Chapters 2: Sequential Experiments

Tree Diagram (2.1)

Sequential experiments consist of many *subexperiments* performed in sequence and an observation taken after each action.

- A **tree diagram** displays the outcome of the subexperiments
  - Label of branches are probabilities and conditional probabilities

- Example: consider a bucket with 3 white balls and 2 red balls. Draw 3 balls in sequence without replacement
  - Events W_f ={“white ball first}, R_s ={“red ball second”}.
  - Tree diagram

- Example: successful detection of plane by a radar system
  - Events $A = \{\text{"plane present"}\}$,
    $B = \{\text{"radar sounds alarm"}\}$
  - $\mathrm{P}[B|A] = 0.99$

    $\mathrm{P}[B|A^c] = 0.1$
  - Determine $\mathrm{P}[B \cap A^c]$
    $\mathrm{P}[B^c \cap A]$

Counting Methods (2.2)

- Introduce methods for determining the number of outcomes in sample space of sequential experiment
  - k-permutation: ordered sequence of k distinguishable objects
  - k-combination: unordered combination of k distinguishable object

- **Experiment 1:** consider a bucket with 5 balls labeled {"A","*B*","*C*","*D*","*E*"}
  - Draw (*or sample*) two balls at random, in succession, and replace ball after observation

- <u>Counting k **ordered** samples of n distinguishable objects with **replacement** (*Thm 2.4*):</u>

$$\text{\# of outcomes} = n^k$$

- Example above:

- <u>**Experiment 2**</u>: consider the bucket with 5 labeled balls
  - Draw (*or sample*) two balls at random, in succession, and <u>do not</u> replace ball after observation

  - Generalize for the choice of k objects out of n distinguishable objects

  - <u>Counting k **ordered** samples of n distinguishable objects **without replacement** (*Thm 2.2*):</u>

$$\text{\# of outcomes} = (n)_k = n(n-1)(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

- <u>**Experiment 3:**</u> consider the bucket with 5 labeled balls
  - Draw (*or sample*) **unordered *combination*** of two balls, at random, and without replacing the ball after each observation

  - Generalize for k number of samples out of n distinguishable objects

- Counting *k* **unordered** samples of *n* distinguishable objects **without replacement** (*Thm 2.3)*:

$$\#\text{ of outcomes} = \binom{n}{k} = \frac{(n)_k}{k!} = \frac{n!}{k!\,(n-k)!}$$

- **Experiment 4:** consider the bucket with 5 labeled balls
  - Draw *(or sample)* **unordered combination** of two balls, at random, and replacing the ball after each observation
  - Generalize for *k* number of samples out of n distinguishable objects
  - Counting *k* **unordered** samples of *n* distinguishable objects with **replacement**

$$\#\text{ of outcomes} = \binom{n+k-1}{k}$$

- *Example*: binary code transmission
  - P["transmitting 2 ones in a sequence of 5 bits"]

- *Example*: binary receiver
  - Transmitter sends 5 zeros in sequence to represent “zero,” and 5 ones to represent “one”
  - Receiver rules that a receiver sequence represent “zero” if 3 or more zeros are detected, otherwise receiver rules sequence represent “one”
  - Receiver makes decision error with probability t
  - Q: what would be the probability of interpreting “zero” if a sequence for “one” is transmitted?

- *Example*: dealing 5 cards from a fair deck of 52 cards.
  - P[“getting 5 hearts”]
  - P[“getting the queen of spade”]
- Example: a committee of 4 people is chosen randomly from 5 men, 7 women and 5 children
  - P[“committee contains 1 woman”]
  - P[“committee contains at least 1 woman”]
  - P[“committee contains a most 1 woman”]

- Example: consider a bucket with 3 white balls and 2 red balls. Draw 2 in succession and define event $A = \{\text{"white on first draw and red on second draw"}\}$.
  - Determine P[A] when replacing the balls after draw

  - Determine P[A] without replacing the balls after draw

  - Determine P["draw combination of a red and a white"]

- We generalize our last example to propose the following experiment
  - We have n_1 red balls and n_2 white balls
  - Event *A* = {"draw k red balls out of n_3 total"}

  - Example: consider bucket with 30 red balls and 70 white balls. Find probability of getting k red balls out a draw of 20 balls.

  - <u>Binomial Probability Law (*Thm 2.8*):</u> the probability of n_0 failures and n_1 successes in $n = n_0 + n_1$ independent trials is

$$\begin{aligned} \text{P}[\text{"}n_0 \text{ fails and } n_1 \text{ success"}] &= \binom{n}{n_1}(1-p)^{n-n_1}p^{n_1} \\ &= \binom{n}{n_0}(1-p)^{n_0}p^{n-n_0} \end{aligned}$$

    where $p = \text{P}[\text{"success"}]$ and $1 - p = \text{P}[\text{"fail"}]$

    <u>*Note:*</u> applies to sequential experiments in which subexperiments are independent and have 2 outcomes (*Bernoulli trials*)