Student Name:

Homework # 7

Instructions: Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and box all your solutions. All Matlab code needs to be published, and all figures needs to have proper axis labeling and legends. Homework assignments will be collected during class time on the due date. *No late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **Homework problems not to be graded**
  - From textbook:
    - Ch 5: 2.2, 3.2, 4.1, 4.2, 5.2, 6.2, 7.12

- **Homework problems to be graded**

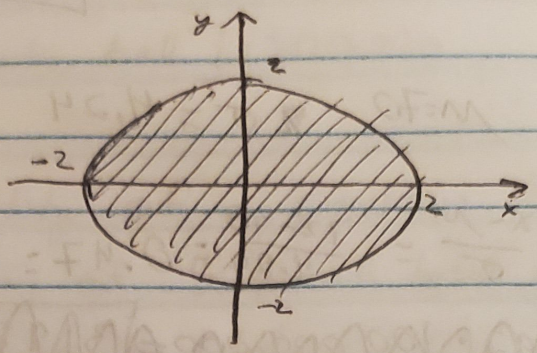
Consider random variables X and Y with join PDF,

$$f_{X,Y}(x,y) = \begin{cases} |xy|/c, & \text{for } x^2 + y^2 \leq 2^2, 0 \leq y \\ 0, & \text{otherwise} \end{cases}$$

a) Sketch the region in the X-Y plane for which the PDF is nonzero.
b) Determine the value of c for $f_{X,Y}$ to be a valid PDF.
c) Determine the marginal PDFs f_X and f_Y.
d) Compute the probability $\mathrm{P}[Y \leq 1]$.

$$f_{X,Y}(x,y) = \begin{cases} |xy|/c & x^2+y^2 \le 2^2,\ 0 \le y \\ 0 & \text{otherwise} \end{cases}$$

a)



b) $c \neq 0$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_{0}^{2} \frac{|xy|}{c}\,dy$$

$$= \pm \frac{x}{c}\, y^2/2 \Big|_0^2 = \pm \frac{2x}{c}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \int_{-\sqrt{4-y^2}}^{+\sqrt{4-y^2}} \frac{|xy|}{2}\,dx$$

c) $$= \frac{(4-y^2)|y|}{c}$$

$$P[Y \le 1] = \int_{-\sqrt{4-y^2}}^{+\sqrt{4-y^2}} \int_0^1 \frac{|xy|}{c}\,dx\,dy$$

$$= \int_0^1 \frac{(4-y^2)|y|}{c}\,dy$$

d) $$= 7/4c$$