ECE 602

1. $\ln(x) = f(x)$

$f'(x) = x^{-1}$ $\quad f''''(x) = -6x^{-4}$

$f''(x) = -x^{-2}$ $\quad f'''''(x) = 24x^{-5}$

$f'''(x) = 2x^{-3}$

$$f(x) = \frac{f(b)}{0!} + \frac{f'(b)(x-b)^1}{1!} + \frac{f''(b)(x-b)^2}{2!} + \frac{f'''(b)(x-b)^3}{3!} + \ldots\ldots$$

$$= \frac{0}{1} + \frac{(x-1)}{1} - \frac{(x-1)^2}{2} + \frac{2(x-1)^3}{6} - \frac{6(x-1)^4}{24} + \ldots$$

$$= \frac{(x-b)^1}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \ldots\ldots$$

$$\boxed{\ln(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-b)^{n+1}}{n+1}}$$

2. $$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \longrightarrow \lim_{n\to\infty} \left| \frac{\dfrac{(-1)^{n+1}(x-1)^{n+1+1}}{n+1+1}}{\dfrac{(-1)^n (x-1)^{n+1}}{n+1}} \right|$$

$$\frac{\cancel{(-1)}^{n+1}\cancel{(x-1)}^{n+1+1}}{n+1+1} \cdot \frac{n+1}{\cancel{(-1)^n}\cancel{(x-1)^{n+1}}} =$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(-1)^1}(x-1)(n+1)}{\underset{n+1}{(n+1+1)}} \right| = \lim_{n\to\infty} \left| (x-1) \cdot \left(\frac{\cancel{n+1}}{\cancel{n+2}}\right)^{1} \right|$$

$$\lim_{n\to\infty} |x-1| < 1 \longrightarrow \boxed{\text{converges when } 0 < x < 2}$$

% 3

```
syms x
f = log(x);

hold on

T1 = taylor(f, 'ExpansionPoint', 1, 'Order', 2);
T5 = taylor(f, 'ExpansionPoint', 1, 'Order', 6);
T10 = taylor(f, 'ExpansionPoint', 1, 'Order', 11);

fplot([T1 T5 T10 f])

xlim([0 2.2])
grid on
xlabel('x')
ylabel('f(x)')
legend('approximation of ln(x) with order 1', ...
       'approximation of ln(x) with order 5', ...
       'approximation of ln(x) with order 10', ...
       'ln(x)','Location','Best')
title('Taylor Series Expansion')

% As x approaches 0 and 2, the approximations fall
% away from ln(x)


% 4
threshold = 0.75   % check error at point here
feval = subs(f,x,threshold);

Teval = subs(T1,x,threshold);
err_T1=feval-Teval
stem(threshold, err_T1)

Teval = subs(T5,x,threshold);
err_T5=feval-Teval
stem(threshold, err_T5)

Teval = subs(T10,x,threshold);
err_T10=feval-Teval
stem(threshold, err_T10)


% 5
threshold = 2.5   % check error at point here
feval = subs(f,x,threshold);

Teval = subs(T1,x,threshold);
err_T1=feval-Teval
stem(threshold, err_T1)

Teval = subs(T5,x,threshold);
err_T5=feval-Teval
stem(threshold, err_T5)

Teval = subs(T10,x,threshold);
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err_T10=feval-Teval
stem(threshold, err_T10)

hold off


% 6
% As the number of terms increase, the error decreases as the lines get
% closer to ln(x). The stem plot didn't really work for me


% 7
f = sin(x);

T1 = taylor(f, 'ExpansionPoint', 0, 'Order', 2);
T5 = taylor(f, 'ExpansionPoint', 0, 'Order', 6);
T10 = taylor(f, 'ExpansionPoint', 0, 'Order', 11);

fplot([T1 T5 T10 f])

xlim([-2*pi 2*pi])
grid on
xlabel('x')
ylabel('f(x)')
legend('approximation of sin(x) with order 1', ...
       'approximation of sin(x) with order 5', ...
       'approximation of sin(x) with order 10', ...
       'sin(x)','Location','Best')
title('Taylor Series Expansion')

% As x approaches infinity, the errors also approache infinity

threshold = 1.5*pi   % check error at point here
feval = subs(f,x,threshold);

Teval = subs(T1,x,threshold);
err_T1=feval-Teval
%stem(threshold, err_T1)

Teval = subs(T5,x,threshold);
err_T5=feval-Teval
%stem(threshold, err_T5)

Teval = subs(T10,x,threshold);
err_T10=feval-Teval
%stem(threshold, err_T10)
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7. $f(x) = \sin x \qquad b=0$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$$f(x) = \frac{f(b)}{0!} + \frac{f'(b)(x-b)^1}{1!} + \frac{f''(b)(x-b)^2}{2!} + \frac{f'''(b)(x-b)^3}{3!} + \cdots$$

$$= \frac{\cancel{\sin(0)}^{0}}{1} + \frac{\cancel{\cos(0)}^{1}(x)^1}{1} + \frac{\cancel{-\sin(0)}^{0}(x)^2}{2} + \frac{\cancel{-\cos(0)}^{-1}(x)^3}{6} + \cdots$$

$$= \frac{0}{1} + \frac{1x}{1} - \frac{0}{2} + \frac{1x^3}{6} + \cdots$$

$$\boxed{\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$$

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} x^{2n+1+1}}{(2n+1+1)!}}{\frac{(-1)^n x^{2n+1}}{(2n+1)!}} \right| = \frac{\cancel{(-1)^{n+1}} \cancel{x}^{2n+1+1}}{(2n+1+1)!} \cdot \frac{(2n+1)!}{\cancel{(-1)^n} \cancel{x^{2n+1}}}$$

$$-x \,/\, 2n+2 \qquad \left|\frac{-x}{2n+2}\right|$$

$$\boxed{= \lim_{n\to\infty} \frac{x}{2n+2} = 0 < 1}$$

8. $f(x) = x^3 - 2x + 4$

$f'(x) = 3x^2 - 2$

$f''(x) = 6x$

$f'''(x) = 6$

$f''''(x) = 0$

$$f(x) = \frac{(x^3 - 2x + 4)}{0!} + \frac{(3x^2+2)(x-b)^1}{1!} + \frac{(6x)(x-b)^2}{2!} + \frac{6(x-b)^3}{3!} + 0 \ldots$$

$$\boxed{f(x) = x^3 - 2x + 4}$$

The Taylor Series of any polynomial is itself