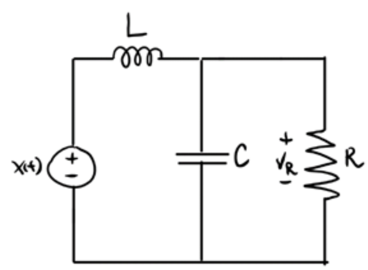
Homework # 3

Instructions: Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and box all your solutions. All Matlab code needs to be published, and all figures needs to have proper axis labeling and legends. Homework assignments will be collected during class time on the due date. *No late homework or submission that do not strictly follow the provided instructions will not be accepted.*

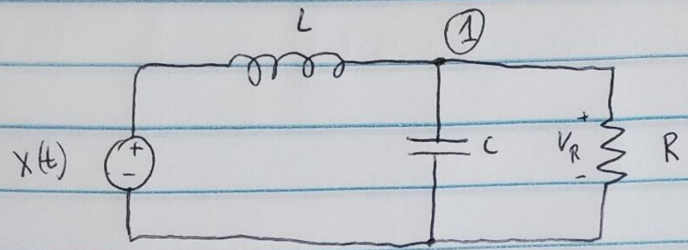
- **Homework problems not to be graded**
  - From textbook (Lathi):
    - Ch 2: 2-4, 2-5, 2-8 2-11

- **Homework problems to be graded**

Consider the following circuit with inductance L, capacitance C, and resistance R. The input is the voltage source $x(t)$, and the output is the voltage across the resistor $y(t) = V_R$.



a) Find the differential equation relating the input $x(t)$ to the output $y(t)$. Show all your work, including the application of the Kirchhoff laws, and equation for each component of the circuit.
b) Let $L = 0.1, C = 0.1$ and $R = 5$. Find the zero input response $y_0(t)$, given that $y_0(0) = 2$ and $\dot{y}_0(0) = 0$. Show all work, including characteristic equation, characteristic roots, characteristic modes, and solve for all unknown coefficients.
c) Plot the zero input response $y_0(t)$. Label all axes and title each figure appropriately. Publish Matlab script and print.



$V_L = L\, I_L'$ $\quad V_c = \frac{Q}{C}$ $\quad V_R = I_R R = y(t)$

$I_c = C V_c'$ $\qquad I_L = I_c + I_R$

$x(t) = L I_L' + I_R R$

$I_L' = \frac{d}{dt}\left(\frac{V_L}{L}\right) = C V_c' + \frac{V_R}{R}$ $\qquad V_c = V_R$

~~$x(t) = L \frac{d}{dt}\left(\frac{V_L}{L}\right) +$~~

$x(t) = L\left(C V_c' + \frac{y(t)}{R}\right)$

a) $\boxed{x(t) = L\left(C y'(t) + \frac{y(t)}{R}\right)}$

$x(t) = 0.1\left(0.1 y'(t) + \frac{y(t)}{5}\right)$

$10x(t) = 0.1 y'(t) + \frac{y(t)}{5}$

$100x(t) = y'(t) + 2y(t)$

$(100)x(t) = (D+2)y(t)$

$Q(D)\, y(t) = P(D)\, x(t)$

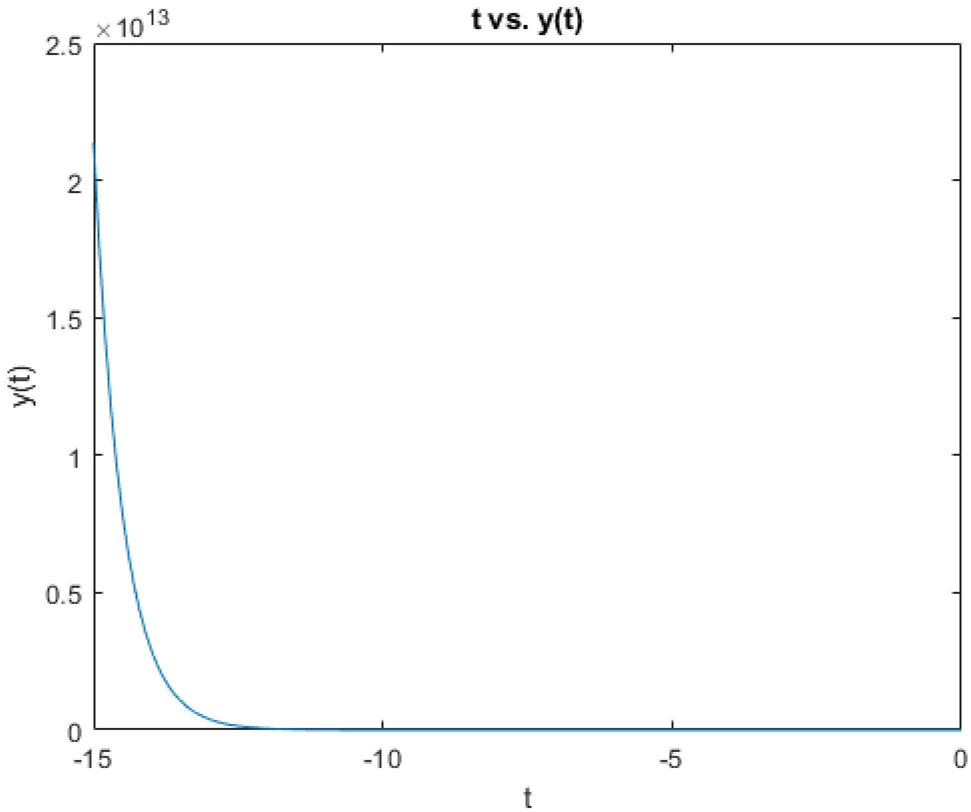
polynomial: $\lambda + 2 = 0$ $\qquad y_0(0) = 2$

roots: $\lambda = -2$ $\qquad y_0'(0) = 0$

modes: e^{-2t}

$y_0(t) = C e^{-2t}$ $\qquad 2 = C e^{-2(0)}$ $\qquad e^0 = 1$ $\qquad 2 = C$

b) $\boxed{ZI = 2e^{-2t}}$




```
>> t=-15:0.1:0;
>> ZI=2*exp(-2*t);
>> plot(t, ZI)
>> xlabel('t')
>> ylabel('y(t)')
>> title('t vs. y(t)')
```