

1. Integrale

1.1. Grundintegrale

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \begin{cases} \arcsin x + c_1 \\ -\arccos x + c_2 \end{cases}$$

$$\int \frac{1}{1+x^2} dx = \begin{cases} \arctan x + c_1 \\ -\operatorname{arccot} x + c_2 \end{cases}$$

$$\int \sinh x dx = \cosh x + c$$

$$\int \cosh x dx = \sinh x + c$$

$$\int \frac{1}{\cosh^2 x} dx = \tanh x + c$$

$$\int \frac{1}{\sinh^2 x} dx = -\coth x + c$$

$$\int \frac{1}{\sqrt{x^2+1}} dx = \operatorname{arsinh} x + c = \ln |x + \sqrt{x^2+1}| + c$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \operatorname{arcosh} |x| + c = \ln |x + \sqrt{x^2-1}| + c \quad (|x| > 1)$$

$$\int \frac{1}{1-x^2} dx = \begin{cases} \operatorname{artanh} x + c_1 = \frac{1}{2} \cdot \ln \left(\frac{1+x}{1-x} \right) + c_1 & |x| < 1 \\ \operatorname{arcoth} x + c_2 = \frac{1}{2} \cdot \ln \left(\frac{1+x}{1-x} \right) + c_2 & |x| > 1 \end{cases}$$

1.2. Substitution

dann ist

$$\int f(g(t)) g'(t) dt = F(g(t)) + C$$

bzw.

$$\int_a^b f(g(t)) g'(t) dt = F(x) \Big|_{x=g(a)}^{x=g(b)}$$

1.3. Standard Substitutionen

f diffbar, $\alpha \in \mathbb{R} \setminus \{-1\}$

$$\int (f(x))^\alpha \underbrace{f'(x) dx}_{du} \quad u = f(x)$$

$$\int u^\alpha du \quad \frac{du}{dx} = f'(x)$$

$$du = f'(x) dx$$

f muß umkehrbar sein!

Fall 1 mit $\alpha = -1$.

$$\int \frac{f'(x)}{f(x)} dx \quad \text{Subst.}$$

$$u = f(x)$$

$$\frac{du}{dx} = f'(x)$$

$$du = f'(x) dx$$

$$\int \frac{1}{u} du$$

$$\ln |u| + C = \ln |f(x)| + C$$

Bsp: $\int \frac{3}{\cos^2(4x-2)} dx$

$$= \frac{1}{4} \int \frac{3}{\cos^2(u)} du$$

Subst
 $u = 4x - 2$
 $du = 4 dx$
 $dx = \frac{1}{4} du$

$u = \tan \frac{x}{2}$ Standard

$$x = 2 \arctan u$$

$$\frac{dx}{du} = 2 \frac{1}{1+u^2}$$

$$dx = \frac{2}{1+u^2} du$$

$$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$= \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2}}$$

$$\sin x = \frac{2u}{1+u^2}$$

$$\cos x = \frac{1-u^2}{1+u^2}$$

$$\tan x = \frac{2u}{1-u^2}$$

$$\cot x = \frac{1-u^2}{2u}$$

1.4. Partielle Integration

$$\int u'(x) v(x) dx = u(x) v(x) - \int u(x) v'(x) dx$$

bestimmtes Integral

$$\int_a^b u'(x) v(x) dx = u(x) v(x) \Big|_a^b - \int_a^b u(x) v'(x) dx$$

1.5. Partialbruchzerlegung

$$\frac{f(x)}{N(x)} = \frac{Z(x)}{N(x)} = \frac{d_1}{x-x_1} + \frac{d_2}{x-x_2} + \dots + \frac{d_n}{x-x_n}$$

PBZ

$$\frac{\textcircled{3x-5}}{x^2+2x-8} = \frac{d_1}{x+4} + \frac{d_2}{x-2}$$

Hauptnenner

$$= \frac{(d_1+d_2)x - 2d_1+4d_2}{(x+4)(x-2)}$$

$$\begin{cases} d_1+d_2=3 \\ 2d_1-4d_2=5 \end{cases}$$

alternativ: $N'(x)=2x+2$

$$d_1 = \left[\frac{3x-5}{2x+2} \right]_{x=-4} = \frac{-17}{-6} = \frac{17}{6}$$

$$d_2 = \left[- \right]_{x=2} = \frac{1}{6}$$

Für fälle höherer ordnung

PBZ

$$\frac{Z(x)}{N(x)} = \frac{d_1}{(x+1)} + \frac{d_2}{(x+2)} + \frac{d_3}{(x+2)^2}$$

Bestimmung der d_i

Hauptnenner

$$Z(x) = d_1(x+2)^2 + d_2(x+1)(x+2) + d_3(x+1)$$

$$x=-1: Z(-1) = \underline{-1} = d_1$$

$$x=-2: Z(-2) = 3 = -d_3 \Rightarrow d_3 = -3$$

$$x^2: 1 = d_1 + d_2 \Rightarrow \underline{d_2 = 1 - d_1 = 2}$$