Beam Algorithm - Software Version (v0.0.5)

- Initialize a groupoid G of minimum size 3x3. Initialize a beam of width w and the set of male terms M equal the set of term variables \(\overline{x}\). Initialize T, the target array of length \(g^k\) where g is the size of the groupoid G and k is the number of term variables in \(\overline{x}\).
- 2. At beam level 0, initialize a set F containing w empty female terms $f_0(\lozenge)$. Let variable H be the current level of a female term in the beam, initialized to 0.
- 3. At beam level 0 mate each female term $f_0(\lozenge)$ with each male term $m(\overrightarrow{x})$ in M and check if the resulting offspring $f_0(m(\overrightarrow{x}))$ is a solution to the target array T. If $f_0(m(\overrightarrow{x}))$ is a solution to T, then return $f_0(m(\overrightarrow{x}))$.
- 4. Start a process $P_{f_H(\overrightarrow{x},\Diamond)}$ for each female term $f_H(\overrightarrow{x},\Diamond)$ in F for a total of w processes. Let each process $P_{f_H(\overrightarrow{x},\Diamond)}$ search for a female term at level H+1 called $f_{H+1}(\overrightarrow{x},\Diamond)$ that is valid with respect to the validity array of $f_H(\overrightarrow{x},\Diamond)$.
- 5. Let variable C be the number of child terms of some term $f_H(\overrightarrow{x}, \lozenge)$ that are required to promote a process $P_{f_H(\overrightarrow{x}, \lozenge)}$ to level H+1 before a higher beam level is full. A beam level is considered full when there are w valid female terms at that level.

START LOOPING CONTINUOUSLY

- 6. Let $f_{H+1}(\overrightarrow{x}, \lozenge)$ be a valid female term returned by a process $P_{f_H(\overrightarrow{x}, \lozenge)}$. Assume $f_{H+1}(\overrightarrow{x}, \lozenge)$ was found using female term candidates chosen at random inside of a process $P_{f_H(\overrightarrow{x}, \lozenge)}$ using the Gamblers Ruin Algorithm (GRA). Add $f_{H+1}(\overrightarrow{x}, \lozenge)$ to beam level H+1.
- 7. Let $f_H(\overrightarrow{x}, \lozenge)$ be the parent term at level H of the valid female term $f_{H+1}(\overrightarrow{x}, \lozenge)$ found in step 6.
- 8. Mate the valid female term $f_{H+1}(\overrightarrow{x}, \lozenge)$ (from 6) with each male term $m(\overrightarrow{x})$ in M and check if the resulting offspring $f_{H+1}(\overrightarrow{x}, m(\overrightarrow{x}))$ has a term operation that is a solution to the validity array of $f_H(\overrightarrow{x}, \lozenge)$. If $f_{H+1}(\overrightarrow{x}, m(\overrightarrow{x}))$ is a solution to the validity array of $f_H(\overrightarrow{x}, \lozenge)$, then break from the loop and proceed to step 12.
- 9. If $f_H(\overrightarrow{x}, \lozenge)$ has produced C children at H+1:
 - Assign process $P_{f_H(\overrightarrow{x},\Diamond)}$ to the child term $f_{H+1}(\overrightarrow{x},\Diamond)$ at level H+1 and have it search for some term $f_{H+2}(\overrightarrow{x},\Diamond)$ that is valid with respect to validity array of $f_{H+1}(\overrightarrow{x},\Diamond)$.

Next let PL be the ordered set of processes running at a level below H+1 and sorted ascending by process level and number of produced child terms. For each process $P_{f_{LH}}(\overrightarrow{x}, \Diamond)$ in PL, kill $P_{f_{LH}}(\overrightarrow{x}, \Diamond)$ and assign $P_{f_{LH}}(\overrightarrow{x}, \Diamond)$ to the next child of $f_H(\overrightarrow{x}, \Diamond)$ that doesn't already have a running process $P_{f_{H+1}}(\overrightarrow{x}, \Diamond)$ associated with it. Have process $P_{f_{LH}}(\overrightarrow{x}, \Diamond)$ search for for some term $f_{H+2}(\overrightarrow{x}, \Diamond)$ that is valid with respect to the validity array of that child of $f_H(\overrightarrow{x}, \Diamond)$ that it was assigned to. Once all child terms of $f_H(\overrightarrow{x}, \Diamond)$ have an assigned running process or there are no more processes running below level H+1, return to step 6.

10. If the beam is full at a level above the level of the lowest running process:

Let PL be the set of processes running at a level below the highest full level. Kill all processes $P_{f_{LH}(\overrightarrow{x},\lozenge)}$ in PL, assign them to terms at the highest full level, and have each of them search for a new female term that is valid with respect to the array of the term that they were respectively assigned to. Return to step 6.

11. If conditions 9 and 10 both were not satisfied, then rerun process $P_{f_H(\overrightarrow{x},\Diamond)}$ for the parent $f_H(\overrightarrow{x},\Diamond)$ of the valid female term $f_{H+1}(\overrightarrow{x},\Diamond)$. Essentially don't reassign any processes to a higher level and continue searching for another female term $f_{H+1}(\overrightarrow{x},\Diamond)$ that is valid with respect to $f_H(\overrightarrow{x},\Diamond)$. Return to step 6.

CONTINUE LOOPING CONTINUOUSLY

12. Some solution term $f_H(\overrightarrow{x}, m(\overrightarrow{x}))$ was found at step 8. Recursively mate $f(\overrightarrow{x}, m(\overrightarrow{x}))$ with each of parent term at $f_{H-1}(\overrightarrow{x}, \diamondsuit)$, until reaching the term that has no parent. The result is a term which has an array that is a solution to the target array T.