ECE 634 Hunter W., Nick S., Andrew P.

Homework 10

5.3-20

Find the following sum $\sum_{k=0}^{n} k^3$

▷ $y(n) = \sum_{k=0}^{n} k^3$ ▷ $y(n) - y(n-1) = n^3$, $y(0) = 0$ ▷ $y(-1) = 0$

$(E-1)y(n) = n^3$ ▷ $\mathcal{Z}\{y(n)\}$ ▷ $z-1\,Y(z) = \frac{z(z^2+4z+1)}{(z-1)^4}$

▷ $\frac{z-1}{z} Y(z) = \frac{z(z^2+4z+1)}{(z-1)^4} = \frac{z(z^2+4z+1)}{(z-1)^5}$ ▷ PFD ▷ $\frac{1}{(z-1)^2} + \frac{7}{(z-1)^3} + \frac{12}{(z-1)^4} + \frac{6}{(z-1)^5}$

▷ $Y(z) = \frac{z}{(z-1)^2} + \frac{7z}{(z-1)^3} + \frac{12z}{(z-1)^4} + \frac{6z}{(z-1)^5}$

▷ $y(n) = n + \frac{7}{2!}n(n-1) + \frac{12}{3!}n(n-1)(n-2) + \frac{6}{4!}n(n-1)(n-2)(n-3)$

$y(n) = n + \frac{7}{2}n(n-1) + 2n(n-1)(n-2) + \frac{1}{4}n(n-1)(n-2)(n-3)$

5.3-21. Find the following sum $\sum_{k=0}^{n} k a^k$ $a \neq 1$

▷ $y(n) - y(n-1) = na^n$, $y(0) = 0$ ▷ $y(-1) = 0$ ▷ $\mathcal{Z}\{y(n)\}$

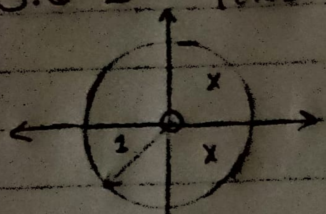
▷ $z-1\,Y(z) = \frac{az}{(z-a)^2}$ ▷ $\frac{z-1}{z} Y(z) = \frac{az}{(z-a)^2}$ ▷ $\frac{Y[z]}{z} = \frac{az}{(z-1)(z-a)^2}$

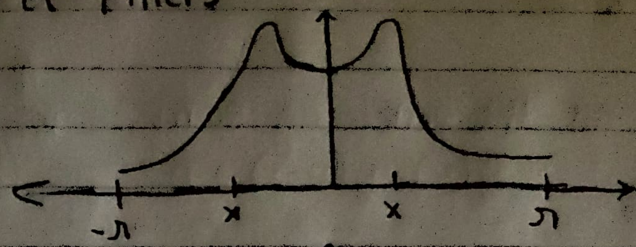
▷ $\frac{Y(z)}{z} = \frac{A}{(z-1)} + \frac{B}{(z-a)} + \frac{C}{(z-a)^2} = \frac{a/(a-1)^2}{z-1} - \frac{a/(a-1)^2}{z-a} + \frac{a^2/a-1}{(z-a)^2}$

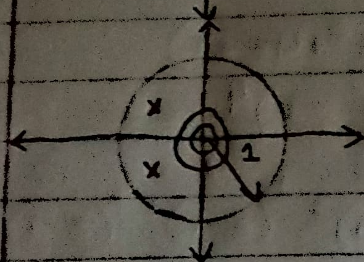
▷ $Y(z) = \frac{a}{(a-1)^2}\left[\frac{z}{z-1} - \frac{z}{z-a} + \frac{(a-1)az}{(z-a)^2}\right]$ ▷ $\mathcal{Z}^{-1}\{Y(z)\}$

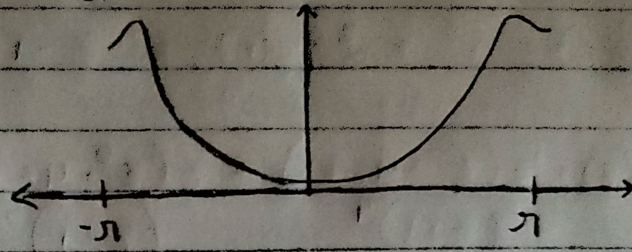
▷ $y(n) = \frac{a}{(a-1)^2}\left[1 - a^n + (a-1)na^n\right]u(n)$

5.6-1 Poles & Zero Config of Filters







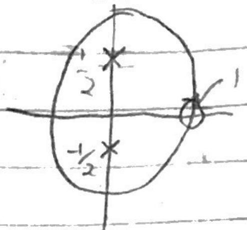


ECE 634

Andrew Pr[illegible] hw10 4/22

P. 5.6-2, 5.6-3

5.6-2) System a.)



find constants and $H(z)$

Poles: $\pm\frac{1}{2}j$

zero: 1

$$H(z) = \frac{z-1}{\underbrace{(z-\frac{1}{2}j)(z+\frac{1}{2}j)}} = \frac{z-1}{z^2+\frac{1}{4}}$$

constant

$$z^2 - \tfrac{1}{2}jz + \tfrac{1}{2}jz + \tfrac{1}{4} = z^2 + \tfrac{1}{4}$$

$$H(z) = \frac{b(z-1)}{z^2+\frac{1}{4}} \rightarrow H(-1) \rightarrow -1 = \frac{b(-1-1)}{1+\frac{1}{4}}$$

$$-1 = \frac{-2b}{5/4} \rightarrow -\frac{5}{4} = -2b \rightarrow b = \frac{5}{8}$$

Now look at $H(z) = \dfrac{b_0 z^2 + b_1 z + b_2}{a_0 z^2 + a_1 z + a_2}$

Using $\dfrac{(5/8)z - 5/8}{z^2 + 1/4}$

$$\boxed{\begin{matrix} b_0 = 0, & b_1 = \frac{5}{8}, & b_2 = 5/8 \\ a_0 = 1, & a_1 = 0, & a_2 = 1/4 \end{matrix}}$$

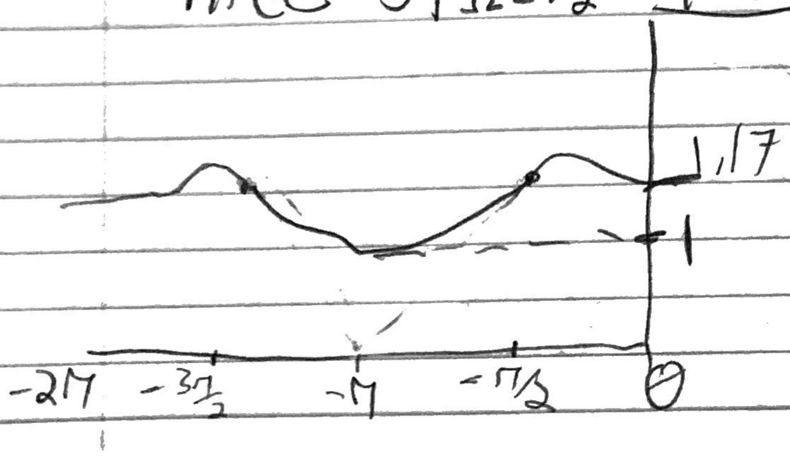
b.) hand sketch the magnitude Response $|H(e^{j\Omega})|$

$$H(e^{j\Omega}) = \frac{\frac{5}{8}e^{j\Omega} - 5/8}{(e^{j\Omega})^2 + 1/4} = \sqrt{\frac{(5/8 e^{j\Omega} - 5/8)^2}{((e^{j\Omega})^2 + \frac{1}{4})^2}}$$

Using ~~la~~ matlab we find

$-2\pi \leq \Omega \leq 0$

$$|H(e^{j\Omega})|_{\Omega=\pi/2} = \underline{1.17}$$



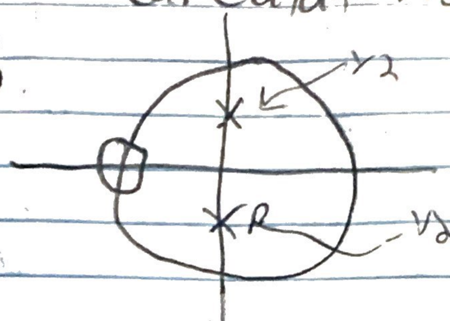
C.) $y(n)$ if $x(n) = \sin(\frac{\pi n}{2})$

$$\angle H(e^{j\pi/2}) = -\frac{\pi}{2} - \frac{\pi}{2} + \frac{3\pi}{4} + 0 = -\frac{\pi}{4}$$

$$x(n) = \sin(\frac{\pi n}{2}) \rightarrow y(n) = |\text{[illegible]}| \sin(\frac{\pi n}{2} - \frac{\pi}{4})$$

Once again using matlab for calculations.

5.6-3



$H(1) = -1$

a.)

$$H(z) = \frac{b(z+1)}{(z^2 + 1/4)}$$

← Same as last time except for the zero which gets turned positive when expanded

$$H(1) = \frac{b(1+1)}{1 + 1/4} = \frac{2b}{5/4} = -1$$

$$2b = -\frac{5}{4} \rightarrow b = -\frac{5}{8}$$

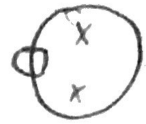
if $H(z)$ $\frac{b_0 z^2 + b_1 z + b_2}{a_0 z^2 + a_1 z + a_2}$

$b_0 = 0$ $\qquad$ $a_0 = 1$

$b_1 = -5/8$ $\qquad$ $a_1 = 0$

$b_2 = -5/8$ $\qquad$ $a_2 = 1/4$

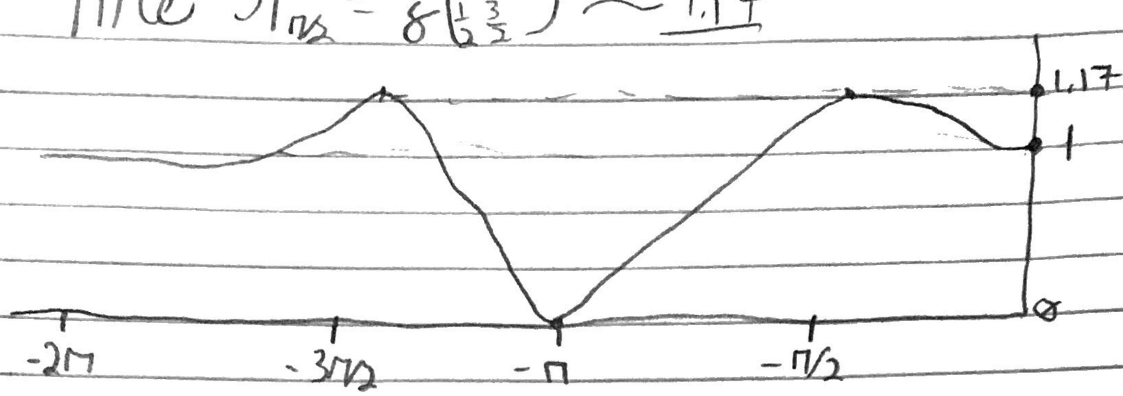
$$H(z) = \frac{0 + (-5/8 z) - 5/8}{1z^2 + 0 + \frac{1}{4}}$$



b) $\Omega_0 \to 1$ $\quad$ $\Omega_{\pi} \to \emptyset$

Once again using matlab for $|H(e^{j\Omega})|$ at $\Omega = \pi/2$

$$|H(e^{j\Omega})|_{\pi/2} = \frac{5}{8}\left(\frac{\sqrt{2}}{\frac{1}{2}\,\frac{3}{2}}\right) \approx \underline{1.17}$$



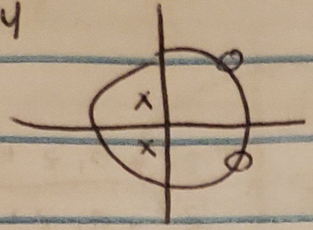
c) Using Matlab to compute response to $\sin(\frac{\pi n}{2})$

$$\angle H(e^{j\pi/2}) \to -\pi + \frac{5\pi}{4} = \frac{\pi}{4}$$

$$\underline{y(n) = 1.17 \sin\left(\frac{\pi n}{2} + \frac{\pi}{4}\right)}$$

Makes sense since zero sign flipped.

5.6-4



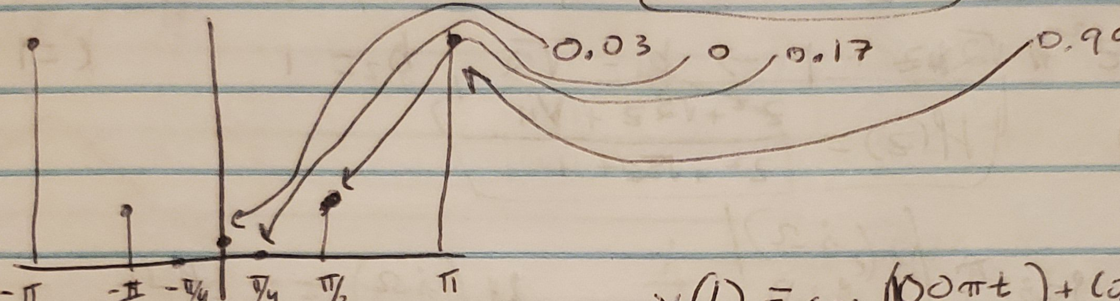
$$H(z) = K\frac{z^2+b_1z+b_2}{z^2+a_1z+a_2}$$

$$= k\frac{(z-e^{j\pi/4})(z-e^{-j\pi/4})}{(z-e^{j\frac{3\pi}{4}})(z-e^{-j\frac{3\pi}{4}})}$$

a) $= \boxed{K\frac{z^2-\sqrt{2}z+1}{z^2+\sqrt{2}z+1}}$ $H(-1) = k\frac{1+\sqrt{2}+1}{1-\sqrt{2}+1} = 1$

$\boxed{k \approx 0.17}$

b)



$x(t) = \cos(100\pi t) + \cos(500\pi t)$

$\boxed{F_s = 1500\ Hz}$ $x(n) = \cos\left(\frac{1}{15}\pi n\right) + \cos\left(\frac{1}{3}\pi n\right)$

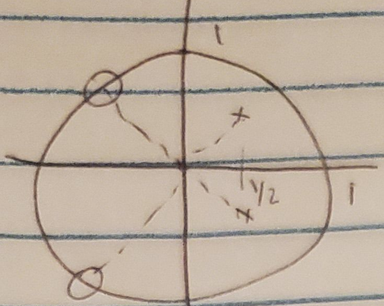
c) $X(z) = \frac{z^2 - z/2}{z^2-z+1} + \frac{z^2 - z\cos\left(\frac{\pi}{15}\right)}{z^2-2z\cos\left(\frac{\pi}{15}\right)+1}$

$Y(z) = 0.17\left(\frac{z^2-\sqrt{2}z+1}{z^2+\sqrt{2}z+1}\right)\left(\frac{z^2-z/2}{z^2-z+1} + \frac{z^2-z\cos\left(\frac{\pi}{15}\right)}{z^2-2z\cos\left(\frac{\pi}{15}\right)+1}\right)$

d) $h(n) = 0.17u(-n) - (0.68i)2^{-\frac{n}{2}-1}\left((-1-i)^n - (-1+i)^n\right)(1-u(-n))$

$\boxed{\text{no, because there are complex numbers}}$

5.6-5



$$H(z) = \frac{\left(z - e^{j\frac{3\pi}{4}}\right)\left(z - e^{-j\frac{3\pi}{4}}\right)}{\left(z - \sqrt{2}e^{j\pi/4}\right)\left(z - \sqrt{2}e^{-j\pi/4}\right)}$$

$$= \frac{z^2 + \sqrt{2}z}{z^2 - 4z + 2}$$

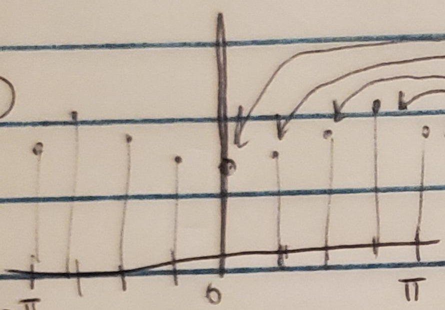
$$H(1) = K\frac{1+\sqrt{2}}{1-4+2} = -1 \rightarrow K(-(2+\sqrt{2})) = -1 \quad K(-2-\sqrt{2}) = -1$$

$$K = \sqrt{2} - 1 \approx 0.41$$

a) $$\boxed{H(z) = (\sqrt{2}-1)\frac{z^2+\sqrt{2}z}{z^2-4z+2}} \qquad H(e^{j\Omega}) = (\sqrt{2}-1)\frac{e^{2j\Omega} + \sqrt{2}e^{j\Omega}}{e^{2j\Omega} - 4e^{j\Omega} + 2}$$

0.14 0.15 0.17 0.21 0.17

b)



$$x(t) = \cos(2\pi f t) \qquad F_s = 1000 \text{ Hz}$$

$$x[n] = \cos(2\pi f n/1000)$$

c) $$X(z) = \frac{z\left(z - \cos\left(\frac{\pi f}{500}\right)\right)}{-2z\cos\left(\frac{\pi f}{500}\right) + z^2 + 1}$$

$$Y(z) = \frac{\sqrt{2}z^4 - \sqrt{2}z^3\cos\left(\frac{\pi f}{500}\right) + 2z^3 - 2z^2\cos\left(\frac{\pi f}{500}\right) - z^4 + z^3\cos\left(\frac{\pi f}{500}\right) - \sqrt{2}z^3 + \sqrt{2}z^2\cos\left(\frac{\pi f}{500}\right)}{-2z^3\cos\left(\frac{\pi f}{500}\right) + z^4 + 3z^2 - 4z^3 + 8z^2\cos\left(\frac{\pi f}{500}\right) - 4z\cos\left(\frac{\pi f}{500}\right) - 4z + 2}$$

$$Y(z)\Big|_{f=500} = \frac{\sqrt{2}z^4 + 2z^3 - z^4 - \sqrt{2}z^3}{z^4 + 3z^2 - 4z^3 - 4z + 2}$$