1. 6.4-9

$$\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix} \Rightarrow \vec{V}_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \vec{V}_2 = \begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix}, \vec{V}_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix}$$

$$\vec{u}_1 = \vec{V}_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}$$

$$\vec{u}_2 = \vec{V}_2 - \frac{\vec{V}_2 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1}\vec{u}_1 = \begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix} - \frac{\begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix} - \frac{\overbrace{-15+1-5-21}^{-40}}{\underbrace{9+1+1+9}_{20}} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix} + \begin{bmatrix} 6 \\ 2 \\ -2 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}$$

$$\vec{u}_3 = \vec{V}_3 - \frac{\vec{V}_3 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1}\vec{u}_1 - \frac{\vec{V}_3 \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2}\vec{u}_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}}{\begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}} \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix} - \frac{\overbrace{3+1+2+24}^{30}}{\underbrace{9+1+1+9}_{20}} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} - \frac{\overbrace{1+3-6-8}^{-10}}{\underbrace{1+9+9+1}_{20}} \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \\ -2 \\ 8 \end{pmatrix} - \begin{bmatrix} 4.5 \\ 1.5 \\ -1.5 \\ 4.5 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1.5 \\ 1.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 3 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ -2 \\ 4 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 3 & 1 & -4 \\ 1 & 3 & -2 \\ -1 & 3 & -2 \\ 3 & -1 & 4 \end{bmatrix}}$$

2. 6.5 − 4

$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \end{bmatrix}$

$A^T A \vec{x} = A^T \vec{b}$

$\left[\begin{array}{cc|c} 3 & 3 & 6 \\ 3 & 11 & 14 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$ $\boxed{\hat{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

6.5−6 $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 7 \\ 2 \\ 3 \\ 6 \\ 5 \\ 4 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 6 & 3 & 3 & 27 \\ 3 & 3 & 0 & 12 \\ 3 & 0 & 3 & 15 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\boxed{\hat{x} = \begin{bmatrix} 5-\alpha \\ \alpha-1 \\ \alpha \end{bmatrix}}$

$x_3 = \alpha \quad x_2 = \alpha - 1 \quad x_1 = 5 - \alpha$

6.5−8 $\vec{b} - A\hat{x} \rightarrow \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

$\sqrt{(1)^2 + (1)^2 + (-2)^2} = \boxed{\sqrt{6}}$

3. 6.6−11

r	θ
3.00	0.88
2.30	1.10
1.65	1.42
1.25	1.77
1.01	2.14
A	$\vec{b}$

$r = \beta + e(r \cdot \cos\theta)$

$A^T = [3.00 \quad 2.30 \quad 1.65 \quad 1.25 \quad 1.01]$

$A^T A = 19.5951 \qquad A^T \vec{b} = 11.8869$

$19.5951 \hat{x} = 11.8869$

$\hat{x} = 0.6066 \rightarrow$ $\boxed{\text{elliptical}}$

4.

a) For the expirement, a ball was dropped from ten seperate heights, starting at 10 cm and ending at 100 cm. The height after one bounce was recorded along with the height at which it was dropped at into a table.

b)

drop height (cm)	10	20	30	40	50	60	70	80	90	100
bounce height (cm)	2	5	8	12	15	18	22	26	30	34

c)

$X/0.3218 = Y$

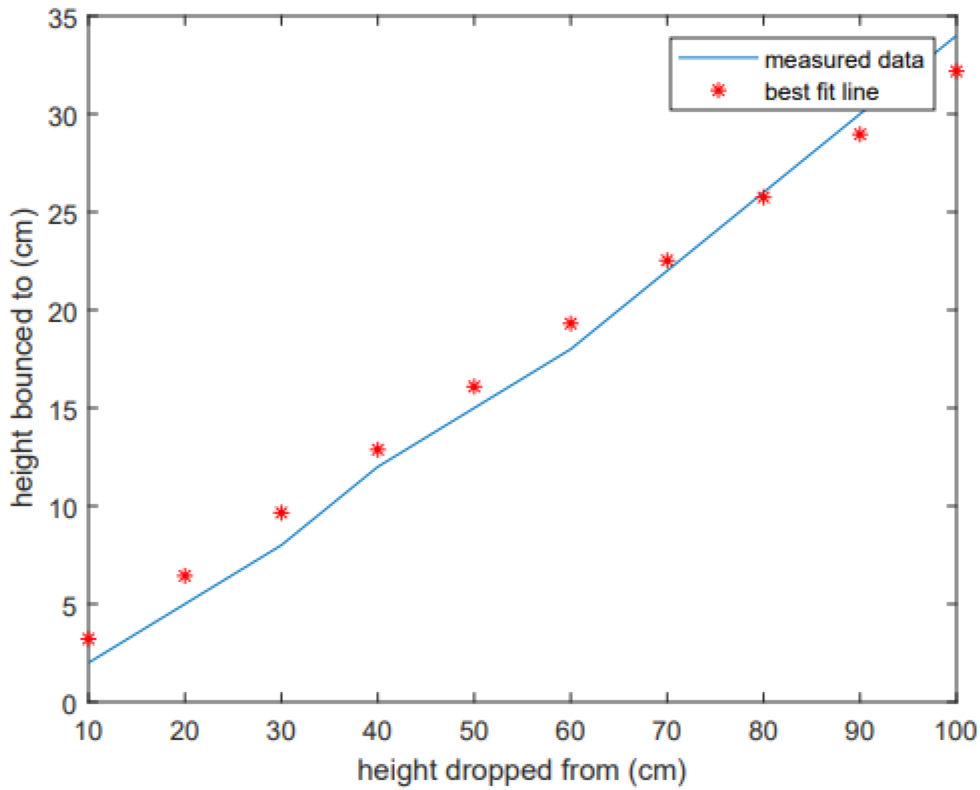
squared error: $[0.0015\ 0.0021\ 0.0027\ 0.0008\ 0.0012\ 0.0017\ 0.0003\ 0.0001\ 0.0011\ 0.0033]$

d)

MATLAB

e)

I couldn't figure out how to do the quadratic model so I only plotted the linear model and thus it fit the best. The softness and weight of the ball could've impacted the height it bounced to.


measured data
best fit line
height bounced to (cm)
height dropped from (cm)
0
5
10
15
20
25
30
35
10
20
30
40
50
60
70
80
90
100

5.

(i) $\vec{s} = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \end{bmatrix}$

$\vec{s} = \beta_0 + \beta_1 \cos(t) + \alpha_1 i \sin(t) + \beta_2 \cos(t) + \alpha_1 i \sin(t) + \ldots\ldots$

$$[F]\,\vec{s} = \vec{c}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ 1 & w^4 & w^8 & w^{12} & w^{16} & w^{20} & w^{24} & w^{28} \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ 1 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ 1 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$