Analytical model of pedestrianized and transit priority zones in a rectilinear grid city

Nicholas Fournier and Eric Gonzales

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1 Concept

Consider an idealized city of dimension R with a rectilinear street network with spacing d, as shown in Figure 1. Demand can be defined by two types of trip patterns, baseline uniform travel demand across the city and monocentric trips to and from the city center. The cumulative effect is increased congestion in the city center. To make the city center more attractive, "livable", and walkable, a square zone of size γ in the city center has been pedestrianized, allowing only pedestrians, bicycles, and transits. The pedestrianized zone forces drivers to divert routes around the zone, increasing traffic density and congestion as a result. To mitigate the congestion's impact on mixed-traffic transit (i.e., buses and streetcars/trams), an area of dimension τ has been designated transit priority, giving transit dedicated lanes. This reduces street capacity for automobiles, but negates the impact of congestion on transit speed.

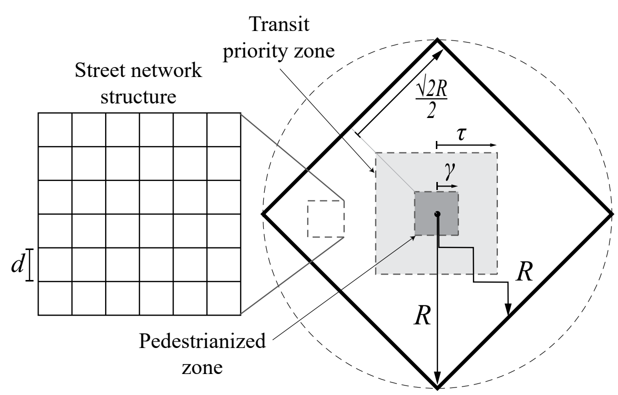


Figure 1: Rectilinear city with pedestrianized corridor

The objective is to model the traffic impact on the surrounding street network in order to determine:

1. *what are the optimal pedestrian and transit zone sizes?*
2. *what are the impacts on travel time?*
3. *what are the resulting shifts in demand?*

2 Demand

Demand is generated in the city area in units of $\frac{trips}{dist^2 \cdot time}$, and can be simplified into two types:

- uniform baseline travel across the network, λ_b, and
- monocentric travel demand going to and from the center of the city, λ_c.

The baseline traffic flows associated with average trip length l generate $q_b = \frac{\lambda_b l}{\delta}$ flow across the network. The average distance for baseline and monocentric trips in a rectilinear city are $l_b = \frac{14}{15}R$ and $l_c = \frac{2}{3}R$, respectively. The additional traffic flow associated with trips to and from the center varies with the distance from the center. Consider a narrow square of infinitesimal width dr at radius r from the center. The total demand for trips crossing this square is $2\lambda_m(R^2 - r^2)$. Each of these vehicles travels a distance dr within the square, and the amount of road infrastructure is $8r\delta$ as oriented perpendicular (as shown), or $4\sqrt{2}r\delta$ as a 45-degree diamond. δ is the density of available road network in $\frac{lane \cdot dist}{dist^2}$. The calculation for network density is derived as the total roadway length $2n(n+1)d$, divided by total area nd^2; both of which can be expressed as a function of the number of city blocks n, and the street spacing dimension d. Since the function is linearly constant, of δ with respect to d, yielding $\delta = \frac{2}{d}$.

The flow across the network at a point r distance from the city center is then calculated as the combined sum of the baseline demand and the monocentric demand, calculated as:

$$q_a(r) = \frac{14R\lambda_b}{15\delta} + \frac{\lambda_c}{4r\delta}\left(R^2 - r^2\right) \tag{1}$$

The pedestrianized zone will cause some trips to be diverted around the perimeter zone. Trips can be categorized into four distinct types (see Figure 2):

1. Unimodal routes entirely within the pedestrianized zone that are unaffected by the zone,
2. Unimodal routes entirely outside the pedestrianized zone that are unaffected by the zone,
3. Bimodal routes taking vehicles around the perimeter of the pedestrianized zone to park nearest the destination and the remaining radial distance is traveled on foot,
4. Unimodal routes that would have passed through the center travels around the perimeter of the pedestrianized zone vertically and horizontally, and
5. Unimodal routes that would have passed through the center travels around the perimeter of the pedestrianized zone diagonally.

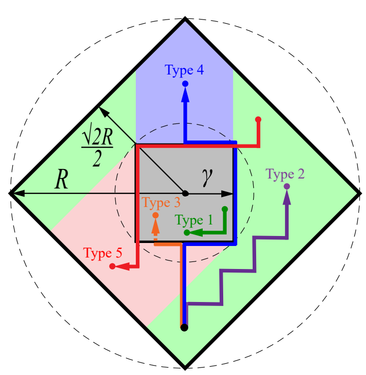


Figure 2: Diverted route types

The flow around the perimeter road is then calculated as the sum of the flow generated by the four trip types:

$$q_1(\gamma) = 0 \tag{2a}$$

$$q_2(\gamma) = \left[2\lambda_b \left(R^2 - \gamma^2\right) \times \frac{\gamma^2}{R^2} \times \frac{\gamma}{2}\right] \frac{4d}{8\gamma(R-2)} \tag{2b}$$

$$q_3(\gamma) = \left[2\lambda_b \left(R^2 - \gamma^2\right) \times \frac{\gamma(2R-3\gamma)}{2R^2} \times 3\gamma\right] \frac{4d}{8\gamma(R-2)} \tag{2c}$$

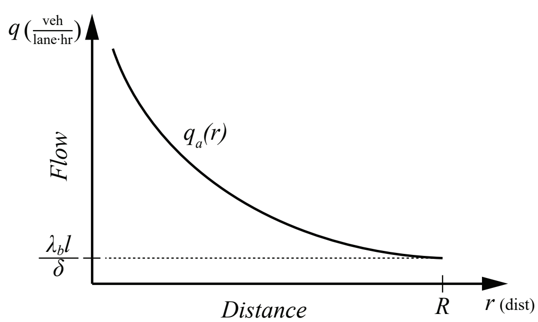
$$q_4(\gamma) = \left[2\lambda_b \left(R^2 - \gamma^2\right) \times \frac{\gamma(\sqrt{2}R-\gamma)}{R^2} \times 3\gamma\right] \frac{4d}{8\gamma(R-2)} \tag{2d}$$

$$\tag{2e}$$

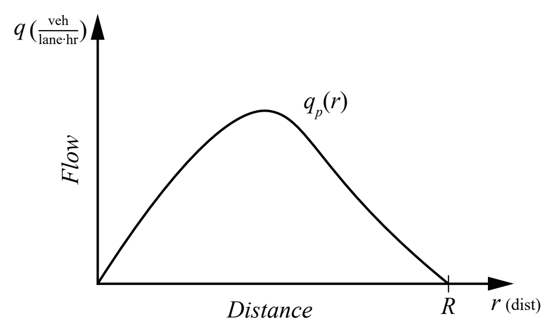
which can be simplified to:

$$q_p(\gamma) = \sum_{i=1}^{4} q_i(\gamma) = \frac{2\lambda_b\gamma(6R + 6\sqrt{2}R - 11\gamma)(R^2-\gamma^2)}{\delta R^2(R-2)} \tag{3}$$

The flow across the network at distance r from the center from the function in Equation (1) possesses the form shown in Figure 3a, and flow around the perimeter of the pedestrian zone from the function in Equation (3) possesses the form shown in Figure 3b. As the distance from the city increases, traffic decreases. When traffic near the center exceeds capacity, one might consider simply pedestrianizing the city center out to the point where capacity if first exceeded. However, as the pedestrian zone size increases so does traffic on the perimeter road.



(a) Trip demand flow at point r distance from center



(b) Trip demand flow on perimeter of pedestrian zone

Figure 3: Macroscopic fundamental diagram and travel time cost function

3 Distance traveled

To determine the travel time, the total average distance traveled must be calculated for the following modes:

3.1 Bicycles and pedestrians

trip is generated by the baseline demand (b) or by the central demand (c). Let D_{ij} be demand amount and l_{ij} be the average walking distance covered in each case ij where i is trip types $\{1,3\}$ and j is demand type $\{b,c\}$. The total distance covered by foot or bike L_P is then:

$$L_P = D_{1b}l_{1b} + D_{1c}l_{1c} + D_{2b}l_{2b} + D_{2c}l_{2c} \tag{4}$$

where:

- $D_{1b} = 2\lambda_b\gamma^2 \times \frac{\gamma^2}{R^2}$: Total baseline demand trips originating in pedestrian zone by the proportion ending in the pedestrian zone.
- $D_{3b} = 4\lambda_b(R^2 - \gamma^2) \times \frac{\gamma^2}{R^2}$: Total baseline demand trips originating inside and ending outside the pedestrian zone, and vice versa. Both cases equal $2\lambda_b(R^2 - \gamma^2) \times \frac{\gamma^2}{R^2}$.
- $D_{1c} = 2\lambda_c\gamma^2$: Total monocentric demand trips originating inside the pedestrian zone.
- $D_{3c} = 2\lambda_c(R^2 - \gamma^2)$: Total monocentric demand trips originating outside the pedestrian zone.
- $l_{1b} = \frac{4}{3}\gamma$: Average distance of baseline trips with uniformly distributed origins and destinations.
- $l_{1c} = \gamma$: Average distance of monocentric trips ending at the center with uniformly distributed origins.
- $l_{3b} = \frac{5}{3}\gamma$: Average distance from uniformly distributed point on the pedestrian zone perimeter to some uniformly distributed destination in the zone.
- $l_{3c} = \gamma$: Average distance from pedestrian zone perimeter to center.

$$L_P = \frac{8\lambda_b\gamma^5 + 20\lambda_b\gamma^3(R^2 - \gamma^2)}{3R^2} + 2\lambda_c\gamma^3 + \gamma(R^2 - \gamma^2) \tag{5}$$

3.2 Driving

The driving distance is the overall average distance, minus the average distance as a pedestrian.

$$L_D = \frac{14R\lambda_b + 10R\lambda_c}{15(\lambda_b + \lambda_c)} - L_P \tag{6}$$

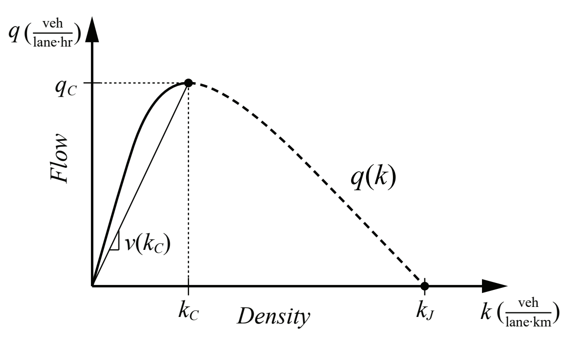
3.3 Transit

3.3.1 in mixed traffic

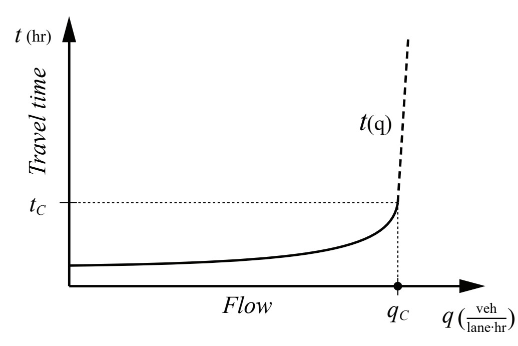
3.3.2 in transit priority zone

4 Traffic flow

Traffic flow through the network is characterized by the macroscopic fundamental diagram (see Figure 4a) as a function of density. A travel time function (see Figure 4b) then depends upon the state of traffic flow through the network as being either "uncongested" (solid line) or "congested" (dashed line) in Figures 4a and 4b.



(a) Macroscopic fundamental diagram



(b) Travel time cost function

Figure 4: Macroscopic fundamental diagram and travel time cost function

A piecewise monotonic cost function for travel time can be defined as:

$$t_D(q) = l\frac{k(q)}{q} \qquad \text{for } q < \mu \tag{7a}$$

$$t_D(q) = t_c\left(\frac{q}{\mu}\right) \qquad \text{for } q \geq \mu \tag{7b}$$

where $t(q)$ is travel time for flow q, $k(q)$ is traffic density for flow q, l is link length, and μ is link capacity. Assuming for this case a parabolic function for the uncongested portion of the flow-density relationship, an expression can be written as

$$q(k) = q_c - (\alpha k - k_c)^2 \tag{8}$$

where k_c is the density at capacity, q_c is the flow at capacity, and α is a fitting parameter. In order to determine travel time using density as a function of flow, $k(q)$, it can be solved for using the quadratic formula

$$k(q) = \frac{k_c - \sqrt{q_c - q}}{\alpha} \tag{9}$$

5 Transit

Transit travel time is conditional upon whether it operates in a transit priority zone (e.g., dedicated lane or right-of-way) or in mixed-traffic (e.g., city bus). In assuming no other source of delay in a transit priority zone, the travel time of transit is then

$$t_{TP} = \frac{l}{v_m} + \frac{l}{s}t_s \qquad \text{with transit priority} \tag{10a}$$

$$t_{TM}(q) = l\frac{k(q)}{q} + \frac{l}{s}t_s \qquad \text{without transit priority when } q \geq \mu \tag{10b}$$

$$t_{TM}(q) = t_c\left(\frac{q}{\mu}\right) + \frac{l}{s}t_s \qquad \text{without transit priority when } q \geq \mu \tag{10c}$$

where v_m is the maximum cruising speed of transit unimpeded by traffic, s is stop spacing, and t_s is stop time. The stop time is essentially the lost time inclusive of acceleration, deceleration and dwell time.

The travel time of transit in mixed-traffic will then always be higher than driving an automobile, eventually converging as traffic conditions reach jam flow. Transit priority provides a constant travel time which will be initially slower than driving in uncongested traffic conditions, but will eventually reach a traffic flow point q_T^*, where the travel time of transit with priority exceeds driving (see Figure 5a).

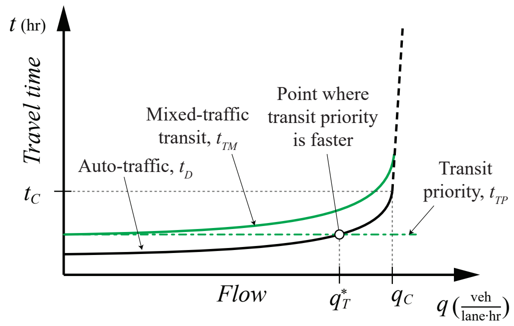
The critical transit priority traffic flow point q_T^*, if found where transit travel time t_{TP} intersects driving travel time $t_D q$. Assuming the parabolic traffic flow function in Equation (9), the critical transit priority traffic flow point is:

$$q_T^* = \alpha^2\left(\frac{1}{v_m} - \frac{t_s}{s}\right)^2 - k_c^2 + q_c \tag{11}$$

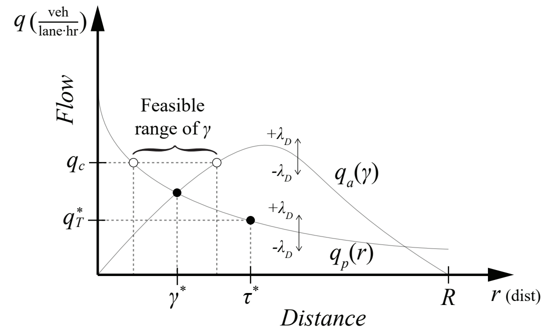
q_T^* could then be used in combination with network flow at distance r in Equation (1) to determine the optimal transit priority zone, τ^*. The solution to this is quadratic, but assuming a non-negative value for the optimal transit zone size, it can be solved for analytically:

$$\tau^* = \frac{2\left(q_t\delta - \lambda_b R\right) - \sqrt{4\left(q_t\delta - \lambda_b R\right)^2 + \lambda_c^2 R^2}}{\lambda_c} \tag{12}$$

Determining an optimal pedestrian zone size is slightly more complex. An appropriate pedestrian zone size can exist anywhere in the range $q_a(\gamma) < q_c > q_p(\gamma)$ where the pedestrian zone is (see Figure 5b):



(a) Transit travel time



(b) Optimal pedestrian and transit priority zone size

Figure 5: Transit travel time and optimal zone sizing

(a) large enough to prevent traffic flow across the network from exceeding capacity $q_a(\gamma) < q_c$, and

(b) small enough to prevent perimeter traffic flow from exceeding capacity $q_p(\gamma) < q_c$.

The intersection point of the two functions may be used as an optimal compromise, ensuring both conditions are satisfied with the least possible traffic flow. However, setting $q_a = q_p$ to find the intersection point results in a quintic function (polynomial to the 5^{th} power):

$$f(\gamma) = \frac{11}{R^2}\gamma^5 - \frac{6(1+\sqrt{2})}{R}\gamma^4 - 11\gamma^3 + \frac{48\lambda_b R(1+\sqrt{2}) + \lambda_c(R-2)}{8\lambda_b}\gamma^2 - \frac{14R(R-2)}{30}\gamma - \frac{\lambda_c R^2(R-2)}{8\lambda_b} \quad (13)$$

yielding multiple points where the function is equal to zero (see Figure 6) and cannot be easily solved analytically.

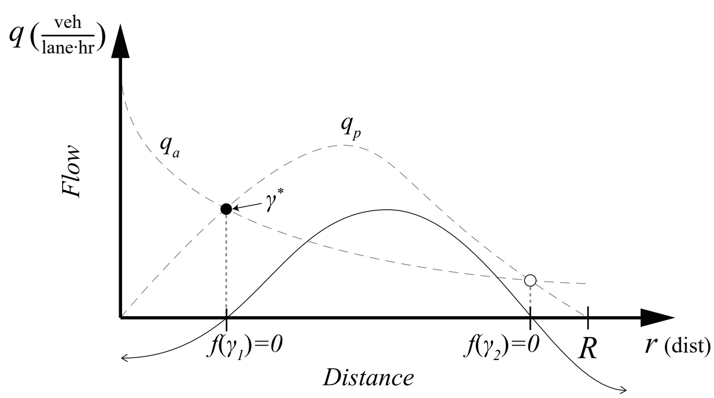


Figure 6: Quintic function for optimal pedestrian zone size, $f(\gamma)$

Given that the pedestrian zone size must exist between $0 \leq \gamma \leq R$ an optimal pedestrian zone dimension γ^*, can be found numerically. Since two intersection points between q_a and q_p exist, the minimum of the two provides the desired optimal γ^*:

$$\gamma^* \Leftarrow \min\left[f(\gamma_1) = 0, f(\gamma_2) = 0\right] \quad (14)$$

$$\text{s.t. } 0 \leq \gamma \leq R \quad (15)$$

6 Mode choice

Assuming unlimited transit capacity relative to driving and that transit priority is unaffected by congestion, a transit system with transit priority in this model effectively serves as a pressure release for demand. As congestion increases with driving trips, transit becomes more appealing, thus drawing trips away from driving. Conversely, a reduction in congestion would attract trips towards driving as the driving travel time has improved. Assuming transit provides a reasonable and reliable alternative to driving, some demand equilibrium exists where the travel time cost of driving matches transit.

The mode choice can be modeled as the probability of choosing to drive P_D or transit P_T, using a simple two-alternative logit model with (dis-)utility as the choice travel cost:

$$P_D = \frac{1}{1 + e^{-\beta(t_D - t_T)}} \tag{16a}$$

$$P_T = 1 - P_D \tag{16b}$$

where t_D and t_T are the travel times associated with automobile and transit, respectively; and β is the a estimated scaling parameter. The probability in Equation (16) can be used to find the proportion of trips made by driving.

7 ...

A two-alternative logit can be simplified to require only the cost difference between the two choices, not the full value, thus logit only requires a travel time differential $\Delta t = t_D - t_T$:

$$\Delta t(q) = l\frac{k(q)}{q} - \left[\frac{\tau}{R}\left(\frac{l}{v_T} + \frac{l}{s}t_s\right) + \frac{R-\tau}{R}\left(l\frac{k(q)}{q} + \frac{l}{s}t_s\right)\right] \tag{17}$$

Since q_T^* is a global constant regardless of demand, we can get τ by setting $q_a(\tau) = q_T^*$. But q_a depends on demand, which depends on probability, which depends on travel time.

$$q_a(\tau, \Delta t(q)) = P_D(\Delta t(q)) \times \left[\frac{\lambda_b l}{\delta} + \frac{\lambda_c}{4\delta}\left(\frac{R^2}{\tau} - \tau\right)\right] \tag{18}$$

$$q_a(\tau, \Delta t(q)) = q_T^* \tag{19}$$

Now there is some sort of dynamic equilibrium with flow and demand where travel time depends on flow and flow depends on travel time.

The pedestrian zone size somewhat less critical since it can exist in a range. However, lets set boundary as the perimeter flow.

$$q_p(\gamma, \Delta t(q)) = P_D(\Delta t(q)) \times \frac{\lambda_b d\gamma(R^2 - \gamma^2)(3R + 3\sqrt{2}R - 7\gamma)}{R^2(R-2)} < q_c \tag{20}$$