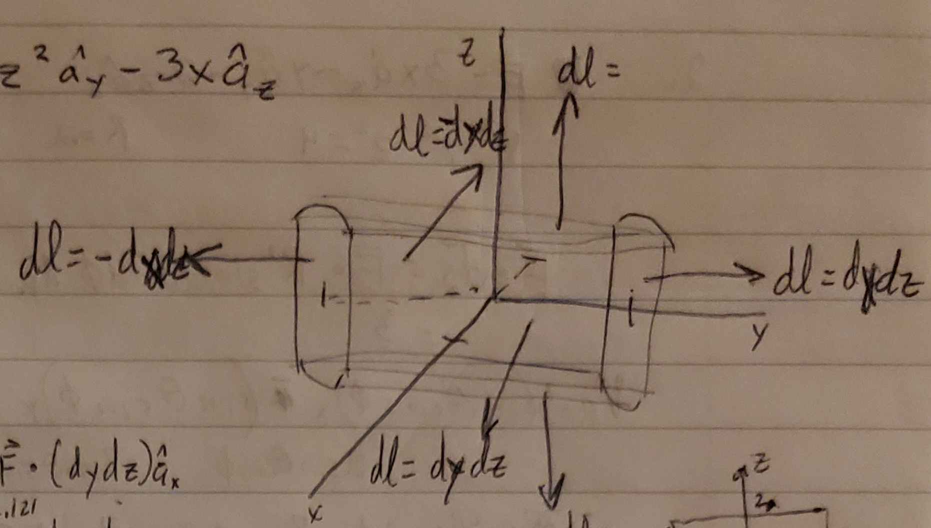
1. $\vec{F} = 2z\hat{a}_x + 12x^2yz^2\hat{a}_y - 3x\hat{a}_z$

$\oiint \vec{F}\cdot d\vec{s} = \iiint \nabla\cdot\vec{F}\,dV$

$3x^2 + 2z^2 = 12$

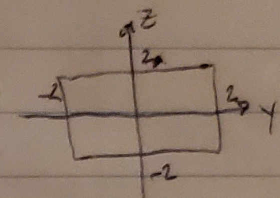


front face: $(x=1)$ $\int \vec{F}\cdot d\vec{s} = \vec{F}\cdot(dydz)\hat{a}_x$

$= 2z\,dydz = 2\int_{-2}^{2}\int_{-2.121}^{2.121} z\,dz\,dy$

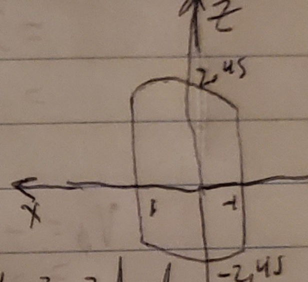
$= \frac{z^2}{2}\Big|_{-2.121}^{2.121} = 2.25$

$= 4.5\int_{-2}^{2} 1\,dy = \quad y\Big|_{-2}^{2} = 4 = 18$



back face $(x=-1)$: $\vec{F}\cdot d\vec{s} = \vec{F}\cdot(-dydz)\hat{a}_x = -2z\,dydz$

$-2\int_{-2}^{2}\int_{-2.121}^{2.121} z\,dz\,dy = -18$



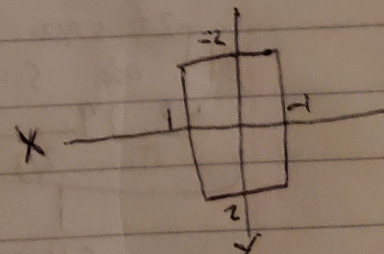
right face $(y=2)$: $\vec{F}\cdot(dx\,dz)\hat{a}_y = 12x^2yz^2\,dxdz = 24x^2z^2\,dxdz$

$24\int_{-1}^{1} x^2\int_{-2.45}^{2.45} z^2\,dz\,dx$ $\quad \frac{z^3}{3}\Big|_{-2.45}^{2.45} = \frac{14.7}{3} - \frac{(-14.7)}{3} = 29.4$

$235.2\int_{-1}^{1} x^2\,dx$ $\quad x^3\Big|_{-1}^{1} = 2$ $\quad \frac{235.2}{3}\times 2 = 156.8$

left face $(y=-2)$: $\vec{F}\cdot(-dxdz)\hat{a}_y = -12x^2yz^2\,dydz = -24x^2z^2\,dxdz$

$-24\int_{-1}^{1} x^2\int_{-2.45}^{2.45} z^2\,dz\,dx = -156.8$

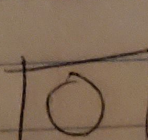


top face: $(z = \sqrt{6 - 3/2\,x^2})$ $\vec{F}\cdot dxdy\,\hat{a}_z = -3x\,dxdy$

$-3\int_{-2}^{2}\int_{-1}^{1} x\,dx\,dy$ $\quad \frac{1}{2}x^2\Big|_{-1}^{1} \to -3\int_{-2}^{2} 1\,dy \to -3y\Big|_{-2}^{2} = -12$

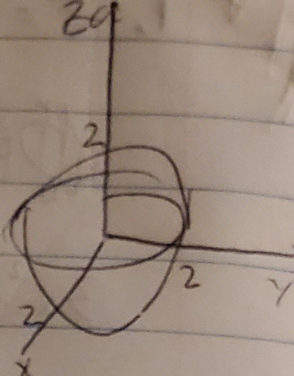
bottom faces $\quad \vec{F}\cdot -dxdy\,\hat{a}_z = 3x\,dx\,dy$

$3\int_{-2}^{2}\int_{-1}^{1} x\,dx\,dy = 12$



2. $\vec{F} = 3x\hat{a}_x - y\hat{a}_y + 2z\hat{a}_z$

$x^2+y^2+z^2=4$ $\quad R=2$



$\vec{F}\cdot\vec{ds} = \vec{F}\cdot R^2\sin\theta\, d\theta\, d\phi\, \hat{a}_R$

$= 3$

$V_R = 3(\sin\theta\cos\phi)x - (\sin\theta\sin\phi)y + 2(\cos\theta)z$

$R\sin\theta\cos\phi \quad R\sin\theta\sin\phi \quad R\cos\theta$

$= 3R\sin^2\theta\cos^2\phi - R\sin^2\theta\sin^2\phi + 2R\cos^2\theta$

$R(3\sin^2\theta\cos^2\phi - \sin^2\theta\sin^2\phi + 2\cos^2\theta) = R(\sin^2\theta(3\cos^2\phi - \sin^2\phi) + 2\cos^2\theta)$

$V_\theta = 3(\cos\theta\cos\phi)x - (\cos\theta\sin\phi)y + 2(-\sin\theta)z$

$= 3R\cos\theta\sin\theta\cos^2\phi - R\cos\theta\sin\theta\sin^2\phi - 2R\cos\theta\sin\theta$

$= R\cos\theta\sin\theta(3\cos^2\phi - \sin^2\phi - 2)$

$V_\phi = -3(\sin\phi)x - (\cos\phi)y$

$= -3R\sin\theta\cos\phi\sin\phi - R\sin\theta\cos\phi\sin\phi$

$= -R(3\sin\theta\cos\phi\sin\phi + \sin\theta\cos\phi\sin\phi)$

$V = (3R\sin^2\theta\cos^2\phi - R\sin^2\theta\sin^2\phi + 2R\cos^2\theta)\hat{a}_R$

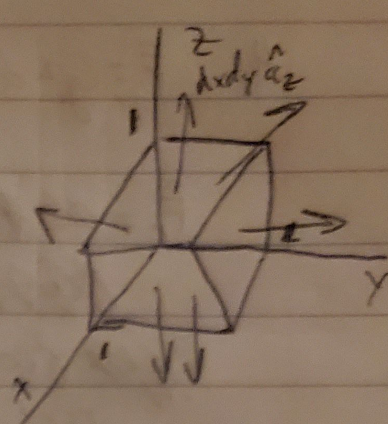
$\vec{F}\cdot\vec{ds} = (3R^3\sin^3\theta\cos^2\phi - R^3\sin^3\theta\sin^2\phi + 2R^3\cos^2\theta\sin\theta)\,d\theta\,d\phi$

$= 81\sin^3\theta\cos^2\phi - 27\sin^3\theta\sin^2\phi + 54\cos^2\theta\sin\theta\,d\theta\,d\phi$

$$\underbrace{81\underbrace{\int_0^{2\pi}\cos^2\phi}_{\pi}\underbrace{\int_0^{\pi/2}\sin^3\theta}_{2/3}\,d\theta\,d\phi}_{54\pi} - \underbrace{27\underbrace{\int_0^{2\pi}\sin^2\phi}_{\pi}\underbrace{\int_0^{\pi/2}\sin^3\theta}_{2/3}\,d\theta\,d\phi}_{-18\pi} + \underbrace{108\pi\underbrace{\int_0^{\pi/2}\cos^2\theta\sin\theta\,d\theta}_{1/3}}_{36\pi}$$

$= \boxed{72\pi}$

3. $\vec{F} = 2xy\,\hat{a}_x + 2yz\,\hat{a}_y + 2xz\,\hat{a}_z$



top (z=1): $\vec{F}\cdot\vec{ds} = 2xz\,dx\,dy = 2x\,dx\,dy$

$2\int_0^1\int_0^1 x\,dx\,dy = 2\int_0^1 \frac{1}{2}x^2\Big|_0^1\,dy$

$= \int_0^1 dy = 1$

bottom (z=0): $\vec{F}\cdot\vec{ds} = -2xz\,dx\,dy = 0$ $\quad \int 0 = 0$

right: (y=1) $\vec{F}\cdot\vec{ds} = 2yz\,dx\,dz = 2z\,dx\,dz$

$2\int_0^1\int_0^1 z\,dz\,dx = \int_0^1 dx = 1$

left: (y=0) $\vec{F}\cdot\vec{ds} = -2yz\,dz\,dx = 0$

front (x=1): $\vec{F}\cdot\vec{ds} = 2xy\,dy\,dz = 2y\,dy\,dz$

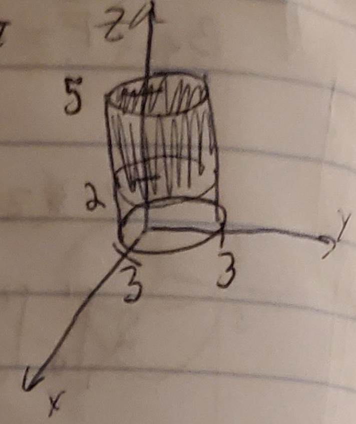
$2\int_0^1\int_0^1 y\,dy\,dz = \int_0^1 dz = 1$

back (x=0): $\vec{F}\cdot\vec{ds} = -2xy\,dy\,dz = 0$

$= \boxed{3}$

4. $\vec{F} = y^2\hat{a}_x + x^2\hat{a}_y + 4z\hat{a}_z$ $\quad r=3,\ z=5,\ \phi=2\pi$

$x^2+y^2=9$



$\vec{F}_r =$

$y^2\cos\phi\,\hat{a}_x + x^2\sin\phi\,\hat{a}_y = r^2\sin^2\phi\cos\phi + r^2\sin\phi\cos^2\phi$

$\quad$ rsinϕ $\quad$ rcosϕ

$\vec{F}_\phi = -y^2\sin\phi\,\hat{a}_x + x^2\cos\phi\,\hat{a}_y = -r^2\sin^3\phi + r^2\cos^3\phi$

$\quad$ rsinϕ $\quad$ rcosϕ

$\vec{F}_z = 4z\hat{a}_z$

$\vec{ds}_{top} = r\,d\phi\,dr\,\hat{a}_z$

$\vec{ds}_{bottom} = -r\,d\phi\,dr\,\hat{a}_z$

$\vec{ds}_{side} = r\,d\phi\,dz\,\hat{a}_r = 3\,d\phi\,dz\,\hat{a}_r$

top: $\vec{F}\cdot\vec{ds} = 4z\,r\,d\phi\,dr = 12\,r\,d\phi\,dr$

$\oiint \vec{F}\cdot\vec{ds} = \iiint \vec{\nabla}\cdot\vec{F}\,dv$

$\vec{\nabla} = \frac{\partial}{\partial x}(y^2)\hat{a}_x + \frac{\partial}{\partial y}(x^2)\hat{a}_y + \frac{\partial}{\partial z}(4z)\hat{a}_z = 4\hat{a}_z$

$\vec{\nabla}\cdot\vec{F} = 16z$

$\int_0^{2\pi}\int_0^3\int_2^5 16z\,r\,dz\,dr\,d\phi = 32\pi\int_0^3 r\int_2^5 z\,dz\,dr$

$\frac{1}{2}z^2\Big|_2^5 \qquad \frac{25-4}{2} = \frac{21}{2} = 21/2$

$\frac{21}{2}\,16\int_0^{2\pi}\int_0^3 r\,dr\,d\phi \rightarrow \frac{9}{2}\,\frac{21}{2}\,16\int_0^{2\pi} d\phi \rightarrow \frac{9}{2}\cdot\frac{21}{2}\cdot 16\cdot 2\pi = \boxed{1512\,\pi}$