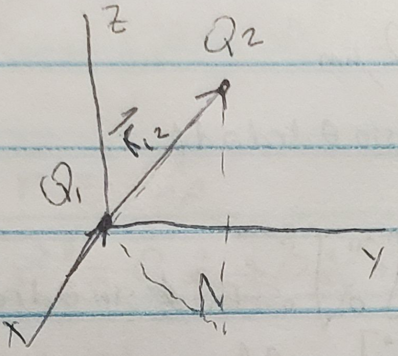
2.2



$Q_1 = 1nC \; @ (0,0,0)$

$Q_2 = -2nC \; @ (1,1,1)$

$\vec{r}_1 = 0 \qquad \vec{r}_2 = 1\hat{a}_x + 1\hat{a}_y + 1\hat{a}_z$

$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1 = -1\hat{a}_x - 1\hat{a}_y - 1\hat{a}_z$

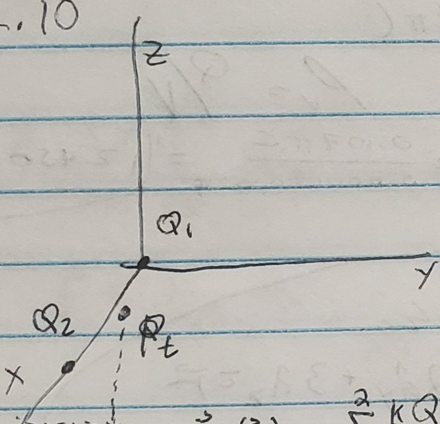
$\vec{F}_2 = k\dfrac{Q_1 Q_2}{\vec{R}_{12}^{\,2}}\hat{a}_{12} \qquad \hat{a}_{12} = \vec{R}_{12}/|\vec{R}_{12}| = -1/\sqrt{3}\,\hat{a}_x - 1/\sqrt{3}\,\hat{a}_y - 1/\sqrt{3}\,\hat{a}_z$

$|\vec{R}_{12}| = \sqrt{(-1)^2 + (-1)^2 + (-1)^2} = \sqrt{3}$

$= kQ_1Q_2\left(-1/3\sqrt{3}\,\hat{a}_x - 1/3\sqrt{3}\,\hat{a}_y - 1/3\sqrt{3}\,\hat{a}_z\right)$

$= \boxed{3.46\hat{a}_x + 3.46\hat{a}_y + 3.46\hat{a}_z \quad nN}$

2.10



$Q_1 = -1nC \; @ (0,0,0)$

$Q_2 = ? \; @ (2,0,0)$

$P = 0 \; @ (3,1,1)$

$\vec{r}_1 = 0\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$

$\vec{r}_2 = 2\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$

$\vec{r} = 3\hat{a}_x + 1\hat{a}_y + 1\hat{a}_z$

$\vec{E}(\vec{r}) = \sum_{m=1}^{2} \dfrac{kQ_m}{|\vec{r}-\vec{r}_m|^2}\hat{a}_m \qquad \hat{a}_m = \dfrac{\vec{r}-\vec{r}_m}{|\vec{r}-\vec{r}_m|}$

$0 = \dfrac{kQ_1}{|\vec{r}|^2}\hat{a}_1 + \dfrac{kQ_2}{|\vec{r}-\vec{r}_2|^2}\hat{a}_2 \rightarrow \dfrac{-kQ_1}{|\vec{r}|^2}\hat{a}_1 = \dfrac{kQ_2}{|\vec{r}-\vec{r}_2|^2}\hat{a}_2$

$\cancel{0\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z = kQ_1} \qquad Q_2 = \dfrac{-Q_1|\vec{r}-\vec{r}_2|^2\hat{a}_1}{|\vec{r}|^2\,\hat{a}_2}$ 3

11

$\vec{r} - \vec{r}_1 = \vec{r} \qquad |\vec{r}| = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{11} \qquad \hat{a}_1 = 3/\sqrt{11}\,\hat{a}_x + 1/\sqrt{11}\,\hat{a}_y + 1/\sqrt{11}\,\hat{a}_z$

$\vec{r} - \vec{r}_2 = 1\hat{a}_x + 1\hat{a}_y + 1\hat{a}_z \qquad |\vec{r}-\vec{r}_2| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\hat{a}_2 = \frac{1}{\sqrt{3}}\hat{a}_x + \frac{1}{\sqrt{3}}\hat{a}_y + \frac{1}{\sqrt{3}}\hat{a}_z$

$Q_2 = \dfrac{(1nC)(3)\left(\frac{1}{\sqrt{3}}\hat{a}_x + \frac{1}{\sqrt{3}}\hat{a}_y + \frac{1}{\sqrt{3}}\hat{a}_z\right)}{(11)\left(3/\sqrt{11}\,\hat{a}_x + 1/\sqrt{11}\,\hat{a}_y + 1/\sqrt{11}\,\hat{a}_z\right)} = \dfrac{(3nC)}{11}\cdot\left(0.64\hat{a}_x + 1.91\hat{a}_y + 1.91\hat{a}_z\right)$

$3/11\,(0.69 + 1.91 + 1.91)(nC) = \boxed{1.22\,nC}$

2.15 $\rho_v = 10E^{15}$ C/m^3 $r = 2\ \mu m$

$$Q_{enclosed} = \underbrace{\int_0^{2\pi}\int_0^{\pi}\int_0^{2\mu m}}_{\text{uniform}} 10E^{15}\ r^2 \sin\theta\, dr\, d\theta\, d\phi$$

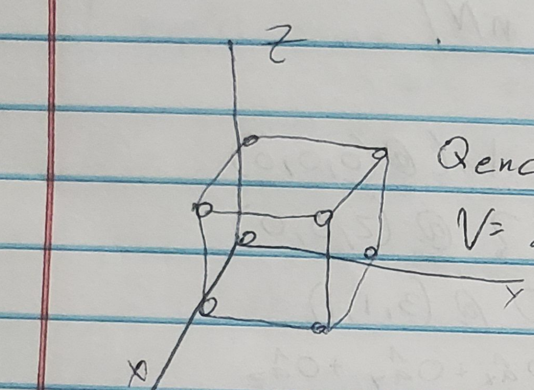
$$= 20\pi E^{15}\int_0^{\pi}\int_0^{2\mu m} r^2 \sin\theta\, dr\, d\theta$$

$$= 20\pi E^{15}\int_0^{\pi}\left[\frac{r^3}{3}\Big|_0^{2\mu m}\right]\sin\theta\, d\theta$$

$$= 53.33\pi E^{-3}\underbrace{\int_0^{\pi}\sin\theta\, d\theta}_{2}$$

$$= 106.667\pi E^{-3}$$

a) $= \boxed{0.107\pi\ C}$



$Q_{enc} = 0.107\pi C$

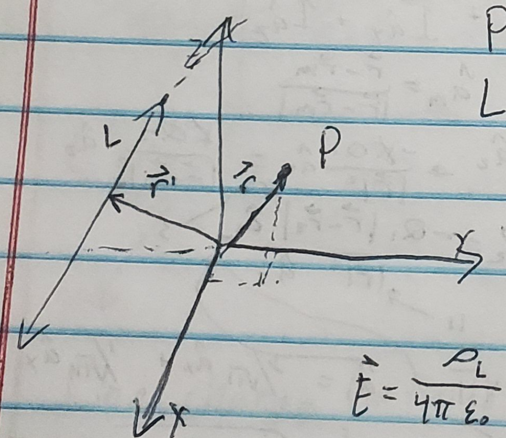
$V = 27 nm^3$ $\rho_v = Q/V$

$$\rho_v = \frac{0.107\pi C}{0.000000000027} = \boxed{12450015.33\ C/m^3}$$

2.17 $Q = 16 nC/m$ $\varepsilon = \varepsilon_0$

$P = 1\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z = \vec{r}$

$L = \cancel{x}\hat{a}_x - 2\hat{a}_y + 5\hat{a}_z = \vec{r}'$



$$\vec{E} = \int_{-\infty}^{\infty}\frac{dQ}{4\pi\varepsilon_0}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \quad dQ = \rho_L\, dx$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}dx \quad \vec{r}-\vec{r}' = (1-x)\hat{a}_x + 4\hat{a}_y - 2\hat{a}_z$$

$$|\vec{r}-\vec{r}'| = \sqrt{(1-x)^2 + (4)^2 + (-2)^2} = \sqrt{x^2 - 2x + 1 + 20}$$

$$|\vec{r}-\vec{r}'|^3 = (x^2 - 2x + 21)^{3/2}$$

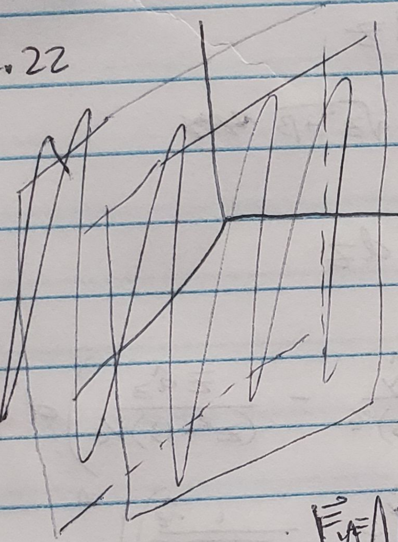
$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\left(\frac{1-x}{(x^2-2x+21)^{3/2}}\hat{a}_x + \frac{4}{(x^2-2x+21)^{3/2}}\hat{a}_y + \frac{-2}{(x^2-2x+21)^{3/2}}\hat{a}_z\right)dx$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\left(\underbrace{\int_{-\infty}^{\infty}\frac{1-x}{(x^2-2x+21)^{3/2}}dx\,\hat{a}_x}_{\frac{1}{\sqrt{x^2-2x+21}}\Big|_{-\infty}^{\infty}} + \underbrace{\int_{-\infty}^{\infty}\frac{4}{(x^2-2x+21)^{3/2}}dx\,\hat{a}_y}_{4\left(\frac{x-1}{20\sqrt{x^2-2x+21}}\Big|_{-\infty}^{\infty}\right)} + \underbrace{\int_{-\infty}^{\infty}\frac{-2}{(x^2-2x+21)^{3/2}}dx\,\hat{a}_z}_{-2\left(\frac{x-1}{20\sqrt{x^2-2x+21}}\Big|_{-\infty}^{\infty}\right)}\right)$$

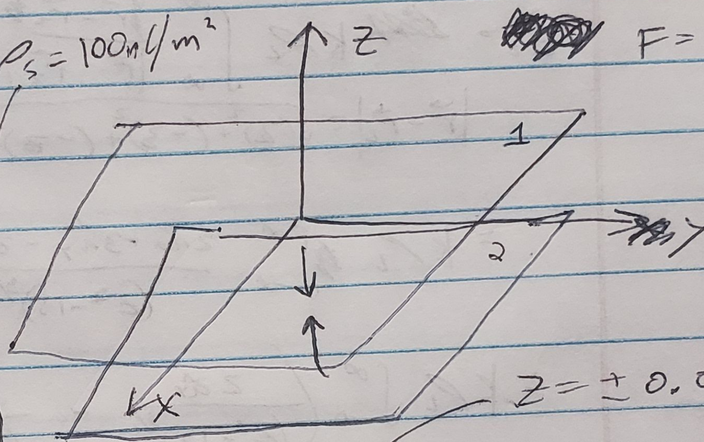
$$144 \qquad 0\,\hat{a}_x \qquad 0.4\,\hat{a}_y \qquad -0.2\,\hat{a}_z$$

$$\boxed{\vec{E} = 0\,\hat{a}_x + 57.6\,\hat{a}_y - 28.8\,\hat{a}_z \ \text{nC/m}}$$

2.22



$\rho_s = 100\,\text{nC/m}^2$



$F = ?$

$Z = \pm 0.02\,\text{m}$

$\hat{a}_z$

$\hat{a}_{12}$

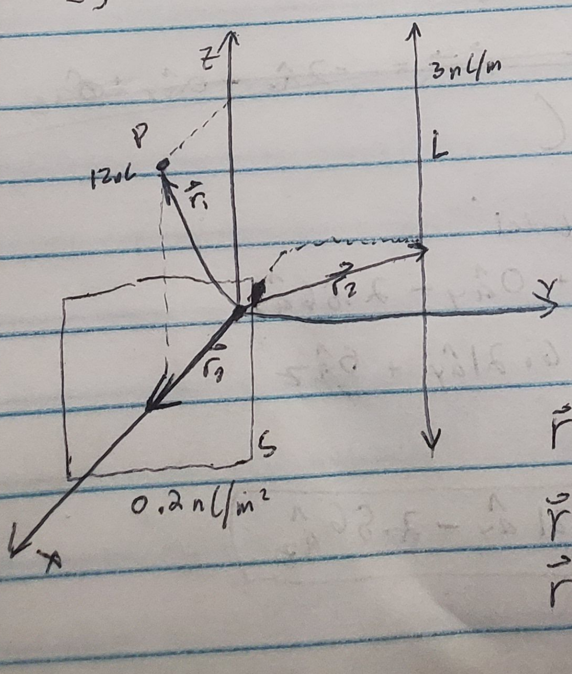
x^2

$A = \infty\,\text{m}^2 \qquad Q = \infty \cdot 100\,\text{nC/m}^2$

$\rho_s = \frac{Q}{A}$

$\vec{F} = k\frac{Q_1 Q_2}{\vec{R}_{12}}\hat{a}_{12} \qquad Q_1 = Q_2 = \infty \qquad \boxed{\vec{F} = \infty\,\text{N}}$

2.25



$\vec{r} = 0\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$

$\vec{r}_1 = 2\hat{a}_x + 0\hat{a}_y + 6\hat{a}_z,\ 12\,\text{nC}$

$\vec{r}_2 = -2\hat{a}_x + 3\hat{a}_y + z\hat{a}_z,\ 3\,\text{nC/m}$

$\vec{r}_3 = 2\hat{a}_x + \cancel{y}^{0}\hat{a}_y + \cancel{z}^{0}\hat{a}_z,\ 0.2\,\text{nC/m}^2$

$\vec{r} - \vec{r}_1 = -2\hat{a}_x + 0\hat{a}_y - 6\hat{a}_z$

$\vec{r} - \vec{r}_2 = 2\hat{a}_x - 3\hat{a}_y - z\hat{a}_z$

$\vec{r} - \vec{r}_3 = -2\hat{a}_x - 0\hat{a}_y - 0\hat{a}_z$

$$\vec{E}(\vec{r}) = K\, Q_1 / |\vec{r}-\vec{r}_1|^2 \ \hat{a}_1$$

$$|\vec{r}-\vec{r}_1| = \sqrt{(-2)^2+(0)^2+(-6)^2} = \sqrt{40}$$

$$\hat{a}_1 = \frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|}$$

$$\vec{E}_1 = (k\,Q_1/40)\left(-2/\sqrt{40}\,\hat{a}_x + 0\,\hat{a}_y - 6/\sqrt{40}\,\hat{a}_z\right)$$

$$= 2.7\left(-2/\sqrt{40}\,\hat{a}_x + 0\,\hat{a}_y - 6/\sqrt{40}\,\hat{a}_z\right)$$

$$= -0.85\,\hat{a}_x + 0\,\hat{a}_y - 2.56\,\hat{a}_z$$

$$\vec{E}_2 = k\rho_L \int_{-\infty}^{\infty} \frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3}\, dz$$

$$|\vec{r}-\vec{r}_2| = \sqrt{(2)^2+(-3)^2+(-z)^2} = \sqrt{z^2+13}$$

$$= k\rho_L \int_{-\infty}^{\infty} \frac{2\hat{a}_x + 3\hat{a}_y - z\hat{a}_z}{(z^2-13)^{3/2}}\, dz$$

$$= k\rho_L \int_{-\infty}^{\infty} \left(\frac{2\hat{a}_x}{(z^2-13)^{3/2}} - \frac{3\hat{a}_y}{(z^2-13)^{3/2}} - \frac{z\hat{a}_z}{(z^2-13)^{3/2}}\right) dz$$

$$k\rho_L \left(\frac{-2\cancel{z}}{13\sqrt{z^2-13}}\hat{a}_x + \frac{3z}{13\sqrt{z^2-13}}\hat{a}_y + \frac{1}{\sqrt{z^2-13}}\hat{a}_z\right)\Bigg|_{-\infty}^{\infty}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$27 \qquad -0.15\,\hat{a}_x \qquad 0.23\,\hat{a}_y \qquad 0\,\hat{a}_z$$

$$= -4.05\,\hat{a}_x + 6.21\,\hat{a}_y + 0\,\hat{a}_z$$

$$\vec{E}_3 = \frac{\rho_S}{2\varepsilon_0} \quad \cancel{\hat{a}_3 = \frac{\vec{r}-\vec{r}_3}{|\vec{r}-\vec{r}_3|} \quad \vec{r}-\vec{r}_3 = -2\hat{a}_x + 0\hat{a}_y + 6\hat{a}_z}$$

$$+0.11\,\hat{a}_x \ \text{nC}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{E}_{total}$$

$$= -0.85\,\hat{a}_x + 0\,\hat{a}_y - 2.56\,\hat{a}_z$$

$$+ -4.05\,\hat{a}_x + 6.21\,\hat{a}_y + 0\,\hat{a}_z$$

$$+ 0.11\,\hat{a}_x$$

$$= \boxed{-4.79\,\hat{a}_x + 6.21\,\hat{a}_y - 2.56\,\hat{a}_z}$$