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Homework 8

5.3-1 $y(n) - \frac{1}{3}y(n-1) = x(n-1)$; $y(-1) = 2$; $x(n) = -u(n)$

▷ $x(-1) = 0$

$\mathcal{Z}\{5.3\text{-}1\}$ ▷ $Y(z) - \frac{1}{3}\left(z^{-1}Y(z) + y(-1)\right) = z^{-1}X(z)$

▷ $Y(z)\left(1 - \frac{1}{3}z^{-1}\right) - \frac{2}{3} = z^{-1}X(z)$ ▷ $X(z) = \frac{-z}{z-1}$

▷ $Y(z)\left(1 - \frac{1}{3}z^{-1}\right) = z^{-1}\left(\frac{-z}{z-1}\right) + \frac{2}{3}$ ▷ $Y(z) = \frac{2/3}{\left(1 - \frac{1}{3}z^{-1}\right)} - \frac{z}{z-1}\frac{z^{-1}}{\left(1 - \frac{1}{3}z^{-1}\right)}$

▷ $Y(z) = \frac{2/3}{(z - 1/3)} - \frac{1}{(z-1)(z-1/3)}$ ▷ partial fraction expansion

▷ $Y(z) = \frac{3/2}{(z-1/3)} - \frac{3/2}{(z-1)} + \frac{2/3}{(z-1/3)} = \frac{13/6}{(z-1/3)} - \frac{3/2}{(z-1)}$

▷ $\mathcal{Z}^{-1}\{Y(z)\} = \frac{13}{6}(1/3)^n - \frac{3}{2}(1)^n = \boxed{\left(\frac{13}{6}(1/3)^n - 3/2\right)u(n)}$

5.3-7 $2y(n+2) - 3y(n+1) + y(n) = 4x(n+2) - 3x(n+1)$

▷ go to delay form ▷ $2y(n) - 3y(n-1) + y(n-2) = 4x(n) - 3x(n-1)$

▷ $2Y(z) - 3\left(z^{-1}Y(z) + \cancel{y(-1)}^{0}\right) + \left(z^{-2}Y(z) - 0 + 1\right) = 4X(z) - 3z^{-1}X(z)$

▷ $Y(z)\left(2 - 3z^{-1} + z^{-2}\right) + 1 = X(z)\left(4 - 3z^{-1}\right)$

▷ $Y(z)\left(2 - 3z^{-1} + z^{-2}\right) = X(z)\left(4 - 3z^{-1}\right) - 1 \Rightarrow x(n) = (4)^{-n}u(n) - (1/4)^n u(n)$

▷ $Y(z)\, z^{-1}\left(2z^2 - 3z^1 + 1\right) = -1 + (4z - 3)$

▷ $Y(z) = \boxed{\underbrace{\frac{-z}{(2z^2 - 3z^1 + 1)}}_{\text{Zero - Input}} + \underbrace{\frac{z(4z-3)}{(2z^2 - 3z^1 + 1)}}_{\text{Zero state}}} = \frac{0.5z}{z-0.5} - \frac{1z}{z-1} + \frac{2z}{z-0.5} + \frac{4/3z}{z-1} - \frac{4/3z}{z-0.25}$

b)

▷ $y(n) = 1/2(1/2)^n - 1 + 2(1/2)^n + 4/3 - 4/3(1/4)^n$

▷ $y(n) = \left(\underbrace{5/2(1/2)^n}_{\text{transient}} + \underbrace{1/3}_{\text{steady state}} - \underbrace{4/3(1/4)^n}_{\text{transient}}\right)u(n)$

5.3-9 $\qquad y(-1)=0\ ,\ y(-2)=1$

▷ $4y(n+2)+4y(n+1)+y(n)=x(n+1) \qquad x(n)=u(n)$

▷ $4y(n)+4y(n-1)+y(n-2)=x(n-1)$

▷ $4Y(z)+4z^{-1}Y(z)+z^{-2}Y(z)+1=z^{-1}X(z)$

▷ $Y(z)\left(4+4z^{-1}+z^{-2}\right)=\dfrac{z^{-1}z}{z-1}=\dfrac{1}{z-1}-1$

▷ $\dfrac{Y(z)}{z}=\dfrac{z}{(z-1)(4z^2+4z+1)}-\dfrac{z}{(4z^2+4z+1)}=\dfrac{-1/4}{z+0.5}+\dfrac{1/8}{(z+0.5)^2}+\dfrac{1/9}{z-1}-\dfrac{1/9}{z+0.5}+\dfrac{1/12}{(z+0.5)^2}$

▷ $Y(z)=\boxed{\underbrace{\dfrac{-1/4\,z}{z+0.5}+\dfrac{1/8\,z}{(z+0.5)^2}}_{\text{Zero input}}+\underbrace{\dfrac{1/9\,z}{z-1}-\dfrac{1/9\,z}{z+0.5}+\dfrac{1/12}{(z+0.5)^2}}_{\text{Zero state}}}$ ▷ $\dfrac{1/4\ 1/2\,z}{(z+0.5)^2}$

▷ $y(n)=\left(\underbrace{-1/4(-1/2)^n-1/4\,n(-1/2)^n}_{\text{transient}}+\underset{\uparrow\ \text{steady state}}{1/9}-\underbrace{1/9(-0.5)^n+1/6\,n(-0.5)^n}_{\text{transient}}\right)u(n)$

5.3-25 a)

$X[z] = \frac{ez}{z-e}$ $\qquad Y(z) = \frac{ez^2}{(z-e)(z+0.2)(z-0.8)}$

$$\frac{Y(z)}{z} = \frac{132}{z-e} - \frac{0.186}{z+0.2} - \frac{1.13}{z-0.8}$$

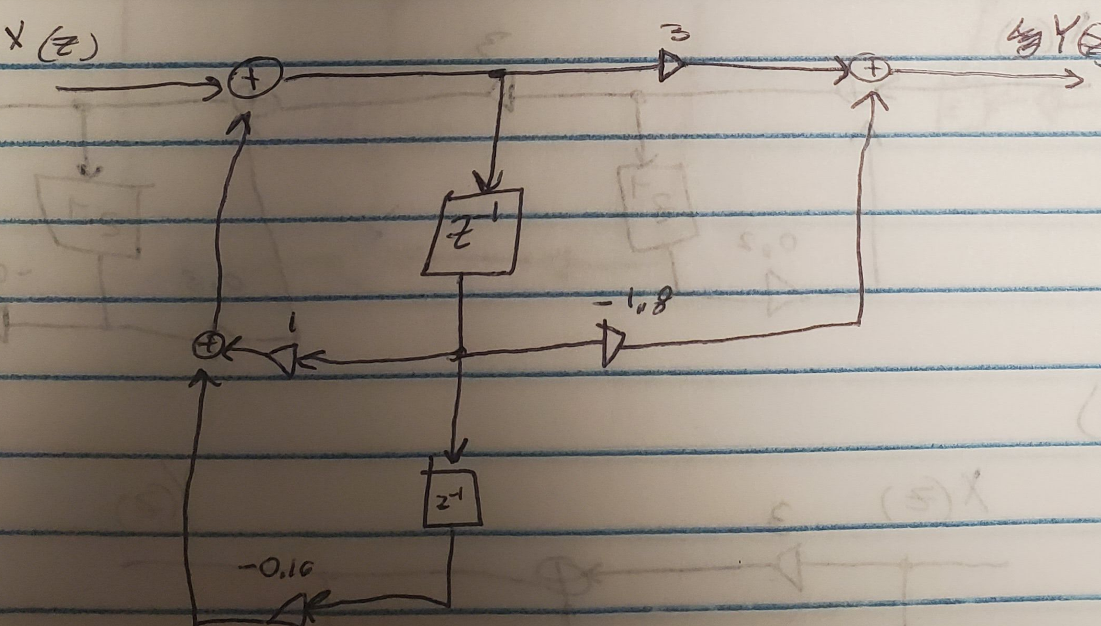
$$\boxed{y(n) = \left(1.32\,(e)^n - 0.186\,(-0.2)^n - 1.13\,(0.8)^n\right)u(n)}$$

b)

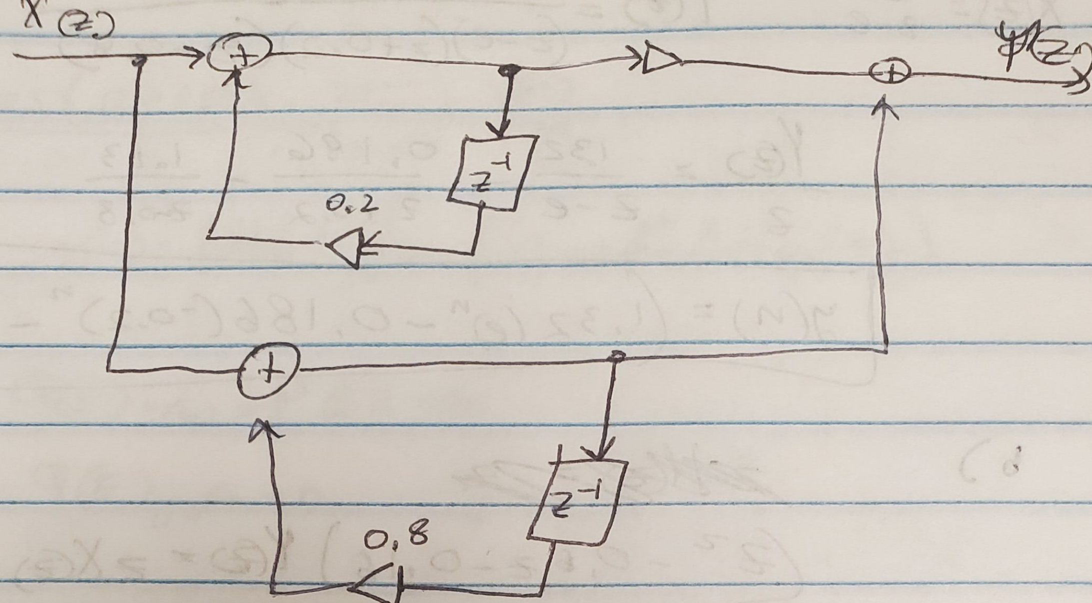
$$(z^2 - 0.6z - 0.16)\,Y(z) = z\,X(z)$$

$$\boxed{y(n) - 0.6\,y(n-1) - 0.16\,y(n-2) = x(n - [illegible])}$$

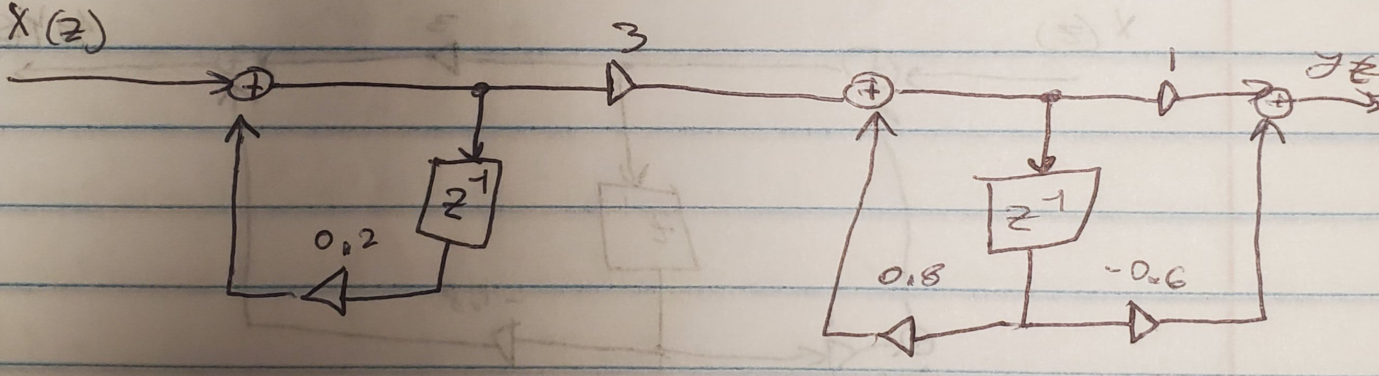
5.4-2 a)



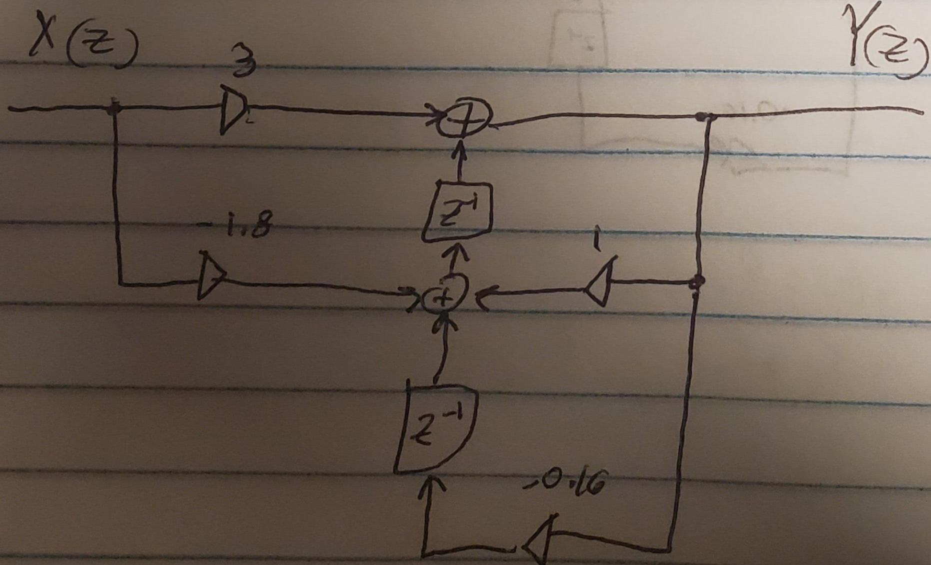
$$H(z) = \frac{2z}{z-0.2} + \frac{z}{z-0.8}$$

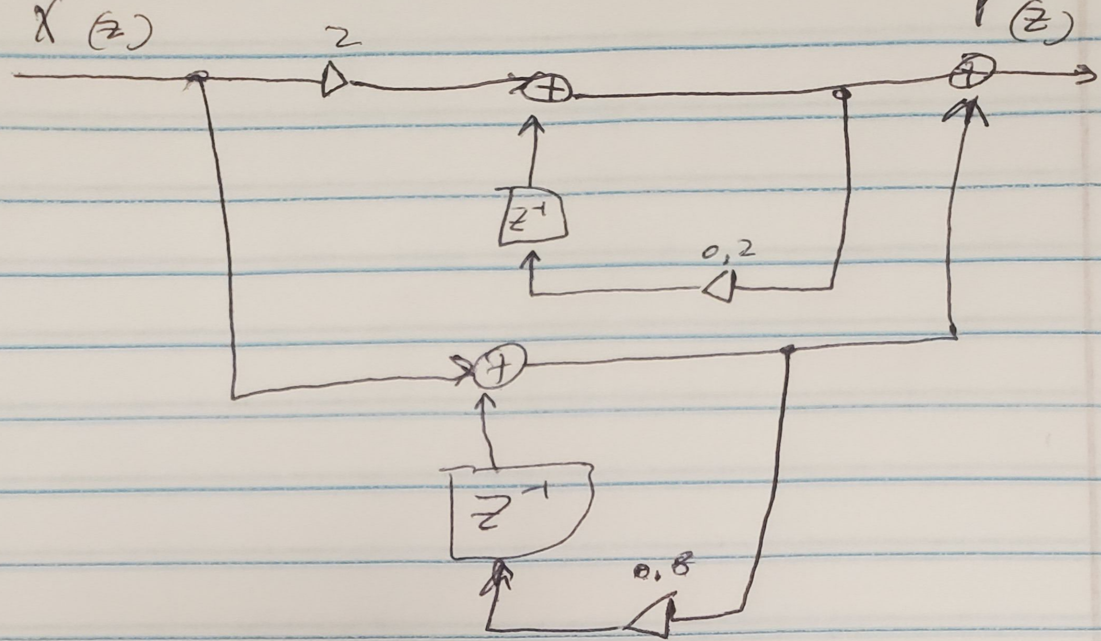


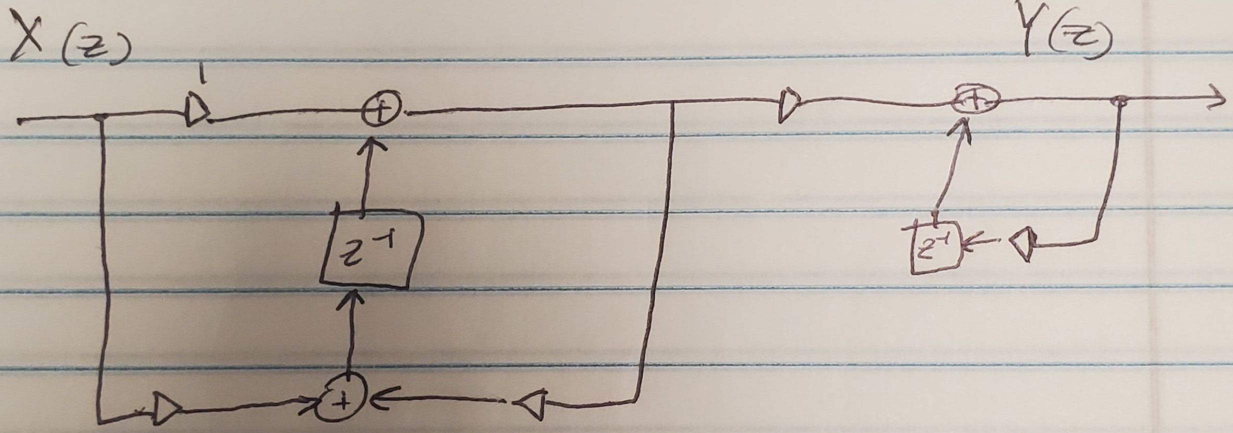
$$H(z) = \left(\frac{3z}{z-0.2}\right)\left(\frac{z-0.6}{z-0.8}\right)$$



b)




X (z)
Y (z)
2
z^{-1}
0.2
z^{-1}
0.8


X (z)
Y (z)
1
z^{-1}
z^{-1}

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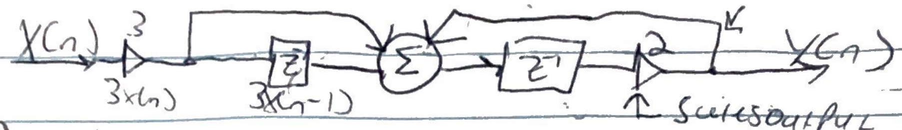
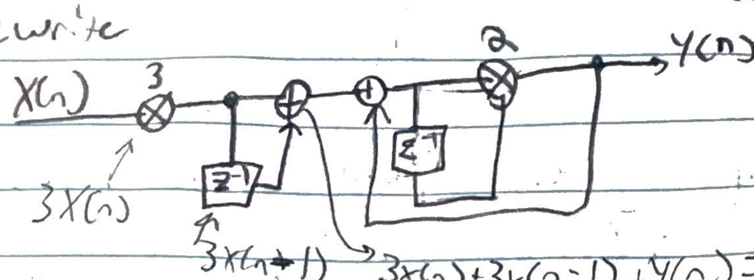
5.4-7 a.)



Rewrite



$3x(n)+3x(n-1)+Y(n) = \frac{Y(n+1)}{-2}$

$\frac{Y(n+1)}{2} - Y(n) = 3x(n)+3x(n-1)$ $\cdot 2$

$2Y(n+1) - 2Y(n) = 6x(n)+6x(n-1)$ ← must adjust delay for Y(n+1)

$\underline{Y(n) - 2Y(n-1) = 6x(n-1)+6x(n-2)}$ ← correct form

b.) need Z transform

$Z\{Y(n) - 2Y(n-1) = 6x(n-1)+6x(n-2)\}$

$Z\{Y(n)\} - 2Z\{Y(n-1)\} = Z\{6x(n-1)\} + 6Z\{x(n-2)\}$

$Y(z) - 2Y(z)z^{-1} = 6X(z)z^{-1} + 6X(z)z^{-2}$ factor out X, Y

$Y(z)(1-2z^{-1}) = X(z)(6z^{-1}+6z^{-2})$

$\frac{Y(z)}{X(z)} = \frac{6(z+1)}{z(z-2)}$ } partial fractions $\frac{a}{z} + \frac{b}{z-2}$

$a(z-2) + b(z) = 6z+6$ $z=0$

$-2a = 6 \to a = -3$

$-3(0) + -2(b) = 12+6 \xrightarrow{z=2} +2b = 18 \to \underline{b=9}$

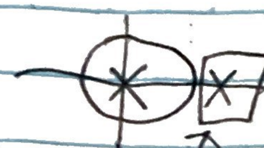
$$\frac{-3}{z} + \frac{9}{z-2} \rightarrow Z^{-1} \rightarrow -3\delta(n-1) + 9(2)^{n-1}u(n-1)$$

All delayed by $n-1$

c.) due to how thee feed back Loops are made there are exactly 2 delay components which is equivilent to $n-2$, the largest delay, the system is canonical

d.) $\frac{6(z+1)}{z(z-2)}$ $\rightarrow -1$

$2, 0$ poles



2 is out of Unit circle

Unstable System

e.) if $h(n) = 0$ for $n < 0$

$$h(n) = -3\delta(n-1) + 9(2)^{n-1}u(n-1)$$

Any input less then 0 has to be 0 because of $u(n)$, $\delta(n)$

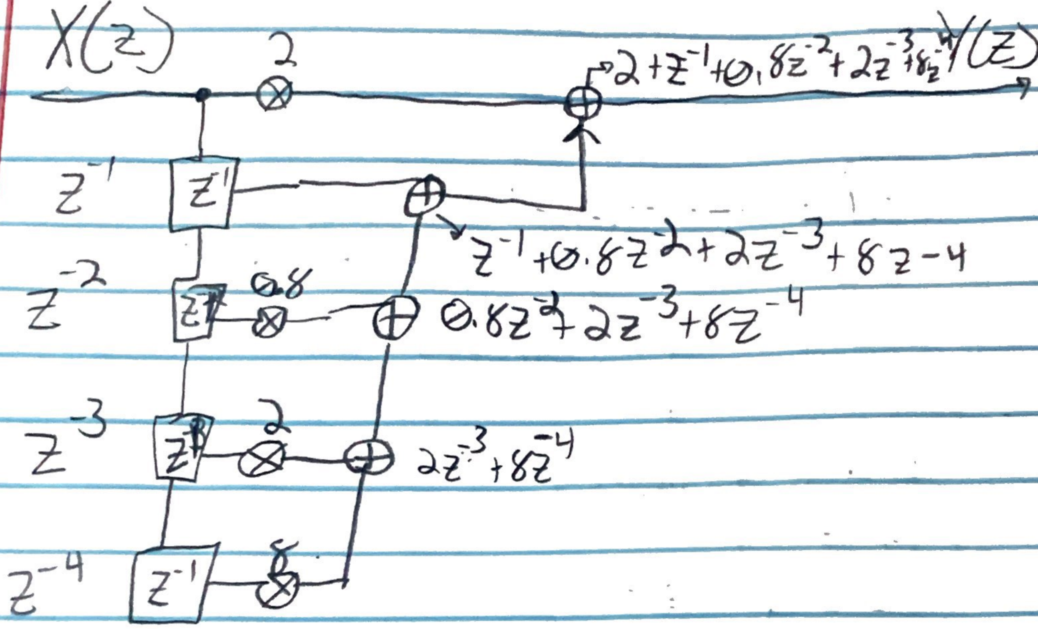
5.4-8) Realize System $H(z) = \dfrac{2z^4 + z^3 + 0.8z^2 + 2z + 8}{z^4}$

$$\frac{2z^{+4} + z^3 + 0.8z^2 + 2z + 8}{z^4} \cdot \left(\frac{z^{-4}}{z^{-4}}\right)$$

$$\rightarrow 2z^0 + z^{-1} + 0.8z^{-2} + 2z^{-3} + 8z^{-4}$$

$\uparrow$ 2

Now the model



5.4-9 Realize model $H(z) = \sum_{n=0}^{6} n z^{-n}$ expand

$$0z^{-0} + 1z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 5z^{-5} + 6z^{-6}$$