

7.5 4.

$$\int \frac{\sin^3 x}{\cos x} dx$$

~~$$u = \sin x \quad du = \cos x dx$$~~

$$dx = \frac{1}{\cos x} du$$

$$\int \sin^2 x \tan x dx \quad \int (1 - \cos^2 x) \tan x dx \quad \int \tan x - \cos x \sin x dx$$

$$\int \tan x - \int \sin x \cos x dx \quad -\ln |\cos x| - \frac{\sin^2 x}{2} + C$$

$$u = \sin x \quad du = \cos x dx \quad dx = \frac{du}{\cos x} \quad \int u du \rightarrow \frac{u^2}{2}$$

$$18. \int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}} dt \quad u = \sqrt{t} \rightarrow t = u^2 \quad du = \frac{1}{2} t^{-\frac{1}{2}} dt \quad dt = 2\sqrt{t} du$$

$$2 \int e^u du \quad 2e^u \quad 2e^{\sqrt{t}} \Big|_1^4 \quad 2e^2 - 2e$$

38.

$$\int_{\pi/6}^{\pi/3} \frac{\sin \theta \cot \theta}{\sec \theta} d\theta \quad \int \frac{\sin \theta \cos^2 \theta}{\sin \theta} d\theta \quad \int \cos^2 \theta d\theta$$

$$\frac{1}{2} \int 1 d\theta + \frac{\cos \theta \sin \theta}{2} \quad \frac{1}{2} \theta + \frac{\cos \theta \sin \theta}{2} \Big|_{\pi/6}^{\pi/3} \quad \frac{4\pi + 3^{3/2}}{24} - \frac{2\pi + 3^{3/2}}{24} = \frac{\pi}{12}$$

$$7.8 \ 28. \int_0^5 \frac{1}{\sqrt[3]{5-x}} dx \quad u = 5-x \quad du = -1 dx \quad dx = -du$$

$$\int \frac{-1}{\sqrt[3]{u}} du \quad -\int u^{-\frac{1}{3}} du \quad -\frac{3}{2} u^{\frac{2}{3}} \quad -\frac{3}{2} (5-x)^{\frac{2}{3}} \Big|_0^5 \quad \frac{2}{3} \sqrt[3]{5}$$

$$30. \int_{-1}^2 \frac{x}{(x+1)^2} dx \quad \int \frac{x}{x^2+2x+1} dx \quad u = x+1 \quad du = 1 dx$$

$$\int \frac{u-1}{u^2} du \quad \int \frac{u}{u^2} - \int \frac{1}{u^2} du \quad \int \frac{1}{u} - \int \frac{1}{u^2} du$$

$$\ln u + \frac{1}{u} \quad \ln |x+1| + \frac{1}{x+1} \Big|_{-1}^2 \quad (\ln 3 + \frac{1}{3}) - (\ln 0 + 0) \quad \text{DNE}$$

34.

$$\int_0^5 \frac{w}{w-2} dw$$

~~$$u = w \quad v = \ln(w-2) \quad du = 1 dw \quad dv = \frac{1}{w-2} dw$$~~

$$u = w-2 \quad du = 1 dw$$

$$\int \frac{u-2}{u} du \rightarrow \int \frac{u}{u} - \int \frac{2}{u} du$$

$$\int 1 du - 2 \int \frac{1}{u} du \quad u - 2 \ln |u| \quad w-2 - 2 \ln |w-2| \Big|_0^5$$

$$(3 - 2 \ln 3) - (-2 + 2 \ln 2) \quad 3 - 2 \ln 3 + 2 - 2 \ln 2 \quad 5 - 2(\ln 3 + \ln 2)$$

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38.

$$\int_0^{\frac{\pi}{2}} \frac{\cos \theta}{\sqrt{\sin \theta}} d\theta$$

$$u = \sin \theta \quad du = \cos \theta d\theta \quad dx = \frac{du}{\cos \theta}$$

$$-\frac{1}{2} + \frac{2}{2} = \frac{1}{2}$$

$$\int \frac{1}{\sqrt{u}} du \rightarrow \int u^{-\frac{1}{2}} du$$

$$2u^{\frac{1}{2}} \quad 2\sqrt{\sin \theta} \Big|_0^{\frac{\pi}{2}}$$

$$2\sqrt{\frac{\sqrt{2}}{2}}$$

40.

$$\int_0^1 \frac{e^{y_x}}{x^3} dx$$

$$u = 1/x \quad du = -1/x^2 dx \quad dx = -x^2 du$$

~~Integration by parts~~

$$\int \frac{e^u}{x} du \rightarrow -\int u e^u du$$

$$u = 1/x \quad v = e^u$$

$$du = -1/x^2 \quad dv = e^u$$

$$u e^u - \int e^u \quad u e^u - e^u \quad \frac{e^{y_x}}{x} - e^{y_x} \Big|_0^1$$

$$(e - e) - \left(\frac{e^{y_0}}{0} - e^{y_0}\right) \text{ DNE}$$

50.

$$\int_1^{\infty} \left(\frac{1}{\sqrt{x}} + \frac{\sin^2 x}{\sqrt{x}} \right) dx$$

$$0 \leq \sin^2 \leq 1$$

$$\frac{1}{x^{\frac{1}{2}}} \geq \frac{\sin^2 x}{\sqrt{x}}$$

$$P < 1$$

diverges

54.

$$\int_0^{\pi} \frac{\sin^2 x}{\sqrt{x}} dx$$

$$\lim_{t \rightarrow 0^+} 2\sqrt{\pi} - 2\sqrt{t}$$

$$2\sqrt{\pi}$$

converges