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11.2

32. $\sum_{n=1}^{\infty}\left[(-0.2)^n + (0.6)^{n-1}\right]$

$a_1\ 0.8$ $\quad a_2\ 0.64$ $\quad a_3\ 0.352$

$(-0.2+1)+(0.04+0.6)+(-0.008+0.36)$

$s_1\ 0.8$ $\quad s_2\ 1.44$ $\quad S_3\ 1.792$

$b_n = 0.6^{n-1}$ positive, decreasing

$\lim_{n\to\infty} 0.6^{n-1}$ geometric $a=1$ $r=0.6$

$\int_\infty = \frac{1}{1-0.6} = 2.5$ converges to 2.5

40. $\sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)$ $\quad a_2\ 1.12$ $\quad a_3\ 0.69$ $\quad a_4\ 0.5$

$\left(\frac{3}{5}+\frac{2}{1}\right)+\left(\frac{3}{25}+\frac{2}{2}\right)+\left(\frac{3}{125}+\frac{2}{3}\right)+\left(\frac{3}{625}+\frac{2}{4}\right)$

$S_1\ 2.6$ $\quad S_2\ 3.72$ $\quad S_3\ 4.41$ $\quad S_4\ 4.91$

$\int_1^\infty \frac{3}{5^x}dx + \int \frac{2}{n}dx \to 3\int_1^\infty 5^{-x}dx + 2\ln x + C$

$3\sum_{n=1}^{\infty}\frac{1}{5^n} + 2\sum_{n=1}^{\infty}\frac{1}{n}$ divergent

divergent

$\frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{1}{\cancel{5}}\times\frac{\cancel{5}}{4} = \frac{1}{4}$

54. $10.1\overline{35}$ $\quad 10.1+\left[\frac{35}{10^3}+\frac{35}{10^5}+\frac{35}{10^7}\right]\ldots$

$\frac{a}{1-r}$ $\quad \frac{\frac{35}{1000}}{1-\frac{1}{100}}\cdot\frac{1000}{1000}$ $\quad \frac{35}{1000-10}$ $\quad \frac{10.1}{1}+\frac{35}{990} = \frac{10034}{990}$

11.3

6. $\sum_{n=1}^{\infty}\frac{1}{(3n-1)^4}dn$ $\quad u=3n-1$ $\quad du=3\,dn$ $\quad dn=\frac{1}{3}du$

$\int_1^\infty \frac{1}{(3n-1)^4}dn$ $\quad \int\frac{1}{3u^4}$ $\quad \frac{1}{3}\int\frac{1}{u^4}du$ $\quad \frac{u^{-4+1}}{-4+1}$ $\quad \frac{u^{-3}}{-3}$

$\frac{1}{3}\left(\frac{-1}{3u^3}\right) = \frac{-1}{9u^3}$ $\quad \frac{-1}{9(3n-1)^3}$ ~~Convergent~~

divergent

$\lim_{t\to\infty}\frac{-1}{9(3n-1)^3}\Big|_1^t$ $\left[\frac{-1}{9(3t-1)^3}\right]$ $-\left[\frac{-1}{9(3(1)-1)^3}\right] = \frac{1}{72} \neq 0$

(9) 2^3 $9\cdot 8$

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14. $\frac{1}{1\sqrt{1}}+\frac{2}{2\sqrt{2}}+\frac{3}{3\sqrt{3}}+\frac{4}{4\sqrt{4}}+\frac{5}{5\sqrt{5}}\ldots$ $\quad \sum_{n=1}^{\infty}\frac{n}{n\sqrt{n}}$ $\quad \frac{1}{\sqrt{n}}\to n^{-1/2}$

diverges $\quad$ p-test $p<1$

20. $\sum_{n=1}^{\infty}\frac{3n-4}{n^2-2n}\left(\frac{1/n^2}{1/n^2}\right)$ $\quad \frac{\frac{3}{n}-\frac{4}{n^2}}{1-\frac{2}{n}}$ $\quad \lim_{n\to\infty}$ $\quad \frac{0-0}{1-0}=\frac{0}{1}=0$

Integral test $\int_3^\infty \frac{3x-4}{x^2-2x}dx$ $\quad u=x^2-2x$ $\quad \cancel{du=2x-2\,dx}$ $\quad \cancel{dx=\frac{1}{2x-2}du}$

$\int\frac{3x-4}{u(2x-2)}du$

$3\int\frac{2n-2}{2n^2-2n} - \int\frac{1}{n^2-2n}$ $\quad 3\int\frac{n-1}{n^2-2n}dn$ $\quad u=n^2-2n$ $\quad du=2n-2\,dn$

$dn=\frac{1}{2n-2}du$

$\frac{1}{2}\int\frac{1}{u}du \to \frac{1}{2}\ln|u| \to \frac{1}{2}\ln|n^2-2n|$

$\int\frac{1}{(n-2)n}dn \to \frac{1}{2}\int\frac{1}{n-2}dn$ $\quad \frac{1}{2}\int\frac{1}{n}dn$ $\quad \frac{3\ln|n^2-2n|}{2}+\frac{\ln|n|}{2}-\frac{\ln|n-2|}{2}\Big|_3^\infty$

$\frac{1}{2}\ln|n-2|-\frac{1}{2}\ln|n|$

$[\infty]\to$ divergent

24. $\sum_{k=1}^{\infty}ke^{-k^2}$ $\quad \frac{k}{e^{k^2}}\to\frac{\infty}{\infty}$ L'H $\quad \frac{k'}{e^{k^2}{}'}$ $\quad \frac{1}{2ke^{k^2}}\to$ converges

36. S_{10} of $\sum_{n=1}^{\infty}1/n^4$ $\quad \int_n^\infty\frac{1}{x^4} = \lim_{t\to\infty}\left[-\frac{1}{3x^3}\right]_n^t$

$=\lim_{t\to\infty}\left(\frac{-1}{3t^3}+\frac{1}{3n^3}\right)=\frac{1}{3n^3}$ $\quad \frac{1}{1}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}\approx 1.08$

a). $R_{10}=\int_{10}^\infty\frac{1}{x^4}dx=\frac{1}{3(10)^3}=\frac{1}{3000}=0.000\overline{3}$

$s_{10}+\int_{11}^\infty\frac{1}{x^4}dx \le s \le s_{10}+\int_{10}^\infty\frac{1}{x^3}dx \to s_{10}+\frac{1}{3(11)^3}\le s\le s_{10}+\frac{1}{3000}$

$s_{10}+\frac{1}{3993}\le s\le s_{10}+\frac{1}{3000}$ $\quad 1.080250\le s\le 1.080333$

b). <0.000077 error

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11.4

2. a) Σa_n is divergent because b_n is smaller and divergent

b) nothing because there isn't enough information

6. $\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$ $\lim_{n \to \infty} = 0$

converges $\frac{n-1}{n^3+1} \sim \frac{1}{n^2}$

8. $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$

$\cancel{\lim_{n \to \infty} \frac{\frac{6^{n+1}}{5^{n+1}}}{6}}$ diverges