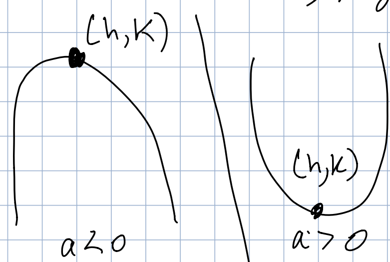
Math 418

Warm Up: Find the range of $g(x) = -3x^2 + 4x - 2$

Note $Dom(g) = (-\infty, \infty)$



Range $= (-\infty, K]$ Range $= [K, \infty)$

$a = -3 < 0$

$b = 4$

$h = \frac{-b}{2a} = \frac{-4}{2(-3)} = \frac{-4}{-6}$

$= \frac{+2}{3} = \frac{2}{3}$

$K = g(h) = g(2/3)$

$= -3\left(\frac{2}{3}\right)^2 + \frac{4}{1}\left(\frac{2}{3}\right) - 2$

$= \frac{-3}{1}\left(\frac{4}{9}\right) + \frac{8}{3} - 2$

$= \frac{-12}{9} + \frac{8}{3} - \frac{2}{1}\cdot\frac{3}{3}$

$= \frac{-4}{3} + \frac{8}{3} - \frac{6}{3}$

$K = \frac{-2}{3}$

$$\boxed{Range(g) = \left(-\infty, \frac{-2}{3}\right]}$$

Conic Sections: Circles, Ellipses, Hyperbolas, Parabolas

Power Functions

Def'n: A function $f(x)$ is a Power Function if it can be written in the form:

$f(x) = Ax^B$ $A \neq 0$, B positive int.
A real #

Ex: $f(x) = x^2$ ✓
$g(x) = -3x^{12}$ ✓
$h(x) = \pi x^7$ ✓

Non-Ex: $\frac{1}{x}$ ✗
$\sqrt{x}$ ✗

$x+4$ ✗

x^3-2 ✗

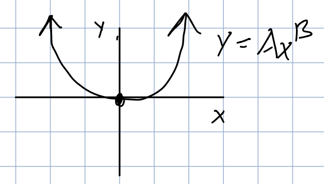
Graphs of Power Functions: (4 options)

Note: As B gets big Power functions get flatter close to 0.

$$f(x) = Ax^B$$

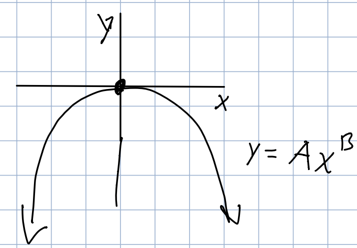
$A > 0$

B even



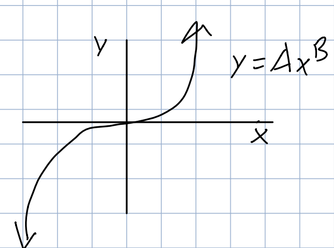
$A < 0$

B even



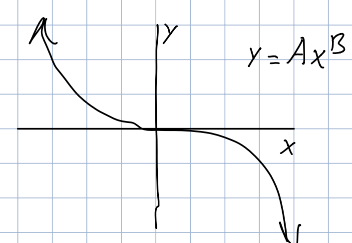
$A > 0$

B odd



$A < 0$

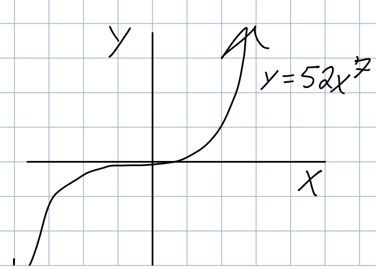
B odd



Sketch $f(x) = 52x^7$

$A=52 > 0$

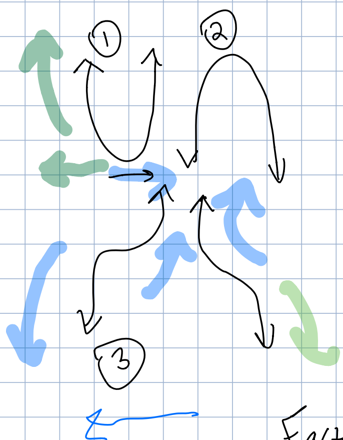
$B=7$ odd $\rightarrow$



Ax^B

$A(0) = A \cdot 0 = 0$

End Behavior: What happens to a Power function as x gets really big (to ∞) or as x gets really small (to $-\infty$)



Notation: x gets really big: x goes to ∞

$x \to \infty$ "x approaches infinity"

x gets really small: x goes to $-\infty$

$x \to -\infty$ "x approaches $-\infty$"

$f(x) = Ax^B$

Facts:

1. If $A > 0$, B even
   as $x \to -\infty$, $f(x) \to +\infty$
   as $x \to +\infty$, $f(x) \to +\infty$
2. If $A < 0$, B even
   as $x \to -\infty$, $f(x) \to -\infty$
   as $x \to +\infty$, $f(x) \to -\infty$

If B is even then end behavior the same on both sides

3. $A > 0$, B odd
   as $x \to -\infty$, $f(x) \to -\infty$
   as $x \to +\infty$, $f(x) \to +\infty$
4. $A < 0$, B odd
   as $x \to -\infty$, $f(x) \to +\infty$
   as $x \to +\infty$, $f(x) \to -\infty$

If B is odd the end behavior opposite each side

Fun Problems

1. Sketch $P(x) = -7x^3$ and $g(x) = 2x^4$
2. Find the end behavior for $g(x) = -3x^5$
3. Find a Power function $f(x) = Ax^B$ such that $f(2) = 12$ and $f(3) = 27$ (Find A and B)

$3x + y = 1$
$2x - y = 5$

Hint:

$$\frac{A(2)^B = 12}{A(3)^B = 27}$$

$$\frac{2^B}{3^B} = \frac{4}{9}$$

$$\left(\frac{2}{3}\right)^B = \frac{4}{9}$$

$5x = 6$

"Sums of Power Functions"

$x^3 - 3x^5 + x$

Def'n: A function $P(x)$ is a Polynomial if it can be written in the form:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

With: n pos. Integer
$a_n \neq 0$
$a_1, \ldots, a_n$ real #'s

Note $n = \text{degree}(P) = \deg(P)$

$a_0, a_1, \ldots, a_n$ = Coefficients

a_2 Coefficient x^2 (and so on)

Ex: $P(x) = x^{12} - \pi x^3 + 2x - 8$

$g(x) = 900x^{900}$

$h(x) = -34x^{1000} - x^{12} + 7x^7 - 13$

$f(x) = 3(x+4)^2(x-8)^3(x+1)(x-7)$

Problem $\longrightarrow$ $z(x) = 12(x-1)^2(x+3)$

Write in $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$