Nick Snyder
MATH 527.00
HW6

1 a. $\mathcal{L}\{(2t+3)\mathcal{U}(t-4)\} =$

$\boxed{e^{-4s}\mathcal{L}\{2(t+4)+3\}}$

b. $\mathcal{L}\{e^{-3t}(t-5)^2\} =$

$\boxed{\mathcal{L}\{(t-5)^2\}(s+3)}$

c. $\mathcal{L}^{-1}\left\{\frac{2s+3}{s^2+6s+34}\right\} =$

$\frac{2s+3}{(s+3)^2+25} = 2\cdot\frac{s+3}{(s+3)^2+25} - 3\cdot\frac{1}{(s+3)^2+25}$

$2\mathcal{L}^{-1}\left\{\frac{s+3}{(s+3)^2+25}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+25}\right\}$

$\left\{\frac{s}{s^2+5^2}\right\}\Big|_{s\to s+3}$ $\left\{\frac{1}{s^2+5^2}\right\}\Big|_{s\to s+3}$

$\frac{s}{s^2+5^2} = \cos(5t)\cdot e^{-3t}$ $\frac{1}{5}\sin(5t)\cdot e^{-3t}$

$\boxed{2e^{-3t}\cos(5t) - \frac{3}{5}e^{-3t}\sin(5t)}$

d. $\mathcal{L}^{-1}\left\{\frac{se^{-\pi s/2}}{s^2+4}\right\} =$

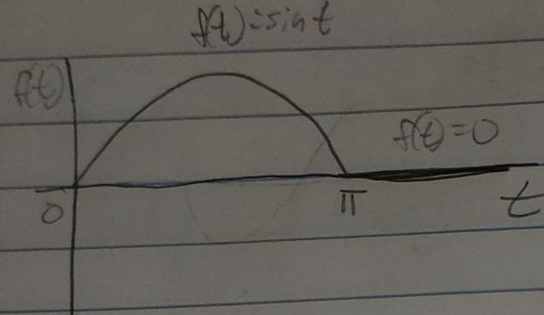
$e^{-\frac{\pi s}{2}}\cdot\frac{s}{s^2+4}$

$\mathcal{H}(t-\frac{\pi}{2})$ $\cos(2t)(t-\frac{\pi}{2})$

$\cos(2t-\pi)$

$\boxed{\mathcal{H}(t-\frac{\pi}{2})\cos(2t-\pi)}$

2a



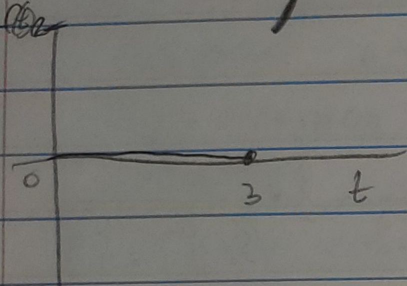
$f(t) = \mathcal{U}(\sin t) \cancel{+} \mathcal{U}(t-\pi)$

$f(t) = -(\sin(t))(t-\pi)\left[\mathcal{U}(\sin(t)) - \mathcal{U}(t-\pi)\right]$

$\mathcal{L}\{f(t)\} = -\mathcal{L}\{\sin(t)(t-\pi)\,\mathcal{U}(\sin t)\} + \mathcal{L}\{\sin(t)(t-\pi)\,\mathcal{U}(t-\pi)\}$

$-e^{-\pi s}\mathcal{L}\{\sin(t)(t-\pi)\big|_{t \to t+\pi}\} + e$

b



$f(t) = t^2\,\mathcal{U}(t-3)$

3 $y' + 2y = f(t) \quad y(0) = 0 \quad f(t) = \begin{cases} t & 0<t<1 \\ 0 & 1<t \end{cases}$

$f(t) = u(t)t - u(t-1)t$

$\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{f(t)\}$

$sY(s) + 2Y(s) \Rightarrow Y(s)(s+2) = \mathcal{L}\{f(t)\} = \mathcal{L}\{u(t)t\} - \mathcal{L}\{u(t-1)(t-1+1)\}$

$\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} = \mathcal{L}\{u(t-1)(t-1)\} - \mathcal{L}\{u(t-1)\}$

$Y(s)(s+2) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$

$\frac{1-e^{-s}}{s^2} - \frac{e^{-s}}{s}\cdot\frac{s}{s} = \frac{1-e^{-s}-se^{-s}}{s^3}$

$Y(s) = \frac{1-e^{-s}-se^{-s}}{s^3(s+2)}$

5. $y'' + 4y' + 5y = \delta(t-2\pi) \quad y(0) = y'(0) = 0$

$s^2 Y(s) + 4s\,Y(s) + 5\,Y(s) = e^{-2\pi s}$

$Y(s)(s^2 + 4s + 5) = e^{-2\pi s}$

$Y(s) = \frac{1}{s^2+4s+5} \cdot e^{-2\pi s}$

$Y(s) = \frac{1}{\underbrace{(s+2)^2}_{\text{s-translation}}+1} \cdot \underbrace{e^{-2\pi s}}_{\text{t-translation}}$

$Y(s) = e^{4\pi} \left. \frac{e^{-2\pi s}}{s^2+1} \right|_{s \to s+2}$

$\mathcal{L}^{-1}\left\{ e^{4\pi} \left. \frac{e^{-2\pi s}}{s^2+1} \right|_{s\to s+2} \right\} = e^{4\pi} e^{-2t} \mathcal{L}^{-1}\left\{ \frac{e^{-2\pi s}}{s^2+1} \right\}$

$e^{-2(t-2\pi)} \mathcal{U}(t-2\pi) \sin(t-2\pi) = y(t)$