Math 418 Quadratic Functions

$f(x) = ax^2 + bx + c$ a, b and c #'s $a \neq 0$

Ex: $x^2 - x + \pi$

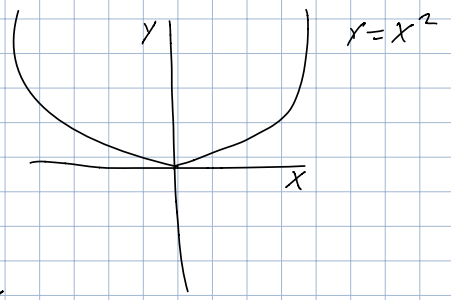
x^2

$(x-3)^2$

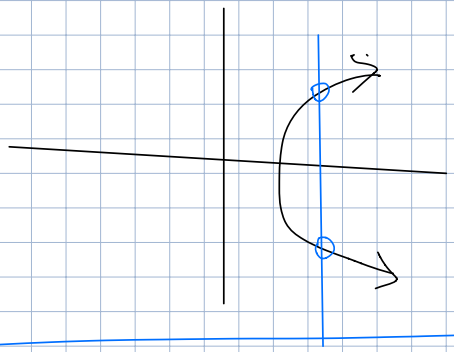
$(2x+2)(3x+1)$

$(x+1)(x-1) = x^2 - 1$

Fact: The Graph of a Quadratic Function is a Parabola

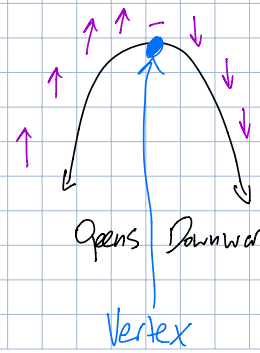


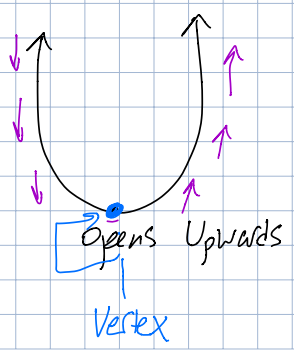
Not A Quad. Function:



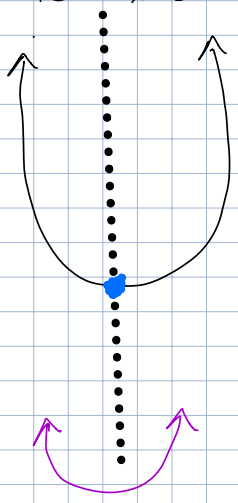
Defn: The Vertex of a Parabola is the Point where it changes direction.

Ex





Def'n: The Axis of Symmetry of a parabola is the line the parabola is symmetric about.

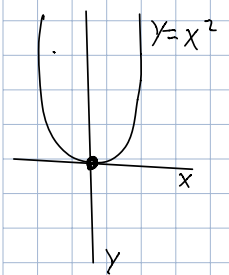


Fact*: If a parabola has vertex at (h,k) then the line of symmetry is

$$\boxed{x=h}$$

* = In our Course

How to find Vertex?



Fact: Every Parabola is a transformation of $f(x)=x^2$

Fact: The Vertex of $y=x^2$ is $(0,0)$

We can write every parabola as $g(x)$ where $g(x)$ is the following Transformation of $f(x)$:

Quadratic ax^2+bx+c

1. Vertical Stretch / maybe Vertical Flip (a)
2. Vertical Shift: k
3. Horizontal Shift: h

$$g(x)=a(x-h)^2+k$$

$(0,0) \longrightarrow (h,k)$

Fact: Vertex of $g(x)=a(x-h)^2+k$ is $\boxed{(h,k)}$

Ex: (1) Vertex of $-2(x-3)^2-4$ is $(3,-4)$

(2) Vertex of $8(x+1)^2+9$ is $(-1,9)$

Note $(x+1) = (x-(-1))$

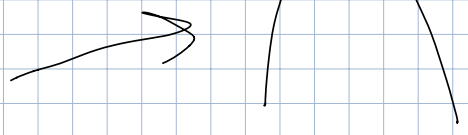
(3) Vertex of x^2+4 is: $(0,4)$

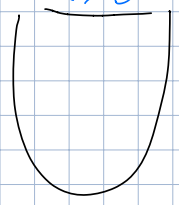
$x^2 = (x-0)^2$

$(x-0)^2 + 4$

Fact: If $f(x) = a(x-h)^2 + k$ OR ax^2+bx+c

Then: $a<0$ $a>0$





Opens Down
"Sad" Parabola

Opens Up
"Happy" Parabola

Note: ax^2+bx+c : General Form

$a(x-h)^2+k$: Vertex Form / Standard Form

Two Methods To get Vertex:

(1) Completing the □

(2) Easier.

$f(x) = ax^2+bx+c$ $\quad a \neq 0$

$\downarrow$

Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Ex: $f(x) = 3x^2+6x+1$

$a=3$
$b=6$

$\frac{-b}{2a} = \frac{-6}{2(3)} = \frac{-6}{6} = -1$

Vertex: $(-1,-2)$

$f(-1) = 3(-1)^2 + 6(-1) + 1 = 3 - 6 + 1$

Ex: $g(x) = -2x^2 + 8x - 10$ $= -2$

$a = -2$
$b = 8$

$\frac{-b}{2a} = \frac{-8}{2(-2)} = \frac{-8}{-4} = 2$

$g(-2) = -2(2)^2 + 8(2) - 10$

$= -2(4) + 8(2) - 10$

$= -8 + 16 - 10 = -2$

Vertex: $(2, -2)$

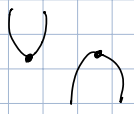
Do This before recitation Th:

Find Vertex of $f(x) = 5x^2 + 3x - 2$

Sketch the Graph of $f(x) = 3x^2 + 2x$

$a = 3$
$b = 2$

① Vertex

② Determine 

$\frac{-b}{2a} = \frac{-2}{2(3)} = \frac{-2}{6} = \frac{-1}{3}$

$f(\frac{-1}{3}) = 3(\frac{-1}{3})^2 + 2(\frac{-1}{3})$

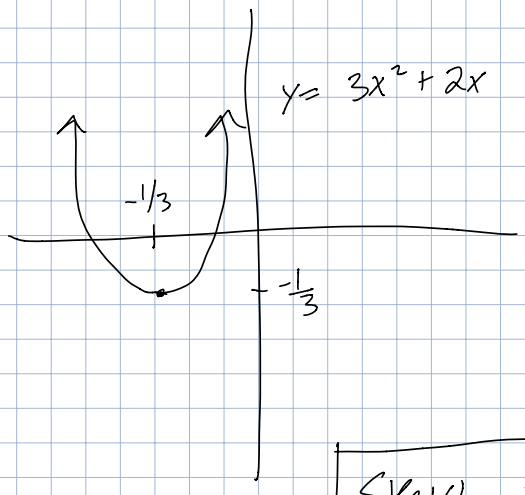
$= 3(\frac{1}{9}) + \frac{-2}{3}$

$= \frac{3}{9} - \frac{2}{3} = \frac{1}{3} - \frac{2}{3}$

$= \frac{-1}{3}$

Vertex: $(\frac{-1}{3}, \frac{-1}{3})$

$a = 3 > 0$ 



Sketch: $f(x) = 2x^2 - x + 3$

Reminder

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a \neq 0$$