Chapter 4

SCIENTIFIC NOTATION AND METRIC PREFIXES

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4.1 Scientific notation

In many disciplines of science and engineering, very large and very small numerical quantities must be managed. Some of these quantities are mind-boggling in their size, either extremely small or extremely large. Take for example the mass of a proton, one of the constituent particles of an atom's nucleus:

Proton mass = 0.00000000000000000000000167 grams

Or, consider the number of electrons passing by a point in a circuit every second with a steady electric current of 1 amp:

1 amp = 6,250,000,000,000,000,000 electrons per second

A lot of zeros, isn't it? Obviously, it can get quite confusing to have to handle so many zero digits in numbers such as this, even with the help of calculators and computers.

Take note of those two numbers and of the relative sparsity of non-zero digits in them. For the mass of the proton, all we have is a "167" preceded by 23 zeros before the decimal point. For the number of electrons per second in 1 amp, we have "625" followed by 16 zeros. We call the span of non-zero digits (from first to last), plus any zero digits *not* merely used for placeholding, the "significant digits" of any number.

The significant digits in a real-world measurement are typically reflective of the accuracy of that measurement. For example, if we were to say that a car weighs 3,000 pounds, we probably don't mean that the car in question weighs *exactly* 3,000 pounds, but that we've rounded its weight to a value more convenient to say and remember. That rounded figure of 3,000 has only one significant digit: the "3" in front – the zeros merely serve as placeholders. However, if we were to say that the car weighed 3,005 pounds, the fact that the weight is not rounded to the nearest thousand pounds tells us that the two zeros in the middle aren't just placeholders, but that all four digits of the number "3,005" are significant to its representative accuracy. Thus, the number "3,005" is said to have *four* significant figures.

In like manner, numbers with many zero digits are not necessarily representative of a real-world quantity all the way to the decimal point. When this is known to be the case, such a number can be written in a kind of mathematical "shorthand" to make it easier to deal with. This "shorthand" is called *scientific notation*.

With scientific notation, a number is written by representing its significant digits as a quantity between 1 and 10 (or -1 and -10, for negative numbers), and the "placeholder" zeros are accounted for by a power-of-ten multiplier. For example:

1 amp = 6,250,000,000,000,000,000 electrons per second

. . . *can be expressed as* . . .

1 amp = 6.25 x 10^{18} electrons per second

10 to the 18th power (10^{18}) means 10 multiplied by itself 18 times, or a "1" followed by 18 zeros. Multiplied by 6.25, it looks like "625" followed by 16 zeros (take 6.25 and skip the decimal point 18 places to the right). The advantages of scientific notation are obvious: the number isn't as unwieldy when written on paper, and the significant digits are plain to identify.

But what about very small numbers, like the mass of the proton in grams? We can still use scientific notation, except with a negative power-of-ten instead of a positive one, to shift the decimal point to the left instead of to the right:

Proton mass = 0.00000000000000000000000167 grams

. . . can be expressed as . . .

Proton mass = 1.67 x 10^{-24} grams

10 to the -24th power (10^{-24}) means the inverse (1/x) of 10 multiplied by itself 24 times, or a "1" preceded by a decimal point and 23 zeros. Multiplied by 1.67, it looks like "167" preceded by a decimal point and 23 zeros. Just as in the case with the very large number, it is a lot

easier for a human being to deal with this "shorthand" notation. As with the prior case, the significant digits in this quantity are clearly expressed.

Because the significant digits are represented "on their own," away from the power-of-ten multiplier, it is easy to show a level of precision even when the number looks round. Taking our 3,000 pound car example, we could express the rounded number of 3,000 in scientific notation as such:

car weight = 3 x 10^3 pounds

If the car actually weighed 3,005 pounds (accurate to the nearest pound) and we wanted to be able to express that full accuracy of measurement, the scientific notation figure could be written like this:

car weight = 3.005 x 10^3 pounds

However, what if the car actually did weigh 3,000 pounds, exactly (to the nearest pound)? If we were to write its weight in "normal" form (3,000 lbs), it wouldn't necessarily be clear that this number was indeed accurate to the nearest pound and not just rounded to the nearest thousand pounds, or to the nearest hundred pounds, or to the nearest ten pounds. Scientific notation, on the other hand, allows us to show that all four digits are significant with no misunderstanding:

car weight = 3.000 x 10^3 pounds

Since there would be no point in adding extra zeros to the right of the decimal point (place-holding zeros being unnecessary with scientific notation), we know those zeros *must* be significant to the precision of the figure.

4.2 Arithmetic with scientific notation

The benefits of scientific notation do not end with ease of writing and expression of accuracy. Such notation also lends itself well to mathematical problems of multiplication and division. Let's say we wanted to know how many electrons would flow past a point in a circuit carrying 1 amp of electric current in 25 seconds. If we know the number of electrons per second in the circuit (which we do), then all we need to do is multiply that quantity by the number of seconds (25) to arrive at an answer of total electrons:

(6,250,000,000,000,000,000 electrons per second) x (25 seconds) =
156,250,000,000,000,000,000 electrons passing by in 25 seconds

Using scientific notation, we can write the problem like this:

(6.25 x 10^{18} electrons per second) x (25 seconds)

If we take the "6.25" and multiply it by 25, we get 156.25. So, the answer could be written as:

156.25 x 10^{18} electrons

However, if we want to hold to standard convention for scientific notation, we must represent the significant digits as a number between 1 and 10. In this case, we'd say "1.5625" multiplied by some power-of-ten. To obtain 1.5625 from 156.25, we have to skip the decimal point two places to the left. To compensate for this without changing the value of the number, we have to raise our power by two notches (10 to the 20th power instead of 10 to the 18th):

1.5625 x 10^{20} electrons

What if we wanted to see how many electrons would pass by in 3,600 seconds (1 hour)? To make our job easier, we could put the time in scientific notation as well:

(6.25 x 10^{18} electrons per second) x (3.6 x 10^{3} seconds)

To multiply, we must take the two significant sets of digits (6.25 and 3.6) and multiply them together; and we need to take the two powers-of-ten and multiply them together. Taking 6.25 times 3.6, we get 22.5. Taking 10^{18} times 10^{3}, we get 10^{21} (exponents with common base numbers add). So, the answer is:

22.5 x 10^{21} electrons

. . . or more properly . . .

2.25 x 10^{22} electrons

To illustrate how division works with scientific notation, we could figure that last problem "backwards" to find out how long it would take for that many electrons to pass by at a current of 1 amp:

(2.25 x 10^{22} electrons) / (6.25 x 10^{18} electrons per second)

Just as in multiplication, we can handle the significant digits and powers-of-ten in separate steps (remember that you subtract the exponents of divided powers-of-ten):

(2.25 / 6.25) x (10^{22} / 10^{18})

And the answer is: 0.36 x 10^{4}, or 3.6 x 10^{3}, seconds. You can see that we arrived at the same quantity of time (3600 seconds). Now, you may be wondering what the point of all this is when we have electronic calculators that can handle the math automatically. Well, back in the days of scientists and engineers using "slide rule" analog computers, these techniques were indispensable. The "hard" arithmetic (dealing with the significant digit figures) would be performed with the slide rule while the powers-of-ten could be figured without any help at all, being nothing more than simple addition and subtraction.

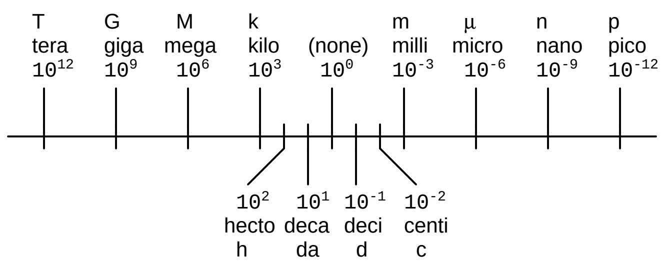
- **REVIEW:**

- Significant digits are representative of the real-world accuracy of a number.
- Scientific notation is a ”shorthand” method to represent very large and very small numbers in easily-handled form.
- When multiplying two numbers in scientific notation, you can multiply the two significant digit figures and arrive at a power-of-ten by adding exponents.
- When dividing two numbers in scientific notation, you can divide the two significant digit figures and arrive at a power-of-ten by subtracting exponents.

4.3 Metric notation

The metric system, besides being a collection of measurement units for all sorts of physical quantities, is structured around the concept of scientific notation. The primary difference is that the powers-of-ten are represented with alphabetical prefixes instead of by literal powers-of-ten. The following number line shows some of the more common prefixes and their respective powers-of-ten:

METRIC PREFIX SCALE



Looking at this scale, we can see that 2.5 Gigabytes would mean 2.5 x 10^{9} bytes, or 2.5 billion bytes. Likewise, 3.21 picoamps would mean 3.21 x 10^{-12} amps, or 3.21 1/trillionths of an amp.

Other metric prefixes exist to symbolize powers of ten for extremely small and extremely large multipliers. On the extremely small end of the spectrum, *femto* (f) = 10^{-15}, *atto* (a) = 10^{-18}, *zepto* (z) = 10^{-21}, and *yocto* (y) = 10^{-24}. On the extremely large end of the spectrum, *Peta* (P) = 10^{15}, *Exa* (E) = 10^{18}, *Zetta* (Z) = 10^{21}, and *Yotta* (Y) = 10^{24}.

Because the major prefixes in the metric system refer to powers of 10 that are multiples of 3 (from ”kilo” on up, and from ”milli” on down), metric notation differs from regular scientific notation in that the significant digits can be anywhere between 1 and 1000, depending on which prefix is chosen. For example, if a laboratory sample weighs 0.000267 grams, scientific notation and metric notation would express it differently:

2.67 x 10^{-4} grams (scientific notation)

267 μgrams (metric notation)

The same figure may also be expressed as 0.267 milligrams (0.267 mg), although it is usually more common to see the significant digits represented as a figure greater than 1.

In recent years a new style of metric notation for electric quantities has emerged which seeks to avoid the use of the decimal point. Since decimal points (".") are easily misread and/or "lost" due to poor print quality, quantities such as 4.7 k may be mistaken for 47 k. The new notation replaces the decimal point with the metric prefix character, so that "4.7 k" is printed instead as "4k7". Our last figure from the prior example, "0.267 m", would be expressed in the new notation as "0m267".

- **REVIEW:**
- The metric system of notation uses alphabetical prefixes to represent certain powers-of-ten instead of the lengthier scientific notation.

4.4 Metric prefix conversions

To express a quantity in a different metric prefix that what it was originally given, all we need to do is skip the decimal point to the right or to the left as needed. Notice that the metric prefix "number line" in the previous section was laid out from larger to smaller, left to right. This layout was purposely chosen to make it easier to remember which direction you need to skip the decimal point for any given conversion.

Example problem: express 0.000023 amps in terms of microamps.

0.000023 amps (has no prefix, just plain unit of amps)

From UNITS to micro on the number line is 6 places (powers of ten) to the right, so we need to skip the decimal point 6 places to the right:

0.000023 amps = 23. , or 23 microamps (μA)

Example problem: express 304,212 volts in terms of kilovolts.

304,212 volts (has no prefix, just plain unit of volts)

From the *(none)* place to *kilo* place on the number line is 3 places (powers of ten) to the left, so we need to skip the decimal point 3 places to the left:

304,212. = 304.212 kilovolts (kV)

Example problem: express 50.3 Mega-ohms in terms of milli-ohms.

50.3 M ohms (mega = 10^6)

From mega to milli is 9 places (powers of ten) to the right (from 10 to the 6th power to 10 to the -3rd power), so we need to skip the decimal point 9 places to the right:

50.3 M ohms = 50,300,000,000 milli-ohms (mΩ)

- **REVIEW:**
- Follow the metric prefix number line to know which direction you skip the decimal point for conversion purposes.
- A number with no decimal point shown has an implicit decimal point to the immediate right of the furthest right digit (i.e. for the number 436 the decimal point is to the right of the 6, as such: 436.)

4.5 Hand calculator use

To enter numbers in scientific notation into a hand calculator, there is usually a button marked "E" or "EE" used to enter the correct power of ten. For example, to enter the mass of a proton in grams (1.67 x 10^{-24} grams) into a hand calculator, I would enter the following keystrokes:

```
[1]  [.]  [6]  [7]  [EE]  [2]  [4]  [+/-]
```

The [+/-] keystroke changes the sign of the power (24) into a -24. Some calculators allow the use of the subtraction key [-] to do this, but I prefer the "change sign" [+/-] key because its more consistent with the use of that key in other contexts.

If I wanted to enter a negative number in scientific notation into a hand calculator, I would have to be careful how I used the [+/-] key, lest I change the sign of the power and not the significant digit value. Pay attention to this example:

Number to be entered: -3.221 x 10^{-15}:

```
[3]  [.]  [2]  [2]  [1]  [+/-]  [EE]  [1]  [5]  [+/-]
```

The first [+/-] keystroke changes the entry from 3.221 to -3.221; the second [+/-] keystroke changes the power from 15 to -15.

Displaying metric and scientific notation on a hand calculator is a different matter. It involves changing the display option from the normal "fixed" decimal point mode to the "scientific" or "engineering" mode. Your calculator manual will tell you how to set each display mode.

These display modes tell the calculator how to represent any number on the numerical readout. The actual value of the number is not affected in any way by the choice of display modes – only how the number appears to the calculator user. Likewise, the procedure for entering numbers into the calculator does not change with different display modes either. Powers of ten are usually represented by a pair of digits in the upper-right hand corner of the display, and are visible only in the "scientific" and "engineering" modes.

The difference between "scientific" and "engineering" display modes is the difference between scientific and metric notation. In "scientific" mode, the power-of-ten display is set so that the main number on the display is always a value between 1 and 10 (or -1 and -10 for negative numbers). In "engineering" mode, the powers-of-ten are set to display in multiples of 3, to represent the major metric prefixes. All the user has to do is memorize a few prefix/power combinations, and his or her calculator will be "speaking" metric!

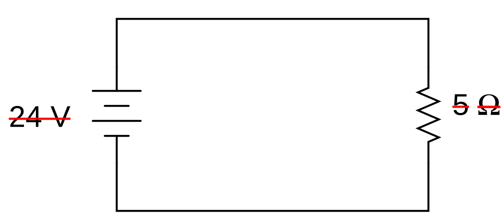
```
POWER       METRIC PREFIX
-----       -------------
12 ......... Tera  (T)
9 .......... Giga  (G)
6 .......... Mega  (M)
3 .......... Kilo  (k)
0 .......... UNITS (plain)
-3 ......... milli (m)
-6 ......... micro (u)
-9 ......... nano  (n)
-12 ........ pico  (p)
```

- **REVIEW:**
- Use the [EE] key to enter powers of ten.
- Use "scientific" or "engineering" to display powers of ten, in scientific or metric notation, respectively.

~~4.6 Scientific notation in SPICE~~

~~The SPICE circuit simulation computer program uses scientific notation to display its output information, and can interpret both scientific notation and metric prefixes in the circuit description files. If you are going to be able to successfully interpret the SPICE analyses throughout this book, you must be able to understand the notation used to express variables of voltage, current, etc. in the program.~~

~~Let's start with a very simple circuit composed of one voltage source (a battery) and one resistor:~~



~~To simulate this circuit using SPICE, we first have to designate node numbers for all the distinct points in the circuit, then list the components along with their respective node numbers so the computer knows which component is connected to which, and how. For a circuit of this simplicity, the use of SPICE seems like overkill, but it serves the purpose of demonstrating practical use of scientific notation:~~