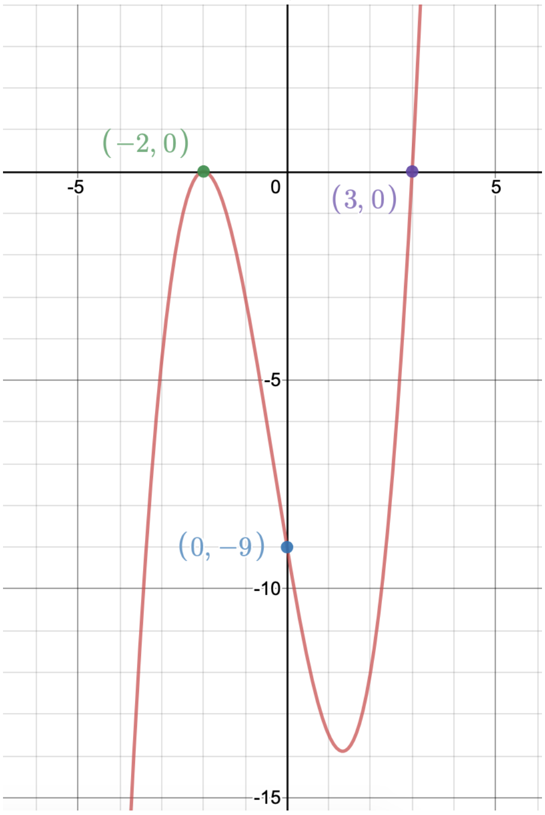
Math 418 Worksheet 6

Instructions: Have students work in groups on the following problems. Students should present their solutions at the board at the end of class.

1. Give an example of a polynomial $f(x)$ such that $f(x)$ has degree 3, $f(3) = 0$, $f(-1) = 0$, $f(4) = 0$ and $f(1) = 8$.
2. Give an example of a polynomial $f(x)$ such that $f(x)$ has degree 4, $f(5) = 0$, $f(-10) = 0$, $f(2) = 0$ and $f(3) = 5$.
3. Give an example of a polynomial $f(x)$ such that $f(x)$ has degree 5, f has two x-intercepts, $(-3, 0)$ and $(2, 0)$ and passes through the point $(0, 4)$.
4. Find a value for a such that $p(x) = a^2x^5 - ax^3 - 1$ has a root when $x = 1$.
5. Use polynomial long division and the fact that $x = 1$ and $x = -2$ are two roots for $p(x) = x^4 - 15x^2 - 10x + 24$ to fully factor $p(x)$.
6. Consider the graph of $y = p(x)$ shown below. Note that $p(x)$ has degree three. Use the graph below to find $p(x)$.



7. Use polynomial long division and the fact that $x = 1$ is a root with multiplicy two to sketch the graph of $f(x) = x^4 + 3x^3 - 33x^2 + 52x - 24$ using the end behavior and the behavior at the roots.

8. Find all intercepts (x and y) for $r(x) = \frac{2x-1}{4x+5}$

9. Find the domain of $f(x) = \frac{5x+2}{x^2-1}$

10. Classify $x = 2, -5$ and 7 as a vertical asymptote, root or hole for $g(x) = \frac{4(x-2)^3(x+5)^4(x-7)}{(x-2)^3(x+5)^7(x-7)^8}$

11. Give an example of a rational function $r(x)$ with two vertical asymptotes, $x = 3$ and $x = -2$, a single root of multiplicity two at $x = 1$ and two holes when $x = -5$ and $x = 4$ such that $r(-1) = 1$.

12. Sam and Andy are playing chess. So far Sam has won 12 out of 27 rounds of chess. How many rounds must Andy win in a row to have won 70% of the matches?

13. A shipping company finds that the cost of building boxes depends on the number of boxes they make in a day. The cost per box for making x thousand boxes in a day is given by $f(x) = \frac{400}{x} + x$. As it turns out there is a single value for x that will minimize the cost. The shipping company would like to find this x value to minimize their costs. Follow the steps below to find the number of boxes they need to produce to achieve the minimal cost.

    I) Let the minimal cost be c. Set $\frac{400}{x} + x = c$. We are going to solve this for both c (the minimum cost) and x (the number of thousands of boxes to make in a day to achieve this minimal cost).

    II) Rewrite the above equation as a quadratic and use the quadratic formula to solve for x in terms of c.

    III) As it stands now we have an equation in two unknowns. Use the fact there is only ONE value for x that minimizes the cost to solve for c. What is the minimum cost? Hint: Think about the discriminant in the quadratic formula.

    IV) Now that you have a value for c, how many thousands of boxes should the company make in a day?

14. In this problem we will find the range for $r(x) = \frac{8x+8}{x^2+8}$ and we will see this is a nontrivial problem to find the range of rational functions in general.

I) Can you find a value for x such that $r(x) = 0$? Is 0 in the range of r?

II) Can you find a value for x such that $r(x) = 1$? Is 1 in the range of r?

III) Can you find a value for x such that $r(x) = 10$? Is 10 in the range of r?

IV) We certainly don't have enough time to test this for each and every real number as there are far too many real numbers and not enough time in this semester to test each real number so we need to approach this problem more intelligently. We will find the range of $r(x)$ directly. Let a be an arbitrary real number...don't actually pick one. Set $r(x) = a$ and solve for x in terms of a.

V) Hopefully you now have a quadratic equation in terms of x and a and you have used the quadratic formula to solve for x. If you haven't yet, do this now.

VI) In order for us to be able to solve for x we must have the discriminant non-negative. What must be true about a for the discriminant to be non-negative? You should have an inequality.

VII) The inequality we have just obtained should be a quadratic inequality. In order to solve this let's pretend is it a quadratic *equation* by replacing the $\leq$ or $\geq$ with $=$. Solve this quadratic equation.

VIII) You now have two solutions to the quadratic equation, this divides the real number line into three regions:

- The numbers less than both solutions,
- The numbers inbetween the two solutions
- The numbers larger than both solutions.

Pick a test number from each of these three regions (be sure to NOT pick the solutions themselves) and plug these test values into the inequality to see where the inequality holds.

IX) Which values for a satisfy the inequality?

X) What is the range of $r(x)$?