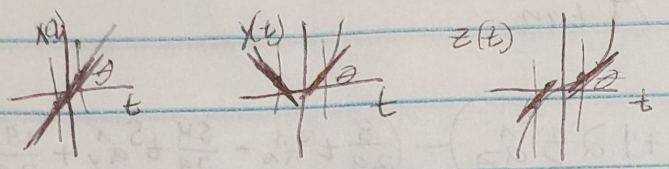
10/11

HW 6

#1.

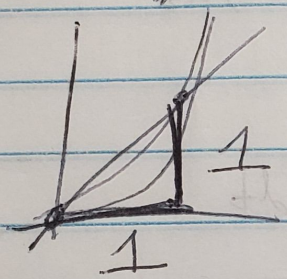


$x = t$ $\quad y = t^2$ $\quad z = t^3$

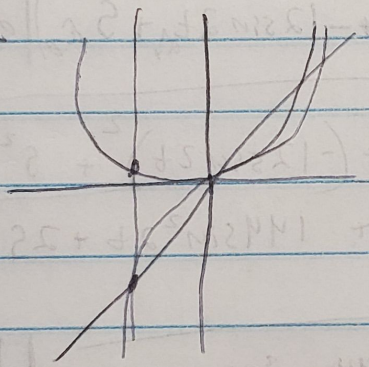
$(1,1)$ $(-1,-1)$ $\quad (1,1)$ $(-1,1)$ $\quad (1,1)$ $(-1,-1)$

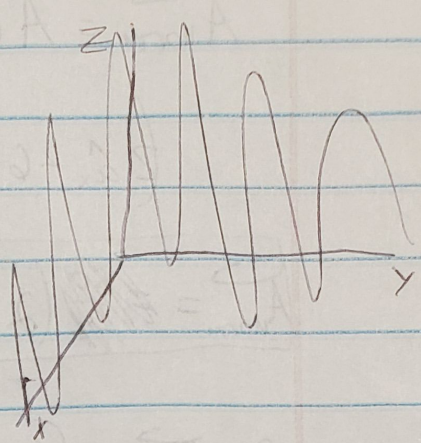
$\frac{d}{dt}[t] = 1$ $\quad \frac{d}{dt}[t^2] = 2t$ $\quad \frac{d}{dt}[t^3] = 3t^2$

$m = 1$ $\quad m = \pm 2$ $\quad m = 3$



$\tan^{-1}(1) = 45°$





$\vec{P}(t) = t\hat{a}_x + t^2\hat{a}_y + t^3\hat{a}_z$

$\vec{V}(t) = \hat{a}_x + 2t\hat{a}_y + 3t^2\hat{a}_z$

#2. $\vec{P}(t) = t^2\hat{a}_x - t^3\hat{a}_y + t^4\hat{a}_z$

$\vec{V}(t) = 2t\hat{a}_x - 3t^2\hat{a}_y + 4t^3\hat{a}_z = \vec{V}_a$

$\vec{V}(1) = 2\hat{a}_x - 3\hat{a}_y + 4\hat{a}_z$

$\vec{V}_b = 8\hat{a}_x - \hat{a}_y + 4\hat{a}_z$

$|\vec{V}_b| = \sqrt{8^2 + (-1)^2 + 4^2} \quad \sqrt{81} = 9$

$|\vec{V}| = \sqrt{2^2 + (-3)^2 + 4^2} \quad \sqrt{4+9+16} = \sqrt{29}$

Scalar Projection $\vec{V}_a$ on $\vec{V}_b$

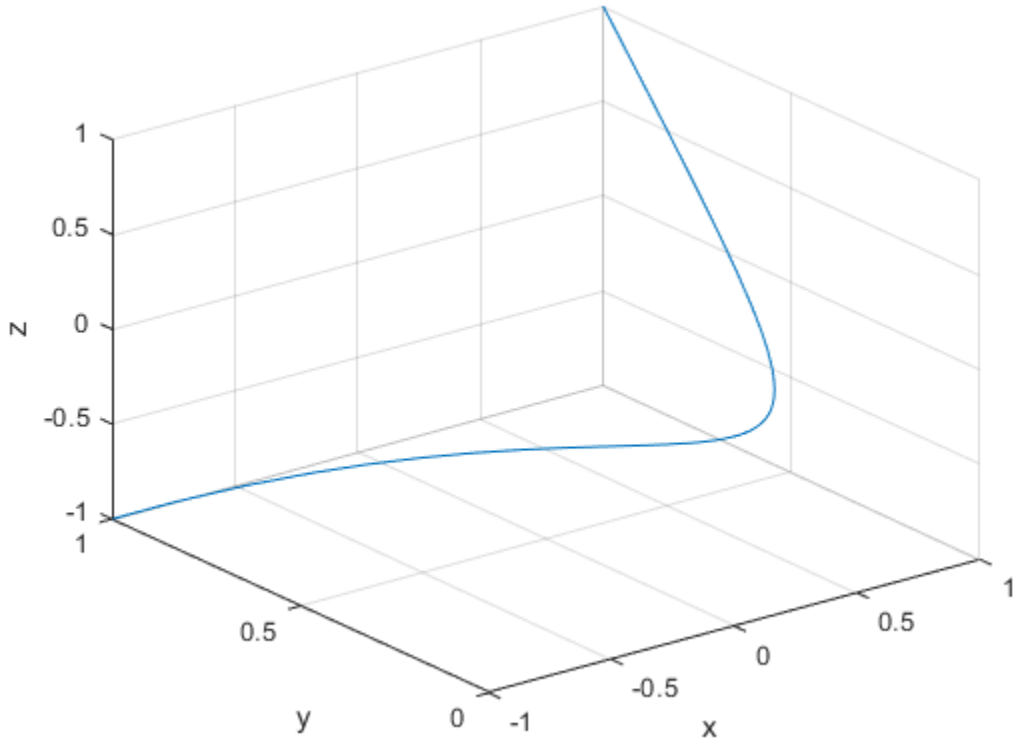
$\frac{16t + 3t^2 + 16t^3}{9}$

$\vec{V}_{sp} = \frac{16}{9}t\hat{a}_x + \frac{1}{3}t^2\hat{a}_y + \frac{16}{9}t^3\hat{a}_z$

$\vec{A}(t) = 2\hat{a}_x - 6t\hat{a}_y + 12t^2\hat{a}_z$

$\frac{\vec{A}(t)\cdot\vec{V}(t)}{|\vec{V}_t|^2}\vec{V}(t) = \frac{4t + 18t^3 + 48t^5}{29}(2t\hat{a}_x - 3t^2\hat{a}_y + 4t^3\hat{a}_z)$

$\vec{A}_{tan} = \frac{8t^2}{29}\hat{a}_x - \frac{54t^5}{29}\hat{a}_y + \frac{192t^8}{29}\hat{a}_z$




```
>> syms t
>> xt=t;
>> yt=t^2;
>> zt=t^3;
>> fplot3(xt,yt,zt,[-1, 1])
>> xlabel('x')
>> ylabel('y')
>> zlabel('z')
```

$$\vec{A}_{norm} = \vec{A}(t) - \vec{A}_{tan}$$

$$\left(2\hat{a}_x - 6t\hat{a}_y + 12t^2\hat{a}_z\right) - \left(\frac{8}{29}t^2\hat{a}_x - \frac{54}{29}t^5\hat{a}_y + \frac{192}{29}t^8\hat{a}_z\right)$$

$$\vec{A}_{norm} = \left(2 - \frac{8}{29}t^2\right)\hat{a}_x + \left(-6 + \frac{54}{29}t^5\right)\hat{a}_y + \left(12 - \frac{192}{29}t^8\right)\hat{a}_z$$

3. $\vec{P}(t) = (6\sin 2t)\hat{a}_x + (6\cos 2t)\hat{a}_y + (5t)\hat{a}_z \quad 0 \le t \le \pi$

$$L = \int_0^{\pi} \left| \frac{d}{dt}\left[\vec{P}(t)\right] \right| dt$$

$$= \int_0^{\pi} \left| (12\cos 2t\,\hat{a}_x - 12\sin 2t\,\hat{a}_y + 5\hat{a}_z) \right| dt$$

$$|\vec{V}_t| = \sqrt{(12\cos 2t)^2 + (-12\sin 2t)^2 + 5^2}$$

$$|\vec{V}_t| = \sqrt{144\cos^2 2t + 144\sin^2 2t + 25}$$

$$L = \int_0^{\pi} \sqrt{144\cos^2 2t + 144\sin^2 2t + 25}\, dt = \boxed{40.84}$$