

$$A = \begin{bmatrix} 25 & 12 & 0 \\ -18 & -5 & 0 \\ 6 & 6 & 13 \end{bmatrix} \quad \det(A - \lambda I) = \begin{bmatrix} 25-\lambda & 12 & 0 \\ -18 & -5-\lambda & 0 \\ 6 & 6 & 13-\lambda \end{bmatrix} = 0$$

$$\cancel{-\lambda^3 + 33\lambda^2 - 135\lambda - 1625} + \cancel{234} - 18\lambda \cancel{70} = \cancel{\lambda^3 + 33\lambda^2 - 153\lambda - 1341}$$

eigen values: $\lambda_1 = 7, \lambda_2 = 13, \lambda_3 = -4$

$$(25-\lambda)(-5-\lambda)(13-\lambda) - 12(-18)(13-\lambda) = 0$$

eigen values: 7, 13 mult 2

for $\lambda = 7$:

$$\begin{bmatrix} 18 & 12 & 0 \\ -18 & -12 & 0 \\ 6 & 6 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 2x_3$

$x_2 = -3x_3$

$x_3 = x$

$\vec{v}_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

for $\lambda = 13$:

$$\begin{bmatrix} 12 & 12 & 0 \\ -18 & -18 & 0 \\ 6 & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = -x$

$x_2 = x$

$x_3 = \beta$

$\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \vec{v}_3$

$$D = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 2 & -1 & -1 \\ -3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(\xi_1 - x)(\xi_1 + x)(\xi_1 + x^2) = \dots$$

$$6. A = \begin{bmatrix} 5 & 1 \\ -4 & 1 \end{bmatrix} \quad \det(A - \lambda I) = \begin{bmatrix} 5-\lambda & 1 \\ -4 & 1-\lambda \end{bmatrix} = 0$$

$$(5-\lambda)(1-\lambda) + 4 = 0 = \lambda^2 - 6\lambda + 9 = 0$$

Eigenvalues: 3 mult 2

for $\lambda = 3$:

$$\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad x_1 = \frac{1}{2}x_2 \quad x_2 = \alpha$$

$$\vec{v}_1 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad V \text{ is not invertible}$$

$$7. A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \quad \det(A - \lambda I) = \begin{bmatrix} 1-\lambda & 2 \\ -1 & 3-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(3-\lambda) + 2 = 0 = \lambda^2 - 4\lambda + 5 = 0 = (\lambda - 5)(\lambda + 1)$$

Eigenvalues: ~~1, 1, 2~~ ~~1, 1, 2~~ ~~2 ± i~~

for $\lambda = -1$:

$$\begin{bmatrix} -2 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad x_1 = x_2 = 0 \quad \vec{v}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for $\lambda = 5$:

$$\begin{bmatrix} -1 & -2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad x_1 = \frac{1}{2}x_2 = 0 \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for $\lambda = 2 + i$:

$$\begin{bmatrix} -1+i & 2 \\ -1 & 1+i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1+i \\ -1+i & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1-i \\ 0 & 2-2i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \vec{v}_1 = \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$8. A = \begin{bmatrix} -3 & 0 & -4 \\ -1 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \quad \det(A - \lambda I) = \begin{bmatrix} -3-\lambda & 0 & -4 \\ -1 & -1-\lambda & -1 \\ 1 & 0 & 1-\lambda \end{bmatrix} = 0$$

$$(-3-\lambda)(-1-\lambda)(1-\lambda) + (-1-\lambda)^3 = 0$$

$$(3+4\lambda+\lambda^2)(1-\lambda) \rightarrow 3+4\lambda+\lambda^2 - 3\lambda - 12\lambda^2 - \lambda^3 - 4\lambda = 0$$

$$\cancel{\lambda^3 - 11\lambda^2 + 22\lambda} = 0 \quad (\lambda+1)^3 \quad \boxed{\text{eigenvalues: } -1 \text{ mult 3}}$$

for $\lambda = -1$

~~$$\begin{bmatrix} -2 & 0 & -1 \\ -1 & 0 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$~~

$$x_1 = -2x_3 = 0 \quad \vec{v}_1 = \vec{v}_2 = \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_3 = 0 \quad x_2 = \alpha$$

V is not invertible

$$1. A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 5 & 9 & -2 \\ 0 & 0 & -5/2 & 25/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ -2 \\ 15 \\ 6 \end{bmatrix}$$

$$4x_1 + 5x_2 + 9x_3 - 2x_4 = 0 \quad x_1 = -\frac{9}{4}x_2 - \frac{9}{4}x_3 + \frac{1}{2}x_4$$

$$-\frac{5}{2}x_2 + \frac{25}{2}x_3 + 15x_4 = 0 \quad x_2 = 5/4x_3 + 3/2x_4$$

$$x_3 = x_3 = \alpha$$

$$x_4 = x_4 = \beta$$

$$x_1 = -\frac{25}{16}x_3 - \frac{15}{8}x_4 - \frac{36}{16}x_3 + \frac{45}{8}x_4$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \alpha \begin{bmatrix} -\frac{61}{16} - \frac{11\beta}{8} \\ \frac{5\alpha}{4} + \frac{3\beta}{2} \\ \alpha \\ \beta \end{bmatrix} = \alpha \begin{bmatrix} -61/16 \\ 5/4 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -11/8 \\ 3/2 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Null}(A) = \left\{ \begin{bmatrix} -61/16 \\ 5/4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -11/8 \\ 3/2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Rank = 2

$$\dim \text{Null}(A) = 4 \times 2$$

$$\dim \text{Col}(A) = 3 \times 2$$

$$\dim \text{Row}(A) = 2 \times 4$$

5.6 #18

$$\vec{x}_{k+1} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.6 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} c_k \\ y_k \\ a_k \end{bmatrix}$$

$$3. A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{det}(A - \lambda I) = \begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(2-\lambda)(2-\lambda) - (-1)(-1)(2-\lambda) + 0 = 0$$

$$(x^2 - 4x + 4)(2-\lambda) - 8x^2 + 12x^2 - 8x^2 + 8x - 8 = x^3 + 6x^2 - 8$$

$$-x^3 + 6x^2 - 11x + 6 = 0 \rightarrow \cancel{(x-1)}(x-2)(x-3) = 0$$

eigen values: 1, 2, 3

for $\lambda = 1$:

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= x_2 - x_3 \\ x_2 &= \alpha \\ x_3 &= 0 \end{aligned}$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

for $\lambda = 2$:

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 0 \\ x_2 &= x_3 = \alpha \\ x_3 &= \alpha \end{aligned}$$

$$\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

for $\lambda = 3$:

$$\begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= -x_2 + x_3 \\ x_2 &= \alpha \\ x_3 &= 0 \end{aligned}$$

$$\vec{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad V^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 \\ -1/2 & 1/2 & -1/2 \end{bmatrix} \quad A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -2 & 1 & -1 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1/2 & 1/2 & -1/2 \end{array} \right]$$

$$V^{-1}A = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 0 & 0 & 2 \\ -3/2 & 3/2 & -3/2 \end{bmatrix} \quad *V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4. $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \quad \det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -1 & 1 \\ 1 & 1-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$

$$(1-\lambda)(1-\lambda)(2-\lambda) - (-1)(2-\lambda) + 0 = 0$$

$$-\lambda^3 + 4\lambda^2 - 5\lambda^2 + 2 + 2 - \lambda = 0 \rightarrow -\lambda^3 + 4\lambda^2 - 6\lambda + 4 = 0 \quad \boxed{\text{EigenValues: } 2, 1 \pm i}$$

for $\lambda=2$:

$$\left[\begin{array}{ccc|c} -1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{aligned} x_1 &= -x_2 + x_3 = 0 \\ x_2 &= x_3 = \alpha \\ x_3 &= \alpha \end{aligned}$$

for $\lambda=1+i$:

$$\left[\begin{array}{ccc|c} i & -1 & 1 & 0 \\ 1 & i & 1 & 0 \\ 0 & 0 & 1+i & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} i & -1 & 1 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 1+i & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1+i & -1-i & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \quad \begin{aligned} x_1 &= (1+i)\alpha \\ x_2 &= \alpha \\ x_3 &= 0 \end{aligned}$$

for $\lambda=1-i$:

$$\left[\begin{array}{ccc|c} -i & -1 & 1 & 0 \\ 1 & -i & 1 & 0 \\ 0 & 0 & 1-i & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} -i & -1 & 1 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 1-i & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1-i & 1+i & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \quad \begin{aligned} x_1 &= (1-i)\alpha \\ x_2 &= \alpha \\ x_3 &= 0 \end{aligned}$$

$$\vec{v} = \begin{bmatrix} 0 & 1+i & 1-i \\ 1-i & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \vec{v}' = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1+i & 0 \\ 0 & 0 & 1-i \end{bmatrix}$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1+i & 1-i & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 1 \\ 0 & 1+i & 1-i & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1+i & 1-i \end{array} \right]$$

5. $A = \begin{bmatrix} 25 & 12 & 0 \\ -18 & -5 & 0 \\ 6 & 6 & 13 \end{bmatrix}$ $\det(A - \lambda I) = \begin{bmatrix} 25-\lambda & 12 & 0 \\ -18 & -5-\lambda & 0 \\ 6 & 6 & 13-\lambda \end{bmatrix} = 0$

$$(25-\lambda)(-5-\lambda)(13-\lambda) + (-1)(-18)(13-\lambda) + (-18)(6)(6)(5-\lambda) = 0$$

$$(-125 - 25\lambda + 5\lambda^2 + \lambda^3) + 234 - 18\lambda - 108 + 30 + 6\lambda = 0$$

$$13x^3 - 260x^2 - 1625 - 3x^3 + 20x^2 + 125x + 234 - 18x - 108 + 30 + 6x = 0$$

$$-x^3 + 33x^2 - 147x - 1469 = 0 = (x^2 + 20x + 113)(x - 13)$$

eigen values: 13, -4.6, 24.6

for $\lambda = 13$:

$$\left[\begin{array}{ccc|c} 12 & 12 & 0 & 0 \\ -18 & -17 & 0 & 0 \\ 6 & 6 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad x_1 = -x_2 = 0 \quad x_3 = 1$$

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda = -4.6$:

$$\left[\begin{array}{ccc|c} 29.6 & 12 & 0 & 0 \\ -18 & -0.4 & 0 & 0 \\ 6 & 6 & 17.6 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 148 & 0 & 0 & 0 \\ 0 & 1276 & 0 & 0 \\ 0 & 0 & 88 & 0 \end{array} \right] \quad x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$$

$$\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$9. A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix} \quad \det(A - \lambda I) = \begin{bmatrix} 3-\lambda & -1 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & -1 & 3-\lambda \end{bmatrix} = 0$$

$$(3-\lambda)(3-\lambda)(2-\lambda) + \cancel{18\lambda^2} + 2\lambda = 0$$

$$(9 - 6\lambda + \lambda^2)(2-\lambda) = 18 - 12\lambda + 2\lambda^2 - 9\lambda + 6\lambda^2 - \lambda^3 + 2 - \lambda = 0$$

$$-\lambda^3 + 8\lambda^2 - 22\lambda + 20 = 0 = (-\lambda^2 + 6\lambda - 10)(\lambda - 2)$$

eigenvalues: $2, 3 \pm i$

$$\lambda = \frac{-6 \pm \sqrt{36 - 4(-1)(-40)}}{-2} = 3 \pm i$$

for $\lambda = 2$:

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} x_1 = x_2 - x_3 \\ x_2 = \alpha \\ x_3 = \beta \end{array}$$

3. Values are the same, vectors are similar

V =

$$\begin{matrix} 0.707106781186547 & 0.707106781186547 & 0 \\ -0.707106781186547 & 0.707106781186547 & 0.707106781186547 \\ 0 & 0 & 0.707106781186548 \end{matrix}$$

D =

$$\begin{matrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix}$$

4. Values are the same, vectors are similar

V =

Column 1
0.707106781186547 + 0.000000000000000i
0.000000000000000 - 0.707106781186547i
0.000000000000000 + 0.000000000000000i

Column 2

0.707106781186547 + 0.000000000000000i
0.000000000000000 + 0.707106781186547i
0.000000000000000 + 0.000000000000000i

Column 3

0.000000000000000 + 0.000000000000000i
0.707106781186547 + 0.000000000000000i
0.707106781186547 + 0.000000000000000i

D =

Column 1
1.000000000000000 + 1.000000000000000i
0.000000000000000 + 0.000000000000000i
0.000000000000000 + 0.000000000000000i

Column 2

$0.00000000000000 + 0.00000000000000i$

$1.00000000000000 - 1.00000000000000i$

$0.00000000000000 + 0.00000000000000i$

Column 3

$0.00000000000000 + 0.00000000000000i$

$0.00000000000000 + 0.00000000000000i$

$2.00000000000000 + 0.00000000000000i$

5. Values are the same, most vectors are similar. Possible error on my part

$V =$

$0 \ 0.534522483824849 \ -0.707106781186547$

$0 \ -0.801783725737273 \ 0.707106781186548$

$1.00000000000000 \ 0.267261241912424 \ 0$

$D =$

$13 \ 0 \ 0$

$0 \ 7 \ 0$

$0 \ 0 \ 13$

6. Values are similar, vectors are slightly off. Possible error on my part

$V =$

Column 1

$0.447213595499958 + 0.00000006664002i$

$-0.894427190999916 + 0.00000000000000i$

Column 2

$0.447213595499958 - 0.00000006664002i$

$-0.894427190999916 + 0.00000000000000i$

$D =$

Column 1

$3.00000000000000 + 0.00000029802322i$

$0.00000000000000 + 0.00000000000000i$

Column 2

$0.00000000000000 + 0.00000000000000i$

$3.00000000000000 - 0.000000029802322i$

7. Values are similar, vectors are slightly off. Possible error on my part

$V =$

Column 1

$0.816496580927726 + 0.00000000000000i$

$0.408248290463863 + 0.408248290463863i$

Column 2

$0.816496580927726 + 0.00000000000000i$

$0.408248290463863 - 0.408248290463863i$

$D =$

Column 1

$2.00000000000000 + 1.00000000000000i$

$0.00000000000000 + 0.00000000000000i$

Column 2

$0.00000000000000 + 0.00000000000000i$

$2.00000000000000 - 1.00000000000000i$

8. Values are the same, vectors are similar

$V =$

$0 \quad -0.00000000000000 \quad -0.00000000000000$

$1.00000000000000 \quad -1.00000000000000 \quad -1.00000000000000$

$0 \quad 0.00000000000000 \quad 0.00000000000000$

$D =$

$-1 \quad 0 \quad 0$

$0 \quad -1 \quad 0$

$0 \quad 0 \quad -1$

9. Values are not similar, vectors are not similar. Possible error on my part

$V =$

$0.707106781186547 \quad -0.707106781186547 \quad 0.408248290463863$

0 0 0.816496580927726

0.707106781186547 0.707106781186547 0.408248290463863

D =

4 0 0

0 2 0

0 0 2