7.3 4.

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx \Rightarrow \int \frac{x^2}{3-x}\,dx$$

$$\begin{array}{r} -x+3 \\ \hline \end{array}$$

$$\begin{array}{r} -x-3 \\ \hline 1x^2+0x+0 \end{array}$$

$$\Rightarrow -x-3+\frac{9}{3-x}$$

$$\underline{1x^2-3x}\downarrow$$
$$0\;+3x+0$$
$$\underline{3x-9}$$
$$0\;+9$$

$$\int -x-3+\int \frac{9}{3-x}\,dx$$

$$\boxed{-\tfrac{1}{2}x^2-3x+\;9\ln|3-x|+C}$$

8.

$$\int \frac{dt}{t^2\sqrt{t^2-16}} = \int \frac{1}{t^2(t-4)}\,dt = \int \frac{1}{t^3-4t^2}\,dt$$

$$\int (t^3-4t^2)^{-1}\,dt \qquad \boxed{\ln(t^3-4t^2)+C}$$

16.

$$\int_{\sqrt{2/3}}^{2/3} \frac{1}{x^5\sqrt{9x^2-1}}\,dx = \int \frac{1}{x^5(3x-1)}\,dx = \int \frac{1}{3x^6-x^5}\,dx$$

$$u=3x^6 \qquad du=18x^5\,dx \qquad dx=\frac{du}{18x^5} \qquad \frac{du}{18}=x^5\,dx$$

$$\int (u-1)^{-1}\,du \qquad \int \frac{1}{u-1}\,du \qquad \ln|u-1| \qquad \boxed{\ln|3x^6-1|+C}\Big|_{\sqrt{2/3}}^{2/3}$$

20.

$$\int \frac{x}{\sqrt{1+x^2}}\,dx \qquad \int \frac{x}{\sqrt{1+x}}\,dx \qquad u=\sqrt{x^2+1} \qquad \frac{du}{dx}=2x$$

$$dx=\frac{1}{2x}\,du \qquad =\int \frac{1}{2\sqrt{u}}\,du = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-\frac{1}{2}}\,du \qquad \frac{1}{2}\cdot 2u^{\frac{1}{2}} \qquad \frac{1}{2}\sqrt{u}$$

$$\frac{u^{-\frac{1}{2}+\frac{2}{2}=\frac{1}{2}}}{-\frac{1}{2}+\frac{2}{2}=\frac{1}{2}} \qquad \frac{u^{1/2}}{\tfrac{1}{2}} \qquad 2\sqrt{u} \qquad \tfrac{1}{2}(2\sqrt{u}) \qquad \sqrt{u}=\boxed{\sqrt{1+x^2}}$$

26. $\displaystyle\int \frac{x^2}{(3+4x-4x^2)^{3/2}} \, dx$

$\dfrac{x^2}{(\sqrt{3+4x-4x^2})^{3}}$

$3+(4x-4x^2)$

$3+4(x-x^2)$

$3+4(1+(x-x^2+1))$

$\displaystyle\int \frac{x}{3+4x-4x^2} \, dx$

$-4x\left(x + \tfrac{1}{2}\right)\left(x - \tfrac{3}{2}\right)$

$-4x^2+4x+3 \qquad -4\left(x^2-x-\tfrac{3}{4}\right)$

$-4(x-0.5)^2 + 9$

$\cancel{(4x^2-4x+3)}$

$\cancel{-4x^2}+4x+3 = -4\left(x-\tfrac{1}{2}\right)^2 + 9$

$u = x - \tfrac{1}{2} \quad du = dx$

$x = u + \tfrac{1}{2}$

$\displaystyle\int \frac{x}{3+4x-4x^2} \, dx = \int \frac{u+\tfrac{1}{2}}{-4u^2+9} \, du$

30. $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}} \, dt = \int \frac{\cos t}{1+\sin t} \, dt$

$\cos^2 + \sin^2 = 1 \qquad \cos^2 = 1 - \sin^2$

$u = \sin x \quad dx = \frac{1}{\cos x} \, du$

$\displaystyle\int \frac{1}{\sqrt{u^2+1}} \, du$

7.4 2.

$$\frac{x-6}{x^2+x-6} \qquad \frac{x-6}{(x+3)(x-2)}$$

$$\frac{Ax-6}{x+3} + \frac{Bx-6}{x-2}$$

8. $\int \frac{3t-2}{t+1} \, dt$