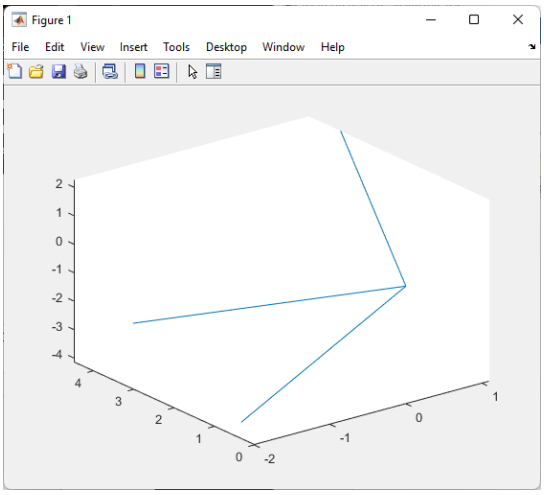
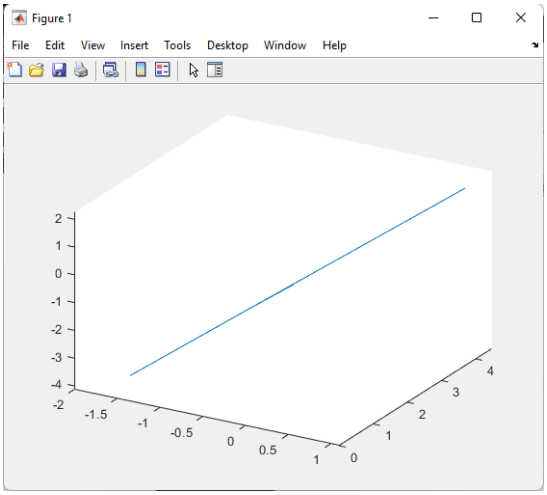
1.

a)





b)

set up an augmented matrix and row-reduce to find pivot points

>> A=[1.1;3.7;2.25]; B=[-1.7;.89;-4.13]; C=[-1.22;4.487;-3.344];

>> Z=[0;0;0];

>> Aug=[A B C Z]

Aug =

1.1000	-1.7000	-1.2200	0
3.7000	0.8900	4.4870	0
2.2500	-4.1300	-3.3440	0

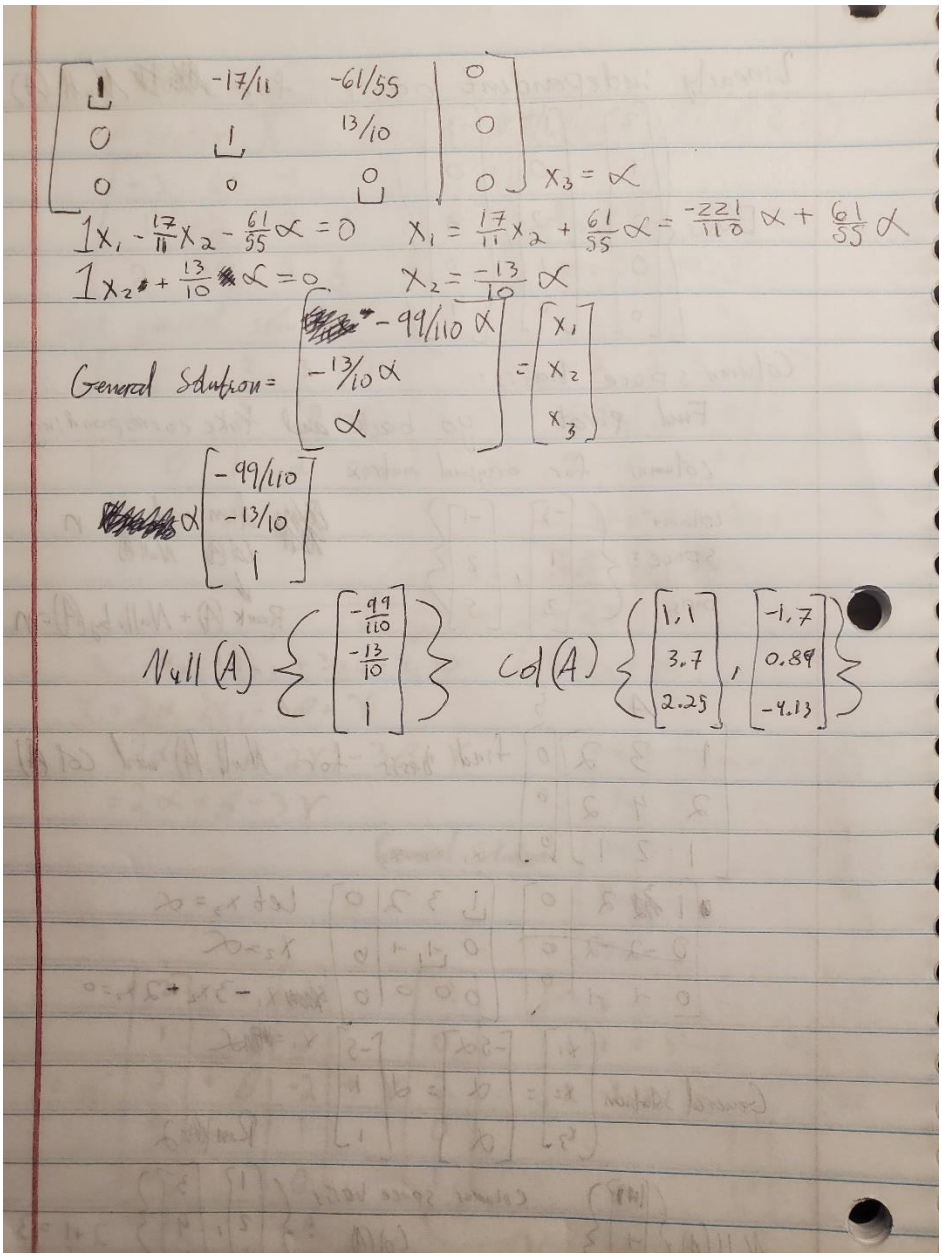
After reducing:

A =

11/10	-17/10	-61/50	0
0	1130/171	1469/171	0
0	*	*	0

This proves x_3 is a free variable

c)


$\begin{bmatrix} 1 & -17/11 & -61/55 & 0 \\ 0 & 1 & 13/10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x_3 = \alpha$
$1x_1 - \frac{17}{11}x_2 - \frac{61}{55}\alpha = 0$ $x_1 = \frac{17}{11}x_2 + \frac{61}{55}\alpha = \frac{-221}{110}\alpha + \frac{61}{55}\alpha$
$1x_2 + \frac{13}{10}\alpha = 0$ $x_2 = \frac{-13}{10}\alpha$
General Solution = $\begin{bmatrix} -99/110\,\alpha \\ -13/10\,\alpha \\ \alpha \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
$\alpha \begin{bmatrix} -99/110 \\ -13/10 \\ 1 \end{bmatrix}$
$Null(A) \left\{ \begin{bmatrix} -\frac{99}{110} \\ -\frac{13}{10} \\ 1 \end{bmatrix} \right\}$ $Col(A) \left\{ \begin{bmatrix} 1.1 \\ 3.7 \\ 2.25 \end{bmatrix}, \begin{bmatrix} -1.7 \\ 0.89 \\ -4.13 \end{bmatrix} \right\}$

2.

$$\begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & 3 \\ -4 & 12 & 2 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & 0 & 11 \\ 0 & 0 & 10 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 2 & -4 & 0 \\ 0 & 0 & 5 & -7 & 0 \\ 0 & 0 & 1 & -9/10 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$x_2 = \alpha$

$x_1 - 3\alpha + 2x_3 - 4x_4 = 0$

$x_3 - 9/10 x_4 = 0$

$x_4 = 0$

$x_1 = 3\alpha - 2x_3 + 4x_4$

$x_3 = 9/10 x_4$

$x_4 = 0$

$$\text{General solution} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3\alpha \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Null}(A) = \left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$\text{Nullity}(A) = 1$

$$\text{Col}(A) = \left\{ \begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ 3 \\ 7 \end{bmatrix} \right\}$$

$\text{Rank } A = 3$

$3 + 1 = 4$