Chapters 5: Multiple Random Variables

Introduction (5.0)

- **We consider** experiments that produce a collection of random variables $X_1, X_2, \cdots, X_n$
  - Often interested in the relationship between the RVs (covariance & correlation)
  - *Note*: We will mainly focus on $n = 2$ (bivariate), but results can be generalized to higher n's
  - Probability model is in the ***joint cumulative distribution function*** (joint PMF or joint PDF)

Joint Cumulative Distribution Function (5.1)

Definition: the *joint cumulative distribution function* of random variables *X* and *Y* is

$$F_{X,Y}(x, y) =$$

- Properties of joint CDF: for random variables X, Y

  a)

  b)

  c)

  d)

  e)

  f)

  g) If $x \leq x_1$ and $y \leq y_1$, then

Joint PMF and Joint PDF (5.2,5.4)

Joint PDF and PMF as more commonly used to study probability models.

<u>Definition:</u> the joint probability mass function for discrete RVs X and Y is

<u>Definition:</u> the joint probability density function for continuous RVs X and Y is a function $f_{X,Y}(x, y)$ with the property

- <u>Probability of events</u>: for an event B, $(X, Y) \in B$,
  - Discrete:
  - Continuous
- <u>Example</u>:
  - $P[x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2]$
  - $P[x_1 \leq X \leq x_2, Y \leq y_2]$
  - $P[X + Y \leq 3]$
  - $P[X^2 + Y^2 \leq 9]$

- Properties of joint PMFs & PDFs:
  - Axiom 1:
  - Axiom 2:
- Example: roll 2 dice and define $X = \#\text{ of die }1$ and $Y = \#\text{ of die }2$. Construct the bivariate PMF
- Example: consider an experiment that can be described by the following bivariate PDF f_{XY}. What is $P[1 \leq X \leq 3, 1 \leq Y \leq 3]$?

- Example: consider the following bivariate PDF f_{XY}. Determine $P[X > Y]$

  - Determine the CDF

- <u>Marginal CDF & PDF</u>: individual PMFs & CDFs can be extracted from the <u>joint</u> PMFs and PDFs
  - Discrete:
  - Continuous

- Example: find the marginal PDF f_X and f_Y for the previous example

Independent Random Variable (5.6)

Definition: random variables X and Y are independent if and only if

- Discrete: $P_{X,Y}(x, y) =$
- Continuous $f_{X,Y}(x, y) =$

- Example: consider the following joint PDF. Determine of the random variables are independent
  - $f_{X,Y} = e^{-(x+y)}, x \geq 0, y \geq 0$

- $f_{X,Y}(x,y) = 2e^{-x}e^{-y}, 0 \le y \le x \le \infty$

Note: the region in which the PDF is defined will play a critical role in the assessment of independence

Recall: integral over dependent variables. E.g., calculate $\mathrm{P}[X + Y \le 1]$

- $f_{X,Y}(x,y) = \frac{1}{4}, -1 \leq x, y \leq 1$

- $f_{X,Y}(x,y) = \frac{1}{\pi}, x^2 + y^2 \leq 1$

Expected Value of Function of two RV (5.7)

Definition: for random variables X and Y, the expected value of W=g(X,Y) is

- Discrete: $\mathrm{E}[W] =$

- Continuous: $\mathrm{E}[W] =$

- Expected value of $X + Y$ (*Thm 5.11*): for two random variables *X* and *Y*

$$\mathrm{E}[X + Y] =$$

- Variance of $X + Y$ *(Thm 5.12)*: for two random variables *X* and *Y*

$$\mathrm{Var}[X + Y] =$$

Covariance of X and Y (5.8)

Definition: *covariance* of two random variables X and Y is

$$\mathrm{Cov}[X, Y] =$$

- By Thm 5.9:

- Note: $\mathrm{Cov}[X, Y]$ is a parameter of the joint distribution
  Typical calculation:

- Example: find the covariance for RVs with the following joint PDF

- Note: covariance describes the distribution of the random variables about their expected values