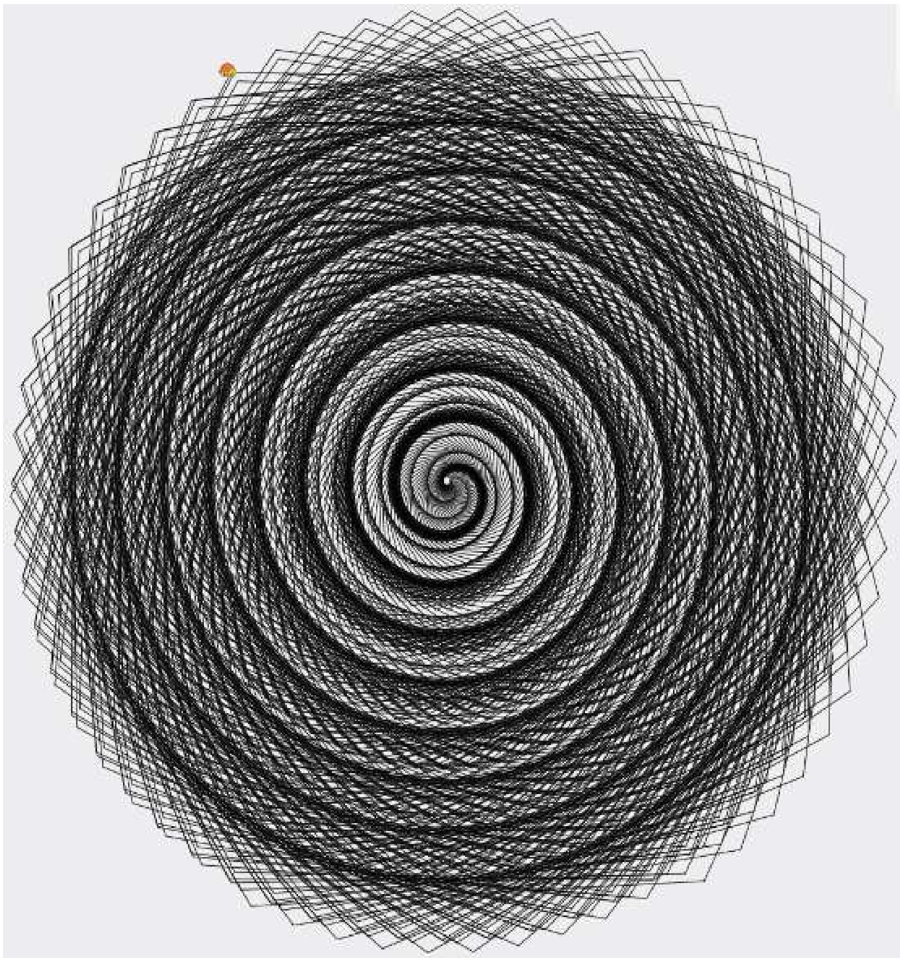
Studio Physics I Activity Book



Spring 2019

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University of New Hampshire

“Make everything as simple as possible. But not simpler.” – Albert Einstein

“Research is what I’m doing when I don’t know what I’m doing” – Werner Von Braun

“Not only is the universe stranger than we imagine. It is stranger than we *can* imagine” – Sir Arthur Eddington

Complex Problem Solving

In this class, “problem solving” will likely be very different from problem solving you have done in any prior class. Too often, students come into physics class viewing “problem solving” as an equation-hunting and number-crunching process, with these general steps:
1. Skim over a problem description, focusing on what numbers are being given – and what variables those correspond to. Write down those numbers (i.e. m=10 kg, etc.).
2. Identify the variable being asked for (i.e. a=?)
3. Hunt for an equation in your notes, the textbook, or elsewhere that combines all of the variables you are given in the description, and also the unknown variable.
4. Plug the known values into the equation, and solve for the unknown.

That is not really “problem solving” – it is just number crunching. When we are first learning new material, the activities will start out with one-step problems to guide you to a conclusion – but ultimately we are always going to work towards solving complex, multi-step problems for which a plug-and-chug approach will not work.

Our primary goals in this course are twofold: (1) for you to gain a solid understanding of classical mechanics, and (2) to be able to apply that understanding to new problems that you have not seen before, which may be fairly complicated. Many problems will require multiple steps to solve the problem, such that you will not be able to just find one equation to plug numbers into to get your answer.

My view of problem solving overall is "figuring out what to do when you have no idea what to do" - it is a process of struggling through confusion. A big reason I place so much emphasis on this is to help you build confidence in your ability to struggle through a complex problem. If you encounter something that at first you can not figure out - that does not mean you have to just give up.

When we get to complex problem solving, a good strategy would be the following:

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1. READ the problem ***carefully – don’t skim!***

2. THINK about what is going on in the problem. Visualize what is happening, and figure out what exactly you are being asked to find. Draw a picture if it helps you visualize it. Don’t write down any numbers – in fact, it may be better to initially ignore any numbers given in the problem (if there are any). Write down in clear sentences what you think the problem is, what your approach to solving the problem will be and why you chose it, and what you think the outcome will be.

3. DETERMINE what physical laws are involved in the processes taking place in the problem. ***That*** is what must guide you as far as determining what equations to use – ***not*** just finding an equation that has the same variables in it.

4. PLAN – for a multi-step problem, it can be very helpful to come up with a formal plan on how you are going to solve the problem. In some of the activities this semester, you will be required to write out a formal step-by-step plan before solving it, to help you develop this skill – it can be invaluable when working with very complex situations.

4. ANALYZE the problem analytically (no numbers!), working through your plan. Depending on the type of problem, this can mean different things (e.g. drawing a free body diagram, then applying conservation of energy to part of a problem, conservation of momentum to another part, and using kinematic equations in a third part, and working through an algebraic mess to get to a final answer).

5. THINK about your answer – does it make sense? In some cases, you may be able to apply a “limiting case” to your answer (see Activity 22) to test its validity. Do the units work out?

Keep in mind that a big emphasis in this class is getting a lot of problem solving practice, to help you build general problem solving skills – not just learning the mechanical laws we will be working with.

In all of the activities in this workbook, SHOW ALL OF YOUR WORK. Presenting your work in a coherent and legible format that someone else can understand is extremely important for all engineers. Just knowing something isn't useful if you can not communicate your knowledge and designs to others.

Table of Contents

Blue activities are extra credit that will be due on test days.
~~Crossed-out activities~~ will be skipped during this course.

1. Checks

Name___________________________

On Canvas, under the Files tab, you should find three images labelled "Checks 1" through "Checks 4". DO NOT OPEN THEM ALL YET! These images are scans of checks that were found in an old desk I bought at a flea market. The family who sold it to me seemed nice enough, but I guess they forgot to clear out some things. The checks paint an interesting picture, but I'm unsure exactly what they mean, and that's where you all come in! Here is your challenge:

A. Open "Checks 1" and look over the four checks. As a group, I want you to hypothesize the circumstances around the writing of these checks. In other words, I want you to come up with a narrative for the people involved that would have led to the writing of these check.

Hypothesis #1:

B. Now, open "Checks 2" and amend your first hypothesis. See if anything can be added or removed

Hypothesis #2:

C. Open “Checks 3” and update your hypothesis again.

Hypothesis #3:

D. Open “Checks 4” and update your hypothesis. This will be your final hypothesis, and it should encompass the events across all the checks.

Final hypothesis:

Error! Reference source not found.. Error! Reference source not found.

Follow-up:

1.1. How confident are you that your narrative is completely correct? (i.e. 80%, for example.)

1.2. Talk to your neighboring groups and compare your story with theirs. How similar are your story structures?

1.3. How similar are your neighbors' stories to yours? (previously I asked about how similar their *structure* is) Are any of them able to get fairly similar events but with a fairly different structure? Do you think that is possible?

1.4. Let's say that you work on this long enough that your story seems to perfectly connect the events of all the checks. Can you be sure that your narrative exactly represents the real-life events? Discuss and summarize your discussion.

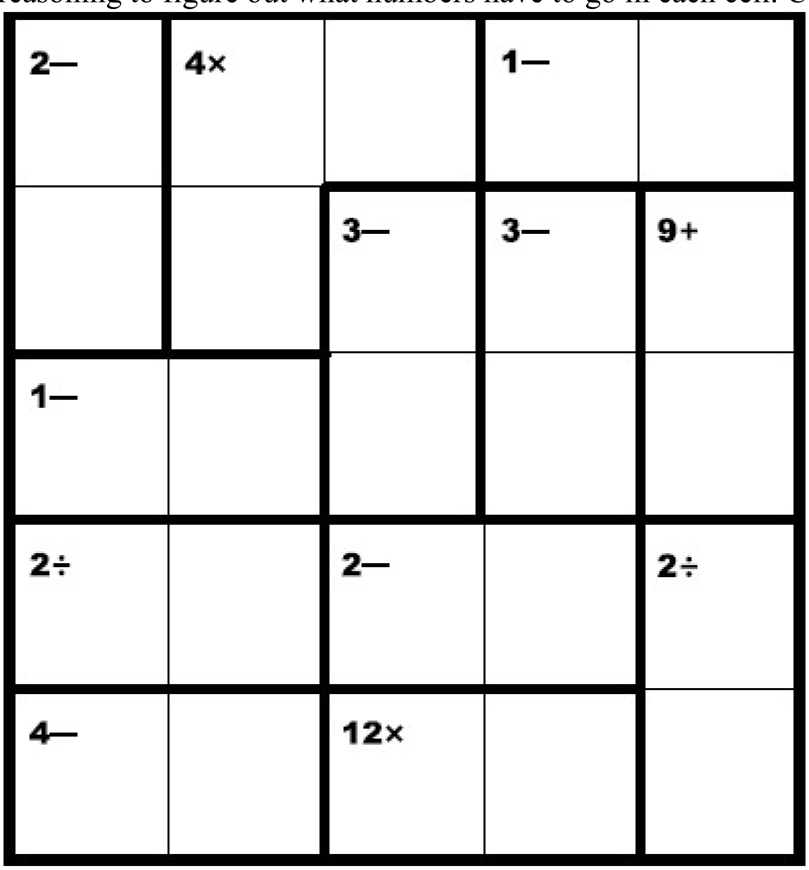
1.5. How is what you have done with these checks similar to the way science works. Are there examples in science where we make conclusions (to a reasonable degree of certainty) about something that we cannot directly observe?

1.6. Is there a key difference between the checks and normal scientific inquiry?

1.7. Experimentally determine a value for π. You may use anything circular you can find in the room, but no other lab supplies (i.e. no ruler, tape measure, etc.). Show your data, explain how you did it, and give your result. Pi was first measured in ancient Babylon and Egypt. Pretend you are alive in that time – you can't go down to the hardware store and buy a ruler or measuring tape because they don't exist. Come up with a way of measuring pi without any conventional means of measuring distances – you need to devise your own. Explain what you did, and how you used your measurements to determine pi. Also, try to be as precise you can, and explain what you could do to improve your precision.

1.8. Throughout the semester, I will give you various types of puzzles to help build various types of reasoning abilities - deductive reasoning, inductive reasoning, spatial reasoning, etc.. These reasoning abilities have been shown to be very important for success in STEM fields. They have also been shown to be buildable abilities in the same way that we can improve our physical strength and stamina. We just need to exercise those abilities.

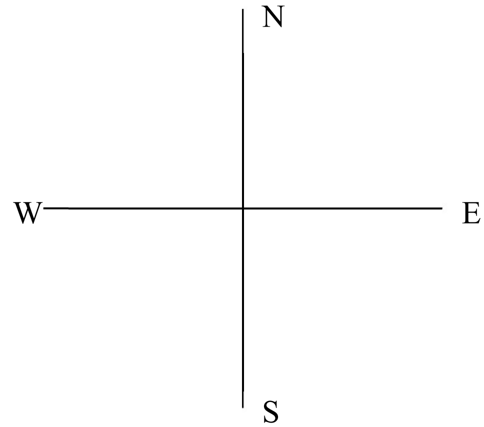
A “kenken” puzzle is similar to a Sudoku puzzle. In the 5x5 kenken shown below, each row must have one (and only one) of each number 1 through 5, and each column must also have one (and only one) of each number 1 through 5. Likewise, a 7x7 kenken would need to have one of each number 1 through 7 in each row and column. Unlike a Sudoku, a kenken provides clues by telling you how numbers within a certain region are related through a math operation. Groups of cells are grouped together with dark walls, with a mathematical operation shown relating the numbers in the cell. For example, the two cells grouped together in the top left have a difference of 2, so there is a “2-” in the upper left hand corner of that group. The three cells in the rotated L shape to the right of that group multiply together to make 4, so there is a “4x” in the top left of that group. Each clue can let you figure out what the possible options are for the cells in a group. By combining that knowledge with the limit that each row and column can only have one of each number 1 through 5, you can use deductive reasoning to figure out what numbers have to go in each cell. Complete the kenken.



2. Vectors

*Name*___________________________

2.1. Bob is standing in a field next to a tree stump. He walks 200 meters due east, then 100 meters due north. Draw an overhead view of his journey below, assuming the stump is at the origin.



2.2. How far away is he from the tree stump?

2.3. If Bob had wanted to go directly from the tree stump to where he is now, what direction should he have walked in to go straight there? (measured with respect to north and east)

You may not realize it, but you just did a vector addition problem. A *vector* is a quantity that has both magnitude and direction.

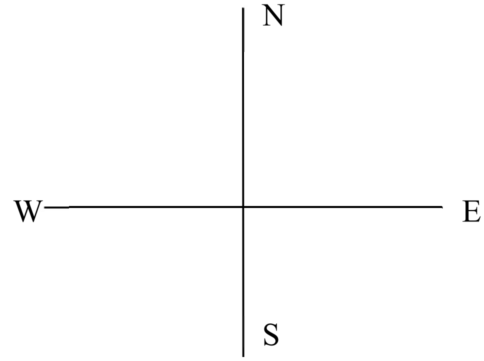
Scalars, by contrast, only have magnitude - no direction. If we know that a car is traveling at a speed of 50 miles per hour, that speed is just a scalar - since we do not know what direction the car is going into. But, if we know the direction, then the quantity can be represented as a vector.

Vectors can be drawn as arrows - the direction the arrow points in obviously indicates the direction of the vector, and the length of the arrow indicates the magnitude of the vector. This is tends to be the way that we naturally draw displacements - as you may have done in problem 1. It turns out though that we can do this for any vectors - whether displacements, velocities, electric fields, or anything else that is a vector.

By drawing vectors out graphically, we can see how multiple vectors can be combined together.

2. Vectors

Homer runs 20 feet to the east and then 10 feet to the west.

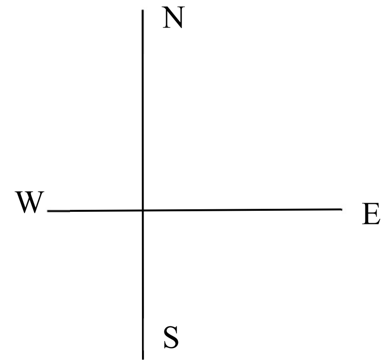


2.4. Draw his path on the plot to the right.

2.5. How far is Homer from where he started?

Stan and Kyle are playing football. They are initially standing right next to each other. Then, Kyle runs 10 feet due north, then 15 feet 30º north of east, while Stan stays put.

2.6. Draw Kyle's route on the plot to the right.



2.7. How far from Stan is Kyle? Don't use the law of cosines – while that would work for this problem, it wouldn't work well if you are combining a lot of vectors (instead of just two) – so we need to develop a method that works well no matter how many vectors you are combining.

2.8. Try explaining in words and formulas the *general* process you use for solving that problem.

2.9. With the path Kyle took in problem 2.6, what direction relative to north and east would Stan need to throw a football for it to go directly at Kyle?

Vector Components – Formal Explanation

To solve the problems from the previous page, you have to break the vectors down into "components". In the case of the problem above, your components are the portion of each vector that is easterly, and the portion that is northerly. For many vectors though, we may not be dealing with the cardinal directions (N, E, S, and W), but instead somewhat arbitrary x and y axes.

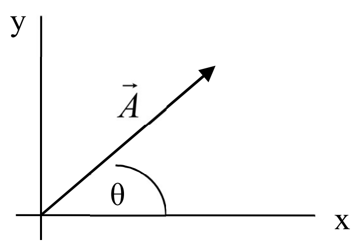
The good news is that even when dealing with x and y axes rather than cardinal directions, we can break vectors up in the same way - any vector can be broken into a component in the x direction and a component in the y direction.

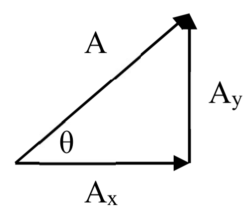
This then allows us to write a vector out in "vector component form". In this form, a vector is written as

$$\vec{A} = A_x \hat{x} + A_y \hat{y}$$

Note that when using a letter to symbolize a vector, we draw an arrow over the top of the letter, like this: $\vec{A}$. The above equation is a form for breaking a vector into some component in the x direction and a component in the y direction. We use "unit vectors" to indicate direction, but not magnitude (unit vectors have a magnitude of one). In the above equation, $\hat{x}$ is a unit vector pointing purely in the x direction, and $\hat{y}$ points purely in the y direction.

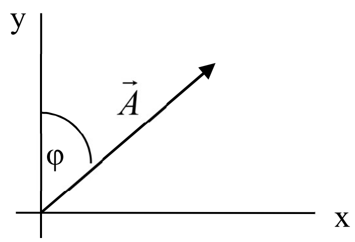
When broken into components, $\vec{A}$ has a portion A_x that points in the x direction, and A_y that points in the y direction, as shown below (next page).

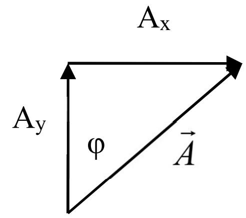




2.10. In the diagram above, the angle θ is measured from the x-axis. This is a common convention, but you may not always be given an angle measured from that axis. When this is the case though, what are A_x and A_y in terms of the magnitude of A and the angle θ? (for example, knowing that sin(θ) equals opposite over hypotenuse, what does sin(θ) equal in this case? Write that out as an equation, and then from that equation solve for either A_y or A_x, whichever shows up in the equation. Do the same with cos(θ))

The angle of the vector could be given measured from either axis (and not necessarily from the positive x or y axis). How you break the vector into its components depends on where an angle is measured from. Suppose that we know the angle of the vector A relative to the y-axis, as shown below (where the angle is now labeled as φ).





2.11. When the angle (φ)is measured from the y-axis, as above, what are A_x and A_y in terms of the magnitude of A and the angle φ?

Notice that the *magnitude* of a vector (how big it is – i.e. 20 meters, 300 mph, etc.) is the *hypotenuse* of the right triangle formed by the components of the vector.

In the following problems, treat east as the +x direction, and north as the +y direction.

2.12. A plane is flying at 500 miles per hour, 60º north of east. Write its velocity, $\vec{v}$, in vector component form - e.g., break its velocity into a component to the east (the x direction) and to the north (y direction).

2.13. If the plane were instead flying *south* of east rather than *north* of east, what would the velocity be written in component form?

Adding Vectors – Formal Explanation

In problem 10, in order to combine the two legs of Kyle's journey, you had to break the second leg of his journey up into components. The first leg of his route (10 feet due north) was essentially already broken into component form, since the component in the east direction was zero. To add the first and second legs of his route, you figure out how much each leg took him to the north – and combine those amounts, and figure out how much each leg took him to the east – and combine those amounts. This then gives you the net displacement to the north and the net displacement to the east (so the y and x components of the net displacement, $\vec{C}$), which you can combine with the Pythagorean theorem to get the magnitude of the net displacement.

This same process can be used to combine any vectors, whether they are displacements, velocities, forces, electric fields, or anything else. The key is to break each vector into components along our axes, and add up the components in each direction.

If we are adding two vectors, $\vec{A}$ and $\vec{B}$, to get a resultant vector $\vec{C}$,

$$\vec{C} = \vec{A} + \vec{B}$$

we could write this out in vector component form as follows:

$$C_x \hat{x} + C_y \hat{y} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y}$$

This looks very complex – because it's a condensed way of writing out a multi-step process.

To find vector $\vec{C}$, we actually need to find the x and y components of vector $\vec{C}$, C_x and C_y. These components are found by summing the components of the vectors $\vec{A}$ and $\vec{B}$:

$$C_x = A_x + B_x \qquad \text{and} \qquad C_y = A_y + B_y$$

If we are combining more than two vectors, additional vectors would just add in the same way,

$$C_x = A_x + B_x + D_x + E_x + F_x + \ldots.$$

2.14. A rotting zombie pulls himself out of his grave, staggers 30 feet due east, turns a little, and staggers 40 feet at 30° north of east. To get back to his grave, what direction should he go in, and how far?

2.15. A commercial airliner is flying at 450 mph, 20° east of north, relative to the surrounding air. The surrounding air though is blowing at 90 mph, 40° south of east, relative to the ground. The relative humidity of the air is 70%. How fast is the airliner flying relative to the ground, and in what direction? (hint: Draw out vectors representing the speed of the plane, and the direction it is being blown by the wind)

2.16. Kenny and Cartman are playing paintball blindfolded. They are initially standing back-to-back, facing away from each other. Treat it as if they are standing on exactly the same spot. Kenny then runs 8 meters due *west.* Meanwhile, Cartman runs 2 meters due *east*, turns left, and runs 6 meters 30° *east of north*. They then turn around and start shooting paintballs. Of course, they are both blindfolded, so they aren't coming anywhere close to hitting each other. How far apart are they when they start shooting? (tip: drawing an overhead picture of their paths might be very helpful)

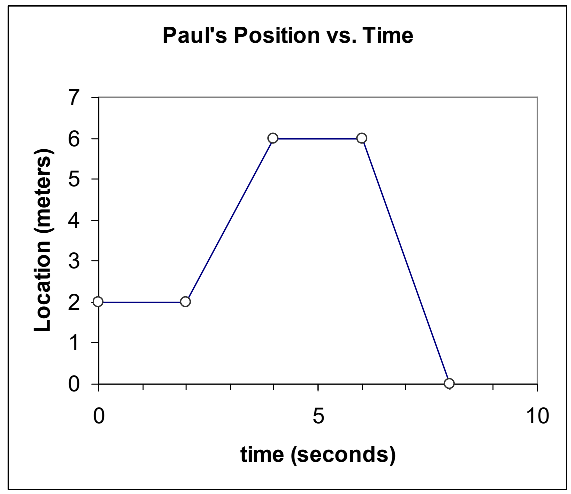
2.17. Cartman decides to cheat, and peaks through his blindfold to see where Kenny is. He then turns and aims at Kenny. What direction does he need to aim in to hit Kenny? (relative to the cardinal axes, i.e. something like 40° south of west)

3. Motion

Name___________________________

In physics, we typically define our own coordinate system for each individual situation. This includes not only a physical coordinate system – perhaps defined with x, y, and z coordinates – but also a time coordinate. By defining a time coordinate, we are defining when t=0 seconds is. If you imagine our time coordinate being measured with a stopwatch, by "defining when t=0 is", we are essentially saying when a stopwatch gets started. Negative times exist – they are just moments that happened before we started our stopwatch (before our t=0).

Paul is moving purely in one-dimension – along an east-west line. We can label this direction as anything we want. For convenience, let's call it the x direction. Let the positive x-axis point east, and the negative x-axis point west. The plot to the right shows his location along this x-axis as a function of time from t=0 to 8 seconds.

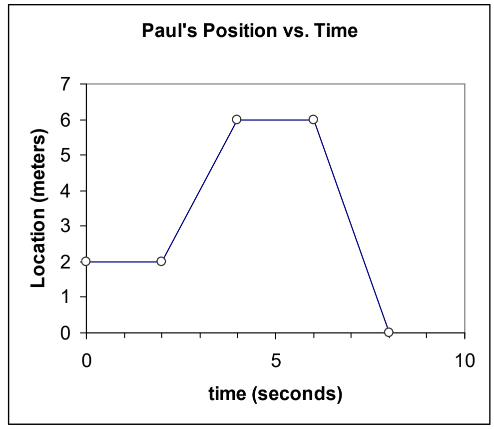


3.1. What would the slope of this plot tell us? (remember, slope of a line is the ratio of the rise over the run)

3.2. How fast was Paul moving at time t=3 seconds?

3.3. How fast was Paul moving at time t=5 seconds?

3.4. How fast was Paul moving at time t=7 seconds?



Speed vs. Velocity

You have learned that scalars are quantities that only have magnitude, and no direction, while vectors have both magnitude and direction. *When describing motion, the term "speed" is used to refer to the scalar measure of how "fast" something is moving, while "velocity" is used for a vector quantity – describing both how fast something is moving, and in what direction.*

So "speed" is the magnitude of the velocity, independent of direction. Velocity also specifies direction, by adding a unit vector (such as $\hat{x}$ to indicate in the x-direction, or $\hat{y}$ for the y-direction), and needs to be negative if it is in the opposite direction from the axis (so -$\hat{x}$ would indicate something in the negative x direction).

3.5. What were Paul's ***velocities*** at t=3 and t=7 seconds, written in vector component form (i.e. $\vec{v} = 10\hat{x}\, {}^{m}\!/\!{}_{s}$ would be a velocity, since the $\hat{x}$ is telling us the direction)

Displacement

3.6. At t=8 seconds, how far is Paul away from where he was at t=0 seconds?

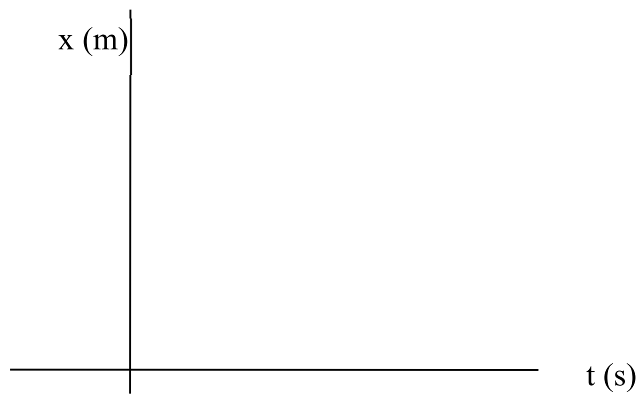
What you found above is called his "displacement", a measure of his *net* change in position. This is commonly written as $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$, the change in position from some initial location ($\vec{x}_i$) to some final location ($\vec{x}_f$). Like position, displacement is also a vector.

3.7. Is that the same as the *total* distance he walked? In other words, if he had a pedometer (like an odometer in your car) measuring how far he walks, what would it tell him for how far he walked in those 8 seconds?

3. Motion

Stan and Kyle are riding their big-wheel bikes. They are both moving purely in one-dimension (the x-direction, with +x being to the east). Stan's location (in meters) obeys the equation $x_S(t) = 3t + 4$, while Kyle's location is given by this equation: $x_K(t) = 2t^2 - 2$. They both start moving at $t=0$, but from different locations – prior to t=0 they are just standing still at those locations.

3.8. Plot both Stan and Kyle's locations as a function of time (plot x vs. t) on the same plot. Do not use a graphing calculator or plotting program - figure out what the plots look like yourself!



3.9. At what time(s) are they at the same place?

3.10. Who is going faster initially, just after $t=0$? Explain how you can tell.

3.11. Who is going faster at $t=3$ *seconds*? Explain how you can tell.

3.12. Is either of them traveling at a constant velocity? If so, who? Again, explain how you can tell.

3.13. At what time is Kyle's speed the slowest? Explain.

3.14. In problem 3.9, you needed to use the quadratic formula to find the time Kyle catches up to Stan. The quadratic formula gave you two solutions – are they both valid for this problem? If not, how did you determine which is valid?

4. Understanding Motion

Name________________________

In several experiments that you do this semester, you will be using an "Ultrasonic Motion Detector" (UMD) connected to a computer running "Logger Pro" software. Before you begin using this though, you need to understand how it works.

The UMD sends out sound pulses above our range of hearing (thus "ultrasonic"), similar to the echolocation that bats do. Those sound pulses travel at a speed of 343 $^{m}/_{s}$. Based on the amount of time it takes a pulse to return to the detector, the UMD can determine how far away an object is.

1. If an object is 1 meter away from the UMD, how long after sending a sound pulse will the detector receive the return pulse?

2. Why do you think it is important that a sound pulse returns to the detector before the next pulse is sent? (think of how the detector works, and how the software interprets the data)

3. You can control how frequently the UMD sends out pulses. If it sends out pulses at a rate of 20 Hz (20 pulses per second), how far away would an object have to be so that the return pulse from the object would not arrive until *after* the *next* pulse is sent from the UMD?

Part I

The UMD can not directly measure how fast an object is moving – it can only measure its location. It can determine how fast an object is moving though based on how the position is changing in time.

The Logger Pro software actually does some data smoothing by using more than just two data points for each velocity and acceleration calculation, but for now let's assume that it calculates the velocity at each moment by comparing the last two position readings, and calculates acceleration by comparing the last two velocities.

Let's assume that Logger Pro calculates velocities and accelerations using only the two most recent data points. The velocity at time 2 would then be given by $v_2 = \dfrac{x_2 - x_1}{t_2 - t_1}$

Notice that if the time interval is very small, this can be a reasonable approach for calculating an instantaneous velocity– since essentially this is average velocity taken over a very small time interval. Since the time intervals between data points are not infinitesimally small, however, this is really an average velocity over that time interval.

Logger Pro calculates acceleration in the same way, using the last two velocity calculations:

$$a_2 = \frac{v_2 - v_1}{t_2 - t_1}$$

4. To get a feel for how these are actually calculated, you are going to go through the process yourself. Do so for the two sets of data in the two tables below:

Time(s)	Position (m)	v ($^m/_s$)	a_{avg} (m/s^2)
0.01	0.5405	N/A	N/A
0.02	0.582		N/A
0.03	0.6245		
0.04	0.668		
0.05	0.7125		

Time (s)	Position (m)	v ($^m/_s$)	a (m/s^2)
0.01	0.541	N/A	N/A
0.02	0.582		N/A
0.03	0.625		
0.04	0.668		
0.05	0.713		

5. Take a closer look at the two sets of position vs. time data. Notice that they are very similar. How exactly do the two sets of data differ?

6. Assume the two sets of data were collected from the same moving object, but with two different measurement devices. Which measurement device has a greater precision, the one that gave the data in the first or second table? Explain.

7. The data for the two tables was made assuming constant acceleration, but with one of the tables having rounded data. ALL measurement devices have limits on their precision – this limit effectively rounds data off, as was done to one of the sets of data above. What does this rounding do to the velocity and acceleration calculations?

Rounding of data – such as what effectively happens due to limitations on the precision of a measurement device – can shift data points up or down, skewing the rate of change of the data.

Because of this, there is error introduced when Logger Pro creates the velocity plot from the position data, and especially when creating the acceleration plot from the calculated velocity data.

Part II

In this part you will be exploring how to create all sorts of position, velocity, and acceleration graphs using The Moving Man.

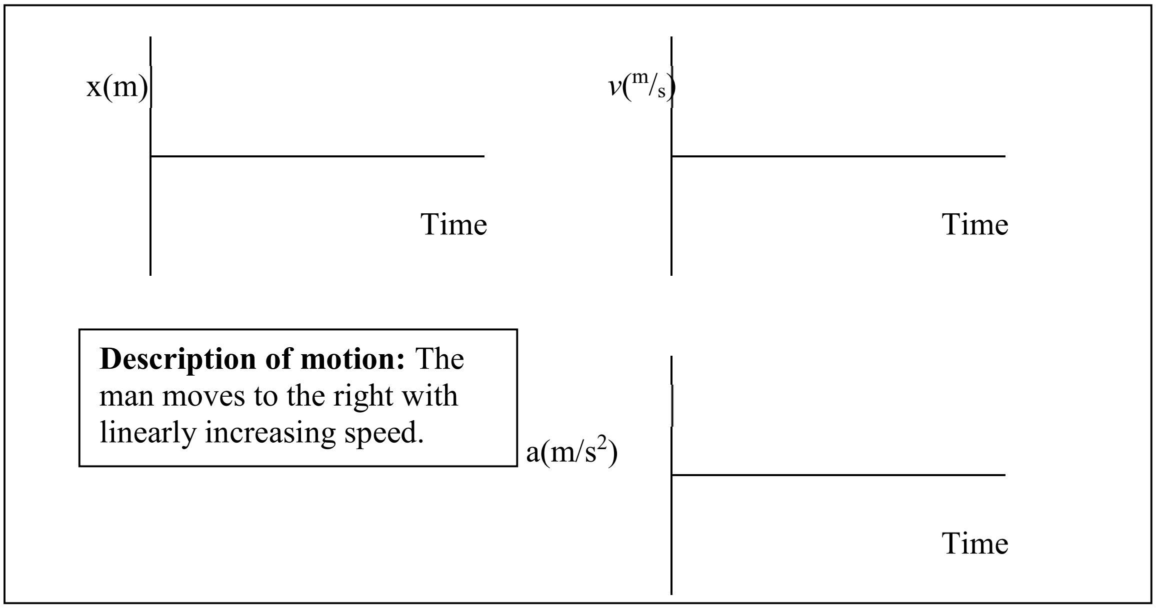
1. Go to Canvas, look under Files, and save "Moving Man.jar". You may need to install a Java Runtime Environment (download from the link here: https://www.java.com/en/download/)
2. Open up Moving Man.jar on your computer.
3. In the top left, click the Charts tab to display the graphs for position, velocity, and acceleration.
4. Along the left side, you can input starting values for the man's position, velocity, and acceleration. Try several different values and see how the plots come out. Feel free to remove the walls that are in the Moving Man's way if you want to plot further distances.

Part III

In this part you will be given one representation of motion and will be asked to predict three other representations. The four types of representations are plots of x vs. t, v vs. t, a vs. t, and a verbal description of the motion. ***After*** predicting three of the representations from the one that you are given, you will use the Moving Man to recreate the motion itself and *check* your predictions. The values that you put in for the Moving Man are not important, as long as the described movement and the graphs match with what has been given.

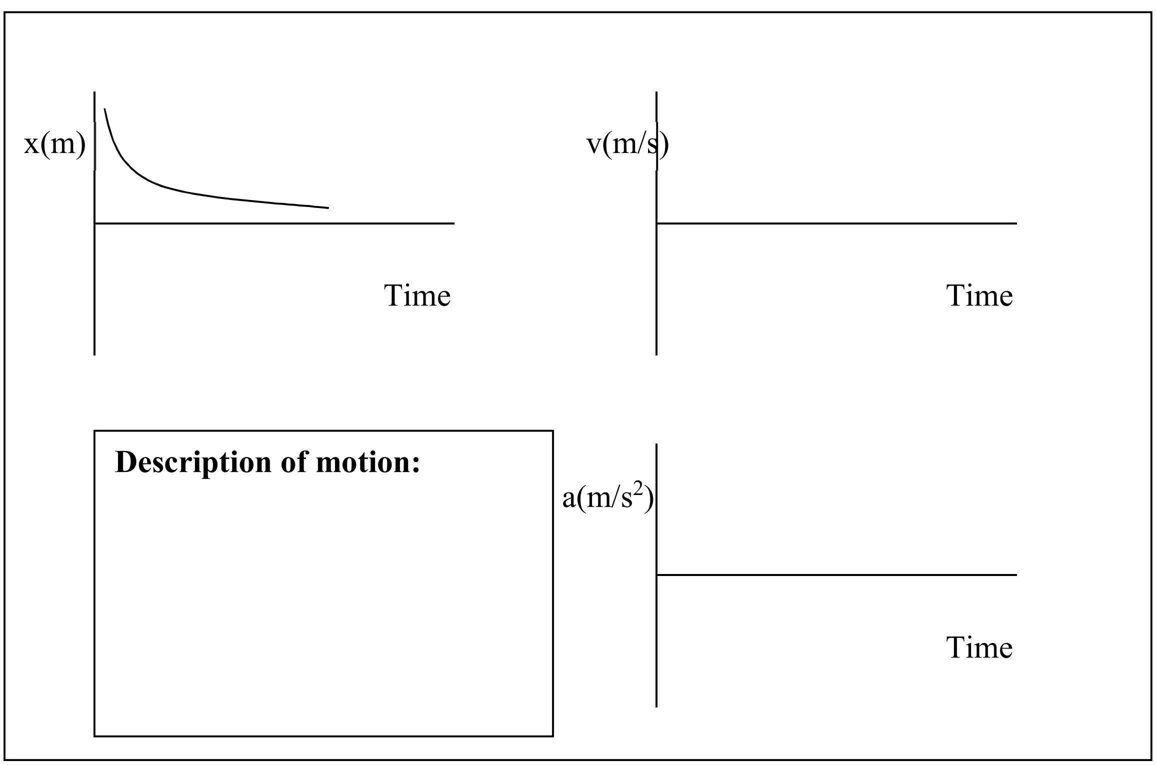
For each set of x, v, and a plots, keep in mind how velocity is related to position, and how acceleration is related to velocity. ***The +x direction is to the right, the negative x-direction is to the left.***

8.

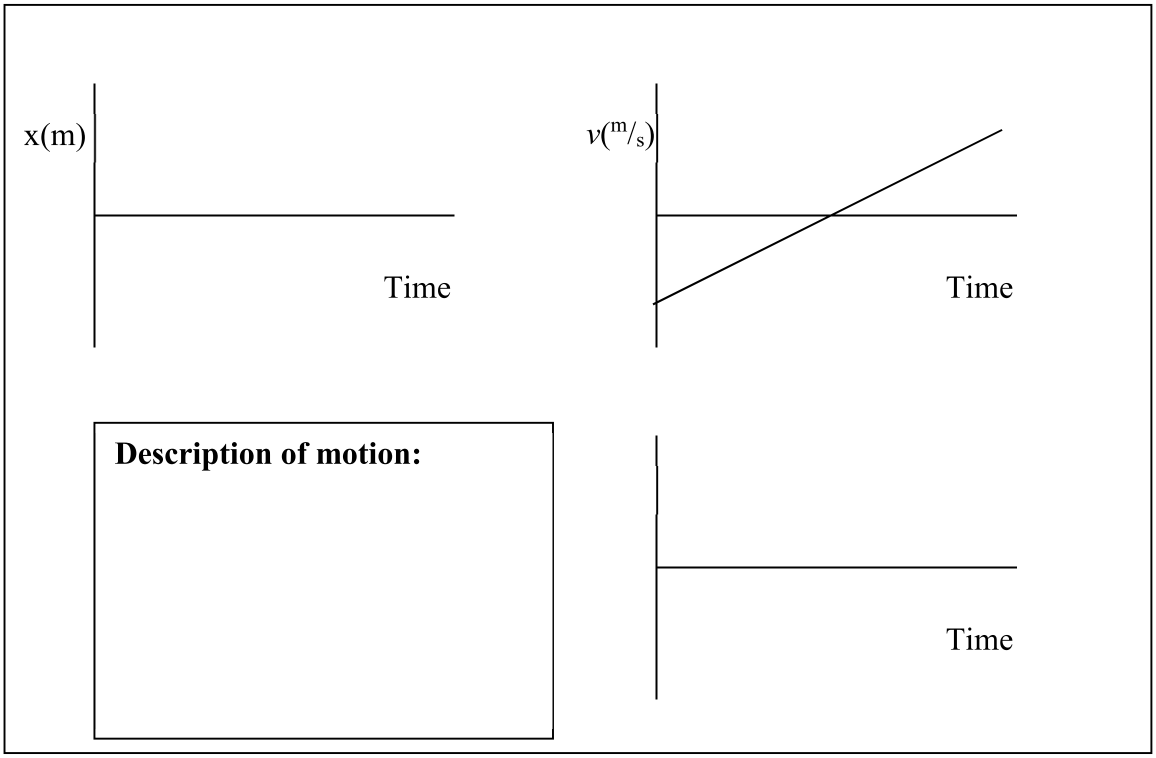


Do the graphs closely match your predictions? If not, why not?

9.



10.



11. Is it possible to have a positive position and a negative velocity? Explain (and if this is possible, also give an example - a hypothetical situation in which this is happening).

12. Is it possible to have a negative velocity and positive acceleration? Explain (and if this is possible, also give an example – a hypothetical situation in which this is happening).

13. Does positive acceleration mean something must be speeding up, and negative acceleration means something must be slowing down? Discuss and explain. Refer to specific plots from the last two pages, and also give a real life example to illustrate your explanation (in your real life example, make sure to specify what direction you are calling positive).

14. Is it possible to have a velocity of zero and a non-zero acceleration? Explain (and if this is possible, also give an example).

15. You are in a boat that goes at a constant speed relative to a river. You travel 15 km down the river in a time of 20 minutes, and back up the river in a time of 40 minutes. How fast is the river going relative to the land?

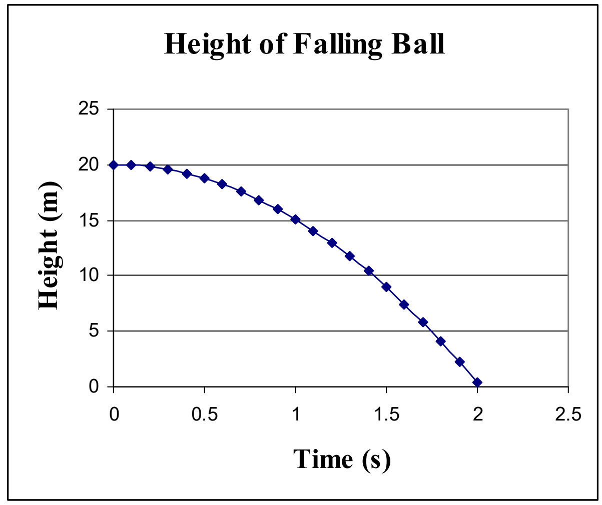
5. Instantaneous vs. Average

*Name*__________________________

In the last activity, you learned that average velocity is defined as $\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t}$.

You also learned that the *instantaneous* velocity (or speed) of an object equals the *slope* of the position vs. time (x vs. t) graph at that particular time.

Consider the plot to the right of the height of a falling ball as a function of time (note: vertical axis is height in meters, horizontal axis is time in seconds)



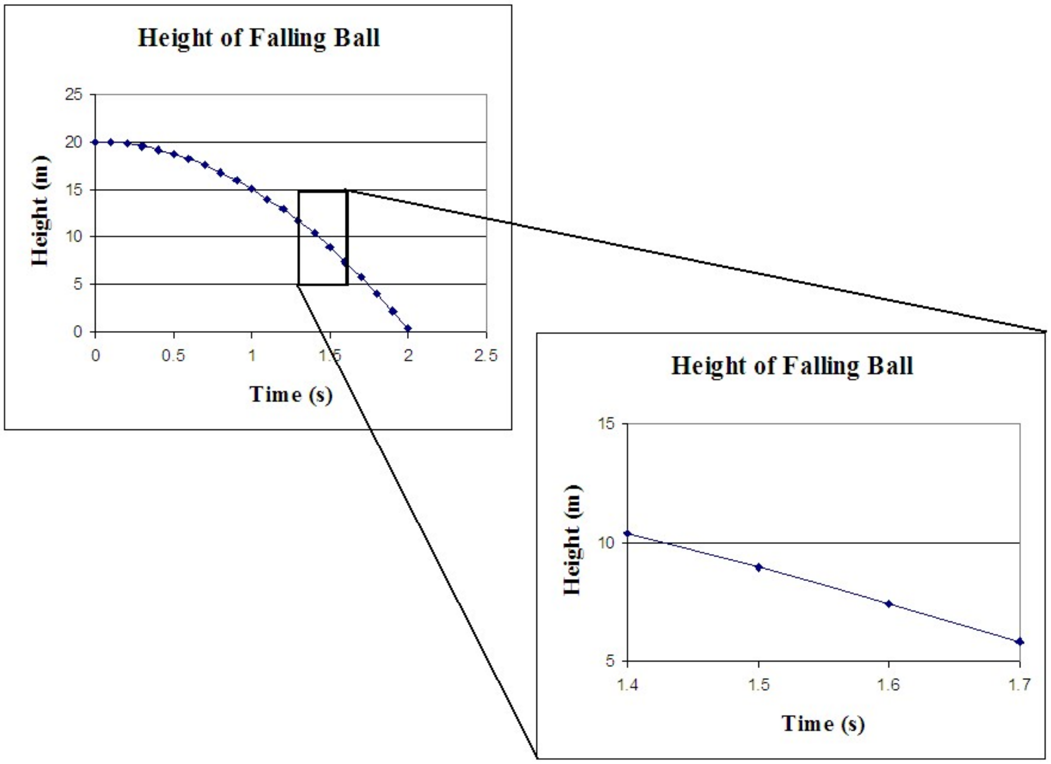
5.1. Is the velocity of the object constant? How can you tell from the plot?

5.2. What is the *average* velocity from the time the ball is released (t=0 seconds) until it lands (t=2 seconds)?

5.3. Would the average velocity be useful for predicting where the ball is at different times? In other words - is the average velocity all that we would need to predict the location at any point in time? Explain.

Lecture Notes:

If we zoom in on a portion of this graph, what we find is that on a small scale the curved shape of the graph gives way to a straight line:



This tells us something very important – if we want to know the *instantaneous velocity* for an object whose velocity is not constant, we can just find the *average velocity* over a very small time interval.

This revelation helped spur the development of calculus. What the above sentence is saying, is that in the limit that the time interval Δt→0, the average velocity (v_{avg}) over that tiny time interval equals the instantaneous velocity, v:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

This is how a derivative is defined. In our case of velocity, it is the change in position over the change in time measured over an extremely small time interval. ***This makes velocity the derivative of position with respect to time.*** A derivative tells us what the slope of a function is at ***any point*** on the graph.

An object whose velocity is changing is said to be accelerating. Acceleration is defined as the change in velocity with respect to time, and thus is related to velocity in the same way that velocity is related to position:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} \qquad \text{and} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Derivatives of polynomials: instantaneous velocity

You know that the average velocity of an object is its net change in position over some time interval, $v_{avg}(t) = \frac{\Delta x}{\Delta t}$. We could write this out formally as

5. Instantaneous vs. Average

$$v_{avg}(t) = \frac{\Delta x}{\Delta t} = \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Instantaneous velocity tells us what the velocity is at exactly one moment in time. So, we could use the same definition as for the average velocity, but we have to assume that the time interval Δt is infinitesimally small. Note that $x(t+\Delta t)$ means the x position at a time $t+\Delta t$.

Effectively, the instantaneous velocity is the average velocity over a time interval of zero.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Consider the case of the falling ball, whose position is given by $x(t) = 20 - 4.9t^2$.

5.4. What is the instantaneous velocity at some moment in time t? (use the limit definition above – do not use the short-cut for taking a derivative. Hint: Wait until after dividing by Δt to let $\Delta t=0$)

Derivatives

A derivative tells us what the slope of a function is at one specific location on its graph. If our graph is a function of time, that means the derivative would tell us precisely what the slope is at one instant in time.

Therefore, the derivative of position with respect to time tells us the *instantaneous* velocity – which you just found. We would write that like this (Leibniz notation):

$$v(t) = \frac{dx}{dt}$$

This says that the velocity at some moment t equals the derivative of position (x) with respect to time, meaning the change in x divided by the change in time, but over an infinitesimally small time interval.

5.5. Let's consider a more general case though – a position function $x(t)=A+Bt^n$, where A, B, and n are constants, and t is the variable that the position $x(t)$ is a function of. Use the limit definition of a derivative to find the derivative of this general polynomial function. Keep in mind that for the derivative, we will want Δt to go to zero. Show your work clearly below.

At this point, check with an instructor or TA.

5.6. Through the previous problem, you should have found a general rule for taking the derivative of a polynomial. Use that general rule now to take the derivative of the height of the ball as a function of time, $x(t) = 20 - 4.9t^2$. Do you get the same thing for the instantaneous velocity by using this "rule" for taking a derivative as when you used the limit definition?

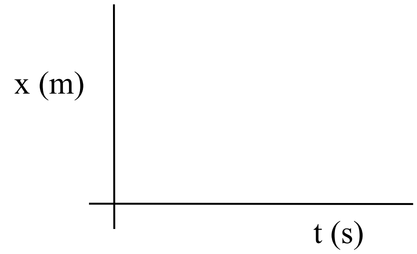
5.7. What is the instantaneous acceleration of the falling ball? Show your work.

5.8. Is the acceleration constant, or does it change in time?

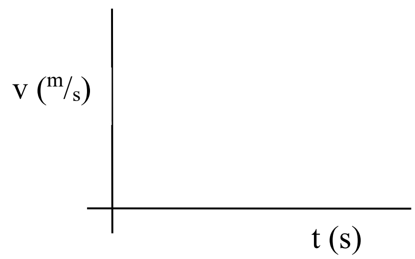
Graphical and Analytical x, v, and t

Let's say we are in a rocket car with a non-constant acceleration. For the first 4 seconds of the car's motion, the position of the car (measured in meters) as a function of time is given by $x(t) = 2t^3$.

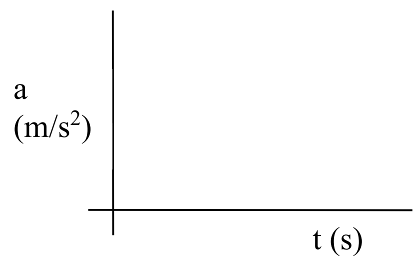
5.9. Graph the position as a function of time on the plot to the right.



5.10. Now plot the velocity as a function of time to the right. You can do this either by taking the derivative of x(t) and plotting that, or focusing on velocity being the slope of the position vs. time graph.



5.11. Now do the same process to plot the acceleration as a function of time.



5. Instantaneous vs. Average

6. Motion and Integrals

*Name*__________________________

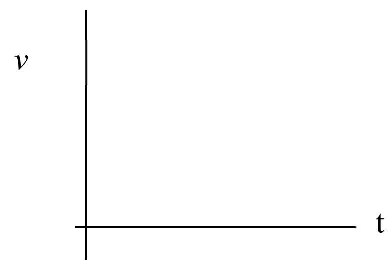
In the first part of this activity, you will further examine the relationship between position, velocity, and acceleration, now using anti-derivatives (aka integrals). In the second part, you will become more familiar with Logger Pro.

To date, you have learned that the instantaneous velocity is the derivative of position with respect to time, and that acceleration is the derivative of velocity with respect to time.

$$v = \frac{dx}{dt} \qquad\qquad a = \frac{dv}{dt}$$

Graphically, this means that the instantaneous velocity is the slope of the position vs. time plot, and the instantaneous acceleration is the slope of the velocity vs. time plot. Now we want to work in the reverse direction. If we have an acceleration vs. time plot, what does that tell us about the velocity? If we have a velocity vs. time plot, what does that tell us about the position?

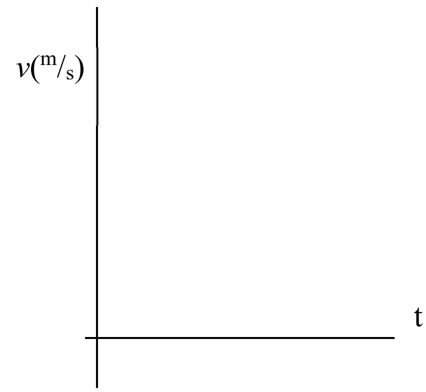
1. A plane flies due north at a constant speed of 500 mph for 30 minutes. Graph the plane's velocity vs. time on the plot to the right.



2. How far does the plane fly in those 30 minutes?

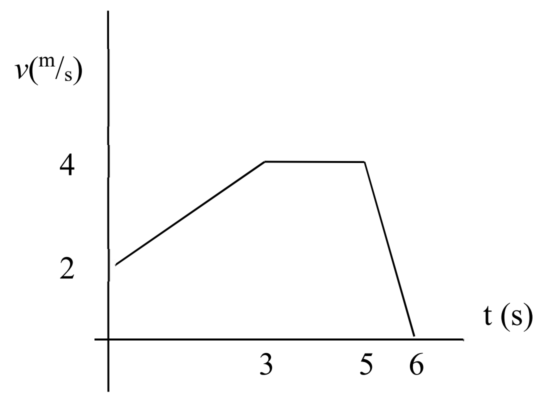
3. The previous problem you can figure out fairly easily by using the definition of average velocity ($v_{avg} = \frac{\Delta x}{\Delta t}$), since the plane is moving at constant velocity – so at every moment in time it has the same velocity. But, before any numbers are put in, look at what that equation gives you for the plane's displacement (Δx) in terms of velocity and time: $\Delta x = v_{avg}\Delta t$. Now, look at your graph – what does that quantity correspond to pictorially for your graph? In other words, could you figure out what the plane's displacement is *from the velocity vs. time graph, rather than from that equation?* How?

4. A different plane starts from rest, and accelerates at 3 m/s^2 for half a minute. Graph this plane's velocity vs. time on the plot to the right.



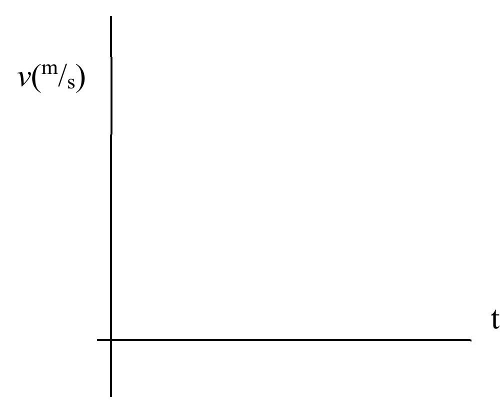
5. Now try to figure out *from the graph* how far this plane has traveled in that half minute. ***Do not use a "kinematic" equation!! Show (and explain) your work!***

6. If a runner has a velocity vs. time plot as shown to the right, what would his net displacement be from t=0 to t=6 seconds?



7. Let's consider a general case. A car is initially driving at an initial velocity (greater than zero) of v_i at time t=0, and accelerating at a constant positive rate of a. Graph its velocity as a function of time to the right.

8. At some time t, how fast is the car going? (think about how average acceleration is defined, and determine based on that)



9. Using your graph, and your answer to 8, what is the car's net displacement from $t=0$ to t? (in terms of a, t, and v_i. Determine this *graphically* by the area under the v vs. t curve. Show your work)

Your answer to problem 9 (if done correctly) is one of our fundamental kinematic equations – for finding displacement (Δx) when there is a ***constant acceleration***. It can be derived graphically, as you did, or through calculus (which you will do also).

Definite Integrals

Note: This material will be (or already was) lectured on, but is also provided here for your reference

Mathematically, the relationships between position, velocity, and acceleration can be written as follows (where for now we will assume only 1-d, so no vector notation):

$$v = \frac{dx}{dt} \qquad\qquad a = \frac{dv}{dt}$$

You have found that *graphically*, if you want to relate position (or rather *change* in position) to velocity, the relationship is that the area under the velocity vs. time graph gives you the displacement – the ***change*** in position. The same should be true for the relationship between velocity and acceleration then – that the area under the acceleration vs. time graph should give the change in velocity.

If we solve the first equation for dx, it becomes

$$dx = vdt$$

If we have a constant velocity, and are dealing with finite intervals (Δx and Δt rather than the infinitesimal dx and dt), this would be $\Delta x = v\Delta t$. In this form, this is more clearly saying that a displacement (Δx) equals the area under a plot of constant velocity vs. time (Δt). The infinitesimal form though ($dx=vdt$) is saying the same thing, but allowing for the possibility that velocity may not be constant. So, v could really be $v(t)$, a function of time.

This equation: $dx=vdt$, still says that the displacement (dx) is the area under the velocity vs. time plot, but over some infinitesimal time interval (dt). From this equation, we would also say that to find Δx over some time interval we need to take the *integral* of velocity with respect to time. We write this as

$$\Delta x = \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} vdt$$

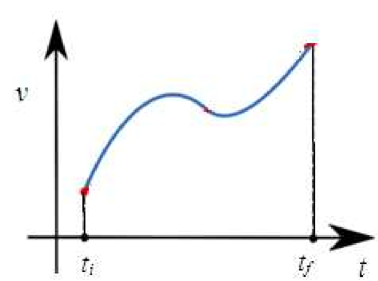


Figure 6-1

where v may be a function of time. Note that this is what is referred to as a *definite integral* when we are specifically looking at it over a certain range (such as from t_i to t_f). The definite integral on the right-hand side of the above equation equals the area under the velocity vs. time graph from t_i to t_f.

The integration process allows us to calculate the area under the v vs. t plot, while accounting for the possibility that v changes in time in such a way that the area may not be easy to figure out geometrically. The integral is summing up the product of v multiplied by very small time intervals (dt), over the range from t_i to t_f.

For example, let's say that we have a v vs. t plot that looks like that shown in Figure 6-1.

The net displacement from t_i to t_f should still be the area under the curve – but now the shape is not something that we can easily find the area of geometrically. For resolving this issue, Issac Newton and Gottfried Leibniz independently developed what we now call anti-derivatives, or integrals (note that for most purposes an anti-derivative is the same as an *indefinite* integral. A definite integral though is evaluated over a particular range. I will leave it up to the math department to focus on the distinction between anti-derivative and indefinite integral).

Bernhard Riemann later developed the idea of a Riemann sum, which is a non-calculus means of finding the area under a curve – but essentially doing the same thing as an integral.

Based on Riemann's approach, we could find the area under the curve in Figure 6-1 by breaking it up into short intervals in the time axis, finding the average velocity over those short intervals, and adding up the areas of all of the rectangular shapes produced by the short time intervals. This is drawn in Figure 6-2.

Breaking it up into smaller and smaller time intervals will increase our precision, but as long as we are using time intervals of some finite (non-zero) size, this is still ultimately just making an approximation.

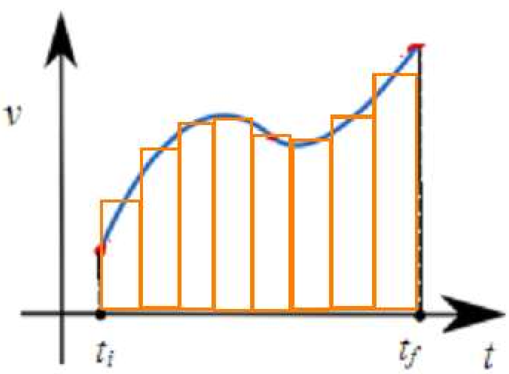


Figure 6-2

By integrating, on the other hand, we are finding a sum of the area of all the rectangles in Figure 6-2 – but with the time intervals being infinitesimally small. This is a similar concept used when taking derivatives – that if we zoom in close enough on a curved line, it becomes a straight line – so the slope becomes constant when looking at the slope over an infinitesimally small portion of the curve.

Integration though is the reverse process as that of taking a derivative. Whereas velocity is the slope of the position vs. time graph, $v = \dfrac{dx}{dt}$, the reverse of that says that position – or rather a change in position – is the *integral* of the velocity vs. time graph (to be precise with the terminology, we should say change in position is the *definite integral* of velocity over time).

$$\Delta x = \int v dt$$

Since integrating is the reverse process of taking a derivative, if we are only dealing with polynomial equations, then the reverse of our derivative rule should apply. For derivatives, our rule was as follows. If a function (*f*) can be written as

$$f = At^n$$

then the derivative is $\dfrac{df}{dt} = Ant^{n-1}$

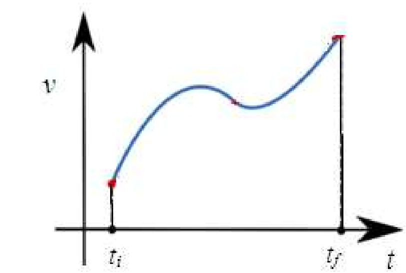
10. By finding the derivative of a few polynomial functions, and knowing that taking an integral is the reverse operation, you should be able to figure out a general rule for taking the integral of a polynomial.

For example, the derivative of the function $f(t)=t^2$ is $2t$. That means that the integral of $2t$ should equal t^2, since they are reverse processes. Write out the derivative of t^3, and perhaps one more function. By looking at those, come up with a general rule for taking the integral of a polynomial such as Bt^m over time: meaning determine $\int Bt^m dt$. Do your work on the next page.

Note that what you are figuring out there is a rule for the *indefinite* integral of a polynomial. It is *indefinite* because you are not evaluating it over a particular range. For clarification on this, ask for assistance from an instructor or assistant, since it is easier to explain verbally.

Back to Kinematics

Consider the plot of v vs. t to the right. You have seen that the integral of the velocity vs. time graph (which gives you the area under that curve) over some time interval tells you the net displacement over that time period. In mathematical terms, $\Delta x = \int v dt$.



11. Let's say that the v vs. t graph to the right is for a ball rolling. If you do ***not*** know where the ball started at time t_i (so you don't know x_i), will knowing its net displacement (Δx) be enough for you to figure out *exactly* where the ball is at t_f? Explain.

12. Acceleration was defined as $a = \frac{dv}{dt}$. The change in velocity should be the area under the a vs. t plot, the same as change in position is the area under the v vs. t plot. Integrate the equation $\Delta v = \int a dt$ to find an equation for the velocity as a function of time, assuming that acceleration is *constant* (not varying in time). You can assume that the initial velocity (at t=0) is v_i, and your limits of integration are t=0 to t=t (if you're not sure what that means, ask for assistance. This is a tricky issue, especially if the terminology is new to you).

13. You know that displacement is $\Delta x = \int v dt$. If velocity is a function of time, then you have to integrate that velocity over time to get the displacement. In the previous problem, you found an equation for the change in velocity over time when there is constant acceleration. Let that Δv equal v-v_i, and solve for v. That will give you an equation for the velocity as a function of time. Put that in for v into the equation $\Delta x = \int v dt$, and integrate to find the displacement as a function of time with constant acceleration.

At this point check with an instructor before going on. In particular, we want to compare what you got in problem 13 to what you got in problem 9. Both problems focus on finding an equation for a change in position (Δx) with a constant acceleration and initial velocity v_i. Problem 9 uses a graphical and geometric approach though, while problem 13 relies on calculus.

14. An object's acceleration may not be constant. If the acceleration is not constant, then can you use the kinematic equation $x_f = \frac{1}{2}at^2 + v_i t + x_i$ to describe its position as a function of time? Explain.

15. If a rocket car's acceleration is given by $a = (8 - 0.1t) m/s^2$, how far has it gone by the time the acceleration drops to zero?

7. 1-D Kinematic Problem Solving

*Name*__________________________

(SHOW ALL OF YOUR WORK, USE ADDITIONAL PAPER IF NECESSARY)

The Road Runner is cruising along in a straight line at a constant 30 m/s (*very* fast for a land bird). He runs right past the Coyote, who has strapped himself onto a rocket. Two seconds after the Road Runner passes the Coyote, the rocket ignites, and accelerates (in the direction the Road Runner went) at a rate of 8 m/s^2.

7.1. On a graph of position vs. time, plot both the Road Runner's (x_{RR}) and the Coyote's (x_C) position as a function of time (on the same graph).

7.2. How long after the Road Runner passes the Coyote will the Coyote catch up to the Road Runner? (SOLVE THIS ON A WHITEBOARD AND HAVE ONE OF US CHECK IT. You can transcribe everything for your own records if you want, but we won't require it)

7.3. How far will the Road Runner go past the Coyote's starting point before the Coyote catches him?

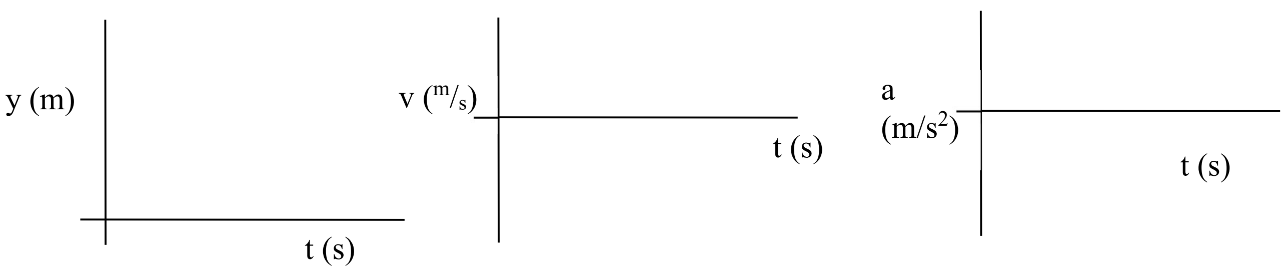
7. 1-D Kinematic Problem Solving

7.4. You toss a ball straight up in the air. If we define our positive y-axis to point upwards, indicated the sign (+, -, or zero) of both the velocity and acceleration at these moments:
a) just after throwing it upwards

b) when the ball is at the peak of its flight

c) when the ball is falling back downwards

7.5. Draw plots of the ball's vertical position (y) vs. time, velocity vs. time, and acceleration vs. time below. Treat up as the +y direction.



At this point, talk with an instructor about your answers thus far.

7.6. Kenny and Butters are standing next to each other at the top of a 10 meter tall cliff. Simultaneously, Kenny throws a ball straight upwards (but such that it will go down the cliff when it comes back down) with an initial speed of 30 m/s, and Butters throws a ball straight downward with the same initial speed. Assume gravity accelerates each ball downward at 9.8 m/s^2. How fast is each ball going when they land at the bottom of the cliff? Which (if either) is going faster?

7.7. How much longer after Butter's ball lands does Kenny's land? (SOLVE ON WHITEBOARD)

7.8. What is the peak height Kenny's ball achieves above him?

In some parts of the world, weather rockets loaded with silver-iodide are used to break hail apart in the atmosphere before it falls to the ground, in order to protect agricultural crops. A particular weather rocket launches completely vertically with an acceleration of 15 m/s^2 for 30 seconds, and then the engines cut out. To disperse the silver-iodide as much as possible, the rocket should be detonated at its peak height. You are in charge of the remote detonation of the rocket.

7.9. Make plots of the rocket's vertical acceleration, velocity, and position as a function of time (for an amount of time necessary to be able to answer the questions below).

7.10. How long after the launch should you detonate the rocket (keep in mind you want to detonate it at the highest possible point)? (SOLVE ON WHITEBOARD)

7.11. At what height will the rocket be when it blows up?

7.12. To try to win the $25 first place prize in South Park's annual sled competition, Cartman has constructed a $15,000 rocket sled. He is doing a test run down Main Street to see how well it works. As the rocket engine fires and he sleds towards downtown, the sled has a constant acceleration for 8.0 seconds, until the engine shuts off. At the moment the engine shuts off a parachute deploys, which begins slowing the sled at a rate of 5 m/s^2. Exactly 12 seconds after the rocket sled started moving, Cartman passes in front of the City Wok restaurant, 600 meters from his starting point.

What was the acceleration of the rocket sled for the first 8 seconds, while the engine was on? (SOLVE ON WHITEBOARD)

7.13. Cartman is unaware that Officer BarBrady is outside the City Wok, measuring the speed of cars passing by, just as they pass in front of the City Wok. The speed limit by the restaurant is 35 mph. The penalty for speeding is $50 for every 5 mph over the speed limit (so if you go between 40 and 45 mph you would get a $50 ticket, between 45 and 50 would be a $100 ticket, etc.). Will Officer BarBrady give Cartman a ticket? If so, what will his penalty be? (note: 1 mph = 0.447 $^{m}/_{s}$)

8. Reaction Time

*Name*______________________

*Group*____________

Part I

In this activity, you will use your understanding of kinematics to figure out your reaction time. We will describe to you the experimental setup you will use, but you will need to figure out how to use the data you gather to determine your reaction time.

Your setup should consist of a large sheet of poster board (or cardboard) hanging from a horizontal rod. The bottom of the poster board must be at least 3 feet above any surface (such as a table).

On one side of the poster you should make a mark with a pencil exactly 25 centimeters up from the bottom. A long piece of string should be taped to the bottom of the poster board directly below the pencil mark.

One of your group members (member A) will hold a ping pong ball at that mark, and they will eventually drop it from that point. Another group member (member B) should be on the other side of the poster board, such that they will not be able to see when the ball is dropped. Group member B should hold one hand flat horizontally (palm up) about 2 centimeters away from the hanging string (2 cm away in the direction such that their hand is further away from where the ball will be dropping).

Group member B should start with his or her hand only an inch below the bottom of the poster board. Without letting member B know that he/she is dropping the ball, member A should drop the ball. Member B should try to swat the ball with the hand that is being held next to the string.

If member B can not consistently hit the ball, (s)he should lower that hand another inch or two, and try again. Keep trying this process, until you find a height at which each member of the group can consistently swat the ball (the height will be different for each member).

Figure out how to use that height to determine the "reaction time" for each member. For this, we are defining the reaction time as the time it takes for the person to react and move their hand that short distance necessary to hit the ball.

Show all of your work on this and the next page, explaining how you are doing your calculation.

8. Reaction Time

8. Reaction Time

Part II

Now let’s consider a more important issue of reaction time. Imagine that you are traveling on the highway at 70 mph, with a car in front of you traveling the same speed. You need to determine the minimum distance behind the car you should be so that you will not run into it. The car in front of you slams on the breaks, you take time to notice that it is breaking (your reaction time), some time to process your observations and realize that you need to slam on the breaks, and time to move your foot from the accelerator to the brake pedal and push it to the floor.

In your reaction time experiment, you knew that you needed to move your hand to swat the ball – so there was no mental processing necessary as far as figuring out what to do based on an observation. If you observe the brake lights of the car in front of you, it may take some time for you to process that information – so you may want to consider adding some time to account for that (you will have to estimate). Likewise you will need to estimate the time to move your foot to the brake pedal, or find online the results of any experiments that attempted to measure that.

You can assume that both vehicles will decelerate at the same rate.

Based on all of that, determine how many feet you should leave between the front of your car and the back of the front car so that you do not run into it. Also figure out approximately how many average car lengths this works out to, explaining how you figured that out.

Hint: drawing plots of the velocity vs. time for both cars (on the same plot) can be a very useful tool in solving this.

8. Reaction Time

9. Projectile Motion

*Name*_________________________

This activity involves multiple parts. In this first part, I have written up solutions to two projectile motion problems. Your task is to evaluate my solutions. Is it completely right? If it's wrong, ***where did I make a mistake***? If it is wrong, don't just tell me what the right answer should be – you need to find where I made a mistake, and tell me what I did wrong. And perhaps I made more than one mistake – ***find them all, and show what I should have done instead!***

The point of this is twofold – to get a little more practice with 2-d kinematic problem solving, and also to get a feel for how to present your work so that it is coherent to someone else.

Note that the acceleration due to gravity is $g=9.8\ m/s^2$ *or* $32\ ft/s^2$, and 1 foot = 0.3048 meters.

After these two problems where you grade me, you will then proceed to an activity involving an experiment.

Part I

1. A golf ball is hit with an initial speed of 80 mph at an angle of 30°above horizontal. It lands on top of a hill, 10 feet above where it was originally hit. How far away from where it started does the ball land? (tip: if you can't figure out what I did in this one, try the second problem and then come back to this one)

$$80\left(\frac{5280}{3600}\right) = 117$$

$-16t^2 + 101t - 10$ $\qquad$ $-16t^2 + 58t - 10$

0.1 $\qquad$ 0.18

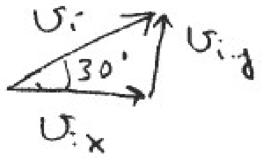
$-16t^2+$ $\qquad$ $101(0.18) = 18.2$

2. An angry monkey is sitting in a tree 20 feet above the ground. He throws a banana at another monkey, named Burt. He threw the banana with a speed of 20 $^{m}/_{s}$, at an angle of 30° above horizontal. The banana misses Burt, and continues on its flight until it eventually lands on the ground. How far away from the angry monkey (horizontally) does the banana land?

I'll first convert the monkey's elevation to meters

$$y_i = 20\,ft\left(\frac{12\,in}{1\,ft}\right)\left(\frac{2.54\,cm}{1\,in}\right)\left(\frac{m}{100\,cm}\right) = 6.096\ m$$

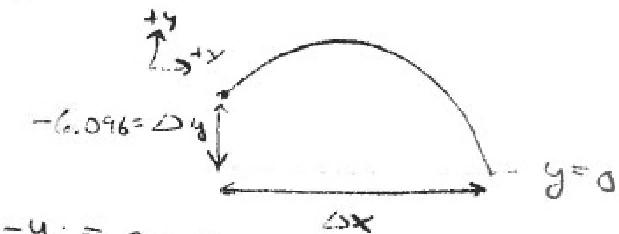
I can treat the x and y motion separately.
The y-direction can tell me how long the banana is in the air, from $\Delta y = \frac{1}{2}a_y t^2 + v_{iy}t$



$\sin 30° = \frac{v_{iy}}{v_i}$, so $v_{iy} = v_i \sin 30°$

$\cos 30° = \frac{v_{ix}}{v_i}$, so $v_{ix} = v_i \cos 30°$

$a_y = -9.8\,\frac{m}{s^2}$, negative since I'm calling up positive y



The banana lands when $\Delta y = y_f - y_i = 0 - 6.096 = -6.096\,m$

so $\Delta y = -6.096 = \frac{1}{2}a_y t^2 + v_{iy}t = \frac{1}{2}(-9.8)t^2 + \underbrace{20\sin 30°}_{10}\,t$

So $-4.9t^2 + 10t + 6.096 = 0$

$$t = \frac{-10 \pm \sqrt{10^2 - 4(-4.9)(6.096)}}{-2(4.9)} = \begin{matrix} -0.49\,s \leftarrow \text{not valid} \\ 2.53\,s \end{matrix}$$

so the banana lands 2.53 seconds after he tosses it

Now that I know how long the banana is in the air, I can look at the x-direction to figure out how far it has travelled horizontally

$\Delta x = \frac{1}{2}a_x t^2 + v_{ix}t$, where $a_x = 0$ since gravity only acts vertically

$\Delta x = v_{ix}t = v_i \sin 30° t = 20 \sin 30° t = 10\frac{m}{s}(2.53\,s) = 25.3\ m$

Part II

To get some proper practice with projectile motion, ideally we would be launching some projectiles of our own. Since I don't want any of you making a mess in your rooms, we'll stick to virtual projectiles. Use the following link: https://phet.colorado.edu/en/simulation/projectile-motion. Click the Play button, then click "Lab". Make sure the projectile is set to **cannonball** and gravity is at 9.81 m/s^2. Set the cannon height to 10 m (this is done by dragging the cannon up).

Before class, I used the simulator to launch a cannonball at 0°. I've recorded the range of the cannonball for all of you, but you need to determine what value I had for the velocity. This will be the velocity you need to keep the cannon set to for the rest of the activity.

Ask one your instructor or a TA for a launch angle – which is the angle that your group will launch the ball at. You will need to determine how far away from the launch point the ball will land when fired at that launch angle. After you have finished your calculation, you will test your result in the pHet simulator. DO NOT USE THE SIMULATOR UNTIL AFTER YOU HAVE FINISHED YOUR CALCULATIONS.

First, record the data we give you (which should be on a whiteboard).

How far did the ball go when launched horizontally? ________________

How high was the launch point above the landing point? _______________

What is your group's angle? ___________________

Using the information above, determine how far the ball should go with your given launch angle. Show all of your work on these two pages, and make sure it is clear enough that a student new to projectile motion could easily follow what you did and why (i.e. so put in brief statements saying why you are doing what you are doing. Drawing pictures, using brief comments will also help make things clearer). DO OUT YOUR WORK ON A WHITEBOARD, AND SUMMARIZE BELOW (you can transcribe everything if you want for your own records, but we won't require it).

9. Projectile Motion

More work space for this problem (there is another problem on the next page)

Part III

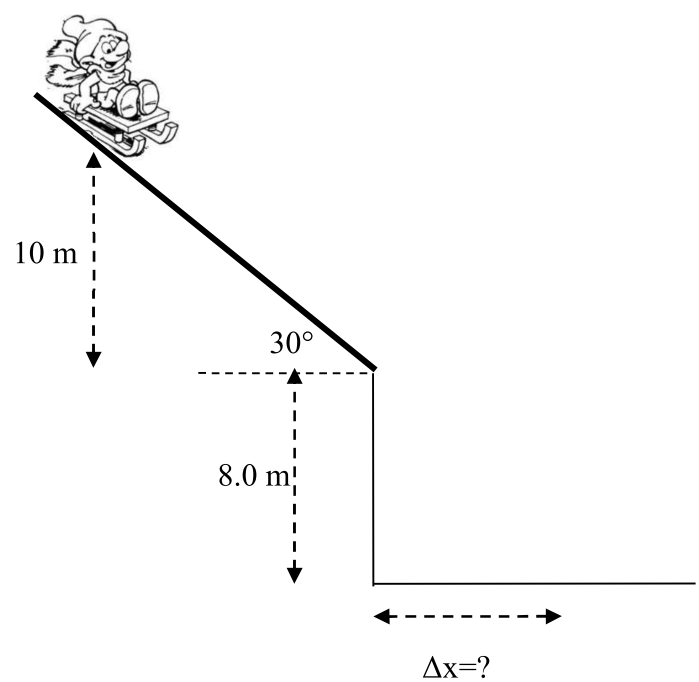
Now try to put this all together with a much more complex problem.

A smurf is sitting on a sled at the top of a 10 meter tall hill, as shown below. Once he starts sliding, he accelerates down the slope at a constant rate of 5 m/s^2, continuing to accelerate at that rate until the moment he reaches the edge of a cliff. If the cliff is 8.0 meters tall, how far away from the base of the cliff does the smurf land?

This problem must be solved in steps – you can not do it all in one step. It can also be helpful to identify important moments in the picture, to help keep track of different moments in time during the problem.

First, solve the problem. Show your work clearly. On the next page is a solution that I wrote. Again I did something wrong in my solution, and you need to figure out what.

WORK ON THE WHITEBOARD, summarize below



Now figure out what I did wrong. I did something very, very wrong. To further emphasize the importance of presenting your work clearly, my solution is not written with the intent of making it easy to follow (to illustrate what ***not*** to do). It would be a lot easier to figure out what I was thinking if I had put in a sentence or two, wouldn't it?

$$v_f^2 - v_i^2 = 2a\Delta x$$

$$v_f = \sqrt{v_i^2 + 2a\Delta x} = \sqrt{0 + 2(5)(20)} = 14.14\,\tfrac{m}{s}$$

$$v_f^2 - v_i^2 = 2a\Delta x$$

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - 14.14^2}{2(-9.8)} = 10.2\,m$$

10. Minimization and Maximization Problems

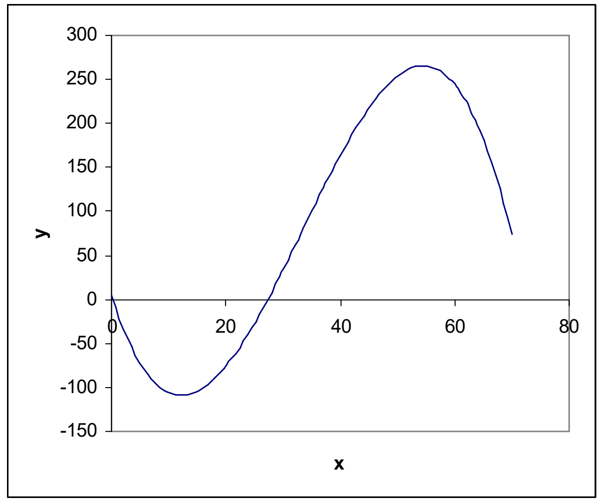
*Name*______________________

Note: This is the extra credit activity for the first exam

Often in science and engineering, it is desirable to determine what value(s) of one parameter will maximize or minimize another variable. An example that we will work on later this semester is figuring out what angle a projectile should be launched at to maximize its range.

Such "maximization/minimization" problems require calculus, and understanding exactly what a derivative is. Before introducing physics into this task, let's first just look at pure math. Consider the equation $y = -0.01x^3 + x^2 - 20x + 4$, which is plotted below.

Notice that at least in the range of x values plotted, there is a minimum value of y, and a maximum value of y. These are actually a "local minimum" and "local maximum". As x continues to get larger above 80 or so, y drops to values lower than the "local minimum" that occurs around somewhere between x=10 and 20. Likewise, for sufficiently negative values of x, y will rise above the local maximum of y (~250) that occurs around x=55 or so. That is why they are referred to as *local* minimums and maximums.



These local minimums and maximums occur at precise locations that we can find. How? Consider a few things:

1. What is the slope, dy/dx, at the local minimum and local maximum?

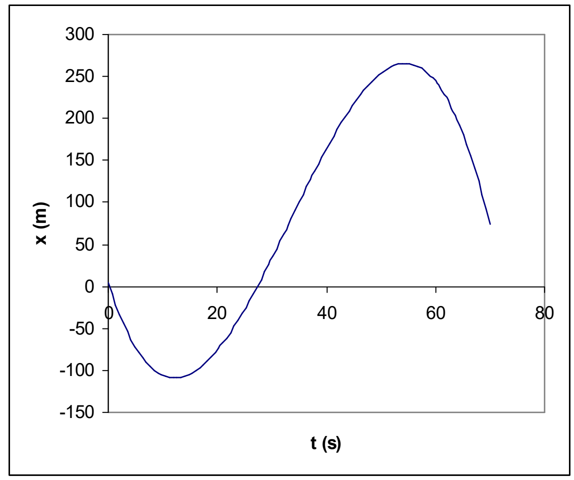
2. Based on your answer to the previous question, you should be able to find the exact values of x for the local minimum and maximum shown above? Do so below.

10. Minimization and Maximization Problems

Both local minimums and local maximums have the same condition on the slope (which in this case is dy/dx). So, finding a value of x that satisfies that condition does not tell us whether y is a maximum or a minimum at that value of x.

In this case, since you already have a plot of y(x) to look at, you can tell which of the two values of x that give y a slope of zero is the local minimum and which is the local maximum. But, what if we did not have that plot? How would we tell whether we have found a local maximum or minimum by setting the slope equal to zero?

To figure that out, let's start introducing some physics. Let's say that plot above is not a plot of y vs. x, but rather of position (x) vs. time, as now shown below.



3.a. What is the velocity of the object at the local minimum and at the local maximum?

3.b. Does this make sense in the context of the requirement that the slope (which in this case is dx/dt) should be zero at a local maximum or minimum? Explain.

4. Is the acceleration of the object at the local *minimum* positive or negative? (determine this based on looking at the graph, and thinking about how the velocity is changing) Explain.

5. Based on the plot, is the acceleration of the object at the local *maximum* positive or negative? Explain.

6. To verify your last two answers, find the formula for the acceleration, and calculate the acceleration at the local minimum and maximum. The plot is for the equation $x(t) = -0.01t^3 + t^2 - 20t + 4$.

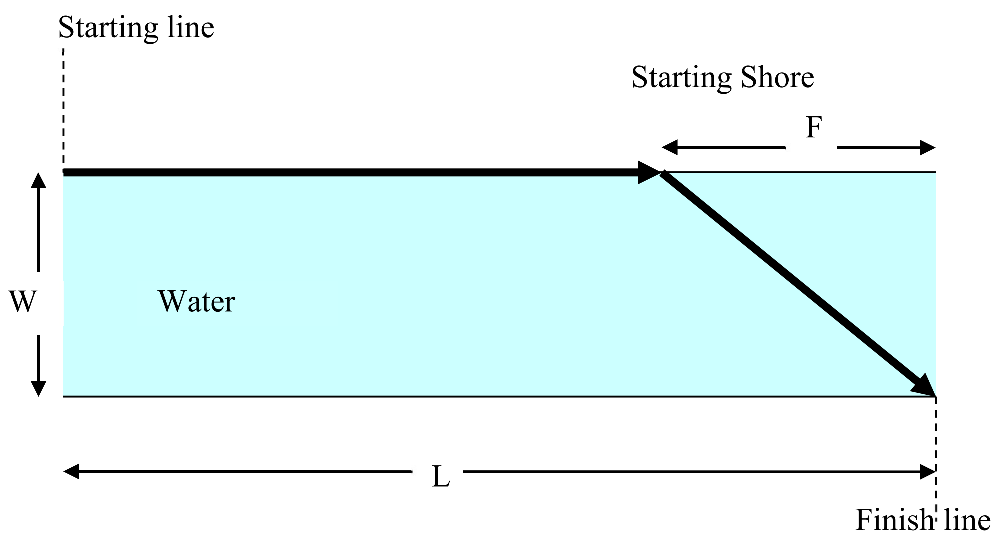
7. Let's say you have some arbitrary function, y(x). You find that the slope of y vs. x is zero at x=3. Based on your answers to problems 4 and 5, how would you determine if x=3 is a local maximum or minimum (without plotting the function)?

8. Consider the equation $y = 3x^2 - 30x + 20$. For x values in the range $0 \le x \le 8$, what are the minimum and maximum values of y?

9. I've invented a new kind of race, which I call "The Drowning Farmer". The race starts on the shore of a river, and is broken into two stages. Ultimately, racers must try to get to a finish line on the opposite side of the river. When the race starts, racers must first run along the shore doing a "farmer's walk" – which means they must carry 40 pound weights in each hand. At any point they choose, they can stop doing the farmer's walk (drop the dumbbells), and swim across the river directly towards the finish line. They are not allowed to come ashore on the other side of the river before the finish line – so once in the river, their quickest path will be a straight line from the point they jump in the river directly over to the finish line (the current in the river is negligible).

Kenny is competing in the race, and wants to figure out his best strategy. What he wants to know is at what point he should enter the river. He knows that he can do a farmer's walk at a speed (v_f) of 6 km/hour, and can swim at a speed (v_s) of 3 km/hour. If the length (L) between the start and finish lines is 2 km, and the river has a width (W) of ¼ km, at what point should Kenny enter the river (measured as F in the diagram below, measured from the end of the race)? (of course, in reality his arms would get increasingly tired the further he does the farmer's walk, which would make the swimming portion quite difficult – but we will ignore that)

Hints: Don't put any numbers in until the very end. You want to find the value of F that minimizes the time he takes for the total race. So, develop an equation for the total race time (T) in terms of F and other parameters, and find what value of F minimizes the time. See the diagram of the race below. You can work on the next page too.



10. What is the total time Kenny will take with the value of F you found?

11. Sketch a graph of the total time (T) vs. F for values of F from 0 to 2 m. You can use *Graphical Analysis*, *Matlab*, or a Graphing calculator. This should let you test your result, to see if T does in fact it reach a minimum at your value of F. (sketch the result below)

12. A projectile is thrown straight up. What is its vertical speed, ${}^{dy}\!/_{dt}$, at the "turning point"? Discuss the relevance of this to the maximum/minimum of a function.

13. You are part of a student engineering team competing in a flywheel-car race. When the race starts, each team can begin winding a flywheel that will then propel their car towards a finish line. Your team has spent weeks building its car. The flywheel is capable of giving the car a constant acceleration of 0.5 m/s^2, but only for as long as you wind the flywheel for (so if you spend 10 seconds winding the flywheel, the car will accelerate at 0.5 m/s^2 for 10 seconds once you release it). Once the flywheel is finished unwinding (after the car has accelerated for the same amount of time as you spent winding it), it will continue rolling at constant velocity.

The "race" starts once you begin winding the flywheel on your car – so the time spent winding it counts towards your total time. You need to figure out how long you should spend winding the flywheel once the clock begins ticking to get your car to the finish line as fast as possible. If the race is a length (L) of 100 meters, how long (t_w) should you spend winding the flywheel to minimize the total race time?

14. To check your answer, check the limiting cases with your equation for the total race time. What does your equation say the total race time will be if you wind the flywheel for 0 seconds, or an infinite amount of time? Does your equation make sense in those limiting cases?

15. Sketch a graph of the total time (T) vs. t_w for values of F from 0 to 30 seconds. You can use *Graphical Analysis*, *Matlab*, or a Graphing calculator. This should let you test your result, to see if T does in fact it reach a minimum at your value of F. (sketch the result below)

16. How long does the total race take, with your ideal winding time?

11. 2-D Kinematic Problem Solving I

*Name*______________________

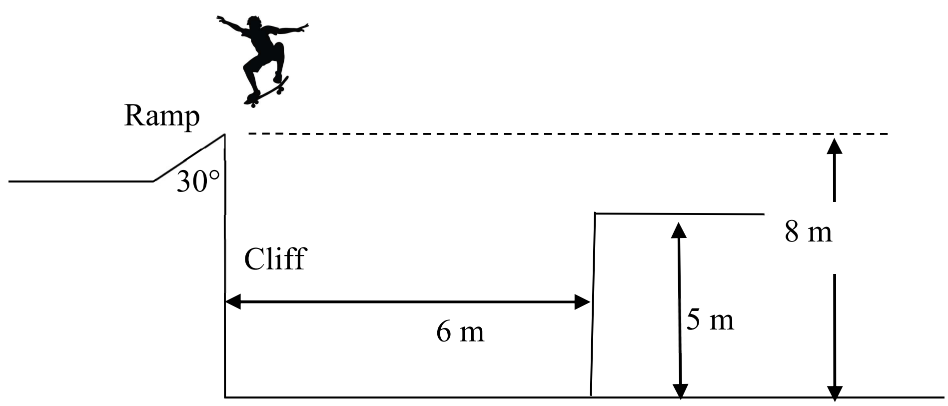
These problems all revolve around 2-d kinematics, and thus will require both an understanding of vectors and kinematics to solve them. Keep in mind that the horizontal and vertical motion of objects can be treated separately (in some situations this is not true, but we will not deal with such situations in this course). (note: some of these problems say to solve on the whiteboard. If you have found that you like doing that, you can do that for all of them, and we will keep track of which problems each group has completed on the boards. For such problems, you can just write a brief summary of how you solved the problem if you do not want to transcribe the entire thing)

1. A baseball player hits a baseball at waist level (about 3 feet above the ground) such that it leaves his bat with an initial speed of 90 mph at an angle of 30° above horizontal. If the outfield fence is 330 feet away and 20 feet high, will his ball clear the fence? If so, by how much? If not, where will it land, or hit the fence? (note that the acceleration due to gravity is 9.8 m/s^2, which equals 32.2 ft/s^2)

2. A field goal kicker is attempting a 40 yard field goal. To make it over incoming defenders trying to block his kick, he kicks the ball at an angle of 45°. If the bottom of the goal post is 10 feet above the ground, what must the initial speed of the football be to clear the uprights? SOLVE ON WHITEBOARD

3. Stan and Kyle are playing football. Kyle is running directly away from Stan at 8 $^{m}/_{s}$ when Stan throws the football at an initial speed of 25 $^{m}/_{s}$, at an angle of 30° above horizontal. Assuming Kyle catches the ball at the same elevation at which it was initially released, how far apart were Stan and Kyle *when Stan threw the football*? (hint: draw a picture of what is happening to help keep track of everything) WHITEBOARD

4. A skateboarder is jumping across a chasm that is 8 meters deep and 6 meters wide, as shown below. He is trying to land on a ledge on the other side of the chasm, which is 5 meters tall. He will be leaving a ramp such that he launches at an initial angle of 30° above horizontal. What should his ***minimum speed*** be so that he can clear the chasm safely?



12. 2-D Kinematic Problem Solving II

*Name*______________________________

1. Stan kicks a soccer ball such that it leaves the ground with an initial velocity of 20 $^{m}/_{s}$ at angle of 30° above horizontal. It lands on a flatbed truck, at an elevation of 3 meters above the ground. How far away (horizontally) from where Stan kicked the ball does it land?

2. For the previous problem, the quadratic formula gave you two times when you were solving for the time at which the ball landed. How did you determine which one was the correct time to use? (explain)

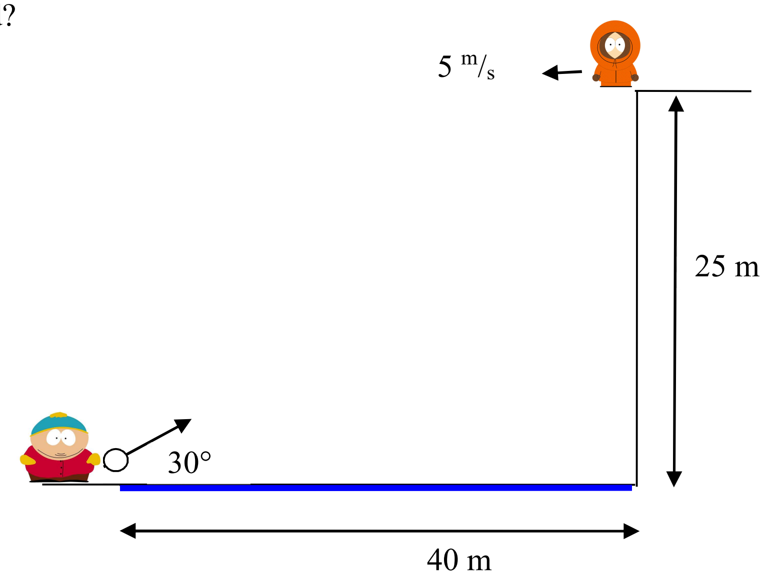
3. Now Stan is standing at the bottom of a hill that is sloped at an angle of 26.57° above horizontal. He kicks a football with an initial velocity of 20 $^{m}/_{s}$, angled 40° above horizontal. How far away from where the ball was kicked does it land? (What is the straight-line distance? Think carefully about what the "landing condition" is. For example, in problem 7, you could say the ball landed when Δy=3 meters. What can you say about Δy for this situation, when the ball lands? Continue on next page)
SOLVE ON WHITEBOARD

4. A cat is chasing a mouse. The mouse runs in a straight line at a speed of 2 $^{m}/_{s}$ away from the cat. If the mouse is initially half a meter in front of the cat, and the cat leaps off the floor at a 30 degree angle (relative to the floor) at what initial speed does the cat need to leap in order to land on the poor mouse?

5. Kenny and Cartman are playing a game. Kenny is running off the edge of a 25 meter tall cliff above a pond, while Cartman launches water balloons at Kenny as he falls. Cartman is standing at the edge of the pond, 40 meters away from the edge of the cliff. Cartman is launching the water balloons with a slingshot that launches them at an initial angle of 30° above horizontal. Assume the balloons leave the slingshot at the same elevation as the pond.

How long after Kenny runs off the cliff should Cartman launch the balloon so that the balloon hits Kenny just as he splashes into the pond?

SOLVE ON WHITEBOARD



Stump your neighbor!

6. Make up your own two-dimensional kinematics problem. It must be a problem that requires more than one step to solve it. Write out your problem on a separate page, and your solution.

Also write your problem (without solution) on another separate piece of paper. Put both in your folder. Ideally, also have us check your problem over to make sure it is solvable. Before the next class I will go through all of the problems and make sure they are solvable (and adjust if not). In the next class we will pass them out to other groups, who will try to solve them. You are going to have to check the solution from the group who gets your problem (and we will arbitrate). You will. Put your own problem and both your solution and your neighbor's solution to your problem in your folder when you are all finished.

If you can create a problem that your neighbor can't solve correctly, but you can, and you also correctly solve the problem created by and given to you by the third group at your table, then your group gets 5 bonus points on the upcoming exam.

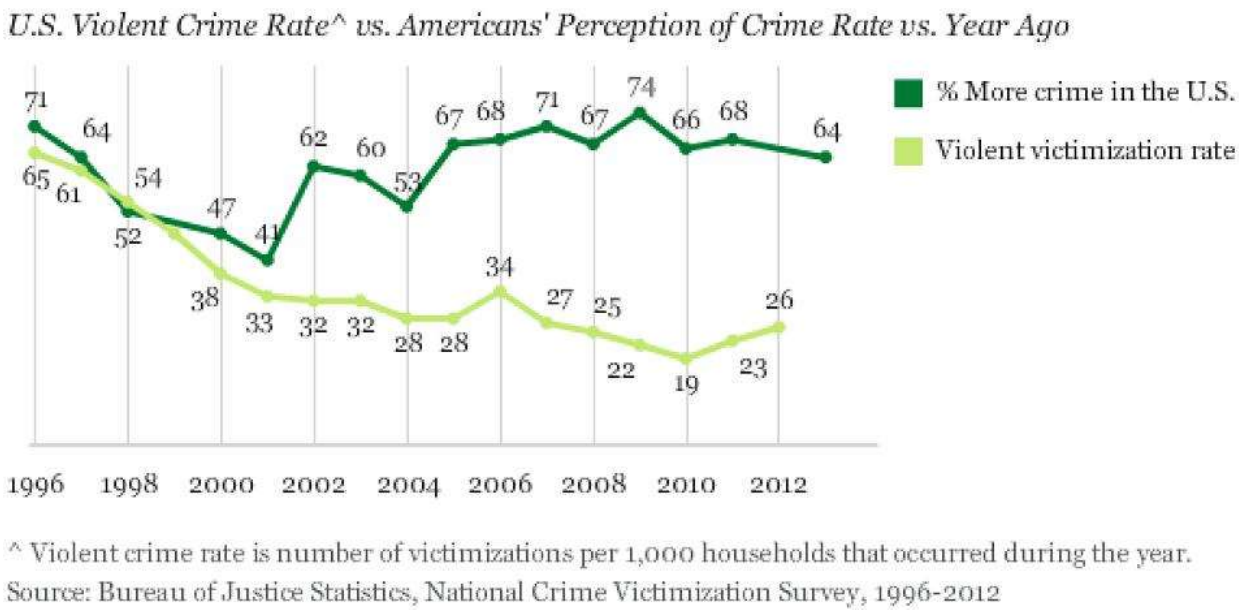
12. 2-D Kinematic Problem Solving II

13. Critical Thinking I

Name________________________

Group_______________

Look at this chart, which was compiled with data from the Department of Justice and the National Crime Victimization Survey. The light line ("Violent victimization rate") represents the actual rate of victimizations of violent crime per 1,000 households. The dark line represents the percent of the public that ***thinks*** violent crime rates are rising ("% More crime in the US").



1. Does the public's perception match reality? If not, why do you suppose that is?

2. Without googling or anything, how would you answer this question: "Does sugar make kids hyperactive?" Discuss with your group members. Do you all agree? Do you have any first hand evidence to support your view?

3. Now use the internet to assess the validity of your answer to 2. What did you find?

4. Assess the validity of this statement: "The total carbon emissions (as CO2) by humans amounts to 8 gigatons annually, compared to 210 gigatons annually from natural sources. Since our emissions amount to only 4% of the natural sources, it is highly unlikely that humans are contributing to climate change." After you discuss this in your group, and write down your thoughts, then talk with one of us and explain your line of reasoning.

5. Assess this statement: "The air humans exhale has 10 times as much CO_2 in it as the air we inhale. The average person (assuming sedentary) exhales around 2-3 pounds of CO_2 a day. With our population now being around 7 billion people, that means that the total annual output of all the humans is around 3 billion tons of CO2. That is contributing nearly as much to climate change as any possible contribution from burning fossil fuels". After you discuss this in your group, and write down your thoughts, talk with one of us and explain your line of reasoning.

14. Forces and Free Body Diagrams

*Name*______________________

We can look at forces in many ways. To start with, forces can be either pushes or pulls, acting to try to make an object move in one direction or another. So, forces act in a particular direction, and also have a magnitude - the *strength* of the force, commonly measured in *pounds* in the US, although scientists generally measure forces in units of *Newtons*. Since forces have both magnitude and direction, that means they must be represented as vectors.

Eventually, we will start looking at how forces affect motion. For now, we just want to get comfortable with the concept of a force, and pictorial representations of forces known as “free body diagrams” – these will become very important as the course progresses.

1. Consider a book sitting stationary on a table (with nothing touching it except for the table). What forces are acting on it? (describe verbally)

2. Draw a free body diagram of the book, indicating all forces acting on it. Represent the forces as vectors, using the direction of the vector to indicate the direction of the force, and the length of the vector to represent the strength of the force.

3. Based on your free body diagram for the stationary book, can you conclude anything *in general* about the net force acting on an object that is “at rest” (stationary)? If so, what?

4. Now, consider the case of a man trying to push a large, heavy crate across a rough floor. Let's assume that he is not strong enough to get the crate to move. When he is pushing the crate though, *trying* to make it move, what forces are acting on it? (verbally)

5. Draw a free body diagram representing the forces acting on the crate.

6. Thinking about your answer to problem 3, what can you conclude about the forces acting on the crate?

A *stationary* object that has no net force on it is said to be in *static equilibrium* – the study of systems in static equilibrium is referred to as "Statics" (a course that civil and mechanical engineering majors will all take).

To begin to analyze static systems, we will use Force Tables. Ultimately, these will allow us to get more practice with vector addition, and check our results. A force table has a ring in the center with three strings attached, with mass hangars on the other ends of strings. The strings can be suspended over pulleys, such that the weight of the hanging mass (the hangar plus any slotted masses put on it) equals the tension in the string. The tensions in the string then pull the ring in different directions, depending on where the pulleys are set. If the forces acting on the ring (due to the tensions in the string) balance, the ring will not be pulled away from the center of the table. There is some friction in the pulleys, such that if our hanging masses are off by a slight amount (a gram or so), the ring should still remain stationary.

These force tables will also allow us to get some more practice with vector addition, which is going to become very important.

9. Put one pulley at 0° on the force table, and the two other at 150° and 210°, with hangars suspended from the strings going over each pulley. Put 150 grams on the hangar at 0°.

a.) How much mass must be hung from the pulleys at 150° and 210° for the ring to be in static equilibrium? (In the space below and on the next page, draw the free body diagram for the ring, and calculate what weights are needed to balance the weight at 0º)

10. Leave one hanger with 150 g at 0°. Position the other two hangers at 150° and 225°.

a.) In the space below, draw the free body diagram for the ring and calculate what weights are needed on the other two hangars to balance the 150 grams at 0°.

15. Forces and Motion

*Name*______________________

So far in this course, we have analyzed the motion of objects, but have not yet discussed what might make an object move – or change how it is moving. In this activity, you will try to figure this out for yourself – how forces relate to motion – and then test your predictions. ***You will not be graded on whether your initial model is correct or not*** – we want you to feel free to try to analyze the situation in your own way, and develop your own model for explaining forces and motion. *Do not search for "the correct answer" in a textbook, online, etc..*

You will have time to carefully discuss your ideas amongst your group (and between groups if you want), and build your own model for how forces relate to motion. In the next activity, you will test your model to see if you are correct or not. There is no penalty though if the model you develop in this activity turns out to not be correct.

What we want is for you to experience the scientific process – to *think* about what is going on, to come up with your own model to explain it, and to try to justify your model. In the next class, when you carry out experiments to test your model, we want you to be able to analyze the data to figure out *for yourself* whether your model is correct or not. That is our goal here – we want to focus on the *process* - i.e. the scientific method.

Many experiments involve numerical measurements to collect data – that will not be the case in this activity (yet). We want you to develop a model out of your own experiences, sensations, and conceptions about motion.

Remember that we represent forces as vectors – they have magnitude (the strength of the force) and direction. We will start out by considering the case of a block being pushed by a hand such that the block moves at constant velocity. Each group should have a block to carry out tests for themselves.

I. Observations

Try to push the block at *constant velocity*. Let each member of your group take turns doing that. While doing so, pay attention to how hard you are pushing the block.

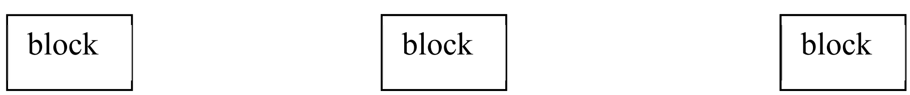
1. Think about all of the forces acting on the block – both from objects touching the block, and objects that may be exerting a force without touching it. Discuss amongst your group how the forces should relate to the constant velocity motion. Then, draw free body diagrams below representing each of the forces acting on the block at three successive times, t_1, t_2 and t_3 (so t_3 comes after t_2, etc.), with the assumption that the block is moving at constant velocity the entire time.

block	block	block
t_1	t_2	t_3

2. Explain your thought process in deciding what forces are acting on the block, and how big they should be at each time. Do all of your group members agree with this? It is perfectly ok to have disagreement within your group. If you do not all agree, what are the different ideas being considered? (use extra paper if necessary)

Now, let's consider a second case – the case of constant acceleration. First, try to make the block accelerate at a constant rate with your hand (make sure it doesn't go flying off the end of the table!). Let each group member take a turn.

3. Again, think about all of the forces acting on the block. Discuss amongst your group how the forces should relate to each other, in order to produce a constant acceleration. Draw free body diagrams of the forces acting on the block at three successive times, indicating the magnitude and direction of each force.



t_1 t_2 t_3

4. Explain your thought process in deciding what forces are acting on the block, and how big they should be at each time. Again, do all of your group members agree with this? If you do not all agree, what are the different ideas being considered, and why?

Now let's consider a third and final case. Consider the case of a block given a brief push and then released (so your hand will not continue pushing the block). Think about forces acting on the block during the push, and then after the block has left your hand. Again, let each group member push the block, and then discuss what is going on.

5. Think about all of the forces acting on the block. Discuss amongst your group how the forces should relate to each other during and after the push, and also discuss what the motion of the block is (i.e. is it constant velocity? Constant acceleration? Neither?). Draw free body diagrams of the forces acting on the block at three successive times during the push, and then two times after the push.

block	block	block
t_1 – during push	*t_2 – after push*	*t_3 – later after push*

6. Explain your thought process in deciding what forces are acting on the block, and how big they should be at each time. Again, do all of your group members agree with this? If you do not all agree, what are the different ideas being considered, and why?

7. How would you describe the velocity and acceleration of the block after it has left your hand?

8. Based on your free body diagrams, create and describe an overall model that relates the forces acting on an object to the motion of the object. If there is disagreement within your group, describe the different models being discussed (you can use the next page). You may want to ask one of the instructors for guidance in how to formalize your model into words.

15. Forces and Motion

16. Testing your Hypothesis

Today you will carry out some experiments to test your predictions from the last class – your predictions of how forces on an object relate to the object's motion. To gather meaningful data, you will not only use a motion detector to measure the position (and then calculate velocity and acceleration) of an object, but also a force probe to measure the force acting on the object. This will allow us to make synchronized plots of force, position, velocity, and acceleration vs. time.

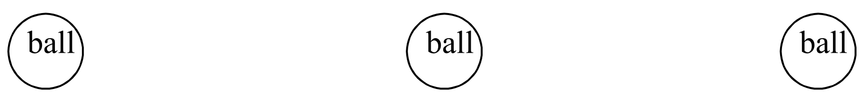
We will start though by bringing the focus to your hypothesis.

---

1. Explain the model you came up with at the end of your last activity, to relate the forces acting on an object with the motion of the object.

Before carrying out any experiments, let's consider another situation – that of dropping a ball. Think about the forces acting on the ball while you are holding it in your hand, and then at different times after you have dropped it. What should the motion of the object be, ***based on your model of how forces and motion relate***?

2. Draw free body diagrams identifying the forces (represent magnitude by length) acting on a ball first while it is in your hand, and then at two successive times after you have dropped it, as it is falling.



t_1 – while holding *t_2 – after dropping* *t_3 – later after dropping*

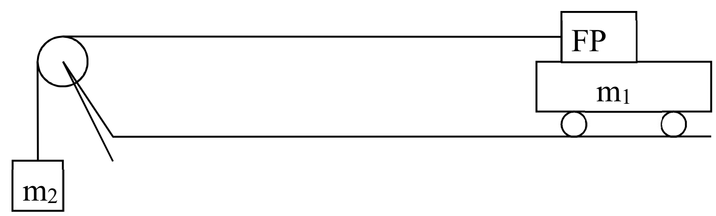
3. Below, draw a plot of the velocity vs. time graph for the ball after it is dropped, and verbally describe its motion. Also draw a plot of the acceleration vs. time, and the net force acting on the ball vs. time. On each plot, clearly indicate the time at which you remove your hand.

16. Testing your Hypothesis

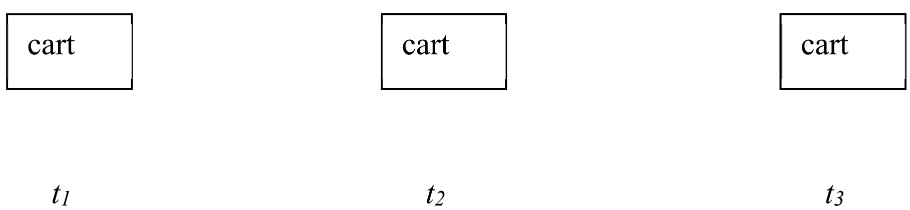
4. Why do you think the ball's motion will be as you described it? Does this match with your model explaining how forces and motion relate that you created in the last class?

To test your model, you will look at the case of a cart attached to a mass by a string hung over a pulley, as shown below (FP is the force probe). We will be using a virtual simulation of this setup, since I assume none of you have spare force sensors lying around your rooms.

Before beginning your experiment though, let's consider what is going on. Since the force probe (FP) and cart (with mass m_1) are attached together, we can treat them as one object (henceforth "the cart" will refer to the cart and force probe together).



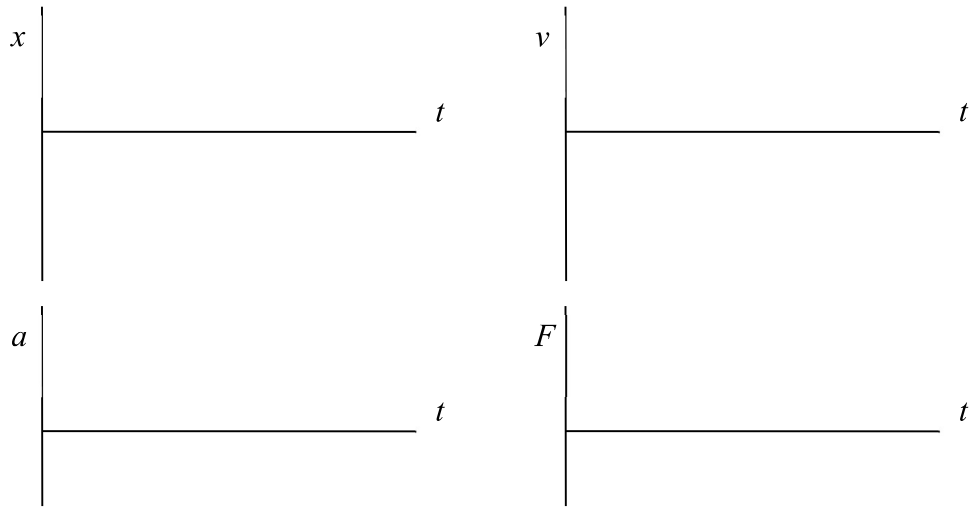
5. Draw a free body diagram of the cart at three different points in time.



6. Do you expect the pulling force on the cart from the string be constant, or change in time? Discuss.

7. On the next page, sketch what you *expect* the force, position, velocity, and acceleration vs. time graphs to look like. Note that the force probe will only be measuring the pulling force the string exerts on the cart – there is also friction in the horizontal direction, but it is a much smaller force.

16. Testing your Hypothesis

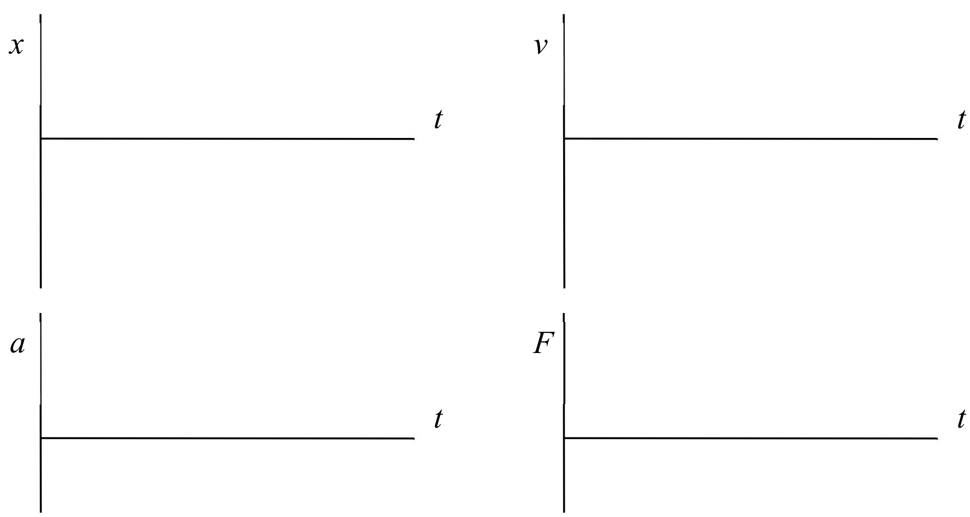


8. Explain why you expect to see the graphs that you drew in the previous problem. Focus on your hypothesized relationship between force and motion.

Okay, time to carry out the experiment. Use the following link for the simulation: https://iwant2study.org/lookangejss/02_newtonianmechanics_3dynamics/ejss_model_AtwoodMachine2wee/AtwoodMachine2wee_Simulation.xhtml. Make sure the static friction, kinetic friction, and added mass are set to 0. You can keep the hanging mass at 0.30 kg. To display all the graphs, first select “x vs t” from the drop menu. This should bring up a graph with a blue indicator for position. Do the same for “v vs t” and “a vs t”. This should give you three indicators (blue, pink, red) on one graph. Next, click “both” from the drop menu, so you can see both the pulley setup and the graph together.

Once your window is properly set up, you can hit the Play button to collect a data set.

9. Draw the position, velocity, and acceleration vs. time graphs produced by the simulation. For the force vs. time graph, watch the “Net Force” value in the top right of the setup and graph its values.



10. How do those plots compare to your predictions? Are there any surprises? If so, discuss these with your group members, and summarize that discussion below.

11. Based on the experiments that you have done, let's reconsider some of the cases from the previous activity. If an object has a constant acceleration, what does that mean about the net force acting on it? Is it increasing or staying the same?

12. What did you "predict" in the previous activity? Does that match with what you found in this activity?

13. Now consider the case of pushing a box at constant velocity. Was your prediction from the previous activity that you have to continually push with a force the same as the opposing friction force? Or did you say that you have to push harder than friction in order to keep it moving?

14. Based on the results of this experiment, what do you think you could say now about something moving at constant velocity? Should there be any net force on it? Explain.

17. Riding an Elevator

Name________________________

Consider an elevator hanging from a single steel cable, attached to a winch at the top of the elevator shaft. The weight of the elevator is *mg*. Let the up direction be positive, and down negative.

For all problems, be as specific as possible, and justify/explain your answer with a free body diagram (FBD), Newton's 2nd Law, etc.. If the FBD is the same from one problem to the next, you do not need to re-draw it each time, but your answer should be based on the FBD and Newton's 2nd Law.

1. If the elevator is moving *up* at constant velocity, what can you say about the tension in the cable? (Is the tension constant? Increasing? Decreasing? How does it compare to the weight?)

2. If the elevator is moving *down* at constant velocity, what can you say about the tension in the cable?

3. If the elevator is moving *up* with constant *positive* acceleration (i.e. accelerating upwards), what can you say about the tension in the cable?

4. If the elevator is moving *up* with constant *negative* acceleration, what can you say about the tension in the cable?

5. If the elevator is moving *down* with constant *negative* acceleration (i.e. accelerating faster in the downward direction), what can you say about the tension in the cable?

6. If the elevator is moving *down* with constant *positive* acceleration, what can you say about the tension in the cable?

7. If the elevator is moving *up* with a *positive* acceleration that gets larger in time (the acceleration is becoming more and more positive), what can you say about the tension in the cable?

8. If the elevator is moving *down* with a *negative* (downward) acceleration that is growing larger in magnitude (the acceleration is becoming more and more negative), what can you say about the tension in the cable? Can you say anything about the magnitude of the elevator's acceleration, related to g?

9. Are all of your answers consistent with Newton's 2nd Law? Consider this, and when you are done, talk to an instructor/TA.

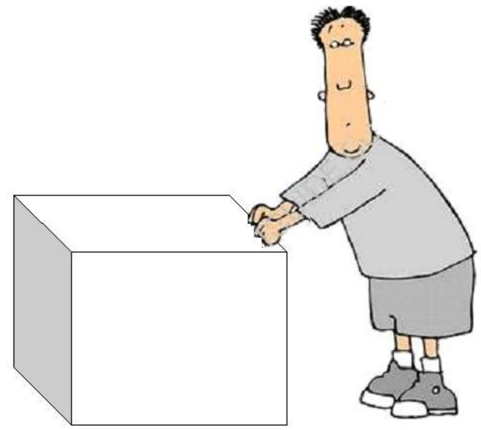
18. Dynamics

Name_______________________

A man is pushing a heavy wooden box, as shown below. He is pushing with a force of F, directed slightly downward at an angle of 30° with respect to the horizontal.

The box itself has a mass of 10 kg, and he has 8 kg of pudding in the box. The static and kinetic coefficients of friction between the box and the ground are 0.4 and 0.2, respectively.

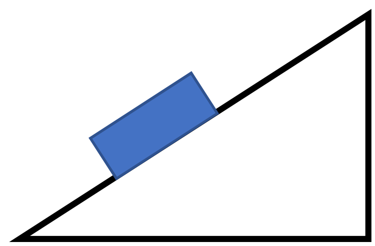
1. Draw a free body diagram showing all forces acting on the box, assuming it is initially at rest. Then determine what minimum force the man must push with for the box to start moving. DO THIS ON THE WHITEBOARD. Since this is your first time solving a problem like this, transcribe your entire solution below, putting in a sentence here or there explaining what you are doing (i.e. "I am going to apply Newton's 2nd Law in the y-direction"). For future whiteboard problems, you can just summarize what you did if you do not want to transcribe everything, particularly if there is a lot of messy algebra involved.



2. Considering the same situation as the previous problem: once the box starts moving, what will happen if he keeps pushing with that same magnitude force? Will the box move at constant velocity, or will it accelerate? If it accelerates, at what rate?

A block with mass m is on a plane inclined at an angle of 30°. The static and kinetic coefficients of friction between the block and the plane are 0.3 and 0.1, respectively. The block is sliding *up* the inclined plane with an initial speed of 3 $^{m}/_{s}$.

3. Draw a free body diagram for the block.



4. How long will it take for the block's speed to drop to zero?

5. Considering the same situation as the previous problem - will the block come back down the inclined plane, or will it stop and stay still? Explain and prove your answer.

6. A 10 kg block is initially stationary at the base of a plane inclined at 15°, with coefficient of friction of 0.2. The wind starts blowing, exerting a constant, horizontal force of 70 Newtons on the block. How long will it take for the block to reach a speed of 8 $^{m}/_{s}$? DO ON WHITEBOARD

7. Based on these problems, is it safe to say that the normal force exerted on an object by the surface the object is on is always equal to the object's weight, *mg*? If not, why not?

19. The Perfect Angle

Name___________________________

Note: This is the extra credit activity for Exam 2.

Since optimizing a system is a common design task in the science and engineering world, we will periodically return to minimization-maximization problems throughout this course. Min/Max activities will be used both to provoke a deeper level of analysis and understanding of physics being covered in the course, and also to bring in aspects of calculus as it is being covered in Math 425.

In the first minimization-maximization activity, the focus was purely on kinematics – and in such a way that angles did not come into play. Now that derivatives of trigonometric functions have been covered in calculus, we can look at minimization-maximization problems that involve angles. Your overall process with any min/max problem should involve three primary steps:

1. ***Think*** about what is going on in the problem (this should be your first step with *any* problem). Try to visualize what is happening, draw a mental picture – and then an actual one on the paper. Then think about what is going on in terms of the underlying physics. What relevant physics concepts or laws are involved? What are you being asked to actually solve for? What does that depend on?

2. Develop an equation for the parameter you want to minimize or maximize in terms of the parameter you want to find the ideal value of. For example, to find the optimal distance (F) that a racer should do the farmer's walk for before swimming across the river in order to minimize the total time the race takes (T), you needed to develop an equation for the total race time (T) in terms of that parameter F.

3. Take the derivative of the quantity you want to minimize or maximize with respect to the parameter you are finding the ideal value of (dT/dF) and set that derivative equal to zero. This is because the slope of the curve relating those quantities (T vs. F) should be zero at a local minimum or maximum. Once you find a value of the parameter (F) that gives a slope of zero, it may be necessary to see if the second derivative is positive or negative at that location (value of F) in order to determine whether it is a local minimum or maximum. Or, you may be able to tell in other ways (such as by thinking about the physics involved).

The minimization-maximization problems in this activity will focus on 2-d kinematics, but now finding the optimal angle to launch projectiles at in various situations.

19. The Perfect Angle

1. Find the optimal angle to launch a projectile at (assuming no wind resistance) to maximize the range of the projectile over horizontal ground. (show your work!)

19. The Perfect Angle

You are at a golf course, competing in a longest drive competition. But, there is a twist to this competition – you are not hitting your ball over flat land. Instead, you and the other competitors are standing on a hill sloped at a steep 45 degrees. The hill is so big that there is no hope of hitting over the hill – so you are certain that your shot will land somewhere on that hill. Also, the hill is completely covered in mud, so balls will not bounce or roll at all after landing. The impulse you can provide to a golf ball when hitting it is limited, so the initial velocity you can provide to it is fixed. But, you can choose among different clubs to affect the angle the ball leaves the ground at.

You will need to determine the optimal angle *relative to the hill* that the golf ball should leave the ground at (this would be the angle of the pitch on the head of the golf club).

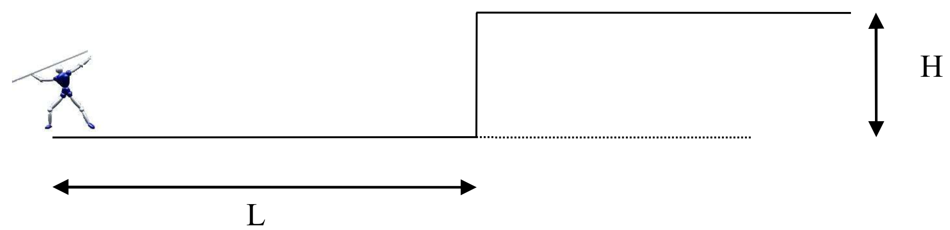
2. In the first problem, your "landing condition" was that $y_f = y_i$ (both of which you likely treated as zero), since the projectile lands when it comes back to its original height. Now, the ball won't be landing when the height of the ball falls back to its original value. If the hill the ball is going to land on is at a 45° angle, and the ball is being launched from that hill, what is your condition on y going to be for when the ball lands? Explain.

3. Use that landing condition to find the optimal angle for the golf ball to be hit at (relative to the hill) for maximizing its range (you can work on the next page as well)

19. The Perfect Angle

19. The Perfect Angle

You are competing in a modified version of a javelin toss. Instead of just throwing javelins on perfectly flat ground, competitors throw towards a cliff, as shown below.



The cliff is a distance (L) of 20 meters away, and is 10 meters high (H). Treat the javelin as if it leaves your arm from ground height initially, for simplicity.

4. With what minimum initial speed must you be able to throw the javelin to be able to reach the base of the cliff?

5. Let's assume that you are actually capable of throwing the javelin hard enough to make it to the top of the cliff. Would you expect that the optimal angle for throwing the javelin at in this situation (it is going to land 10 meters higher than the elevation that you are throwing it from) is the same angle that maximizes the range over purely horizontal ground? Would you expect the optimal angle to be the same as, higher than, or lower than the optimal angle for over horizontal ground? Explain.

6. In the previous cases, the optimal angle was independent of the initial speed of the projectile. Do you expect that to still be true in this situation? For example, if you can just barely throw it hard enough to make it to the top, maximizing your horizontal distance means throwing it at whatever angle will allow the javelin to just barely make it to the top of the cliff. Would that angle be the same as the angle that maximizes your range if you can throw the javelin much faster?

7. Use the same general approach as you did in the first two problems to try to find the angle that maximizes the range of the javelin when landing ten meters above your elevation. You may not be able to get an exact numerical value, if the optimal angle depends on the launch speed. If it turns out to be that way, for full credit, try to develop a plot (using a computer) of the optimal angle as a function of launch speed.

20. Newton's Third Law

Newton's third law says that every force has an equal and opposite force. Ultimately, what this means is that whenever object A exerts a force on object B, then object B must exert an equal and opposite force on A. The forces are equal in magnitude, but opposite in direction. These forces are sometimes referred to as "third law pairs", indicating that they are the equal and opposite forces the two objects are exerting on each other. When examining the forces acting in a system of objects, it is extremely important to keep track of third law pairs between objects within our system.

When using Newton's Second Law ($\sum \vec{F} = m\vec{a}$), we commonly apply the law to individual objects (although we could apply it to a collection of objects as an entire system). Often, we may need to apply the law to multiple objects to fully understand what is going on in a system. If some of these objects are exerting forces on one another, we need to make sure that the equal and opposite forces show up. To help keep track of these "third law pairs", it can be helpful to use a particular notation for forces being exerted on one object by another object. To see how this notation works, assume that Paul is pushing Jane. We could label the force Paul exerts on Jane as F_{PJ}, where the subscripts indicate that this is a force that *Paul* exerts *on Jane*. According to Newton's 3rd law, Jane must exert an equal and opposite force back on Paul, which we could label F_{JP}. In our free body diagram for Jane, we would need to include all forces acting on her – which must include F_{PJ}.

When we draw a free body diagram for Paul, we would have to include F_{JP}, which must point in the opposite direction of F_{PJ}.

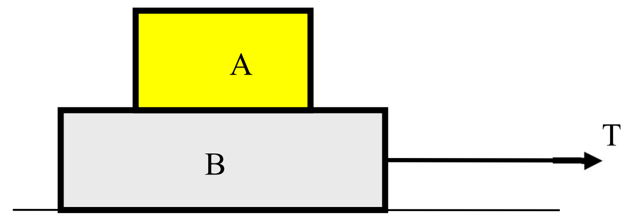
1. A book sits stationary on a table. Is the normal force that the table exerts upward on the book the third law pair of the force of gravity on the book? (note – this is not just asking if the forces are equal in magnitude. Are they actually third law pairs) Explain.

2. Draw a free body diagram of the book, indicating all forces acting on it.

3. What is the third law pair for each of the forces acting on the book?

20. Newton's Third Law

Block A is stacked on top of block B, which is on a table. The coefficients of static and kinetic friction between block B and the table are 0.5 and 0.2, respectively. The coefficients of friction between blocks A and B are also 0.5 and 0.2. Blocks A and B have masses of 2 kg and 5 kg, respectively (don't put numbers in though until you have worked out your answers in symbolic form!).



4. Draw free body diagrams of both blocks, identifying each force. Have one of us check your FBDs before proceeding to the next problem.

5. Let's assume for the moment that block A is not slipping on block B (they are moving together). If block B is accelerating to the right at an acceleration of a_B, what can you say about block A's acceleration? Explain.

6. What is the maximum possible acceleration that block B can undergo before block A starts slipping?

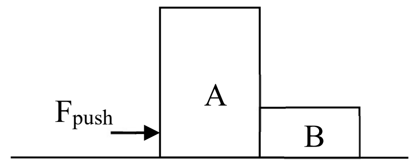
7. What is the maximum tension that can be applied to the string (T) before block A starts slipping? DO THIS ON THE WHITEBOARD, summarize here.

8. Let's say that the rope is pulled with twice the tension at which block A starts to slip on top of block B (so twice the tension that you found in the previous problem). If block A is slipping, does that mean that it is stationary relative to the table? Or does it move in some direction relative to the table? If so, which direction? Explain.

9. At some moment t=0, the left edge of block A is 20 cm to the right of the left edge of block B. At t=0, someone starts pulling the rope with twice the tension you calculated in problem 7 (so block A starts slipping). How long will it take for the left edge of the two blocks to line up?
WHITEBOARD, summarize here

20. Newton's Third Law

Blocks A and B are next to each other as shown. Block A is thrice as heavy as block B. Assume that friction between the blocks and the surface is negligible, and the pushing force is continually applied to A (i.e. it is not a brief impact, but rather a constant push).



10. How does the acceleration of block A compare to that of block B? Explain.

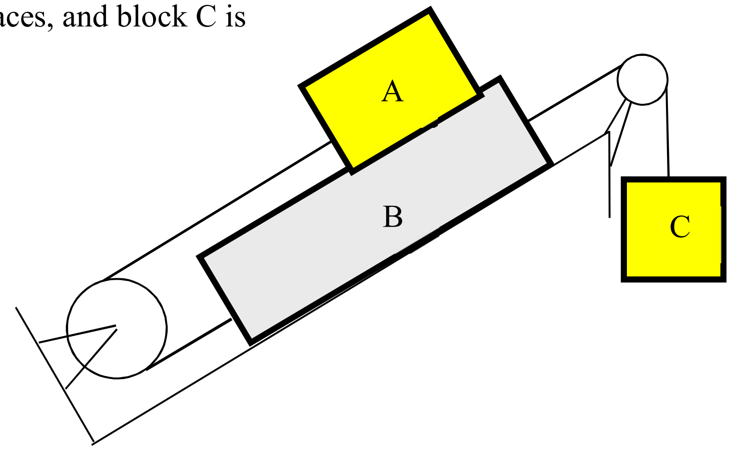
11. How does the *net* force on block A compare to the net force on block B? Explain your reasoning, and be as specific as possible.

12. How does the horizontal force that A exerts on B compare to the force that B exerts on block A? Explain.

13. If you push on block A with a force of $F_{push}=10\ N$, how hard does block A push on block B? (hint: start by drawing FBDs!)

Consider the collection of blocks shown below.

13. Draw free body diagrams of the three blocks (A, B, and C) and clearly identify all forces acting on each block. Assume that there is friction at all surfaces, and block C is moving downward.



14. You are considering the following scenario: a happy monkey is pushing a crate full of bananas across the floor at constant speed. A fellow student – Odin - makes the following statement:

Odin: *"If the crate is moving, that means the monkey must be pushing on the crate harder than the crate is pushing back on the monkey*"

In response to that statement, another student – Bhodie – says:

Bhodie: *"I think the monkey would only be pushing harder on the crate than the crate pushes back if the crate is accelerating."*

Discuss the statements of these two students, and summarize your discussion below (and on the next page). Is one of them right? If so, which one? Explain why each one is right or wrong.

20. Newton's Third Law

21. Smashing Cars

All forces in nature ultimately come from two objects interacting. This is true both on the large scale (two people pushing each other, the force of gravity between the sun and earth, etc.) and the microscopic scale (the electrical forces between molecules, the nuclear forces between nuclei, etc.).

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1. When two objects interact, is it possible for one to exert a force on the second, without the second object exerting a force back on the first? Can you think of any situation in which that happens, or appears to happen?

2. Consider this situation: the earth exerts a gravitational force on a ball; does the ball exert a gravitational force back on the earth? Explain. If the ball does exert a force on the earth, does it make the earth move much?

3. Consider two cars colliding. It makes sense that they are each exerting forces on each other – but how does the magnitude of those forces compare? Is it always equal in magnitude? Is it possible for the forces they exert on each other to be different? Explain. If it is possible, describe a situation in which the forces two cars (A and B) exert on each other are different.

21. Smashing Cars

A 4,000 kg truck collides with a 2,000 kg car. Inside each vehicle is a 100 kg driver (the driver's masses are included in the vehicles' masses already). Let's assume that the drivers of each vehicle are seat-belted in similarly. The truck is driving towards the car at 20 $^{m}/_{s}$, while the car is driving towards the truck at 30 $^{m}/_{s}$.

4. During the collision, do you expect that the *drivers* of the two vehicles will experience equal magnitude forces? Explain. If the forces on them differ, in what way do you think they should differ? (i.e. "the car's driver should experience 8 times the force that the truck's driver experiences")

5. Do the drivers exert "3rd law pair" forces on each other? In other words, the force pushing on the car's driver due to the collision – is that force the 3rd law pair of a force on the truck driver? Explain.

6. Even though the collision is very fast, it is not instantaneous; the cars decelerate over time. Let's assume that after the first 0.1 seconds of the collision, the car has decelerated from its initial speed down to 5 $^{m}/_{s}$ (still traveling in the same direction though). What is the *average* force that the truck exerts on the car (call it F_{TC}) over that time?

7. What is the *average* force that the car exerts on the truck (call it F_{CT}) over that same time?

8. What will the speed of the truck be after that first 0.1 seconds?

9. How does the change in speed of the truck in this time interval compare to the change in speed of the car? Does that match your intuition about what would happen?

10. Consider the quantity mass times velocity, mv. How much does that quantity *change* for the car during the collision?

11. How much does the quantity mv change for the truck during the collision?

Check with an instructor at this point. There is an important issue that starts to arise in the previous problems, which we will focus on much more later.

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12. Now consider the drivers. Let's assume that since the drivers are seat-belted in tightly, they undergo the same rate of acceleration as the vehicles they are driving. So, do the drivers of the two vehicles undergo the same acceleration?

13. How does the force exerted on the driver of the car by his seat belt compare to the force exerted on the driver of the truck? Explicitly calculate the force on each driver. Do the results match your intuition?

14. In this situation, the force causing each driver to decelerate was exerted on them by their seat-belts. Does that mean that they would be better off not wearing seat-belts? If they *weren't* wearing seat belts, would they be likely to experience lesser forces? Explain in terms of physics, and what would be expected to happen. (consider that if you are not wearing your seat belt, when your car suddenly starts decelerating, you may go flying forward and smash into the dashboard, but not actually hit the dashboard until the collision is finished)

15. Modern cars are designed with "crumple zones" – points at which vehicles are designed to fail and crumple during impact, rather than remaining stiff and solid. Why would this make accidents safer during collisions?

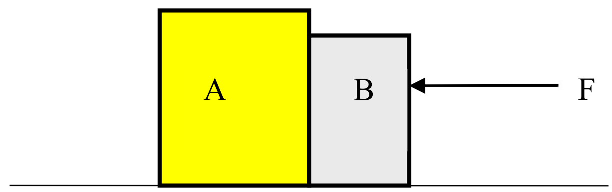
16. What is another safety device that most modern cars have, and why does it help (in physics terminology)?

22. Limiting Cases

This activity will give you more practice with drawing free body diagrams, and applying Newton's laws – with particular focus on properly accounting for third law pairs. You will also get some experience with looking at a limiting case, or a "special case" of a problem, to see if your answer makes sense.

Note: pushing blocks around isn't a common situation for actual physicists or engineers to spend their time analyzing – but this activity and the previous one involving blocks are designed to force you to carefully apply Newton's laws.

Two blocks, A and B, are back-to-back on the floor, as shown below. Block B is scuffed up on the bottom, while block A is smooth – the result is that block B has a higher coefficient of kinetic friction on the floor than block A.



1. Will the two blocks have the same acceleration?

2. What will the acceleration of the two blocks be? (first identify and name any unknown parameters that you will need to include in your analysis. For example, let m_A = *mass of block A*)

22. Limiting Cases

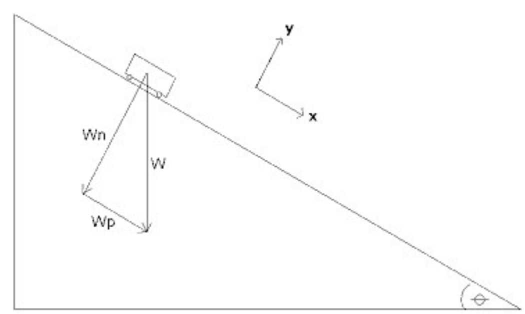
Often in physics, it can help to look at a “limiting case”, which is a special case of a problem to which you know what the answer *should* be. By seeing if your answer works out to that answer in that special case, you can see if your answer makes sense.

3. Assume that the two blocks are really one block (with mass equal to the two block’s individual masses combined), and with the same coefficients of friction. What should the acceleration of that one block be in that situation?

4. Does your solution for the acceleration of the two blocks work out to that acceleration in this “limiting case” (the “limiting case” being that they have the same coefficients of friction)? (show)

Let’s look at another situation in which limiting cases can be useful – that of figuring out which trig function (sin or cos) is the appropriate one for breaking the force of gravity on an object into components normal and parallel to an inclined plane.

5. Don’t go through the trig to determine which component needs sin or cos – instead just guess. You know that one of the components (W_N or W_P) will be $W\cos\theta$ and the other will be $W\sin\theta$. We can look at two limiting cases – the angle of incline being zero or 90°. What should W_N and W_P be in these limiting cases?



6. Based on that, what are W_N and W_P in terms of $\sin\theta$ and $\cos\theta$? Explain.

22. Limiting Cases

Limiting cases can be particularly helpful for checking the validity of our solution, if we can think of a special case for which we know what the answer should be without having to go through any lengthy calculations. If we know what the answer *should be* in a particular situation, then we can check the validity of a solution method by seeing if we get that answer when we apply the method to that particular situation. Consider an example:

Let's assume you have been assigned the following homework problem:

"A frog is squatting on the ground, and notices that a predatory snake is about to lunge for him. He hops off the ground with an initial speed of 5 $^m/_s$ at an angle of 30° above the ground. What is the maximum height the frog will reach?"

A friend works on the problem, and comes up with the following solution:

I first found just the y component of the initial velocity:

$$v_{yi} = v_i \cos(30°) = 5\cos(30°) = 4.33\, {}^{m}\!/_{s}$$

I then used the kinematic equation $v_{yf}^2 - v_{yi}^2 = 2a\Delta y$ *to find the maximum height, by knowing that at the peak of his hop his speed in the y-direction should be zero. Solving for* Δy,

$$\Delta y = \frac{v_{yf}^2 - v_{yi}^2}{2a} = \frac{0 - 4.33^2}{2(-9.8)} = 0.957m$$

Your task is to evaluate your friend's solution. But, you need to do it by focusing on a limiting case, or a "special case".

7. First, is there any special case of the frog jumping (i.e. a particular angle, for example) for which his maximum height is much easier to calculate than other cases? Perhaps a case for which virtually no calculation is required at all? Discuss and explain what a useful limiting case would be in this situation.

8. Test your friend's solution in that limiting case, and see if it gives you the answer you would expect.

9. Based on that – is your friend's solution valid? If not, why not?

22. Limiting Cases

23. Coupled Systems

*Name*______________________

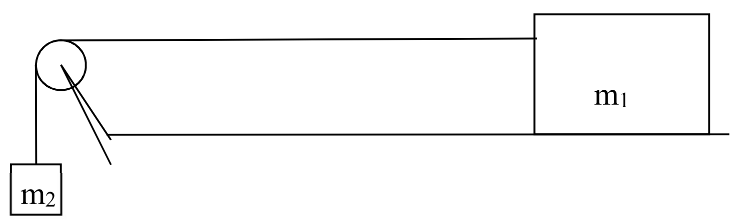
Often, we deal with systems of multiple objects that interact in various ways. There can be forces between the systems, and their motion may be connected somehow (objects moving together, against each other, etc.). This activity will give you practice analyzing systems that involve a few objects coupled together in some way.

For now, we will assume that all pulleys are "ideal pulleys" – this means that the pulleys have no mass themselves and no friction. Towards the end of the semester we will see what happens with non-ideal pulleys. For now, the key is that having ideal pulleys means that the tension in a rope is the same on both sides of the pulley.

When we first started looking at how forces affect motion, we used a setup essentially like the one below (but where we used a cart with a force probe attached instead of a block). We have not yet used Newton's laws to analyze this setup though. You will now analyze this setup, and consider how your analysis compares to what you observed.

Do your analysis completely symbolically: label the masses m_1 and m_2, the coefficient of friction μ, the acceleration due to gravity g, etc.

1. Draw free body diagrams (FBDs) of both blocks, indicating your choice of axes. Next to each FBD, also draw an arrow indicating the direction you expect the object to accelerate in.



2. Are the accelerations of the two blocks related? If so, how are they related, and why?

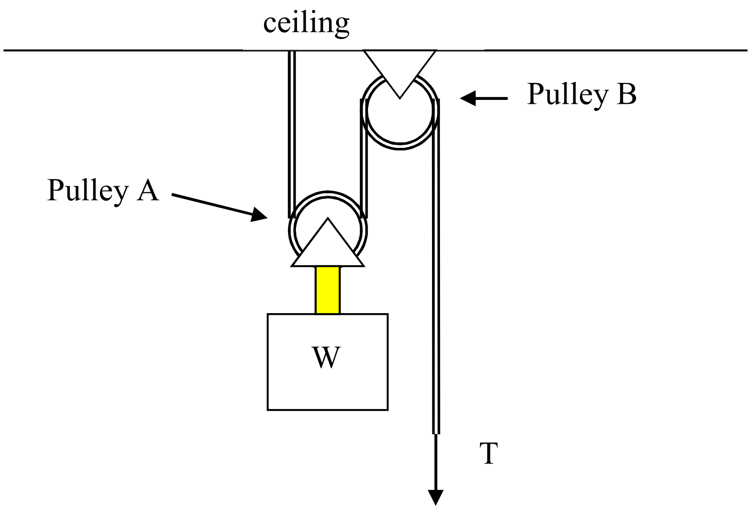
3. Write out Newton's 2nd Law in both directions for both blocks, putting all of the forces in symbolically. Do not start solving for anything yet, just write out Newton's 2nd Law for both blocks, putting in the forces as they show up in your FBDs. (continue on next page)

4. Keeping in mind that the tension in the rope is the same on both sides of the pulley, use your equations above (from Newton's 2nd Law applied to both masses) to solve for the acceleration of the falling block, to see how it depends on the two masses.

5. After going through an analysis in which it is easy to make a mathematical mistake or forget to take something into account, a good check can be to see if your answer makes sense in some particular situation. For example, let's consider this situation: you are doing this experiment, measuring the acceleration with some hanging mass (m_2), then making the hanging mass bigger, measuring it again, and repeating. As the amount of mass you are hanging from the string gets larger, would you expect that the acceleration would keep increasing indefinitely? (i.e. could it eventually become 30 m/s^2? Then maybe 50 m/s^2?) Or do you think there would be some maximum acceleration that the system should approach, and never exceed? Explain thoroughly.

6. Now, let's consider whether your solution (your answer to problem 4) agrees with your thoughts on the maximum acceleration. What does your predicted acceleration (your answer to problem 4) become in the limit that $m_2 \gg m_1$? Show whether this agrees with what you think the acceleration should be in this situation (your answer to problem 5). You can work on the next page….

A rope runs over two pulleys, as shown below. Pulley B is firmly attached to the ceiling, and can not move. Pulley A is free to move up and down. You are trying to hoist a weight (W) attached to pulley A, by pulling down on the rope where it hangs off of pulley B. You are pulling with tension T.

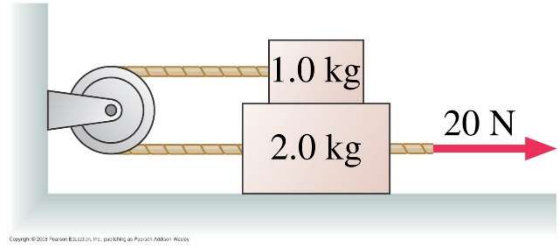


7. How does the tension you have to pull with in order to raise the weight at a constant velocity compare to the weight of the box? Prove your answer (in particular, draw a free body diagram of pulley A).

8. Pulley B is attached to the ceiling. What upward force is the ceiling mount applying to pulley B (in terms of W)? Prove your answer.

23. Coupled Systems

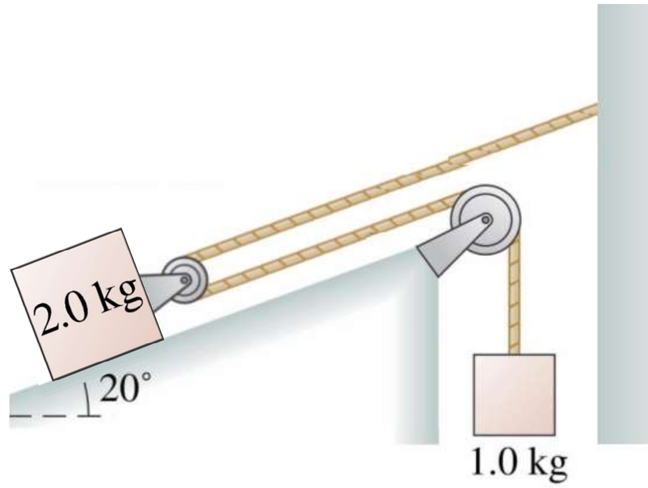
See the figure to the right. The coefficient of kinetic friction between all surfaces is 0.3. The left edge of the upper block is initially 0.5 meters to the right of the left edge of the lower block.



9. How long after you start pulling on the rope will the left edges of the two blocks line up? (it starts sliding as soon as you start pulling) DO ON THE WHITEBOARD, draw your FBDs here, including the axes you chose, and your Newton's 2nd Law equations. You don't need to transcribe all of the algebra, but do show how you used the acceleration you found to solve for the time.

23. Coupled Systems

Consider the arrangement shown below. The coefficient of friction between the 2.0 kg block and the table is 0.4. The system starts from rest, and we want to know how long it will take for the 2.0 kg block to slide 1 meter to the right.



10. In the same amount of time that the 2.0 kg block slides 1 meter up the incline, how far does the 1.0 kg block fall downward? (hint: this is an important point for solving the rest of the problem)

11. Determine how long it will take the 2.0 kg block to slide 1 meter up the incline. Show all your work.

24. Measuring the acceleration due to gravity

Come up with a method of determining the acceleration due to gravity (g) using Logger Pro and the ultrasonic motion detector (UMD), and a block sliding on an inclined plane. You need to figure out how you are going to do the analysis. Note that there is friction between the block and the incline, so you will have two unknowns – g and the coefficient of friction. You need to determine experimental values for both.

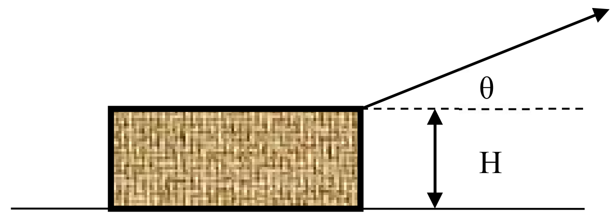
Explain everything you are doing and why you are doing it.

25. Minimizing the Force

Name________________________

The Problem:

Imagine that you have tied a rope to a heavy box laying on concrete, and are trying to drag the box. Both the wooden box and concrete have rough surfaces, so the coefficient of friction between them is very high. You pull on the rope such that the tension force is at an angle θ relative to the horizontal, as indicated in the drawing.



1. What angle (θ) should the rope be at in order to minimize the amount of force you have to apply to drag the box at ***constant speed***? (assume that the box is heavy enough that you can't actually lift it)

25. Minimizing the Force

26. Measuring Friction

Part I

Come up with a method of determining coefficients of static and kinetic friction between a wooden block and some other surface. You need to design and analyze your own experiment, collect whatever data you need, and use that to determine the coefficients. With whatever experiments you do, repeat it at least three times, calculate the coefficients from your data, and see if your result is well reproducible (i.e. if you determine the static coefficient of friction three times, and get values of 0.9, 0.5, and 0.2, then something is off – your results should be closer together each time).

Part II

Find some rough and smooth objects and surfaces around the room. Measure the coefficient of friction of at least 5 different pairs (a certain material sliding on a certain surface is a pair), and rank the pairs from smoothest (smallest coefficient of friction) to roughest.

Bring your data home, and type up a brief report (~ 2 pages) presenting your experiment, data, calculations, and results, for both parts.

26. Measuring Friction

27. Circular Motion

*Name*______________________________

A key idea for circular motion: an object moving along a circular path ***must*** have a particular acceleration, which we call "centripetal acceleration", with a value of $a_c = \frac{v_t^2}{r}$ where v_t is the tangential speed of the object (the speed tangent to the circle), and r is the radius of the circle.

Eric rockets down a hill on his Big Wheel action bike. At the bottom of a hill is a curve on flat ground, which goes through a quarter-circle over a linear length of 10 meters (the distance Eric would cover as he goes through the quarter circle is 10 meters). The coefficients of static and kinetic friction between the bike's plastic tires and the road are 0.4 and 0.3, respectively.

1. How fast can Eric can go (in meters per second) around the curve without sliding?

2. Eric is going around the curve at just barely under the maximum speed that he can make the corner (as you found in problem 12). He suddenly hits a patch of ice, and the coefficient of friction between his tires and that ice is a mere 0.1. Explain what will happen.

27. Circular Motion

Jack and Sue are playing on an icy pond. Sue has smooth shoes on, so she can slide around essentially without friction. Jack is wearing boots with good traction though. Jack and Sue are holding hands, and Jack is swinging Sue around him in a circle (so that she is sliding on the ice in a circular path around him). Jack and Sue have masses of 80 kg and 55 kg, respectively. Jack is swinging Sue around him such that Sue makes a full revolution in 2 seconds. Jack and Sue's arms are long enough that Sue is sliding around on the ice 1.3 meters away from Jack.

We want to know what the coefficient of static friction must be between Jack's shoes so that he doesn't slip.

3. Sketch a picture of what is going on.

4. You are given various numbers in the problem – now give names for those quantities, so you can identify exactly what you have as "knowns". For example, you could define Jack's mass as $m_J = 80\ kg$. Until the end of the problem, you should work purely in terms of the names you give things (such as m_J), don't put numbers in until the end.

5. The problem says that Sue makes one full revolution every 2 seconds. We want to know how fast she is moving in meters per second, along her circular path (that is the "tangential velocity" that shows up in the centripetal acceleration equation). Knowing the equation for circumference, and that Sue is traveling around a circular path of radius 1.3 meters, determine how fast she is moving in meters per second (the "tangential velocity").

6. Draw FBDs of Jack and Sue at some moment in time, clearly labeling all forces. Note that you probably don't want to draw FBDs from a perspective viewed from above, since then you wouldn't be able to show the weight.

7. Why will Jack slip if the coefficient of friction between his boots and the ice isn't big enough? In other words – there must be some force other than friction acting on him horizontally such that he would slide if friction is too small. What force is that? (explain in words, and indicate how you are symbolizing that force in your FBD)

8. For Sue to move in her circular path around Jack, what radial acceleration does she need? (put numbers in to get an exact value)

9. In what direction is Sue's acceleration? Explain. Using a picture might help.

10. Apply Newton's 2^{nd} Law to Sue in the radial direction. Put in what her radial acceleration is, and determine how hard Jack must be pulling on Sue in order to keep her moving in the circle (just in terms of the names you gave quantities – don't put the numbers in!).

11. Now apply Newton's 2^{nd} Law to Jack in the radial direction (where "radial" points along his arms toward Sue). From this, figure out what the static coefficient of friction between his shoes and the ice must be so that he doesn't slip. (put numbers in at the very end)

12. If Jack suddenly loses his grip on Sue, what will happen? Will Sue keep spinning around Jack in a circle? Will she spiral away from him? Draw an overhead picture depicting how Sue would move.

Experimental verification of centripetal acceleration

Now you will verify that the centripetal acceleration of an object traveling in a circular path is in fact $\frac{v^2}{r}$. You should have a metal straw with a string going through it. One end of the string is attached to a rubber stopper, and the other end has a loop for hanging masses from.

We can demonstrate to you how to use the apparatus. You will try to swing it around such that the rubber stopper spins around in a roughly horizontal circle of constant radius, while the hanging mass on the other end neither rises nor falls. You will need to measure the angular velocity and radius of the rubber stopper's circular path, to see what the net force it needs to hold it in its circular path. Do this with a few different hanging masses to try to verify that an object moving in a circular path needs an inward acceleration of $\frac{v^2}{r}$. (hint: in this scenario, where is that force coming from?)

27. Circular Motion

28. Weighing Planets

*Name*_______________________________

Isaac Newton figured many things out, not just his laws of motion. By observing the motion of objects falling here on earth, and also the motion of the moon and other celestial bodies, Newton figured out that the attractive gravitational force between two objects must be inversely proportional to the square of the distance between the objects (center to center distance), and proportional to the product of the two masses multiplied together. Newton's law of gravitation can be written like this:

$$F = G\frac{m_1 m_2}{r^2}$$

The G in the equation is known as the Universal Gravitational Constant, which accounts for the type of units we choose to measure masses, distances, and forces in. Newton did not actually determine what the value of G is though. It was determined 71 years after Newton's death by Henry Cavendish, using a very precise torsion balance. The currently accepted value of G, to four significant figures, is $6.674x10^{-11}\ \frac{m^3}{kg^{\ 2}}$.

From understanding Newton's Law of Gravity as well as how to analyze circular motion, we can actually determine the mass of any object that has a "satellite" orbiting it in a circular orbit. Astronomically, a satellite is any smaller object (man-made or not) orbiting a significantly larger object. If the satellite is much much lighter than the object it's orbiting, then the satellite's force of gravity won't have a significant effect on the large object, so the large object can be viewed as stationary.

1. We want to be able to determine the mass of some object (like the sun) by knowing how long it takes some "satellite" orbiting it to go through one full revolution around it (the period). In astronomy, a "satellite" is any smaller object (man-made or not) that orbits a significantly larger object. Assume the orbit is circular (in reality, the planets' orbits are slightly elliptical). Determine an equation for the mass of the object being orbited in terms of the radius of the satellite's orbit, the period of its orbit, and the universal gravitational constant G (your answer should not depend on anything else, such as the mass of the satellite, the speed of the satellite, etc.).

2. Using your equation, determine the mass of the sun. The distance from the earth to the sun is 149,600,000 km (93 million miles). Note that you should probably know the period (don't forget that we have a "leap year" once every four years).

3. Compare your mass of the sun to the accepted mass (google it). Calculate a percent difference

4. Our moon is 384,400 km away, and goes all the way around the earth once every 27.322 days. Determine the mass of earth, and compare your mass to the accepted value.

5. Is the calculation as accurate for the earth as it was for the sun? If not, why do you suppose it isn't quite as accurate?

6. Mercury and Venus have no natural satellites – no moons. So, how do you think you might be able to determine their mass?

7. Develop an equation for the tangential velocity (v) of a satellite orbiting a large mass. It should only depend on the mass being orbited (m), the constant G, and the radial distance between the mass being orbited and the satellite (r). If you were to plot v as a function of r, what would it look like? This calculation is of great importance in cosmology, and we will discuss it more soon.

Isaac Newton's laws of motion (in particular his second law, $\sum \vec{F} = m\vec{a}$) apply only in "inertial" reference frames - that means frames that aren't accelerating. For example, let's say you have a pie resting on top of your dashboard in your car. As you drive along at a constant speed in a straight line, the pie remains motionless. Even though you are moving, you are not accelerating, so everything behaves the same as it would if the car were sitting still.

But what would happen if you suddenly turned the car to the right? The pie would slide to the left across the dashboard. Did something push the pie to the left? No, there was no new force acting on it. You are observing the pie from an accelerating reference frame. You are accelerating to the right. The pie "wants" to keep moving forward, in the direction it's been moving – and as long as no force acts on it, it will continue doing so. Since you are accelerating to the right though, from your point of view (your frame of reference), the pie appears to accelerate to the left. In fact, not only is no force pushing the pie to the left, but the only horizontal force acting on it (friction) is pushing it to the right, trying to keep it from sliding across the dashboard.

So, clearly there is a problem. If we drew a Free Body Diagram of the pie, the only horizontal force is pushing it to the right – so it should be accelerating to the right. But we observe it accelerating to the left. That is because our frame of reference is accelerating – skewing Newton's laws. In fact, the pie is accelerating to the right, if observed from a non-accelerating reference frame (such as a person in a helicopter directly above your car, who continues flying straight).

8. The Earth is rotating – pretty quickly in fact. Since the Earth is roughly a sphere, that means a point on the surface of the earth is moving in circular motion – and therefore undergoing a centripetal acceleration towards Earth's axis of rotation (a line between the north and south poles). So, in principle, the surface of the Earth is not really an inertial reference frame, because we are accelerating. Let's see how significant this is though. First, without using numbers, determine what the centripetal acceleration would be for a point on the equator, in terms of the radius of the earth (r) and the period of one rotation (T). Do this by putting the speed of the surface of the earth's motion (v) in terms of the radius and period.

9. Now put in numbers (look up any on google as needed), to calculate what the centripetal acceleration for a point on the equator would be.

10. We are not on the equator – we're quite a distance away from it. Do we have the same centripetal acceleration as somebody who is at the equator? Explain.

11. How does this compare to the accepted acceleration due to gravity? Do you think that it's large enough to cause a problem, such that we can't treat the surface of the earth as an inertial reference frame (at least for most purposes)?

12. How short would a day have to be so that a person on the equator would feel weightless?

28. Weighing Planets

Einstein figured out a fairly simple way of handling accelerating reference frames. He realized that if you are in an accelerating reference frame, everything behaves the same as if there is a gravitational acceleration pointing opposite the acceleration. For example, let's say you are in an elevator with no windows. If the elevator starts accelerating upwards, you feel like you are being pushed down into the floor, as if gravity suddenly got stronger. If the elevator accelerates upwards at 3 m/s^2, then in your frame of reference, everything behaves as if Newton's Laws apply, but the acceleration due to gravity has increased from 9.8 m/s^2 to 12.8 m/s^2.

If the elevator suddenly somehow accelerates horizontally, due north at 3 m/s^2 (and no longer accelerates upwards), then from your point of view inside the elevator, everything appears to be pushed "backwards", towards the south, the same as if there were a gravitational acceleration of 3 m/s^2 pointing south – in addition to the actual gravitational acceleration of 9.8 m/s^2 pointing downwards.

This is known as Einstein's Equivalence Principle – that in an accelerating reference frame, that acceleration appears to impart a force to objects, and within that accelerating reference frame this force is indistinguishable from a gravitational force. This idea led to the development of his Theory of General Relativity.

Note that the "acceleration due to gravity" that we observe here on earth is actually a combination of the acceleration due to the force of gravity, minus the centripetal acceleration. In Einstein's General Theory of Relativity (GR), gravity is viewed as a fictitious force. In GR, what we normally think of as the force of gravity is instead the result of how matter bends space-time, with the curvature of space-time exerting a force on other objects with mass.

28. Weighing Planets

29. Dynamics II

Name___________________

1. The 4,900 pound Tesla Model S P100D electric car can accelerate from 0-60 mph in 2.28 seconds. Assuming that the acceleration is constant over that time (fairly reasonable due to the single-speed gearbox), we want to know what the the minimum coefficient of friction must be between the tires and the ground so that the tires won't slip as the car is accelerating. First though, does it matter if the car is two wheel drive versus four-wheel drive? Explain.

Is this static or kinetic friction? Explain.

2. Determine the minimum coefficient of friction to allow it to accelerate that fast (note that it is 4-wheel drive).

3. Why do race tracks bank the curves? Discuss and explain

4. You are driving a Lamborghini Countach LP500s around Texas Motor Speedway. Including you, the car weighs 3,500 pounds, and its high performance tires have a static coefficient of friction of 0.9 on dry asphalt. The curves on this track are banked at 24°, with a radius of curvature of 750 ft. How fast (in mph) can you drive around this curve before the tires start slipping? (note: definitely draw a free body diagram, and be careful about what you choose for axes. What direction is it accelerating in?) Solve the problem symbolically first – don't put numbers in until you have a formula for the maximum speed in terms of the bank angle, g, etc..
DO ON WHITEBOARD, summarize below (FBDs, Newton's 2nd Law equations, answer)

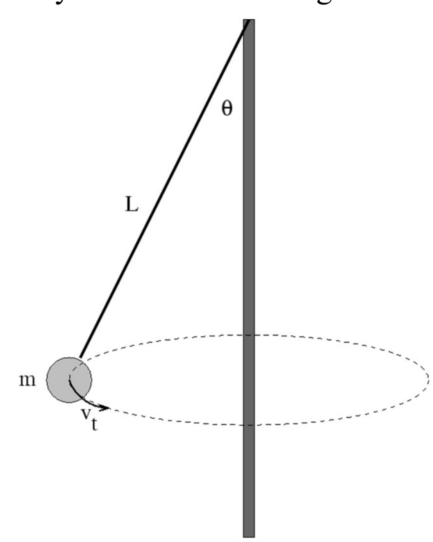
5. Is there a bank angle above which there would be no maximum speed? Look at your symbolic equation for the top speed – is there an angle at which the top speed would be infinite? If so, what angle?

6. Why do you think if the bank angle is equal to or above that value it is not possible for a car to go fast enough that it slides off the curve?

7. Going back to problem 4 - how many "g's" will your body experience as you take the corner at the maximum speed found in that problem?

8. The rear wing of the Countach creates "down force" – a force directed "down" perpendicular to the roadway, due to the pressure difference above and below the wing. This is similar to the "lift" on an airplane's wings, but pointing in the opposite direction (the car's rear wing is essentially an upside down airplane wing). Will this "down force" allow the car to go around the corner faster? Why or why not?

9. A tetherball of mass *m=1.5 kg* is attached to a rope *2 meters* long, suspended from the top of a *3 meter* tall pole as shown. If the tetherball is moving around in a purely horizontal circle at constant speed, and the rope makes an angle of 30° with respect to the pole, how many times will the ball go around the pole in 1 minute?



10. If the rope suddenly breaks, how far away from the pole will the ball land? (more room on the next page)

Time to play "stump your neighbors"!
Make up your own two-dimensional kinematics problem. It must be a problem that requires more than one step to solve it. Write out your problem on a separate page, and your solution.

Also write your problem (without solution) on another separate piece of paper. Put both in your folder. Ideally, also have us check your problem over to make sure it is solvable. Before the next class I will go through all of the problems and make sure they are solvable (and adjust if not). In the next class we will pass them out to other groups, who will try to solve them. You are going to have to check the solution from the group who gets your problem (and we will arbitrate). You will. Put your own problem and both your solution and your neighbor's solution to your problem in your folder when you are all finished.

If you can create a problem that your neighbor can't solve correctly, but you can, and you also correctly solve the problem created by and given to you by the third group at your table, then your group gets 5 bonus points on the upcoming exam.

30. Centrifugapalooza

Stan (30 kg) and Kyle (28 kg) have gone to a local amusement park, and are going on a ride known as the “Centrifugapalooza” (aka “Turkish Twist”). The ride is a large, cylindrical room 7 meters high and 7 meters across (diameter). The walls of the room are covered in carpet, while the floor is a grate (to allow any regurgitated cotton candy to fall through). The static and kinetic coefficients of friction between the carpeted walls and the boys are 0.4 and 0.3, respectively. Stan and Kyle stand on opposite sides of the room, with their backs against the wall. As the ride starts, the room begins to spin faster and faster.

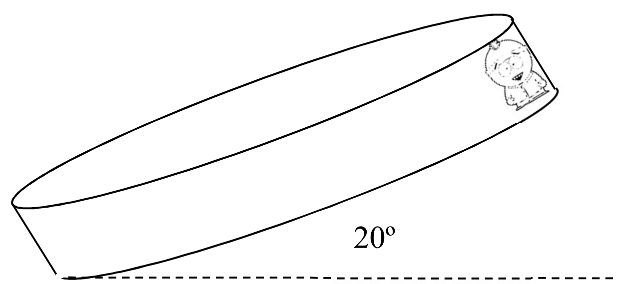
1. The boys feel like they are being pushed outward against the wall. Is there actually a force pushing them outward? If so, what is that force. If not, why do they feel like they are being pushed outward? In what direction are they being pushed, if any? Discuss in your group, and summarize your discussion below.

2. Consider a similar situation – you are a passenger in a car driving straight, when the driver suddenly turns the steering wheel to the left. You *feel* as if you are pushed to the right – but are you actually pushed that way?

3. As the Centrifugapalooza spins faster and faster, eventually Stan discovers that he is able to pull his feet up off the floor, and essentially “float” on the wall without falling down. What force is keeping him from falling down, without needing to touch the floor? Explain.

4. What is the minimum speed of the Centrifugapalooza in revolutions per second for Stan to be able to “float” on the wall?

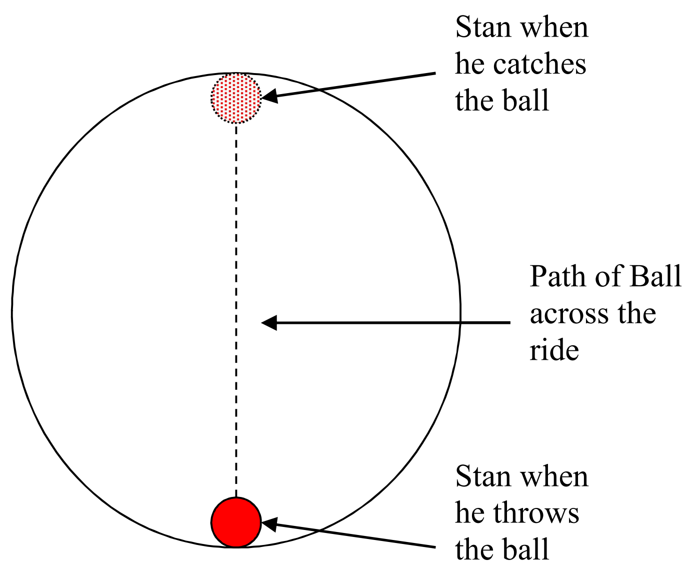
5. After spinning for a while, the room starts to tilt, as shown to the right. What is the minimum speed the Centrifugapalooza must have so that Stan will stay in place against the wall (without slipping) when he is at the highest point (the spot where he is in the picture)?



6. If the Centrifugapalooza is spinning that fast (as found in the previous problem), will that also be fast enough to keep him from slipping when he is at the lowest point of the swing? Explain.

7. The Centrifugapalooza returns to horizontal, and is spinning at 0.5 revolutions per second – it's top speed. After years of overstimulation from constantly playing video games on their Okama Gamesphere, Stan and Kyle get easily bored, and don't find the Centrifugapalloza interesting enough on its own. So, they decide to try tossing a ball back and forth to each other while the ride is going on. They are directly across from each other on the spinning ride. If Stan throws the ball straight out from his body (from his perspective), will the ball fly straight across to Kyle? Discuss and summarize your thoughts.

8. Is it possible for Stan to throw the ball such that he himself can catch it after having made half a revolution? (i.e. so he throws it, and then catches the ball when he is exactly opposite where he was when he threw it) Again discuss and summarize your discussion.



9. Let's assume for the moment that it is possible for Stan to do what is described in the previous problem. If the Centrifugapalloza is spinning at a constant 0.5 revolutions per second, can we figure out at what speed and angle Stan would have to throw the ball for him to be able to catch it after half a revolution? Consider the diagram above showing what is happening. Assume the Centrifugapalloza is rotating clockwise. Figure out what angle (as seen from above, measured relative to the dashed line) Stan would have to throw the ball at (from his perspective) so that it travels straight across the ride as shown. Keep in mind that he is spinning with the ride (i.e. so if he were to just drop the ball, would it fall straight down from where he was when he dropped it?).

9. At what angle *relative to horizontal* should Stan throw the ball so that he can catch it at the same height on the other side of the ride?

10. At what speed should Stan throw the ball (relative to *him*) so he can catch it on the other side of the ride?

11. Here's a real puzzler – think about the path the ball appears to follow according to Stan as he moves around the ride (ignore the vertical motion, going up and then down – focus on the horizontal motion). Does it look *to him* like the ball travels in a straight line? To help, try drawing the ride with Stan on it and the ball at various points as he moves around the ride (draw the situation from overhead, and at at least five different times – just as he throws the ball, when he has gone ¼ of the way around to the other side, ½ way around to the other side, ¾ of the way around, and all the way to the other side). He is always facing towards the center of the ride. Draw the pictures, and describe how it looks to him below.

30. Centrifugapalooza

31. Momentum

You have probably heard – and even used – the term "momentum" in common conversation. In physics, this term has a particular meaning. In fact, Newton's 2nd Law in its pure form is actually a statement about momentum. We have been writing Newton's 2nd Law as

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

but this is actually a simplification, from the assumption that the mass of our system (object) in question does not change. But, for some systems, the mass does change. For example, a rocket propels itself by shooting combusted fuel out the back end – so the total mass of the rocket (including fuel) continually decreases. In such situations, we need to use the pure form of Newton's second law, which allows for the mass to change:

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt}$$

where the product rule has been used to expand the derivative of *mv*. This quantity, *mv*, is what we define as momentum in physics, labeling it with the letter *p*.

$$\vec{p} = m\vec{v}$$

Notice that momentum is a vector, since it is a scalar (mass) multiplied by another vector (velocity). Newton's Law actually says that the net force acting on an object equals its change in momentum with respect to time.

$$\sum \vec{F} = \vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Notice that we could multiply the equation by dt and integrate to get

$$\int \vec{F}_{net}dt = \int d\vec{p} = \vec{p}_f - \vec{p}_i$$

This quantity on the left-hand side, the integral of the net force acting on an object over a period of time, is called an "impulse", symbolized with the letter *J*. Newton's 2nd Law, as written in this form, says that an impulse causes a change in an object's momentum.

---

1. Consider an isolated star sitting by itself somewhere in the universe. Another star is approaching the first one. The two stars exert a gravitational force on each other, pulling them towards one another. Can we say momentum is conserved in this situation? What should our system be, for us to say that momentum is conserved? (i.e. can we say the momentum of one of the stars alone is conserved?)

2. Considering the same situation as the previous problem; the second star is twice as massive as the first, and is initially approaching the first star with a speed of 10^7 $^{m}/_{s}$ relative to the first star (which you can consider stationary). After some amount of time, the second star is traveling at 3×10^7 $^{m}/_{s}$ towards the first star. How fast is the first star moving at that point in time, and in what direction?

4. Stan (40 kg) and Eric (90 kg) are standing on a 150 kg raft floating in a lake. Simultaneously, Stan runs due south and dives straight off the raft at a speed of 6 $^{m}/_{s}$, while Eric dives southwest off the raft at 4 $^{m}/_{s}$. At what speed and direction will the raft move?

A cue ball is rolling without friction across a pool table with a velocity of $\vec{v}_i = 4\hat{x}\, {}^{m}\!/\!_{s}$. The cue ball then hits the eight-ball, not striking it dead-center though. After the collision, the eight-ball rolls away with a velocity of $(2\hat{x} + 2\hat{y})\, {}^{m}\!/\!_{s}$. Assume the balls have the same mass (a cue ball is actually slightly lighter)

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5. Draw two overhead pictures of the collision – a before drawing, and an after drawing. Indicate the velocity of each ball with an arrow in the drawings (the same way we indicate forces – we draw the vectors as arrows). Make an initial estimate of how you think the cue ball will be moving after the collision based on your intuition (or experience playing pool).

6. Now determine exactly what the cue ball's velocity should be after the collision. Does your answer agree with your intuition?

7. A frog sits on the end of a long board of length L. The board rests on a *frictionless* horizontal table. The frog wants to jump to the opposite end of the board. As he jumps though, the board also starts sliding – since there is no friction between the board and the table to hold it stationary. If the frog jumps at an angle of 45° above horizontal, and the board has twice the mass of the frog, how fast does the frog need to jump so that he lands on the end of the board? Your answer should be in terms of L. Does your answer seem reasonable?

Hints: Draw a picture! The board moves when the frog jumps – is there some way you can figure out how fast the board goes backwards in relation to how fast the frog moves forwards?

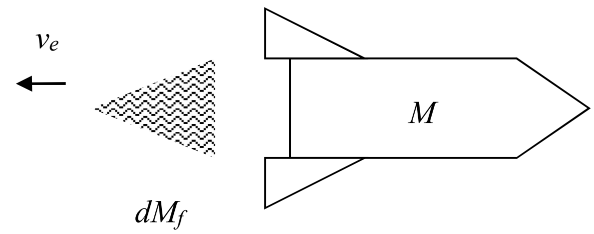
DO THIS ON THE WHITEBOARD, summarize below.

32. Rockets

Name____________________________

Note: This is the extra credit activity for the third exam

Consider a rocket way out in space, far enough away from any nearby planets that the force of gravity on it is negligible. There is essentially no air around the rocket. So, how is a rocket able to propel itself by burning fuel? It is a pure example of conservation of linear momentum – some small amount of mass (burned fuel) is shot at high velocity out the back of the rocket, and the rocket must therefore gain some speed in the other direction, for momentum to be conserved (since there are no external forces acting on the rocket + fuel system). This is depicted in the (high quality) sketch to the right.



So, how do we analyze such a system? And what exactly should our system be? Just the rocket, or the rocket and the fuel it is constantly expelling? If we want to know how the rocket's velocity varies with its decreasing mass, then we should need to count both the rocket and the exhausted fuel as our system – with this view, there is no external force (what we normally think of as the "thrust" from the rocket's engine is really an internal force within our system, arising from conservation of momentum).

Since there is no external force, that means momentum must be conserved. We can analyze this system, with the assumption that the rocket starts out at rest, and as it expels a mass of burned fuel dM_f out the back at an exhaust velocity v_e (this is relative to the rocket), this propels the rocket (initial mass M) forward some additional velocity dv.

1. Assuming fuel is burned at a rate R_B (mass of fuel burned per unit time), what is the acceleration of the rocket? (start with the conservation of momentum, divide that equation by dt, and then use this burn rate condition, $\frac{dM_f}{dt} = R_B$. No integration necessary!)

What though if your rocket is taking off from earth? (let's assume the force of earth's gravity is Mg. Later we will learn that the further you get from earth, the weaker the force gets) Do you expect that the acceleration due to gravity will just combine with your acceleration above? Let's find out. Recall that Newton's 2nd Law can be written as $F_{net}dt = dp$. The right-hand side of that equation is just Δp in differential form (so you have already figured out what that is in problem 1. While it must add up to zero in total, you have already put dp in terms of the rocket's mass, exhaust mass, etc..). Now there is also an external force, that of gravity.

2. What is the acceleration of the rocket as it is firing straight upwards from earth's surface? Does it just work out to the acceleration you found in problem 1, minus the acceleration due to gravity? (No integration necessary!)

Let's return to the rocket out in space, far away from anything else. The exhaust is propelled at a velocity of 500 $^{m}/_{s}$, and your rocket's initial mass is ¾ fuel.

3. Assuming your rocket is initially at rest, how fast will it be going when all the fuel is burned up? First figure out the velocity of the rocket as a function of the mass, rather than as a function of time. Go back to where you started in problem 1, but don't divide by *dt.* Instead, separate terms involving v onto one side of the equation, and involving m onto the other side. Integrate each side, using appropriate limits of integration for each integral.

4. If you want your rocket to be able to attain a speed of 1,000 $^m/_s$, and the rockets mass not counting fuel is 2,000 kg, how much fuel must you start out with? (assuming the same exhaust velocity)

5. You are designing a self-propelled rocket to be launched from a roof-mount system on a HUMVEE for the military. The rocket is intended to be used against enemy tanks – and to try to optimize the "penetrating power" of the rocket, your task is to figure out how much of the rocket's initial mass should be fuel to maximize its final momentum. In rocket design, the ratio of the initial mass of the rocket to the final mass defined as the mass ratio, R. Find what value of this mass ratio will result in the maximum momentum of the rocket upon impact, assuming no wind resistance and that all of the fuel has been burned before impact (you can ignore any vertical motion – assume the rocket impacts at the same elevation that it started at).

32. Rockets

33. Momentum in 3-Dimensions

*Name*______________________________

A 2-stage rocket is blasting off, accelerating upwards due to the thrust of the first-stage engines for the first two minutes after launch. After the first two minutes, the rocket's velocity (relative to the ground) is $\left(30\hat{x}+20\hat{y}+200\hat{z}\right)\frac{m}{s}$ (where the z direction is up, x and y are east and north, respectively).

At that moment, the first stage engines shut off, and detach from the rest of the rocket. A miniature explosion (part of this two-stage rocket design) ejects the first stage engines away from the rest of the rocket before the single second stage engine ignites. This explosion happens *between* the rocket and each engine, which makes it an internal force if we consider the entire rocket (including engines) our "system".

During the explosion, the rocket and the two first-stage engines are presumably still experiencing the force of gravity. Does this mean you can not assume momentum is conserved during the explosion? This is a tricky question. In principle, if there is a net external force (such as the force of gravity in this case), momentum is not conserved. But, consider the time frame. Newton's 2nd Law says $\vec{F}_{net}\Delta t=\Delta\vec{p}=m\Delta\vec{v}$.

1. Will the force of the explosion cause a change in momentum of the system as a whole? Explain.

2. Gravity is an external force – so it could potentially cause the momentum of the system to change. Consider how long an explosion lasts though. If we consider the momentum of the entire system just before the collision (p_i) and just after the explosion (p_f), do you think that over that length of time (however long the explosion takes) gravity will produce a *significant* change in momentum of the system? Explain.

3. Based on this, do you think it is reasonable to say that momentum is conserved during the explosion? If you want to calculate the final velocity of the rest of the rocket, what should your "system" be (in other words – for what collection of objects can you say that the total momentum is conserved)?

4. You will now determine the velocity of the rest of the rocket after the explosion that detaches the two first-stage engines. Just before the explosion the entire rocket had a velocity of $\left(30\hat{x} + 20\hat{y} + 200\hat{z}\right)\frac{m}{s}$. The mass of each of the first-stage engines is 1/5$^{\text{th}}$ of the total mass of the rocket (so together they make up 40% of the rocket's mass). Just after the explosion, the left first-stage engine moves (relative to the ground) at $\left(-40\hat{x} - 20\hat{y} + 50\hat{z}\right)\frac{m}{s}$, while the right first-stage engine moves (relative to the ground) at $\left(35\hat{x} + 30\hat{y} + 50\hat{z}\right)\frac{m}{s}$. After the explosion, what is the velocity of the rest of the rocket relative to the ground?

5. After the explosion (before the second engine starts), what is the velocity of the rocket relative to the left engine?

6. If the rocket was 3,000 meters high at the moment of the explosion, and the second-stage engines don't ignite (so after the explosion the only force acting on each piece is the force of gravity – since we are again ignoring wind resistance), how far away from the left engine does the rest of the rocket land?

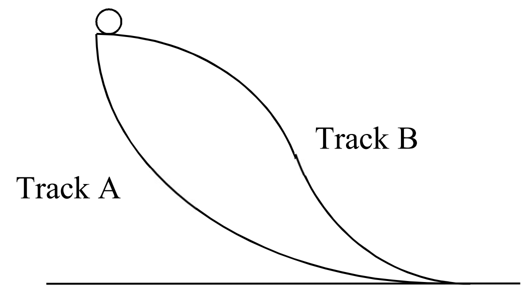
33. Momentum in 3-Dimensions

34. Work and Energy

*Name*____________________________

We will revisit these first two questions again later – but I want you to think about them before we begin this next section. You will not be penalized for being wrong on these questions (this time) – we do need to see some good discussion though of what you think will happen.

Two identical balls sit at the top of two tracks, starting at the same elevation. The two tracks have very different shapes though. Track A starts out very steep, while track B starts out less steep. This is shown in the picture to the right.



1. Which ball will get to the bottom first? The ball on track A or B? Or will they get there at the same time? Discuss and explain.

2. When each ball is at the bottom of its track, which one will be going faster? Or will they have the same speed? Discuss and explain.

Lecture Notes:

If we integrate Newton's 2nd Law over a displacement, we get what is referred to as the "work-energy" theorem: $\int_{y_i}^{y_f} F_{net}\,dy = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

The right-hand side of the equation has a special significance – and this is where the concept of energy comes from.

We choose to define the quantity $\frac{1}{2}mv^2$ as an object's "*kinetic energy*" (*K*), a measure of the amount of "energy" the object has due to its mass and speed.

We define the quantity force multipled by distance as "work". The left-hand side of the equation above (the "work-energy theorem") is the net work done on an object, $W_{net} = \int_{y_i}^{y_f} F_{net}\,dy$.

So, this work-energy theorem says that the *net* work done on something equals the change in its kinetic energy, $W_{net} = \int_{y_i}^{y_f} F_{net}\,dy = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta KE$.

This result – that the net work done on an object equals its change in kinetic energy – is known as the "work-energy theorem". It is important to understand though that ultimately this is not a new fundamental law – it is just a different form of Newton's 2nd Law. Or rather, it is what Newton's 2nd Law tells us must be true for the net result of a net force applied on an object as it moves through some displacement.

An important point is that the "work" in this equation is the *net* work done on the object – since it comes from adding up the *net force* multiplied by distance. If the net force is constant, then the integral simplies: $W_{net} = \int_{y_i}^{y_f} F_{net} dy = F_{net} \Delta y$ (for constant F_{net}).

If many forces are acting on an object, then we could say that each of those forces is doing some amount of work as the object moves. The net work done would equal the sum of all of the individual works – which is the same thing we get by adding up the net forces times the displacement.

This may sound very confusing initially – so we are going to get a lot of practice with this idea to help clarify it. The most important thing to keep in mind is the next sentence, in bold.

For a constant force, the work done is just $W = \pm F \Delta y$, where it is positive if the force is "winning" (in the same direction as motion), and negative if the force is losing (opposite the motion).

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3. You are pushing a block to the left across the ground at ***constant velocity*** (nobody else is pushing it – it's just you pushing the block over the ground, with friction opposing the motion). You are pushing with a constant force of P as the block moves to the left a distance of L. How much work do YOU do on the block?

4. What is the *net* work done on the block by all forces acting on it? Keep in mind that the block is moving at constant velocity – what does that tell you about how much work is done?

5. Is friction doing any work? If so, how much? Explain your answer.

6. In the previous problem, we focused on the force of friction and the force that you exert on the box. But there is also a gravitational force and a normal force on the box. Are those forces doing any work? Explain.

7. If I throw a ball up in the air, is gravity doing positive or negative work as the ball is going upwards?

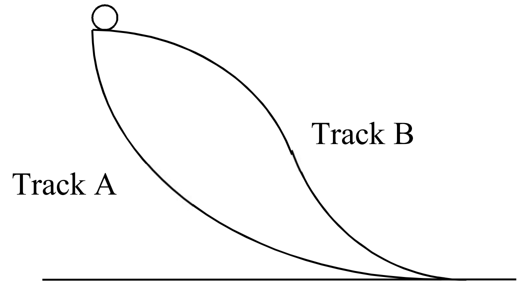
8. As the ball falls back down is gravity doing positive or negative work?

9. If I throw a ball straight up, and catch it when it returns back to the same height, has gravity done any net work? Explain.

10. If I throw a ball to a friend (so it has horizontal motion in addition to vertical), and he catches it at the same height, has gravity done any net work?

11. Does the horizontal motion involved in the previous problem have any impact on how much work gravity does? Explain.

12. Let's return to the question that started this activity. Two balls are released from the same height on tracks of different shapes (A and B). Now, think about the work that gravity does as the balls roll down the tracks. Does gravity do the same or a different amount of work on the two balls as they go from the top to the bottom?



Are there any other forces doing work on the ball? Static friction allows balls to roll, but it won't change the speed of a rolling object. There is something called "rolling resistance", which arises due to rolling objects deforming some as they roll, which creates a torque that acts to slow the ball down. The more a ball deforms, the greater the rolling resistance (i.e. when you inflate your car tires more, that air pressure helps keep the tires from deforming as much when they roll, reducing the rolling resistance). Let's assume those balls are steel balls, which hardly deform at all, so rolling resistance is negligible. So there is no work being done due to that.

14. Now what do you think - how should the speed of the two balls compare when they reach the bottom of their tracks? (don't revise your previous answers if you feel differently now, you will not be penalized if your initial thought was wrong)

15. What about how long it takes to get to the bottom? Will they get there at the same time? If not, which one wins? Explain.

35. Energy Problems

*Name*________________________________

$$W_{NC} + mgh_i + {}^1\!/_2\, mv_i^2 = mgh_f + {}^1\!/_2\, mv_f^2$$

Work input + total initial energy = total final energy

This activity involves problems that can't be solved with just one of our "tools" – you will need to break the problem into parts, and figure out which tool to use for each part. Before solving a problem, your first task will be to identify the individual steps you will take in order to get to the final answer, thinking about which "tool" will be helpful (and valid!) for each step.

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1. A girl is on a swing, which has chains that are 4 meters long. When she is at the high point of the swing, the chains make an angle of 60° with the vertical. We want to determine how fast she is going when she is at the bottom of the swing.

a. Draw a sketch of what is going on. You need to pick an initial moment and a final moment. Let's call the moment when she is at the high point our initial moment (i), and at the bottom of the swing our final moment (f). Label those moments on your sketch.

b. Since gravitational potential energy is involved, we need to pick a reference point from which we will measure our heights (for the *mgh* terms). The most convenient spot would be the lowest point of her swing – so label that spot $h=0$ on your sketch, to make it clear where you are measuring heights from. Why is that a convenient choice?

c. Write out the conservation of energy equation: $W_{NC} + mgh_i + {}^1\!/_2\, mv_i^2 = mgh_f + {}^1\!/_2\, mv_f^2$. After writing it out, cross out any terms that are zero.

d. Let's think carefully about the W_{NC} term. Other than gravity, what force is acting on the girl?

e. Does that force do any work on her as she swings? Why or why not? If you aren't sure, ask one of us – this is an important issue.

f. In the problem, I don't tell you how high she initially is. I do give you some information though that you can use to figure that out. Do so below.

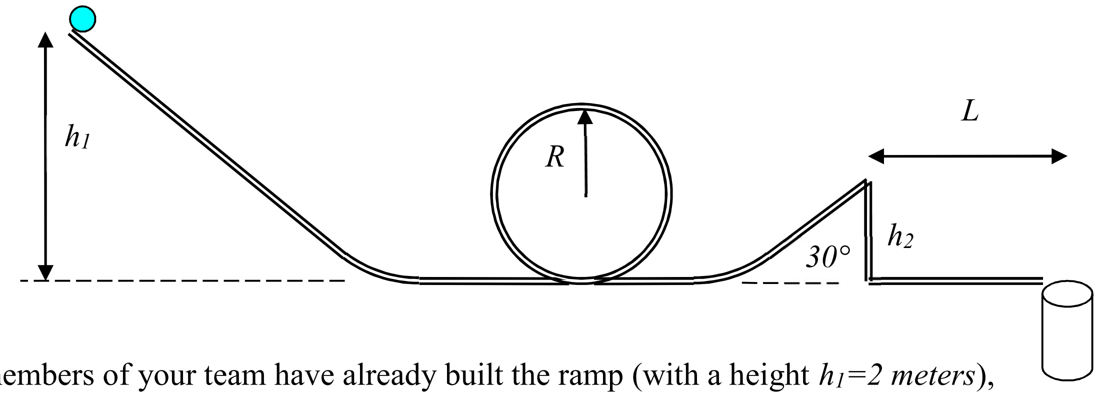
g. Now that you know her initial height, you should be able to use your conservation of energy equation to solve for the speed of the girl at the bottom of the swing. Do so below.

Does that process make sense overall? Make sure everyone in your group is comfortable with it. If anything is unclear, talk to one of us. As we progress, things will get messier – so you want to make sure you start off being comfortable with this. There will be problems that involve multiple tools. For example, what if I had instead asked this question:

On her backswing the chains make an angle of 60° with respect to the vertical. She then swings forward, and as she is swinging up she jumps off the swing when the chains make an angle of 30° with respect to the vertical. If the chains are connected to a pole 4.5 meters above the ground, how far does she fly while in the air before landing?

2. How would you solve that? (you don't need to solve it – just explain what you would do)

3. You are part of an engineering team building a large Rube Goldberg machine for a contest. The part you are working on today will involve a ball (of mass *m=10 grams*) rolling down a track, around a circular loop, and then flying off a ramp and landing in a cup, as shown below.



Other members of your team have already built the ramp (with a height $h_1=2$ *meters*), the loop (radius $R=0.8$ *meters)*, and the ramp (of height $h_2=20$ *cm*). Your task is to determine how far (L) away the cup should be placed so that the ball will land in the cup. The hill is sufficiently high that the ball makes it around the loop without falling off. Note that we will revisit this problem near the end of the semester after we learn how to properly account for the rolling motion of the ball – for now we are treating rolling objects the same as if it is a sliding object, by implicitly treating it as a point mass. SOLVE ON THE WHITEBOARD, summarize your process below.

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4. How fast will the ball be going when it lands? (hint: there is a very quick and easy way to find this)

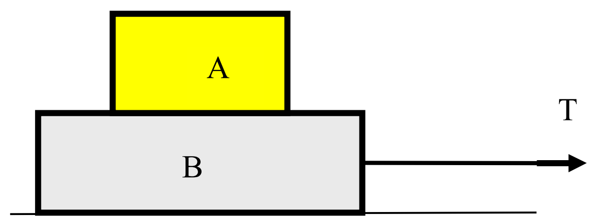
5. What direction will the ball be moving in when it lands? (express as some angle measured with respect to horizontal or vertical)

6. Kinetic friction *usually* does negative work. Can it ever do positive work though? Discuss – if it can do positive work, describe a situation in which it is doing so.

7. Can static friction ever do work, positive or negative? Discuss.

Block A (2 kg) is stacked on top of block B (5 kg), with a rope attached to block B, as shown below. The rope is pulled with a force of 40 N. Between all surfaces, the static and kinetic coefficients of friction are 0.4 and 0.2, respectively. For the tension being applied, block A will ***not*** slip relative to B.

8. Is the net work on block A positive or negative?



9. What force (or forces) is *directly* responsible for the work done on block A (i.e. by directly responsible we mean that that is the force directly acting on block A, causing it to accelerate). Draw a FBD of block A. Your answer to this *may* make you want to revise your answer to problem 7. ☺

10. After the block has slid 10 meters, how fast are the blocks moving? First determine this by using Newton's 2nd Law to find acceleration, and then using kinematics. (hint: note that the two blocks are moving together – does that let you analyze this in a much simpler way than if block A was slipping?)

11. Now determine how fast the blocks are going after 10 meters using the concept of energy.

12. If we wanted to know how fast they are going after 10 *seconds* instead of 10 *meters*, would the concept of energy be the quickest approach? Explain.

35. Energy Problems

36. Variable Forces

Name___________________________

Background

Since the work done by gravity only depends on the change in elevation, we can use the concept of potential energy to quantify how much work gravity does. Another way of looking at this, is that the potential energy in a system is a measure of the capacity to do work. The higher up you lift an object, the more you increase its potential energy - meaning that gravity will be able to do more work on it when you release it, as gravity will continue pushing it over a greater distance.

Forces like gravity, for which the net work done over a path does not depend on the path taken (only on the net displacement), are called conservative forces. Next semester you will encounter another important conservative force – the electric force. A simpler conservative force we can deal with now though is the force a spring exerts on an object due to the spring being compressed or extended.

Presumably by this point in your lives, at some point or another, you have experienced stretching and compressing a spring. If so, you may have noticed that the more you stretch or compress the spring, displacing it from its resting state (equilibrium), the harder the string fights you. We call the force the spring resists with a "restoring force" – because that force is always acting to try to push the spring back to its equilibrium state (neither stretched nor compressed).

In the first part of this activity, you will determine through experiment *how* the restoring force a spring exerts depends on how much it is displaced from equilibrium. In the second part, you will determine what the potential energy stored in a compressed or stretched spring is, based on what the restoring force is.

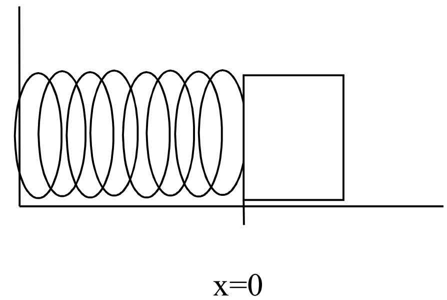
36.1. **Experiment**

Your lab group will have one spring, a set of masses, a meter stick, and rods and clamps sufficient for hanging the spring. Using these, determine how the restoring force exerted by the spring relates to how much the spring is displaced from equilibrium (which we can call x. So $x=0$ would mean the spring is in equilibrium, a negative value of x means it is compressed, and positive x means it is stretched). In other words, is the restoring force proportional to x? Or maybe x^2? Perhaps $^1/_x$? What is the proportionality constant?

Discuss within your group how you are going to collect data, what data is important to collect, and how you are going to analyze that data. On this page and the next, explain how you carry out your experiment, record your data, show your data analysis, and your conclusion. You can use the *Graphical Analysis* software on the computers to plot your data and fit as you think necessary.

36.2. **Potential Energy of a Variable Force**

Consider a spring laying on its side, fixed to a wall at one end (the left). A block of mass *m* is to the right end of the spring, but not attached to the spring. You push the block to the left, compressing the spring. Let *x* define the right end of the spring, where *x=0* when the spring is neither stretched nor compressed. Positive values of *x* indicate it is stretched, negative values indicate that it is compressed. Assume the spring applies a restoring force equal to $\vec{F} = -k\vec{x}$, where the negative sign indicates that the force points opposite the displacement (thus a “restoring” force).
Assume there is no friction.



1. If you compress the spring a distance *x* from equilibrium, how much work has *the spring* done?

2. You release the block, allowing the spring to uncompress. At the moment it gets to the point where it is neither compressed nor stretched, there should be no restoring force. Does that mean the spring will suddenly stop and stay like that? Explain what will happen. If the spring keeps moving, will it maintain contact with the block? Assume there is no friction.

3. How fast should the block be going after it has left the spring?

4. Let's reconsider this hypothetical case, but now with the block being glued to the end of the spring, so they maintain contact. Let's say you initially compressed the spring a distance of 3 cm. You then release the block, and the spring uncompresses, and then starts stretching past equilibrium. By the time it stops, how far past equilibrium should the spring go, if there is no friction?

5. When the block comes to rest at that point, will it remain at rest? Assuming no friction, what do you expect to happen over time?

37. Energy and Momentum

*Name*__________________________

Vacuum Cannon

Hopefully you just saw this demonstration in lecture. A cardboard box was hanging from a string connected to the lecture hall ceiling, making a giant pendulum. We shot a projectile (made of PVC) into the box, out of a vacuum cannon. Assuming everything went according to plan, the box was not blown into tiny pieces, and the projectile was trapped inside the box. After the projectile crashed into the pendulum-box, getting trapped inside, the box swung upwards along its circular path – and we estimated how far horizontally the box went from its initial position. We also hopefully remembered to measure the mass of the box before the experiment, and after the experiment. From all of this, you need to determine how fast the projectile was going before it hit the pendulum-box.

Necessary data: Length of pendulum: ____________________________
Initial mass of pendulum-box: _____________________
Final mass of pendulum-box: ______________________
Horizontal distance travelled by box: ________________

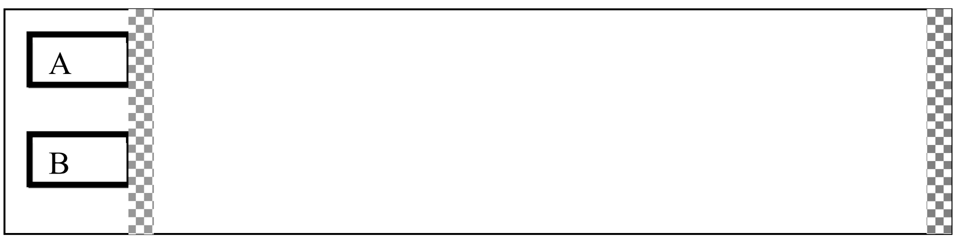
1. Explain in words how you will do this, in a step-by-step manner.

2. Determine the speed of the projectile before it hit the box. (more room on the next page)

37. Energy and Momentum

You have seen this semester how conservation of energy and conservation of momentum are both a consequence of Newton’s Laws. One though looks at the result of a force being applied over time, while the other looks at the result of a force being applied over a displacement. This is a critical distinction, which we will focus on in this activity. ***Unless stated otherwise, assume there is no friction throughout this activity. Also prove all of your answers.***

First, let’s consider this scenario: two carts (A and B) are at the left end of a track, as shown below. Both carts have fans on them, which will exert an equal magnitude force on each cart, propelling them to the right. Cart A though is twice the mass of cart B.



Starting line

Finish line

3. If the cars are pushed with the same magnitude force (each being propelled by an equivalent fan), how will the time it takes cart A to get to the finish line (t_A) compare to the time it takes cart B to get to the end (t_B)? Determine an exact answer.

4. How will the kinetic energies of the two carts compare when each they reach the finish line? Prove your answer.

5. How will the momentums of the two carts compare when they each reach the finish line?

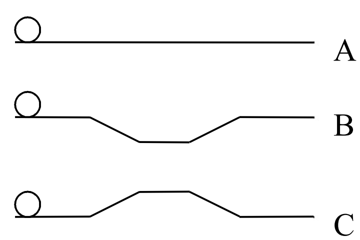
Let's now consider a different scenario: the two carts (A and B) are held at the top of an inclined track, and simultaneously released (this time there is no fan propelling them). Again, cart A is twice as massive as cart B.

6. How will the amounts of time it takes each cart to reach the bottom of the incline compare?

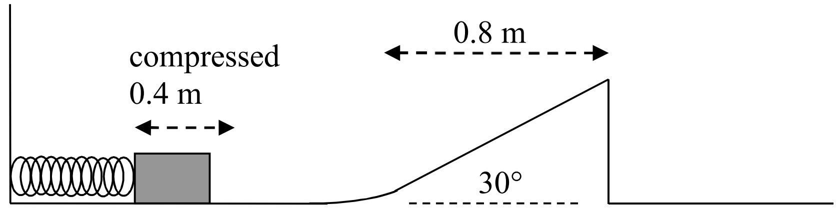
7. How will the kinetic energies of the two carts compare when they reach the bottom?

8. Compare the momentum of the two carts when they reach the bottom of the incline.

9. Three identical balls are rolled to the right with the same initial speed on three different tracks– a purely flat one (A), a track with a brief dip (B), and a track with a brief hill (C). If the horizontal length of each track is the same, which ball will reach the end of its track first? Explain.



10. A block of mass 2 kg is pushed against a spring with spring constant $k=500\ ^{N}/_{m}$, until the spring compresses by 0.4 meters. The block is then released, and pushed to the right by the spring. The block slides, without friction, along a flat surface over a distance of 2 meters (not drawn to scale above). The surface then curves upwards (so the block does not smack into the base of the inclined plane). The block ultimately flies off a plane inclined at 30°, eventually coming back down and landing at the same elevation. How fast is the block going when it lands?



11. For the previous problem, do you *need* to analyze the projectile motion of the block at all for answering the question of how fast it is going when it lands? Discuss and explain.

12. Now we want to know how far from the plane the block lands. Can you solve this purely through conservation of energy? Explain.

13. Find how far the block lands from the end of the plane.

38. Energy Problem Solving I

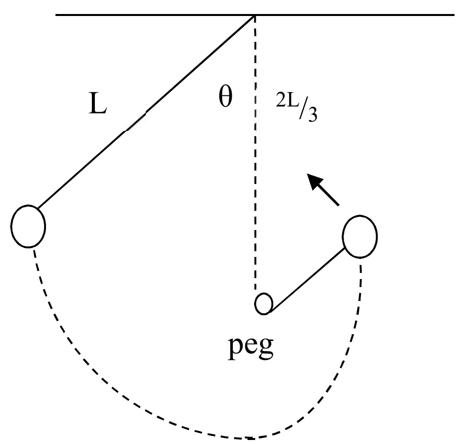
*Name*________________________

At this point in the course, you have seen a few different laws of physics – although they all ultimately come from Newton's Laws. A common difficulty students have in introductory physics courses is reading a problem and knowing how to approach it – generally phrased as "which equation to use". With many physics problems, there are multiple approaches you could take to solve the problem – so just because a problem has a "right solution", that does not mean there is one "right method" to solve it. However, very often one method will be easier or more direct than others.

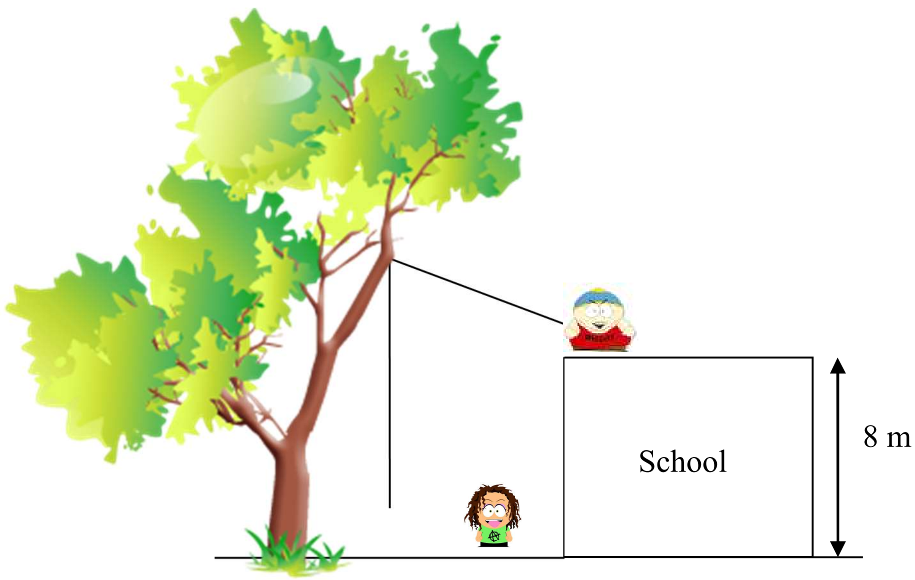
Today's activity will primarily give you practice solving problems of varying levels of difficulty – problems for which multiple approaches may be possible. Perhaps it would be useful to look at it from a Dynamics perspective (Newton's 2nd Law and kinematics), or maybe conservation of momentum, or perhaps conservation of energy. Most problems will require a combination of approaches.

1. A ball hangs on a string of length L, suspended from the ceiling, forming a pendulum. A peg is mounted a distance of ⅔ L directly below the point on the ceiling where the string is attached. The pendulum is pulled back, and released. When it gets to the bottom, the string hits the peg, and the pendulum then pivots around that point instead of the ceiling.

What is the minimum angle (θ) you must pull the pendulum back to, so that after the string hits the peg, the ball on the end of the string will be able to swing all the way around, without the string going slack at the top? SOLVE ON WHITEBOARD, summarize here.



An 8 meter long braided rope with a "breaking force" of 800 Newtons (180 pounds) hangs from a large tree next to an elementary school. Kenny, Stan, Kyle, and Cartman have climbed to the top of the school, and are taking turns swinging from the top of the school on the rope, buzzing by other kids on the playground, and then swinging back up to the top of the school. The kids each have masses of 28, 32, 20, and 50 kg, respectively. If the elementary school is 8 meters high, and the rope is attached to the tree 9 m off the ground, which of the kids (if any) will snap the rope when they swing?

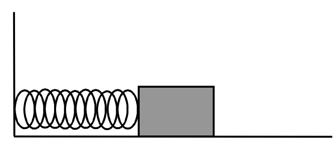


2. WHY might the rope snap while the kids are swinging? Does it matter that the kids are swinging on the rope instead of just hanging from it (with the rope vertical)? Explain.

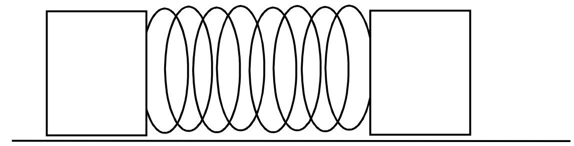
3. Determine which of the four kids (if any) can swing safely without breaking the rope. Clearly show and explain your work. SOLVE ON WHITEBOARD, summarize here.

8. A block (M=0.5 kg) is connected to a horizontal spring (k=400 N/m), and resting on a rough table (μ=0.3). This makes up a new ballistics setup at a local gun manufacturer. The company plans to shoot the block with bullets, and measure how far the spring compresses (before it uncompresses, and oscillates back and forth). They want you to tell them how to use this setup to calculate the speed of the bullet before it hit the block.

They do a test-fire with a 5 gram bullet, and observe that the block moves a maximum distance of 15 cm after being shot by the bullet (which stays in the block). Based on this, calculate how fast the bullet was moving *before* it hit the block.

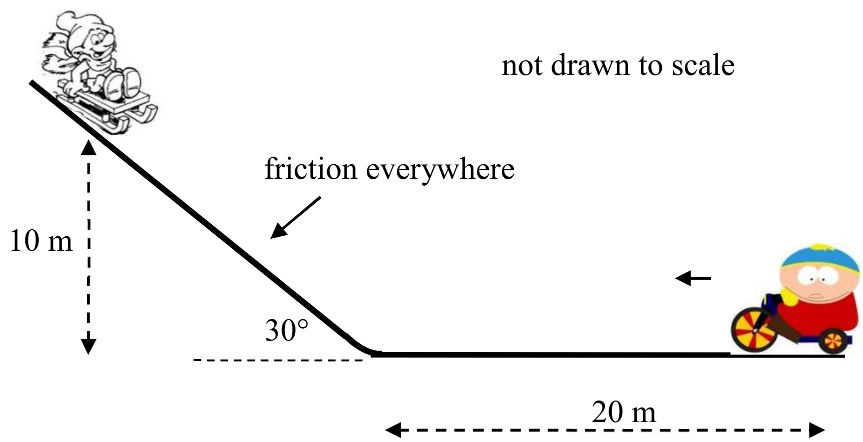


9. A spring with spring constant of 1,000 N/m is squished between two blocks, which each have a mass of 100 grams. The spring is initially squished such that it is 10 cm shorter than normal, and then released such that both blocks go flying apart. How fast will each block be going after leaving the spring? Assume there is no friction.
WHITEBOARD, summarize



10. Let's now consider the same scenario, but where the block on the right has a mass of 200 grams, twice that of the block on the left. Now how fast is each block going after leaving the spring?

11. An evil smurf has developed an addiction to running over South Park characters while on his sled. He has found a 30° slope that is 10 meters tall, and begins sledding down it while Cartman is riding his Big Wheel towards the bottom of the hill. When the Smurf begins sledding, Cartman is 20 meters away from the bottom of the hill, riding towards it at a constant speed of 4 $^{m}/_{s}$. The coefficient of friction between the Smurf's sled and the snow is 0.2. How far away from the bottom of the hill does the Smurf run into Cartman?
WHITEBOARD, summarize



38. Energy Problem Solving I

39. Determining a Spring Constant

*Name*________________________

We have carts in the classroom that have spring-loaded “plungers”. You can compress the plunger to different catch points, and then push a button on top to release it. There is an issue though - the springs do not fully uncompress. When the plungers come out as far as they can, there is something stopping them – keeping the spring from fully uncompressing.

1. Does that matter? If I want to know how much energy is released from a spring as it uncompresses, is it equivalent to just treat the final positions as having no energy, and measuring the compression for the initial spring potential energy based on how much the spring was compressed relative to its final position? (I don’t feel like I worded this well…. Feel free to ask for clarification)

Fortunately we have a cart whose plunger detached from the spring. So we can determine where the actual equilibrium position is, so you can measure the initial and final compression relative to that. Hopefully by the time we get to this activity I will have figured out a good way to explain how to do this. ☺

2. Come up with an experiment to determine the spring constant of the spring that pushes the plunger. Explain your procedure below, show any data, and your determined result. With any science experiment, it is important to assess the validity of your method by repeating your experiment and seeing if you get the same result (or fairly close). You should also change something that you can vary (such as the mass being pushed by the spring) to see if that has an effect on the value of the spring constant you determine. If you get wildly different values for the spring constant under different circumstances, then that is an indication that something is amiss.

Part II:

Put two carts against each other, with the plungers fully compressed in. If you release the plunger of one of the carts, both should go flying apart. Try to calculate how fast they will initially be going, using the value of the spring constant you determined, and any other measurements you need to make (such as the mass of the carts).

Then do that experimentally, using a motion detector to measure the initial velocity, and see how close your calculation was. Show your calculations below, and your experimental value. Then describe any factors that you think could account for a difference between your calculations and experimental value.

Part III:

Repeat the experiment done in part II, but now make one of the carts twice as heavy as the other. Determine what the final speed of each cart should be on paper. Then experimentally measure one of the cart's speeds after the plunger pushes the two carts apart, and see how closely it matches your calculation.

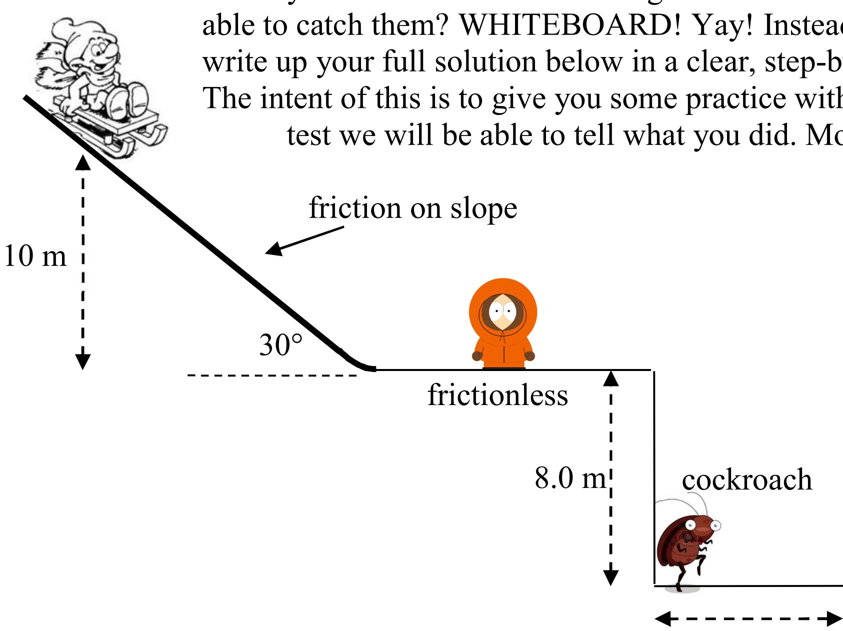
40. Energy Problem Solving II

*Name*________________________

1. A smurf sits on a sled on top of a 10 meter tall hill inclined at 30° above horizontal. The smurf and sled together have a mass of 45 kg, and the coefficient of friction between the sled and the snow on the slope is 0.4. At the bottom of the hill, the ground is all ice due to melted snow having frozen – such that the flat area shown in the picture is frictionless.

The smurf starts at the top of the hill from rest, slides down the hill, then across the frictionless flat ground for a little ways, and runs into Kenny, who has a mass of 20 kg. As the smurf collides with Kenny, Kenny falls onto the sled, and he and the smurf then slide together across the rest of the frictionless flat surface, until they reach a cliff. They fly off the edge of the cliff, which is 8.0 meters above the ground below.

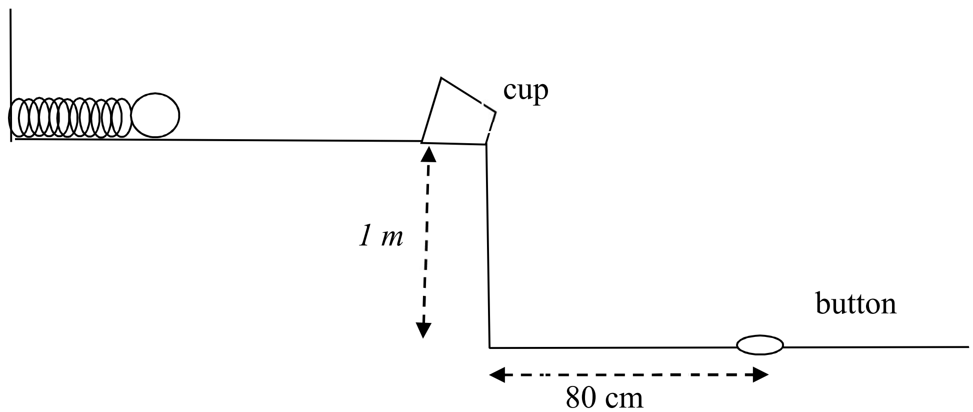
A giant, but friendly 15 kg cockroach happens to be standing at the bottom of the cliff, directly under the edge. He sees the smurf and Kenny fly off the cliff, and starts running to try to catch them. He sees the smurf and Kenny fly off the cliff, and starts running to try to catch them. If he gets there in time, the cockroach can catch them at a height of 50 cm above the ground. If he accelerates at 15 m/s^2, how soon after Kenny and the smurf leave the edge of the cliff must the cockroach begin running to be able to catch them? WHITEBOARD! Yay! Instead of just summarizing for this one, write up your full solution below in a clear, step-by-step manner that is easy to follow. The intent of this is to give you some practice with how to present things so that on the test we will be able to tell what you did. More room on the next page.



2. Kenny is on a swing. The chains supporting the swing are 3 meters long, and at the lowest point Kenny is 0.4 meters above the ground. Kenny starts swinging as fast as he can, and on the "backswing" he swings up such that the chains are only 20° below horizontal. On the forward swing, he "jumps" off of the swing (he just lets go of the swing) when the chains are 50° below horizontal. How far (horizontally) does he fly in the air before landing? WHITEBOARD, summarize

3. Cartman is now swinging on the same swing. He notices that Kenny trips and falls after landing, and is laying down on the ground 5 meters in front of the swing (in front of the spot where the swing hangs). Cartman starts swinging back and forth, and is going to "jump" off the swing (let go) when the swing has swung up 30° past vertical (past where it just hangs normally). How close to horizontal does he have to swing on his backswing, so that he will land on Kenny after letting go of the swing? WHITEBOARD, summarize

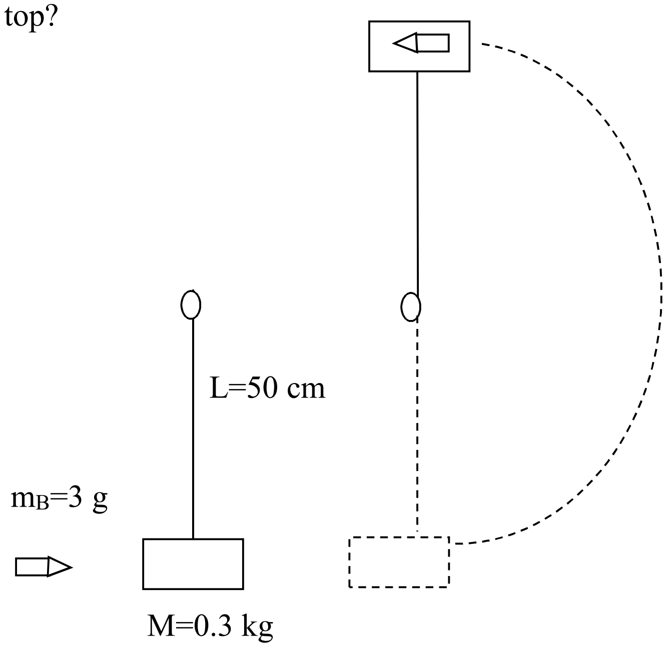
4. Part of your Rube Goldberg machine you are building involves using a spring to shoot a ball across a table, roll into a heavy metal cup, and the ball and cup together fly off the end of the table (horizontally, initially). The cup (with the ball in it) needs to land on a button to trigger the next stage of the machine. The ball has a mass of 50 grams, and the cup has a mass of 30 grams. The spring has a spring constant of 40 $^N/_m$. The table is 1 meter high, and the button needs to be placed 80 cm away from the edge of the table, as shown. How much should you compress the spring so that the cup (with the ball in it) will land on the button? WHITEBOARD, summarize



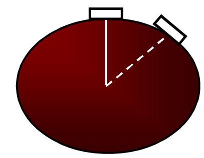
5. A wooden block of mass M=0.3 kg is hanging vertically from a string 50 cm long. A 3 gram bullet is fired at the block of wood as shown below (bottom left drawing). The bullet embeds itself in the wood, and the block of wood swings around in a circular path, as shown in the right drawing.

How fast does the bullet need to be going in order for the block to make it all the way around the circular path, without the string going slack at the top?

WHITEBOARD, summarize



7. An ice cube is placed on top of a bowling ball, just slightly off-center, and it starts sliding down the side of the ball. At what moment will the ice cube leave the surface of the bowling ball? (i.e. what angle will it have fallen through, as shown in the picture, at the moment it loses contact. Assume the bowling ball does not move)
WHITEBOARD, summarize



Time to play "stump your neighbors" again!

8. Make up your own two-dimensional kinematics problem. It must be a problem that requires more than one step to solve it. Write out your problem on a separate page, and your solution.

Also write your problem (without solution) on another separate piece of paper. Put both in your folder. Ideally, also have us check your problem over to make sure it is solvable. Before the next class I will go through all of the problems and make sure they are solvable (and adjust if not). In the next class we will pass them out to other groups, who will try to solve them. You are going to have to check the solution from the group who gets your problem (and we will arbitrate). You will. Put your own problem and both your solution and your neighbor's solution to your problem in your folder when you are all finished.

If you can create a problem that your neighbor can't solve correctly, but you can, and you also correctly solve the problem created by and given to you by the third group at your table, then your group gets 5 bonus points on the upcoming exam.

41. Torque and Static Equilibrium

Name________________________

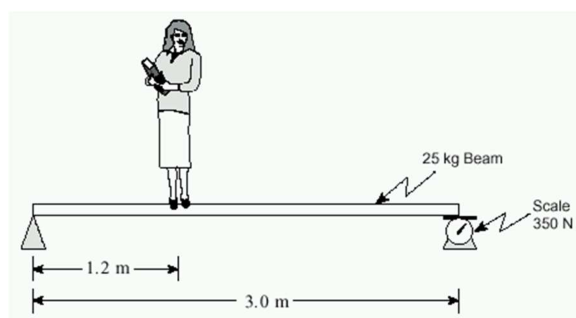
For an object to be in static equilibrium, not only must the net force on it be zero (so it has no acceleration), but also the net torque on it *about any point* must be zero (so it will have no angular acceleration).

1. A meter stick is balancing on a pivot placed at the 50 cm mark (right in the middle of the meter stick). A 100 gram mass is hung from the 10 cm mark. You also have a 250 gram mass – where should you place it so that the meter stick will balance perfectly? (draw a picture, then show your work)

A librarian is for some reason standing on a 3.0 meter long 25 kg beam that is supported on the left end by a triangular piece of wood, and on the other end by a scale. The scale reads how much force is being exerted downward on it by the beam – which by Newton's 3rd law means it is also telling us how much force the scale is exerting upwards on that end of the beam. When the librarian is standing 1.2 meters in from the left end, as shown, the scale reads 350 Newtons. Determine the weight of the librarian in Newtons, and also how much force the triangular piece of wood is exerting upwards on the left end of the beam.

I will break the problem into parts for you.

2. Draw a free body diagram (FBD) of the beam. Since we care about where forces act, your FBD should show the beam roughly how it is shaped, and show the forces where they act. You should have four forces acting on the beam (gravity, librarian, left support and scale).



3. For something to be in static equilibrium, two things must be true: there must be no net force on it (since it is not accelerating linearly), and there also must be no net torque on it (since it is not accelerating angularly). We have two unknown forces – from the librarian and from the left support. Since we have two unknown vertical forces, we won't be able to find both of them by summing forces in the y-direction alone. It may still end up being worthwhile to sum forces though. Go ahead and apply Newton's 2nd law to the vertical direction and see what it gives you.

You should have two unknowns in your force equation, which means we need another equation. So let's try applying Newton's 2^{nd} law in angular form – which means summing torques. When summing torques, you must pick some point to serve as your pivot point. That is the point around which you are summing torques, so all of your lever arms are measured from there (the r in the $\tau = rF_{\perp}$ equation). If something is actually rotating, then it is rotating around a certain point – so your pivot point is already chosen for you. But if it is not rotating, then it is in static equilibrium around EVERY point. So the net torque around ANY point must be zero. This gives you the freedom of choosing any point you want as your pivot point for static equilibrium.

3. What I wrote above is an important point – the most important issue in fact right now. Discuss it with your group members, and if anything is unclear about it, please ask us to help out. Then explain below in your own words what I wrote above.

4. Where we choose our pivot point can make the problem much harder or easier to solve. Each time we sum torques, the lever arm for every force is measured from our chosen pivot point (for that equation) to where each force acts. That is a very powerful tool. If a force acts directly at the point we choose as our pivot, how much torque does that force make around that point? (this is again a critical point – if you are unsure, ask for help)

Since we have two unknowns, we can make our work easier by choosing as our pivot a spot where one of those unknown forces acts, because that will effectively remove that force from our equation (your answer to the previous problem is why).

5. Pick the spot where one of your unknown forces acts as your pivot, and sum torques around that point (and since it is in equilibrium, what does the net torque have to add up to?). To do this, you will need to pick some direction as positive (i.e. clockwise or counterclockwise), and measure all of your lever arms from the pivot point you chose. Do this, and use that to solve for one of your unknowns.

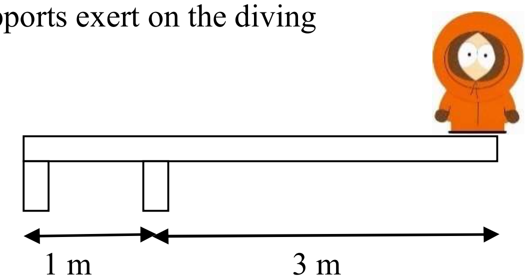
6. Once you have one of your unknowns, you can find the other one either by using your equation from summing forces in the y-direction, or by picking another spot to sum torques around. Do one of these to determine the other unknown force.

41. Torque and Static Equilibrium

Kenny walks out to the end of a diving board, as shown to the right. The diving board has two supports, L and R (for left and right supports), separated by a distance of 1.0 m, as shown in the diagram. For now, assume the diving board itself is massless, and Kenny has a mass of 20 kg. The system is stationary.

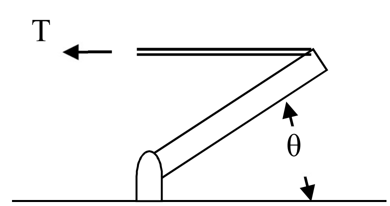
We want to know what the forces on the diving board from the two supports are – and also what direction those forces are in (i.e. are both supports pushing up? Or maybe one is pushing up, while one pulls down?) This problem is very similar to the previous one, so I won't walk you through it. One tip though – if you are not sure what direction a force acts in, just make a guess. If your guess is wrong, you will just get a negative number, indicating it is in the opposite direction (i.e. if a force is upwards and equal to 40 Newtons, but you guess downwards initially, you will calculate a force equal to -40 Newtons, telling you it is actually upwards).

7. What forces (and in what direction) do the left and right supports exert on the diving board?



Now let's try one with an angle. Remember – it's only the portion of a force ***perpendicular*** to the lever arm that produces any torque. If you push on the very end of a door, pushing directly towards the hinge (as shown below), you produce no torque. So when calculating the torque made by a force, it's $\tau = rF_{\perp}$, where $F_{\perp}$ is the portion of the force that is perpendicular to r, the lever arm.

8. As shown below, a solid steel beam of mass m is attached to a pivot at the bottom, and a steel cable at the top. For the beam to be held stationary at some angle θ, what must the tension (T) in the cable be? (in terms of m and θ) Start by drawing a FBD of the beam, with it shaped the way it actually is, showing all the forces acting on it in the appropriate places.

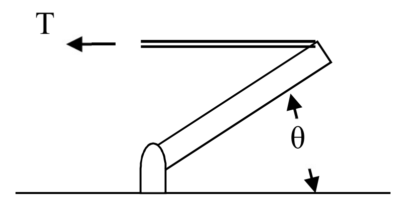


41. Torque and Static Equilibrium

Looking at "limiting cases" can often help you check to see if you made any mistakes or not. This is one of the values of solving for things symbolically – we can check our answer at different values, and see if it gives us what we would expect. Very often, in problems with angles, we can intuitively tell what the answer should be at certain angles or extremes. We can then check to see if our symbolic answer gives us the same thing in those cases.

9. Think about the previous problem, and how much tension you would need to have in the cable to keep the beam held in equilibrium if the angle were two extreme cases: 0° and 90°. First explain what tension you would expect to need in each case (and whether it would be possible for the beam to be held in equilibrium in each case – where at 0° the beam would be parallel to the ground but not resting on it) – and then use your equation to see what tension it says would be required at those angles.

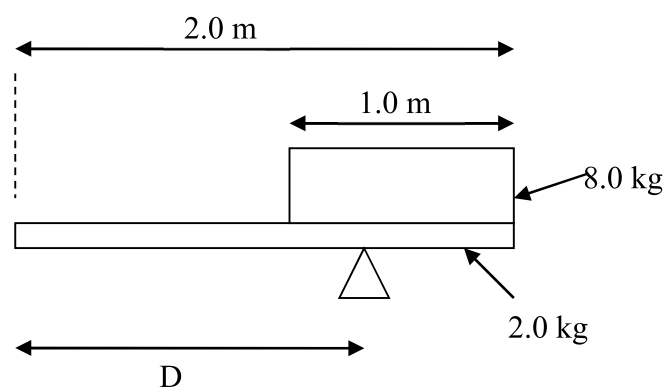
0° (beam is horizontal):



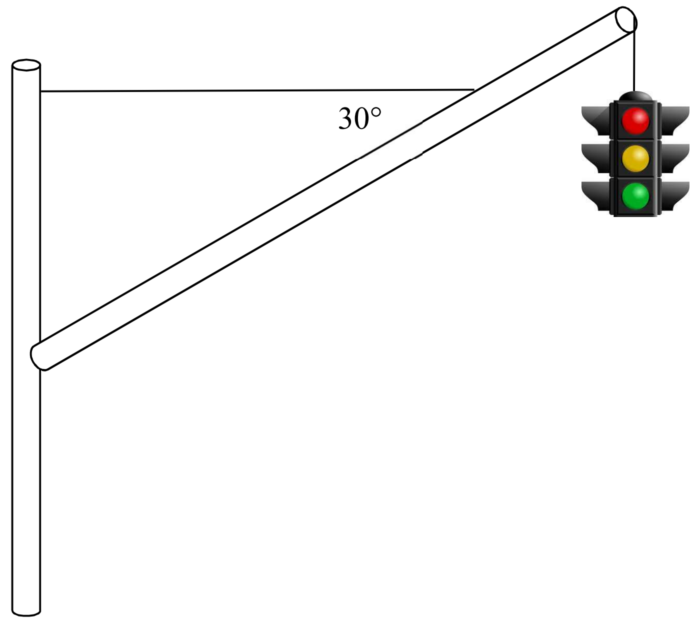
90° (beam is vertical)

10. The two blocks shown to the right are resting on a pivot. The lower block is 2 meters long and has a mass of 2.0 kg. The upper block is 1 meter long and has a mass of 8.0 kg. The right edges of each block are lined up. How far in from the left edge (distance D in the diagram) should the pivot be placed, so that the blocks balance?

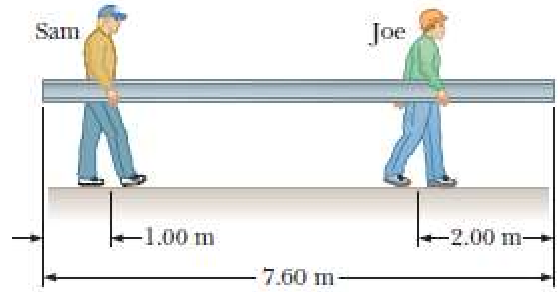
WHITEBOARD, write solution below (it's tricky, but not long)



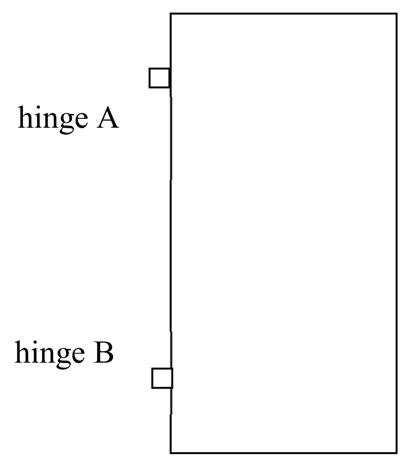
11. A 15 kg traffic light hangs from the end of a 10 kg metal pole that is 7 meters long, as shown below. A massless cable attaches to the angled pole and also to the main vertical pole. The cable is perfectly horizontal, and makes an angle of 30° with respect to the tilted pole. The cable is 4 meters long. What is the tension in the cable?



12. A uniform beam of length 7.60 meters and mass 50 kg is carried by two workers, Sam and Joe, as shown to the right. What vertical force is each worker exerting?



13. A 10 kg rectangular door hangs from two hinges, as shown to the right. The door is 1 meter wide and 2.2 meters tall. The hinges are attached 0.5 m from the top (hinge A) and 0.5 m from the bottom (hinge B). What is the horizontal force that each hinge exerts on the door (also indicate direction)? (don't worry about finding the vertical force each hinge applies to the door)



42. Moment of Inertia

*Name*____________________

1. Does a hammer have the same moment of inertia (aka "rotational inertia") regardless of whether you swing it from its "handle" end or from the opposite end? Explain.

2. Suppose you have a solid sphere and a hula hoop of the *same radius and same total mass*. Which should have a higher moment of inertia? Discuss with your group (don't use the book!), and justify your answer qualitatively (in words).

3. A merry-go-round is essentially a large disk – and the moment of inertia of a flat disk is $I = \frac{1}{2}mr^2$. A merry-go-round with a mass of 200 kg and diameter of 6 meters is initially stationary, until Kenny starts pushing it. Kenny, who has a mass of 30 kg, pushes with a constant force of 30 N as he runs around with the spinning merry-go-round. How fast will he have to run to keep up with the merry-go-round after 20 seconds, assuming the merry-go-round was initially stationary?

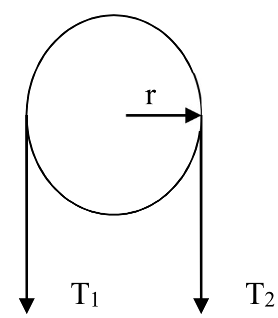
4. How many revolutions will the merry-go-round go through in those 20 seconds?

5. If Cartman had been sitting on the edge of the merry-go-round while Kenny was pushing it (pushing with the same force), would the merry-go-round still have the same angular velocity after 20 seconds as you found with Cartman not sitting on it? Explain in terms of the moment of inertia, and how it would affect things.

Up to this point, we treated pulleys as if they had no mass, and thus no moment of inertia. That meant that no *net* torque was required to give a pulley an angular acceleration. That is what allowed us to say that the tension in a rope was the same on both sides of a pulley.

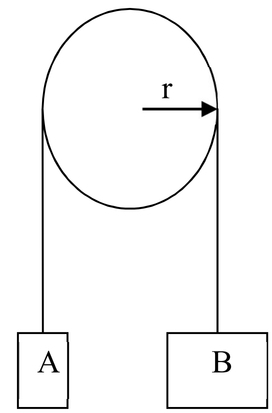
However, now we are treating pulleys as having mass – thus having a moment of inertia. That means that if a rope going over a pulley is accelerating – and therefore the pulley is undergoing angular acceleration – there must be a net torque on the pulley.

6. If a pulley that DOES have mass (and therefore moment of inertia) is undergoing an angular acceleration, can the tension in the rope going over the pulley be the same on both sides? Explain. (Consider the diagram to the right of a pulley with a rope going over it, with the rope on the left having tension T_1 and the right having T_2. If $T_1=T_2$, is there a net torque on the pulley?)



42. Moment of Inertia

A disk-shaped pulley of radius r=10 cm and mass M=2 kg has a massless string going over it, attached to masses A and B, as shown to the right. If B has a mass of 100 grams and A has a mass of 50 grams, how fast will B accelerate?



7. Draw Free Body Diagrams of each block and of the pulley, giving each unique force a unique name

8. Apply Newton's 2nd Law linearly to the blocks and angularly to the pulley. Consider how the accelerations of the blocks relate to each other, and how they relate to the angular acceleration of the pulley. Find the acceleration of block B. *Do not put numbers in until you have a symbolic answer for the acceleration.*

9. If we solved the same problem, but made the assumption that the pulley has no mass (so we would say the tension is the same on both sides, as we did earlier in the semester), we would have found an acceleration of $a = g\left(\frac{m_2-m_1}{m_2+m_1}\right)$. If you let the mass of the pulley (M) be zero, does your solution simplify to this? Should it?

10. If the blocks started out at the same elevation and stationary, how long will it take until block A is 2 meters above block B?

11. How fast is each block going when they are 2 meters apart?

12. Let's consider energy now. If we were only focusing on block A, would the tension in the rope do work on it? Explain.

13. If we look at the system as a whole though (both blocks and pulley), is there any net work done by non-conservative forces (such as tension)? Explain.

13. Let's call the initial time when the blocks are stationary and at the same elevation – so let's call that starting elevation $h_i=0\ m$. What is the initial energy of the entire system at that time? (you don't need to count the gravitational potential energy of the pulley itself, since the pulley only rotates – it doesn't change elevation) If your answer to this seems odd, talk to one of us.

14. When the blocks are 2 meters apart, what is the total energy of the system combined? Calculate it to confirm that energy is conserved. Don't forget about the spinning pulley!

42. Moment of Inertia

43. Angular Problems

Name________________________________

1. An ant is resting on a pottery wheel, 20 cm from the center, and a scorpion is 40 cm from the center. Somebody turns on the pottery wheel, and it begins to accelerate at a rate of 0.3 rad/s^2.

a.) How many full revolutions has the ant gone through after 30 seconds (since the pottery wheel was turned on)?

b.) What total distance has the ant traveled in meters in that time?

c.) How many revolutions has the scorpion gone through in the same amount of time, and what total distance has it traveled in meters?

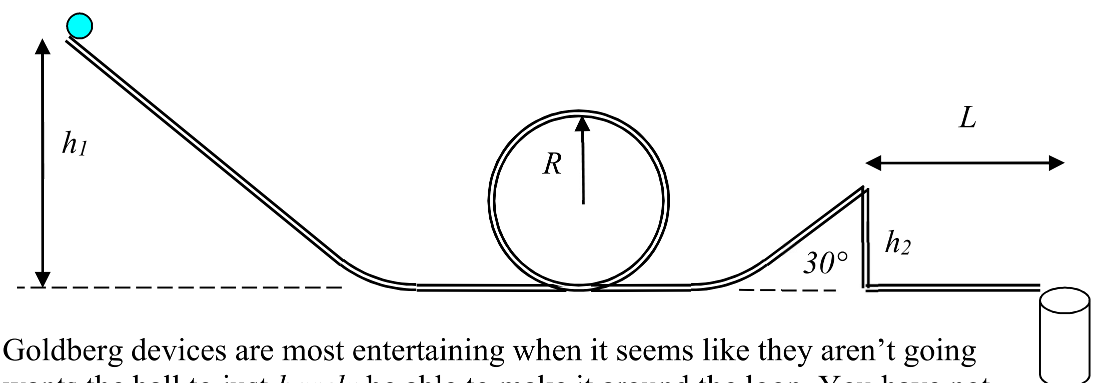
d.) What is the angular velocity of the ant 30 seconds after the pottery wheel was turned on?

e.) What is the linear velocity of the ant (the "tangential velocity") at that time?

f.) What are the angular velocity and tangential velocity of the scorpion at that time?

g.) If the coefficient of static friction between the ant's feet and the pottery wheel is 2 (yes, it is higher than 1), at what time would the ant start slipping on the wheel? (note that the tangential acceleration is very very small compared to the centripetal acceleration by the time he slips, so it can be ignored in the final analysis. For extra credit, solve it including the tangential acceleration and show that you get essentially the same answer)

You are part of an engineering team building a large Rube Goldberg machine for a contest. The part you are working on today will involve a ball rolling down a track, around a circular loop, and then flying off a ramp and landing in a cup, as shown below.



Since Rube Goldberg devices are most entertaining when it seems like they aren't going to work, your team wants the ball to just *barely* be able to make it around the loop. You have not yet built any of the above components – you first need to calculate what the maximum radius of the loop should be for a given hill height (h_1), such that the ball will just *barely* be able to make it around the loop without falling off. Assume there is no friction.

2. If the ball is just *barely* able to make it around the loop, find the maximum radius (R) of the loop you should build as a function of the hill height, h_1.

Part II

Devise an experiment to determine the moment of inertia of a metal pulley. You will then carry out this experiment, to determine the moment of inertia of a large aluminum pulley. To assess the validity of your answer, also make measurements of the dimensions of the pulley, look up the density of aluminum, and calculate the moment of inertia through that route – and compare that with the value you get through an experiment.

43. Angular Problems

44. Conservation Laws (Angular)

*Name*______________________________

Moment of Inertia of a hoop rotating about its axis = mr^2
Moment of Inertia of a solid disk (a cylinder) rotating about its axis = $\frac{1}{2} mr^2$

1. A ball is released from rest at the top of a hill, and starts rolling down the hill. Explain what is going on as the ball rolls down the hill in terms of the ball's energy (i.e. as the ball rolls down the hill, what is happening to the gravitational potential energy? Where is that energy going?).

2. As an object rolls down a hill, how does its moment of inertia effect the translational speed the object gains? In other words, consider this situation: A solid disk and hoop of the same radius and mass are both released an elevation h above the bottom of an inclined plane. The hoop has a larger moment of inertia than the solid disk. Which will be going faster when they each get to the bottom of the plane? Discuss and explain.

3. Using the moments of inertia at the top of this page, determine the translational speed of the hoop and disk described above in terms of the parameters given, to check your answer to the previous problem.

4. Considering the problem of the hoop and disk rolling down the plane - which of the two items has more rotational kinetic energy when they reach the bottom? Explain, and verify by finding their rotational kinetic energies in terms of the parameters given.

5. Which of the two items – the hoop or disk – will get to the bottom quicker? Discuss within your group, and give a qualitative answer and explanation.

Thought Experiments

The setup for these experiments revolves (figuratively and literally) around a chair that can rotate relatively free of friction, and a bicycle wheel with handles on the axle. Some of these thought experiments you will physically carry out after thinking about them – others we will do during the next lecture.

6. Someone sits in the chair and is initially stationary. If a member of your group pushes the person in the chair so as to start him or her spinning, can the person in the chair be considered an isolated system? (i.e. is there any *net* external force or torque acting on the person while being pushed? Has the angular momentum and/or rotational kinetic energy of the person-chair system changed or remained constant as he or she was pushed and started spinning? Explain what happens in this scenario in terms of our new concepts (rotational kinetic energy, angular momentum, and torque).

7. Again consider a hypothetical situation (which you will later test). A group member is sitting in the chair, holding weights in his or her hands, with arms stretched straight out to the side. Another group member pushes the seated group member so as to start the chair spinning. If the seated group member then pulls his or her arms inward, will the angular velocity with which he or she is rotating change? Discuss in your group, and explain. Consider whether the seated group member is an isolated system, whether his or her moment of inertia changes, what should remain constant, etc. Summarize your discussion below.

Now we will move on to some thought experiments (which you will then test yourself) involving a bicycle wheel with the axle extended, so that there are handles on each side. A picture of this apparatus is shown to the right.

Imagine that you hold the bicycle wheel by the handles on the axle, with the wheel *not* spinning. You flip the wheel over (i.e. you flip the axle of the wheel, such that your right hand is now to the left of your left hand). You then flip it back. Consider how much torque you might have to apply to flip the wheel over.

8. You then start the wheel spinning very fast. If you now flip the wheel over, should it feel any different than when the wheel *wasn't* spinning? (i.e. is any additional force or torque required to flip the wheel when it is spinning, compared to when it is not) Discuss and explain.

9. A student is sitting in the chair that can rotate without friction, but it is not rotating. You have the bicycle wheel, and start it spinning. With one of the handles pointing upward, you hand the bicycle wheel to the seated student. If the seated student flips the bicycle wheel over, such that that handle that was pointing upwards now points down – what else must happen? Consider whether the student-chair system is an isolated system, and what the implications of that are? Discuss and explain.

When you reach this point, have an instructor/TA look over and discuss your answers with you. You can then carry out experiments to test the conclusions from your "thought experiments".

10. For each experiment – were your predictions correct? If not, what were you wrong about?

45. Angular problem solving

Name_________________________

Note: Moment of inertia of a long rod rotating about one end is $\frac{1}{3}mL^2$

Moment of inertia of solid sphere rotating around center is $\frac{2}{5}mr^2$

Moment of inertia of solid disk rotating around its central axis is $\frac{1}{2}mr^2$

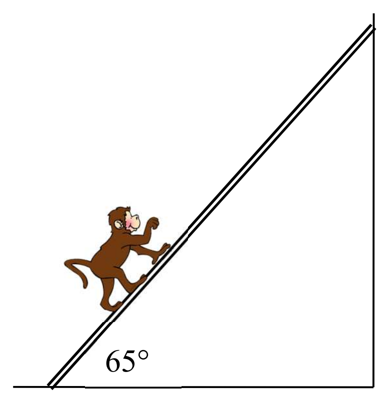
1 pound = 4.45 Newtons

1. Stan, who weighs 60 pounds, pushes a 200 kg merry-go-round (6-meter diameter) with a force of 40 N as he runs with it for 10 seconds. He then hops onto the edge of the merry-go-round – without yanking on the merry-go-round as he hops on, and hopping in a purely radial direction (towards the center). Should the angular velocity of the merry-go-round change due to him hopping on? Discuss and explain your answer.

2. What will be the angular velocity of the merry-go-round after Stan hops on?

3. Kenny (who weighs the same as Stan) comes along and decides to try to stop the merry-go-round. He grabs onto the outer edge, and holds on tight – but is pulled off of his feet. He begins getting dragged around by the merry-go-round, with the coefficient of friction between his pants and the dirt being 0.3. By the time they merry-go-round has stopped, how much energy has been converted to "heat" via friction, heating up Kenny's pants as friction slows down the merry-go-round? Find that value, and then use it to determine how many rotations the merry-go-round goes through by the time it stops. There is more room on the next page.

4. A 3 meter long wooden plank is leaning against a wall, and a monkey is climbing up the plank to be able to reach some bananas hanging from a tree near the wall (see the sketch to the right). The plank and monkey have masses of 20 and 30 kg, respectively. Assuming there is no friction between the plank and the wall, and the coefficient of static friction between the plank and the floor is 0.3, how far along the plank can the monkey climb before it starts to slip (on the floor)? (make sure to draw a free body diagram and identify *all* forces acting on the system. Express your answer as the fraction of the ladder he will be able to climb before it starts slipping) WHITEBOARD, summarize.



45. Angular problem solving

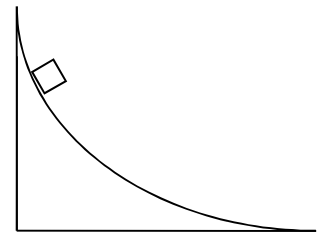
A construction company is about to install a new telephone pole, and has temporarily set the 10 meter pole on its end, standing upright (perfectly vertical). Suddenly, a slight breeze provides just enough of a kick to tilt the pole slightly, and it begins to fall. As it falls, the very tip of the pole smashes into the roof of a house, after the pole has fallen enough that it makes a 45° angle with respect to the ground. As the pole falls, the bottom end of the pole stays stuck in place in the ground, and does not slip (the ground is muddy, and the pole sunk in just enough to keep the bottom from moving). How fast (in $^{m}/_{s}$) was the top tip of the pole moving just before it hit the roof? (make up labels for any important parameters that you need to define)

5. First, consider the above problem from the standpoint of Torque, and the angular version of Newton's 2nd Law.
a.) What is causing a torque on the pole, causing it to rotate (fall)?

b.) Is the net torque on the pole constant as it falls? Explain.

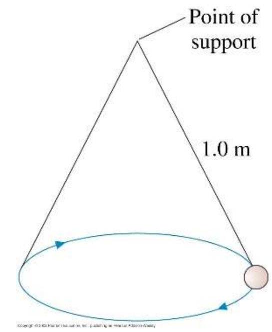
c.) Does Newton's 2nd Law seem like a good way to solve this problem? Try it and find out, and explain whether this is the *simplest* approach or not. Keep in mind that the angular acceleration, α, is the second derivative of the angular position with respect to time, $\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2}$.

6. Consider another problem – a block is sliding down a plane, curved as shown to the right. The elevation of the curve, y, is given by the equation $y = \frac{3m}{x+1}$. We want to know how fast the block is going when it gets to the bottom of the plane. Is this problem in any way analogous to the falling telephone pole problem? If so, how? If not, why not? Discuss and explain your answer.



7. Now, figure out how fast the end of the telephone pole was moving just before it hit the roof (in the problems on the previous page). Keep problem 8 in mind.

8. A conical pendulum is formed by attaching a 500 gram ball to a 1.0 meter long string, then allowing the mass to move in a horizontal circle of radius 20 cm.
a.)What is the tension in the string?



b.) What is the ball's angular speed in rpm?

c.) The top of the pole is 2 meters above the ground. If the rope suddenly breaks, how far away from the base of the pole will the ball land?

Time to play "stump your neighbors" again!
Make up a dynamics question, and give it to the group to your right to try to solve (while the group to your left will give you their problem).

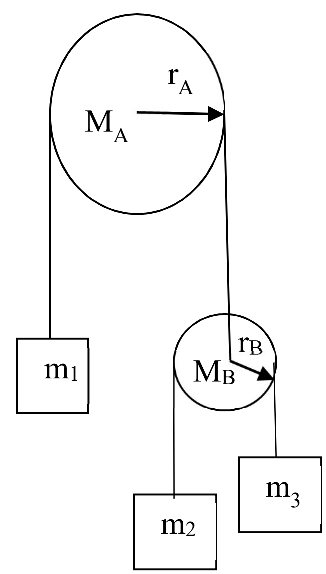
45. Angular problem solving

46. Review

Name_________________________

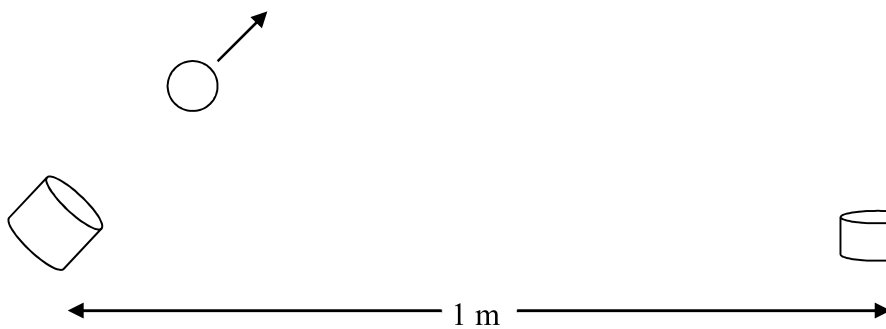
This is a collection of problems of varying levels of difficulty to test your knowledge of all the material covered this semester, and help prepare you for the final exam. Note that some of these problems are quite difficult – don't assume that the exam must be as difficult as these problems. My general philosophy is that it is better to have you prepare by doing problems that are more difficult than actual exam problems, rather than having review problems be easier than the actual exam (in which case the review would not be very helpful).

46.1. For no reason other than to watch you suffer, I've hypothetically built the creation shown to the right. The right-most mass, m_3, is heavier than m_2. The combination of the pulley M_B and masses m_2 and m_3 is sufficiently heavy that those will accelerate downward while m_1 accelerates upwards. Mass m_3 starts 1 meter off the ground, with the entire system at rest. Determine how long after the system is released m_3 will hit the ground. (tips: carefully define all of your accelerations, and think hard about how they relate to or affect each other. If you can get your FBDs right and define your accelerations properly, this problem isn't as bad as it looks. Well, the algebra is still bad…. You'll probably need to work on extra paper)



46.2. A field goal kicker kicks a football off the turf and it clears the uprights (which are 10 feet off the ground) by 6 feet. The kicker kicked the ball from the 30 yard line, and the field goal posts are at the back end of the endzone, which is 10 yards deep. An NFL football has a mass of 0.40 kg. The moment of inertia of an NFL football rotating in a perfect spiral (so rotating around the long axis) is 0.00194 kgm^2. The moment of inertia for a football rotating around an axis perpendicular to the long axis (so flipping end over end) is 0.00321 kg/m^2. The kicker's foot hit the ball 1 cm to the side of the long axis (inducing spiraling motion) and 3 cm away from the center of the football along the long axis (inducing end-over-end motion). When the ball cleared the uprights, how fast was it spiraling and how fast was it flipping end over end, in revolutions per minute?

46.3. You are continuing to build a Rube Goldberg device, and the stage you are working on now is going to involve a 100 gram ball bearing being shot out of a spring-loaded cannon. Compressing the spring to the locking point requires compressing it by 10 cm (the locking point is the point where a trigger-released lock catches the spring, so it can be release by pulling a trigger – allowing the ball to be shot). You tested the spring mechanism in the cannon, and discovered that the spring compresses to the locking point if you put 3.5 kg of mass on top of the spring. You want to shoot the ball out of the cannon such that it lands inside a cup 1 meter away. What angle should the cannon be angled at, relative to horizontal? (clearly show your work, and explain each step)



46.4. A 12 kg model rocket is blasting off, zooming straight upwards (in reality, the mass of a rocket decreases as its engine propels it, since the engine essentially ejects mass out the back. Let's assume that lost mass though is negligible compared to the total rocket mass, so we can assume its mass remains constant). Its engine provides a constant thrust of 300 N. After 7 seconds, a malfunction causes an internal explosion that blows the rocket into two pieces. Right after the explosion, one piece has a velocity of $40\ \hat{x} + 160\ \hat{y}$ (assume the y direction points up, and x is horizontal). The second piece has twice the mass of that first piece. When the second piece eventually lands, how far away will it land from the spot it launched from?

Note: This is not an easy problem! Draw a picture of what is going on. Break the problem down into steps

46.5. A ball is held next to one end of a thin, horizontal rod, with the rod fixed to a pivot at the other end. The rod has a square cross-section, 1 cm by 1 cm. It is 1 meter long, and is made of white ash wood (density of 900 kg/m^3). The ball is also made of white ash wood, and has a radius of 1 cm. The end of the rod opposite the pivot is released at the same time that the ball is dropped. At the moment the end of the rod is released, how will the end of the rod's acceleration compare to that of the ball? Will the ball start out accelerating downward faster or slower than the end of the rod? (determine exactly how their accelerations compare)

46.6. When the rod in the previous problem has pivoted 90°, such that the rod is vertical, what will be its angular velocity? (hint: Starting from Newton's 2^{nd} Law on this one may not be the best way to go. Instead, think about another way of solving the analogous problem for the ball – how fast the ball is going after it has fallen some height)

46. Review

Most nuclear fusion research revolves around what is referred to as "DT fusion" – forcing deuterium (D) and tritium (T) nuclei together, such that they fuse into one nucleus. The result is a ^4He nucleus and a "spare" neutron that separates from the ^4He (helium) nucleus. Neutrons and protons both have roughly the same mass, 1.67×10^{-27} kg. A deuterium nucleus is a hydrogen atom with one neutron, while a tritium nucleus is a hydrogen atom with two neutrons.

For fusion to happen, the D and T nuclei must be traveling towards each other very fast, to overcome the electrical repulsion they exert on each other (since both nuclei are positively charged). Assume the D nucleus is traveling to the right (+x direction) at 10^6 $^m/_s$, and the T nucleus is traveling to the left with the same magnitude momentum (these are realistic speeds for this fusion reaction). When they collide, the fusion reaction releases $2.82\text{x}10^{-12}$ Joules of energy, which goes into the kinetic energy of the products of the fusion reaction (the helium nucleus and the bare neutron). Let's assume that after a particular fusion reaction the helium nucleus flies off at 45° above the +x axis. Can we find the speed of both the helium nucleus and the neutron after the fusion reaction, and determine the direction of the neutron?

46.7. What is the *net* initial momentum before the collision? What is the total initial energy of the system before the reaction? (note we are ignoring some forms of potential energy that we have not discussed yet for simplicity)

46.8. Ignore gravity (it is negligible on this scale). Are there any *external* forces acting on the particles before, during, or after the collision? Based on that, should momentum be conserved?

46.9. There is energy released during the fusion reaction – this is ultimately due to the conversion of potential energy (primarily nuclear potential energy) into kinetic energy. Is there any additional outside source of energy input? Based on that, what should the final kinetic energy of the system equal?

46.10. What are the speeds of the helium nucleus and neutron after the fusion reaction? What direction does the neutron go in?

46. Review

47. Mathematical Messiness

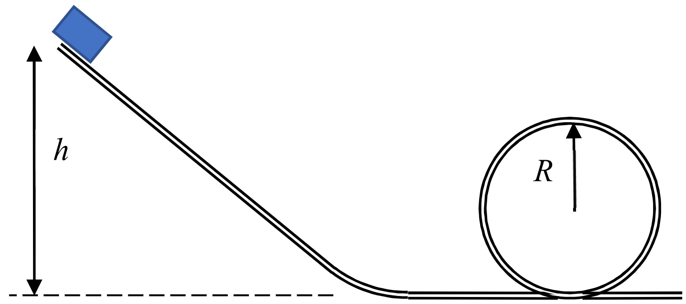
Name________________________

Most students in this course are concurrently taking calculus I or II, which restricts the mathematical complexity of problems we can do. Because of that, we often consider very "idealized" situations – not because we can't solve problems that haven't been idealized, but because without those idealizations the math can become very messy. In particular, differential equations show up very easily. For example, by just not ignoring wind resistance, projectile motion problems shift from being algebraic problems to coupled differential equations.

In this activity, we will look at some other problems where the math becomes very messy. In general, the expectation is not for you to be able to work through all of the messy math to get a final answer, but to start solving the problem and get as far as you can – generally to the point at which your current math skills begin to fail. Work on extra paper as necessary.

1. A girl is on a swing (seems simple so far). On her backswing, she swings back to be the same elevation as the bar supporting the swing, which is 3 meters above the ground. At the low point of her swing, she is 0.3 meters above the ground (you can make the idealization of treating her as a point mass, and the chains supporting the swing as massless). She wants to jump off of the swing, and land as far away from the structure as possible. At what angle past vertical should she jump off, so that she lands as far away as possible from the spot on the ground right underneath the supporting pole?

2. Let's revisit the problem of determining how high you need to start an object up a ramp in order for it to make it around a loop without falling off. But, let's throw in a little twist. Now, instead of it being a ball, it is a block – and friction is present. Assuming there is some coefficient of friction μ between the block and the surface, try to determine how high up the ramp (h) you would need to start it so that it just barely makes it around the loop without falling off.



48. Critical Thinking II

Name_____________________________

Go to the website for “The Flat Earth Society” (www.theflatearthsociety.org). Read through their FAQ at https://theflatearthsociety.org/home/index.php/about-the-society/faq
Don’t google for arguments for or against their positions – I want you to have to use your own head here for a little while.

1. You presumably believe the earth is round. Why? We tell kids that in elementary school, but we offer no proof, and we don’t expect them to come to that conclusion themselves. So, why believe it? Maybe you have been duped your entire life. Do you have any reason to believe one over the other? If so, what?

2. Can you come up with any experiments that you could feasibly do to prove whether the earth is round or flat? Discuss with your group, and explain.

3. Discuss the following statement in your group, and explain below how accurate you feel it is: "In science, we formulate models and hypotheses, and then design experiments to try to confirm them".

Now speak to one of us about that statement.

4. Read the article at the link below (I'll also put it on Canvas so you can click on it rather than typing it out). Discuss it with your group. Each member of your group should try to come up with at least one example from your own life (other than things discussed in the article) where you feel confirmation bias has had a significant impact on how you viewed something.
https://www.verywellmind.com/what-is-a-confirmation-bias-2795024