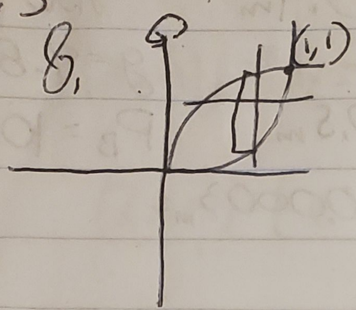
6.2

54. $\int_0^{180} (2\sin\theta)^2 \, d\theta$

6.3

8. $x=0$



$V = \pi\int_0^1 (x^2)^2 - (\sqrt{x})^2 dx$

$\pi\int x^4 - x \, dx$

$\pi\left(\frac{x^5}{5} - \frac{x^2}{2}\right)\Big|_0^1 = \frac{3\pi}{5}$

$V = 2\pi\int_0^1 (\sqrt{x} - x^2)x \, dx \quad 2\pi\int x^{3/2} - x^3 \, dx$

$2\pi\left(\frac{2x^{5/2}}{5} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{3\pi}{5}$

12. $x = -3y^2 + 12y - 9 \quad x=0 \quad$ about $y=0$

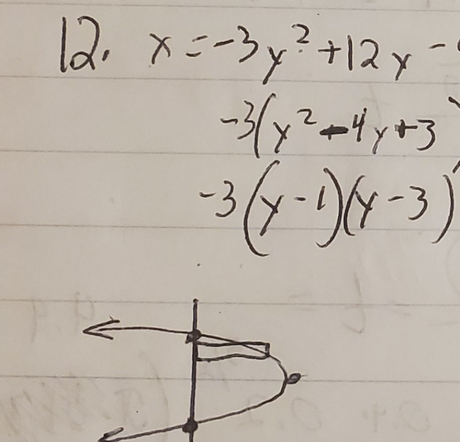
$-3(y^2 - 4y + 3)$

$-3(y-1)(y-3) \quad y = 1, 3$

$-3 + 12 - 9$

$-12 + 24 - 9$

$12 - 9 = 3$



$2\pi\int_1^3 (-3y^2 + 12y - 9)y \, dy$

$-3y^3 + 12y^2 - 9y$

$2\pi\left(\frac{-3y^4}{4} + \frac{12y^3}{3} - \frac{9y^2}{2}\right)\Big|_1^3$

$2\pi\left(\left[\frac{-3(81)}{4} + \frac{324}{3} - \frac{81}{2}\right] - \left[\frac{-3}{4} + \frac{12}{3} - \frac{9}{2}\right]\right) \quad 2\pi\left(\frac{514}{12}\right) = \frac{257\pi}{3}$

$\frac{729}{12} + \frac{216}{12} - \frac{486}{12} = \frac{459}{12} \qquad -\frac{9}{12} + \frac{8}{12} - \frac{54}{12} = \frac{-55}{12}$

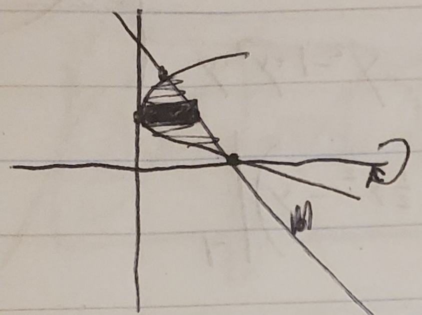
14. $x+y=4 \quad x=y^2-4y+4 \quad y=0$

$x=4-y$

$(y-2)(y-2) \quad 4-y=y^2-4y+4$

$0=y^2+3y$

$y(y+3)=0 \quad y=0,+3$



$r=4-y$

$(y^2-4y+4)-(y-4)$

$2\pi\int (4-y)(y^2-4y+4-y+4)\,dy$

$-y^3+9y^2-52y+32 \quad (4-y)(y^2-5y+8)$

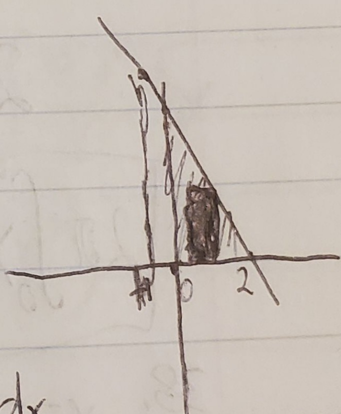
$4y^2-y^3-20y+32+5y^2-32y$

$2\pi\left(-\frac{1}{4}y^4+3y^3-26y^2+32y\right)\Big|_0^3$

$2\pi\left(-\frac{81}{4}+81-234+96\right)$

$-\frac{81}{4}-\frac{228}{4}=\frac{-309}{4}\cdot\frac{2}{1} \quad \frac{-618}{4}=\boxed{\frac{-309\pi}{2}}$

16. $y=4-2x,\ y=0,\ x=0$ about $x=-1$



$2\pi\int_0^2 (1+x)(4-2x)\,dx$

$4-2x+4x-2x^2$

$-2(x^2-x-2) \quad -2x^2+2x+4$

$-2(x-2)(x+1)$

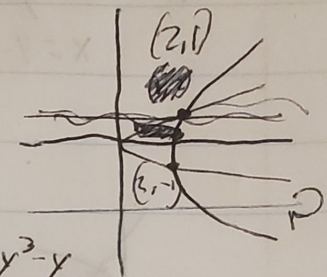
$\int -2x^2+2x+4\,dx$

$2\pi\left(-\frac{2}{3}x^3+x^2+4x\right)\Big|_0^2$

$2\pi\left(\frac{-16}{3}+4+8\right)$

$\frac{-16}{3}+\frac{12}{3}+\frac{24}{3}=\frac{20}{3}=\boxed{\frac{40\pi}{3}}$

20. $x=2y^2 \quad x=y^2+1$ about $y=-2$



$2y^2=y^2+1 \qquad y^2=1 \qquad y=\pm 1$

$2\pi\int_{-1}^{1}(1+y)(y^2-1)\,dy \qquad y^2-1+y^3-y$

$2\pi\int y^3+y^2-y-1\,dy \qquad 2\pi\left(\frac{1}{4}y^4+\frac{1}{3}y^3-\frac{1}{2}y^2-y\right)\Big|_{-1}^{1}$

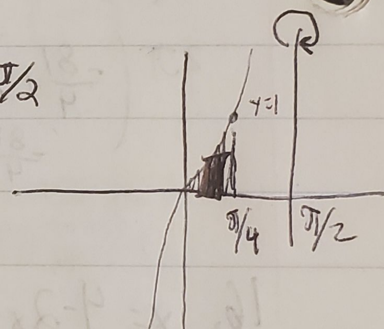
$\pi\left(\frac{1}{2}y^4+\frac{2}{3}y^3-y^2-2y\right)\Big|_{-1}^{1}$

$\left[\pi\left(\frac{1}{2}+\frac{2}{3}-\frac{1}{1}-\frac{2}{1}\right)\right]-\left[\pi\left(\frac{1}{2}-\frac{2}{3}-\frac{1}{1}+\frac{2}{1}\right)\right]$

$\frac{3}{6}+\frac{4}{6}-\frac{6}{6}-\frac{12}{6} \qquad \frac{3}{6}-\frac{4}{6}-\frac{6}{6}+\frac{12}{6}$

$-\frac{11\pi}{6}-\frac{5\pi}{6} \qquad \frac{5\pi}{6} \qquad -\frac{16\pi}{6}\rightarrow\boxed{\frac{-8\pi}{3}}$

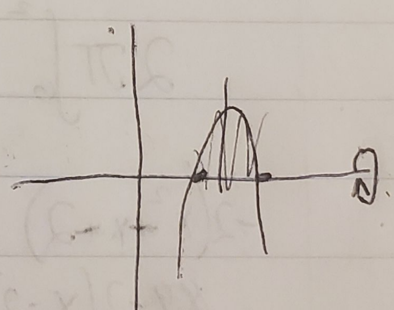
22a. $y=\tan x \quad y=0 \quad x=\pi/4$ about $x=\pi/2$



$2\pi\int_0^{\pi/4}\left(y+\pi/2\right)(\tan y)\,dy$

$\boxed{2\pi\int_0^{\pi/4} y\tan y+\frac{\pi}{2}\tan y\,dy}$

38. $y=-x^2+6x-8 \quad y=0$ about $y=0$



$-(x^2-6x+8)\rightarrow-(x-2)(x-4) \qquad x=2,4$

$\pi\int_2^4 -x^2+6x-8\,dx \qquad \pi\left(-\frac{1}{3}x^3+3x^2-8x\right)_2^4$

$\pi\left[\frac{-256}{3}+48-32\right]-\pi\left[\frac{-8}{3}+12-16\right]$

$\frac{-256}{3}+\frac{144}{3}-\frac{96}{3} \qquad \frac{-8}{3}+\frac{36}{3}-\frac{48}{3}$

$\frac{-208}{3} \quad + \quad \frac{20}{3} \qquad \frac{-188\pi}{3}$

7.1 2. $\int \sqrt{x} \ln x \, dx$ $\quad u = \ln x \quad dv = \sqrt{x}\,dx$

$du = 1/x \qquad x^{1/2} \to \frac{2}{3}x^{3/2} = v$

$\int u\,dv = uv - \int v\,du$

$\int \ln x \sqrt{x}\,dx = \ln x \, \frac{2}{3}x^{3/2} - \int \frac{2}{3}x^{3/2} \cdot 1/x$

$\cancel{\int \ln x \sqrt{x}\,dx = \ln x \, \frac{2}{3}x^{3/2} - \int \frac{2x^{3/2}}{3x}}$ $\qquad \frac{2}{3}x^{1/2}$

$\ln x \, \frac{2}{3}x^{3/2} - 2/3\left(2/3\, x^{3/2}\right) \to \boxed{\ln x \, \frac{2}{3}x^{3/2} - 4/9\, x^{3/2} + C}$

4. $\int y e^{0.2y}\,dy \qquad u = y \quad dv = e^{0.2y}\,dy \quad du = 1\,dy$

$e^{1/5} = e^{0.2y} = \sqrt[5]{e}\; y \to \sqrt[5]{e}\int y\,dy \to \sqrt[5]{e}\left(\frac{1}{2}y^2\right)$

$\frac{\sqrt[5]{e}\,y^2}{2} + C \qquad \int y e^{0.2y}\,dy = y\left(\frac{\sqrt[5]{e}\,y^2}{2}\right) - \int \frac{\sqrt[5]{e}\,y^2}{2}\,dy$

$\frac{y^3\sqrt[5]{e}}{2} - \frac{\sqrt[5]{e}}{2}\int y^2\,dy \to \frac{y^3\sqrt[5]{e}}{2} - \frac{\sqrt[5]{e}}{2}\left(\frac{1}{3}y^3\right)$

$$\boxed{\frac{y^3\sqrt[5]{e}}{2} - \frac{y^3\sqrt[5]{e}}{6} + C}$$