1. Integrale

1.1. Grundintegrale

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$$\int e^x\,dx = e^x + c$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \frac{1}{\cos^2 x}\,dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x}\,dx = \cot x + c$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \begin{cases} \arcsin x + c_1 \\ -\arccos x + c_2 \end{cases}$$

$$\int \frac{1}{1+x^2}\,dx = \begin{cases} \arctan x + c_1 \\ -\operatorname{arccot} x + c_2 \end{cases}$$

$$\int \sinh x\,dx = \cosh x + c$$

$$\int \cosh x\,dx = \sinh x + c$$

$$\int \frac{1}{\cosh^2 x}\,dx = \tanh x + c$$

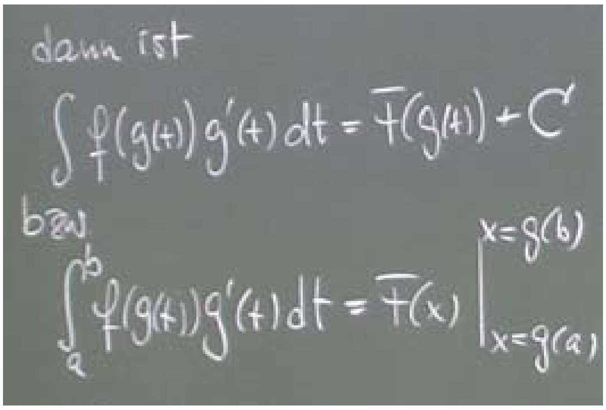
$$\int \frac{1}{\sinh^2 x}\,dx = -\coth x + c$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \operatorname{arsinh} x + c = \ln|x + \sqrt{x^2+1}| + c$$

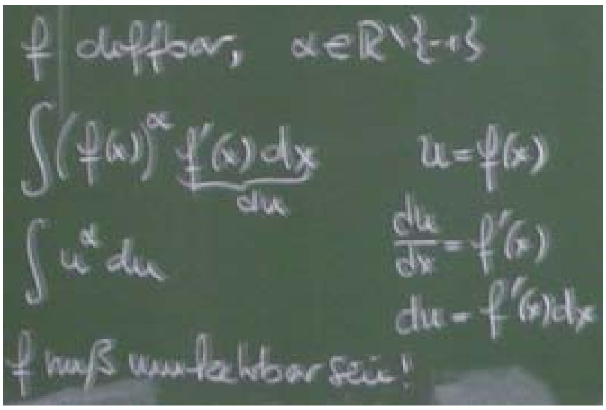
$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \operatorname{arcosh}|x| + c = \ln|x + \sqrt{x^2-1}| + c \quad (|x|) > 1)$$

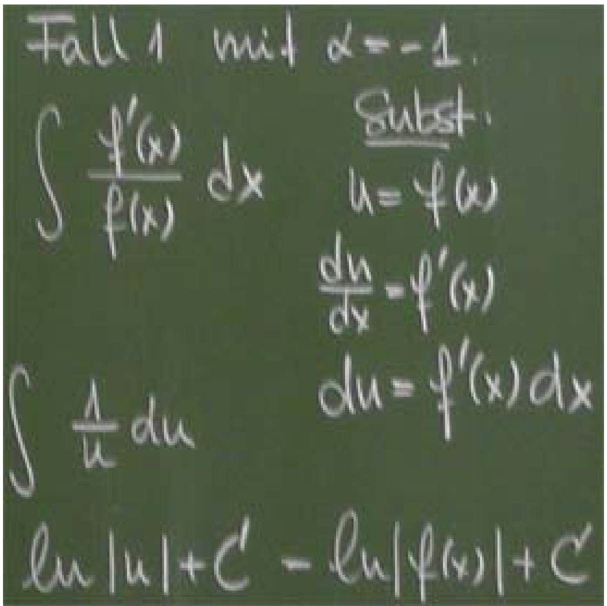
$$\int \frac{1}{1-x^2}\,dx = \left\{ \begin{array}{lcl} \operatorname{artanh} x + c_1 = \frac{1}{2}\cdot\ln\left(\frac{1+x}{1-x}\right) + c_1 & & |x| < 1 \\ & \text{für} & \\ \operatorname{arcoth} x + c_2 = \frac{1}{2}\cdot\ln\left(\frac{1+x}{1-x}\right) + c_2 & & |x| > 1 \end{array} \right\}$$

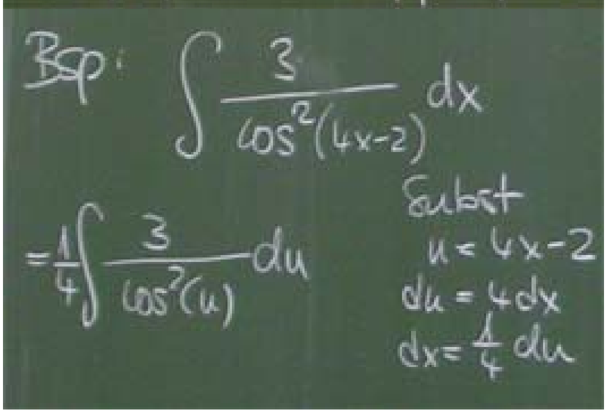
1.2. Substitution

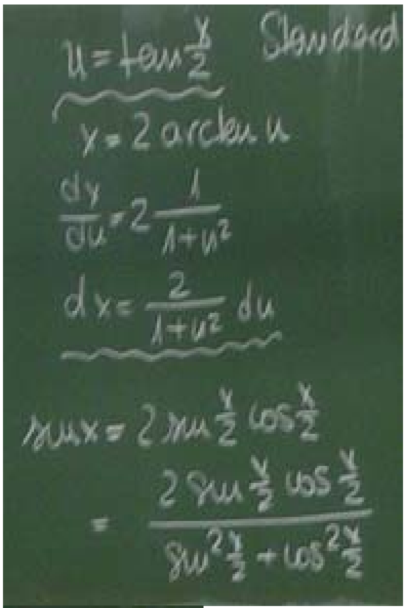


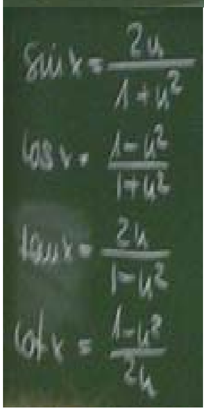
1.3. Standart Substitutionen



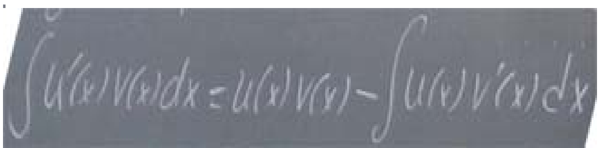


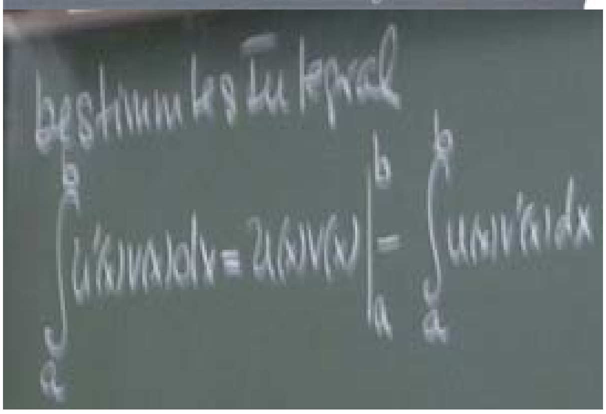






1.4. Partelle Integration





1.5. Partialbruchzerlegung

$$f(x)=\frac{Z(x)}{N(x)}=\frac{d_1}{x-x_1}+\frac{d_2}{x-x_2}+\dots+\frac{d_n}{x-x_n}$$

PBZ

$$\frac{3x-5}{x^2+2x-8}=\frac{d_1}{x+4}+\frac{d_2}{x-2}$$

Hauptnenner

$$=\frac{(d_1+d_2)x-2d_1+4d_2}{(x+4)(x-2)}$$

$d_1+d_2=3$

$2d_1-4d_2=5$

alternativ: $N'(x)=2x+2$

$$d_1=\left[\frac{3x-5}{2x+2}\right]_{x=-4}=\frac{-17}{-6}=\frac{17}{6}$$

$$d_2=\left[\quad -\text{''}- \quad\right]_{x=2}=\frac{1}{6}$$

Für fälle höherer ordnung

PBZ

$$\frac{Z(x)}{N(x)}=\frac{d_1}{(x+1)}+\frac{d_2}{(x+2)}+\frac{d_3}{(x+2)^2}$$

Bestimmung der di

Hauptnenner

$Z(x)=d_1(x+2)^2+d_2(x+1)(x+2)+d_3(x+1)$

$x=-1: \quad Z(-1)=\underline{-1=d_1}$

$x=-2: \quad Z(-2)=3=-d_3 \Rightarrow d_3=-3$

$x^2: \quad 1=d_1+d_2 \Rightarrow \underline{d_2}=1-d_1=\underline{2}$