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7.4 16. $\int_1^2 \frac{x^3+4x^2+x-1}{x^3+x^2}\,dx$

$$\begin{array}{r} 1 \\ x^3+x^2 \overline{) x^3+4x^2+x-1} \\ -\ x^3+x^2+0x+0 \\ \hline 0\ \ 3x^2+x-1 \end{array}$$

$\int\left(1+\frac{3x^2+x-1}{x^3+x^2}\right)dx$

$\int_1^2 1 + \int_1^2 \frac{3x^2+x-1}{x^3+x^2}\,dx$ $\qquad \frac{3x^2+x-1}{x^2(x+1)}$

$\frac{A}{x+1} + \cancel{\frac{B}{x}} + \frac{B}{x^2} = \frac{3x^2+x-1}{(x^2)(x+1)}$

$A(x^2) + B(x+1) = 3x^2+x-1$

$x=0$ $\quad A(0)+B(1) = 0+0-1$, $\quad B=-1$

$x=-1$ $\quad A(1)+B(0) = 3-1-1$ $\quad A=1$

$x^{-2} \quad \frac{1}{-2+1}x^{-1} \quad -x^{-1}$

$\int \frac{1}{x+1} - \frac{1}{x^2}\,dx \qquad x+\ln|x+1| - \frac{1}{x} + C \Big|_1^2$

$\frac{3}{2}$

$\left[\cancel{2}+\ln 3 - \frac{1}{2}\right] - \left[\cancel{x}+\ln 2 - \cancel{x}\right] = \frac{3}{2} + \ln 3 - \ln 2$

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20. $\int_2^3 \frac{x(3-5x)}{(3x-1)(x-1)^2}\,dx \qquad \frac{A}{3x-1} + \frac{B}{(x-1)^2} = \frac{x(3-5x)}{(3x-1)(x-1)^2}$

$A(x^2-2x+1) + B(3x-1) = x(3-5x)$

$x=\frac{1}{3}$ $\quad A\left(-\frac{2}{3}\right)^2 + B(0) = \frac{1}{3}\left(\frac{9}{3}-\frac{5}{3}\right)$ $\quad \frac{4}{9}$ $\quad -\frac{4}{9}A = \frac{4}{9}$ $\quad A=-1$

$x=1$ $\quad A(0) + B(2) = 2(3-10)$ $\quad -14$ $\quad 2B=-14$ $\quad B=-7$

$\frac{-1}{3x-1} - \frac{7}{(x-1)^2} \to -\frac{1}{3}\int\frac{1}{\cancel{3}x-1} - 7\int\frac{1}{(x-1)^2}\,dx$ $\qquad u=x-1$ $\quad du=1dx$ $\quad dx=du$

$-\frac{1}{3}\ln|x-1| + 7/x-1 \Big|_2^3$ $\qquad \int\frac{1}{u^2}\,du \to \frac{-1}{u} \to \frac{-1}{x-1}$

$\left[-\ln 2 + \frac{7}{2}\right] - \left[-\frac{2}{3}\ln 1 + 7\right]$ $\quad -\ln 2 + \frac{7}{2} - \frac{14}{2} = -\ln 2 - \frac{7}{2}$

22. $\int \frac{x^4+9x^2+x+2}{x^2+9}\,dx$

$$\begin{array}{r} x^2-9x+90 \\ x^2+9 \overline{) x^4+0x^3+9x^2+x+2} \\ x^4+9x^3 \\ \hline 0\ \ -9x^3+9x^2+x+2 \\ -9x^3-81x^2 \\ \hline 0\ \ 90x^2+x+2 \\ 90x^2+810x \\ \hline 0-809x+2 \end{array}$$

$\int x^2-9x+90\,dx - \int\frac{809x+2}{x^2+9}\,dx$

$\frac{1}{3}x^3 - \frac{9}{2}x^2 + 90x$

$\int\left(\frac{809x}{x^2+9} + \frac{2}{x^2+9}\right) \to 809\int\frac{x}{x^2+9} + 2\int\frac{1}{x^2+9}$

$u=x^2+9$ $\quad du=2x\,dx$ $\quad dx=\frac{1}{2x}du$ $\qquad u=\frac{x}{3}$ $\quad du=\frac{1}{3}dx$ $\quad dx=3du$

$\frac{1}{2}\int\frac{1}{u}\,du \to \frac{1}{2}\ln u \to \frac{1}{2}\ln(x^2+9)$ $\qquad \frac{1}{3}\int\frac{3}{9u^2+9}\,du$ $\qquad \frac{1}{3}\int\frac{1}{u^2+1}\,du$

$\frac{1}{3}\arctan\left(\frac{x}{3}\right)$

$\frac{1}{3}x^3 - \frac{9}{2}x^2 + 90x - \left(\frac{809\ln(x^2+9)}{2} + \frac{2\arctan\left(\frac{x}{3}\right)}{3}\right) + C$

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24.

$$\int \frac{x^2-x+6}{x^3+3x}\,dx \qquad \frac{x^2-x+6}{x(x^2+3)}$$

$$\frac{x^2-x+6}{(x)(x^2+3)} = \frac{A}{x} + \frac{B}{(x^2+3)} \qquad x^2-x+6 = A(x^2+3) + B(x)$$

$x=0 \quad A(0+3) + B(0) = 6 \quad 3A=6 \quad A=2$

$x=\sqrt{3} \quad x^2+3=0 \quad x^2=-3 \quad x=\sqrt{-3}$ bad

$$\int\left(\frac{-x-1}{x^2+3}\right) + \int\left(\frac{2}{x}\right) \rightarrow 2\ln x$$

$$\frac{-x}{x^2+3} \qquad \frac{-1}{x^2+3}$$

$u=x^2+3$

$du=2x\,dx$

$$\int \frac{1}{2u} \qquad \frac{1}{2}\int\frac{1}{u}\,du \rightarrow -\frac{1}{2}\ln(u) \rightarrow -\frac{1}{2}\ln(x^2+3)$$

$$\rightarrow -\frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right)$$

$$2\ln|x| - \frac{1}{2}\ln(x^2+3) - \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

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$$\int_1^\infty \frac{1}{(2x+1)^3}\,dx \qquad u=2x+1 \quad du=2dx \quad dx=\frac{1}{2}du$$

u^{-3}

$$\frac{1}{2}\int \frac{1}{u^3}\,du$$

$$\frac{1}{2}\left(\frac{-1}{4}u^{-4}\right)du \quad -\frac{1}{8}u^{-4} \rightarrow \frac{1}{8(2x+1)}\Bigg|_1^\infty$$

$$\frac{1}{\infty} \rightarrow [0] \quad -\left[\frac{1}{8(2+1)}\right] \quad = 1/24$$

12.

$$\int_{-\infty}^{\infty} (y^3-3y^2)\,dy \qquad \int_{-\infty}^{\infty} y^3\,dy - \int_{-\infty}^{\infty} 3y^2\,dy$$

$$\int_0^\infty + \int_{-\infty}^0 \qquad \int_0^\infty \quad \int_{-\infty}^0$$

$$\frac{1}{4}y^4\Big|_0^\infty \qquad \frac{1}{4}y^4\Big|_{-\infty}^0$$

$$\frac{1}{4}\infty^4 - 0 \qquad 0 - \frac{1}{4}\infty^4 \qquad 3\int_0^\infty y^2 \qquad 3\int_{-\infty}^0 x^2$$

$$\infty \qquad -\infty \qquad y^3\Big|_0^\infty + y^3\Big|_{-\infty}^0$$

0 divergent $\infty^3 \quad 0 \quad -\infty$

14.

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx \rightarrow \int\frac{1}{x^2e^{1/x}} \qquad u=-1/x \quad du=\frac{1}{x^2} \quad dx=x^2du$$

$$\int e^u\,du \rightarrow e^u \rightarrow e^{-1/x}\Big|^\infty$$

$$\left[e^{-1/\infty}\right]^0 - \left[e^{-1}\right] \qquad -\frac{1}{e}$$

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16.

$$\int_0^\infty \sin\theta\, e^{\cos\theta}\,d\theta \qquad u=\cos\theta \quad du=-\sin\theta\,d\theta$$

$$dx=\frac{du}{-\sin\theta}$$

$$-\int e^u\,du \rightarrow -e^u \rightarrow -e^{\cos\theta}$$

$$\left[-e^{\cos\infty}\right] - \left[-e^{\cos 0}\right] \qquad \text{undefined}$$