

3.2 $\vec{E} = 5z^2/\epsilon_0 \hat{a}_z$ cube

$\phi = \int \vec{E} \cdot d\vec{s}$ $x=y=z=\pm 2$

$\phi = \int \vec{E} \cdot d\vec{s}_{top} + \int \vec{E} \cdot d\vec{s}_{bottom}$

$Q = \epsilon_0 \phi$

$$\begin{aligned} & \int_{-2}^2 \int_{-2}^2 5(z)^2/\epsilon_0 dx dy \\ &= \frac{20}{\epsilon_0} \int_{-2}^2 \int_{-2}^2 dx dy \\ &= \frac{20}{\epsilon_0} \int_{-2}^2 dx \\ &= 320/\epsilon_0 \end{aligned}$$

$$\begin{aligned} & \int_{-2}^2 \int_{-2}^2 5(z)^2/\epsilon_0 dx dy \\ &= 320/\epsilon_0 \end{aligned}$$

$\phi_{top} + \phi_{bottom} = \frac{640}{\epsilon_0}$

$Q = \epsilon_0 \cdot \frac{640}{\epsilon_0} = 640$

3.3 $\rho = 0.08 \text{ m}$ $\rho_s = 5e^{-20|z|} \text{ nC/m}^2$

$Q = \int \rho_s$

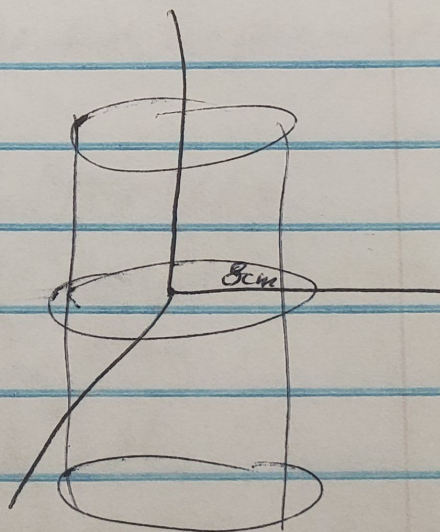
$= \int_{-\infty}^{\infty} \int_0^{2\pi} \rho_s r d\phi dz$

$= 0.08 \int_{-\infty}^{\infty} \int_0^{2\pi} 5e^{-20|z|} d\phi dz$

$= 0.4 \int_{-\infty}^{\infty} e^{-20|z|} \int_0^{2\pi} d\phi$

$= 0.8\pi \int_{-\infty}^{\infty} e^{-20|z|} dz$

$= 0.8\pi (1/10) \approx 0.25 \text{ nC}$



$Q = \int_{0.01}^{0.05} \int_{\pi/6}^{\pi/2} \rho_s r d\phi dz$

$= 0.08(\pi/3) \int_{0.01}^{0.05} 5e^{-20|z|} d\phi dz$

$= 0.4\pi/3 \int_{0.01}^{0.05} e^{-20|z|} d\phi dz$

$= \frac{0.4\pi}{3} (0.0225) = 9.42 \text{ pC}$

3.7

$$\rho_v = 2e^{-1000r} \text{ nC/m}^3$$

$$0 \leq r \leq 0.001$$

$$A = 4\pi r^2$$

$$r = 0.001 \text{ m}$$

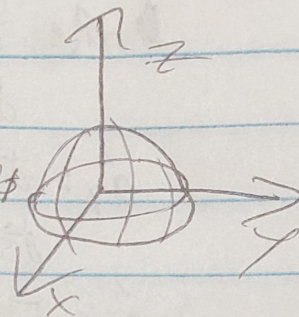
$$Q = \int_0^{2\pi} \int_0^\pi \int_0^{0.001} \rho_v r^2 \sin\theta dr d\theta d\phi$$

$$= 4\pi (2) \int_0^{0.001} r^2 e^{-1000r} dr$$

$$= 8\pi (1.606 \times 10^{-10}) \approx \boxed{4 \text{ nC}}$$

$$D_r = Q/A = 4 \times 10^{-9} / 4\pi r^2, \quad r = 0.001$$

$$= \boxed{0.32 \text{ pC}}$$



3.15

$$\rho_v = 0 \text{ for } \rho < 0.001 \quad \rho_v = 0 \text{ for } \rho > 0.002$$

$$\rho_v = 4\rho \text{ nC/m}^3 \text{ for } 0.001 < \rho < 0.002$$

$$0 < \rho < \rho_1$$

$$0 < z < L$$

$$Q = \int_0^L \int_0^{2\pi} \int_{0.001}^{\rho_1} \rho_v \rho^2 d\rho d\phi dz$$

$$= \int_0^L \int_0^{2\pi} \int_{0.001}^{\rho_1} 4\rho^2 d\rho d\phi dz$$

$$= 8\pi L \int_{0.001}^{\rho_1} \rho^2 d\rho$$

$$= \frac{8\pi L}{3} [\rho_1^3 - 10^{-9}] \mu\text{C}$$

$$\approx \boxed{\frac{8\pi L \rho_1^3}{3} \mu\text{C}}$$

$$D_\rho = \frac{Q}{2\pi \rho_1 L}$$

$$= \boxed{\frac{4\rho_1^2}{3}} \mu\text{C/m}^2$$

$$D_\rho (0.8 \text{ mm}) = \boxed{0}$$

$$0.0008 < 0.001$$

$$D_\rho (1.6 \text{ mm}) = \boxed{3.4 \mu\text{C/m}^2}$$

$$D_\rho (2.4 \text{ mm}) = \boxed{3.9 \mu\text{C/m}^2}$$