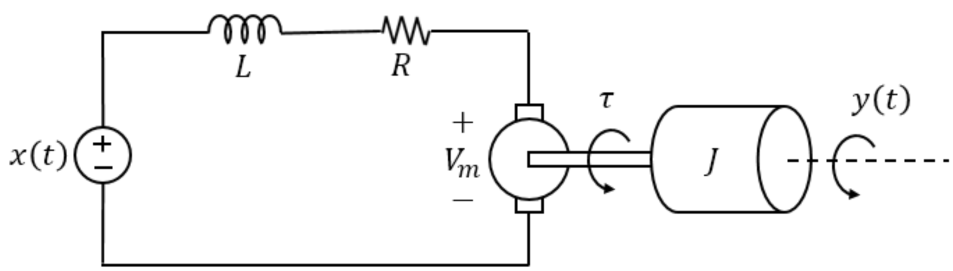
Homework # 2

Instructions: Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and box all your solutions. All Matlab code needs to be published, and all figures needs to have proper axis labeling and legends. Homework assignments will be collected during class time on the due date. *No late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **Homework problems not to be graded**
  - From textbook (Lathi):
    - Ch 1: 7-1, 7-2, 8-1, 8-3

- **Homework problems to be graded**

Electromechanical systems are interconnections of electrical and mechanical subsystems. Electric motors are common example of electromechanical systems. Below is the schematic diagram of a simple DC motor. The motor generates a back electromotive force voltage $v_m(t)$ on the electrical circuit, and a torque $\tau(t)$ on the mechanical load.

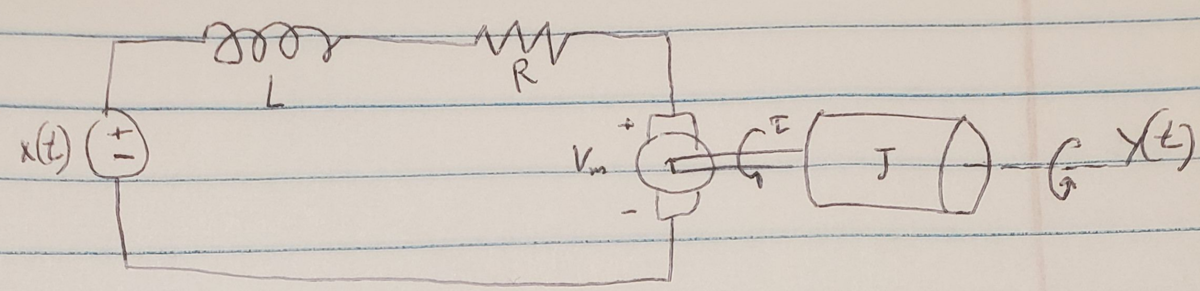


a) Use the Kirchhoff's Voltage Law to find the differential equation relating the input voltage $x(t)$, electromotive force voltage $v_m(t)$, and motor input current $i(t)$

b) Find the differential equation relating torque $\tau(t)$ and load angular displacement $y(t)$

c) Let motor torque $\tau(t) = k_{\mathrm{m}} i(t)$ for a given motor torque constant k_{m}, and back electromotive force voltage $v_m(t) = k_e \dot{y}(t)$ for an electric constant k_e. Combine the differential equations in part (a) and (b) to find a differential equation relating input voltage $x(t)$ and load angular displacement $y(t)$.
Hint: the final equation is a 3rd order ordinary differential equation

a)



$\cancel{V(t) = L \frac{di}{dt}}$ $\cancel{V_m = x(t)}$ $V_L = L\, d/dt[I_L]$

$$\boxed{V_m = x(t) - I_R R - L\, d/dt[I_L]}$$

b) $\tau = \frac{d}{dt}[L]$ $L = I\omega$ $I = 1/4\, \pi r^4$

$\omega = d/dt\, \theta$

$$\boxed{\tau(t) = d/dt[I\, d/dt[y(t)]]}$$

c) $V_m(t) = K_e \dot{y}(t)$ $\tau(t) = K_m i(t)$ $i(t) = \frac{\tau(t)}{K_m}$

$$V_m(t) = K_e \dot{y}(t) = x(t) - R\left(\frac{\tau(t)}{K_m}\right) - L\, d/dt\left[\frac{\tau(t)}{K_m}\right]$$

$$K_e \dot{y}(t) = x(t) - R\left(\frac{d/dt[I\, \cancel{d/dt[y(t)]}^{\,y'(t)}]}{K_m}\right) - L\, d/dt\left[\frac{d/dt[I\, \cancel{d/dt[y(t)]}^{\,y'(t)}]}{K_m}\right]$$

$$\boxed{X(t) = K_e \dot{y}(t) + R\left(\frac{d/dt[I\, y'(t)]}{K_m}\right) + L\, d/dt\left[\frac{d/dt[I\, y(t)]}{K_m}\right]}$$