Math 418 Exam 3 Review Sheet

These practice problems are meant to give you a guide to study for the exam. There will not be this many problems on the exam. Note that there may be problems on the exam that are not represented among these practice problems. It is impossible to provide enough practice problems to represent all of the ideas and skill that we are assessing through this exam. That said, if you are able to do all of these problems by yourself, without any resources other than your brain, you can rest assured that you have prepared well for the exam. The distribution of topics on this review sheet is NOT indicative of the distribution of topics on the exam

1. Evaluate:

   a) $\log_{16} 64$
      Answer: $\frac{3}{2}$

   b) $\log_9 \frac{1}{\sqrt{3}}$
      Answer: $\frac{-1}{4}$

2. Suppose $\ln a = 3.2$ and $\ln b = 1.1$. Evaluate $\ln(a^2 b^3)$ and $\frac{a}{b^2}$
   Answer: $\ln(a^2 b^3) = 9.7$, $\frac{a}{b^2} = e$

3. Suppose Samantha starts out with \$200 in a magical purse that doubles her money every day. Exactly how long will it take Samantha to have \$20000?
   Answer: $\log_2 100$ days.

4. Solve for x:

   a) $3^x = 12$
      Answer: $x = \log_3 12$

   b) $\log(x+1) + \log(x-1) = 2$
      Answer: $x = \sqrt{101}$

   c) $\frac{\log(2x^2)}{\log(3x)} = -1$
      Answer: $x = \frac{1}{\sqrt[3]{6}}$

   d) $\ln x - \ln(3x^2) = 1$
      Answer: $x = \frac{1}{3e}$

   e) $\log_3(x) + \log_9(x^2) = 2$
      Answer: $x = 3$

5. Find the domain and range of $f(x) = 3e^{2x} + 1$
   Answer: **Dom**$(f) = (-\infty, \infty)$, **Range**$(f) = (1, \infty)$

6. Find the domain of $h(z) = \frac{\ln(4-z^2)}{\sqrt{z-1}}$
   Answer: **Dom**$(h) = (1, 2)$

7. Suppose $f(x) = 3(2^{4x})$. Prove that $y = \ln(f(x))$ is a linear function. What is the slope?
   Answer: $m = 4\ln 2$

8. Expand and simplify: $\log_\pi \left(\frac{\pi^3 x^4 y^5 \sqrt{z}}{(x-1)^2 z} \right)$
   Answer: $3 + 4\log_\pi x + 5\log_\pi y - \frac{1}{2}\log_\pi z - 2\log_\pi(x-1)$

9. Find all points that lie on the unit circle and:

   a) The line $y = -12x$
      Answer: $\left(-\frac{1}{\sqrt{145}}, \frac{12}{\sqrt{145}}\right)$ and $\left(\frac{1}{\sqrt{145}}, \frac{-12}{\sqrt{145}}\right)$

   b) The parabola $y = 3x^2$.
      Answer: $\left(-\sqrt{\frac{-1+\sqrt{37}}{6}}, \frac{-1+\sqrt{37}}{6}\right)$ and $\left(\sqrt{\frac{-1+\sqrt{37}}{6}}, \frac{-1+\sqrt{37}}{6}\right)$

10. Find all y values so that $\left(\frac{1}{5}, y\right)$ lies on the unit circle.
    Answer: $y = \frac{-\sqrt{24}}{5}, \frac{\sqrt{24}}{5}$

11. Find all x values so that $\left(x, \frac{-2}{3}\right)$ lies on the unit circle.
    Answer: $x = \frac{-\sqrt{5}}{3}, \frac{\sqrt{5}}{3}$

12. Evaluate:

$\cos\frac{-3\pi}{4} = \frac{-\sqrt{2}}{2}$	$\sin 0 = 0$	$\tan\frac{\pi}{4} = 1$
$\sec\frac{-\pi}{6} = \frac{2}{\sqrt{3}}$	$\csc\frac{11\pi}{3} = \frac{-2}{\sqrt{3}}$	$\cot\frac{2\pi}{3} = \frac{-1}{\sqrt{3}}$

13. An ant walks counterclockwise around a circle of radius 6 centered at the origin. If the ant starts walking at $(-6, 0)$ and stops walking the second time they reach $(-3\sqrt{2}, 3\sqrt{2})$, how far did the ant walk?

    Answer: $\frac{45\pi}{2}$ units.

14. Suppose $0 < \psi < \pi$ and $\cot\psi = \frac{3}{4}$. Evaluate $\sin\psi$.
    Answer: $\frac{4}{5}$

15. What is the domain of $\tan\theta$?
    Answer: $\{\theta | \theta \neq \frac{(2k+1)\pi}{2}$ with k an integer$\}$

16. Suppose $\pi < \alpha < 2\pi$ and $\sec\alpha = 10$. Evaluate $\sin\alpha$
    Answer: $\frac{-\sqrt{99}}{10}$

17. Suppose $\cos\beta > 0$ and $\sin\beta = \frac{1}{6}$. Evaluate $\sec\beta$.
    Answer: $\frac{6}{\sqrt{35}}$

18. Find all values for θ such that $\cos^2\theta - 3\cos\theta + 2 = 0$
    Answer: $\theta = 2k\pi$ with k an integer

19. Find all values for ω in $[0, 2\pi]$ such that $(\tan(\omega) + 1)\left(\cos(\omega) - \frac{1}{2}\right) = 0$
    Answer: $\omega = \frac{\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{3}$ and $\frac{7\pi}{4}$

20. Suppose $\pi < \theta < \frac{3\pi}{2}$ and $\tan\theta = \frac{10}{3}$. Evaluate $\sin\theta$ and $\cos\theta$.
    Answer: $\sin\theta = \frac{-10}{\sqrt{109}}$, $\cos\theta = \frac{-3}{\sqrt{109}}$

21. Tommy and Sally are having a mathemagical debate about the problem below. Tommy claims that $\sin\alpha = 5$ and Sally doesn't agree. Who is right? Justify your answer by explaining why Tommy is correct or by finding a different value for $\sin\alpha$.

    Suppose $0 < \alpha < \frac{\pi}{2}$ and $\tan\alpha = \frac{5}{7}$. Evaluate $\sin\alpha$.
    Answer: Tommy is wrong. $\sin\alpha = \frac{5}{\sqrt{74}}$.