A. $\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}_{\vec{u}_1}$ $\begin{bmatrix} 2 \\ -8 \\ -6 \end{bmatrix}_{\vec{u}_2}$ $\rightarrow$ $\overset{\vec{A}}{\begin{bmatrix} \vec{u}_1 & u_2 \end{bmatrix}} \overset{\vec{x}}{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}} = \overset{\vec{b}}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$

$-1\alpha_1 + 2\alpha_2 = 0 \Rightarrow 2\alpha_2 = \alpha_1$

$4\alpha_1 - 8\alpha_2 = 0 \Rightarrow 4(2\alpha_2) - 8\alpha_2 = 0$

$3\alpha_1 - 6\alpha_2 = 0 \Rightarrow$

they are not linearly independent because they are multiples of one another

B. $\begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$

$-\alpha_1 + 2\alpha_2 + \alpha_3 = 0$

$6\alpha_1 - 5\alpha_2 + \alpha_3 = 0$

$4\alpha_1 + \alpha_2 + 5\alpha_3 = 0$

they are linearly independent because there is only a trivial solution.

1A.

	N	S	P
Noodle	10%	10%	80%
Sausage	40%	20%	40%
Pig	30%	50%	20%

B.

$0.9N - 0.4S - 0.3P = 0$

$-0.1N + 0.8S - 0.5P = 0$

$-0.8N - 0.40S + 0.8P = 0$

$$\left[\begin{array}{ccc|c} 0.9 & -0.4 & -0.3 & 0 \\ -0.1 & 0.8 & -0.5 & 0 \\ -0.8 & -0.4 & 0.5 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -0.6471 & 0 \\ 0 & 1 & -0.7059 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{\text{c) no solution}}$

2.1.7/21

a) True, that is the definition of linear independence

b) True, a set is linearly dependent if the vectors are linear combinations

c) False, it possible but not likely for the columns to be linearly independent

d) True, the set of $\{x, y, z\}$ is linearly dependent

22.

a) False, all vectors go through the origin

b) False, they could still be linear combinations

c) False, x and y are linearly independent

d) False, ~~possible~~ number of vectors and entries do not determine linear dependense

9.

$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$ $v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}$ $v_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$ span $\{v_1, v_2\}$

$x_1\vec{v_1} + x_2\vec{v_2} = \vec{b} = \vec{v_3}$

$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -3x_1 + 9x_2 &= -7 \\ 2x_1 - 6x_2 &= h \end{aligned} \Rightarrow \left[\begin{array}{cc|c} 1 & -3 & 5 \\ -3 & 9 & -7 \\ 2 & -6 & h \end{array}\right]$$

$x_1 = 3x_2 + 5$ $\quad 9x_2 = 3x_1 - 7 \Rightarrow x_2 = \frac{1}{3}x_1 - 7/9$

$x_1 = 3\left(\frac{1}{3}x_1 - 7/9\right) \Rightarrow x_1 \neq x_1 - \frac{7}{3}$

a) no values, there is no solution to $\begin{bmatrix} 1 & -3 \\ -3 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$

$\underset{-3}{(1)(-3)} + \underset{-27}{(-3)(9)} + \underset{-12}{(2)(-6)}$

b) no values, v_3 in span $\{v_1, v_2, v_3\}$ is not possible

12. $\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -6 \\ 7 \\ -3 \end{bmatrix}$ $\begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix}$ $\quad 2x_1 - 6x_2 = 8$

$1x_1 - 3x_2 = 4$

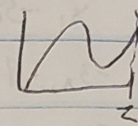
all real numbers, x_3 is a free variable

~~3~~/3.1.8/29

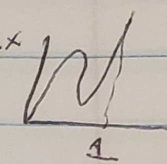
a) $f(x)=mx+b \quad f(y)=my+b \qquad b=0$

$f(x)+f(y)=mx+my=m(x+y)=f(x+y)$

$f(x)=x$



$f(x)=2x$



b) $f(x)+f(y)=mx+b+my+b \qquad b\neq 0$

$=m(x+y)+2b$

$=f(x+y)+2b \qquad T(x+y)=T(x)+T(y)$ is violated

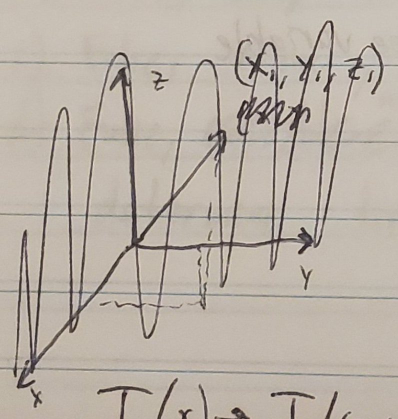
c) f is a polynomial

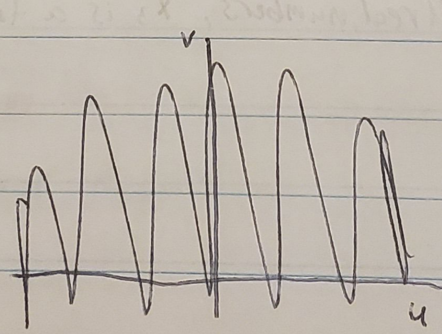
30. The property $T(x+y)=T(x)+T(y)$ is violated

$T(x)=A\vec{x}+b \qquad T(x)+T(y)=Ax+b+Ay+b$

$=T(x+y)+2b$

26. $X=su+tv \qquad T:\mathbb{R}^3\to\mathbb{R}^3$





$T(x)\to T(su+tv)=T(su)+T(tv)=sT(u)+tT(v)$

T(u) and T(v) must satisfy the law of superposition

5.1.9/4 $\quad T(e_1) = (-1/\sqrt{2},\ 1/\sqrt{2})$

find T $\quad$ rotates $-\pi/4$

$T(x) = \begin{bmatrix} 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Ax$

$T(x) = x_1 T(e_1) + x_2 T(e_2)$

$T(e_1) = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

6. $e_1 = e_1 \quad e_2 = e_2 + 3e_1$

horizontal shear

$\boxed{T = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}}$

11. reflects through x_1-axis, reflects through x_2-axis

$\boxed{T = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}$

angle would be 180° or π

15. $\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_3 \\ 4x_1 \\ x_1 - x_2 + x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 & -2 \\ 4 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

~~16.~~ $\cancel{\begin{bmatrix} ? & ? \\ ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ -2x_1 + x_2 \\ x_1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix}}$

19. $T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3,\ x_2 - 6x_3)$

$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - 5x_2 + 4x_3 \\ x_2 - 6x_3 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} 1 & -5 & 4 \\ 0 & 2 & -6 \end{bmatrix}}$

23.

a) True, $T(x)$ is a matrix transformation $x \mapsto Ax$ and important properties of T are intimately related to properties of A

b) ~~[illegible]~~ False, rotation is not a linear transformation

c) True, reflection ~~[illegible]~~ over x_1 then over x_2 resembles a rotation

d) True, A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ is said to be onto $\mathbb{R}^m$ if each b in $\mathbb{R}^m$ is the image of at least one x in $\mathbb{R}^n$