Math 418 Exam 2 Review Sheet

These practice problems are meant to give you a guide to study for the exam. There will not be this many problems on the exam. Note that there may be problems on the exam that are not represented among these practice problems. It is impossible to provide enough practice problems to represent all of the ideas and skill that we are assessing through this exam. That said, if you are able to do all of these problems by yourself, without any resources other than your brain, you can rest assured that you have prepared well for the exam. The distribution of topics on this review sheet is NOT indicative of the distribution of topics on the exam

1. Find values for A and B so that $p(x) = Ax^B$ passes through $(-3, -3)$ and $\left(4, \frac{-16}{3}\right)$
   Answer: $A = \frac{-1}{3}, B = 2$

2. Find all the x and y intercepts for $p(x) = x^3 + 11x^2 - 12x$.
   Answer: y-int: $(0, 0)$, x-ints: $(-12, 0), (0, 0), (1, 0)$.

3. Give an example of a polynomial $p(x)$ with precisely three roots, at $x = 3, x = -4$ and $x = 1$ such that $p(-1) = 4$ and the degree of p is 5.
   Answer: $p(x) = \frac{-1}{36}(x-3)(x+4)^2(x-1)^2$ **or** $p(x) = \frac{1}{24}(x-3)(x+4)(x-1)^3$**, etc.**

4. Is it possible to have a polynomial $p(x)$ of degree two such that $p(4) = 0, p(-2) = 0, p(1) = 0$ and $p(3) = 8$? Why or why not? If it is possible give an example of such a polynomial.
   Answer: No. A polynomial of degree two can have at most two roots.

5. Solve: $x^8 - x^4 - 12 = 0$.
   Answer: $x = \pm\sqrt{2}$

6. Sketch the following polynomials using the end behavior and behavior at the roots:

   I) $f(x) = -2(x+3)^3(x-1)^2(x+4)^7$
   Answer: Your answer should start down and to the left and come up to cross the x-axis at $(-4, 0)$ then cross at $(-3, 0)$ and then come up and bounce off at $(1, 0)$ before heading down and to the right.

   II) $p(x) = 4(x+1)^9(x-1)^{101}x^2$
   Answer: Your answer should start up and to the left and come down to cross the x-axis at $(-1, 0)$ then go up to cross at $(0, 0)$ and bounce at $(0, 0)$ and crossing at $(1, 0)$ before going up and to the right.

   III) $s(t) = -3(t^2 - 4)(t^2 + 10t + 16)$
   Answer: Your answer should start down and to the left and come up to cross at $(-8, 0)$ then come down to bounce at $(-2, 0)$ and then cross at $(2, 0)$ before moving down and to the right.

7. Give an example of a rational function $r(x)$ with precisely two vertical asymptotes, at $x = 1$ and $x = -1$ and one x-intercept at $(3, 0)$.
   Answer: $r(x) = \frac{x-3}{(x-1)(x+1)}$

8. Suppose $r(x) = \frac{2(x+3)^2(x-4)^4(x+7)^3}{(x-1)^4(x+3)^5(x+7)^3(x-4)^2}$. Find all vertical asymptotes, roots and holes of $r(x)$ and specify how you know which is which.
Answer: V.A.'s: $x = -3$, $x = -1$. Holes: $x = 4$, $x = -7$. The multiplicities determine which is which, factors with a larger multiplicity in the denominator lead to vertical asymptotes and factors with a larger mult. in the numerator lead to holes.

9. Use the fact that $x = 1$ and $x = -5$ are both roots of $p(x) = -x^4 - 5x^3 + 7x^2 + 29x - 30$ to factor $p(x)$.
Answer: $p(x) = -(x+3)(x-2)(x-1)(x+5)$

10. Use the fact that $x = -1$ is a triple root of $f(x) = x^7 - 10x^6 + 70x^4 + 95x^3 + 36x^2$ to fully factor $f(x)$.
Answer: $f(x) = x^2(x-9)x-4)(x+1)^3$

11. Write $r(x) = \frac{x^4-3x^2+2x-1}{x+2}$ in the form $p(x)+\frac{q(x)}{x+2}$ with p a polynomial and $Deg(q) < 1$.
Answer: $r(x) = x^3 - 2x^2 + x + \frac{-1}{x+2}$

12. Suppose $r(x) = \frac{2x^4+1}{3x^2+5x^4-8x+4}$. Find the horizontal asymptote of r or explain why there is no horizontal asymptote.
Answer: $y = \frac{2}{5}$.

13. Find the domain, holes, vertical asymptotes and roots for $r(x) = \frac{x^4+2x^3-15x^2}{x^3-25x}$
Answer: Domain$(r) = \{x | x \neq 0, x \neq \pm 5\}$

14. Is $f(x) = \frac{3}{x-1} - \frac{4x}{x^2+2}$ a rational function? Why or why not?
Answer: Yes. It can be written as a ratio of two polynomials.

15. Find a value of a such that $p(x) = ax^3 - 2ax + 13$ has a root at $x = 2$.
Answer: $a = \frac{-13}{4}$

16. Tina Student and Joe Learner have been playing chess and keeping track of their wins and losses. So far Joe has won 13 out of 43 games. How many games will Joe have to win in a row so that he will have won 70% of the games?
Answer: 57 games.

17. Give an example of a function $h(x)$ so that $(f \circ h)(x) = \frac{x^2-1}{x^4+4}$ with $f(x) = \frac{x-1}{x^2+4}$.
Answer: $h(x) = x^2$.

18. Suppose f and g are function defined by the tables below. Find $g \circ f$.

x	$f(x)$
1	3
2	9
π	π
8	12

x	$g(x)$
12	19
9	7
π	4
3	2^π
1	10

Answer:

x	$(g \circ f)(x)$
1	2^π
2	7
π	4
8	19

19. Suppose $f(x) = 1 + x^2$ and $g(x) = \frac{x+1}{x-1}$. Evaluate and simplify$(f \circ g)(x)$ and $(g \circ f)(x)$. Find the domain of $(f \circ g)$. **Answer:** $(f \circ g)(x) = 1 + (\frac{x+1}{x-1})^2$**,** $(g \circ f)(x) = \frac{x^2+2}{x^2}$**,** **Dom**$(f \circ g) = \{x | x \neq 1\}$**.**

20. Find the inverse of $g(x) = 2x^{15} - 3$
Answer: $g^{-1}(x) = \sqrt[15]{\frac{x+3}{2}}$

21. Find the domain, range and inverse of $f(x) = \frac{2x+3}{4x-4}$
Answer: $\text{Dom}(f) = \{x|x \neq 1\}$, $\text{Range}(f) = \{x|x \neq \frac{1}{2}\}$, $f^{-1}(x) = \frac{4x+3}{4x-2}$.