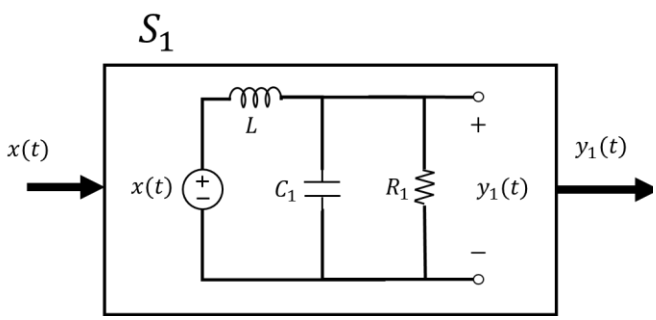
Homework # 4

<u>Instructions:</u> Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and <u>box all your solutions</u>. All Matlab code needs to be published, and <u>all figures needs to have proper axis labeling and legends</u>. Homework assignments will be collected during class time on the due date. *No late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **<u>Homework problems not to be graded</u>**
  - From textbook (Lathi):
    - Ch 2: 3.2, 3.3, 4-16, 4-20

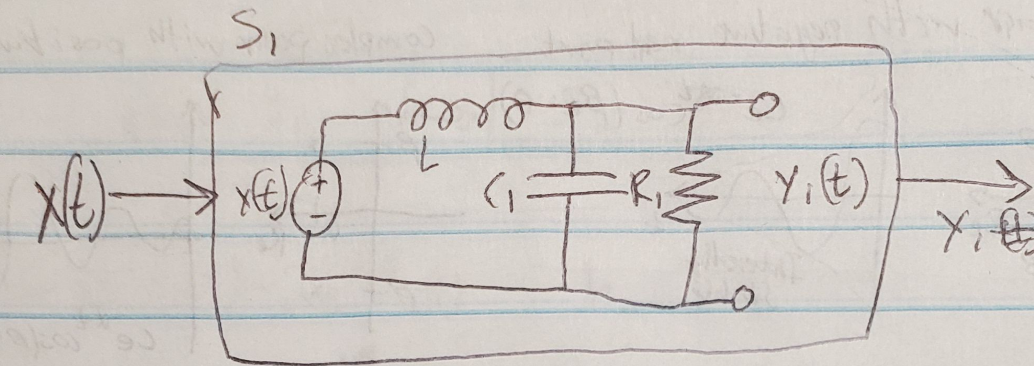
- **<u>Homework problems to be graded</u>**

Consider the following circuit system S_1 with input voltage $x(t)$ and output voltage $y_1(t)$. The differential equation relating the input $x(t)$ to the output $y_1(t)$ was found in Homework #3.



Let $L = 0.01, C_1 = 0.01, R_1 = 100.$

a) Find the impulse responses of systems S_1.
b) Find the zero-state response $y_1(t)$ of system S_1 to the unit step input $x(t) = u(t)$. Show all your work.
c) Plot the zero-state response $y_1(t)$ in Matlab. In one or two sentences, what solutions could you offer to mitigate/damp the oscillation of the system response?



$L = 0.01H$

$C_1 = 0.01F$

$R_1 = 100\,\Omega$

a) find the impulse responses of systems S_1

$$\left(D^2 + \frac{1}{CR}D + \frac{1}{LC}\right)y(t) = \left(\frac{1}{LC}\right)x(t)$$

$$\hookrightarrow \left(D^2 + D + 10000\right)y(t) = (10000)x(t)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{\left(\frac{1}{CR}\right)^2 - 4\left(\frac{1}{LC}\right)} \qquad x = \frac{-1 \pm \sqrt{1 - 40000}}{2} \approx \frac{-1 \pm 200i}{2} = \frac{-1}{2} \pm 100i$$

roots: $\underset{\alpha}{\frac{-1}{2}} \pm \underset{\beta}{100i}$ modes: $e^{-1/2\,t}\cos(100t + \theta)$

$$y(t) = \left(Ce^{-t/2}\cos(100t + \theta)\right)$$

$$y(0) = C\cos(\theta) = 2$$

$$y'(t) = -\frac{1}{2}Ce^{-t/2}\cos(100t + \theta) - \frac{1}{2}Ce^{-t/2}\sin(100t + \theta)$$

$$y'(0) = \frac{-1}{2}C\cos(\theta) - \frac{1}{2}C\sin(\theta) = 0$$

$$\cancel{C\cos\theta - C\sin\theta = -2}$$

$$\cancel{= C(\cos\theta - \sin\theta) = -2}$$

$$\cancel{C = \frac{-2}{\cos\theta - \sin\theta}}$$

$$\cancel{\left(\frac{-2}{\cos\theta - \sin\theta}\right)\cos\theta = 2} \qquad \cancel{\theta = 14.137}$$

$$\frac{-1}{2}C\cos\theta = \frac{1}{2}C\sin\theta \qquad \frac{-1}{2}\cos\theta = \frac{1}{2}\sin\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = -1 \qquad \theta = -0.79\text{ rad}$$

$$C = 2/\cos(-0.79) = 2.84$$

$$\boxed{h(t) = \left(2.84e^{-t/2}\cos(100t - 0.79)\,u(t)\right)\cdot 10000}$$

b) find the zero-state response $y_1(t)$ of system S_1 to the unit step input $x(t) = u(t)$. Show all your work.

$$h(t-\tau) = 28400 e^{-(t-\tau)/2} \cos\left(100(t-\tau) - 0.79\right) u(t-\tau)$$

$$y(t) = \int_0^t \cancel{x(\tau)}^{\,1} h(t-\tau)\, d\tau \quad t \geq 0$$

$$= \int_0^t 28400 e^{-\frac{1}{2}(t-\tau)} \cos\left(100(t-\tau) - 0.79\right) d\tau$$

$$= 28400 \int_0^t e^{-\frac{1}{2}(t-\tau)} \cos\left(100(t-\tau) - 0.79\right) d\tau$$

$$e^{-t/2} / e^{-\tau/2}$$

$$= 28400 e^{-t/2} \int_0^t e^{\tau/2} \cos\left(100(t-\tau) - 0.79\right) d\tau$$

CI #12: $\alpha = \frac{1}{2}$ $\beta = 100$ $\theta = -0.79$ $\lambda = 0$

$$= \cancel{28400}^{\,2.84} \left(\frac{\cos(-0.79 + \phi) - e^{-\frac{1}{2}t} \cos(100t - 0.79 - \phi)}{\cancel{\sqrt{\frac{1}{2}^2 + 10000^2}}_{\,\cancel{10000}}} \right) u(t)$$

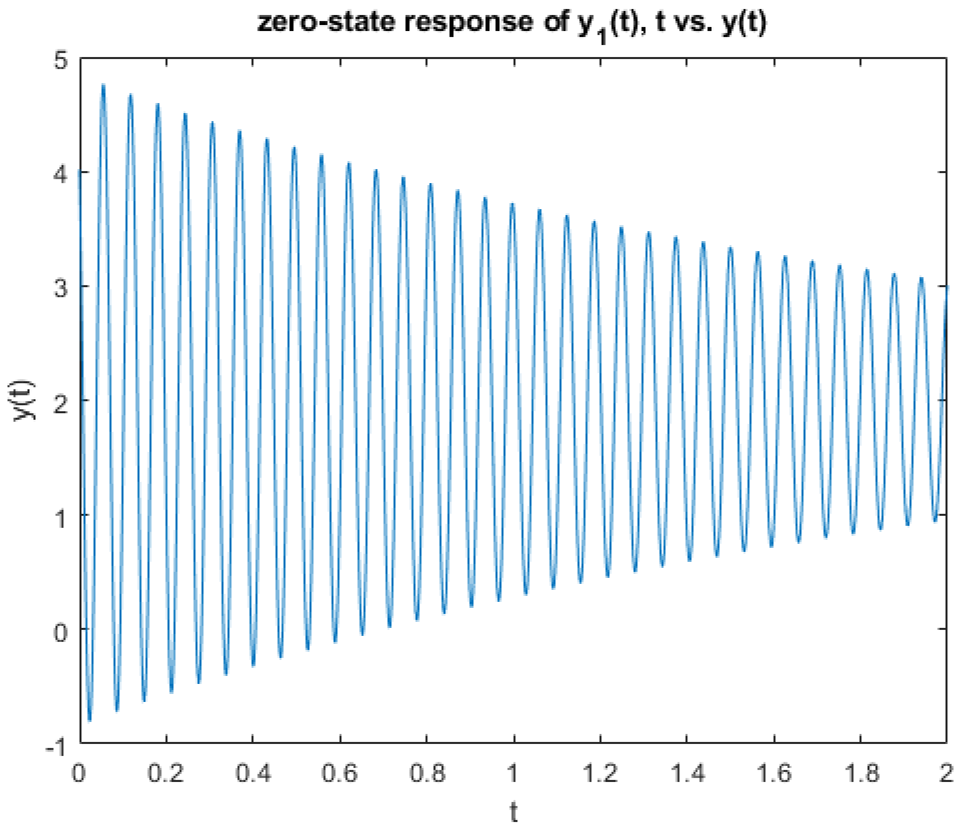
$$= 2.84 \left(\cos(-0.79 - \phi) - e^{-\frac{1}{2}t} \cos(100t - 0.79 - \phi) \right) u(t)$$

$$\phi = \tan^{-1}\left[-\cancel{100/\tfrac{1}{2}} \right]^{-200}$$

$$= \tan^{-1}(-200) = -1.57$$

$$= 2.84 \left(\underbrace{\cos(-2.36)}_{-0.71} - e^{-\frac{1}{2}t} \cos(100t - 2.36) \right) u(t)$$

$$\boxed{y(t) = \left(2 - 2.84 e^{-t/2} \cos(100t - 2.36) \right) u(t)}$$



c) To mitigate the oscillation of the response, one could decrease the frequency of the cosine function by increasing the capacitance of the C1. The result of this change would still decay at the same rate, but now with a longer period.