MATH 426

7.1 10. $\int \ln\sqrt{x}\,dx$ $\quad u=\ln x \quad v=x$

$du = 1/x\,dx \quad dv = 1\,dx$

$\frac{1}{2}\int \ln\sqrt{x}\,dx = x\ln x - \int x/x\,dx \quad \frac{x\ln x - x}{2} + C$

18. $\int e^{-\theta}\cos 2\theta\,d\theta$ $\quad u=\cos 2\theta \quad v=-e^{-\theta}$

$du=-2\sin 2\theta \quad dv = e^{-\theta}\,d\theta$

$-\cos(2\theta)e^{-\theta} - \int 2\sin(2\theta)e^{-\theta}\,d\theta$

$u=-2\sin 2\theta \quad v=-e^{-\theta}$

$-2e^{-\theta}\sin 2\theta - \int -4e^{-\theta}\cos 2\theta\,d\theta \quad du=-4\cos 2\theta\,d\theta \quad dv=-e^{-\theta}\,d\theta$

$-e^{-\theta}\cos 2\theta - \left(-2e^{-\theta}\sin 2\theta + 4\int e^{-\theta}\cos 2\theta\,d\theta\right)$

$\frac{2e^{-\theta}\sin 2\theta - e^{-\theta}\cos 2\theta}{5} + C$

$\frac{e^{-\theta}(2\sin 2\theta - \cos 2\theta)}{5} + C$

26. $\int_0^1 w^2 \ln w\,dw$ $\quad u=\ln w \quad v=\frac{1}{3}w^3$

$du=\frac{1}{w}\,dw \quad dv=w^2\,dw$

$\frac{1}{3}w^3\ln w - \int \frac{\frac{1}{3}w^3}{w}\,dw$

$\frac{1}{3}w^3\ln w - \frac{1}{3}\int w^2\,dw \quad \left(\frac{1}{3}w^3\ln w - \frac{1}{3}\left(\frac{1}{3}w^3\right)\Big|_0^1\right)\Big|_0^1$

$\cancel{\frac{1}{3}}\cancel{\ln(1)} - \frac{1}{9} = -\frac{1}{9}$

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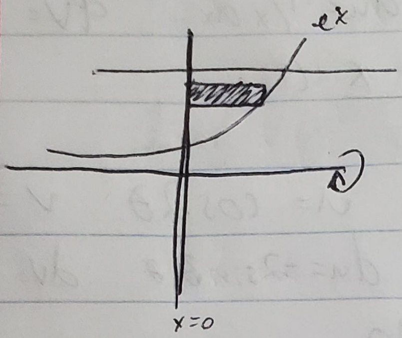
64. $y = e^x$ $x=0$ $y=3$ about $y=0$



$$2\pi \int_1^3 R(x)\, h(x)\, dx$$

$h(x) = e^x$ $\quad 2\pi \int_1^3 e^{2x}\, dx$

$R(x) = e^x$ $\quad u = 2x \quad du = 2$

$\frac{1}{2} du = dx$

$\frac{1}{2}\int e^u\, du$

$\frac{1}{2}(e^u)$

$$2\pi \left(\frac{e^6}{2} - \frac{e^2}{2}\right) \qquad 2\pi \left(\frac{e^{2x}}{2}\right)\Big|_1^3$$

$\pi(e^6 - e^2)$

7.2 2. $\int \sin^3\theta \cos^4\theta\, d\theta$

$$\int \sin^2\theta \cos^2\theta \cos^2\theta \sin\theta\, d\theta$$

$u = \cos\theta \quad du = -\sin\theta\, dx \quad -du = \sin\theta\, dx$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x) = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x) = \frac{1 + \cos 2x}{2}$

$\frac{1 - \cos 2x}{2} \cdot \frac{1 + \cos 2x}{2} \cdot \frac{1 + \cos 2x}{2} \cdot \sin \cancel{x}$

$- \frac{1-u}{2} \cdot \frac{1+u}{2} \cdot \frac{1+u}{2} \cdot du$

$- \frac{(1-u)(1+u)^2}{8}$ $\quad (1 + 2u + u^2)\cancel{(1-u)}$ $\quad 1 + 2u + u^2 - u - 2u + u^3$

$- \frac{(-u^3 + 1)}{8}$

$-\frac{1}{8}\int 1 + u^3$

$-\frac{1}{8}\left(u + \frac{u^4}{4}\right) \quad \frac{-u}{8} - \frac{u^4}{32} \quad \frac{\left(\cos\theta + \frac{1}{4}\cos^4\theta\right)}{-8} + C$

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10. $\int_0^{\pi} \sin^2 t \cos^4 t \, dt$

$\int \frac{1-\cos 2x}{2} \cdot \frac{1+\cos 2x}{2} \cdot \frac{1+\cos 2x}{2} \, dx$

$\frac{1-\cos^2 2x}{4} \cdot \frac{1+\cos^2 2x}{2}$

$\frac{1+\cos 2x - \cos^2 2x - \cos^3 2x}{8}$

$\frac{1}{8} \int 1 + \cos 2x - \cos^2 2x - \cos^3 2x \, dx$

$\int 1 + \cos 2x - \frac{1+\cos 2x}{2} - \cos 2x \, \frac{1+\cos 2x}{2} \, dx$

$\frac{1+\cos 2x}{1} \cdot \frac{1}{2}$

$-\cos^2 2x - \cos^2 2x$ $\quad \frac{-\cos 2x}{2} - \frac{\cos^2 2x}{2}$

$\int 1 + \int \cos 2x - \frac{1}{2}\int \cos 2x - \frac{1}{2}\int 1 - \frac{1}{2}\int \cos 2x - \int 1 - \int \cos 4x \, dx$

$x + \frac{2\sin 2x}{4} - \frac{\sin 2x}{4} - \frac{x}{2} - \frac{\sin 2x}{4} - x - \frac{\sin 4x}{4}$

$\frac{x}{2} + \frac{\cancel{2\sin 2x + \sin 2x + \sin 2x} - \sin 4x}{4}$ $\quad \frac{x}{2} + \frac{-\sin 4x}{4}$ $\quad \frac{\pi}{2} - \cancel{\frac{\sin 4\pi}{4}}$ 0

$\pi/2$

16. $\int \tan^2 x \cos^3 x \, dx = \int \tan^2 x \cancel{\cos^2} x \cos x \, dx$

$\frac{\sin^2}{\cos^2}$

$\int \sin^2 x \cos x$ $\quad \cancel{u = \cos x} \quad \cancel{du = -\sin x dx}$

$[1-\cos^2 x]^2 \cos x$ $\quad \cancel{[1-u^2]}$ $\quad -\cancel{\int (1-\cos)} \, du$

$u = \sin x$ $\quad du = \cos x \, dx$ $\quad \frac{du}{\cos x} = dx$

$\int u^2 \, du$ $\quad \frac{1}{3}u^3 \rightarrow \frac{1}{3}\sin^3 x$

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26.

$$\int_0^{\pi/4} \sec^6 x \tan^4 x \, dx$$

$$\int \sec^2 x \left(\tan^2 x + 1\right)^2 \tan^2 x \, dx$$

$u = \tan x \quad du = \sec^2 x \quad \frac{du}{\sec^2 x} = dx$

$$\int \left(u^2+1\right)^2 \left(u^4\right) du$$

$$\left(u^4 + 2u^2 + 1\right)\left(u^4\right) \quad \int u^8 + 2u^6 + u^4 \, du$$

$$\int u^8 + 2\int 2u^6 + \int u^4 \, du$$

$$\frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5 \Big|_0^{\pi/4}$$

$$\frac{\tan^9 x}{9} + \frac{2\tan^7 x}{7} + \frac{\tan^5 x}{5}$$

$$\frac{1}{9} + \frac{2}{7} + \frac{1}{5}$$

$$\frac{35}{315} + \frac{90}{315} + \frac{63}{315}$$

$$= \frac{188}{315}$$

28.

$$\int \tan^5 x \sec^3 x \, dx$$

$u = \sec x \quad du = \sec x \tan x$

$\frac{du}{\sec x \tan x} = dx$

$$\cancel{\int \tan^5 x \left(1 - \tan^2 x\right) \sec x \, dx}$$

$$\cancel{\left(\tan^5 x - \tan^7 x\right) \sec x}$$

$$\int \sec^2 x \left(\sec^2 x - 1\right)^2 \sec x \tan x \, dx$$

$$\int u^2 \left(u^2 - 1\right)^2 du$$

$$u^2\left(u^4 - 2u^2 + 1\right) \quad \int u^6 - 2u^4 + u^2$$

$$\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}$$

$$\frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$$

MATH 129

56. a.

$\int \sin x \cos x \, dx$ $\qquad u = \cos x \quad du = -\sin x$

$-\frac{du}{\sin x} = dx$

$-\int u \, du \qquad -\frac{u^2}{2} \rightarrow -\frac{\cos^2 x}{2}$

b. $u = \sin x \quad du = \cos x \, dx \rightarrow \frac{du}{\cos x} = dx \qquad \int u \, du$

$\frac{u^2}{2} \rightarrow \frac{\sin^2 x}{2}$

c.

$\rightarrow \int \frac{\sin 2x \, dx}{2} \quad \frac{1}{2}\int \sin 2x \, dx \qquad u = 2x \qquad du = 2$

$\frac{1}{2}\int \sin u \, du \qquad \frac{-\cos u}{2} \qquad \frac{-\cos 2x}{2}$

$u = \sin x \qquad v = \sin x$

$du = \cos x \qquad dv = \cos x$

$\sin^2 x - \int \sin x \cos x \, dx \rightarrow \sin^2 x - \frac{\sin^2 x}{2}$

answers are 90° apart