

## Problem 1: Multiples of 3 or 5

Published on Friday, 5th October 2001, 06:00 pm / Solved by 969573 / Difficulty rating: 5%

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

## Problem 2: Even Fibonacci numbers

Published on Friday, 19th October 2001, 06:00 pm / Solved by 773841 / Difficulty rating: 5%

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

## Problem 3: Largest prime factor

Published on Friday, 2nd November 2001, 06:00 pm / Solved by 557297 / Difficulty rating: 5%

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143 ?

## Problem 4: Largest palindrome product

Published on Friday, 16th November 2001, 06:00 pm / Solved by 493412 / Difficulty rating: 5%

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is  $9009 = 91 \times 99$ .

Find the largest palindrome made from the product of two 3-digit numbers.

## Problem 5: Smallest multiple

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2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is *evenly divisible* by all of the numbers from 1 to 20?

## Problem 6: Sum square difference

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The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \dots + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \dots + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is  $3025 - 385 = 2640$ .

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

## Problem 7: 10001st prime

Published on Friday, 28th December 2001, 06:00 pm / Solved by 428050 / Difficulty rating: 5%

By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13.

What is the 10 001st prime number?

## Problem 8: Largest product in a series

Published on Friday, 11th January 2002, 06:00 pm / Solved by 359043 / Difficulty rating: 5%

The four adjacent digits in the 1000-digit number that have the greatest product are  $9 \times 9 \times 8 \times 9 = 5832$ .

73167176531330624919225119674426574742355349194934  
96983520312774506326239578318016984801869478851843  
85861560789112949495459501737958331952853208805511  
12540698747158523863050715693290963295227443043557  
66896648950445244523161731856403098711121722383113  
62229893423380308135336276614282806444486645238749  
30358907296290491560440772390713810515859307960866  
70172427121883998797908792274921901699720888093776  
65727333001053367881220235421809751254540594752243  
52584907711670556013604839586446706324415722155397  
53697817977846174064955149290862569321978468622482  
83972241375657056057490261407972968652414535100474  
82166370484403199890008895243450658541227588666881  
16427171479924442928230863465674813919123162824586  
17866458359124566529476545682848912883142607690042  
2421902267105562632111109370544217506941658960408  
07198403850962455444362981230987879927244284909188  
84580156166097919133875499200524063689912560717606  
05886116467109405077541002256983155200055935729725  
71636269561882670428252483600823257530420752963450

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

## Problem 9: Special Pythagorean triplet

Published on Friday, 25th January 2002, 06:00 pm / Solved by 364048 / Difficulty rating: 5%

A Pythagorean triplet is a set of three natural numbers,  $a < b < c$ , for which,

$$a^2 + b^2 = c^2$$

For example,  $3^2 + 4^2 = 9 + 16 = 25 = 5^2$ .

There exists exactly one Pythagorean triplet for which  $a + b + c = 1000$ .

Find the product  $abc$ .

## Problem 10: Summation of primes

Published on Friday, 8th February 2002, 06:00 pm / Solved by 333788 / Difficulty rating: 5%

The sum of the primes below 10 is  $2 + 3 + 5 + 7 = 17$ .

Find the sum of all the primes below two million.

## Problem 11: Largest product in a grid

Published on Friday, 22nd February 2002, 06:00 pm / Solved by 240044 / Difficulty rating: 5%

In the  $20 \times 20$  grid below, four numbers along a diagonal line have been marked in red.

08	02	22	97	38	15	00	40	00	75	04	05	07	78	52	12	50	77	91	08
49	49	99	40	17	81	18	57	60	87	17	40	98	43	69	48	04	56	62	00
81	49	31	73	55	79	14	29	93	71	40	67	53	88	30	03	49	13	36	65
52	70	95	23	04	60	11	42	69	24	68	56	01	32	56	71	37	02	36	91
22	31	16	71	51	67	63	89	41	92	36	54	22	40	40	28	66	33	13	80
24	47	32	60	99	03	45	02	44	75	33	53	78	36	84	20	35	17	12	50
32	98	81	28	64	23	67	10	26	38	40	67	59	54	70	66	18	38	64	70
67	26	20	68	02	62	12	20	95	63	94	39	63	08	40	91	66	49	94	21
24	55	58	05	66	73	99	26	97	17	78	78	96	83	14	88	34	89	63	72
21	36	23	09	75	00	76	44	20	45	35	14	00	61	33	97	34	31	33	95
78	17	53	28	22	75	31	67	15	94	03	80	04	62	16	14	09	53	56	92
16	39	05	42	96	35	31	47	55	58	88	24	00	17	54	24	36	29	85	57
86	56	00	48	35	71	89	07	05	44	44	37	44	60	21	58	51	54	17	58
19	80	81	68	05	94	47	69	28	73	92	13	86	52	17	77	04	89	55	40
04	52	08	83	97	35	99	16	07	97	57	32	16	26	26	79	33	27	98	66
88	36	68	87	57	62	20	72	03	46	33	67	46	55	12	32	63	93	53	69
04	42	16	73	38	25	39	11	24	94	72	18	08	46	29	32	40	62	76	36
20	69	36	41	72	30	23	88	34	62	99	69	82	67	59	85	74	04	36	16
20	73	35	29	78	31	90	01	74	31	49	71	48	86	81	16	23	57	05	54
01	70	54	71	83	51	54	69	16	92	33	48	61	43	52	01	89	19	67	48

The product of these numbers is  $26 \times 63 \times 78 \times 14 = 1788696$ .

What is the greatest product of four adjacent numbers in the same direction (up, down, left, right, or diagonally) in the  $20 \times 20$  grid?

## Problem 12: Highly divisible triangular number

Published on Friday, 8th March 2002, 06:00 pm / Solved by 226664 / Difficulty rating: 5%

The sequence of triangle numbers is generated by adding the natural numbers. So the 7<sup>th</sup> triangle number would be  $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$ . The first ten terms would be:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Let us list the factors of the first seven triangle numbers:

**1:** 1

**3:** 1,3

**6:** 1,2,3,6

**10:** 1,2,5,10

**15:** 1,3,5,15

**21:** 1,3,7,21

**28:** 1,2,4,7,14,28

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

## Problem 13: Large sum

Published on Friday, 22nd March 2002, 06:00 pm / Solved by 231993 / Difficulty rating: 5%

Work out the first ten digits of the sum of the following one-hundred 50-digit numbers.

37107287533902102798797998220837590246510135740250  
46376937677490009712648124896970078050417018260538  
7432498619952474105947423309513058123726617309629  
91942213363574161572522430563301811072406154908250  
23067588207539346171171980310421047513778063246676  
89261670696623633820136378418383684178734361726757  
28112879812849979408065481931592621691275889832738  
44274228917432520321923589422876796487670272189318  
47451445736001306439091167216856844588711603153276  
70386486105843025439939619828917593665686757934951  
62176457141856560629502157223196586755079324193331  
64906352462741904929101432445813822663347944758178  
92575867718337217661963751590579239728245598838407  
58203565325359399008402633568948830189458628227828  
80181199384826282014278194139940567587151170094390  
35398664372827112653829987240784473053190104293586  
86515506006295864861532075273371959191420517255829  
71693888707715466499115593487603532921714970056938  
54370070576826684624621495650076471787294438377604  
53282654108756828443191190634694037855217779295145  
36123272525000296071075082563815656710885258350721  
45876576172410976447339110607218265236877223636045  
17423706905851860660448207621209813287860733969412  
81142660418086830619328460811191061556940512689692  
51934325451728388641918047049293215058642563049483  
62467221648435076201727918039944693004732956340691  
15732444386908125794514089057706229429197107928209  
55037687525678773091862540744969844508330393682126  
18336384825330154686196124348767681297534375946515  
80386287592878490201521685554828717201219257766954  
78182833757993103614740356856449095527097864797581  
16726320100436897842553539920931837441497806860984  
48403098129077791799088218795327364475675590848030  
87086987551392711854517078544161852424320693150332  
59959406895756536782107074926966537676326235447210  
69793950679652694742597709739166693763042633987085  
41052684708299085211399427365734116182760315001271  
65378607361501080857009149939512557028198746004375  
35829035317434717326932123578154982629742552737307  
94953759765105305946966067683156574377167401875275  
88902802571733229619176668713819931811048770190271  
25267680276078003013678680992525463401061632866526

36270218540497705585629946580636237993140746255962  
24074486908231174977792365466257246923322810917141  
91430288197103288597806669760892938638285025333403  
34413065578016127815921815005561868836468420090470  
23053081172816430487623791969842487255036638784583  
11487696932154902810424020138335124462181441773470  
63783299490636259666498587618221225225512486764533  
67720186971698544312419572409913959008952310058822  
95548255300263520781532296796249481641953868218774  
76085327132285723110424803456124867697064507995236  
37774242535411291684276865538926205024910326572967  
23701913275725675285653248258265463092207058596522  
29798860272258331913126375147341994889534765745501  
18495701454879288984856827726077713721403798879715  
38298203783031473527721580348144513491373226651381  
34829543829199918180278916522431027392251122869539  
40957953066405232632538044100059654939159879593635  
2974615218502371307642255121183693803580388584903  
41698116222072977186158236678424689157993532961922  
62467957194401269043877107275048102390895523597457  
23189706772547915061505504953922979530901129967519  
86188088225875314529584099251203829009407770775672  
11306739708304724483816533873502340845647058077308  
82959174767140363198008187129011875491310547126581  
97623331044818386269515456334926366572897563400500  
42846280183517070527831839425882145521227251250327  
55121603546981200581762165212827652751691296897789  
32238195734329339946437501907836945765883352399886  
75506164965184775180738168837861091527357929701337  
62177842752192623401942399639168044983993173312731  
32924185707147349566916674687634660915035914677504  
99518671430235219628894890102423325116913619626622  
73267460800591547471830798392868535206946944540724  
7684182252467441716151403642798227334805556214818  
97142617910342598647204516893989422179826088076852  
87783646182799346313767754307809363333018982642090  
10848802521674670883215120185883543223812876952786  
71329612474782464538636993009049310363619763878039  
62184073572399794223406235393808339651327408011116  
66627891981488087797941876876144230030984490851411  
60661826293682836764744779239180335110989069790714  
85786944089552990653640447425576083659976645795096  
66024396409905389607120198219976047599490197230297  
64913982680032973156037120041377903785566085089252  
16730939319872750275468906903707539413042652315011  
94809377245048795150954100921645863754710598436791  
78639167021187492431995700641917969777599028300699  
15368713711936614952811305876380278410754449733078  
40789923115535562561142322423255033685442488917353

44889911501440648020369068063960672322193204149535  
41503128880339536053299340368006977710650566631954  
81234880673210146739058568557934581403627822703280  
82616570773948327592232845941706525094512325230608  
22918802058777319719839450180888072429661980811197  
77158542502016545090413245809786882778948721859617  
72107838435069186155435662884062257473692284509516  
20849603980134001723930671666823555245252804609722  
53503534226472524250874054075591789781264330331690

## Problem 14: Longest Collatz sequence

Published on Friday, 5th April 2002, 06:00 pm / Solved by 232238 / Difficulty rating: 5%

The following iterative sequence is defined for the set of positive integers:

$$\begin{aligned}n &\rightarrow n/2 \text{ (} n \text{ is even)} \\n &\rightarrow 3n + 1 \text{ (} n \text{ is odd)}$$

Using the rule above and starting with 13, we generate the following sequence:

$$13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

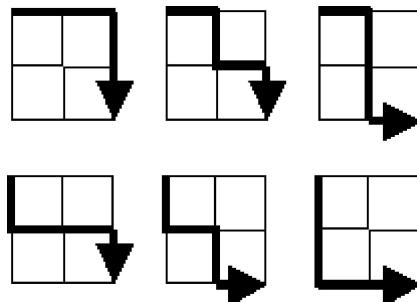
Which starting number, under one million, produces the longest chain?

**NOTE:** Once the chain starts the terms are allowed to go above one million.

## Problem 15: Lattice paths

Published on Friday, 19th April 2002, 06:00 pm / Solved by 191850 / Difficulty rating: 5%

Starting in the top left corner of a  $2 \times 2$  grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.



How many such routes are there through a  $20 \times 20$  grid?

## Problem 16: Power digit sum

Published on Friday, 3rd May 2002, 06:00 pm / Solved by 234472 / Difficulty rating: 5%

$2^{15} = 32768$  and the sum of its digits is  $3 + 2 + 7 + 6 + 8 = 26$ .

What is the sum of the digits of the number  $2^{1000}$ ?

## Problem 17: Number letter counts

Published on Friday, 17th May 2002, 06:00 pm / Solved by 155893 / Difficulty rating: 5%

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are  $3 + 3 + 5 + 4 + 4 = 19$  letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

**NOTE:** Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

## Problem 18: Maximum path sum |

Published on Friday, 31st May 2002, 06:00 pm / Solved by 149246 / Difficulty rating: 5%

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3			
7	4		
2	4	6	
8	5	9	3

That is,  $3 + 7 + 4 + 9 = 23$ .

Find the maximum total from top to bottom of the triangle below:

75														
95	64													
17	47	82												
18	35	87	10											
20	04	82	47	65										
19	01	23	75	03	34									
88	02	77	73	07	63	67								
99	65	04	28	06	16	70	92							
41	41	26	56	83	40	80	70	33						
41	48	72	33	47	32	37	16	94	29					
53	71	44	65	25	43	91	52	97	51	14				
70	11	33	28	77	73	17	78	39	68	17	57			
91	71	52	38	17	14	91	43	58	50	27	29	48		
63	66	04	68	89	53	67	30	73	16	69	87	40	31	
04	62	98	27	23	09	70	98	73	93	38	53	60	04	23

**NOTE:** As there are only 16384 routes, it is possible to solve this problem by trying every route. However, **Problem 67**, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

## Problem 19: Counting Sundays

Published on Friday, 14th June 2002, 06:00 pm / Solved by 138686 / Difficulty rating: 5%

You are given the following information, but you may prefer to do some research for yourself.

- 1 Jan 1900 was a Monday.
- Thirty days has September,  
April, June and November.  
All the rest have thirty-one,  
Saving February alone,  
Which has twenty-eight, rain or shine.  
And on leap years, twenty-nine.
- A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

## Problem 20: Factorial digit sum

Published on Friday, 21st June 2002, 06:00 pm / Solved by 203073 / Difficulty rating: 5%

$n!$  means  $n \times (n - 1) \times \dots \times 3 \times 2 \times 1$

For example,  $10! = 10 \times 9 \times \dots \times 3 \times 2 \times 1 = 3628800$ ,  
and the sum of the digits in the number  $10!$  is  $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$ .

Find the sum of the digits in the number  $100!$

## Problem 21: Amicable numbers

Published on Friday, 5th July 2002, 06:00 pm / Solved by 150528 / Difficulty rating: 5%

Let  $d(n)$  be defined as the sum of proper divisors of  $n$  (numbers less than  $n$  which divide evenly into  $n$ ).

If  $d(a) = b$  and  $d(b) = a$ , where  $a \neq b$ , then  $a$  and  $b$  are an amicable pair and each of  $a$  and  $b$  are called amicable numbers.

For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore  $d(220) = 284$ . The proper divisors of 284 are 1, 2, 4, 71 and 142; so  $d(284) = 220$ .

Evaluate the sum of all the amicable numbers under 10000.

## Problem 22: Names scores

Published on Friday, 19th July 2002, 06:00 pm / Solved by 138528 / Difficulty rating: 5%

Using **names.txt** (right click and 'Save Link/Target As...'), a 46K text file containing over five-thousand first names, begin by sorting it into alphabetical order. Then working out the alphabetical value for each name, multiply this value by its alphabetical position in the list to obtain a name score.

For example, when the list is sorted into alphabetical order, COLIN, which is worth  $3 + 15 + 12 + 9 + 14 = 53$ , is the 938th name in the list. So, COLIN would obtain a score of  $938 \times 53 = 49714$ .

What is the total of all the name scores in the file?

## Problem 23: Non-abundant sums

Published on Friday, 2nd August 2002, 06:00 pm / Solved by 107652 / Difficulty rating: 5%

A perfect number is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be  $1 + 2 + 4 + 7 + 14 = 28$ , which means that 28 is a perfect number.

A number  $n$  is called deficient if the sum of its proper divisors is less than  $n$  and it is called abundant if this sum exceeds  $n$ .

As 12 is the smallest abundant number,  $1 + 2 + 3 + 4 + 6 = 16$ , the smallest number that can be written as the sum of two abundant numbers is 24. By mathematical analysis, it can be shown that all integers greater than 28123 can be written as the sum of two abundant numbers. However, this upper limit cannot be reduced any further by analysis even though it is known that the greatest number that cannot be expressed as the sum of two abundant numbers is less than this limit.

Find the sum of all the positive integers which cannot be written as the sum of two abundant numbers.

## Problem 24: Lexicographic permutations

Published on Friday, 16th August 2002, 06:00 pm / Solved by 118205 / Difficulty rating: 5%

A permutation is an ordered arrangement of objects. For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it lexicographic order. The lexicographic permutations of 0, 1 and 2 are:

012 021 102 120 201 210

What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9?

## Problem 25: 1000-digit Fibonacci number

Published on Friday, 30th August 2002, 06:00 pm / Solved by 159661 / Difficulty rating: 5%

The Fibonacci sequence is defined by the recurrence relation:

$$F_n = F_{n-1} + F_{n-2}, \text{ where } F_1 = 1 \text{ and } F_2 = 1.$$

Hence the first 12 terms will be:

$$F_1 = 1$$

$$F_2 = 1$$

$$F_3 = 2$$

$$F_4 = 3$$

$$F_5 = 5$$

$$F_6 = 8$$

$$F_7 = 13$$

$$F_8 = 21$$

$$F_9 = 34$$

$$F_{10} = 55$$

$$F_{11} = 89$$

$$F_{12} = 144$$

The 12th term,  $F_{12}$ , is the first term to contain three digits.

What is the index of the first term in the Fibonacci sequence to contain 1000 digits?

## Problem 26: Reciprocal cycles

Published on Friday, 13th September 2002, 06:00 pm / Solved by 86931 / Difficulty rating: 5%

A unit fraction contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

$$\frac{1}{2} = 0.5$$

$$\frac{1}{3} = 0.(3)$$

$$\frac{1}{4} = 0.25$$

$$\frac{1}{5} = 0.2$$

$$\frac{1}{6} = 0.1(6)$$

$$\frac{1}{7} = 0.(142857)$$

$$\frac{1}{8} = 0.125$$

$$\frac{1}{9} = 0.(1)$$

$$\frac{1}{10} = 0.1$$

Where  $0.1(6)$  means  $0.16666\dots$ , and has a 1-digit recurring cycle. It can be seen that  $\frac{1}{7}$  has a 6-digit recurring cycle.

Find the value of  $d < 1000$  for which  $\frac{1}{d}$  contains the longest recurring cycle in its decimal fraction part.

## Problem 27: Quadratic primes

Published on Friday, 27th September 2002, 06:00 pm / Solved by 90371 / Difficulty rating: 5%

Euler discovered the remarkable quadratic formula:

$$n^2 + n + 41$$

It turns out that the formula will produce 40 primes for the consecutive integer values  $0 \leq n \leq 39$ . However, when  $n = 40$ ,  $40^2 + 40 + 41 = 40(40 + 1) + 41$  is divisible by 41, and certainly when  $n = 41$ ,  $41^2 + 41 + 41$  is clearly divisible by 41.

The incredible formula  $n^2 - 79n + 1601$  was discovered, which produces 80 primes for the consecutive values  $0 \leq n \leq 79$ . The product of the coefficients,  $-79$  and  $1601$ , is  $-126479$ .

Considering quadratics of the form:

$$n^2 + an + b, \text{ where } |a| < 1000 \text{ and } |b| \leq 1000$$

where  $|n|$  is the modulus/absolute value of  $n$

e.g.  $|11| = 11$  and  $|-4| = 4$

Find the product of the coefficients,  $a$  and  $b$ , for the quadratic expression that produces the maximum number of primes for consecutive values of  $n$ , starting with  $n = 0$ .

## Problem 28: Number spiral diagonals

Published on Friday, 11th October 2002, 06:00 pm / Solved by 111295 / Difficulty rating: 5%

Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

21	22	23	24	25
20	7	8	9	10
19	6	1	2	11
18	5	4	3	12
17	16	15	14	13

It can be verified that the sum of the numbers on the diagonals is 101.

What is the sum of the numbers on the diagonals in a 1001 by 1001 spiral formed in the same way?

## Problem 29: Distinct powers

Published on Friday, 25th October 2002, 06:00 pm / Solved by 108767 / Difficulty rating: 5%

Consider all integer combinations of  $a^b$  for  $2 \leq a \leq 5$  and  $2 \leq b \leq 5$ :

$$2^2=4, 2^3=8, 2^4=16, 2^5=32$$

$$3^2=9, 3^3=27, 3^4=81, 3^5=243$$

$$4^2=16, 4^3=64, 4^4=256, 4^5=1024$$

$$5^2=25, 5^3=125, 5^4=625, 5^5=3125$$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

$$4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125$$

How many distinct terms are in the sequence generated by  $a^b$  for  $2 \leq a \leq 100$  and  $2 \leq b \leq 100$ ?

## Problem 30: Digit fifth powers

Published on Friday, 8th November 2002, 06:00 pm / Solved by 113346 / Difficulty rating: 5%

Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits:

$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$

$$8208 = 8^4 + 2^4 + 0^4 + 8^4$$

$$9474 = 9^4 + 4^4 + 7^4 + 4^4$$

As  $1 = 1^4$  is not a sum it is not included.

The sum of these numbers is  $1634 + 8208 + 9474 = 19316$ .

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

## Problem 31: Coin sums

Published on Friday, 22nd November 2002, 06:00 pm / Solved by 88004 / Difficulty rating: 5%

In the United Kingdom the currency is made up of pound (£) and pence (p). There are eight coins in general circulation:

1p, 2p, 5p, 10p, 20p, 50p, £1 (100p), and £2 (200p).

It is possible to make £2 in the following way:

$$1 \times £1 + 1 \times 50p + 2 \times 20p + 1 \times 5p + 1 \times 2p + 3 \times 1p$$

How many different ways can £2 be made using any number of coins?

## Problem 32: Pandigital products

Published on Friday, 6th December 2002, 06:00 pm / Solved by 73805 / Difficulty rating: 5%

We shall say that an  $n$ -digit number is pandigital if it makes use of all the digits 1 to  $n$  exactly once; for example, the 5-digit number, 15234, is 1 through 5 pandigital.

The product 7254 is unusual, as the identity,  $39 \times 186 = 7254$ , containing multiplicand, multiplier, and product is 1 through 9 pandigital.

Find the sum of all products whose multiplicand/multiplier/product identity can be written as a 1 through 9 pandigital.

HINT: Some products can be obtained in more than one way so be sure to only include it once in your sum.

## Problem 33: Digit cancelling fractions

Published on Friday, 20th December 2002, 06:00 pm / Solved by 74184 / Difficulty rating: 5%

The fraction  $\frac{49}{98}$  is a curious fraction, as an inexperienced mathematician in attempting to simplify it may incorrectly believe that  $\frac{49}{98} = \frac{4}{8}$ , which is correct, is obtained by cancelling the 9s.

We shall consider fractions like,  $\frac{30}{50} = \frac{3}{5}$ , to be trivial examples.

There are exactly four non-trivial examples of this type of fraction, less than one in value, and containing two digits in the numerator and denominator.

If the product of these four fractions is given in its lowest common terms, find the value of the denominator.

## Problem 34: Digit factorials

Published on Friday, 3rd January 2003, 06:00 pm / Solved by 97180 / Difficulty rating: 5%

145 is a curious number, as  $1! + 4! + 5! = 1 + 24 + 120 = 145$ .

Find the sum of all numbers which are equal to the sum of the factorial of their digits.

Note: As  $1! = 1$  and  $2! = 2$  are not sums they are not included.

## Problem 35: Circular primes

Published on Friday, 17th January 2003, 06:00 pm / Solved by 87439 / Difficulty rating: 5%

The number, 197, is called a circular prime because all rotations of the digits: 197, 971, and 719, are themselves prime.

There are thirteen such primes below 100: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, and 97.

How many circular primes are there below one million?

## Problem 36: Double-base palindromes

Published on Friday, 31st January 2003, 06:00 pm / Solved by 91573 / Difficulty rating: 5%

The decimal number,  $585 = 1001001001_2$  (binary), is palindromic in both bases.

Find the sum of all numbers, less than one million, which are palindromic in base 10 and base 2.

(Please note that the palindromic number, in either base, may not include leading zeros.)

## Problem 37: Truncatable primes

Published on Friday, 14th February 2003, 06:00 pm / Solved by 76038 / Difficulty rating: 5%

The number 3797 has an interesting property. Being prime itself, it is possible to continuously remove digits from left to right, and remain prime at each stage: 3797, 797, 97, and 7.

Similarly we can work from right to left: 3797, 379, 37, and 3.

Find the sum of the only eleven primes that are both truncatable from left to right and right to left.

NOTE: 2, 3, 5, and 7 are not considered to be truncatable primes.

## Problem 38: Pandigital multiples

Published on Friday, 28th February 2003, 06:00 pm / Solved by 65261 / Difficulty rating: 5%

Take the number 192 and multiply it by each of 1, 2, and 3:

$$192 \times 1 = 192$$

$$192 \times 2 = 384$$

$$192 \times 3 = 576$$

By concatenating each product we get the 1 to 9 pandigital, 192384576. We will call 192384576 the concatenated product of 192 and (1,2,3)

The same can be achieved by starting with 9 and multiplying by 1, 2, 3, 4, and 5, giving the pandigital, 918273645, which is the concatenated product of 9 and (1,2,3,4,5).

What is the largest 1 to 9 pandigital 9-digit number that can be formed as the concatenated product of an integer with (1,2, ..., n) where  $n > 1$ ?

## Problem 39: Integer right triangles

Published on Friday, 14th March 2003, 06:00 pm / Solved by 75650 / Difficulty rating: 5%

If  $p$  is the perimeter of a right angle triangle with integral length sides,  $\{a,b,c\}$ , there are exactly three solutions for  $p = 120$ .

$$\{20,48,52\}, \{24,45,51\}, \{30,40,50\}$$

For which value of  $p \leq 1000$ , is the number of solutions maximised?

## Problem 40: Champernowne's constant

Published on Friday, 28th March 2003, 06:00 pm / Solved by 82662 / Difficulty rating: 5%

An irrational decimal fraction is created by concatenating the positive integers:

0.123456789101112131415161718192021...

It can be seen that the 12<sup>th</sup> digit of the fractional part is 1.

If  $d_n$  represents the  $n^{\text{th}}$  digit of the fractional part, find the value of the following expression.

$$d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000}$$

## Problem 41: Pandigital prime

Published on Friday, 11th April 2003, 06:00 pm / Solved by 70459 / Difficulty rating: 5%

We shall say that an  $n$ -digit number is pandigital if it makes use of all the digits 1 to  $n$  exactly once. For example, 2143 is a 4-digit pandigital and is also prime.

What is the largest  $n$ -digit pandigital prime that exists?

## Problem 42: Coded triangle numbers

Published on Friday, 25th April 2003, 06:00 pm / Solved by 76786 / Difficulty rating: 5%

The  $n^{\text{th}}$  term of the sequence of triangle numbers is given by,  $t_n = \frac{1}{2}n(n+1)$ ; so the first ten triangle numbers are:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is  $19 + 11 + 25 = 55 = t_{10}$ . If the word value is a triangle number then we shall call the word a triangle word.

Using **words.txt** (right click and 'Save Link/Target As...'), a 16K text file containing nearly two-thousand common English words, how many are triangle words?

## Problem 43: Sub-string divisibility

Published on Friday, 9th May 2003, 06:00 pm / Solved by 62008 / Difficulty rating: 5%

The number, 1406357289, is a 0 to 9 pandigital number because it is made up of each of the digits 0 to 9 in some order, but it also has a rather interesting sub-string divisibility property.

Let  $d_1$  be the 1<sup>st</sup> digit,  $d_2$  be the 2<sup>nd</sup> digit, and so on. In this way, we note the following:

- $d_2d_3d_4=406$  is divisible by 2
- $d_3d_4d_5=063$  is divisible by 3
- $d_4d_5d_6=635$  is divisible by 5
- $d_5d_6d_7=357$  is divisible by 7
- $d_6d_7d_8=572$  is divisible by 11
- $d_7d_8d_9=728$  is divisible by 13
- $d_8d_9d_{10}=289$  is divisible by 17

Find the sum of all 0 to 9 pandigital numbers with this property.

## Problem 44: Pentagon numbers

Published on Friday, 23rd May 2003, 06:00 pm / Solved by 60678 / Difficulty rating: 5%

Pentagonal numbers are generated by the formula,  $P_n=n(3n-1)/2$ . The first ten pentagonal numbers are:

$$1, 5, 12, 22, 35, 51, 70, 92, 117, 145, \dots$$

It can be seen that  $P_4 + P_7 = 22 + 70 = 92 = P_8$ . However, their difference,  $70 - 22 = 48$ , is not pentagonal.

Find the pair of pentagonal numbers,  $P_j$  and  $P_k$ , for which their sum and difference are pentagonal and  $D = |P_k - P_j|$  is minimised; what is the value of D?

## Problem 45: Triangular, pentagonal, and hexagonal

Published on Friday, 6th June 2003, 06:00 pm / Solved by 73248 / Difficulty rating: 5%

Triangle, pentagonal, and hexagonal numbers are generated by the following formulae:

Triangle       $T_n = n(n+1)/2$     1, 3, 6, 10, 15, ...

Pentagonal     $P_n = n(3n-1)/2$     1, 5, 12, 22, 35, ...

Hexagonal     $H_n = n(2n-1)$     1, 6, 15, 28, 45, ...

It can be verified that  $T_{285} = P_{165} = H_{143} = 40755$ .

Find the next triangle number that is also pentagonal and hexagonal.

## Problem 46: Goldbach's other conjecture

Published on Friday, 20th June 2003, 06:00 pm / Solved by 63801 / Difficulty rating: 5%

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

$$9 = 7 + 2 \times 1^2$$

$$15 = 7 + 2 \times 2^2$$

$$21 = 3 + 2 \times 3^2$$

$$25 = 7 + 2 \times 3^2$$

$$27 = 19 + 2 \times 2^2$$

$$33 = 31 + 2 \times 1^2$$

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

## Problem 47: Distinct primes factors

Published on Friday, 4th July 2003, 06:00 pm / Solved by 60080 / Difficulty rating: 5%

The first two consecutive numbers to have two distinct prime factors are:

$$14 = 2 \times 7$$

$$15 = 3 \times 5$$

The first three consecutive numbers to have three distinct prime factors are:

$$644 = 2^2 \times 7 \times 23$$

$$645 = 3 \times 5 \times 43$$

$$646 = 2 \times 17 \times 19.$$

Find the first four consecutive integers to have four distinct prime factors each. What is the first of these numbers?

## Problem 48: Self powers

Published on Friday, 18th July 2003, 06:00 pm / Solved by 115160 / Difficulty rating: 5%

The series,  $1^1 + 2^2 + 3^3 + \dots + 10^{10} = 10405071317$ .

Find the last ten digits of the series,  $1^1 + 2^2 + 3^3 + \dots + 1000^{1000}$ .

## Problem 49: Prime permutations

Published on Friday, 1st August 2003, 06:00 pm / Solved by 59971 / Difficulty rating: 5%

The arithmetic sequence, 1487, 4817, 8147, in which each of the terms increases by 3330, is unusual in two ways: (i) each of the three terms are prime, and, (ii) each of the 4-digit numbers are permutations of one another.

There are no arithmetic sequences made up of three 1-, 2-, or 3-digit primes, exhibiting this property, but there is one other 4-digit increasing sequence.

What 12-digit number do you form by concatenating the three terms in this sequence?

## Problem 50: Consecutive prime sum

Published on Friday, 15th August 2003, 06:00 pm / Solved by 64624 / Difficulty rating: 5%

The prime 41, can be written as the sum of six consecutive primes:

$$41 = 2 + 3 + 5 + 7 + 11 + 13$$

This is the longest sum of consecutive primes that adds to a prime below one-hundred.

The longest sum of consecutive primes below one-thousand that adds to a prime, contains 21 terms, and is equal to 953.

Which prime, below one-million, can be written as the sum of the most consecutive primes?