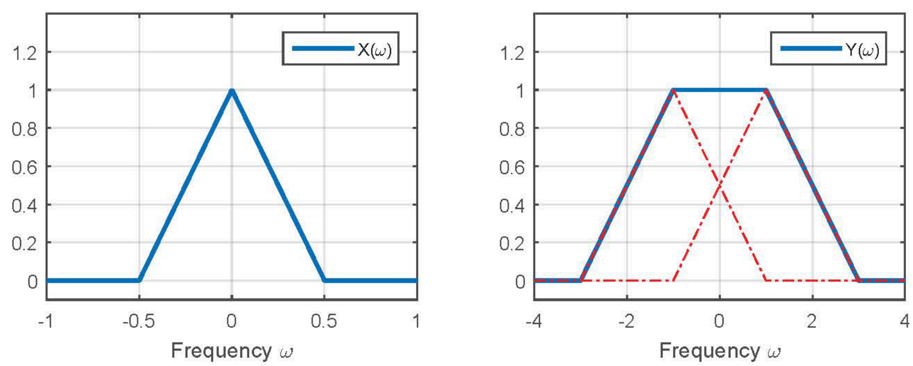
Homework #10

<u>Instructions:</u> Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and <u>box all your solutions</u>. All Matlab code needs to be published, with your name and date at the top of the script, and <u>all figures needs to have proper axis labeling and legends</u>. Homework assignments will be collected during class time on the due date. *Late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **<u>Homework problems not to be graded</u>**
    - From textbook (Lathi):
        - Ch 7: 3-1, 3-4, 6-2, 6-9

    - Let $X(\omega) = \Delta(\omega)$ be the Fourier transform of a signal $x(t)$, where $\Delta(t)$ is a triangle function, and $Y(\omega)$ be given as the figure below



a) Find $x(t) = \mathcal{F}^{-1}[X(\omega)]$

b) Find $y(t) = \mathcal{F}^{-1}[X(\omega)]$. Note that $Y(\omega)$ is the combination of 2 triangle functions (in red).

c) Use Parseval's Theorem to calculate the energy of $y(t)$.

d) Determine the bandwidth W of $y(t)$ such that 95% of the energy is contained within the frequencies $\omega \in [-W, W]$

a) find $x(t) = \mathcal{F}^{-1}[X(\omega)]$ when $X(\omega) = \Delta\left(\frac{\omega}{2W}\right)$, $W = 1/2$

$$\frac{W}{2\pi}\,\text{sinc}^2\left(\frac{Wt}{2}\right) = \boxed{\frac{1}{4\pi}\,\text{sinc}^2(t/4)}$$

b) find $y(t) = \mathcal{F}^{-1}[Y(\omega)]$ when $Y(\omega) =$

$$X(\omega - \omega_0) + X(\omega + \omega_0)$$

$$\boxed{\frac{1}{4\pi}\,\text{sinc}^2\left(\frac{t}{4}\right)e^{-jt} + \frac{1}{4\pi}\,\text{sinc}^2\left(\frac{t}{4}\right)e^{jt}}$$

$$= \left(\frac{1}{4\pi}\,\text{sinc}^2\left(\frac{t}{4}\right)\right)\left(e^{-jt} + e^{jt}\right)$$

$$\cos(-t) + i\sin(-t) + \cos t + i\sin t$$

$$2\cos t + i2\sin t =$$

c) $E_y = \frac{1}{\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2\, d\omega$ $\qquad w = 1/2$

$X(\omega - \omega_0) = \Delta\,\frac{\omega - \omega_0}{2W}$

$$\int_{-\infty}^{\infty}\left|\left(X(\omega - \omega_0) + X(\omega + \omega_0)\right)\right|^2 d\omega$$

$\omega_0 = 1$

$$|\Delta(\omega - \omega_0) + \Delta(\omega + \omega_0)|^2$$

$$= \frac{4}{\pi\omega^2}\sin^2 \omega/4$$

$$\frac{1}{\pi}\Bigg(\underbrace{\int_0^1 1\, d\omega}_{1} + \underbrace{\int_1^3 \left(1 - \tfrac{1}{2}\omega\right) d\omega}_{} \Bigg) = \boxed{\frac{1}{\pi}}$$

$$\int_1^3 1\, d\omega - \frac{1}{2}\int_1^3 \omega\, d\omega$$

$$\underbrace{\underbrace{2}_{} - \underbrace{1/2}_{}\ \underbrace{4}_{}}_{0}$$

d) $0.95\, E_y = \frac{1}{\pi}\int_0^W |Y(\omega)|^2\, d\omega$

$$= \frac{1}{\pi}\int_0^1 1\, d\omega + \frac{1}{\pi}\int_1^W \left(1 - \frac{\omega}{2}\right) d\omega$$

$$\frac{1}{\pi}\left((W) + \left(W - \tfrac{1}{12}W^3 - \tfrac{1}{2}W^2\right)\right) = \frac{0.95}{\pi}$$

$$2W - \tfrac{1}{12}W^3 - \tfrac{1}{2}W^2 = 0.95$$

$$\boxed{W = 2.29}$$