$

Applying this result to the unconditional expectation of equation (48) immediately obtains the following corollary.

[22] This is essentially a one-factor asset pricing model. Miranda-Agrippino and Rey (2022) show that a one-factor model performs well in explaining international asset prices.

Proposition 7 (FX share)**.** *The average risk premium on currency j is delivered entirely through interest rate differentials:*

$$\mathrm{E}\big[rx_{ijt+1}\big] = \mathrm{E}\big[r^f_{jt} - r^f_{it}\big], \tag{50}$$

with no contribution from expected appreciations of j's currency.

Hassan, Mertens and Wang (2024) show that models in which discount factor shocks drive exchange rate dynamics imply that risky currencies appreciate on average over time so as to deliver a risk premium to investors, whereas in the data, risky currencies depreciate on average over time, and risk premia are delivered as interest rate differentials. Hence, equation (50) shows that relative demand shocks more closely match the data in this dimension.

6.3 Quantification

To isolate the effect of currency safety, I model two countries that are totally symmetric except in the way their currency responds to global shocks. As exchange rate dynamics are mainly driven by relative demand ω_{it} in this model, for exchange rates to be correlated with asset prices, the global shock must affect relative demand.[23] I model this like I model productivity: relative demand ω_{it} has a global component ω_{Gt} and a country-specific idiosyncratic component ω_{Iit}:

$$\omega_{it} = \Gamma^\omega_i \omega_{Gt} + \omega_{Iit},$$

where Γ^ω_i denotes the loading of country i on the global component ω_{Gt}. The two processes ω_{Gt} and ω_{Iit} are AR(1) with persistence ρ_ω, and driven by shocks ε_{Gt+1} and $\varepsilon^\omega_{Iit+1}$, respectively. I set $\Gamma^\omega = (-6, 0)$ to match the -0.43 correlation between global asset prices and the US dollar exchange rate documented by Miranda-Agrippino and Rey (2022).[24] I set the annualized volatility of the idiosyncratic shock $\sigma_{I\omega}$ to 5.6%. I explicitly introduce Harrod-neutral trend growth in global productivity:

$$a_{Gt+1} = \frac{\bar{g}}{1-\alpha} + \rho_a a_{Gt} + \varepsilon_{Gt+1}.$$

[23] Maggiori (2017) considers a similar mechanism.

[24] This has the implication that the stream of capital income produced by country i's firms is safer, carries a lower risk premium, and hence enjoys a higher valuation. Assuming that each country's households are initially endowed with claims to domestic capital income, country i's households have a lower marginal utility of wealth: $\mu_i \mathcal{E}_{ij}(s^0)/\mu_j = 72\%$.

I calibrate $\bar{g} = 1.8\%$ (annualized) to match growth in US labor productivity from 1973 onwards.[25] The other new parameters are the persistence of log surplus consumption ρ_s and the steady state surplus consumption $\bar{S}$. Following Campbell and Cochrane (1999), I set $\rho_s = 0.87$ to match the persistence of the US equity price-dividend ratio, and $\bar{S} = \sigma_G \times \sqrt{\gamma/(1-\rho_s)}$ to have a stable risk-free rate in the safe country.

I keep all other parameters the same as in the business cycle model in Section 4, except for the discount factor β, which I re-calibrate downwards to match the same target for the capital-to-output ratio.[26] I compute the equilibrium by approximating the price functions with Chebyshev polynomials over a Smolyak sparse grid (Judd, Maliar, Maliar and Valero, 2014).

6.4 Findings

The risk premium on currency j is simply solved in the stochastic steady state ($\tilde{s}_t = \bar{s}$). The stochastic volatility of the surplus consumption process is $\lambda(\bar{s}) = 1/\bar{S} - 1 \approx 1/\bar{S}$, with the approximation following as $\bar{S} \approx 0.03$ is small. Substituting $\bar{S} = \sigma_G \times \sqrt{\gamma/(1-\rho_s)}$ into the risk premium in Proposition 6, and decomposing the covariance term, the expression becomes

$$\begin{aligned}
\mathrm{E}_t[rx_{ijt+1} \mid \tilde{s}_t = \bar{s}] &\approx \frac{\sqrt{\gamma(1-\rho_s)}}{\sigma_G} \times \sigma_G \times \sigma_t[\Delta e_{ijt+1}] \times \operatorname{corr}_t\left[\varepsilon_{Gt+1}, \Delta e_{ijt+1}\right] \\
&= \underbrace{\sqrt{\gamma(1-\rho_s)}}_{\approx 1/2} \times \underbrace{\sigma_t\left[\Delta e_{ijt+1}\right]}_{\approx 7\%} \times \underbrace{\operatorname{corr}_t\left[\varepsilon_{Gt+1}, \Delta e_{ijt+1}\right]}_{\approx 0.4} \approx 1.4\%.
\end{aligned}$$

These results imply that US interest rates have been 1.4 percentage points lower relative to other countries than they otherwise would have been in the period where the US dollar has been seen as a safe currency.

7 Conclusion

A central question in international macroeconomics is the extent to which international financial markets allow countries to mitigate aggregate shocks, with the Backus-Smith puzzle—increases in a country's relative consumption are correlated with its exchange rate appreciating—being the key

[25]The level of growth $\bar{g}$ has little effect on cross-country allocations but brings the level of asset prices up in line with data. If total factor productivity grows at $\bar{g}/(1-\alpha)$, then macroeconomic aggregates grow at rate $\bar{g}$ due to trend growth in the capital-to-labor ratio. In particular, the disutility of labor $v_t(L)$ also grows at rate g, so $v_t(L) = \exp(gt) \times v_0(L)$.

[26]Households are more risk-averse under Campbell-Cochrane preferences, and would accumulate more capital in the stochastic steady state given the same β. Conversely, introducing trend growth makes households more impatient, which makes them accumulate less capital.

piece of evidence in favor of limited risk sharing. I show that in a model with international financial frictions, with foreign portfolio holdings as they are in the data, relative demand shocks resolve the Backus-Smith puzzle. The mechanism operates via changes in wealth: when demand for American goods increases, American firms become more profitable and American securities pay investors higher returns. Under home portfolio bias, American households own most American securities, so they become relatively wealthier and consume more. Hence, American households increase their relative consumption following a US real exchange rate appreciation, resolving the Backus-Smith puzzle.

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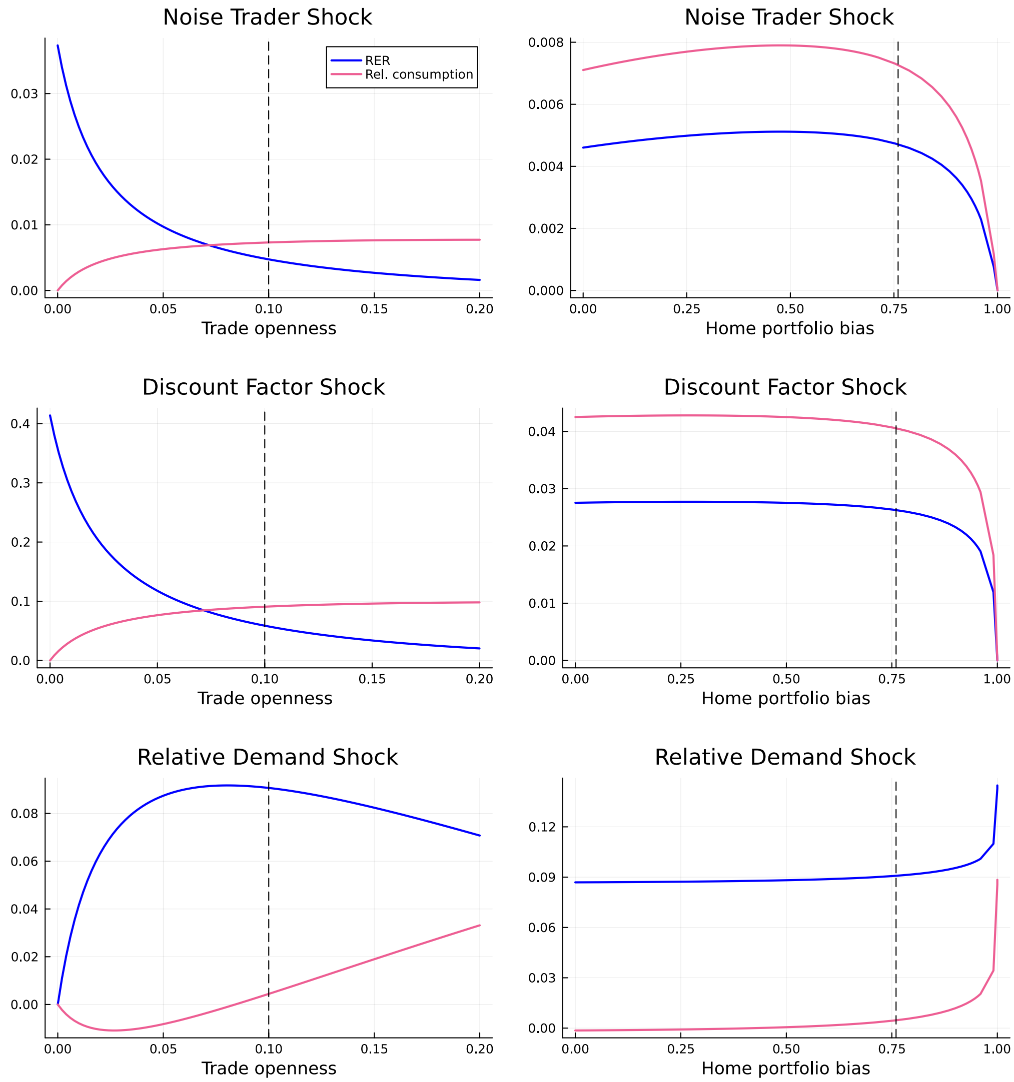

Figure 8: Alternative Shocks: Impulse Response of Real Exchange Rate and Relative Consumption
Left column: variation in trade openness $1 - \Omega^F_{ii}$. Right column: variation in home portfolio bias from varying Z_{ij} $(j \neq i)$. First row: a 10ppt increase in Z_{ij} $(j \neq i)$. Second row: a 20% increase in β_i. Third row: a 20% increase in ω_i. I turn off the investment channel in this exercise.

A Derivations

A.1 Proof of Proposition 1 (Aggregation)

Re-statement of proposition: household ι's problem is to choose paths of consumption of goods $C_i^g(\iota, s^t)$, labor $L_i(\iota, s^t)$, and asset portfolio $B_{ij}(\iota, s^t)$ to maximize its expected utility

$$\sum_t \sum_{s^t} \beta^t \pi(s^t) u\big(C_i^g(\iota, s^t) - v(L_i(\iota, s^t))\big), \tag{51}$$

subject to cash-in-advance constraint

$$P_i^F(s^t) C_i^g(\iota, s^t) = \sum_j \frac{D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{Z_{ij}(\iota)} + \frac{W_i(s^t)L_i(\iota, s^t)}{Z_i^L} \tag{52}$$

and a sequence of budget constraints

$$\sum_{s_{t+1}} Q_j(s_{t+1} \mid s^t) B_{ij}(\iota, s^{t+1}) + D_{ij}(\iota, s^t) \leq B_{ij}(\iota, s^t) \tag{53}$$

$$\sum_j Q_j(s^0)\mathcal{E}_{ij}(s^0) B_{ij}(\iota, s^0) \leq \bar{B}_i(\iota). \tag{54}$$

Assume that $Z_{ij}(\iota)$ is distributed according to Assumption 1:

$$\frac{1}{Z_{ij}(\iota)} \sim \text{Fréchet}\left(\frac{\kappa_0}{Z_{ij}}, \frac{1}{\zeta}\right),$$

where the constant $\kappa_0 = (\frac{\Gamma(\zeta/(\zeta-1))}{\Gamma((\zeta-1/\gamma)/(\zeta-1))})^\gamma$, and that the initial wealth of households are such that all households in country i have the same marginal utility of wealth μ_i.

Then the economy admits a representative household representation: the representative household chooses consumption of goods $C_i^g(s^t)$, labor $L_i(s^t)$, and portfolio of claims $D_{ij}(s^t)$ to maximize

$$\sum_t \sum_{s^t} \beta^t \pi(s^t) u\big(C_i^g(s^t) - v(L_i(s^t))\big), \tag{55}$$

subject to cash-in-advance constraint

$$P_i^F(s^t) C_i^g(s^t) = \kappa_1 D_i^{agg}(s^t) + \frac{W_i(s^t)L_i(s^t)}{Z_i^L}, \tag{56}$$

where $\kappa_1 = \kappa_0/\Gamma(\frac{\zeta}{\zeta-1})$ and the dividend income aggregator is defined as

$$D_i^{agg}(s^t) \equiv \left[\sum_j \left(\frac{D_{ij}(s^t)\mathcal{E}_{ij}(s^t)}{Z_{ij}}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{57}$$

subject to a sequence of budget constraints

$$\sum_{s_{t+1}} Q_j(s_{t+1} \mid s^t) B_{ij}(s^{t+1}) + D_{ij}(s^t) \leq B_{ij}(s^t), \tag{58}$$

and time-0 budget constraint

$$\sum_j Q_j(s^0)\mathcal{E}_{ij}(s^0) B_{ij}(s^0) \leq \bar{B}_i, \tag{59}$$

where the initial endowment is $\bar{B}_i = \int \bar{B}_i(\iota) d\iota$.

Step 1: Reformulate household problem in real terms. For ease of exposition, I rewrite the problem of household ι of country i in real terms, so that all quantities are expressed in terms of country i's final goods. In this section, let lowercase variables denote real variables:

$$b_{ij}(\iota, s^t) = \frac{B_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)} \tag{60}$$

$$d_{ij}(\iota, s^t) = \frac{D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)} \tag{61}$$

$$q_{ij}(s^t) = \frac{Q_j(s^t)\mathcal{E}_{ij}(s^0)}{\mathcal{E}_{ij}(s^t)} \frac{P_i^F(s^t)}{P_i^F(s^0)} \tag{62}$$

$$w_i(s^t) = \frac{W_i(s^t)}{P_i^F(s^t)} \tag{63}$$

Using this notation, the household's cash-in-advance constraint (52) becomes

$$C_i^g(\iota, s^t) \leq \sum_j \frac{d_{ij}(\iota, s^t)}{Z_{ij}(\iota)} + \frac{w_i(s^t)L_i(\iota, s^t)}{Z_i^L}.$$

I assume that monetary settings are such that this constraint holds with equality. Before proceeding with the budget constraints, recursively substitute the time-t budget constraint (53) into the time-0 budget constraint (54) to obtain an intertemporal budget constraint:

$$\begin{aligned}
\bar{B}_i(\iota) &\geq \sum_j Q_j(s^0)\mathcal{E}_{ij}(s^0)\Big[\sum_{s_1} Q_j(s_1 \mid s^0)B_{ij}(\iota, s^1) + D_{ij}(\iota, s^0)\Big] \\
&\geq \sum_j Q_j(s^0)\mathcal{E}_{ij}(s^0)\Big[D_{ij}(\iota, s^0) + \sum_{s_1} Q_j(s_1 \mid s^0)\Big[\sum_{s_2} Q_j(s_2 \mid s^1)B_{ij}(\iota, s^2) + D_{ij}(\iota, s^1)\Big]\Big] \\
&\geq \dots \\
&\geq \sum_t \sum_{s^t} \sum_j Q_j(s^t)\mathcal{E}_{ij}(s^0)D_{ij}(\iota, s^t),
\end{aligned}$$

where $Q_j(s^t) = Q_j(s_t \mid s^{t-1}) \times Q_j(s_{t-1} \mid s^{t-2}) \times \cdots \times Q_j(s_1 \mid s^0) \times Q(s^0)$. Rewrite this intertemporal budget constraint as

$$\sum_t \sum_{s^t} \sum_j \Big[\frac{Q_j(s^t)\mathcal{E}_{ij}(s^0)}{\mathcal{E}_{ij}(s^t)} \frac{P_i^F(s^t)}{P_i^F(s^0)} \times \frac{D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)}\Big] \leq \frac{\bar{B}_i(\iota)}{P_i^F(s^0)} \equiv \bar{b}_i(\iota).$$

Substituting the real notation in equations (61) and (62), the intertemporal budget constraint becomes

$$\sum_t \sum_{s^t} \sum_j q_{ij}(s^t)d_{ij}(\iota, s^t) \leq \bar{b}_i(\iota).$$

Substituting the new cash-in-advance constraint for $C_i^g(\iota, s^t)$ into the household's utility function (6), the household maximizes

$$\sum_t \sum_{s^t} \beta^t \pi(s^t) u\Big(\sum_j \frac{d_{ij}(\iota, s^t)}{Z_{ij}(\iota)} + \frac{w_i(s^t)L_i(\iota, s^t)}{Z_i^L} - v(L_i(\iota, s^t))\Big) \tag{64}$$

subject to budget constraint

$$\sum_t \sum_{s^t} \sum_j q_{ij}(s^t)d_{ij}(\iota, s^t) \leq \bar{b}_i(\iota) \tag{65}$$

and short-sale constraints

$$d_{ij}(\iota, s^t) \geq 0, \quad \forall \iota, s^t.$$

Step 2: Labor supply. The household's first-order condition for $L_i(\iota, s^t)$ is

$$0 = \beta^t \pi(s^t) u_i'(\iota, s^t) \left(\frac{w_i(s^t)}{Z_i^L} - v'(L_i(\iota, s^t)) \right),$$

so

$$v'(L_i(\iota, s^t)) = \frac{w_i(s^t)}{Z_i^L}. \tag{66}$$

Note that all households ι supply the same amount of labor, as the RHS is independent of ι:

$$L_i(\iota, s^t) \equiv L_i(s^t).$$

Let the net contribution of labor to the term inside the u function be denoted by

$$u_i^L(s^t) \equiv \frac{w_i(s^t) L_i(s^t)}{Z_i^L} - v(L_i(s^t)), \tag{67}$$

where $L_i(s^t)$ satisfies the aforementioned FOC.

Step 3: Portfolio choice. In state s^t, household ι chooses to hold claims to capital income from the country $j = J(\iota, s^t)$ that provides the highest return after paying for financial services:

$$J(\iota, s^t) = \arg\max_j \frac{1}{q_{ij}(s^t) Z_{ij}(\iota)}. \tag{68}$$

Recall that

$$\frac{1}{Z_{ij}(\iota)} \sim \text{Fréchet}\left(\frac{\kappa_0}{Z_{ij}}, \zeta - 1 \right).$$

Multiplying a Fréchet random variable by a scalar returns a new Fréchet distribution:

$$\frac{1}{q_{ij}(s^t) Z_{ij}(\iota)} \sim \text{Fréchet}\left(\frac{\kappa_0}{q_{ij}(s^t) Z_{ij}}, \zeta - 1 \right) \tag{69}$$

Then

$$\Pr(J(\iota, s^t) = j) = \Pr\left(j = \arg\max_{\hat{j}} \frac{1}{q_{i\hat{j}}(s^t) Z_{i\hat{j}}(\iota)} \right).$$

Using the max-stability of the Fréchet distribution, the probability becomes:

$$\Pr(J(\iota, s^t) = j) = \frac{(q_{ij}(s^t) Z_{ij})^{-(\zeta-1)}}{\sum_{\hat{j}} (q_{i\hat{j}}(s^t) Z_{i\hat{j}})^{-(\zeta-1)}} \tag{70}$$

Substituting $j = J(\iota, s^t)$ into the household's utility function (64), the household's problem is to choose a portfolio $\{b_{iJ(\iota,s^t)}(\iota, s^t)$ to maximize

$$\sum_t \sum_{s^t} \beta^t \pi(s^t) u\left(\frac{d_{iJ(\iota,s^t)}(\iota, s^t)}{Z_{iJ(\iota,s^t)}(\iota)} + u_i^L(s^t) \right)$$

subject to budget constraint

$$\sum_t \sum_{s^t} q_{J(\iota,s^t)}(s^t) d_{iJ(\iota,s^t)}(\iota, s^t) \leq \bar{b}_i(\iota).$$

The household's first-order condition for $d_{ij}(\iota, s^t)$, given $j = J(\iota, s^t)$, is

$$\beta^t \pi(s^t) u'\left(\frac{d_{ij}(\iota, s^t)}{Z_{ij}(\iota)} + u_i^L(s^t)\right) \frac{1}{Z_{ij}(\iota)} = \mu_i q_{ij}(s^t)$$

where μ_i is the Lagrange multiplier on the budget constraint, and is assumed to be the same across all households in a country. Rearranging:

$$u'\left(\frac{d_{ij}(\iota, s^t)}{Z_{ij}(\iota)} + u_i^L(s^t)\right) = \left(\frac{\beta^t \pi(s^t)}{\mu_i q_{ij}(s^t) Z_{ij}(\iota)}\right)^{-1}$$

With power utility, $u'(x) = x^{-\gamma}$, this becomes

$$\frac{d_{ij}(\iota, s^t)}{Z_{ij}(\iota)} + u_i^L(s^t) = \left(\frac{\beta^t \pi(s^t)}{\mu_i q_{ij}(s^t) Z_{ij}(\iota)}\right)^{\frac{1}{\gamma}} \tag{71}$$

where $j = J(\iota, s^t)$.

Step 4: Aggregate portfolio choice. This section derives the aggregate portfolio holdings of country i. Aggregate country i holdings of country j's bonds are

$$d_{ij}(s^t) \equiv \int d_{ij}(\iota, s^t) d\iota = \mathrm{E}\big[d_{ij}(\iota, s^t) \mid J(\iota) = j\big] \times \Pr\big(J(\iota) = j\big), \tag{72}$$

where the last expression makes use of the law of iterated expectation. First, consider $\mathrm{E}[d_{ij}(\iota, s^t) \mid J(\iota) = j]$, which is the average amount of j's claims held by households who choose to hold j's claims. Multiplying equation (71) through by $Z_{ij}(\iota)$ and taking the conditional expectation:

$$\begin{aligned} &\mathrm{E}\big[d_{ij}(\iota, s^t) \mid J(\iota) = j\big] + u_i^L(s^t)\, \mathrm{E}\big[Z_{ij}(\iota) \mid J(\iota) = j\big] \\ &= \left(\frac{\beta^t \pi(s^t)}{\mu_i q_{ij}(s^t)}\right)^{\frac{1}{\gamma}} \mathrm{E}\big[Z_{ij}(\iota)^{1-\frac{1}{\gamma}} \mid J(\iota) = j\big] \end{aligned} \tag{73}$$

To evaluate these conditional expectations, I need the obtain the conditional distributions. The max-stability of the Fréchet distribution, combined with the decision rule for $J(\iota, s^t)$ in (68) and the distribution of $1/q_{ij}(s^t)Z_{ij}(\iota)$ in (69), obtains the following conditional distribution:

$$\frac{1}{q_{ij}(s^t) Z_{ij}(\iota)} \,\Bigg|\, J(\iota, s^t) = j \sim \text{Fréchet}\left(\kappa_0 \left(\sum_{\hat{j}} (q_{i\hat{j}}(s^t) Z_{i\hat{j}})^{-(\zeta-1)}\right)^{\frac{1}{\zeta-1}},\ \zeta - 1\right).$$

Multiplying a Fréchet random variable by the scalar $q_{ij}(s^t)$ returns a new Fréchet distribution:

$$\frac{1}{Z_{ij}(\iota)} \,\Bigg|\, J(\iota, s^t) = j \sim \text{Fréchet}\left(\kappa_0 q_{ij} \left(\sum_{\hat{j}} (q_{i\hat{j}}(s^t) Z_{i\hat{j}})^{-(\zeta-1)}\right)^{\frac{1}{\zeta-1}},\ \zeta - 1\right) \tag{74}$$

Taking the reciprocal of a Fréchet random variable obtains a Weibull random variable:

$$Z_{ij}(\iota) \mid J(\iota, s^t) = j \sim \text{Weibull}\left(\frac{1}{\kappa_0 q_{ij}} \left(\sum_{\hat{j}} (q_{i\hat{j}}(s^t) Z_{i\hat{j}})^{-(\zeta-1)}\right)^{-\frac{1}{\zeta-1}},\ \zeta - 1\right)$$

$$Z_{ij}(\iota)^{1-\frac{1}{\gamma}} \mid J(\iota, s^t) = j \sim \text{Weibull}\left(\frac{1}{(\kappa_0 q_{ij})^{1-\frac{1}{\gamma}}} \left(\sum_{\hat{j}} (q_{i\hat{j}}(s^t) Z_{i\hat{j}})^{-(\zeta-1)}\right)^{-\frac{1}{\zeta-1}(1-\frac{1}{\gamma})},\ \frac{\zeta - 1}{1 - \frac{1}{\gamma}}\right)$$

Evaluating the expectations:

$$\mathrm{E}\big[Z_{ij}(\iota) \mid J(\iota, s^t) = j\big] = \frac{1}{\kappa_0 q_{ij}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{1}{\zeta-1}} \Gamma(1 + \tfrac{1}{\zeta-1})$$

$$\mathrm{E}\big[Z_{ij}(\iota)^{1-\frac{1}{\gamma}} \mid J(\iota, s^t) = j\big] = \frac{1}{(\kappa_0 q_{ij})^{1-\frac{1}{\gamma}}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{1}{\zeta-1}(1-\frac{1}{\gamma})} \Gamma\big(1 + \tfrac{1}{\zeta-1}(1 - \tfrac{1}{\gamma})\big)$$

Simplify: $\Gamma(1 + \frac{1}{\zeta-1}) = \Gamma(\frac{\zeta}{\zeta-1})$ and $\Gamma(1 + \frac{1}{\zeta-1}(1 - \frac{1}{\gamma})) = \Gamma(\frac{\zeta-1/\gamma}{\zeta-1})$. Now substitute these expressions back into equation (73):

$$\begin{aligned}
&\mathrm{E}[d_{ij}(\iota, s^t) \mid J(\iota) = j] + \frac{u_i^L(s^t)}{q_{ij}(s^t)} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{1}{\zeta-1}} \times \frac{\Gamma(\frac{\zeta}{\zeta-1})}{\kappa_0} \\
&= \left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{\frac{1}{\gamma}} \frac{1}{q_{ij}(s^t)} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{1}{\zeta-1}(1-\frac{1}{\gamma})} \times \frac{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}{\kappa_0^{1-\frac{1}{\gamma}}}
\end{aligned}$$

Multiplying this expression by $\Pr(J(\iota) = j)$, using the expression derived in equation (70), and using the expression for aggregate holdings of j's claims in equation (72) to simplify the first term on the left-hand side:

$$\begin{aligned}
&d_{ij}(s^t) + \frac{u_i^L(s^t)}{q_{ij}(s^t)^{\zeta} Z_{ij}^{\zeta-1}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{\zeta}{\zeta-1}} \times \frac{\Gamma(\frac{\zeta}{\zeta-1})}{\kappa_0} \\
&= \left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{\frac{1}{\gamma}} \frac{1}{q_{ij}(s^t)^{\zeta} Z_{ij}^{\zeta-1}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{1}{\zeta-1}(1-\frac{1}{\gamma})-1} \times \frac{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}{\kappa_0^{1-\frac{1}{\gamma}}}
\end{aligned}$$

Separate out the terms involving j from the terms that do not involve j:

$$\begin{aligned}
d_{ij}(s^t) &= \frac{1}{q_{ij}(s^t)^{\zeta} Z_{ij}^{\zeta-1}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{\zeta}{\zeta-1}} \times \\
&\quad \left[\left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{\frac{1}{\gamma}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{\frac{1}{\zeta-1}\frac{1}{\gamma}} \times \frac{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}{\kappa_0^{1-\frac{1}{\gamma}}} - u_i^L(s^t) \times \frac{\Gamma(\frac{\zeta}{\zeta-1})}{\kappa_0} \right] \\
&= \frac{1}{q_{ij}(s^t)^{\zeta} Z_{ij}^{\zeta-1}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{\zeta}{\zeta-1}} A_{0i}(s^t)
\end{aligned}$$

where I denote the bracketed term on the second line by $A_{0i}(s^t)$:

$$A_{0i}(s^t) = \left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{\frac{1}{\gamma}} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{\frac{1}{\zeta-1}\frac{1}{\gamma}} \times \frac{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}{\kappa_0^{1-\frac{1}{\gamma}}} - u_i^L(s^t) \times \frac{\Gamma(\frac{\zeta}{\zeta-1})}{\kappa_0}. \tag{75}$$

Divide by Z_{ij}:

$$\frac{d_{ij}(s^t)}{Z_{ij}} = (q_{ij}(s^t) Z_{ij})^{-\zeta} \left(\sum_{\hat{\jmath}} (q_{i\hat{\jmath}}(s^t) Z_{i\hat{\jmath}})^{-(\zeta-1)} \right)^{-\frac{\zeta}{\zeta-1}} A_{0i}(s^t). \tag{76}$$

Take the power of $(\zeta-1)/\zeta$:

$$\left(\frac{d_{ij}(s^t)}{Z_{ij}}\right)^{\frac{\zeta-1}{\zeta}} = (q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}\left(\sum_{\hat{j}}(q_{i\hat{j}}(s^t)Z_{i\hat{j}})^{-(\zeta-1)}\right)^{-1} A_{0i}(s^t)^{\frac{\zeta-1}{\zeta}}.$$

Sum over j:

$$\sum_j\left(\frac{d_{ij}(s^t)}{Z_{ij}}\right)^{\frac{\zeta-1}{\zeta}} = \sum_j(q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}\left(\sum_{\hat{j}}(q_{i\hat{j}}(s^t)Z_{i\hat{j}})^{-(\zeta-1)}\right)^{-1} A_{0i}(s^t)^{\frac{\zeta-1}{\zeta}} = A_{0i}(s^t)^{\frac{\zeta-1}{\zeta}}.$$

Define the bond aggregator d_i^{agg} as the LHS of the preceding expression taken to the power of $\zeta/(\zeta-1)$:

$$d_i^{agg}(s^t) \equiv \left[\sum_j\left(\frac{d_{ij}(s^t)}{Z_{ij}}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}} = A_{0i}(s^t). \tag{77}$$

Dividing equation (76) by (77) obtains:

$$\frac{d_{ij}(s^t)/Z_{ij}}{d_i^{agg}(s^t)} = (q_{ij}(s^t)Z_{ij})^{-\zeta}\left(\sum_{\hat{j}}(q_{i\hat{j}}(s^t)Z_{i\hat{j}})^{-(\zeta-1)}\right)^{-\frac{\zeta}{\zeta-1}}$$

Hence, aggregate holdings of j's bonds by i's households are most easily represented as shares of the aggregator d_i^{agg}. The previous expression rearranges to:

$$\frac{d_{ij}(s^t)/Z_{ij}}{d_i^{agg}(s^t)} = \left(\frac{(q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}}{\sum_{\hat{j}}(q_{i\hat{j}}(s^t)Z_{i\hat{j}})^{-(\zeta-1)}}\right)^{\frac{\zeta}{\zeta-1}}. \tag{78}$$

Step 5: Representative household's problem. As with the heterogeneous households, it is convenient to reformulate the problem in real terms, using the variables defined in (60)–(63), and

$$d_i^{agg}(s^t) \equiv \frac{D_i^{agg}(s^t)}{P_i^F(s^t)} = \left[\sum_j\left(\frac{d_{ij}(s^t)}{Z_{ij}}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}. \tag{79}$$

Using this notation, the household's cash-in-advance constraint (56) becomes

$$C_i^g(s^t) \le \kappa_1 d_i^{agg}(s^t) + \frac{W_i(s^t)L_i(s^t)}{Z_i^L}.$$

I assume that monetary settings are such that this constraint holds with equality. Substituting this expression for $C_i^g(s^t)$ into the household's utility function (55), the household maximizes

$$\sum_t\sum_{s^t}\beta^t\pi(s^t)u\left(\kappa_1 d_i^{agg}(s^t) + \frac{w_i(s^t)L_i(s^t)}{Z_i^L} - v(L_i(s^t))\right) \tag{80}$$

subject to budget constraint

$$\sum_t\sum_{s^t}\sum_j q_{ij}(s^t)d_{ij}(s^t) \le \bar{b}_i.$$

The sequential budget constraints (58) and (59) are collected into an intertemporal budget constraint in the same manner as the heterogeneous agent case in Step 1.

Step 6: Representative household's labor supply. The representative household's first-order condition for $L_i(s^t)$ is

$$0 = \beta^t \pi(s^t) u_i'(s^t) \left(\frac{w_i(s^t)}{Z_i^L} - v'(L_i(s^t)) \right)$$

so the FOC is the same as the heterogeneous agent problem in equation (66):

$$v'(L_i(s^t)) = \frac{w_i(s^t)}{Z_i^L}. \tag{81}$$

As with the heterogeneous household case, let the net contribution of labor to the term inside the u function be denoted by

$$u_i^L(s^t) \equiv \frac{w_i(s^t) L_i(s^t)}{Z_i^L} - v(L_i(s^t)).$$

Step 7: Representative household's bond holdings. The representative household's first-order condition for $d_{ij}(s^t)$ is

$$\beta^t \pi(s^t) u'\big(\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t)\big) \left(\frac{d_i^{agg}(s^t)}{d_{ij}(s^t)/Z_{ij}} \right)^{-\frac{1}{\zeta}} \frac{1}{Z_{ij}} = \mu_i q_{ij}(s^t) \tag{82}$$

where μ_i is the Lagrange multiplier on the budget constraint. Using the power utility form $u'(x) = x^{-\gamma}$, and rearranging:

$$\left(\frac{d_{ij}(s^t)/Z_{ij}}{d_i^{agg}(s^t)} \right)^{\frac{1}{\zeta}} = \frac{\beta^t \pi(s^t)}{\mu_i} \frac{1}{q_{ij}(s^t) Z_{ij}} \big(\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t)\big)^{-\gamma} \tag{83}$$

so

$$\left(\frac{d_{ij}(s^t)}{Z_{ij}} \right)^{\frac{1}{\zeta}} = \frac{\beta^t \pi(s^t)}{\mu_i} \frac{1}{q_{ij}(s^t) Z_{ij}} \big(\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t)\big)^{-\gamma} d_i^{agg}(s^t)^{\frac{1}{\zeta}}$$

Taking the power of $\zeta - 1$:

$$\left(\frac{d_{ij}(s^t)}{Z_{ij}} \right)^{\frac{\zeta-1}{\zeta}} = \left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{\zeta-1} (q_{ij}(s^t) Z_{ij})^{-(\zeta-1)} \big(\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t)\big)^{-\gamma(\zeta-1)} d_i^{agg}(s^t)^{\frac{\zeta-1}{\zeta}}$$

Sum over j:

$$\sum_j \left(\frac{d_{ij}(s^t)}{Z_{ij}} \right)^{\frac{\zeta-1}{\zeta}} = \left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{\zeta-1} \left(\sum_j (q_{ij}(s^t) Z_{ij})^{-(\zeta-1)} \right) (\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t))^{-\gamma(\zeta-1)} d_i^{agg}(s^t)^{\frac{\zeta-1}{\zeta}}$$

Take the power of $\zeta/(\zeta - 1)$ and use the definition of d_i^{agg} in equation (79):

$$d_i^{agg}(s^t) = \left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{\zeta} \left(\sum_j (q_{ij}(s^t) Z_{ij})^{-(\zeta-1)} \right)^{\frac{\zeta}{\zeta-1}} \big(\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t)\big)^{-\gamma\zeta} d_i^{agg}(s^t)$$

The terms $d_i^{agg}(s^t)$ cancel out on the LHS and RHS. Rearranging and taking the power of $1/\zeta$:

$$\big(\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t)\big)^{-\gamma} = \left(\frac{\beta^t \pi(s^t)}{\mu_i} \right)^{-1} \left(\sum_j (q_{ij}(s^t) Z_{ij})^{-(\zeta-1)} \right)^{-\frac{1}{\zeta-1}}. \tag{84}$$

Substituting this into equation (83):

$$\left(\frac{d_{ij}(s^t)/Z_{ij}}{d_i^{agg}(s^t)}\right)^{\frac{1}{\zeta}} = \frac{1}{q_{ij}(s^t)Z_{ij}}\left(\sum_j (q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}\right)^{-\frac{1}{\zeta-1}}. \tag{85}$$

Taking the power of ζ, the following expression shows that the share of capital income the representative household in i receives from country j:

$$\frac{d_{ij}(s^t)/Z_{ij}}{d_i^{agg}(s^t)} = \left(\frac{(q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}}{\sum_j (q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}}\right)^{\frac{\zeta}{\zeta-1}} \tag{86}$$

is identical to equation (78) for the aggregated heterogeneous households.

Take the expression for the household's marginal utility in equation (84) to the power of $-1/\gamma$:

$$\kappa_1 d_i^{agg}(s^t) + u_i^L(s^t) = \left(\frac{\mu_i}{\beta^t \pi(s^t)}\right)^{-\frac{1}{\gamma}} \left(\sum_j (q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}\right)^{\frac{1}{\zeta-1}\frac{1}{\gamma}}$$

Therefore,

$$d_i^{agg}(s^t) = \left(\frac{\beta^t \pi(s^t)}{\mu_i}\right)^{\frac{1}{\gamma}} \left(\sum_j (q_{ij}(s^t)Z_{ij})^{-(\zeta-1)}\right)^{\frac{1}{\zeta-1}\frac{1}{\gamma}} \frac{1}{\kappa_1} - \frac{u_i^L(s^t)}{\kappa_1} \tag{87}$$

Recall that in the heterogeneous households case, by combining equations (77) and (75), the bond aggregator was

$$d_i^{agg}(s^t) = \left(\frac{\beta^t \pi(s^t)}{\mu_i}\right)^{\frac{1}{\gamma}} \left(\sum_{\hat{j}} (q_{i\hat{j}}(s^t)Z_{i\hat{j}})^{-(\zeta-1)}\right)^{\frac{1}{\zeta-1}\frac{1}{\gamma}} \times \frac{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}{\kappa_0^{1-\frac{1}{\gamma}}} - u_i^L(s^t) \times \frac{\Gamma(\frac{\zeta}{\zeta-1})}{\kappa_0}. \tag{88}$$

Step 8: Solve for the constants. By matching coefficients, we can get the expressions for $d_i^{agg}(s^t)$ to align exactly between the two cases:

$$\frac{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}{\kappa_0^{1-\frac{1}{\gamma}}} = \frac{1}{\kappa_1}$$

$$\frac{\Gamma(\frac{\zeta}{\zeta-1})}{\kappa_0} = \frac{1}{\kappa_1}$$

Combining the two equations obtains

$$\frac{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}{\kappa_0^{1-\frac{1}{\gamma}}} = \frac{\Gamma(\frac{\zeta}{\zeta-1})}{\kappa_0}$$

Cancelling terms and rearranging:

$$\kappa_0^{\frac{1}{\gamma}} = \frac{\Gamma(\frac{\zeta}{\zeta-1})}{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}$$

so

$$\kappa_0 = \left(\frac{\Gamma(\frac{\zeta}{\zeta-1})}{\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})}\right)^{\gamma} \tag{89}$$

and

$$\kappa_1 = \frac{\kappa_0}{\Gamma(\frac{\zeta}{\zeta-1})} = \frac{\left[\Gamma(\frac{\zeta}{\zeta-1})\right]^{\gamma-1}}{\left[\Gamma(\frac{\zeta-1/\gamma}{\zeta-1})\right]^{\gamma}} \tag{90}$$

Comparing equations (66) and (81) shows that labor supply $L_i(s^t)$ is the same across the heterogeneous and representative household economies. Comparing equations (87) and (88), along with (89) and (90), show that the aggregator $d_i^{agg}(s^t)$ is the same up to the marginal utility of wealth term μ_i. Comparing equations (78) and (86) shows that portfolio dividend shares $(d_{ij}(s^t)/Z_{ij})/d_i^{agg}(s^t)$ are the same. Given that the representative agents have the same wealth as the aggregate of the heterogeneous agents ($\bar{B}_i = \int \bar{B}_i(\iota)d\iota$), for the time-0 budget constraints to hold, it must be that the portfolios are the same, so μ_i and $d_i^{agg}(s^t)$ are the same. *QED*

A.2 Shipping Costs

Say if the export of intermediate goods involves an iceberg cost, so that if one unit of j's good is shipped to country i, country i's final goods producer receives $1/Z_{ij}^X < 1$ units of the good. Fixing the state s^t, the final goods producer's problem becomes to maximize

$$P_i^F\left[\sum_j (\Omega_{ij}^F)^{\frac{1}{\theta}} \left(\frac{X_{ij}^F}{Z_{ij}^X}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} - \sum_j P_j^X \mathcal{E}_{ij} X_{ij}^F$$

Now define $\hat{\Omega}_{ij}^F$ as follows:

$$(\hat{\Omega}_{ij}^F)^{\frac{1}{\theta}} = (\Omega_{ij}^F)^{\frac{1}{\theta}} \left(\frac{1}{Z_{ij}^X}\right)^{\frac{\theta-1}{\theta}} \implies \hat{\Omega}_{ij}^F = \frac{\Omega_{ij}^F}{(Z_{ij}^X)^{\theta-1}} \tag{91}$$

Then the final goods producer's problem becomes to maximize

$$P_i^F\left[\sum_j (\hat{\Omega}_{ij}^F)^{\frac{1}{\theta}} (X_{ij}^F)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} - \sum_j P_j^X \mathcal{E}_{ij} X_{ij}^F.$$

This has the same form as the final goods producer's problem without trade costs (see (18)), with Ω_{ij}^F replaced by a composite parameter of the pure input weights Ω_{ij}^F and the iceberg trade cost.

A.3 Hansen-Jagannathan Bound on Labor

Consider reformulating the household's problem with preferences that are separable between consumption and labor. The household ι of country i maximizes

$$\sum_t \sum_{s^t} \beta^t \pi(s^t)\left[u(C_i(\iota, s^t)) - v(L_i(\iota, s^t))\right]$$

subject to the same cash-in-advance constraint:

$$P_i^F(s^t)C_i(\iota, s^t) \leq \sum_j \frac{D_{ij}(\iota, s^t)\mathcal{E}_{ij}(s^t)}{Z_{ij}(\iota)} + \frac{W_i(s^t)L_i(\iota, s^t)}{Z_i^L}$$

and the same budget constraint. Combining the first-order conditions for consumption $C_i(\iota, s^t)$ and labor $L_i(\iota, s^t)$ obtains

$$\frac{v'(L_i(\iota, s^t))}{u'(C_i(\iota, s^t))} = \frac{W_i(s^t)}{P_i^F(s^t) Z_i^L},$$

which can be rearranged to

$$\frac{v'(L_i(\iota,s^t))}{W_i(s^t)/Z_i^L}=\frac{u'(C_i(\iota,s^t))}{P_i^F(s^t)}.$$

This allows the stochastic discount factor $\tilde{M}$ to be written in terms of the disutility of labor:

$$\tilde{M}(s^t,s^{t+1})\equiv\beta\frac{u'(C(s^{t+1}))/P^F(s^{t+1})}{u'(C(s^t))/P^F(s^t)}=\beta\frac{v'(L(s^{t+1}))/W(s^{t+1})}{v'(L(s^t))/W(s^t)}$$

Hansen and Jagannathan (1991) show that the volatility of $\tilde{M}$ must be higher than the Sharpe ratio of any asset:

$$\sigma[\tilde{M}]\geq\max_{\{R_t\}}\left\{\frac{\mathrm{E}[R_t-R_t^f]}{\sigma[R_t-R_t^f]}\right\},$$

The Sharpe ratio of the aggregate US stock market is on the order of 0.5. Hence, it must be that

$$\sigma\left[\beta\frac{v'(L(s^{t+1}))/W(s^{t+1})}{v'(L(s^t))/W(s^t)}\right]\geq0.5.$$

Now substitute in the constant elasticity specification for the disutility of labor: $v'(L)=\chi_0L^{\chi_1}$:

$$\beta\times\sigma\left[\left(\frac{L(s^{t+1})}{L(s^t)}\right)^{\chi_1}\frac{W(s^t)}{W(s^{t+1})}\right]\geq0.5,$$

where $1/\chi_1$ is the Frisch elasticity of labor supply. Using the delta method and the Cauchy-Schwarz inequality, the volatility term can be bounded from above by

$$\chi\,\mathrm{E}\left[\left(\frac{L(s^{t+1})}{L(s^t)}\right)^{\chi_1-1}\frac{W(s^t)}{W(s^{t+1})}\right]\times\sigma\left[\frac{L(s^{t+1})}{L(s^t)}\right]+\mathrm{E}\left[\left(\frac{L(s^{t+1})}{L(s^t)}\right)^{\chi_1}\left(\frac{W(s^t)}{W(s^{t+1})}\right)^2\right]\times\sigma\left[\frac{W(s^{t+1})}{W(s^t)}\right]$$

The expectation terms are approximately 1 at an annual frequency. The volatility of annual growth in hours worked is below 3 percent and the volatility of annual growth in average wages is below 2 percent in the US. The Frisch elasticity of labor supply is generally estimated to be above 0.5, which puts an upper bound of 2 on χ. The discount factor β is bounded from above by 1. Putting this into the bounding term, it must be that

$$\beta\times\sigma\left[\left(\frac{L(s^{t+1})}{L(s^t)}\right)^{\chi_1}\frac{W(s^t)}{W(s^{t+1})}\right]<0.08,$$

which is one order of magnitude too low for the Hansen-Jagannathan bound to be satisfied. Hence, in a setting with endogenous labor supply, the mechanism that makes marginal utility of consumption volatile must operate on a composite of consumption and labor.

A.4 Proof of Proposition 6 (Currency Risk Premium)

Step 1: Euler equation for contingent claims. Recall the first-order condition for country-j claims, in equation (30)

$$\beta^t\pi(s^t)u'\big(C_i(s^t)-v(L_i(s^t))\big)\left[d_{ij}^{sh}(s^t)\right]^{-\frac{1}{\zeta}}\frac{\mathcal{E}_{ij}(s^t)}{P_i^F(s^t)Z_{ij}}=\mu_iQ_j(s^t)\mathcal{E}_{ij}(s^0)$$

The same equation for state s^{t+1} is

$$\beta^{t+1}\pi(s^{t+1})u_i'(s^{t+1})\left[d_{ij}^{sh}(s^{t+1})\right]^{-\frac{1}{\zeta}}\frac{\mathcal{E}_{ij}(s^{t+1})}{P_i^F(s^{t+1})Z_{ij}}=\mu_iQ_j(s^{t+1})\mathcal{E}_{ij}(s^0)$$

Taking the ratio obtains an Euler equation:

$$\pi(s^{t+1} \mid s^t)\beta \frac{u_i'(s^{t+1})/P_i^F(s^{t+1})}{u_i'(s^t)/P_i^F(s^t)} \left(\frac{d_{ij}^{sh}(s^{t+1})}{d_{ij}^{sh}(s^t)}\right)^{-\frac{1}{\zeta}} \frac{\mathcal{E}_{ij}(s^{t+1})}{\mathcal{E}_{ij}(s^t)} = \frac{Q_j(s^{t+1})}{Q_j(s^t)}. \tag{92}$$

The nominal stochastic discount factor (SDF) of i's representative household $\tilde{M}_i$ is the change in marginal utility per unit of home currency across periods:[27]

$$\tilde{M}_i(s^{t+1} \mid s^t) = \beta \frac{u_i'(s^{t+1})/P_i^F(s^{t+1})}{u_i'(s^t)/P_i^F(s^t)}. \tag{93}$$

Denote the change in portfolio shares by

$$\exp(\eta_{ij}(s^{t+1})) = \left(\frac{d_{ij}^{sh}(s^{t+1})}{d_{ij}^{sh}(s^t)}\right)^{-\frac{1}{\zeta}}$$

Then the Euler equation can be rewritten as

$$\pi(s^{t+1} \mid s^t)\tilde{M}_i(s^{t+1} \mid s^t)\exp(\eta_{ij}(s^{t+1}))\frac{\mathcal{E}_{ij}(s^{t+1})}{\mathcal{E}_{ij}(s^t)} = \frac{Q_j(s^{t+1})}{Q_j(s^t)}. \tag{94}$$

Symmetrically, for $i = j$, the corresponding Euler equation is

$$\pi(s^{t+1} \mid s^t)\tilde{M}_j(s^{t+1} \mid s^t)\exp(\eta_{jj}(s^{t+1})) = \frac{Q_j(s^{t+1})}{Q_j(s^t)}.$$

Taking the ratio of these two Euler equations, and rearranging, an expression for exchange rate changes in terms of the SDF and the η terms emerges:

$$\frac{\mathcal{E}_{ij}(s^{t+1})}{\mathcal{E}_{ij}(s^t)} = \frac{\tilde{M}_j(s^{t+1} \mid s^t)}{\tilde{M}_i(s^{t+1} \mid s^t)} \frac{\exp(\eta_{jj}(s^{t+1}))}{\exp(\eta_{ij}(s^{t+1}))}. \tag{95}$$

In logarithms, the change in Euler equation

$$\Delta e_{ijt+1} = (\tilde{m}_{jt+1} + \eta_{jjt+1}) - (\tilde{m}_{it+1} + \eta_{ijt+1}). \tag{96}$$

Step 2: Euler equation for risk-free bonds. For a bundle of j-originated claims to pay 1 unit of i's currency in each state s^{t+1}, the bundle must pay $\mathcal{E}_{ji}(s^{t+1}) = 1/\mathcal{E}_{ij}(s^{t+1})$ units of j's currency in each state. At state s^t, putting this bundle together costs $\sum_{s_{t+1}} \mathcal{E}_{ji}(s^{t+1})Q_j(s^{t+1})/Q_j(s^t)$ units of j's currency, which is equivalent to

$$\sum_{s_{t+1}} \frac{\mathcal{E}_{ij}(s^t)}{\mathcal{E}_{ij}(s^{t+1})} \frac{Q_j(s^{t+1})}{Q_j(s^t)}$$

units of i's currency. As this is the cost to obtain 1 unit of i's currency in each state s^{t+1}, this expression must be $1/R_j^f(s^t)$, the reciprocal of the gross risk-free rate on currency i. Substituting the Euler equation (94) and rearranging obtains

$$1 = \sum_{s_{t+1}} \pi(s^{t+1} \mid s^t)\tilde{M}_i(s^{t+1} \mid s^t)\exp(\eta_{ij}(s^{t+1}))R_i^f(s^t).$$

[27] I add the tilde to $\tilde{M}$ to disambiguate from money holdings, which is denoted M.

Note that the right-hand side is an expectation:

$$1 = \mathrm{E}_t\Big[\tilde{M}_i(s^{t+1} \mid s^t)\exp(\eta_{ij}(s^{t+1}))\Big] R_i^f(s^t). \tag{97}$$

Assuming that the global shock follows a normal distribution, the conditional SDF is log-linear. Taking logarithms:

$$r_{it}^f = -\,\mathrm{E}_t\big[\tilde{m}_{it+1} + \eta_{ijt+1}\big] - \tfrac{1}{2}\,\mathrm{var}_t\big[\tilde{m}_{it+1} + \eta_{ijt+1}\big]. \tag{98}$$

Symmetrically, currency j's risk-free rate is

$$r_{jt}^f = -\,\mathrm{E}_t\big[\tilde{m}_{jt+1} + \eta_{jjt+1}\big] - \tfrac{1}{2}\,\mathrm{var}_t\big[\tilde{m}_{jt+1} + \eta_{jjt+1}\big].$$

Step 3: Currency risk premium for generic SDF. Combining the two preceding equations obtains the interest rate differential between the two currencies:

$$\begin{aligned} r_{jt}^f - r_{it}^f = &- \mathrm{E}_t\big[\tilde{m}_{jt+1} + \eta_{jjt+1} - \tilde{m}_{it+1} - \eta_{ijt+1}\big] \\ &- \tfrac{1}{2}\big(\mathrm{var}_t\big[\tilde{m}_{jt+1} + \eta_{jjt+1}\big] - \mathrm{var}_t\big[\tilde{m}_{it+1} + \eta_{ijt+1}\big]\big) \end{aligned} \tag{99}$$

Recall the definition of the excess return on j's one-period nominal risk-free bonds from equation (48):

$$rx_{ijt+1} = \big(r_{jt}^f - r_{it}^f\big) + \Delta e_{ijt+1}.$$

Substituting in the expression for the interest rate differential from equation (99) and the expression for the exchange rate appreciation from equation (96), the expected excess return, which is j's currency risk premium, is

$$\mathrm{E}_t[rx_{ijt+1}] = -\tfrac{1}{2}\big(\mathrm{var}_t\big[\tilde{m}_{jt+1} + \eta_{jjt+1}\big] - \mathrm{var}_t\big[\tilde{m}_{it+1} + \eta_{ijt+1}\big]\big) \tag{100}$$

Step 4: Currency risk premium for habit-formation preferences. Let $\tilde{C}_i$ denote the consumption-leisure aggregator:

$$\tilde{C}_i \equiv C_i - v(L_i).$$

The expression for marginal utility (47) under habit-formation preferences becomes:

$$u_i'(s^t) = \big[\tilde{S}_i(\iota, s^t)\tilde{C}_i(s^t)\big]^{-\gamma}.$$

Substituting this into the definition of the SDF (equation (93)):

$$\tilde{M}_i(s^{t+1} \mid s^t) = \beta\left(\frac{\tilde{S}_i(\iota, s^{t+1})\tilde{C}_i(s^{t+1})}{\tilde{S}_i(\iota, s^t)\tilde{C}_i(s^t)}\right)^{-\gamma} \times \left(\frac{P_i^F(s^{t+1})}{P_i^F(s^t)}\right)^{-1}.$$

Taking logarithms:

$$\tilde{m}_{it+1} = \log\beta - \gamma\Delta\tilde{c}_{it+1} - \gamma\Delta\tilde{s}_{it+1} - \Delta p_{it+1}^F.$$

Recall from equation (46) that log surplus consumption evolves according to

$$\tilde{s}_{t+1} = (1-\rho_s)\bar{s} + \rho_s\tilde{s}_t + \lambda(\tilde{s}_t)\varepsilon_{Gt},$$

so that its first difference is

$$\Delta\tilde{s}_{t+1} = -(1-\rho_s)(\tilde{s}_t - \bar{s}) + \lambda(\tilde{s}_t)\varepsilon_{Gt+1}.$$

Therefore, the log SDF becomes

$$\tilde{m}_{it+1} = \log\beta + \gamma(1-\rho_s)(\tilde{s}_t - \bar{s}) - \gamma\Delta\tilde{c}_{it+1} - \Delta p^F_{it+1} - \gamma\lambda(\tilde{s}_t)\varepsilon_{Gt+1}. \tag{101}$$

From equation (96), the exchange rate movement is

$$\Delta e_{ijt+1} = -\gamma(\Delta\tilde{c}_{jt+1} - \Delta\tilde{c}_{it+1}) - (\Delta p^F_{jt+1} - \Delta p^F_{it+1}) + (\eta_{jjt+1} - \eta_{ijt+1}) \tag{102}$$

From equation (100), the term of interest for the risk premium is $\mathrm{var}_t\big[\tilde{m}_{it+1}+\eta_{ijt+1}\big]$, which decomposes to:

$$\begin{aligned}
\mathrm{var}_t\big[\tilde{m}_{it+1} + \eta_{ijt+1}\big] &= \mathrm{var}_t[\tilde{m}_{it+1}] + 2\,\mathrm{cov}_t[\tilde{m}_{it+1}, \eta_{ijt+1}] + \mathrm{var}_t[\eta_{ijt+1}] \\
\mathrm{var}_t[\tilde{m}_{it+1}] &= \gamma^2\,\mathrm{var}_t[\Delta\tilde{c}_{it+1}] + \mathrm{var}_t[\Delta p^F_{it+1}] + \gamma^2\lambda(\tilde{s}_t)^2\sigma_G^2 \\
&\quad + 2\gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\big[\gamma\Delta\tilde{c}_{it+1} + \Delta p^F_{it+1}, \varepsilon_{Gt+1}\big] + 2\gamma\,\mathrm{cov}_t[\Delta\tilde{c}_{it+1}, \Delta p^F_{it+1}].
\end{aligned}$$

As shown by Mehra and Prescott (1985), the variance terms are small; the only terms that matter quantitatively are those that interact with the stochastic volatility term $\lambda(\tilde{s}_t)$:

$$\mathrm{var}_t[\tilde{m}_{it+1}] \approx \gamma^2\lambda(\tilde{s}_t)^2\sigma_G^2 + 2\gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\big[\gamma\Delta\tilde{c}_{it+1} + \Delta p^F_{it+1}, \varepsilon_{Gt+1}\big],$$

and hence

$$\mathrm{var}_t\big[\tilde{m}_{it+1} + \eta_{ijt+1}\big] \approx \gamma^2\lambda(\tilde{s}_t)^2\sigma_G^2 + 2\gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\big[\gamma\Delta\tilde{c}_{it+1} + \Delta p^F_{it+1} - \eta_{ijt+1}, \varepsilon_{Gt+1}\big].$$

Substituting this expression into the risk premium (100) obtains

$$\begin{aligned}
\mathrm{E}_t[rx_{ijt+1}] &\approx -\gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\big[\gamma\Delta\tilde{c}_{jt+1} + \Delta p^F_{jt+1} - \eta_{jjt+1}, \varepsilon_{Gt+1}\big] \\
&\qquad + \gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\big[\gamma\Delta\tilde{c}_{it+1} + \Delta p^F_{it+1} - \eta_{ijt+1}, \varepsilon_{Gt+1}\big] \\
&= \gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\big[-\gamma(\Delta\tilde{c}_{jt+1} - \Delta\tilde{c}_{it+1}) - (\Delta p^F_{jt+1} - \Delta p^F_{it+1}) + (\eta_{jjt+1} - \eta_{ijt+1}), \\
&\qquad\qquad \varepsilon_{Gt+1}\big]
\end{aligned}$$

Substituting equation (102) for the expression in the covariance term, the desired expression is achieved:

$$\mathrm{E}_t[rx_{ijt+1}] \approx \gamma\lambda(\tilde{s}_t)\,\mathrm{cov}_t\big[\Delta e_{ijt+1}, \varepsilon_{Gt+1}\big].$$

QED

B Data Appendix

B.1 Data Construction

International data come from International Financial Statistics (IMF, 2025), except for market capitalization data used to calculate the denominator of the home bias measure in equation (42), which are compiled by World Federation of Exchanges (2025) and published by the World Bank. Quarterly US hours worked, capital stock, and total factor productivity are from Fernald (2014). US consumption of financial services is from the Bureau of Economic Analysis (2025), Table 2.4.5. Pre-Euro currency exchange rates are from Chari, Kehoe and McGrattan (2002).

B.2 Plots

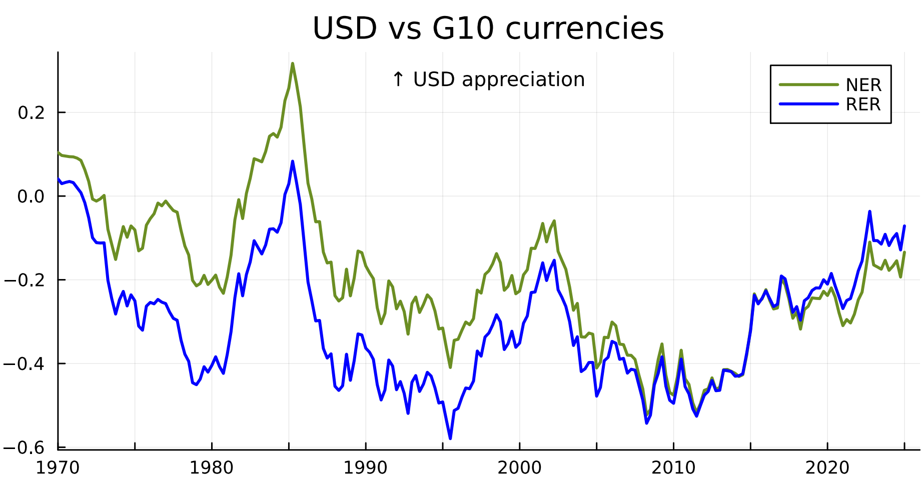

Figure 9: US Dollar Nominal and Real Exchange Rates vs G10 Currencies

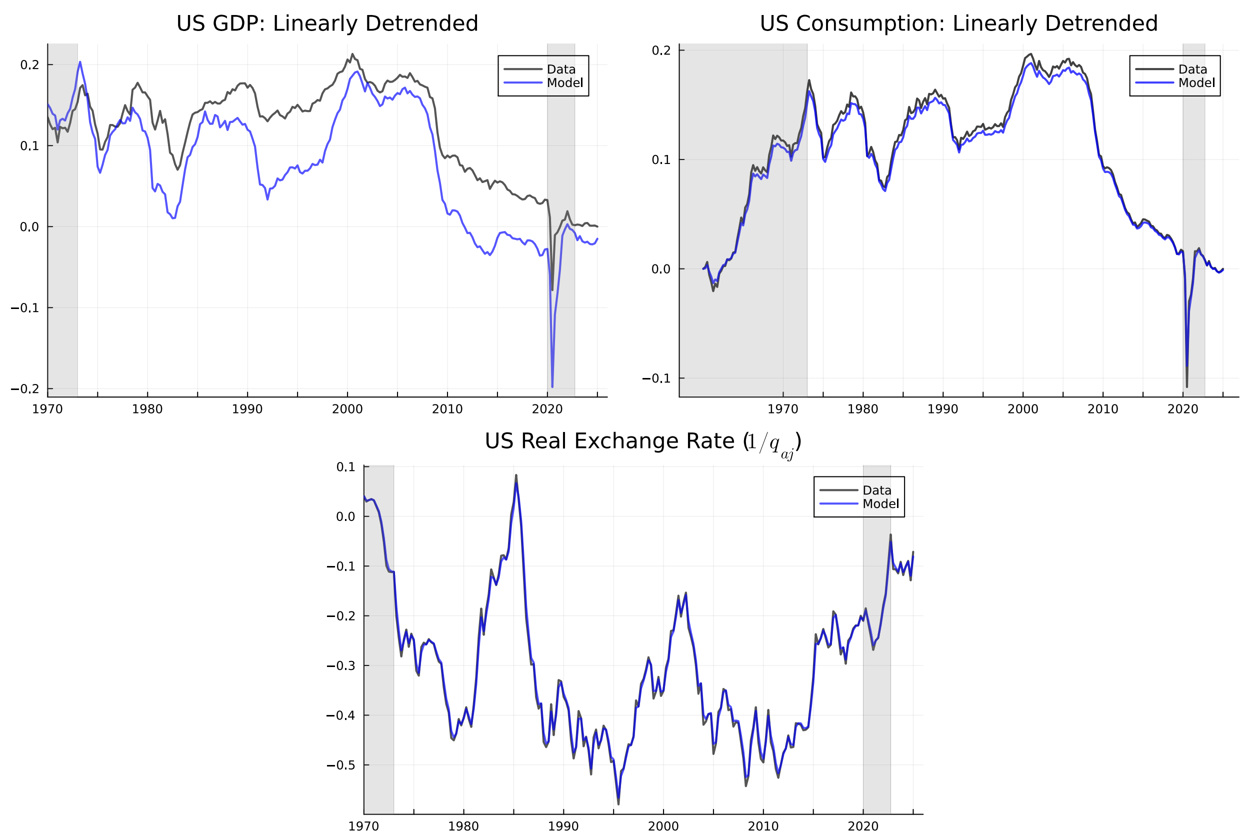

Figure 10: Kalman Filter: Model vs Data