Homework # 7

Instructions: Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and box all your solutions. All Matlab code needs to be published, with your name and date at the top of the script, and all figures needs to have proper axis labeling and legends. Homework assignments will be collected during class time on the due date. *Late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **Homework problems not to be graded**
  - From textbook (Lathi):
    - Ch 4: 8-2

- **Homework problems to be graded**

Consider a system with transfer function

$$H(s) = \frac{s(s+10)}{(s^2+50s+2500)}$$

a) Determine the output of the system to an everlasting sinusoidal input

$$x(t) = 1 + 0.2\cos(12t) + 0.4\sin\left(24t + \frac{\pi}{4}\right).$$

b) Use MATLAB to obtain the Bode plots of $H(s)$ (hint: use the "bode" command). In your plots, mark the amplification and phase values corresponding to the DC frequency ($\omega = 0$), $\omega = 12$ rad/s, and $\omega = 24$ rad/s. Do these values match your solution to Part (a)?

$$H(i\omega) = \frac{i\omega(i\omega+10)}{-\omega^2+50i\omega+2500}, \quad i\omega(i\omega+10) = -\omega^2+10i\omega$$

$$|H(i\omega)| = \frac{\sqrt{\omega^4-100\omega^2}}{\sqrt{\omega^4-2500\omega^2+6250000}}$$

$$\angle H(i\omega) = \angle 10i\omega-\omega^2 - \angle 50i\omega+2500-\omega^2$$
$$= \tan^{-1}\left(\frac{10\omega}{-\omega^2}\right) - \tan^{-1}\left(\frac{50\omega}{2500-\omega^2}\right)$$
$$= \tan^{-1}\left(\frac{-10}{\omega}\right) - \tan^{-1}\left(\frac{50\omega}{2500-\omega^2}\right)$$

$$|H(i12)| = \frac{\sqrt{6336}}{\sqrt{5910736}} \qquad \angle H(i12) = \tan^{-1}\left(\frac{-10}{12}\right) - \tan^{-1}\left(\frac{600}{2356}\right)$$
$$= 0.033 \qquad\qquad = -0.23\text{ rad} - 0.25\text{ rad} = -0.48\text{ rad}$$

$$(0.2)(0.033)\cos(12t \underset{28^\circ}{-0.48\text{rad}})$$

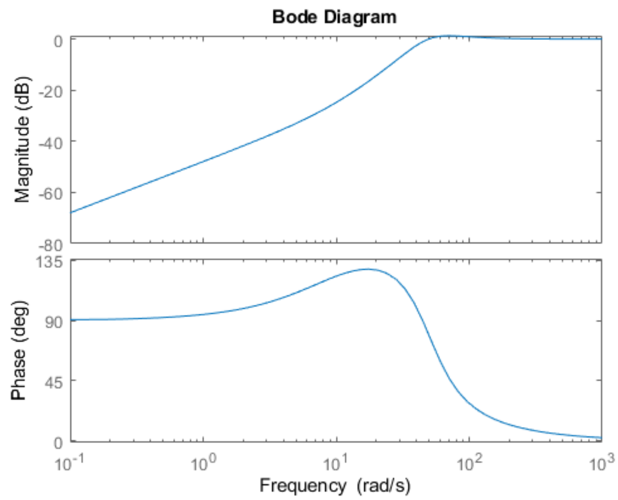
$$|H(i24)| = \frac{\sqrt{234176}}{\sqrt{5141776}} \qquad \angle H(i24) = \tan^{-1}\left(\frac{-10}{24}\right) - \tan^{-1}\left(\frac{1440}{1924}\right)$$
$$= 0.23 \qquad\qquad = -0.11\text{ rad} - 0.0008\text{rad} = -0.1108\text{rad}$$

$$(0.4)(\underset{0.23}{})\cos\left(24t - \underset{45^\circ}{\frac{\pi}{4}} \underset{6^\circ}{-0.1108\text{rad}}\right)$$

$$|H(i0)| = \frac{0}{\sqrt{6250000}} = 0$$

$$\boxed{y(t) = \underset{0.09}{}\cos(24t - \underset{0.84\text{rad}}{51^\circ}) + 0.0066\cos(12t - \underset{0.48\text{rad}}{28^\circ})}$$

b)

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>> H=tf([1 10 0], [1 50 2500])
H =

     s^2 + 10 s
  -----------------
  s^2 + 50 s + 2500

>> [mag,phase]=bode(H,0.0000001)
mag =
   4.0000e-10
phase =
   90.0000
>> [mag,phase]=bode(H,12)
mag =
  0.0771
phase =
  125.9067
>> [mag,phase]=bode(H,24)
mag =
  0.2752
phase =
  125.4283
```

Other than w=0, the magnitudes and phase don't match my answer from part a. The magnitudes are in the ballpark, but the phases are nowhere near it should be. I probably made a mistake in calculating the values. w=0 is spot on though.