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Math 527.02
HW 7

1. $x' = -6x + 5y$ $\quad A = \begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix}$
$y' = -5x + 4y$

$\det\begin{pmatrix} -6-\lambda & 5 \\ -5 & 4-\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad (-6-\lambda)(4-\lambda) + 25 = 0$

$-24 + 6\lambda - 4\lambda + \lambda^2 + 25 = 0$ $\quad \lambda^2 + 2\lambda + 1 = 0$

$\lambda = -1$ $\quad (\lambda+1)(\lambda+1) = 0$

mult. 2

for $\lambda = -1$

$\begin{pmatrix} -6+1 & 5 \\ -5 & 4+1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} =$ $\quad -5a + 5b = 0$ $\quad \underline{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$-5a + 5b = 0$

$\begin{pmatrix} -5 & 5 \\ -5 & 5 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad -5c + 5d = 1$ $\quad \underline{v}_2 = \begin{pmatrix} 0 \\ 1/5 \end{pmatrix}$
$-5c + 5d = 1$

$\boxed{\underline{x}(t) = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-t} + c_2\left(\begin{pmatrix} 0 \\ 1/5 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}t\right)e^{-t}}$

$$\boxed{\begin{aligned} x(t) &= c_1e^{-t} + c_2te^{-t} \\ y(t) &= c_1e^{-t} + c_2\left(\tfrac{1}{5} + t\right)e^{-t} \end{aligned}}$$

2. $x' = \begin{pmatrix} -1 & 3 \\ -3 & 5 \end{pmatrix} x$ $\leftarrow A$

$\det\begin{pmatrix} -1-\lambda & 3 \\ -3 & 5-\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad (-1-\lambda)(5-\lambda) + 9 = 0$

$-5 + \lambda - 5\lambda + \lambda^2 + 9 = 0$ $\quad \lambda^2 - 4\lambda + 4 = 0$

for $\lambda = 2$ $\quad \lambda = 2$ (mult. 2) $\quad (\lambda-2)(\lambda-2) = 0$

$\begin{pmatrix} -1-2 & 3 \\ -3 & 5-2 \end{pmatrix} \rightarrow \begin{pmatrix} -3 & 3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad -3a + 3b = 0$ $\quad \underline{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$-3a + 3b = 0$

$\begin{pmatrix} -3 & 3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad -3c + 3d = 1$ $\quad \underline{v}_2 = \begin{pmatrix} 1/3 \\ 0 \end{pmatrix}$
$-3c + 3d = 1$

$\underline{x}(t) = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{2t} + c_2\left(\begin{pmatrix} 1/3 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}t\right)e^{2t}$

$x(t) = c_1e^{2t} + c_2\left(1/3 + t\right)e^{2t}$

$y(t) = c_1e^{2t} + c_2te^{2t}$

3. $x' = \begin{pmatrix} 0 & -8 \\ 2 & 0 \end{pmatrix} x$ $\quad \det\begin{pmatrix} 0-\lambda & -8 \\ 2 & 0-\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad (0-\lambda)(0-\lambda)+16=0$

$0+\lambda^2+16=0 \quad \lambda^2+16=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad x = \frac{\pm\sqrt{-32}}{2} \quad \frac{\pm 4\sqrt{-2}}{2} \quad \lambda = \pm 4i$

$\lambda = 0 \pm 4i$

for $\lambda = 4i$

$\begin{pmatrix} 0-4i & -8 \\ 2 & 0-4i \end{pmatrix}\begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad (-4i)u - 8w = 0$

$2u + 4iw = 0$

$u = 4i \quad w = 2$

$x(t) = c\begin{pmatrix} 4i \\ 2 \end{pmatrix}e^{4it} + c^*\begin{pmatrix} -4i \\ 2 \end{pmatrix}e^{4it}$

$x(t) = C_1 e^{0}\left[\begin{pmatrix} 0 \\ 2 \end{pmatrix}\cos 2t - \begin{pmatrix} 4i \\ 0 \end{pmatrix}\sin 2t\right] + C_2 e^{0}\left[\begin{pmatrix} 0 \\ 2 \end{pmatrix}\cos 2t + \begin{pmatrix} 4i \\ 0 \end{pmatrix}\sin 2t\right]$

4. $x' = 2x + 4y \quad x(0) = -1 \quad \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ $\quad \det\begin{pmatrix} 2-\lambda & 4 \\ -1 & 6-\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$y' = -x + 6y \quad y(0) = 6$

$(2-\lambda)(6-\lambda)+4=0 \quad 12-2\lambda-6\lambda+\lambda^2+4=0$

$\lambda = \frac{+8 \pm \sqrt{64-4(16)}}{2} \quad \lambda = 4 \text{ mult. } 2 \quad \lambda^2 - 8\lambda + 16 = 0$

$(\lambda-4)(\lambda-4)=0$

for $\lambda = 4$, $\begin{pmatrix} -2 & 4 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad -2a+4b=0 \quad \underline{v_1} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$-a+2b=0$

$\begin{pmatrix} -2 & 4 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad -2c+4d=2 \quad \underline{v_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$-c+2d=-1$

$c = 2d+1 \quad -4d-2+4d=2$

$x(t) = C_1 e^{4t} + C_2(1+2t)e^{4t}$

$y(t) = -C_1 e^{4t} + C_2(0-t)e^{4t}$

$\begin{pmatrix} -1 \\ 6 \end{pmatrix} = C_1\begin{pmatrix} 2 \\ -1 \end{pmatrix} + C_2\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$-1 = 2C_1 + C_2 \quad -1 = -12 + C_2 \quad C_2 = 11$

$6 = -C_1 + 0C_2 \quad C_1 = -6$

$\boxed{\begin{aligned} x(t) &= -12e^{4t} + 11(1+2t)e^{4t} \\ y(t) &= 6e^{4t} - 11te^{4t} \end{aligned}}$

5. $x' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} x$ $\quad x(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ $\quad \det \begin{pmatrix} 6-\lambda & -1 \\ 5 & 4-\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\lambda = \dfrac{10 \pm \sqrt{100-116}}{2}$ $\quad \dfrac{10 \pm 4i}{2}$ $\quad (6-\lambda)(4-\lambda) + 5 = 0$

$24 - 6\lambda - 4\lambda + \lambda^2 + 5 = 0$

$\lambda = 5 \pm 2i$ $\quad \lambda^2 - 10\lambda + 29 = 0$

for $\lambda = 5+2i$ $\quad \begin{pmatrix} 6-(5+2i) & -1 \\ 5 & 4-(5+2i) \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$ $\quad (1-2i)u - w = 0$

$5u + (-1-2i)w = 0$