MATH 527

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2/6
Section 2

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$ sol: $y(x) = e^{3x}$

$r^2e^{rx} - re^{rx} - 6e^{rx} = 0$ interval: $-2 < x < 3$

$e^{rx}(r^2 - r - 6) = 0$

$e^{rx}(r-3)(r+2) = 0$ $r = -2, 3$ $y(x) = e^{rx}$

2. $x^2\frac{d^2y}{dx^2} - 2y = 0$ sol: $y(x) = x^2$

$\frac{y}{y}\frac{d^2y}{dx^2} - \frac{2y}{y} = \frac{0}{y}$

$\frac{d^2y}{dx^2} - 2 = 0$ $\frac{e^{rx}(r^2 - \frac{2}{e^{rx}}) = 0}{e^{rx}}$

$r = \pm\sqrt{\frac{2}{e^{rx}}}$ $e^{rx}\left(r + \sqrt{\frac{2}{e^{rx}}}\right)\left(r - \sqrt{\frac{2}{e^{rx}}}\right) = 0$

3. $\frac{dy}{dt} = (1+t)(1+y)$ $\frac{dy}{dt} = 1^2 + ty + t + y$

$y' + p(t)y = f(x)$ $y' - ty = ty + 1$ $\frac{1}{y}\frac{dy}{dt} - 1 = t + \frac{1}{y}$

$ty + y = y(1+t)$

$\frac{1}{y}\frac{dy}{dt} - \frac{1}{y} - 1 = t$ $\frac{dy}{dt} - 1 - y = ty$

$y^{-1}y' - y^{-1} = t + 1$ $y' - 1 = y(t+1)$ $y' - ty - y - 1 = 0$

$y' - 1 = ty + y$

4. $y' = 1 - t + y^2 - ty^2$ $y' + ty^2 - y^2 = 1 - t$ $y' + y^2(t-1)$

$P(t)$

$r(t) = e^{\int(t-1)dt}$ $e^{\frac{t^2}{2} - t}$ $r(t) = e^{\frac{t^2}{2} - t}$

$y'e^{\frac{t^2}{2} - t} + y(t-1)e^{\frac{t^2}{2} - t} = (1-t)e^{\frac{t^2}{2} - t}$

$y' + \frac{d}{dt}\left[y e^{\frac{t^2}{2} - t}\right] = (1-t)e^{\frac{t^2}{2} - t}$

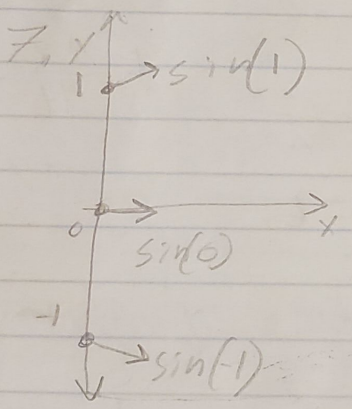
$ye^{\frac{t^2}{2} - t} = \int(1-t)e^{\frac{t^2}{2} - t}dt = \int e^{\frac{t^2}{2} - t}dt - \int te^{\frac{t^2}{2} - t}dt$

5. $y' = e^{x|y|3}$ I don't know what "$x|y|3$" means

6. $\frac{dy}{dt} = \frac{2t}{y+yt^2}$ $y(2) = 3$

$$\int \frac{dy}{dt}\,dt = \int \frac{2t}{y+yt^2}\,dt$$

7.



8. $t=0$ $y(0)=184$ $t=2$ $y(2)=403$

a. $184 = Ce^{k0}$ $C=184$ $403 = 184e^{k2}$

$\frac{403}{184} = e^{2k}$ $\frac{1}{2}\ln\left(\frac{403}{184}\right) = k = 0.39$

b. $y(27) = 184e^{0.39(27)} \approx 6,885,551$

c. $328,000,000 = 184e^{0.39(37)}$ $t = 37$

d. no, ~~measure~~ way too many factors (restrictions, pop. density)