1. 4.1.2.

a) yes, the definition of a vector space (axiom 6)

b) $[-1;1]$ and $[1;-1]$

3. $[0.1;0.1]$ and 1

11. $[0;0;0]$, the definition of a subspace says the zero vector of W is in $\mathbb{R}^3$

14. no, $\mathrm{rref}(A)=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

19. $y(t)=c_1\cos(\omega t)+c_2\sin(\omega t)$

$y(t)=\underset{c}{c_1}\left[\underset{u}{\cos(\omega t)}\right]+\underset{d}{c_2}\left[\underset{v}{\sin(\omega t)}\right]$ $\quad W=\vec{0}=0$

(1) $\cos(\omega t)+\sin(\omega t)\in V$

(2) $\cos(\omega t)+\sin(\omega t)=\sin(\omega t)+\cos(\omega t)$

(3) $\left(\cos(\omega t)+\sin(\omega t)\right)+0=\cos(\omega t)+\left(\sin(\omega t)+0\right)$

(4) $\cos(\omega t)+0=\cos(\omega t)$

(5) $\cos(\omega t)-\cos(\omega t)=0$

(6) $c_1(\cos(\omega t)+\sin(\omega t))=c_1\cos(\omega t)+c_2\sin(\omega t)$

(7) $c_1\cos(\omega t)\in V$

(8) $(c_1+c_2)\cos(\omega t)=c_1\cos(\omega t)+c_2\cos(\omega t)$

(9) $c_1(c_2\cos(\omega t))=(c_1c_2)\cos(\omega t)$

(10) $1\cos(\omega t)=1$

3. a)

$$\left[\begin{array}{ccccc|c} 1 & 0 & 3 & -2 & -3 & 0 \\ 0 & 3 & 1 & 1 & 0 & 0 \\ 1 & 3 & 4 & -1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccccc} 1 & 0 & 3 & -2 & 0 \\ 0 & 1 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{A}{\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{bmatrix}} \underset{\vec{w}}{\begin{bmatrix} -4 \\ -1 \\ 2 \\ 1 \end{bmatrix}} = \underset{\vec{0}}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \rightarrow$$

$$\underset{A}{\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{bmatrix}} \underset{u}{\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{no}$$

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{Yes}$$

b) $A = \begin{bmatrix} 1 & 9 & -3 & 3 & 7 \\ 0 & 2 & 6 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & -30 & 3 & -29 & 0 \\ 0 & 1 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$$\left.\begin{aligned} x_1 &= 30x_3 - 3x_4 + 29x_5 \\ x_2 &= -3x_3 - 4x_5 \\ x_3 &= x_3 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned}\right\} =$$

$$\text{Null}(A) = \left\{ \begin{bmatrix} 30x_3 \\ -3x_3 \\ x_3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3x_4 \\ 0 \\ 0 \\ x_4 \\ 0 \end{bmatrix}, \begin{bmatrix} 29x_5 \\ -4x_5 \\ 0 \\ 0 \\ x_5 \end{bmatrix} \right\}$$

c) (i) 3

$$\begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

(ii) 3

(iii)
$$\begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2.3 \\ 0 & 1 & 0 & -1.4 \\ 0 & 0 & 1 & 0.4 \end{array}\right] \text{ yes}$$

(iv)
$$\begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 11 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 50 \\ 69 \\ 82 \end{bmatrix} \text{ no}$$

d) (i)
$$\begin{bmatrix} 1 & -5 & 7 \\ 1 & -1 & 0 \\ -2 & 2 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ -5 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.7 \\ 0 & 1 & 0 & -1.4 \\ 0 & 0 & 1 & -0.2 \end{array}\right] \text{ yes}$$

(ii)
$$\begin{bmatrix} 1 & -5 & 7 \\ 1 & -1 & 0 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ -5 \\ 15 \end{bmatrix} \text{ no}$$

4.4.2, 33

(1) $\begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} \in V$

(2) $\begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix} + \begin{bmatrix} a \\ c \end{bmatrix}$

(3) $\left(\begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix}\right) + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix} + \left(\begin{bmatrix} b \\ d \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}\right)$

(4) $\begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$

(5) $\begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} -a \\ -c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(6) $c\left(\begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix}\right) = c\begin{bmatrix} a \\ c \end{bmatrix} + c\begin{bmatrix} b \\ d \end{bmatrix}$

(7) $c\begin{bmatrix} a \\ c \end{bmatrix} \in V$

(8) $(c+d)\begin{bmatrix} a \\ c \end{bmatrix} = c\begin{bmatrix} a \\ c \end{bmatrix} + d\begin{bmatrix} a \\ c \end{bmatrix}$

(9) $c\left(d\begin{bmatrix} a \\ c \end{bmatrix}\right) = (cd)\begin{bmatrix} a \\ c \end{bmatrix}$

a) $T(A) = A + A^T$

$T\left(\begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix}\right) = T\left(\begin{bmatrix} a \\ c \end{bmatrix}\right) + T\left(\begin{bmatrix} b \\ d \end{bmatrix}\right)$

$T\left(c\begin{bmatrix} a \\ c \end{bmatrix}\right) = cT\left(\begin{bmatrix} a \\ c \end{bmatrix}\right)$

b) $A = \begin{bmatrix} a & B \\ c & d \end{bmatrix}$

5 (I) $A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \to \left[\begin{array}{ccccc|c} 1 & 0 & 1/3 & 0 & 10/3 & 0 \\ 0 & 1 & 1/3 & 0 & -26/3 & 0 \\ 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = -1/3\, x_3 - 10/3\, x_5$

$x_2 = -1/3\, x_3 + 26/3\, x_5$

$x_3 = x_3 = \alpha$

$x_4 = 4x_5$

$x_5 = x_5 = \beta$

$Null(A) = \left\{ \alpha \begin{bmatrix} -1/3 \\ -1/3 \\ 1 \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} -10/3 \\ 26/3 \\ 0 \\ 4 \\ 1 \end{bmatrix} \right\}$

(II) 4.3, 21.

a) false, what is it dependent on?

b) false, only if $\{b_1 \cdots b_n\}$ are linearly independent and span the space

c) true, if it is invertable, then it's columns are linearly independent and has all pivots

d) false, is as small as possible

e) false, row operations don't affect linear dependence

22. a) false, may not span the space

b) true, the definition of a basis

c) true, if it isn't, it doesn't span

d) false, works all the time

e) ~~[illegible]~~ false, the pivot columns of B represent the columns of A that are the col(A)