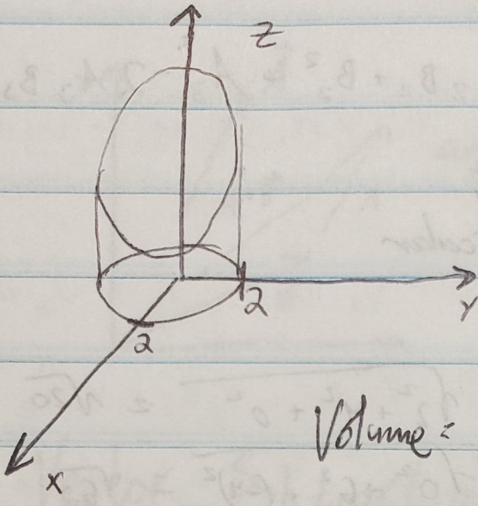
HW 4

1. find volume bounded by $x^2+y^2=4$, $z=0$, $x+z=3$



$x^2+y^2=4 \rightarrow r^2=4,\ r=2$

$z=0 \rightarrow z=0$

$x+z=3 \rightarrow r\cos\phi+z=3$

$$\text{Volume} = \int_0^{2\pi}\int_0^2\int_0^{3-r\cos\phi} r\,dz\,dr\,d\phi$$

$$= \int_0^{2\pi}\int_0^2 (3-r\cos\phi)\,r\,dr\,d\phi$$

$$= \int_0^{2\pi}\int_0^2 (3r-r^2\cos\phi)\,dr\,d\phi$$

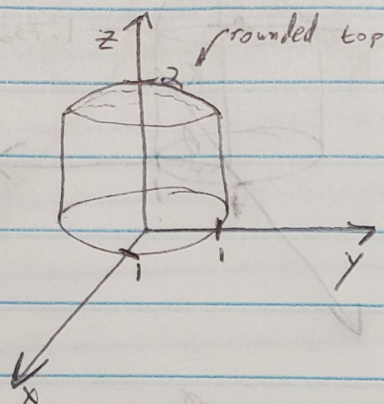
$$= \int_0^{2\pi}\left(\frac{3}{2}r^2-\frac{r^3}{3}(\cos\phi)\right)\Bigg|_0^2 d\phi$$

$$= \int_0^{2\pi} 6-\frac{8}{3}\cos\phi\,d\phi$$

$$= \left(6\phi-\frac{8}{3}\sin\phi\right)\Bigg|_0^{2\pi} = \boxed{12\pi}$$

2. set up integral for volume bounded by $x^2+y^2+z^2=4$, $z=0$, $x^2+y^2=1$

~~evaluate triple integral~~ for a) $dz\,dr\,d\phi$, b) $dr\,dz\,d\phi$, solve for c) $d\phi\,dz\,dr$



$$a)\ V=\int_0^{2\pi}\int_0^1\int_0^{z=\sqrt{4-x^2-y^2}} r\,dz\,dr\,d\phi$$

$$b)\ V=\int_0^{2\pi}\int_0^{\sqrt{4-x^2-y^2}}\left(\int_0^1+\int_1^2\right) r\,dr\,dz\,d\phi$$

$$c)\ V=\left(\int_0^1+\int_0^{r=4-r(\cos\phi+\sin\phi)+z^2}\right)\int_0^2\int_0^{2\pi} r\,d\phi\,dz\,dr$$

$$=\int_0^1+\int_0^{4-r(\cos\phi+\sin\phi+z^2}\int_0^2 2\pi r\,dz\,dr$$

$$=\int_0^1+\int_0^{4-r(\cos\phi+\sin\phi)+z^2} 4\pi r\,dr$$

$$=4\pi\left(\int_0^1 r\,dr+\int_0^{4-r(\cos\phi+\sin\phi)+z^2} r\,dr\right)$$

$$\frac{r^2}{2}\Big|_0^1=\frac{1}{2}$$

$$\boxed{4\pi\left(\frac{1}{2}+\frac{(4-r(\cos\phi+\sin\phi)+z^2)^2}{2}-4\right)}$$

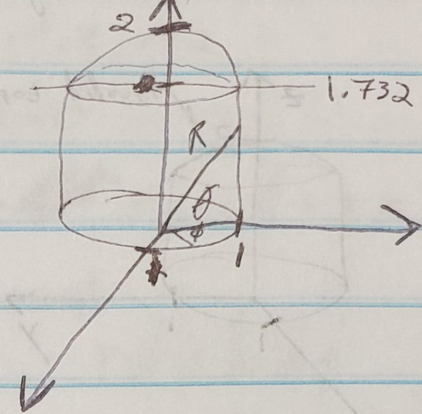
3. $x^2+y^2+z^2=4$, $z=0$, $x^2+y^2=1$

$dV = R^2 \sin\theta \, d\theta \, d\phi \, dR$

$R=\sqrt{x^2+y^2+z^2}$

$\theta = \cos^{-1} \frac{z}{R}$

$\phi = \tan^{-1} y/x$



$$\boxed{V=\int_0^{2\pi}\int_0^{\pi/2}\int_{R=\csc\theta}^{R=\frac{2\csc\theta}{\sqrt{1+\cos^2\theta}}} dR\,d\theta\,d\phi}$$

$x=R\sin\theta\cos\phi \quad y=R\sin\theta\sin\phi \quad z=R\cos\theta$

$(R\sin\theta\cos\phi)^2+(R\sin\theta\sin\phi)^2=1$ $\quad 1$

$1=R^2\sin^2\theta\cos^2\phi + R^2\sin^2\theta\sin^2\phi,\quad R^3\sin^2\theta\left(\cancel{\cos^2\phi+\sin^2\phi}\right)=1$

$R^2\sin^2\theta=1,\ R=\frac{1}{\sqrt{\sin^2\theta}} \rightarrow R=\csc\theta$

$(R\sin\theta\cos\phi)^2+(R\sin\theta\sin\phi)^2+(R\cos\theta)^2=4$

$R^2\sin^2\theta\cos^2\phi+R^2\sin^2\theta\sin^2\phi+R^2\cos^2\theta=4$

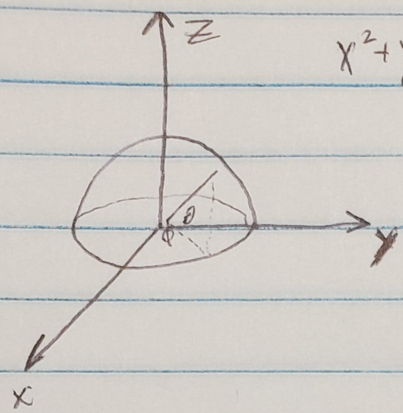
$R^2\sin^2\theta\left(\cancel{\cos^2\phi+\sin^2\phi}+\cos^2\theta\right)=4$

$\sqrt{R^2\sin^2\theta\left(1+\cos^3\theta\right)}=\sqrt{4}$

$R\sin\theta\sqrt{1+\cos^2\theta}=2$

$R=\frac{2\csc\theta}{\sqrt{1+\cos^2\theta}}$

4.



$x^2+y^2+z^2=1$, $z=0$ where f(x,y,z)=2 ???

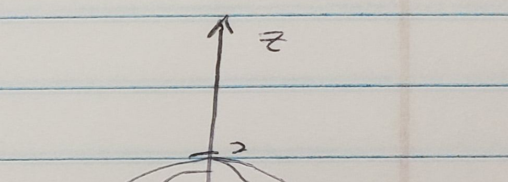
$$V=\int_0^{\phi=2\pi}\int_0^{\theta=\pi/2}\int_0^{R=1} R^2\sin\theta\, dR\, d\theta\, d\phi$$

$$=\int_0^{2\pi}\int_0^{\pi/2}\frac{1}{3}\sin\theta\, d\theta\, d\phi$$

$$-\frac{1}{3}\cos\theta\Big|_0^{\pi/2} \rightarrow 0+1/3$$

$$=\int_0^{2\pi} 1/3\, d\phi \rightarrow \frac{1}{3}\phi\Big|_0^{2\pi} \rightarrow \boxed{2\pi/3}$$

5. $x^2+y^2+z^2=4$, $z=0$, $x^2+y^2+(z-1)^2=1$



$$V=\left(\int_0^{\phi=2\pi}\int_0^{\theta=\pi/2}\int_0^{R=2} R^2\sin\theta\, dR\, d\theta\, d\phi\right)-$$

$$\cancel{\left(\int_0^{\phi=2\pi}\int_0^{\theta=\pi}\int_0^{R=1} R^2\sin\theta\, dR\, d\theta\, d\phi\right)}\ \frac{4\pi}{3}$$

$$\int_0^{2\pi}\int_0^{\pi/2}\frac{\sin\theta R^3}{3}\Big|_0^{2} d\theta\, d\phi=\int_0^{2\pi}\int_0^{\pi/2}\frac{8\sin\theta}{3}\, d\theta\, d\phi=\int_0^{2\pi}-\frac{8}{3}\cos\theta\Big|_0^{\pi/2} d\phi$$

$$=\int_0^{2\pi}\frac{8}{3}\, d\phi=\frac{8}{3}\phi\Big|_0^{2\pi}=\frac{16\pi}{3}-\frac{4\pi}{3}=\frac{12\pi}{3}=\boxed{4\pi}$$