MATH 425

Exam 2

1. $f(x) = \frac{6}{2-x}$ $\qquad \left(\frac{\frac{6}{2-(x+h)} - \frac{6}{2-x}}{h}\right)$

$$\cancel{\frac{\left(\frac{6}{2-(x+h)} - \frac{6}{2-x}\right)}{h} \cdot \frac{\left(\frac{6}{2-(x+h)} + \frac{6}{2-x}\right)}{\left(\frac{6}{2-(x+h)} + \frac{6}{2-x}\right)} = \frac{\left(\frac{6}{2-(x+h)}\right)^2 - \left(\frac{6}{2-x}\right)^2}{h\left(\frac{6}{2-(x+h)} + \frac{6}{2-x}\right)}}$$

$$\cancel{\frac{6}{2-(x+h)} \cdot \frac{6}{2-(x+h)} = \frac{36}{4-(4(x+h))+(x+h)^2}}$$

$$\lim_{h \to 0} \left(\frac{\left(\frac{6}{2-(x+h)}\right) - \left(\frac{6}{2-x}\right)}{h}\right) \cdot \frac{2-(x+h)}{6} + \frac{2-x}{6}$$

$$\left(\frac{\frac{6(2-(x+h))}{(2-(x+h))6} - \frac{6(2-x)}{(2-x)6}}{h\left(\frac{2-(x+h)}{6} + \frac{2-x}{6}\right)}\right) \qquad \frac{0}{\frac{h(2-(x+h))}{6} + \frac{h(2-x)}{6}}$$

$$\frac{2h - h(x+h)}{6} + \frac{2h - hx}{6}$$

$$\frac{2h - hx - h^2 + 2h - hx}{6} \qquad \frac{4h - 2hx - h^2}{6} \text{ plug in}$$

$$0 = \frac{0}{6} = \frac{4(0) - 2(0)x - (0)^2}{6}$$

a. 0
b. 0
c. 0 is the slope of the line tangent to $(1, f(1))$
d. True

Exam 2

2.

$y_f = -4.9t^2 + 30.2t + 0.5$

$x_f = 23.5t$

a. 0.5 meters

b. 3.08 seconds

c. They are lying, the maximum height is 47.03 meters

d. 6.18 seconds

3.

$F(t) = e^t \sin(t) + \frac{\cos(t)}{e^{-t}}$

$\frac{d}{dx}[e^t \sin(t)] + \frac{d}{dx}[\cos(t)/e^{-t}]$

product Quotient

$(e^t)\cdot(\cos(t)) + (\sin(t))\cdot(e^t)$ $\frac{(e^{-t})(-\sin(t)) - (\cos(t))\cdot(e^{-t})}{(e^{-t})^2}$

$\boxed{a.\ e^t \cos t + e^t \sin t - \sin t - \cos t}$

$G(u) = \frac{3-2u^3}{\tan(u)}$ Quotient $\frac{d}{dx}(3-2u^3)$

$\frac{(\tan u)\cdot(-6u^2) - (3-2u^3)\cdot(\sec^2 u)}{\tan^2 u}$

$\boxed{b.\ \frac{(-6u^2) - (3-2u^3)\cdot(\sec^2 u)}{\tan u}}$

$H(x) = -x^{2/3} + 5x^{-2} + e^{2x+1}$

$\frac{d}{dx}[-x^{2/3}] + \frac{d}{dx}[5x^{-2}] + \frac{d}{dx}[e^{2x+1}]$

$-2/3\, x^{-1/3}$ $-10x^{-3}$ $e^{2x+1} \cdot \frac{d}{dx}[2x+1]$

$2e^{2x+1}$

$\boxed{c.\ -\frac{2}{3}x^{-\frac{1}{3}} - 10x^{-3} + 2e^{2x+1}}$

4.

$\boxed{a.\ -2}$

$2f(g(x))$ $x=-1$ $g(-1)=4$ $f(4)=2$

$f'(g(-1))g'(-1)$ $f'(4)\ 1/2$ $-2\cdot 1/2$

$g(f(x))$ $x=1$ $f(1)=e$ $g(e)=\text{Undefined}$

$g'(f(1))f'(1)=$ $\boxed{b.\ \text{undefined}}$ $g'(e)\ 6$

$g(x)/f(x)$ $x=2$ $g(2)=-1$ $f(2)=4$ $-1/4$

$3f(x)g(x)$ $x=4$ $3(f(4)\cdot g'(4) + g(4)\cdot f'(4))$ $2 \cdot -1/2 + \pi \cdot -2$ $(-2\pi-1)3$

$\frac{f(2)\cdot g'(2) - f'(2)\cdot g(2)}{f(2)^2}$ $\frac{(4)\cdot(3) - (-5)\cdot(-1)}{16}$ $\frac{12-5}{16} = \frac{7}{16}$

$\boxed{c.\ 7/16}$

$-f(x) + [g(x)]^2 + \frac{\pi}{2}$ $x=1$ $\boxed{d.\ -6\pi-3}$

$f'(x) + 2g(x)\cdot g'(x) + 0$ $6 + 2\cdot 8 = 6+16 = 22$ $\boxed{e.\ -22}$

5. $xy + x^2y^2 + x^3 = 10$

$\boxed{a.\ (2)(-1) + (4)(1) + 8 = 10 \quad 8+4-2=10}$

$\frac{d}{dx}[xy] + \frac{d}{dx}[x^2y^2] + \frac{d}{dx}[x^3] = 0$

$y' + 2x\,2yy' + 3x^2 = 0$

$y' + 2x2yy' = -3x^2$ $y'(4xy) = -3x^2$

$\boxed{b.\ y' = \frac{-3x^2}{4xy}}$

$m = y' = \frac{3(2)^2}{4(2)(-1)}$ $\frac{12}{-8} = \frac{-3}{2}$ $\boxed{c.\ m = -3/2}$

d. $\boxed{y+1 = -3/2(x-2)}$

point-slope form