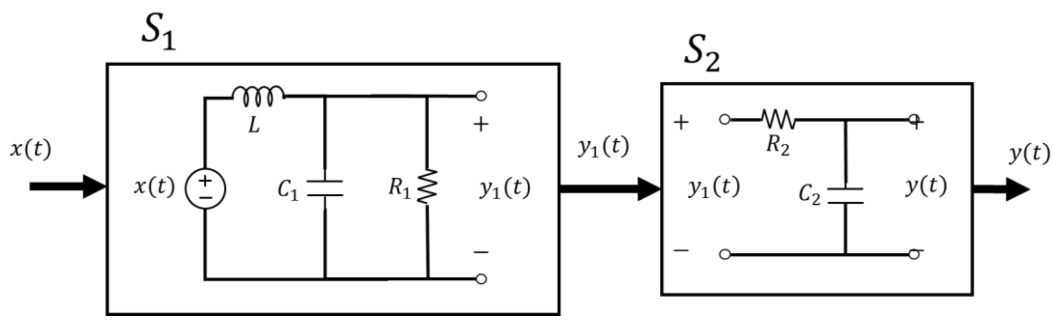
Homework # 5

<u>Instructions:</u> Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and box all your solutions. All Matlab code needs to be published, with your name and date at the top of the script, and all figures needs to have proper axis labeling and legends. Homework assignments will be collected during class time on the due date. *Late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **Homework problems not to be graded**
  - From textbook (Lathi):
    - Ch 2: 5-1
    - Ch 4: 1-3, 1-4
- **Homework problems to be graded**

1) Consider the following circuits connected in series. The input is the voltage $x(t)$, the output to system S_1 is the voltage $y_1(t)$, and the output of system S_2 is the voltage $y(t)$.

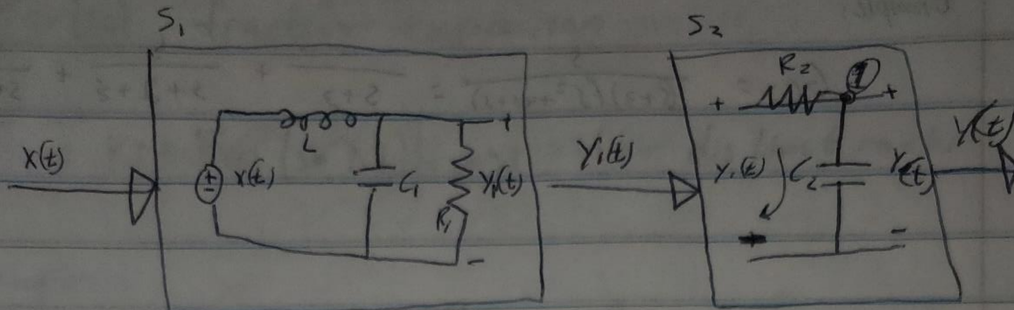


Let $L = 0.01, C_1 = 0.01, R_1 = 100, C_2 = 0.002$, and $R_2 = 50$.

a. Find the impulse response of system S_2.
b. Find the impulse response of the combined system S from $x(t)$ to $y(t)$, using only the impulse response of S_1 and S_2 (the impulse response of system S_1 was found in HW #4).
c. Find the zero-state response $y(t)$ of combined system S to the unit step input $x(t) = u(t)$. Show all your work.
d. Compare the responses $y_1(t)$ and $y(t)$ by plotting the signals in Matlab. Label all axes and include legends as needed to differentiate the signals. Note that S_2 is a low pass filter with bandwidth $^1/_{R_2C_2}$. What is the effect of adding S_2 on the system output $y(t)$ of the combined system S?

2) Determine the internal stability and external (BIBO) stability of the following systems (*do not assume observability and/or controllability*)
(a) $(D^2 + 3D + 2)y(t) = (D + 3)x(t)$
(b) $(D^2 + 3D + 2)y(t) = (D + 1)x(t)$
(c) $(D^2 + D - 2)y(t) = (D - 1)x(t)$

1.



$y_1(t) = u(t) + e^{-t/2}\cos(100t + 3.137)u(t)$

$C_2 = 0.002 \quad R_2 = 50$

$y(t) = V_{C_2} \qquad i_{C_2} = C_2 V'_{C_2} \qquad i_{C_2} = C_2 D V_{C_2}$

$V_{C_2} = \frac{i_{C_2}}{C_2 D} \qquad i_{R_2} = i_{C_2} = \frac{V_{R_2}}{R_2},\ V_{R_2} = i R_2$

KVL: $y_1(t) = iR_2 + V_{C_2}$

KCL: $\frac{V_{R_2}}{R_2} = C D y(t) \rightarrow V_{R_2} = R_2 C_2 D y(t)$

$\frac{y_1(t)}{x(t)} = R_2 C_2 D y(t) + y(t)$

$\left(D + \cancel{\frac{1}{R_2 C_2}}^{10}\right) y(t) = \left(\cancel{\frac{1}{R_2 C_2}}^{10}\right) x(t)$

$(D + 10)y(t) = 10x(t)$

root: -10 mode: e^{-10t} $\qquad h_2(t) = 10e^{-10t}$

poly: $y(t) = Ce^{-10t} \qquad y(0) = C = 1$

a) find impulse response of S_2

b) find impulse response of S_1 and S_2 combined

$h_1(t) = -100e^{-t/2}\cos(100t + \frac{\pi}{2})$

$h_2(t) = 10e^{10t}$

$y(t) = h_2(t) * y_1(t) = h_2(t) * (h_1(t) * x(t))$

$h(t) = \left(-100e^{-t/2}\cos(100t + \frac{\pi}{2})\right)\left(10e^{10t}\right)x(t)$

$= \boxed{-1000e^{10t - t/2}\cos(100t + \pi/2)\,x(t)}$

c) find zero-state response $y(t)$ of S

$$h(t-\tau) = -1000e^{9.5(t-\tau)} \cos\left(100(t-\tau) + \pi/2\right) u(t-\tau)$$

$$y(t) = \int_0^t h(t-\tau)\, d\tau$$

$$= -1000e^{9.5t} \int_0^t e^{9.5\tau} \cos\left(100(t-\tau) + \pi/2\right) d\tau$$

Convolution Integral #12: $\alpha = -9.5$, $\beta = 100$, $\theta = \pi/2$, $\lambda = 0$

$\phi = \tan^{-1}(-\beta/\alpha) = 1.48 \text{ rad}$

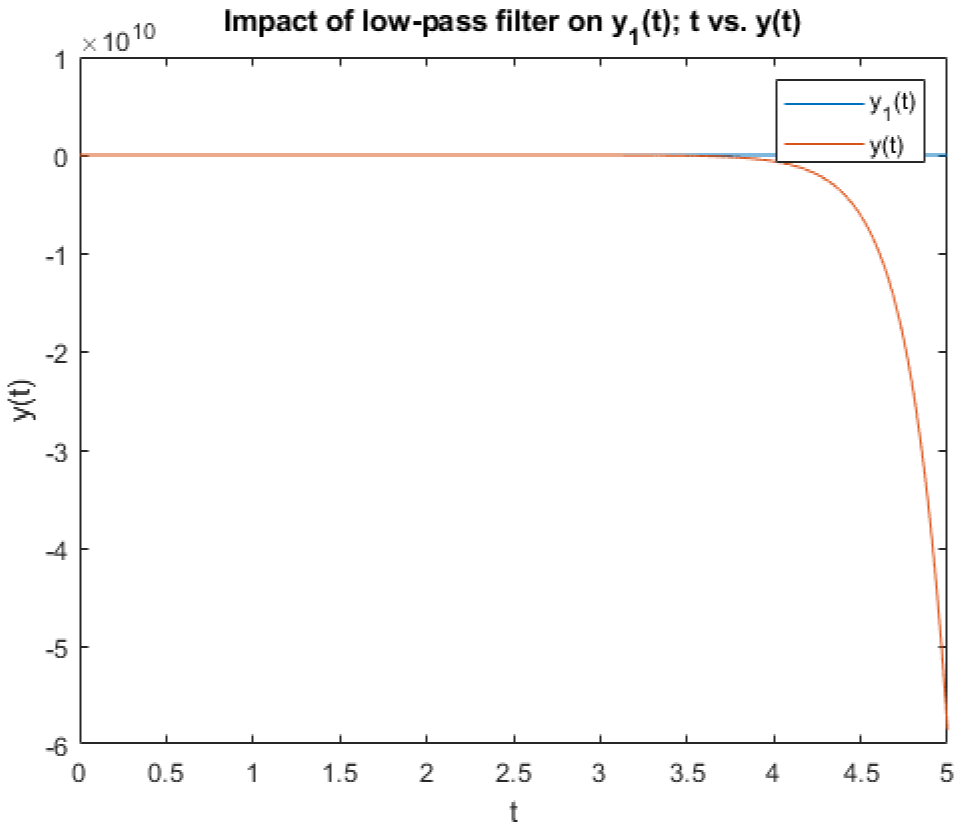
$$\frac{\cos\left(\frac{\pi}{2} - \phi\right) - e^{9.5t} \cos\left(100t + \frac{\pi}{2} - \phi\right)}{\underset{90.25}{\sqrt{\alpha^2}} + \underset{10000}{\beta^2}}$$

$$\left.\left(\frac{\cos(0.09) - e^{-9.5t}\cos(100t + 0.09)}{100.45}\right)\right|_0^t$$

$$\left(0.0099 - \frac{e^{-9.5t}\cos(100t + 0.09)}{100.45}\right)$$

$$y(t) = \left(-9.9e^{9.5t} + 10\cos(100t + 0.09)\right) u(t)$$

d)



The signal doesn't seem to oscillate at all anymore. It seems to oscillate when t<0 but go negative as soon as t>0. It also seems to drop off more starting at around t=4.

2.

2. Determine internal and external stability

a) $(D^2+3D+2)y(t) = (D+3)x(t)$

$(D+1)(D+2)$

$\lambda = -1, -2$ $\boxed{\text{Internally stable}}$

$y(t) = C_1 e^{-t} + C_2 e^{-2t}$ $\quad y(0) = C_1 + C_2 = 0 \quad C_1 = 1$

$y'(t) = -C_1 e^{-t} - 2C_2 e^{-2t}$ $\quad y(0) = -C_1 - 2C_2 = 1 \quad C_2 = -1$

$y(t) = e^{-t} - e^{-2t}$ $\quad h(t) = (e^{-t} - e^{-2t})u(t)$

$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{0}^{\infty} |e^{-t} - e^{-2t}|\,dt$

$0 \quad 0$ $\boxed{\text{BIBO stable}}$

b) $(D^2+3D+2)y(t) = \cancel{(D+1)}x(t)$

$\cancel{(D+1)}(D+2)$ $\quad \lambda = -2$ $\boxed{\text{Internally Stable}}$

$y(t) = Ce^{-2t}$ $\quad y(0) = C = 1$ $\quad y(t) = e^{-2t}$

$h(t) = e^{-2t}u(t)$ $\quad \int_0^\infty |e^{-2t}|\,dt$

0 $\boxed{\text{BIBO stable}}$

c) $(D^2+D-2)y(t) = \cancel{(D-1)}x(t)$

$\cancel{(D-1)}(D+2)$ $\quad \lambda = -2$ $\boxed{\text{Internally stable}}$

$\boxed{\text{BIBO stable}}$