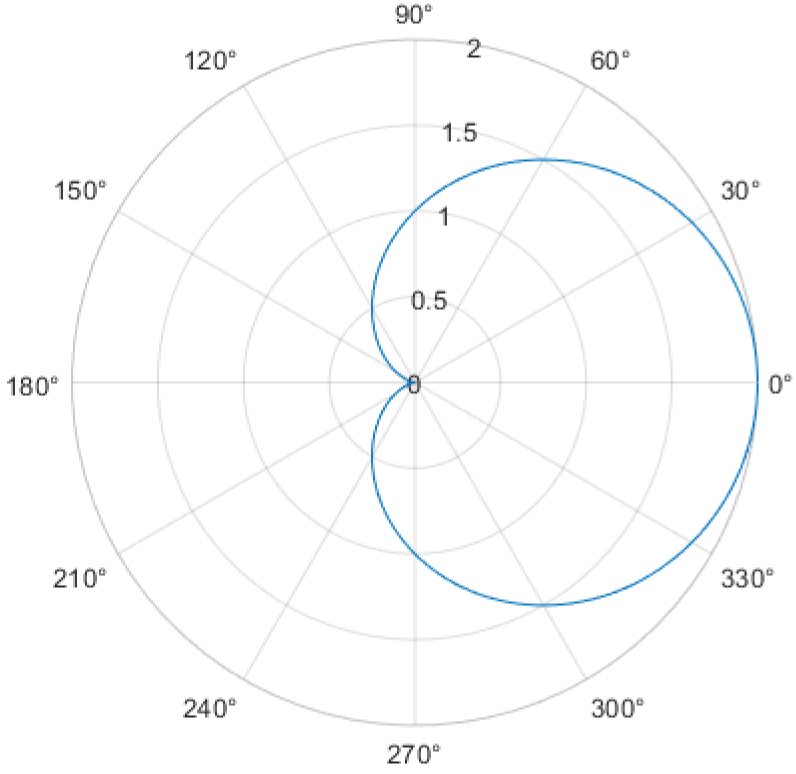


90°
120°
60°
150°
30°
180°
0°
210°
330°
240°
300°
270°
2
1.5
1
0.5
0

HW #2

1.

a.

b. $\iint (1+\cos\theta)\, dr\, d\theta$ $\qquad 1+\cos\theta = r$

$\cancel{\int_0^{2\pi}\int_0^{2}(1+\cos\theta)\, r\, dr\, d\theta \Rightarrow \int_0^{2\pi}\left[\frac{r^2}{2}\Big|_0^2\right](1+\cos\theta)\, d\theta}$

$\cancel{2\left[\theta+\sin\theta\Big|_0^{2\pi}\right] \quad (2\pi+0)-(0+0)}$

$\int_0^{2\pi}\int_0^{2}(r)\, dr\, d\theta \Rightarrow \frac{1}{2}\int_0^{2\pi}(1+\cos\theta)^2 d\theta$

$\frac{1}{2}\int_0^{2\pi}\left(1+2\cos\theta+\frac{1+\cos 2\theta}{2}\right)d\theta$

$\cancel{2\left(\int_0^{2\pi}(1)d\theta+2\int_0^{2\pi}(\cos\theta)d\theta+\frac{1}{2}\int_0^{2\pi}(1)d\theta+\int_0^{2\pi}(\cos 2\theta)d\theta\right)}$

$\cancel{2\theta^{2\pi}+4\sin\theta+\theta^{\pi}+\frac{\sin 2\theta}{2}\Big|_0^{\pi}}$

$\cancel{2\pi+4+\pi+0 \quad 3\pi+4}$

$3/2\,\theta+2\cancel{\sin\theta}+\frac{1}{4}\cancel{\sin 2\theta}\Big|_0^{\pi}$ $\qquad \boxed{3\pi/2}$

c. $dr/d\theta = -\sin\theta \rightarrow dr = -\sin\theta\, d\theta$

$dl = \sqrt{(1+\cos\theta\, d\theta)^2+(-\sin\theta\, d\theta)^2}$

$1+2\cos\theta+\frac{1+\cos 2\theta}{2}+\sin^2\theta$

Length $= \int_0^{2\pi}\sqrt{1+2\cos\theta+\frac{1+\cos 2\theta}{2}+\sin^2\theta}\; d\theta$ $\qquad u = 1+2\cos\theta+\frac{1+\cos 2\theta}{2}+\sin^2\theta$

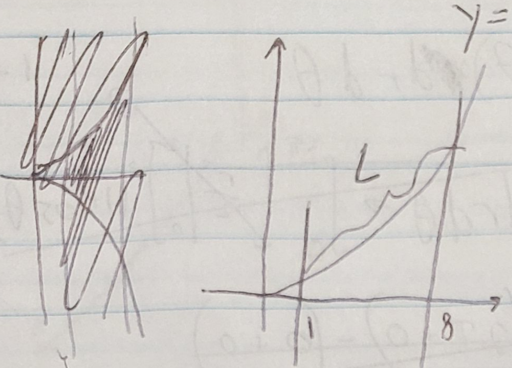
$du = (2\cos(\theta)-2)\sin(\theta)-\sin(2\theta)$

$\frac{1}{2}\int_4^{\theta=\pi} u^{1/2}\, du = \frac{1}{2}u^{3/2}\Big|_4^{\theta} = \frac{1}{2}\left(\sqrt{1+2\cos\theta+\frac{1+\cos 2\theta}{2}+\sin^2\theta}\right)^3\Bigg|_4^{\theta}$

$\frac{1}{2}\left(\left(\sqrt{1+2+1+0}\right)^3-\left(\sqrt{1+(-1.307)+(-0.427)+0.573}\right)^3\right) = \boxed{8}$

14. find length of $x = y^{2/3}$, $1 \le x \le 8$

$y = x^{3/2}$ $dy/dx = \frac{3}{2} \cdot x^{1/2}$



$dy = \frac{3\sqrt{x}}{2}\,dx$

$dl = \sqrt{dx^2 + dy^2}$

$dl = \sqrt{dx^2 + \left(\frac{3\sqrt{x}}{2}\right)^2}$

$dl = \sqrt{1 + \frac{9}{4}x}$

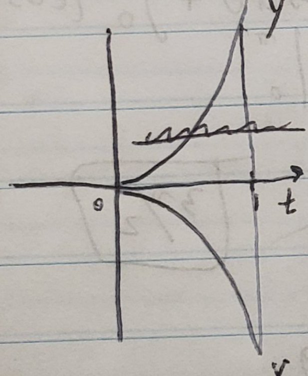
$\text{Length} = \int_1^8 \sqrt{1 + \frac{9}{4}x}\,dx$

$= x\sqrt{4/9 + x} + \frac{4}{9}\ln\left(x + \sqrt{4/9 + x}\right)\Big|_1^8$

$= \left(8\sqrt{\frac{4}{9}+8} + \frac{4}{9}\ln\left(8 + \sqrt{\frac{4}{9}+8}\right)\right) - \left(\sqrt{\frac{4}{9}+1} + \frac{4}{9}\ln\left(1 + \sqrt{\frac{4}{9}+1}\right)\right) = \boxed{22.318}$

18. find length of $x = t^3 - 6t^2$, $y = t^3 + 6t^2$, $0 \le t \le 1$

$dy = 3t^2 - 12t\,dt$ $dx = 3t^2 + 12t\,dt$



$dl = \sqrt{(3t^2 - 12t)^2 + (3t^2 + 12t)^2}$

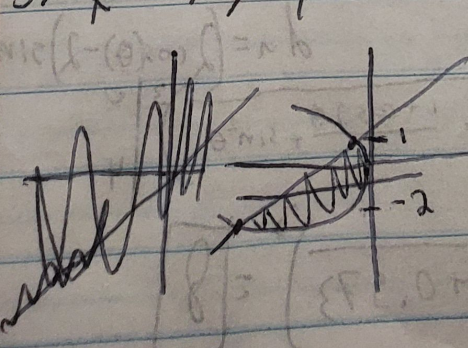
$dl = \sqrt{9t^4 - 72t^3 + 144t^2 + 9t^4 + 72t^3 + 144t^2}$

$= \sqrt{18t^4 + 288t^2}\,dt$

$\text{Length} = \int_0^1 \sqrt{18t^4 + 288t^2}\,dt = \frac{\sqrt{2}\,t \cdot \left((t^2+16)^{3/2} - 64\right)}{|t|}\Bigg]_0^1$

$\frac{\sqrt{2} \cdot \left(17^{3/2} - 64\right)}{1} = \boxed{8.616}$

3. $x = -y^2$, $y = x + 2$ $ds = dx\,dy$



$S = \int_{-2}^{1} \int_{x=y-2}^{x=-y^2} dx\,dy$

$= \int_{-2}^{1} \left((-y^2) - (y-2)\right) dy$

$= \frac{-y^3}{3} - \frac{y^2}{2} + 2y \Big|_{-2}^{1}$

$\left(-1/3 - 1/2 + 2\right) - \left(8/3 - 2 - 4\right) = \boxed{4.5}$