3.17 $1 < x, y, z < 1.2$

$D = 2x^2 y \hat{a}_x + 3x^3 y^2 \hat{a}_y \quad C/m^2$

find flux

$$\int_1^{1.2}\int_1^{1.2}\int_1^{1.2} \nabla\cdot\left(2x^2y + 3x^2y^2\right) dx\,dy\,dz$$

$$= \iiint 4xy + 6x^2y \; dx\,dy\,dz$$

$$= 0.2 \iint 4xy + 6x^2y \; dx\,dy$$

$$= 0.2\int \left[4\cdot\frac{x^2}{2}\cdot y\right]\Bigg|_1^{1.2} + \int\left[6\cdot\frac{x^3}{3}\cdot y\right]\Bigg|_1^{1.2} dy$$

$$= \quad 0.88y \quad + \quad 3.456y$$

$$= 0.2\int_1^{1.2} 0.88y + 3.456y \; dy$$

$$= 0.8672 \int_1^{1.2} y\,dy = 0.8672\left[y^2/2\right]\Bigg|_1^{1.2}$$

a) $= \boxed{0.190784\,C} = \Phi$

b) $\nabla\cdot D = 4xy + 6x^2y \Big|_{x=1.1,\,y=1.1} = 4.84 + 7.986$

$= \boxed{12.826\ C}$

c) $Q = \varepsilon_0\cdot\Phi = 1.7E^{-12}\ C = \boxed{1.7pC}$

3.19 $\quad r = 0.003 \quad P(4,1,5) \quad D = x\hat{a}_x \ C/m^2$

$$\phi = \int_{4.997}^{5.003}\int_{0.997}^{1.003}\int_{3.997}^{4.003}$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{0.003} r\sin\theta\cos\phi \cdot r^2\sin\theta \, dr\,d\theta\,d\phi$$

$$= \underbrace{\int_0^{2\pi}\cos\phi}_{0}\ \underbrace{\int_0^{\pi}\sin^2\theta}_{\pi/2}\ \underbrace{\int_0^{0.003} r^3}_{2E^{-11}} \, dr\,d\theta\,d\phi$$

$$= 0$$

3.21

$$D = 1/z^2\left[10xyz\,\hat{a}_x + 5x^2z\,\hat{a}_y + (2z^3 - 5x^2y)\hat{a}_z\right], \quad x=-2,\ y=3,\ z=5$$

$$\nabla\cdot D = 10y/z + 0 + 2 + \frac{10x^2y}{z^3}$$

a) $$= 10(3)/(5) + 2 + \frac{10(4)(3)}{(125)} = 6 + 2 + 0.96 = \boxed{8.96}$$

$$D = 5z^2\,\hat{a}_\rho + 10\rho z\,\hat{a}_z \quad \rho=3,\ \phi=-45^\circ,\ z=5$$

b) $$\nabla\cdot D = 0 + 10\rho = 10(3) = \boxed{30}$$

$$D = 2r\sin\theta\sin\phi\,\hat{a}_r + r\cos\theta\sin\phi\,\hat{a}_\theta + r\cos\phi\,\hat{a}_\phi \quad r=3,\ \theta=45^\circ,\ \phi=-45^\circ$$

$$\nabla\cdot D = 2\sin\theta\sin\phi \cdot -r\sin\theta\sin\phi \cdot -r\sin\phi$$

$$= 2\cdot\left(\tfrac{\sqrt{2}}{2}\right)\cdot\left(\tfrac{2\sqrt{2}}{2}\right)\cdot -(3)\cdot\left(\tfrac{\sqrt{2}}{2}\right)\left(\tfrac{-\sqrt{2}}{2}\right) - (3)\cdot\left(\tfrac{\sqrt{2}}{2}\right)$$

d) $$-1 + 3/4 + \frac{3\sqrt{2}}{2} = \boxed{1.87}$$

3.23 $\text{div } D = 0$ $\quad \text{div } D = \nabla \cdot D$

~~$\hat{a}_x + \hat{a}_y + \hat{a}_z$~~

$$D = \frac{Q}{4\pi r^2}\hat{a}_r$$

$$\nabla \cdot D = \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{Q}{4\pi r^2}\right) = 0 \quad \text{when } r \neq 0$$

undefined otherwise

3.24

$$D = \begin{cases} \rho_0 (z+2d)\hat{a}_z \ C/m^2, & -2d \le z \le 0 \\ -\rho_0 (z-2d)\hat{a}_z \ C/m^2, & 0 \le z \le 2d \end{cases}$$

a)
$$\boxed{\begin{aligned} \nabla \cdot D &= \rho_0, & -2d \le z \le 0 \\ \nabla \cdot D &= -\rho_0, & 0 \le z \le 2d \\ \nabla \cdot D &= 0, & \text{otherwise} \end{aligned}}$$

$$\int_{-b}^{b}\int_{-a}^{a} \nabla \cdot D \, dx\, dy$$
$$= 4ab \cdot (\nabla \cdot D)$$

b)
$$\boxed{\begin{aligned} &= 4ab\rho_0, & -2d \le z \le 0 \\ &= -4ab\rho_0, & 0 \le z \le 2d \\ &= 0, & \text{otherwise} \end{aligned}}$$

$$\int_{-d}^{d}\int_{-b}^{b}\int_{-a}^{a} \nabla \cdot D \, dx\, dy\, dz$$
$$= 6abd(\nabla \cdot D)$$

c), d)
$$\boxed{\begin{aligned} &= 6abd\rho_0, & -2d \le z \le 0 \\ &= -6abd\rho_0, & 0 \le z \le 2d \\ &= 0, & \text{otherwise} \end{aligned}}$$