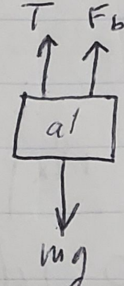
PHYS

1. As you go higher in air, it gets less dense. At some point, the air is as thin as helium and thus the balloon stops climbing

2. a.



$m = 0.054\,kg$ $\quad \Sigma F = 0 = T + F_b - mg$

$\rho_{al} = 2700\,kg/m^3$ $\quad T = mg - F_b$

$\rho_{ea} = 790\,kg/m^3$

$F_b = \rho_{ea} V_{al} g$

$V_{al} = \frac{m}{\rho_{al}}$

$T = mg - \rho_{ea} \frac{m}{\rho_{al}} g = \boxed{0.37\,N}$

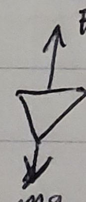
b.

decrease to $0.33\,N$

$\rho_c R^2 L^3 = \rho_w \frac{R^2 H^2}{L^2} H^3$

3.

$\rho_c L^3 = \rho_w H^3$



$\rho_c = \rho_c$ $\quad F_b = mg$ $\quad m = \rho_c V_c$

$V_c = \frac{1}{3}\pi R^2 L$ $\quad F_b = \rho_w V_s g$ $\quad \frac{H}{r} = \frac{L}{R}$

$V_s = \frac{1}{3}\pi r^2 H$ $\quad r = \frac{RH}{L}$

$\rho_w = 1000\,kg/m^3$ $\quad mg = \rho_w \frac{1}{3}\pi r^2 H g$

$\rho_c \frac{1}{3}\pi R^2 L = \rho_w \frac{1}{3}\pi r^2 H$

$\rho_c R^2 L = \rho_w r^2 H \rightarrow \rho_c R^2 L^2 = \rho_w \left(\frac{R^2 H^2}{L^2}\right) H$

$\frac{H^3}{L^3} = \frac{\rho_c}{\rho_w} \rightarrow \boxed{\frac{H}{L} = \sqrt[3]{\frac{\rho_c}{\rho_w}}}$

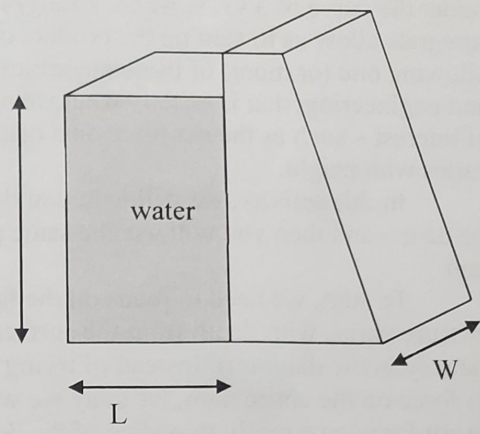
5. Summing Forces and Torques

*Name*____________________________

*Group*__________

A dam has been constructed to hold back water in a lake, and only release it when a port is opened. The water behind the dam has a height of H above the base of the dam, and the length of the lake away from the dam is L. The width of the dam is W.

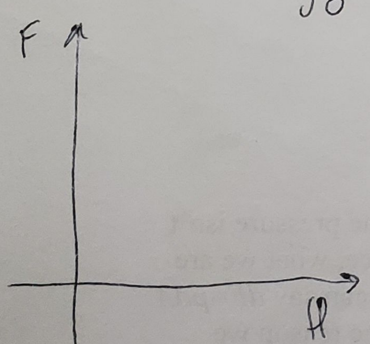
1. Does the force on the dam due to the water depend at all on the dimension L? Explain.



no ~~yes~~ because it only depends on the ~~[illegible]~~ water the dam is ~~[illegible]~~ touching

2. We want to know the net horizontal force on the dam exerted by the water. The area of the dam that has water touching it is HW (the height of the water times the width of the dam). The gauge pressure at the bottom of the water is going to be ρgH. Since force equals pressure times area, I could calculate the horizontal force to be that pressure times that area, or $\rho g H^2 W$. What is wrong with that calculation? Discuss and explain.

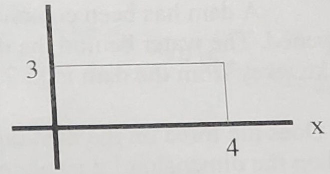
$\int_0^H dF$



$\rho g H^2 W$ only accounts for one height, not the entire dam

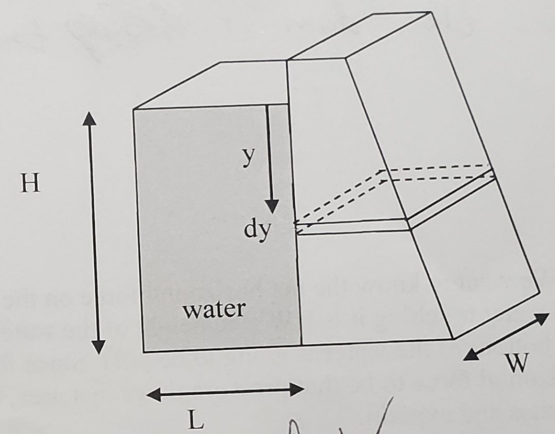
5. Summing Forces and Torques

Hopefully you realized that the problem with calculating the force that way is that the pressure at the bottom of the water is not the same as the pressure at other points on the dam. Because of this, you can't just multiply one particular value for the pressure times the area. We still want to multiply pressure times area, but allow for the fact that the pressure varies with height. To do that, we need to use an integral. This is something that is going to show up repeatedly in this course.



Integrals are really sums. For example, $\int_0^4 3dx$ sums up the product of 3*x, but where x varies from 0 to 4. This is the same as finding the area under the curve of 3 vs. x, where x ranges from 0 to 4 – so the area is 12. Integrals allow us to sum up the product of two (or more) quantities, while allowing one (or more) of those quantities to vary. Very often in science and engineering, that is exactly what we need to do to calculate something of interest – such as the net force on a dam due to the pressure in the water it is holding back, as that pressure varies with height.

In this activity, we will help you through this process of using an integral to calculate the net force on the dam – and then you will use the same process on your own to find the net torque around the base of the dam.



To start, we need to focus on the fact that the pressure varies with depth from the surface of the water (y in the diagram). Instead of trying to find the net force on the entire dam, let's say we want to find the net force on a really thin slice of the dam – a slice of a height dy (an infinitesimally thin height). Let's assume that this slice is thin enough that the pressure can be treated as essentially constant (so the entire slice of height dy can be treated as if it is all at depth y, rather than varying from depth y to y+dy).

3. What is the net horizontal force on the slice of the dam shown in the picture due to the additional pressure from the water (the slice with height dy and width W, a depth y below the surface)?

$\rho g y \cdot dy W$

$\underbrace{\rho g y}_{P}\ \underbrace{w\,dy}_{dA}$

If the pressure were constant, we could just find the net force simply as $F=pA$. But the pressure isn't constant, it increases with depth. This situation happens very often. To figure out the net force, what we are going to do is find the net force (dF) on a small slice of the dam with area dA, for which we can say $dF=pdA$.

That dA should be the area of the slice in problem 3 above, which should be Wdy. The reason we want to break the dam up into horizontal slices of a very small dimension in the vertical direction (dy) is because the pressure changes in the vertical direction (it depends on y) – so we need to integrate over that direction.

The net force on the entire dam, F, would then be found by summing up the forces on every thin little slice of which the dam is composed. So the net force will be $F = \int pdA = \int pWdy$.

Since y is our integration variable, our integration limits should be the range of y values for where the water is pushing against the dam. At the very top, y is zero. At the bottom, y is H.

The integral will sum up the pressure times area on one slice, plus pressure times area on the next slice, and so on.

4. Integrate to find the net force on the dam due to the water pushing against the left side.

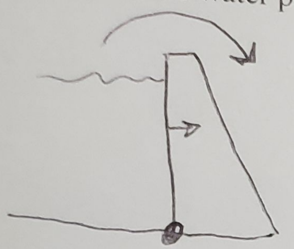
$$\int_0^H \rho g y \, W dy$$

$$\rho g \frac{1}{2} y^2 W \Big|_0^H$$

5. Check to see if your answer makes sense. How does your answer compare to the pressure I initially calculated by treating it as if the pressure over the entire dam was the same as the pressure at the bottom? I treated the pressure everywhere as ρgH and calculated a net force of $\rho g H^2 W$. Is your answer bigger or smaller than that? Does that make sense? Explain.

Its hard to tell without numbers but I think it should be smaller because pressure measured at the bottom is the highest and $\rho g H^2 W$ is for the whole dam meaning the whole dam expirences the highest pressure which $\rho g \frac{1}{2} y^2 W \Big|_0^H$ does not

6. Now use the same kind of process, but to determine the net *torque* on the dam around the base of the dam, due to the water. In other words, treat it as if the dam is pivoting around its base, and find the net torque on it due to the water pressure.



$F = pA$ $\quad r = H$

$\tau = F_{\perp} r$

$\tau = \int_0^H (\rho g y W dy) \cdot H$