3.5 -1 $2y(n) + 2y(n-1) = x(n-1)$

$2y(n+1) + 2y(n) = x(n)$

a) $(2E + 2)\,y(n) = x(n)$

$(E + 1)\,y(n) = \frac{1}{2}x(n)$

$\gamma + 1 = 0 \qquad \gamma = -1$

$y_0(n) = C(1)^n = C$

$y(n) = \frac{1}{2}x(n-1) - y(n-1)$

b)

$h(0) = \frac{1}{2}\delta(-1) - h(-1)$

$h(1) = \frac{1}{2}\delta(0) - h(0)$

$h(2) = \frac{1}{2}\delta(1) - h(1)$

$h(3) = \frac{1}{2}\delta(2) - h(2)$

$h(4) = \frac{1}{2}\delta(3) - h(3)$

c) $x(n) = 2u(n)$

$y(0) = u(-1) - y(-1)$

$y(1) = u(0) - y(0)$

$y(2) = u(1) - y(1)$

$y(3) = u(2) - y(2)$

$y(4) = u(3) - y(3)$

3.5-2

a) $y(n+1) - \frac{1}{2}y(n) = 0 \qquad y(-1) = 10$

$y(n) - \frac{1}{2}y(n-1) = 0$

$y(n) = \frac{1}{2}y(n-1)$

$y(0) = \frac{1}{2}y(-1) = 5$

$y(1) = \frac{1}{2}y(0) = 5/2$

$y(2) = \frac{1}{2}y(1) = 5/4$

b) $y(n+1) + 2y(n) = x(n+1) \qquad x(n) = e^{-n}u(n) \qquad y(-1) = 0$

$y(n) + 2y(n-1) = x(n)$

$y(n) = x(n) - 2y(n-1)$

$y(0) = e^{-(0)}u(0) - 2y(-1) = e$

$y(1) = e^{-(1)}u(1) - 2y(0) = \frac{1}{e} - 2e$

$y(2) = e^{-2}u(2) - 2y(1) = \frac{1}{e^2} - 2\left(\frac{1}{e} - 2e\right)$

3.6-1

$$y(n) + \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = \frac{1}{3}x(n)$$

$$y(n+2) + \frac{1}{6}y(n+1) - \frac{1}{6}y(n) = \frac{1}{3}x(n+2)$$

$$\left(E^2 + \frac{1}{6}E - \frac{1}{6}\right)y(n) = \frac{1}{3}E^2 x(n) = 0$$

$y_0(-1) = 3$

$y_0(-2) = -1$

$\gamma_1 = 1/2 \quad \gamma_2 = -1/3$

$$y_0(n) = C_1\left(\tfrac{1}{2}\right)^n + C_2\left(\tfrac{-1}{3}\right)^n$$

$$y_0(-2) = C_1\left(\tfrac{1}{2}\right)^{(-2)} + C_2\left(\tfrac{-1}{3}\right)^{(-2)} = -1$$

$$y_0(-1) = C_1\left(\tfrac{1}{2}\right)^{(-1)} + C_2\left(\tfrac{-1}{3}\right)^{(-1)} = 3$$

$$4C_1 + 9C_2 = -1 \qquad 9C_2 = -1 - 4C_1$$

$$2C_1 - 3C_2 = 3 \qquad 2C_1 = 3C_2 + 3$$

$$C_1 = \frac{3}{2}C_2 + 3/2$$

$$C_2 = \frac{-1}{9} - \frac{4}{9}C_1$$

$$C_2 = \frac{-1}{9} - \frac{4}{9}\left(\frac{3}{2}C_2 + \frac{3}{2}\right)$$

$$C_2 = \frac{-1}{9} - \frac{12}{18}C_2 - \frac{12}{18}$$

$$C_2 + \frac{12}{18}C_2 = \frac{-1}{9} - \frac{12}{18} \qquad \frac{2}{3} = \frac{6}{9}$$

$$C_2\left(1 + \frac{2}{3}\right) = \frac{-1}{9} - \frac{3}{3}$$

$$\frac{5}{3}C_2 = \frac{-7}{9}$$

$$\left[\begin{array}{cc|c} 4 & 9 & -1 \\ 2 & -3 & 3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0.8 \\ 0 & 1 & -0.4667 \end{array}\right]$$

$$y_0(n) = 0.8\left(\tfrac{1}{2}\right)^n - 0.4667\left(\tfrac{-1}{3}\right)^n$$

3.6-2

$$y(n+2) + 3y(n+1) + 2y(n) = 0$$
$$y(n) + 3y(n-1) + 2y(n-2) = 0$$
$$y(0) + 3y(-1) + 2y(-2) = 0$$
$$y(0) + 0 + 2 = 0$$
$$y(0) = -2$$

$$(E^2 + 3E + 2)\,y(n) = 0$$
$$(E+1)(E+2)\,y(n) = 0 \qquad \gamma_1 = -1 \quad \gamma_2 = -2$$

$$y_0(n) = C_1(-1)^n + C_2(-2)^n$$
$$y_0(-1) = C_1(-1)^{-1} + C_2(-2)^{-1} = 0$$
$$= -C_1 - C_2\left(\tfrac{1}{2}\right) = -C_1 - \tfrac{1}{2}C_2 = 0$$
$$y_0(-2) = C_1(-1)^{-2} + C_2(-2)^{-2} = 1$$
$$= C_1(1) + C_2\left(\tfrac{1}{4}\right) = C_1 + \tfrac{1}{4}C_2 = 1$$

$$\left[\begin{array}{cc|c} -1 & -\frac{1}{2} & 0 \\ 1 & 1/4 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -4 \end{array}\right]$$

$$\boxed{y_0(n) = 2(-1)^n - 4(-2)^n}$$

```
daily_cases = [2034 2671 1460 1371 1556 1807 1738 1924 2611 1590 1386 1385 1829 1816
1837 2108 1379 1340];

x = flip(daily_cases);

y = [];
for n = 6:18
    y(n) = (x(n - 1) + x(n - 2) + x(n - 3) + x(n - 4) + x(n - 5)) / 5;
end

figure;
hold on
plot(y(6:end));
hold on
bar(x(5:end));
hold off

xlabel('days since 9/24/21');
ylabel('new cases');
title('cases in Austria');
```