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Senior Project Report: Short Gamma Ray Bursts And The Intrinsic Nature Of Their Luminosity

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ABSTRACT
Bursts of gamma-rays are emitted from relativistic jets born from neutron star mergers. These are called short Gamma-Ray Bursts (sGRBs). The number of sGRBs can be plotted as a function of luminosity, which we refer to as the “luminosity function.” Using observations from the Neil Gehrels Swift satellite, different studies have shown that the number of sGRBs decreases as their luminosity increases. More specifically, these studies find that this function is a broken power-law and our main goal is to answer why is this the case. Two explanations for this function’s nature were explored: the sGRB jets angular-dependence and possible propagation effects as sGRB jets are ejected, which we hypothesized that would turn a single power-law function into a broken power-law one. Our calculations have shown that these two explanations are not sufficient in describing the observed behavior of the luminosity function. This suggests that the broken power-law luminosity function is an intrinsic property of sGRBs. Our exploration of this option have led us to calculate the effects of mass accretion rates and spin parameters of black holes in neutron star mergers.

Key words: Luminosity Function – GRB – Accretion – Spin

1 INTRODUCTION

1.1 Gamma Ray Bursts

Since GRB170817, an enormous amount of information has been gathered on short gamma ray bursts including the exciting confirmation that NS-NS mergers result in black holes (BH) ejecting GRBs. A GRB is an incredibly powerful polar jet that is launched from a merger event like GRB170817. This jet has an angular structure to it that is most powerful close to the axis of propagation and weakens sharply as the angle from the axis increases. Figure 1 shows the structure of the jet as it is ejected from the BH and spreads out wider in angle with distance while the jet’s core remains smaller than the rest of the jet. This was previously thought to be a "top hat" shape as the luminosity would drop off after 10 degrees and no longer be detectable, but simulations (Kathirgamaraju et al. 2018) and GRB170817 have shown the luminosity to drop off more smoothly. Most GRBs tilt their polar axis away from Earth and are considered to be "off-axis" providing little to no visible luminosity at their normally observed distances, but when near enough the luminosity may be detected off-axis and provide insight to the angular structure of the jet. When the jet is pointed at Earth and considered "on-axis", it is easily detectable and is the most luminous ejection of energy that is known. A single GRB may emit more energy in a second than the Sun will emit over its entire lifetime.

1.2 Luminosity and Filtering

The GRBs discussed in this paper are short GRBs and typically last less than 1-2 seconds, while longer GRBs may last for minutes. Most GRBs are weaker than the average and may be more difficult to observe. Long GRBs are ejected from a star collapsing into a BH and as the jet is released, there is material from the star still collapsing that may have a “filtering" effect on the weaker jets (Petropoulou et al. 2017). This does not happen with short GRBs as they result from a merging of spiralling binary NS system instead of a radial collapse. This filtering goes away with GRBs that have enough luminosity to “push" through the collapsing material and emerge out the other side with enough energy needed to be observed. The number of GRBs as a function of observed gamma-ray luminosity can be seen in figure 2 where the number increases with less energy. The observed data for long GRBs shows this function is “bent" from the expected values and provides a broken power law similar to figure 2 that can be explained with the filtering process of long GRB jets. The function has two different slopes on either side of a threshold energy needed to pierce the material. A similarly “bent" power law is seen with short GRBs that is not simply explained with filtering, see Fig 2. The piece-wise function that describes the broken power law is,

$$\Phi(L) = \frac{dN_{GRB}}{dlog(L)} = \begin{cases} \left(\frac{L}{L_*}\right)^{-\alpha_L} & L_{min} \leq L \leq L_* \\ \left(\frac{L}{L_*}\right)^{-\beta_L} & L \geq L_*. \end{cases} \tag{1}$$

This broken power law can be seen in figure 2 with two different slopes $\propto L^{-\alpha}$ and $\propto L^{-\beta}$ are on either side of a threshold lumi-

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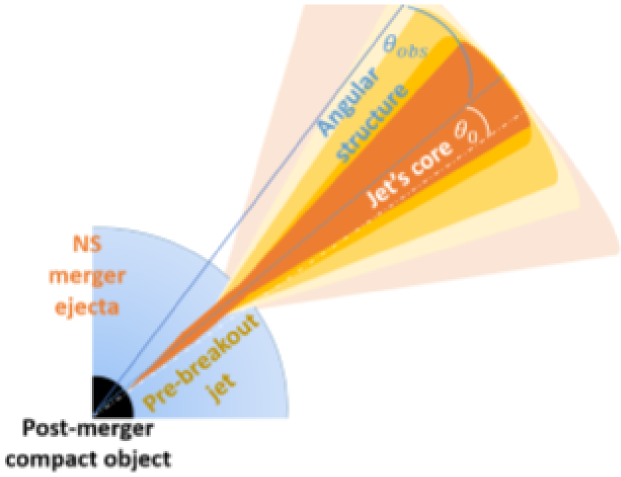

Figure 1. This diagram shows the structure of the jet being launched from the compact object or the BH originating from a NS-NS merger event. The core of the jet is along the jet axis up to about 5 degrees on either side. The angular structure varies with angle from the axis of propagation and diminishes rapidly as the angle increases to 20 degrees. The angular structure angle θ is different from the observation angle θ_{obs} which is specific to the angle the GRB is detected at. Figure from (Beniamini et al. 2019).

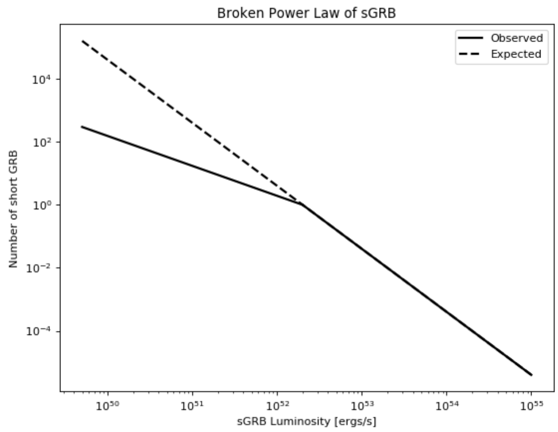

Figure 2. This is a log-scale plot of the piece-wise function describing the broken power law $\Phi(L)$ where red shows the expected slope of the luminosity of short GRBs. This is the luminosity that follows $L^{-\beta_L}$. Similarly there is a “bending" after the threshold luminosity. The blue line shows the shallow slope of the $L^{-\alpha_L}$ part of the piece-wise function describing the power law.

nosity L_*. This threshold luminosity is where the function changes from the expected luminosity to the more shallow luminosities. The minimum luminosity of GRBs is $L_{min} = 5 \times 10^{49}$ erg, the threshold luminosity $L_* = 2 \times 10^{52}$ erg without accounting for relativistic beaming and where $\alpha = 0.95$ and $\beta = 1.95$ (Wanderman & Piran 2015). The broken power law is a function $\Phi(L)$ that describes the relationship between the number of short GRBs with varying luminosity. As the luminosity increases for events there are less of them and much greater number of lower luminosity GRBs.

1.3 Arguments For Broken Power Law of Short GRBs

There are three arguments to explain the broken power law nature of the luminosity, the first is filtering. Like long GRBs there may be some filtering preventing all of the weaker jets from passing through the material and being observed.

All the the jets above the threshold, L_* would be detected, but the lower luminosity jets below L_* would have a various amount of surrounding material to fight through that would prevent them from successfully escaping, therefore, reducing the number of jets observed. This is consistent with the higher luminosity jets having enough power to neglect the material and thus followed the expected power-law.

The angular structure of the jet could be why the luminosity function is “bent", and it is a more complex argument. The structure of the jet changes sharply with angle from the center of the core of the jet. Perhaps if the jet were off-axis and weaker, we would not be able to detect the GRB past a certain threshold angle and luminosity.

The last of these arguments is that it might be possible the luminosity function of GRBs is *intrinsic* and no matter the viewing angle or the material surrounding the jet, the result will always be the same. This would mean that the intrinsic nature of short GRBs is not the same as long GRBs and must be explained with a new theory compensating for the incorrect explanations that disagree with the data significantly. More information on these arguments can be found in section 2.

In an effort to learn why the luminosity function of short GRBs from NS-NS mergers is a broken power law, the cause was believed to be related to the mass accretion rate of the black hole, $\dot{M}_{BH}$. Using the proportionality, $L \propto \dot{M}^{\gamma}_{BH}$, we learned there is a correlation between both the luminosity and mass accretion rate relative to ADAF and NDAF in the power law respectively Kawanaka et al. (2013). Kawanaka Kawanaka et al. (2013) provides a range of regimes of accretion values where the proportionality turns to $L \propto \dot{M}^{\frac{2}{3}}_{BH}$. This may seem like a significant difference but it is simple to show how the power changes by a negligible 0.01 of the original β.

2 THE BROKEN POWER LAW IS INTRINSIC

2.1 Angular Structure

The argument for the angular structure of the jet being the cause for the broken power law depends on the relationship between the angle and luminosity. In the figure showing the number of GRBs to luminosity, the number of GRBs is a function proportional to L which is proportional to θ to some power δ, $L \propto \theta^{-\delta}$, given by

$$\Phi(L) = \frac{dN}{dlog(L)} = \frac{dL}{dlog(L)}\frac{dN}{dL} = L\frac{dN}{dL}$$

$$\Phi(L) = L\frac{dN}{dL} \propto \theta^{-\delta}\frac{dN}{d\theta}\frac{d\theta}{dL}.$$

Using the relationship $L \propto \theta^{-\delta}$ we plug in $\frac{d\theta}{dL}$ and may also use this to find the exponent needed for L with $L^{-\frac{1}{\delta}} \propto \theta$. Taking the distribution of GRBs to be isotropic and using the solid angle of observation the relationship $N \propto \theta^2$ can be used to describe how more GRBs can be found by increasing the angle of observation.

The more of the universe being observed, the more events there are to be detected. This will increase the number of events being detected, but will also increase the number of "off-axis" GRBs that were weaker or not observable. These assumptions would lead to a luminosity function below L_* given by,

$$\Phi(L) \propto \theta^{-\delta} \frac{dN}{d\theta} \frac{d\theta}{dL}$$

$$\Phi(L) \propto \theta^{-\delta} 2\theta (\frac{\theta^{\delta+1}}{-\delta}) \propto \theta^2$$

$$\Phi(L) = L\frac{dN}{dL} \propto L^{-\frac{2}{\delta}}.$$

In order for the exponent of L to be equal to the observed value of $\propto L^{-\alpha} \propto L^{-0.95}$, δ must be a value approximately 2. What we find from simulations Kathirgamaraju et al. (2018) is a value of δ that is closer to $\delta \approx 4 - 5$ instead of $\delta \approx 2$ in this scenario. This is a huge difference that does not agree with observation making the argument for angular structure insufficient in explaining the broken power law of luminosity (Beniamini et al. 2019).

2.2 Filtering

Filtering has a prominent effect on the detectable luminosity of long GRBs caused by collapsing massive stars, but it does not play a significant role in short GRBs. The difference being that collapsing stars have material moving radially inward while the jet is launched radially outward in the opposite direction. If the jet is not strong enough it will be unable to overcome the material and be lost to the collapse. If the jet is above a specific luminosity threshold, it will be strong enough to prevail and pass through the collapsing material allowing it to be observed provided the observation angle is also within 20 degrees from the prompt.

This filtering effect cannot be an option for short GRBs because there is not a prominent amount of surrounding material moving radially inward to oppose the jet launched outward. Most of the material is rotating perpendicular to the jet's axis of propagation and so doesn't have enough collapsing material to filter, or oppose it. This produces a ratio of failed jets and successful jets, equation 2 shows the number of jets that are expected to fail given the NSNS merger rate provided by SWIFT. These numbers are off from observed failed jets by several magnitudes and thus are also insufficient in describing the broken power-law Beniamini et al. (2019).

$$r_{fail} = \frac{R_{merg}}{R_{sGRB}} - 1 \approx (\frac{L_{min}}{L*})^{-\beta_L+\alpha_L} = 540 \tag{2}$$

Numerical simulations predict that Beniamini et al. (2019) if filtering were to be present in short gamma-ray bursts, the fail rate would be as low as $r_{fail} \approx 1$, using the observational data from SWIFT for the number of NSNS mergers and traditional tophat model for the observation angles. Contrary to the filtering regarding short GRBs, the observed rate of failed jets for long GRBs are as high as 10^5, multiple magnitudes greater than the rate found from the observations from SWIFT for short GRBS.

2.3 Intrinsic Property

Without a fourth argument the final option is that the broken power law is itself an intrinsic nature of short GRBs and is in need of an adequate explanation. The intrinsic nature of the luminosity may best be explained with the Blandford-Znajeck Mechanism being the one that powers the jet. In a NS-NS merger event the material spins rapidly around a magnetic field creating a force that launches the jet along the axis of rotation of the accreting material. The mechanism has two different scenarios that change the luminosity of the jets and the change in mass accretion known as Advection Dominated Accretion Flow (ADAF) and Neutrino Dominated Accretion Flow (NDAF) processes. It may be the case that the two different slopes in the broken power law function for luminosity can be related to the two different processes by which the accreting material change. These properties lead to interesting relationships

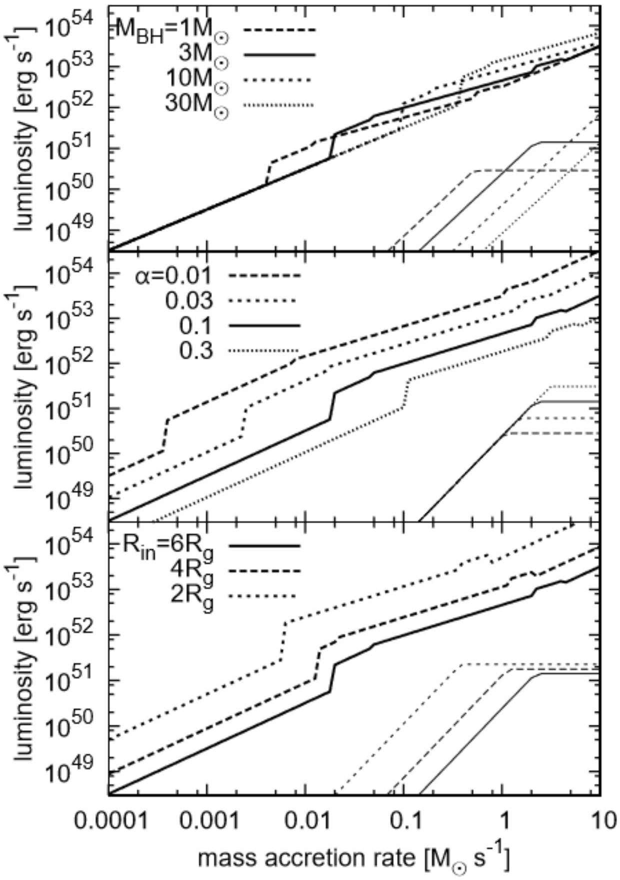

Figure 3. This figure shows the three main properties of the accretion rate of a BH and luminosity of a GRB with respect to the BZ-mechanism. The top plot shows the mass of the BH and the effect it has on luminosity and accretion rate. The BH mass does not seem to effect the luminosity significantly while it does shift $\dot{M}_{ign}$ high as the mass increases. The middle plot shows the viscosity of the accretion disk and as it increases the luminosity does as well, but more significant are the changes in $\dot{M}_{ign}$ as the accreting material becomes more viscous. The bottom plot shows the inner radius of the accretion disk to the compact object and changes luminosity more than the viscosity does, while having little effect on $\dot{M}_{ign}$. Figure from (Kawanaka et al. 2013).

3 ACCRETION RATE AND LUMINOSITY

3.1 Blanford-Znajeck Mechanism

The Blandford-Znajeck mechanism describes the behavior of accreting material around a BH and the magnetic field the material rotates around producing a powerful jet of radiation perpendicular to the axis of rotation. The radius from the event horizon, mass of the accretion disk, and the viscosity of the disk all contribute to the luminosity of the jet produced. Provided the BH is spinning with some velocity and is able to cool by emitting neutrinos, the process resembles the BZ-mechanism closely. Figure 3 shows the three different properties of the accretion disk that may affect the luminosity of the jet. It is shown that the luminosity is related to all three, the mass of the BH, the viscosity of the accretion disk, and the inner radius of the accreting material.

It is interesting to note that the mass accretion rate remains constant for a moment at accretion rate $\dot{M}_{ign}$ as the cooling of the accretion disk switches from one process to another (see Figure 3). Higher accretion rates use neutrino-dominated-accretion-flows (NDAF) and at lower accretion rates after crossing through $\dot{M}_{ign}$ becomes advection-dominated-accretion-flow (ADAF). During this transition the luminosity changes significantly and could be the cause of internal shocks that take place within GRBs. During NDAF, some neutrino-neutrino pair annihilation, $\nu\nu' \rightarrow e^+e^-$ occurs and adds to the luminosity and the efficiency of the accretion rate where ν' is an anti-neutrino (Kawanaka et al. 2013).

According to (Kawanaka et al. 2013), the mass accretion rate at which a transition from ADAF to NDAF occurs is $\dot{M}_{ign} \approx 0.003 - 0.01 M\frac{\odot}{s}$. This corresponds to a jet luminosity of $L_{jet} = \eta \dot{M}_{ign} c^2$, where $\eta = 0.1$ is the efficiency of the accreting process in forming a jet. In this case

$$L_{jet} = 0.1 \dot{M}_{ign} c^2$$

$$L_{jet} = (0.003 - 0.01) \cdot 2 \times 10^{33} \cdot 9 \times 10^{20}$$

$$L_{jet} = 5 \times 10^{50} erg - 2 \times 10^{51} erg.$$

The beaming factor of GRBs is $f_b = \frac{\theta_j^2}{2}$. For $\theta_j = 0.1$ then $f_b = \frac{0.1^2}{2} = \frac{0.01}{2} = 0.005$. The isotropic luminosity is then:

$$L_{iso} = f_b^{-1} L_{jet}$$

$$L_{iso} = (0.005)^{-1} [5 \times 10^{50} erg - 2 \times 10^{51}] erg$$

$$L_{iso} = 10^{53} erg - 4 \times 10^{54} erg.$$

The efficiency in producing gamma-rays is about 0.1, therefore the isotropic gamma-ray luminosity is

$$L_{\gamma,iso} = 0.1 L_{iso}$$

$$L_{\gamma,iso} = 10^{52} erg - 4 \times 10^{53} erg.$$

This number corresponds to the value of $L_* = 2 \times 10^{52} erg$ obtained by Wanderman & Piran (2015). This order of magnitude calculation yields a luminosity in the range observed around the break of the luminosity function, providing motivation for us to consider this scenario in more detail in section 3.2.

3.2 Connections

A connection must be made if the broken power law nature of the GRB luminosity is to be explained by the BZ-mechanism. Using the relationship between the accretion rate and luminosity $L \propto \dot{M}$, we can find what a relationship describing the number of GRBs to the accretion rate would look like just as figure 2 showed. Beginning with the function of Φ, we can again acquire $\Phi(L) = L\frac{dN}{dL}$. Using this function and the chain rule,

$$L\frac{dN}{dL} = L\frac{dN}{d\dot{M}}\frac{d\dot{M}}{dL}$$

It is convenient that $\frac{dN}{dL}$ and $\frac{dN}{d\dot{M}}$ vary proportional to one another. This expression reveals that the final term $\frac{d\dot{M}}{dL}$ is a constant term helping to describe a linear relationship between the luminosity of GRBs and the accretion rate of BHs. A linear relationship, $L = c\dot{M} + L_o$ to show $\frac{dL}{d\dot{M}} = c$ shows that the rate of change between the two is constant and proportional to each other, $L \propto \dot{M}$.

This shows that the function for the number of accretion rates to the rate of accretion, $\Phi(\dot{M})$ is itself a broken power law proportional to $\Phi(L)$ seen in figure 2. $\dot{M}$ will have the same slope on either side of the power law, $\propto \dot{M}^{-\alpha}$ and $\propto \dot{M}^{-\beta}$ and follow the luminosity of GRBs everywhere differing by a constant with the threshold accretion rate $\dot{M}_{ign}$ from one power law slope to the other is the cross over accretion rate describing when the accretion changes from ADAF to NDAF resulting in a change in luminosity simultaneously. Below we will see that the luminosity depends on the mass accretion rate to a specific power calculated by Kawanaka et al. (2013), which will modify our calculations.

3.3 Inner Radius

The inner radius of the accretion disk around a BH is unknown and needs to be approximated for use. A great way to approximate this radius is by using the inner-most stable circular orbit (ISCO) of the celestial body. This is the closest the accreting material can orbit the BH without falling beyond the event horizon. The equation for this is approximated by using Foucart (2012) this radius (R_{ISCO}) in terms of the mass and spin parameter (χ_{BH}) of the BH. Where the spin parameter is a ratio of the angular momentum of the BH compared to the speed needed to tear itself apart.

$$Z_1 = 1 + (1 - \chi_{BH}^2)^{\frac{1}{3}} [(1 + \chi_{BH})^{\frac{1}{3}} + (1 - \chi_{BH})^{\frac{1}{3}}]$$

$$Z_2 = \sqrt{3_{BH}^2 + Z_1^2}$$

$$\frac{R_{ISCO}}{MBH} = 3 + Z_2 - \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)} \tag{3}$$

This equation 3 and figure 4 show that the "inner radius" of the accretion disk decreases as the spin parameter of the BH increase to unity. We have found (Foucart (2012), Shapiro (2017)) that the spin parameters usually stay within the range of $\chi_{BH} \approx 0.65$ and $\chi_{BH} \approx 0.85$. If the spin parameter falls below ≈ 0.65 there will not be an accretion disk, and for values above 0.9 the radius decreases sharply. This R_{ISCO} was also plotted using a log scale as can be seen in figure 5 where the decrease can be seen more clearly. Also the largest change in the radius occurs through the range of spin parameters previously specified.

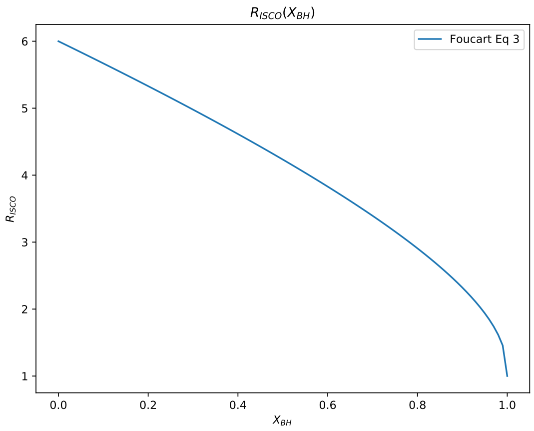

Figure 4. This shows the inner-most stable circular orbit as a function of the spin parameters of the black hole. This is not the inner-most radius of the accretion disk around the BH but it is meant to approximate it since the inner radius is not known. R_{ISCO} tends to decrease to unity as the BH spins faster (approaching unity) with the greatest amount of change taking place between $\chi_{BH} = 0.7$ and $\chi_{BH} = 0.9$.

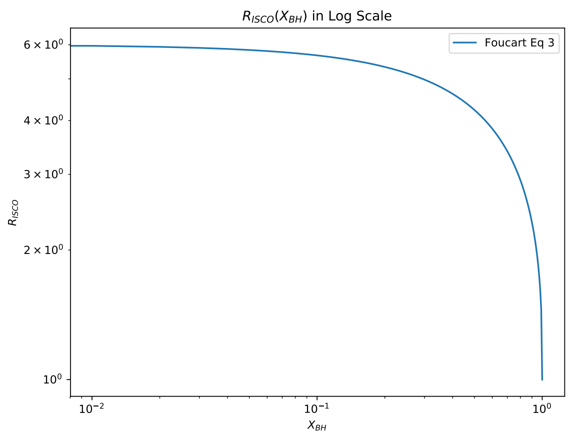

Figure 5. This shows the log scale of the R_{ISCO} and demonstrates that the greatest change occurs in the range of $\chi_{BH} = 0.65$ and $\chi_{BH} = 0.9$ as the radius goes to unity approaching the BH. That range of χ_{BH} is where the greatest sensitivity is as R_{ISCO} dives outside of this range and the lower values of χ_{BH} are to demonstrate more that the accretion disk is not existent when the spin parameters are below $\chi_{BH} = 0.6$ as the mass of the accretion disk goes to unity.

4 CALCULATING THE NUMBER OF EVENTS BASED ON MASS ACCRETION RATE

4.1 Discontinuity and Expanding Proportionality

As shown in Kawanaka et al. (2013), the luminosity is $L \propto \dot{M}^{\gamma}$ for different values of γ is determined by the accretion disk properties. Specifically, the optical thickness of the gases in the accretion disk increases and decreases for various regimes of accretion rate. This is shown to yield two powers of interest for γ. Using $L \propto \dot{M}^{\gamma}$ we can find a generalized proportionality between luminosity and accretion rate, with the values of interest being, $\gamma = 1$ and $\gamma = \frac{2}{3}$ for α and β respectively.

$$\Phi(L) = L\frac{dN}{dL} = L\frac{dN}{d\dot{M}}\frac{d\dot{M}}{dL}$$

Using the chain rule to implement mass accretion rate and using the proportionality above to substitute L with $\dot{M}$ gives,

$$\Phi(L) = L\frac{dN}{dL} = \dot{M}^{\gamma}\frac{dN}{d\dot{M}}(\gamma\dot{M}^{\gamma-1})^{-1}$$

$$\Phi(L) = L\frac{dN}{dL} = \frac{\dot{M}}{\gamma}\frac{dN}{d\dot{M}} = \frac{1}{\gamma}\Phi(\dot{M})$$

Taking the derivative of the proportionality and another substitution reveals the proportion relation between the number of GRBs as a function of luminosity and the number of GRBs as a function of mass accretion rate. This brings us to the proportionality needed for $L^{-\alpha}$ and $L^{-\beta}$.

$$L^{-\alpha} \propto \Phi(\dot{M}) \rightarrow \dot{M}^{-\gamma\alpha} \propto \Phi(\dot{M})$$

$$L^{-\beta} \propto \Phi(\dot{M}) \rightarrow \dot{M}^{-\gamma\beta} \propto \Phi(\dot{M})$$

This raises the question, would there be a "jump" in a $\Phi(\dot{M})$ plot? This can be determined by treating the two functions, $\Phi(L)$ and $\Phi(\dot{M})$ as piece-wise functions. Starting with L, we can use the 5 regimes of mass accretion rate (**?**) and the constants out front to characterise luminosity as a piece-wise function.

By using the proportionality $L \propto \dot{M}$ or when $\gamma = 1$, it was shown above that the relationship between the luminosity and mass accretion rate are one to one for some $\dot{M}$ above and below $\dot{M}_0$. This shows that there is no "jump" in the function while $\gamma = 1$. This covers the transition from ADAF to NDAF in the mass accretion, there is no break from $\Phi(L)$. The next piece to investigate is the transition within NDAF from optically thin to optically thick when γ changes.

Let $\gamma = \frac{2}{3}$ and $\dot{M}_0$ be some arbitrary $\dot{M}$,

$$L = \begin{cases} a\dot{M} & \dot{M} < \dot{M}_0 \\ b\dot{M}^{\frac{2}{3}} & \dot{M} > \dot{M}_0 \end{cases}$$

$$\Phi(L) = L\frac{dN}{d\dot{M}}\frac{d\dot{M}}{dL} = \begin{cases} c\dot{M}\frac{dN}{d\dot{M}}\left(\frac{1}{c}\right) & \dot{M} < \dot{M}_0 \\ d\dot{M}^{\frac{2}{3}}\frac{dN}{d\dot{M}}\left(\frac{1}{d}\frac{3}{2}\dot{M}^{\frac{1}{3}}\right) & \dot{M} > \dot{M}_0 \end{cases}$$

$$\Phi(L) = \begin{cases} \Phi(\dot{M}) & \dot{M} < \dot{M}_0 \\ \frac{3}{2}\Phi(\dot{M}) & \dot{M} > \dot{M}_0 \end{cases}$$

This is a clear "jump" in the function! This shows that when the accretion disk changes from optically thin to optically thick the function is discontinuous by $\frac{1}{3}$. The "jump" happens again when changing from optically thick back to ADAF and returning to a one-to-one proportionality with $\gamma = 1$. Allowing gamma to vary between the two values gives,

$$\Phi(L) = L\frac{dN}{d\dot{M}}\frac{d\dot{M}}{dL} = \begin{cases} a\dot{M}\frac{dN}{d\dot{M}}\left(\frac{1}{a}\right) & \dot{M} < \dot{M}_0 \\ b\dot{M}\frac{dN}{d\dot{M}}\left(\frac{1}{b}\right) & \dot{M} < \dot{M}_0 \\ c\dot{M}^{\frac{2}{3}}\frac{dN}{c\dot{M}}\left(\frac{1}{c}\frac{3}{2}\dot{M}^{\frac{1}{3}}\right) & \dot{M} > \dot{M}_0 \\ d\dot{M}^{\frac{2}{3}}\frac{dN}{d\dot{M}}\left(\frac{1}{d}\frac{3}{2}\dot{M}^{\frac{1}{3}}\right) & \dot{M} > \dot{M}_0 \\ e\dot{M}\frac{dN}{d\dot{M}}\left(\frac{1}{e}\right) & \dot{M} < \dot{M}_0 \end{cases} \quad (4)$$

This can be used for the conversion from the number of GRBs based on luminosity to the number of GRBs based on the mass accretion rate by using the proportionality between luminosity and accretion stated at the beginning of the section.

4.2 Calculating Luminosity

Kawanaka et al. (2013) provides an equation for luminosity of an isotropic jet for each of the five ranges of mass accretion rates. These can be turned into a single expression and calculated using the constants provided in equation 4 and the range of values for $\dot{M}$ related to the different types of cooling in the accretion diskKawanaka et al. (2013).

$$L = L(\dot{M}, R_{ISCO}(X_{BH}), f(X_{BH})) \quad (5)$$

$$f = 1 + 0.035\omega^2 - 0.58\omega^4 \quad (6)$$

Using equation 6 for the efficiency of the BZ mechanism determined by the BH spin parameter, χ_{BH} (Tchekhovskoy & Giannios 2015), a luminosity can be acquired that is built of components external to Kawanaka's calculations. Some differences are the use of R_{ISCO} instead of the inner most radius of the accretion disk, and implementing $f(\chi_{BH})$ as some value other than unity. This can be plotted as a function of $\dot{M}$ with various values of χ_{BH} to look for any significant changes from Kawanaka's plot of luminosity from figure 3. Calculations using equation 5 are shifted by different constants as in figure 7 and the shift around M_o becomes more prominent with higher values of spin. There are different constants used to distinguish between Kawanaka's efficiency function (Kawanaka et al. 2013) and equation 6 from Tchekhovskoy & Giannios (2015). Beyond this, there are no significant differences in luminosity.

The efficiency function of the BH spin can be seen in figure 6 shows how the efficiency of the BZ-mechanism increases exponentially with BH spin. The angular momentum of BHs directly effects the electromagnetic forces involved in propagating jets, but the efficiency asymptotically reaches 0.35 as the BH spin nears the maximum speed physically tolerable by the BH. Numerical simulations (Tchekhovskoy & Giannios 2015) of the spin efficiency are only accurate up to a $\chi = 0.95$ approaching unity as Kawanaka (Kawanaka et al. 2013) assumes.

4.3 Results

An important result found in figure 7 as the luminosity calculated for each of the five regimes of mass accretion rate (Kawanaka et al.

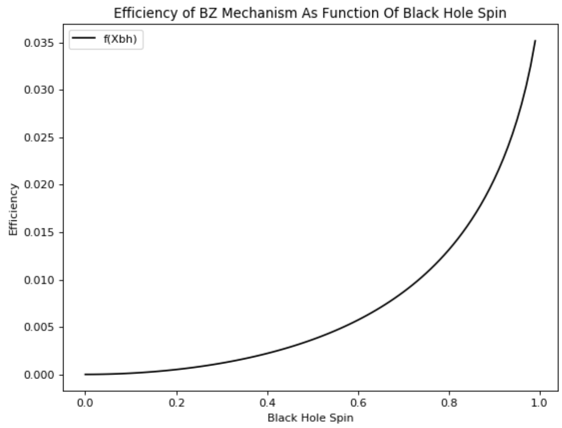

Figure 6. Graph showing the efficiency of BZ-mechanism as a function of spin parameters from the BH. As the BH spins closer to its maximum physically tolerable speed, the efficiency increases exponentially. The function reaches an asymptotic limit at 0.035 efficiency.

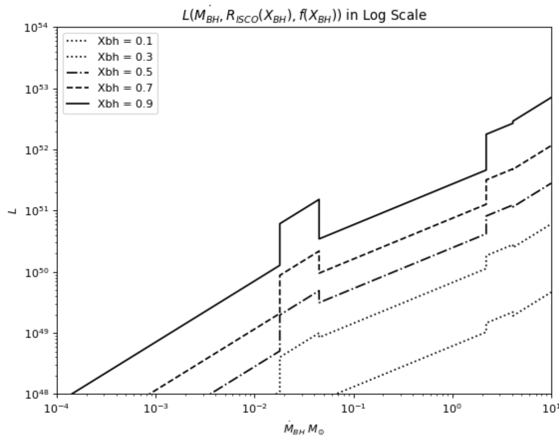

Figure 7. Luminosity calculated using R_{isco}, $\dot{M}$, and $f(X_{BH})$ all as functions of varying X_{BH} values from 0.1 to 0.9 with odd value intervals. Notice that the luminosity increases with spin parameters while the mass accretion rate remains constant. This calculates the luminosity from the inner radius and spin, which is important to include in the equations used by Kawanaka et al. (2013).

2013) shift significantly for different values of BH spin. The functions used to plot this figure were all dependent on χ_{BH} including the "inner radius" and the spin parameter. These were then used to calculate the mass accretion rate and luminosity as discussed above. This result was overlooked in Kawanaka's paper Kawanaka et al. (2013) where the assumption, $f(\chi_{BH} = 1$ is made, thus neglecting the large variability in luminosity caused by BH spin.

This resulting function was found by taking the proportionality of luminosity and mass accretion rate and equation 5 for luminosity as a function of mass accretion rate, inner radius, and spin. Furthermore, finding how to calculate each of the parameters in the luminosity function, after some assumptions and approximations mentioned above, is dependent on the spin efficiency. (Kawanaka et al. (2013), Foucart (2012), Tchekhovskoy & Giannios (2015), Shapiro (2017)). This was a trail in need of several assumptions that led to the realization that BH spin cannot be ignored when calculating the jet luminosity of short GRBs.

Finding the number of events as a function of mass accretion rate, $\dot{M}$ is the important step in learning more about the sophistication of the model being constructed in the effort to answer why the luminosity function is a broken power law. By calculating the merger number as a function of the mass accretion rate, or $\Phi(\dot{M})$ as a power law, the function graphed in figure 8 can be seen. This function is a completely different result than the number of GRBs as a function of luminosity, albeit they follow a similar expected shape. The number decreases as the luminosity, or in this new case, the mass accretion rate increases. This is an expected result as bodies with increasingly large amounts of energy should become more rare.

This new mass accretion rate function was calculated using the piece-wise relationship from section 4.1 to "convert" the 5 regimes of mass accretion rate and the luminosity function into terms of mass. Connecting the number of events to the rate of accretion (Kawanaka et al. 2013) keeping the BH spin parameters inside the functions throughout. In figure 8, only the high values of spin were considered and the discontinuities can still be seen. These "jumps" are acceptable as the 5 regimes of mass accretion rate were selectively picked from a three dimensional plot (Kawanaka et al. 2013) where several parameters were allowed to vary. This was limited to two dimensions and need to be expounded to understand a more complete model of the number of mergers.

In addition to this, a connection can be drawn from the observational data from SWIFT representing the number of GRBs as a function of luminosity to simulations Piro et al. (2017) representing a function of the number of NSNS merger events to the mass of BHs, or $\Phi(M)$, (see figure 9), using the work already done to calculate the number of events as a function of mass accretion rate. If the two functions are similar, then some significant progress has been made in the project prompting further investigation as discussed in the section below.

5 FUTURE WORK

Although important progress has been made in building the model to explain the broken power-law, the model is not complete. It would be beneficial to calculate $\Phi(\dot{M})$ from other means than the ones previously used to test the validity of the assumptions. However, despite the success in plotting $\Phi(\dot{M})$ it is insufficient in answering the prompted question alone. It doesn't explain why the intrinsic properties of the GRB launching mechanism and BH cause the luminosity function to be a broken power-law. It shows how the number of short GRBs would change with the varying mass accretion rate, but a mathematical relationship describing the mechanism behind the luminosity function is not established. This suggests that the question may need to be resolved from a different approach and that this model can be improved upon.

One immediate investigation can point towards the magnetic flux threading the BHs and contributing to the luminosity of the GRBs. The mass accretion rate is only a part of the solution to this broken power-law problem, the other part is driven by the magnetic field of the black hole. Although it is true that the mass accretion rate of the black hole helps drive the luminosity, this is most prominent for low values of accretion. While the mass accretion rate is large, the magnetic flux is the primary mechanism describing the luminosity of GRBs, but when the accretion rate decreases past a threshold (Tchekhovskoy & Giannios 2015), it forces the magnetic flux and the luminosity to decline along with it. The accretion rate thus acts as an "off switch" for the GRB luminosity function over time as all described and shown by Tchekhovskoy and Giannios. A figure is provided that demonstrates the rudimentary idea of these two mechanisms working in tandem to control the luminosity Tchekhovskoy & Giannios (2015).

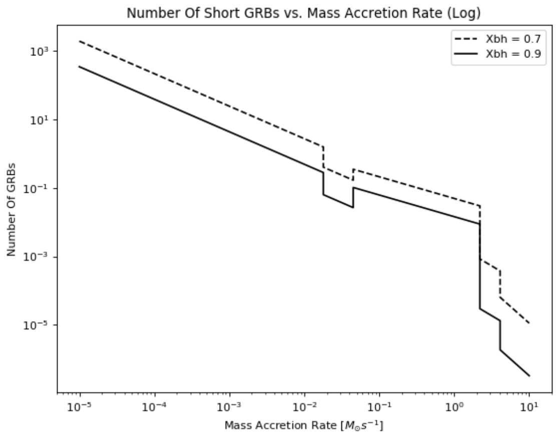

Figure 8. The number of GRBs as a function of mass accretion rate. As the accretion rate increases the number of events drops at varying rates. The slope of the function was decided by calculating a luminosity greater than or less than L_* and varying BH spin parameters. Luminosity does not appear to be significantly affected by BH spin for values below L_* and amplified by spin for values greater than that. BH spin was allowed to vary from 0.1 to 0.9 to observe the effects on $\Phi(\dot{M})$.

There are two main ideas that can be used to attempt to answer the question of the luminosity function in a different way than prior. The options are to explain the broken power law through magnetic flux, or the total mass of the binary primordial neutron stars before merging (Piro et al. 2017), using the variations in total mass has been shown to affect the out come of mergers between NS-NS, NS-BH, and BH-BH mergers, this might have similar affects on short GRBs that are "born" from the events. It has been shown in this paper that the accretion rate of the NS mass varies the luminosity, perhaps a connection can be made between the mass and the luminosity in the reverse direction. It is possible the "fate" of the neutron stars determines the "fate" of the jet luminosity and that their mass and magnetic fields are significant in this model.

REFERENCES

Beniamini P., Petropoulou M., Barniol Duran R., Giannios D., 2019, MNRAS, 483, 840
Foucart F., 2012, Phys. Rev. D, 86, 124007
Kathirgamaraju A., Barniol Duran R., Giannios D., 2018, MNRAS, 473, L121
Kawanaka N., Piran T., Krolik J. H., 2013, ApJ, 766, 31
Petropoulou M., Barniol Duran R., Giannios D., 2017, MNRAS, 472, 2722
Piro A. L., Giacomazzo B., Perna R., 2017, ApJ, 844, L19
Shapiro S. L., 2017, Physical Review D, 95
Tchekhovskoy A., Giannios D., 2015, MNRAS, 447, 327
Wanderman D., Piran T., 2015, MNRAS, 448, 3026

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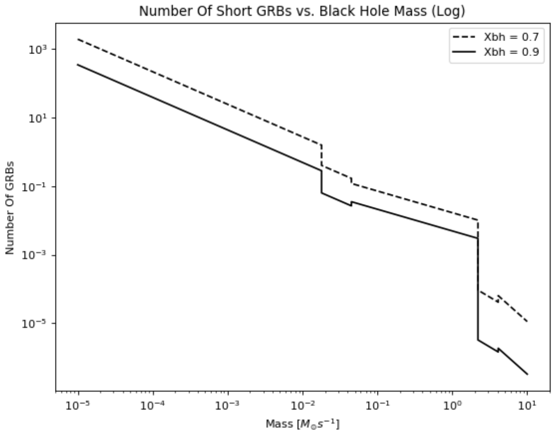

Figure 9. This final graph shows the number of GRBs as a function of black hole mass. Only high values of χ were used as these are the more typical black hole spin parameters. The function is similar to the number of GRBs as a function of mass accretion rate with smoother transitions through the regimes of accretion rate provided by **?**