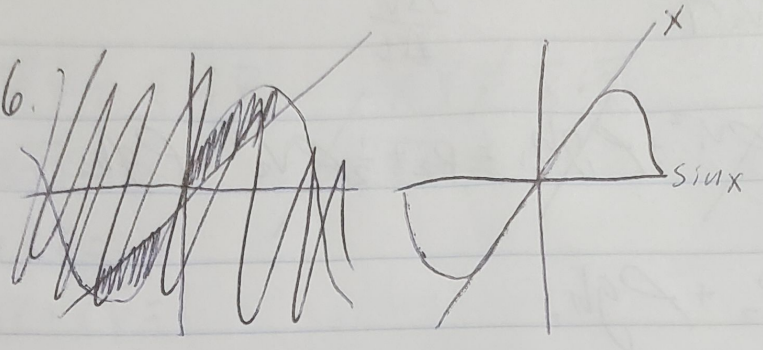
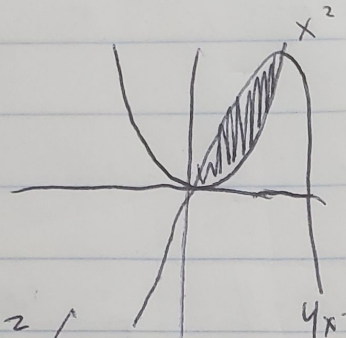
6.1

6. 

14. 

(2,4), (0,0)

$x^2 = 4x - x^2$ $+x^2$ $+x^2$ $\quad +2x^2 = 4x$ $\div x$ $\quad +2x = 4$ $\div 2$ $+2$

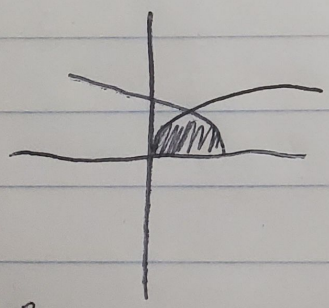
$x = 2$

$$\int_0^2 (4x - 2x^2)\,dx \qquad 2\int_0^2 (2x - x^2)\,dx$$

$$2\left(x^2 - \tfrac{1}{3}x^3\right)\Big|_0^2 \rightarrow \left[2(2)^2 - \tfrac{2}{3}(2)^3\right] - \left[2(0)^2 - \tfrac{2}{3}(0)^3\right]$$

$$8 - \tfrac{2}{3}8 = \tfrac{1}{3}8 = \boxed{8/3}$$

20. $y = \sqrt[4]{x}$ $\quad y = \sqrt{2-x}$ $\quad x = 0$



$$\left(\sqrt[4]{x}\right)^4 = \left(\sqrt{2-x}\right)^4 \qquad x = (2-x)^2$$

$$8 - 4x - 4x - x^3$$

$$(4 - 4x + x^2)(2-x)$$

$$8 - 8x - 2x^2 - 4x + 4x^2 + x^3$$

$$x^3 + 2x^2 - 12x + 8$$

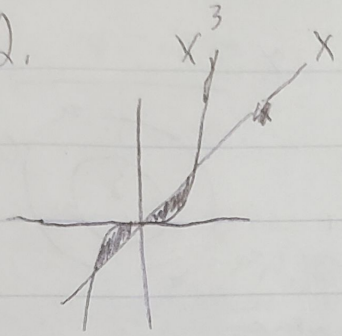
$$8 - 12x + 6x^2 - x^3$$

$$\int_0^2 (8 - 12x + 6x^2 - x^3)\,dx$$

$$8x - 6x^2 + 2x^3 - \tfrac{1}{4}x^4 + C\Big|_0^2$$

$$8(2) - 6(2)^2 + 2(2)^3 - \tfrac{1}{4}(2)^4$$

$$16 - 24 + 16 - 4$$

22. 

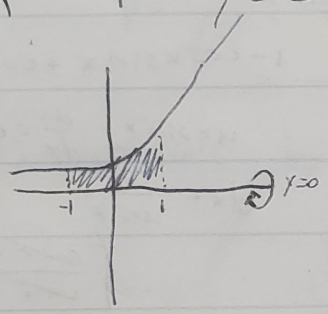
$x = x^3$ $-1, 0, 1$

$(-1,-1), (0,0), (1,1)$

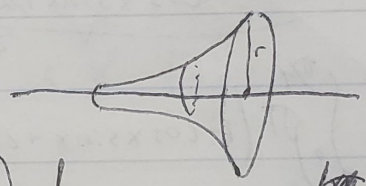
$$\int_0^1 (x - x^3)\,dx + \int_{-1}^0 (x^3 - x)\,dx$$

$\frac{1}{2}x^2 - \frac{1}{4}x^4 \Big|_0^1$ $\quad \frac{1}{2} - \frac{1}{4} = 1/4$ $\quad$ 1/2 $\quad$ $\frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_{-1}^0$ $\quad 1/4 - 1/2 = -1/4$

6.2 4. $y = e^x$ $y = 0$ $x = -1$ $x = 1$ x-axis



$y = r$



$$\int_{-1}^{1} \pi(e^{2x})\,dx$$

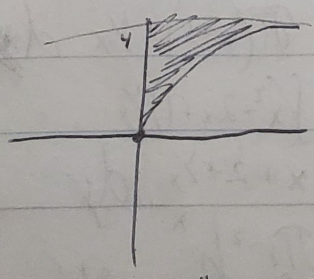
$u = 2x$ $\quad \frac{du}{dx} = 2$ $\quad dx = \frac{1}{2}du$ $\quad \frac{\pi}{2}\int e^u\,du$

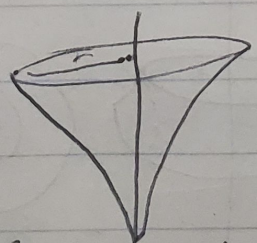
$\frac{\pi}{2} \cdot e^u$

$$\left[\pi/2 \cdot e^{2(1)}\right] - \left[\pi/2 \cdot e^{2(-1)}\right] = \boxed{\pi/2\left(e^2 - e^{-2}\right)}$$

6. $2x = y^2$ $x = 0$ y-axis $\quad \sqrt{2x} = 0$ $\quad x = 0$ $(0,0)$

$y = 4$ $\quad 1, \sqrt{2}$ $\quad x = \frac{y^2}{2}$





$r = x$

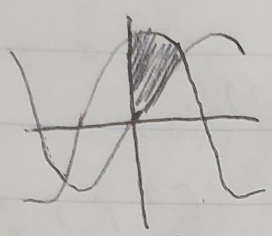
$$\int_0^4 \pi\left(\frac{y^2}{2}\right)^2 dy \to \int_0^4 \pi \cdot \frac{y^4}{4}\,dy$$

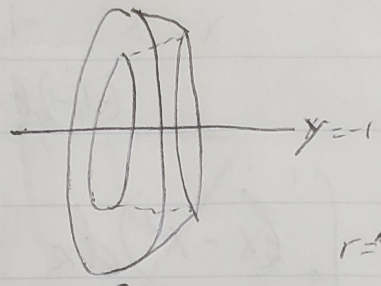
$u = \frac{y^4}{4}$ $\quad \frac{du}{dy} = y^3$ $\quad dy = \frac{1}{y^3}du$ $\quad \int \pi \cdot y^{-3} \cdot 4\,du$ $\quad \pi \cdot -2y^{-2} \cdot \frac{y^2}{2} + C$

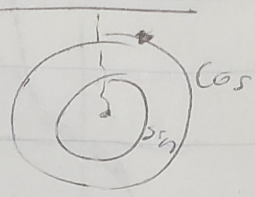
$\pi \cdot -\frac{2}{y^2} \cdot \frac{y^2}{2} = -1\pi \Big|_0^4$ $\quad [-\pi] - [-\pi]$

$= 0$

14. $\sin x$ $\cos x$ $[0, \pi/4]$ $y=-1$







$r = \cos x - \sin x + 1$

$$\int_0^{\pi/4} \pi (\cos x - \sin x + 1)^2 dx$$

$(\cos x - \sin x + 1)(\cos x - \sin x + 1)$

$\cos^2 x - \cos x \sin x + \cos x - \sin x \cos x + \sin^2 x - \sin x + \cos x - \sin x + 1$

$\cos^2 x + \sin^2 x - 2\cos x \sin x + 2\cos x - 2\sin x + 1$

$1 - 2\cos x \sin x + 2\cos x - 2\sin x + 1$

$\frac{2 - 2\cos x \sin x + 2\cos x - 2\sin x}{2}$ $\quad 1 - \cos x \sin x + \cos x - \sin x$

$u = \sin x \quad \frac{du}{dx} = \cos x$

$dx = \frac{1}{\cos x}$

$$\int_0^{\pi/4} \pi (1 - \cos x \sin x + \cos x - \sin x) dx$$

$$\int \frac{\cos x}{\cos x} u \, du$$

$$\int 1 - \int \cos x \sin x + \int \cos x - \int \sin x \qquad \int u \, du$$

$$x - \frac{\sin^2 x}{2} + \sin x + \cos x \qquad u^2/2$$

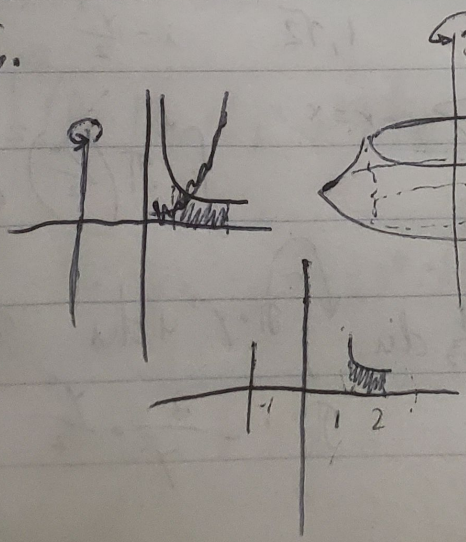
$$\left[\pi/4 - \frac{\sin^2 \pi/4}{2} + \sin \frac{\pi}{4} + \cos \pi/4\right] - \left[0 - \frac{\sin^2 0}{2} + \sin 0 + \cos 0\right]$$

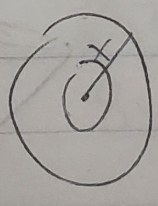
$$0 - 0 + 0 + 1$$

$$\pi/4 - \frac{1}{4} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$\pi/4 - 1/4 + \frac{2\sqrt{2}}{2} \cdot \frac{2}{2} \quad \boxed{\frac{\pi - 1 + 4\sqrt{2}}{4} - 1}$$

16.





$$\int_0^1 \pi (x+1)^2 \, 1/x \, dx$$

$$\pi \int (x^2 + 2x + 1) \, 1/x$$

$$\pi \int x + 2 + 1/x \, dy$$

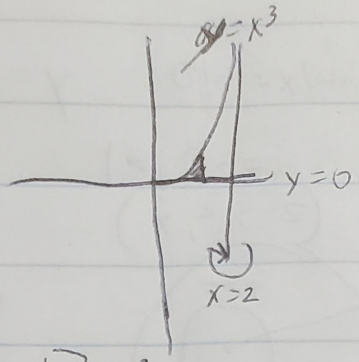
$\pi r^2 h$

$r = x + 1$ $\quad \pi \left(\frac{1}{2}x^2 + 2x + 1\right)\Big|_0^1$

$h = y$ $\quad \pi\left(\frac{1}{2} + 2 + 1\right) - \pi\left(0 + 0 + 1\right) = 2.5\pi$

8.3

4.



$V = \int_0^1 \pi(2-x)^2 x^3\,dx$

$\pi(4 - 4x + x^2)x^3$

$\pi(4x^3 - 4x^4 + x^5)\,dx$

$r = 2 - x$

$h = x^3$

$\pi\left(x^4 - \frac{4}{5}x^5 + \frac{1}{6}x^6\right)\Big|_0^1$

$\left[1 - \frac{4}{5} + \frac{1}{6}\right] - [0]$

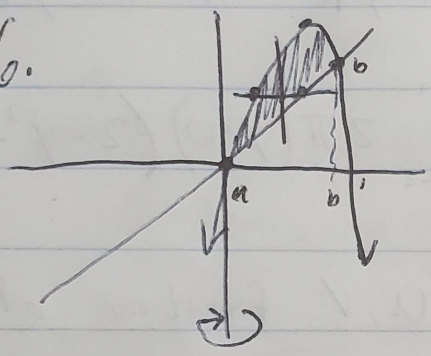
$\frac{30}{30} - \frac{24}{30} + \frac{5}{30} = \frac{11\pi}{30}$

$V = \int_0^2 \pi x^2 x^3\,dx$

$\pi \int x^5\,dx \quad \pi\left(\frac{1}{6}x^6\right)\Big|_0^2$

$\left[\frac{32\pi}{3}\right] - \left[\frac{\pi}{6}\right] = \frac{64\pi}{6} - \frac{\pi}{6} = \frac{64\pi - \pi}{6} \quad \frac{63\pi}{6} \to \frac{21\pi}{2}$

6.



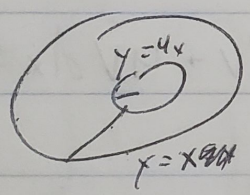
$y = 4x - x^2 \qquad y = x$

$4x - x^2 = x \qquad 4x = x + x^2$

$4 = \frac{x + x^2}{x} \qquad 4 = 1 + x \qquad x = 3, 0$

$-x^2 + 3x + 0 \qquad (x)(3 - x)$

$4(2) - 2^2 \qquad 8 - 4 = 4 \qquad 2, 4$



$V = 2\pi \int_a^b r h\,dx$

$r = x \qquad h = 4x - x^2 - x$

$V = 2\pi \int_0^3 x(4x - x^2 - x)\,dx$

$3x^2 - x^3$

$2\pi\left(x^3 - \frac{1}{4}x^4\right)\Big|_0^3 = \left[2\pi\left(\frac{27}{1} - \frac{81}{4}\right)\right] - [0]$

$\frac{108}{4} - \frac{81}{4} = \frac{27}{4} \to \frac{54\pi}{4}$