$$\left(9(R\sin\theta\cos\phi) + R\sin\theta\sin\phi + 4R\cos\theta\right)dV$$

$$\left(9R^3\sin^2\theta\cos\phi + R^3\sin^2\theta\sin\phi + 4R^3\sin\theta\cos\theta\right)dR\,d\theta\,d\phi$$

1.

$\vec{V} = 3x\hat{a}_x - y\hat{a}_y + 2z\hat{a}_z$

$\vec{\nabla}\cdot\vec{V} = 3 - 1 + 2 = 4$

$dV = R^2\sin\theta\,dR\,d\theta\,d\phi$

$$4\int_0^{2\pi}\int_0^{\pi/2}\int_0^{2} R^2\sin\theta\,dR\,d\theta\,d\phi$$

$$\frac{32}{3}\int_0^{2\pi}\int_0^{\pi/2}\sin\theta\,d\theta\,d\phi$$

$$\frac{32}{3}\int_0^{2\pi}d\phi = \boxed{\frac{64\pi}{3}}$$

$$16\int_0^{2\pi}\int_0^{\pi/2}\int_0^{2}\sin\theta\,dR\,d\theta\,d\phi$$

$$= 32\int_0^{2\pi}\int_0^{\pi/2}\sin\theta\,d\theta\,d\phi$$

2. $\vec{V} = 2xy\hat{a}_x + 2yz\hat{a}_y + 2xz\hat{a}_z$

$dV = dx\,dy\,dz$

$\vec{\nabla}\cdot\vec{V} = 2y + 2z + 2x$

$2\iiint y\,dV + 2\iiint z\,dV + 2\iiint x\,dV$

$2\int_0^1\int_0^1\int_0^1 y\,dx\,dy\,dz$

$= \int_0^1\int_0^1 dx\,dz = 1$

$2\int_0^1\int_0^1\int_0^1 z\,dx\,dy\,dz$

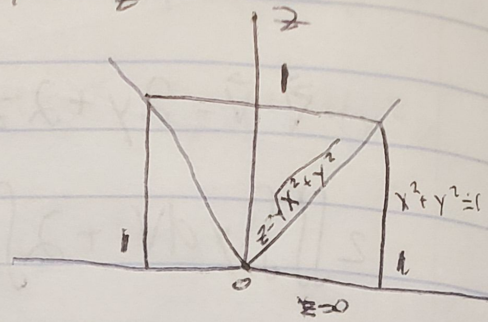
$= \int_0^1\int_0^1 dx\,dy = 1$

$2\int_0^1\int_0^1\int_0^1 x\,dx\,dy\,dz$

$= \int_0^1\int_0^1 dy\,dz = 1$

$1+1+1 = \boxed{3}$

3. $\vec{V} = (6x+y)\hat{a}_x + (-x-z)\hat{a}_y + 4yz\hat{a}_z$

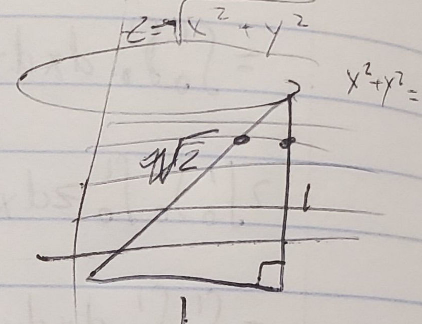


$\nabla \cdot \vec{V} = 6 + 0 + 4y = 4y + 6$

$\iiint (4y+6)\,dV$

$dV = r\,dr\,d\phi\,dz$ $\quad x = r\cos\phi \quad y = r\sin\phi \quad z = z$

$\int_0^1 \int_0^{\pi/4} \int_0 (4r\sin\phi + 6)\, r\,dr\,d\phi\,dz$



$z = \sqrt{r^2\cos^2\phi + r^2\sin^2\phi}$

$z = \sqrt{(\cos^2\phi + \sin^2\phi)r^2}$

$x^2+y^2 - \sqrt{x^2+y^2}$ $\quad z = r$

$1 = r^2\cos^2\phi + r^2\sin^2\phi$

$1 = r^2(\cos^2\phi + \sin^2\phi)$

$1 = r^2 = r$

$4\int_0^1 \int_0^{\pi/4} \int_z^1 r^2 \sin\phi\, dr\, d\phi\, dz + 6\int_0^1 \int_0^{\pi/4} \int_z^1 r\,dr\,d\phi\,dz$

$\left.\frac{r^3}{3}\right|_z^1 = \frac{1}{3} - \frac{z^3}{3}$ $\qquad \frac{3\pi}{2}\int_0^1 \int_z^1 r\,dr\,dz = \left.\frac{r^2}{2}\right|_z^1$

$= \frac{1}{2} - \frac{z^2}{2}$

$4\int_0^{\pi/4} \int_0^1 \frac{1}{3} - \frac{z^3}{3}\,dz \rightarrow \frac{4}{3}\int_0^{\pi/4}\int_0^1 (1-z^3)\,dz$

$\int_0^{\pi/4} \sin\phi\, d\phi = 1 - \frac{1}{\sqrt{2}}$ $\qquad \left.\left(z - \frac{z^4}{4}\right)\right|_0^1 = \left(1-\frac{1}{4}\right) - \left(0 - \frac{0}{4}\right) = 3/4$

$\frac{3}{2}\pi \int_0^1 \frac{1}{2} - \frac{z^2}{2}\,dz$

$\boxed{1 - \frac{1}{\sqrt{2}} + \frac{9\pi}{8}}$ $\qquad \frac{3\pi}{4}\int_0^1 1 - z^2\,dz = \left. z - \frac{z^3}{3}\right|_0^1$

$3\pi/4\,(3/4) = \frac{9\pi}{8}$