a) $r \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \cancel{C} \end{bmatrix} = \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} \cancel{C} \end{bmatrix} \to r \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} B \end{bmatrix}$

$2 \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

b) $\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} C^{-1} \end{bmatrix} \begin{bmatrix} P \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} P^{-1} \end{bmatrix} \begin{bmatrix} C \end{bmatrix}$

$A C^{-1} = C^{-1} P B P^{-1} \cancel{C C^{-1}}^{I}$ $\quad [C^{-1}][P] = [C^{-1}P]$

$A C^{-1} P = C^{-1} P B \cancel{P^{-1} P}^{I}$ $\quad [C^{-1}P]^{-1} = [P^{-1}C]$

$P^{-1} C A C^{-1} P = B$

c) $C(AB + rX) B A^{-1} = I_n$ $\quad AB + rX = C^{-1} I A B^{-1}$

$C(AB + rX) B = IA$ $\quad rX = C^{-1} I A B^{-1} - AB$

$C(AB + rX) = IAB^{-1}$ $\quad X = \frac{1}{r} C^{-1} I A B^{-1} - AB$

d) $\underbrace{(A - AX)^{-1}}_{\text{invertable}} = \underbrace{X^{-1}}_{\text{invertable}} B \to X(A - AX)^{-1} = B$ the product of invertable matrices is invertable

$X = B(A - AX) \to B^{-1} X = A - AX \to B^{-1} X = A(I - X)$

$\to A^{-1} B^{-1} X = I - X \to A^{-1} B^{-1} = (I - X) X^{-1} \to A^{-1} B^{-1} = X^{-1} - \cancel{X X^{-1}}^{I}$

$\to A^{-1} B^{-1} + I = X^{-1} \to X(A^{-1} B^{-1} + I) = I \to X = (A^{-1} B^{-1} + I)^{-1}$

. yes, Theorm 8 g states $Ax = b$ has at least one solution for each b in $\mathbb{R}^n$. This is because the columns of C span $\mathbb{R}^6$

3.

a)

```
>> D = (eye(7) - C)

D =

    0.8412   -0.0064   -0.0025   -0.0304   -0.0014   -0.0083   -0.1594
   -0.0057    0.7355   -0.0436   -0.0099   -0.0083   -0.0201   -0.3413
   -0.0264   -0.1506    0.6443   -0.0139   -0.0142   -0.0070   -0.0236
   -0.3299   -0.0565   -0.0495    0.6364   -0.0204   -0.0483   -0.0649
   -0.0089   -0.0081   -0.0333   -0.0295    0.6588   -0.0237   -0.0020
   -0.1190   -0.0901   -0.0996   -0.1260   -0.1722    0.7632   -0.3369
   -0.0063   -0.0126   -0.0196   -0.0098   -0.0064   -0.0132    0.9988

>> xx_vect = rref([D d])

xx_vect =

   1.0e+05 *

    0.0000         0         0         0         0         0         0    0.9958
         0    0.0000         0         0         0         0         0    0.9770
         0         0    0.0000         0         0         0         0    0.5123
         0         0         0    0.0000         0         0         0    1.3157
         0         0         0         0    0.0000         0         0    0.4949
         0         0         0         0         0    0.0000         0    3.2955
         0         0         0         0         0         0    0.0000    0.1384
```

b)

```
>> d=[98756;83627;17625;12220;76534;5463;9677];
>> xx_vect = rref([D d])

xx_vect =

   1.0e+05 *

    0.0000         0         0         0         0         0         0    1.2735
         0    0.0000         0         0         0         0         0    1.3280
         0         0    0.0000         0         0         0         0    0.7085
         0         0         0    0.0000         0         0         0    1.1673
```

0	0	0	0	0.0000	0	0	1.3229
0	0	0	0	0	0.0000	0	1.0856
0	0	0	0	0	0	0.0000	0.1699

c)

Program:

```
load ps3no1.mat;

C
xk=0;
for k=0:16
    k
    xk=d+C*xk
end
```

Output after 16 iterations:

k =

16

xk =

1.0e+05 *

0.9958	0.9958	0.9958	0.9958	0.9958	0.9958	0.9958
0.9770	0.9770	0.9770	0.9770	0.9770	0.9770	0.9770
0.5123	0.5123	0.5123	0.5123	0.5123	0.5123	0.5123
1.3157	1.3157	1.3157	1.3157	1.3157	1.3157	1.3157
0.4949	0.4949	0.4949	0.4949	0.4949	0.4949	0.4949
3.2955	3.2955	3.2955	3.2955	3.2955	3.2955	3.2955
0.1384	0.1384	0.1384	0.1384	0.1384	0.1384	0.1384

Not sure why xk is shown as a 7x7 matrix instead of a column vector

4.

$$\int_0^\infty dx_1\,dx_2\,dx_3\, e^{-(x_1^2+4x_1x_2+4x_1x_3+7x_2^2+14x_2x_3+19x_3^2)}$$

$$\int_0^\infty\int_0^\infty\int_0^\infty e^{-x^2}\cdot e^{-4xy}\cdot e^{-4xz}\cdot e^{7y^2}\cdot e^{14yz}\cdot e^{19z^2}\,dx\,dy\,dz$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_{z=0}^{z=\infty} e^{19z^2}\int_{y=0}^{y=\infty} e^{7y^2}\cdot e^{14yz}\int_{x=0}^{x=\infty} e^{-x^2}\cdot e^{-4xy}\cdot e^{-4xz}\,dx\,dy\,dz$$

$$e^{-x^2-4xy-4xz} \Rightarrow e^{-x(x+4y+4z)}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}\underbrace{\begin{bmatrix} +1 & +4 & +4 \\ +4 & +7 & +14 \\ +4 & +14 & +19 \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}\begin{bmatrix} +x_1+4x_2+4x_3 \\ +4x_1+7x_2+14x_3 \\ +4x_1+14x_2+19x_3 \end{bmatrix}$$

$$\begin{array}{cccc} U & L & D & L^T \\ \begin{bmatrix} 1 & 4 & 4 \\ 0 & -9 & -2 \\ 0 & 0 & 31/9 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 4 & 2/9 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & 31/9 \end{bmatrix} & \begin{bmatrix} 1 & 4 & 4 \\ 0 & 1 & 2/9 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 4 & 2/9 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & 31/9 \end{bmatrix}\begin{bmatrix} 1 & 4 & 4 \\ 0 & 1 & 2/9 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1+4x_2+4x_3 & x_2+\frac{2}{9}x_3 & x_3 \end{bmatrix}$$

$$1(x_1+4x_2+4x_3)$$
$$-9(x_2+2/9\,x_3)$$
$$31/9(x_3)$$

(i) $\underbrace{(x_1+4x_2+4x_3)^2}_{y_1} \underbrace{-9\left(x_2+\frac{2}{9}x_3\right)^2}_{y_2} + \underbrace{31/9(x_3)^2}_{y_3}$

(ii) $y_1 = x_1+4x_2+4x_3 \quad y_2 = x_2+\frac{2}{9}x_3 \quad y_3 = x_3$

$$\int_0^\infty dy_1\, dy_2\, dy_3\, |J|\, e^{-(\alpha y_1^2 + \beta y_2^2 + \gamma y_3^2)}$$

(iii) $$\int_0^\infty dy_1\, dy_2\, dy_3\, |J|\, e^{-(1 y_1^2 - 9 y_2^2 + 31/9\, y_3^2)}$$

$$\int_0^\infty e^{-y_1^2}\, dy_1 \int_0^\infty e^{+9 y_2^2}\, dy_2 \int_0^\infty e^{-31/9\, y_3^2}\, dy_3$$

$$\frac{1}{2}\sqrt{\frac{\pi}{1}} \cdot \frac{1}{2}\sqrt{-\frac{\pi}{9}} \cdot \frac{1}{2}\sqrt{\frac{\pi}{31/9}}$$

$$\frac{1}{2}\left(\sqrt{\pi} \cdot \sqrt{-\frac{\pi}{9}} \cdot \sqrt{\frac{\pi}{31/9}}\right)$$

(iv) $$= 0.8464\sqrt{-\frac{\pi}{9}}$$