# University of Iceland: Physics 2 Notes

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## Ampere's Law

With electric fields, it was possible to find the electric field due to any collection of charges. We could find some small or differential electric field $d\vec{E}$ due to a single charged particle and integrate along some collection of them to sum the electric field and calculate the total field from all elements or charges.

This can be complicated for distributions of charges that are more realistic and less uniform, but for symmetrical shapes we learned that Gauss' law can be applied. Specifically for distributions of charges that look like a plane, cylinder, or sphere, the charge that is enclosed within a Gaussian shape (one of those three), can help us calculate the electric field and sum the parts more simply.

The same approach can be used in finding the magnetic field! In this case, we can use what is credited as Amepere's Law (Eq 1), which also takes advantage of symmetrical distributions to make our calculations easier. With enclosed current instead of enclosed charge, and integrating along a closed loop instead of along a distribution of charge.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \tag{1}$$

If you've ever used an amp-meter, this works very much the same and can aid you in imagining the loop around an enclosed charge. Figure 1 shows a generalized Ampere loop lacking in symmetry and shaped like a bean. It is important to note that the loop still works.

In the case of the bean shaped loop, the closed path integral is a summation of many tiny segments along the line, $d\vec{s}$ and the direction to integrate is arbitrary. Recall the dot product as the argument of the integral. If the magnetic field and the segment along the line are parallel then we can pull out the magnitudes of the magnetic field and ds, just as before with Gauss' Law. If the magnetic field is not pointing in the same direction and is separated from the direction of the segment along the line by some angle theta, then we'd get a $(B)(ds)\cos(\theta)$.

$$\oint \vec{B} \cos(\theta) d\vec{s} = \mu_0 i_{enc}$$

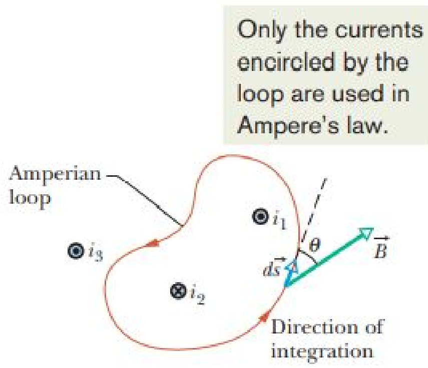



Figure 1: Figure from Halliday and Resnick

Okay, continuing to look at figure 1, the Amperian loop is enclosing two wires with currents moving through them. One wire has a current going into the page and the other has current going out of the page. The third wire is not included in our loop so we won't think about it for now.

The right hand rule is not necessary to solve the integral, but helps us check the sign (+/−) of the expression. If you hold up your right hand and point the thumb towards yourself, that is the direction we (and Halliday and Resnick) are calling positive direction for current, so if a current is flowing into the page the sign is negative.

Using this and including the enclosed current of the loop in the figure, our equation becomes,

$$\oint \vec{B}\cos(\theta)d\vec{s} = \mu_0(i_1 - i_2)$$

Okay, after all that work, Halliday and Resnick finally tell you that this problem is complex and we will move to a simpler example. You can see from the work done so far that this won't give you all the information needed, but it does tell you the magnetic field generated by the two wires enclosed. The two currents within do not cancel each other either, and that is important. There is still a field that exists.

## Simplified Symmetric Loops

Okay, Now that we have justified the usefulness of Ampere's Law and likened it to something we are familiar with, lets test it out on something we've already solved for, the long straight wire.

Figure 2 shows the side view of a long straight wire. We are allowed to use any Amperian loop we choose, but if we're just looking at a wire...why not use a simple circle? So that is what we do, and now the radius is constant from the center of the wire to the loop and the magnetic field is tangent to the segment

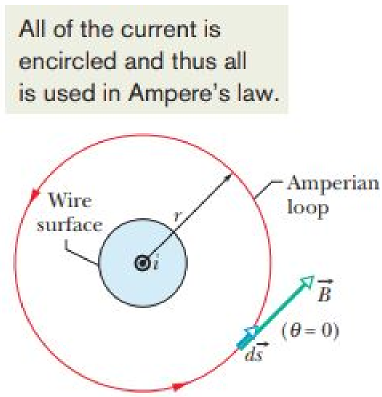



Figure 2: Figure from Halliday and Resnick

of the loop everywhere along it. This means $\theta = 0$ everywhere between the segment and the magnetic field making our $\cos(\theta)$ also zero.

$$\oint \vec{B}\cos(\theta) d\vec{s} = \mu_0 i_{enc}$$

$$\oint B ds = \mu_0 i_{enc}$$

$$B \oint ds = \mu_0 i_{enc}$$

$$B(2\pi r) = \mu_0 i_{enc}$$

$$B_{wire} = \frac{\mu_0 i_{enc}}{2\pi r}$$

Great! This is what we would expect for a long straight wire. We solved for this in our last problem solving session using the Biot-Savart Law! So we have a more intuitive method of calculating the magnetic field along a wire as long as we take advantage of symmetries and use selective shapes.

## Faraday's Law

Okay, so we have looked at how to calculate the magnetic field generated around wires, both straight and curved, that carry a current. We will now look at currents generated by magnetic fields. Faraday's Law can be stated like,

**An emf is induced in a loop when the number of magnetic field lines in another near by loop change.**

The important thing to take away is the rate of change in a magnetic field generates a current. If it is not *changing*, then it won't *induce* a current.

If we're going to care about the *number of magnetic field lines*, then we need a way of calculating the *flux* of the magnetic field. We have done this for electric fields already, and similarly the equation looks like,

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

Same as the electric field, we want to look at a vector $d\vec{A}$ that is normal (perpendicular) to the surface the magnetic field lines "pierce". This will allow us to pull magnitudes out of the integral like before (if the magnetic field is also uniform.

[Recall: $\Phi_B = \int \vec{B} \cdot d\vec{A} = BA$]

So this is the flux of the magnetic field, and when it changes a current in induced. We can now show Faraday's Law mathematically to say,

$$\xi = -\frac{d\Phi_B}{dt}.$$

Halliday and Resnick use a script E, but I don't have that option it seems so I will use $\xi$ or (xi).

We can change the magnetic field through any number of loops just by multiplying the rate of change by $N$ loops,

$$\xi = -N\frac{d\Phi_B}{dt}.$$

**Halliday and Resnick Rules**

1. Change the magnitude B of the magnetic field within the coil.

2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).

3. Change the angle between the direction of the magnetic field and the plane of the coil (for example, by rotating the coil so that field is first perpendicular to the plane of the coil and then is along that plane).

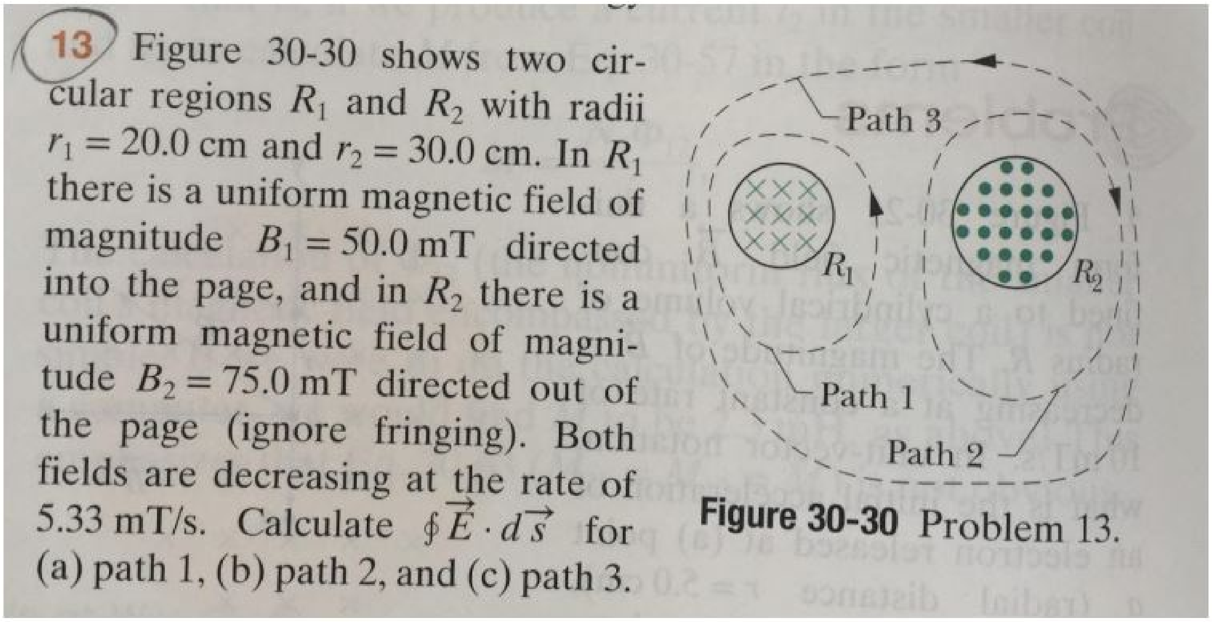
## Lenz Law

Building off the ideas of Faraday's Law, we can talk about Lenz' Law,

**Nature hates a changing magnetic field**.

A changing magnetic field will induce a current in a circuit to oppose the change. Said another way, if a magnetic field is decreasing, then a current will be induced to prevent the decease and boost the magnetic field. This eventually fails as the rate of change in the magnetic field stabilizes over time and slows down, making the induced current also weaker.

Something important to note is that if a current is induced in a circuit...wouldn't that mean there is an electric field in the wire holding a potential difference? Yes! You'd be right to think so! Likewise, a changing electric field induces a magnetic field, and a sort of echoing happens. Each induced field is not as



**13** Figure 30-30 shows two circular regions $R_1$ and $R_2$ with radii $r_1 = 20.0$ cm and $r_2 = 30.0$ cm. In $R_1$ there is a uniform magnetic field of magnitude $B_1 = 50.0$ mT directed into the page, and in $R_2$ there is a uniform magnetic field of magnitude $B_2 = 75.0$ mT directed out of the page (ignore fringing). Both fields are decreasing at the rate of 5.33 mT/s. Calculate $\oint \vec{E} \cdot d\vec{s}$ for (a) path 1, (b) path 2, and (c) path 3.



**Figure 30-30** Problem 13.

Figure 3: Problem from Halliday and Resnick

strong as the original and energy has to go somewhere so these echoing fields die down and the system stabilizes unless more energy is put into it. Cool huh!?

## Example

I wanted to go through the example from our problem solving session with the different paths and calculate the emf and flux for each as I didn't explain it well during the session. So lets hop in!

In figure **??** we see two loops of different radius ($R_1$, $R_2$) and different directions of uniform magnetic field. There are 3 paths we can draw like an "Amperian Loop" using Faraday's Law,

$$\xi = -\frac{d\Phi_B}{dt} = \oint \vec{E} \cdot d\vec{s}$$

This is an equation further in the chapter in Halliday and Resnick on Induction, but notice that it is just an electric potential, which is what our emf is.

That's the thinking behind the equation but we don't need to find the electric field anywhere to solve this. We can focus on the flux of the magnetic fields moving through the loops. Let's start with path 1.

**Path 1**,

Focusing on the parts we know,

$$\xi = -\frac{d\Phi_B}{dt}$$

and,

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

We notice the magnetic field is parallel with the cross sectional area of the loop in the path. "Piercing" through the loop to provide a flux in the circular area only. The magnetic field is uniform everywhere in the loop so we can pull it out of the integration, and the integration of the $dA$ gives us the area of a circle.

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int BdA = B\int dA = B(\pi R_1^2)$$

Okay we found the flux, now lets talk sign! The magnetic field is moving into the page and away from us. If we use our trusty right hand to find the direction a current would be induced as nature tries to fight the decreasing field, we see it would rotate clockwise around the wire or loop to amplify it and keep it from decreasing. This is in the opposite direction of the path however! The path chosen for us is moving in the counterclockwise direction, this inconsistency means we gain a minus sign.

$$\Phi_B = -B(\pi R_1^2)$$

Now for the induced emf!

$$\xi_1 = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(-B\pi R_1^2)$$

$$\xi_1 = -\frac{d\Phi_B}{dt} = -\pi R_1^2 \frac{d}{dt}(-B)$$

These minus signs cancel and we were given the change in magnetic field at the beginning of the problem, which was $dB/dt = -5.33mT/s$.

$$\xi_1 = -\frac{d\Phi_B}{dt} = \pi R_1^2 \frac{d}{dt}(B)$$

$$\xi_1 = \pi R_1^2 \frac{d}{dt}(-B) = \pi(20cm)(-5.33mT/s)$$

Notice that this gives us another negative! That's where all these come and go. There is more "book keeping" in problems like this than the usual ones and it is easy to mix them up. Also notice that the magnitude of the magnetic field $B_1$ wasn't used at all, only the rate of change which we were told was decreasing.

The calculations are very similar for path 2, but keep in mind the radius is different. It should also be a negative value. For path 3, something changes and one of the signs we'd find is not negative as the induced current is in agreement with the path direction, so only one emf is negative in path 3!

I hope this helps!