Math 418 Nov, 24

Exam 3

1a. $\log_{81}\left(\frac{1}{5\sqrt{3}}\right) \Rightarrow 81^x = \frac{1}{5\sqrt{3}}$, $81^{5x} = \frac{1}{\sqrt{3}}$, $5\log_{81}\left(\frac{1}{3}\right) = x$, $\boxed{x=-4}$

b. $\log_{125}(5) \rightarrow 125^x = 5$ $\boxed{x=\frac{1}{3}}$

2. $r=1$ $(\pi/4)+(\pi/6)+(\pi/4)$ $\frac{12\pi}{12}+\frac{2\pi}{12}+\frac{3\pi}{12}=\boxed{\frac{17\pi}{12}}$

3a. $\cos\left(\frac{-7\pi}{6}\right)$ $\boxed{\frac{\sqrt{3}}{2}}$

b. $\csc\left(\frac{-\pi}{3}\right)$ $\sin \rightarrow \frac{\sqrt{3}}{2}$, $\csc \rightarrow \boxed{\frac{-2}{\sqrt{3}} \text{ or } \frac{-2\sqrt{3}}{3}}$

4. $\csc\theta\cos, \cot\theta = \frac{12}{7}, \cos\theta = ?$

$\sin\left(\frac{\cos\theta}{\sin\theta}\right) = \left(\frac{12}{7}\right)\sin$ $\cos\theta = \frac{12\sin\theta}{7}$, $\sin^2\theta + \frac{144}{49}\sin^2\theta = 1$,

$49\left(\frac{144}{49}\sin^2\theta\right) = \left(1-\sin^2\theta\right)49$, $144\sin^2\theta = 49 - 49\sin^2\theta$, $\frac{\sqrt{193}}{193}\sin^2\theta = \frac{\sqrt{49}}{193}$

$\left(\cos\theta = \frac{12\sin\theta}{7}\right)^2$ $+49\sin^2\theta$ $+49\sin^2\theta$ $\sin\theta = \pm\frac{7}{\sqrt{193}}$

$\frac{7\cos\theta}{12} = \frac{12\sin\theta}{12}$ $\cos^2\theta + \frac{49}{144}\cos^2\theta = 1$,

$+144\cos^2\theta$ $\left(144/\frac{49}{144}\right)\left(\cos^2\theta\right) = \left(1-\cos^2\theta\right)144$

$49\cos^2\theta = 144 - 144\cos^2\theta$ $\frac{\sqrt{193}}{193}\cos^2\theta = \frac{\sqrt{144}}{193}$

$\cos\theta = \pm\frac{12}{\sqrt{193}}$ or $\frac{12\sqrt{193}}{193}$

$\boxed{\cos\theta = \frac{12}{-\sqrt{193}}}$

5. $x^2+y^2=1 \Rightarrow \left(e^{4t}\right)^2+\left(e^{2t}\right)^2 = 1 \rightarrow \left(e^{4t}\right)\left(e^{4t}\right)+\left(e^{2t}\right)\left(e^{2t}\right)=1$

$\ln(1)+\ln(1) = 4t$, $2\ln(1) = 4t$, $t=0$, $\ln$

$\log_{10} 10$ $\ln e^x = 10$ $e^{4t} = 1-e^{2t} \rightarrow 1-\ln^4\left(e^{4t}\right) = 2t$, $1-4t\ln(e) = 2t$, $1-4t = 2t$

$+4t$ $+4t$

$\frac{1}{6} = \frac{6t}{6}$ $\boxed{t=\frac{1}{6}}$

6a. $f(x) = 3(2^{7x})$, $y = \log_6(f(x)) \Rightarrow \log_6(3)+\log_6(2^{7x}) = y \rightarrow$

$y = 7x\log_6(2)+\log_6(3)$ $\boxed{m = 7\log_6(2),\ b = \log_6(3)}$

b. $\log_5(x)+\log_5(x-2) = 1 \rightarrow \log_5(x^2-2x) = 1 \rightarrow$

$5^1 = x^2-2x$ $x^2-2x-5=0$ $a=1, b=-2, c=-5$

-5 -5

$\frac{2\pm\sqrt{4-4(1)(-5)}}{2}$ $4+20$ $\boxed{\frac{2\pm\sqrt{24}}{2} = x}$