4.5

$$\int_A^P \vec{G} \cdot \vec{dl} \qquad \vec{G} = 2y\hat{a}_x$$

$A(1,-1,2) \to B(1,1,2)\ P(2,1,2)$

a) $\int_1^2 2y\, dx \Rightarrow \int_1^2 2(1)\, dx \to \boxed{2}$

$A(1,-1,2),\ C(2-1,2),\ P(2,1,2)$

b) $\int_1^2 2y\ dx \to \int_1^2 2(-1)\, dx \to \boxed{-2}$

4,6

$$\vec{E} = x\hat{a}_x + 4z\hat{a}_y + 4y\hat{a}_z \qquad V(1,1,1) = 10V$$

$V(3,3,3) = ?$

$$\vec{E} = -\nabla V \qquad V = -\int \vec{E} \cdot \vec{dl}$$

$$V = -\left(\int_1^3 x\,dx + \int_1^3 4z\,dy + \int_1^3 4y\,dz\right)$$

$$= -\left(\frac{x^2}{2}\Big|_1^3 + 4zy\Big|_1^3 + 4yz\Big|_1^3\right)$$

$$= -(4 + 8z + 8y)$$

$$= -4 - 8(3) - 8(3) = \boxed{-52V}$$

4.12

$\vec{E} = \frac{2r}{(r^2+a^2)^2}\hat{a_r}$ find potential at any point using $V=0(\infty)$, $V=0(0)$, $V=100(a)$

$V = -\int_{ref}^{P(r,\theta,\phi)} \vec{E}\cdot d\vec{l}$

$\cancel{= -\int_{\infty}^{r} \frac{2r}{(r^2+a^2)^2}dr}$

$V(r) = -\int \frac{2r}{(r^2+a^2)^2}dr + C \rightarrow \frac{1}{r^2+a^2} + C$

a) $V(\infty)=0 \rightarrow V(r) = \boxed{\frac{1}{r^2+a^2}}$

$V(0) = 0 \rightarrow V(0) = \frac{1}{a^2} + C = 0$

$C = -\frac{1}{a^2}$

b) $V(r) = \boxed{\frac{1}{r^2+a^2} - \frac{1}{a^2}}$

$V(r=a) = 100 \rightarrow \frac{1}{a^2+a^2} + C = 100$

$C = 100 - \frac{1}{2a^2}$

c) $V(r) = \boxed{\frac{1}{r^2+a^2} + 100 - \frac{1}{2a^2}}$

4.21

$V = 2xy^2z^3 + 3\ln(x^2+2y^2+3z^2)$ $P(3,2,-1)$

$V = 2(3)(2)^2(-1)^3 + 3\ln((3)^2+2(2)^2+3(-1)^2)$

a) $= \boxed{-15\text{ V}}$

b) $\boxed{15\text{ V}}$

c) $E|_p = -\nabla V|_p = \left(2y^2z^3 + \frac{6x}{x^2+2y^2+3z^2}\hat{a_x}\right) + \left(4xyz^3 + \frac{12y}{x^2+2y^2+3z^2}\hat{a_y}\right)$
$+ \left(6xy^2z^2 + \frac{18z}{x^2+2y^2+3z^2}\hat{a_z}\right)\Big|_p = \boxed{7.1\hat{a_x} + 22.8\hat{a_y} - 71.1\hat{a_z}}$

$\vec{E} = 7.1\,\hat{a}_x + 22.8\,\hat{a}_y - 71.1\,\hat{a}_z$ V/m

$\sqrt{(7.1)^2 + (22.8)^2 + (-71.1)^2} = \sqrt{50.41 + 519.84 + 5055.21}$

d) $= \boxed{75 \text{ V/m}}$

$-\left(\frac{7.1}{75}\hat{a}_x + \frac{22.8}{75}\hat{a}_y - \frac{71.1}{75}\hat{a}_z\right)$

e) $\boxed{-0.095\,\hat{a}_x - 0.304\,\hat{a}_y + 0.948\,\hat{a}_z}$

d) $\vec{D} = \varepsilon_0 \vec{E} \rightarrow$ $62.8\,\hat{a}_x + 201.9\,\hat{a}_y - 629.5\,\hat{a}_z$ pC/m^3

4.23

$V = 80\rho^{0.6}$ V

a) $\vec{E} = -\nabla V \rightarrow 80 \cdot \frac{3}{5\rho^{2/5}} = \boxed{48\rho^{-2/5}\,\hat{a}_\rho}$

$\rho = 0.5$ $\quad \rho_v = \nabla \cdot \vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho)\Big|_{0.5}$

$= 96/5\rho^{7/5} = \varepsilon_0 \frac{96}{5}\rho^{-1.4}\Big|_{0.5}$

b) $\varepsilon_0 \cdot 19.2 \cdot (0.5)^{-1.4} = \boxed{448.6 \text{ pC/m}^3}$

$Q = \frac{96}{5}\varepsilon_0 \int_0^1 \int_0^{2\pi} \int_0^{0.6} \rho^{-1.4}\, \rho\, d\rho\, d\phi\, dz$

c) $Q = \frac{96}{5} \cdot \varepsilon_0 \cdot 1 \cdot 2\pi \cdot 1.2207 = \boxed{417.2\pi \text{ pC}}$