Student Name:

Homework # 5

Instructions: Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and box all your solutions. All Matlab code needs to be published, and all figures needs to have proper axis labeling and legends. Homework assignments will be collected during class time on the due date. *No late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **Homework problems not to be graded**
  - From textbook:
    - Ch 3: 5.7, 6.2, 8.3

- **Homework problems to be graded**

In a lottery ticket game, each ticket has four randomly marked numbers from 1 to 20 (no repeated numbers, order is not important). A ticket is a winner if the four marked numbers match four numbers drawn at random. Suppose that there are *n* tickets in the game. Answer the following questions.

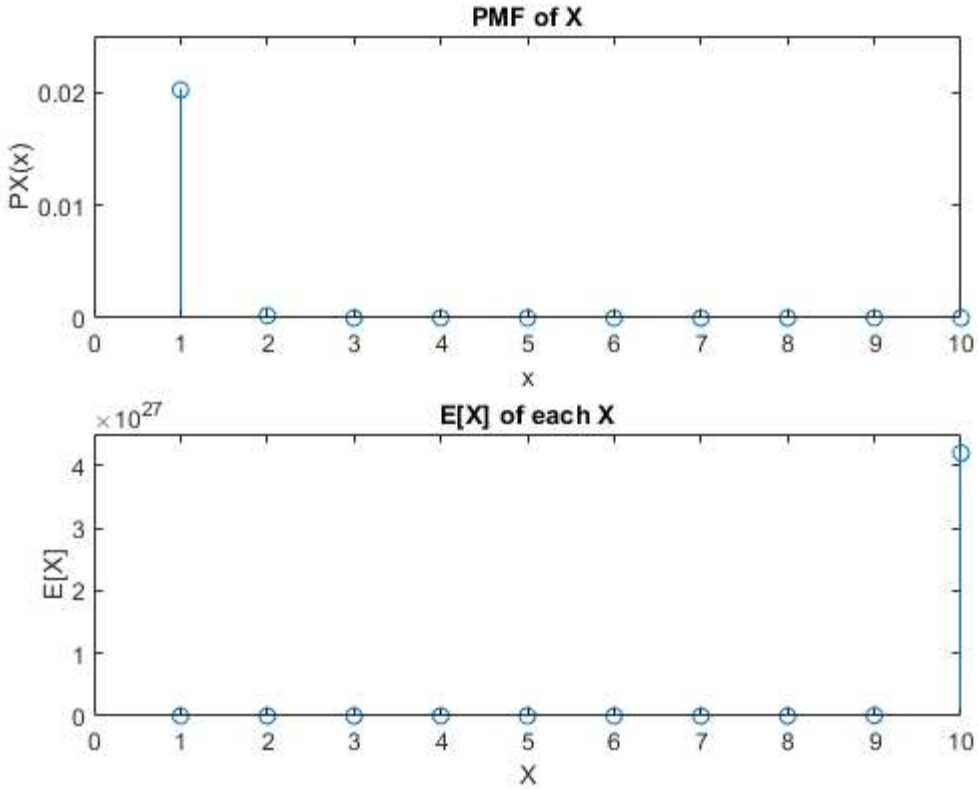
(a) What is the probability *p* that a random ticket will be a winner? Justify your solution

(b) Find the PMF of the random variable *X*={"number of winning tickets in the game"}. Explain your solution, and state what family of random variable *X* belongs to.

(c) Let *n* = 100. Write a Matlab script to find the PMF of *X* and plot the PMF for *X* from 1 to 10. Comment and label your script and figures.
(Hint: check Matlab function *nchoosek)*

d) Assume that all winning tickets receive an equally divided portion of the $10,000 price money. Given that there is at least one winning ticket, what is the expected value of the price money for each winning ticket? Use Matlab to find the answer. Include your commented script as part of your solution.

```
% a)
%       20 possible numbers can appear on a card 4 times. This is equal to
%       20Choose4. 20Choose4 = 4845 possible combinations. A winning combination
%       is only one combination. This means the probability of a random ticket
%       holding the winning combination is 1/4845 = 0.0002064.
% b)
%       Since there is only winners and losers(2 outcomes) and we are trying to
%       find the number of winning tickets, this random variable is a Binomial
%       Random Variable. This can be modelled by
%       PX(x)=(nChoosex)(p^x)(1-p)^(n-x).

figure;

% c)
p = 1/4845; % probability of a winning ticket
n = 100;    % number of available tickets
PMF = zeros(1,10);  % Probability Mass Function vector

for x=1:10
    % Binomial RV formula
    PMF(x)=nchoosek(n,x)*(p^x)*(1-p)^(n-x);
end

subplot(2,1,1);
stem(PMF);
axis([0 10 0 0.025]);
xlabel('x');
ylabel('PX(x)');
title('PMF of X');

% d)
prize_pool = 10000; % prize pool of $10000
E = zeros(1, 10);   % Expected Value vector

for i=1:10
    E(i) = prize_pool / PMF(i);
end

subplot(2,1,2);
stem(E);
axis([0 10 0 4.5*10^27]);
xlabel('X');
ylabel('E[X]');
title('E[X] of each X');
```



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