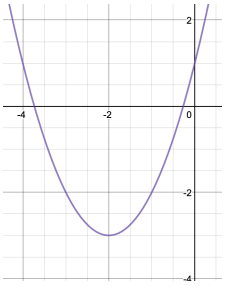
# Math 418 Final Exam Review Sheet

These practice problems are meant to give you a guide to study for the exam. There will not be this many problems on the exam. Note that there may be problems on the exam that are not represented among these practice problems. It is impossible to provide enough practice problems to represent all of the ideas and skills that we are assessing through this exam. That said, if you are able to do all of these problems by yourself, without any resources other than your brain, you can rest assured that you have prepared well for the exam. The distribution of topics on this review sheet is NOT indicative of the distribution of topics on the exam.

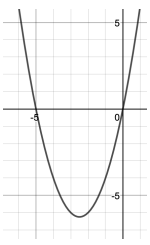
**Final: Thursday, December 17th at 8AM.**

1. Find the equation of the line passing through the point $(\tan\frac{-\pi}{4}, \log_9 3)$ that is parallel to the line given by $\ln 5^{2y} - x\sin\left(\frac{-2\pi}{3}\right) = e^0$.
   **Answer:** $y - 1/2 = -\frac{\sqrt{3}}{4\ln 5}(x+1)$
2. Find the slope and all intercepts of the linear function determined by $12x - 3y + 4 = 0$.
   **Answer:** $m = 4$, y-int: $(0, \frac{4}{3})$
3. Find the vertex of the following parabola and determine if it is facing upwards or facing downwards, then sketch the parabola.

   a) $x^2 + 4x = y - 1$
   **Answer:** Vertex: $(-2, -3)$




   b) $2x^2 + 10x - 2y = 0$
   **Answer:** Vertex: $(-2.5, -\frac{25}{4})$



4. Find a polynomial $p(x)$ of degree 3 such that $p(1) = 0, p(4) = 0, p(-1) = 0$ and $p(2) = 1$.
**Answer:** $p(x) = \frac{-1}{6}(x-1)(x+1)(x-4)$

5. Give an example of two quadratic polynomials such that their sum is a linear function.
**Answer:** $f(x) = x^2 + x, g(x) = -x^2$.

6. Give an example of a rational function $r(x)$ with precisely two vertical asymptotes, at $x = 2$ and $x = -3$ and one x-intercept at $(0, 0)$.
**Answer:** $r(x) = \frac{x}{(x-2)(x+3)}$

7. Find the domain and range and inverse of $r(x) = \frac{x-4}{3x-5}$.
**Answer:** $\text{Dom}(r) = \{x | x \neq \frac{5}{3}\}$, $\text{Range}(r) = \{x | x \neq \frac{1}{3}\}, r^{-1}(x) = \frac{5x-4}{3x-1}$.

8. If I start out with 22 rabbits and the number of rabbits I have doubles every 10 months how many years will it take until I have 1000 rabbits?
**Answer:** $t = \frac{5}{6}\ln\left(\frac{500}{11}\right)$ years

9. How much money would I have to invest in a bank account that gets compounded continuously (with 1.5% per year) to have $20,000 after 5 years?
**Answer:** $\frac{20000}{e^{.075}}$ years

10. Find the domain of $r(x) = \frac{2x+1}{\log(8x+1)-10}$.
**Answer:** $\{x | x > -\frac{1}{8}, x \neq \frac{1}{8}(10^{10} - 1)\}$

11. Find the domain of $g(x) = \frac{3x+4}{\sqrt{\ln x}} - \frac{2\sin(x)}{e^x - 9}$
**Answer:** $\{x | x > 1, x \neq \ln 9\}$

12. Suppose $a$ and $b$ are positive numbers such that $\log_3 a = 1.4$ and $\log_3 b = 1.6$. Evaluate:

    a) $\log_3 ab$
    **Answer:** 3

    b) $\log_3 a^2 b^4$
    **Answer:** 9.2

    c) $\frac{b}{a}$
    **Answer:** $3^{0.2}$

    d) $\log_a 7$
    **Answer:** $\frac{\log_3 7}{1.4}$

13. Solve the following equations for x:

    (a) $\log 4x = 12$
    **Answer:** $x = \frac{10^{12}}{4}$

    (b) $\log(x+1) + \log(x-1) = 2$
    **Answer:** $x = \sqrt{101}$

(c) $\frac{\ln 2x^2}{\ln 3x} = 5$
**Answer:** $x = (\frac{2}{3^5})^{\frac{1}{3}}$

(d) $\ln 3x - \ln x^2 = -2$
**Answer:** $x = 3e^2$

(e) $\ln(\ln(x)) = 4$
**Answer:** $x = e^{e^4}$

14. Suppose $f(x) = 3e^{2x}$. Show that the graph of $y = \log_4 f(x)$ is a line. What is the slope of this line?
**Answer:** Use property of logs. $m = 2\log_4 e$

15. If $f(x) = 3^{2x} - 4$, find $f^{-1}(x)$.
**Answer:** $f^{-1}(x) = \frac{1}{2}\log_3(x+4)$

16. If $f(x) = \log_8(2x+1) + 5$, find $f^{-1}(x)$.
**Answer:** $f^{-1}(x) = \frac{1}{2}(8^{x-5} - 1)$

17. Solve the following equation: $2e^{4x} + 8e^{2x} = -8$
**Answer:** No Solution.

18. Solve the following equation: $\sin^2(2x) + 2\sin(2x) + 5 = 4$
**Answer:** $x = \frac{-\pi}{4} + \pi k$, $k$ an integer

19. Find all $\theta$ such that $-\pi \leq \theta \leq \pi$ and $\sec^2\theta = 4$
**Answer:** $x = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

20. Find all $\psi$ with $\pi \leq \psi \leq 2\pi$ such that $(\sin^2(2\psi) - 1)(\tan(3\psi)) = 0$
**Answer:** $\psi = \frac{5\pi}{4}, \frac{7\pi}{4}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$.

21. Suppose $0 \leq \alpha \leq \pi, -\pi \leq \gamma \leq 0, \sec\alpha = \frac{3}{2}$ and $\tan\gamma = 5$. Evaluate $\sin(\alpha - \gamma)$.
**Answer:** $\sin(\alpha - \gamma) = \frac{-\sqrt{5}+10}{3\sqrt{26}}$

22. Solve: $\sin(3\theta + 1) = \frac{-1}{2}$
**Answer:** $\theta = -\frac{\pi}{18} - \frac{1}{3} + \frac{2\pi}{3}k$, $k$ an integer.

23. Evaluate $\sin\left(\frac{5\pi}{12}\right)$
**Answer:** $\frac{\sqrt{2+\sqrt{3}}}{2}$

24. Evaluate $\cos\left(\frac{-3\pi}{4} - \frac{7\pi}{6}\right)$
**Answer:** $\frac{\sqrt{2+\sqrt{3}}}{2}$

25. Evaluate $\tan\left(\frac{\pi}{3} + \frac{\pi}{8}\right)$
**Answer:** $\frac{\sqrt{2+\sqrt{2+\sqrt{3}}}}{\sqrt{2-\sqrt{2+\sqrt{3}}}}$

26. Suppose $\csc\theta = \frac{-7}{2}$ Evaluate $\cot\theta$ if $\sec\theta > 0$.
**Answer:** $\cot\theta = \frac{-\sqrt{45}}{2}$

27. Suppose $\sin\kappa < 0$ and $\tan\kappa = \frac{10}{11}$. Evaluate $\cos\kappa$
**Answer:** $\cos\kappa = \frac{-11}{\sqrt{221}}$