

~~Math 527.2~~

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MATH 527.2  
HW 7

1.  $2x - y = \frac{1}{2}$   $y = 2x - \frac{1}{2}$   
 $4x + 3y = 5$

$4x + 3(2x - \frac{1}{2}) = 5$   $4x + 6x - \frac{3}{2} = 5$

$10x = 6\frac{1}{2}$   $x = 0.65$   $y = 2(0.65) - \frac{1}{2}$   
 $y = 1.3 - 0.5$   $y = 0.8$

2.  $2x - y = \frac{1}{2}$   $ax + by = p$   $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$   
 $4x + 3y = 5$   $c x + d y = q$

3.  $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 5 \end{pmatrix}$

$A^{-1}A = I$   $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$   $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

b.  $x = A^{-1}b$   $\begin{bmatrix} \frac{3}{10} & \frac{1}{10} \\ -\frac{1}{10} & \frac{2}{10} \end{bmatrix} \begin{pmatrix} \frac{1}{2} \\ 5 \end{pmatrix} = \frac{1}{10} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{pmatrix} \frac{1}{2} \\ 5 \end{pmatrix}$

c.  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} \\ -\frac{1}{10} & \frac{2}{10} \end{bmatrix} \begin{pmatrix} \frac{1}{2} \\ 5 \end{pmatrix}$   $x = 0.65$   $y = 0.8$  ✓

d.  $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \begin{pmatrix} 0.65 \\ 0.8 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 5 \end{pmatrix}$

3.  $A = \begin{pmatrix} -1 & 2 \\ -7 & 8 \end{pmatrix}$   $A - \lambda I = \begin{pmatrix} -1-\lambda & 2 \\ -7 & 8-\lambda \end{pmatrix}$   
 $(-1-\lambda)(8-\lambda) + 14 = 0$

$-8 + \lambda - 8\lambda + \lambda^2 + 14 = 0$

$\lambda^2 - 7\lambda + 6 = 0$

$(\lambda-1)(\lambda-6) = 0$

$\lambda_1 = 1$   $\lambda_2 = 6$

$\lambda = 1$

$\begin{pmatrix} -2 & 2 \\ -7 & 7 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-2u + 2v = 0$

$-7u + 7v = 0$

$\underline{V_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = 6$

$\begin{pmatrix} -7 & 2 \\ -7 & 2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-7u + 2v = 0$

$-7u + 2v = 0$

$\underline{V_2} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$

$\begin{pmatrix} -1 & 2 \\ -7 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

4.  $x' = x + 2y$   $x' = Ax \rightarrow x' = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} x$   
 $y' = 4x + 3y$

$\det \begin{pmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{pmatrix} = 0$

$(1-\lambda)(3-\lambda) - 8 = 0$

$3 - \lambda - 3\lambda + \lambda^2 - 8 = 0$

$\lambda^2 - 4\lambda - 5 = 0$

$\lambda_1 = -1$   $\lambda_2 = 5$   $(\lambda+1)(\lambda-5) = 0$

for  $\lambda_1 = -1$ :

$\begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$2u + 2v = 0$

$4u + 4v = 0$

$\underline{V_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

for  $\lambda_2 = 5$ :

$\begin{pmatrix} -4 & 2 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-4u + 2v = 0$

$4u + 2v = 0$

$2v = 4u$

$\underline{V_2} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$



$$x_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} \quad x_2(t) = \begin{pmatrix} 2 \\ 4 \end{pmatrix} e^{5t}$$

$$\underline{x}(t) = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 2 \\ 4 \end{pmatrix} e^{5t}$$

$$\begin{aligned} x(t) &= C_1 e^{-t} + 2C_2 e^{5t} \\ y(t) &= C_1 e^{-t} + 4C_2 e^{5t} \end{aligned}$$

5.

$$\begin{aligned} x' &= -4x + 2y & \underline{x}' &= A \underline{x} & \Rightarrow \underline{x}' &= \begin{pmatrix} -4 & 2 \\ -5/2 & 2 \end{pmatrix} \underline{x} \\ y' &= -5/2 x + 2y \end{aligned}$$

$$\det \begin{pmatrix} -4-\lambda & 2 \\ -5/2 & 2-\lambda \end{pmatrix} = 0 \quad (-4-\lambda)(2-\lambda) + 5 = 0$$

$$-8 - 4\lambda - 2\lambda + \lambda^2 + 5 = 0 \quad \lambda^2 + 2\lambda - 3 = 0 \quad (\lambda-1)(\lambda+3) = 0$$

$$\lambda_1 = 1, \lambda_2 = -3$$

$$\text{for } \lambda_1 = 1: \begin{pmatrix} -5 & 2 \\ -5/2 & 1 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{aligned} -5u + 2w &= 0 \\ -5/2 u + w &= 0 \end{aligned} \quad \underline{v}_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$\text{for } \lambda_2 = -3: \begin{pmatrix} -1 & 2 \\ -5/2 & 5 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{aligned} -u + 2w &= 0 \\ -5/2 u + 5w &= 0 \end{aligned} \quad \underline{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\underline{x}_1(t) = \begin{pmatrix} 2 \\ 5 \end{pmatrix} e^t \quad \underline{x}_2(t) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t}$$

$$\underline{x}(t) = C_1 \begin{pmatrix} 2 \\ 5 \end{pmatrix} e^t + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t}$$

$$\begin{aligned} x(t) &= 2C_1 e^t + 2C_2 e^{-3t} \\ y(t) &= 5C_1 e^t + C_2 e^{-3t} \end{aligned}$$

$$\begin{aligned} 6. \quad x' &= \frac{1}{2} x & x(0) &= 3 & \underline{x}(0) &= \begin{pmatrix} 3 \\ 5 \end{pmatrix} \\ y' &= x - 2y & y(0) &= 5 \end{aligned}$$

$$\underline{x}' = A \underline{x} \Rightarrow \underline{x}' = \begin{pmatrix} \frac{1}{2} & 0 \\ 1 & -2 \end{pmatrix} \underline{x} \quad \det \begin{pmatrix} \frac{1}{2} - \lambda & 0 \\ 1 & -2 - \lambda \end{pmatrix} = 0$$

$$\begin{aligned} \left(\frac{1}{2} - \lambda\right)(-2 - \lambda) - 0 &= 0 & -1 - \frac{1}{2}\lambda + 2\lambda + \lambda^2 &= 0 \\ \lambda^2 + \frac{3}{2}\lambda - 1 &= 0 & (\lambda - \frac{1}{2})(\lambda + 2) &= 0 \end{aligned}$$

$$\lambda_1 = \frac{1}{2} \quad \lambda_2 = -2$$

$$\text{for } \lambda_1 = \frac{1}{2}: \begin{pmatrix} 0 & 0 \\ 1 & -5/2 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{aligned} 0u + 0w &= 0 \\ u - 5/2 w &= 0 \end{aligned} \quad \underline{v}_1 = \begin{pmatrix} 5/2 \\ 1 \end{pmatrix}$$

$$\text{for } \lambda_2 = -2: \begin{pmatrix} 3/2 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{aligned} 3/2 u + 0w &= 0 \\ u + 0w &= 0 \end{aligned} \quad \underline{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\underline{x}_1(t) = \begin{pmatrix} 5/2 \\ 1 \end{pmatrix} e^{\frac{1}{2}t} \quad \underline{x}_2(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-2t}$$

$$\underline{x}(t) = C_1 \begin{pmatrix} 5/2 \\ 1 \end{pmatrix} e^{\frac{1}{2}t} + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-2t}$$

$$x(t) = -5/2 C_1 e^{\frac{1}{2}t}$$

$$y(t) = C_1 e^{\frac{1}{2}t}$$

$$\begin{pmatrix} 3 \\ 5 \end{pmatrix} = C_1 \begin{pmatrix} -5/2 \\ 1 \end{pmatrix} \quad \begin{aligned} 3 &= -5/2 C_1 \\ 5 &= C_1 \end{aligned}$$