1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\det A = \begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & -1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0$

$(1-\lambda)\begin{bmatrix} -1-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1-\lambda \end{bmatrix} + \begin{bmatrix} 1 & -1-\lambda \\ 0 & 0 \end{bmatrix}$

$(1-\lambda)(-1-\lambda)(1-\lambda) - (1-\lambda) + 0$

$(1-\lambda)((-1-\lambda)(1-\lambda) - 1)$

λ^2-1-1 $(1-\lambda)(\lambda^2-2)$ $\lambda^2 - 2 - \lambda^3 + 2\lambda \Rightarrow -\lambda^3 + \lambda^2 + 2\lambda - 2 = 0$

$\lambda = 1, \pm\sqrt{2}$ = eigenvalues

for $\lambda = 1$, $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_x \\ \updownarrow \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$V_y - 2V_x = 0$ $V_x + V_z = 0$ $V_z = \alpha$ $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} = \vec{V}_1$

$V_y = 2V_x = -2\alpha$ $V_x = -V_z = -\alpha$ -0.414

for $\lambda = \sqrt{2}$ $\begin{bmatrix} 1-\sqrt{2} & 1 & 1 \\ 1 & -1-\sqrt{2} & 0 \\ 0 & 0 & 1-\sqrt{2} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $(1-\sqrt{2})V_x + V_y = 0$ $V_y = 0.414V_x$

$V_x + (-1-\sqrt{2})V_y = 0$ $V_x = 2.414 V_y$

$V_z = 0$

$\vec{V}_2 = \begin{bmatrix} 1 \\ 0.414 \\ 0 \end{bmatrix}$ $(0.414)(2.414)V_y + V_y = 0$ $V_y(1+1) = 0$

$V_y =$

for $\lambda = -\sqrt{2}$ $\begin{bmatrix} 2.414 & 1 & 1 \\ 1 & 0.414 & 0 \\ 0 & 0 & 2.414 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\vec{V}_3 = \begin{bmatrix} 0.414 \\ 1 \\ 0 \end{bmatrix}$

eigenvectors = $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0.414 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.414 \\ 1 \\ 0 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 2 & -1 \\ 0 & 2 & 0 \\ -2 & 2 & 1 \end{bmatrix}$ $\det B = \begin{bmatrix} -\lambda & 2 & -1 \\ 0 & 2-\lambda & 0 \\ -2 & 2 & 1-\lambda \end{bmatrix} = 0$

$-\lambda \begin{bmatrix} 2-\lambda & 0 \\ 2 & 1-\lambda \end{bmatrix} - 2 \cancel{\begin{bmatrix} 0 & 0 \\ -2 & 1-\lambda \end{bmatrix}}^{0} - \begin{bmatrix} 0 & 2-\lambda \\ -2 & 2 \end{bmatrix} = 0$

$(2-2\lambda-\lambda+\lambda^2)(-\lambda)$ $+4-2\lambda$

$(-\lambda)(\lambda^2-3\lambda+2) \Rightarrow -\lambda^3+3\lambda^2-2\lambda+4-2\lambda$

$-\lambda^3+3\lambda^2-4\lambda+4=0$

$(\lambda-2)(-\lambda^2+\lambda-2)$

~~$\lambda-2 \overline{)-\lambda^3-3\lambda^2-4\lambda+4}$ quotient $-\lambda^2-7\lambda-18$~~

~~$-\ -\lambda^3+4\lambda^2$~~

~~$0-7\lambda^2-4\lambda$~~

~~$-\ -7\lambda^2+14\lambda$~~

~~$0-18\lambda+4$~~

~~$-18\lambda+36$~~

~~$0+\boxed{40}$~~

$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1-4(-1)(-2)}}{-2}$

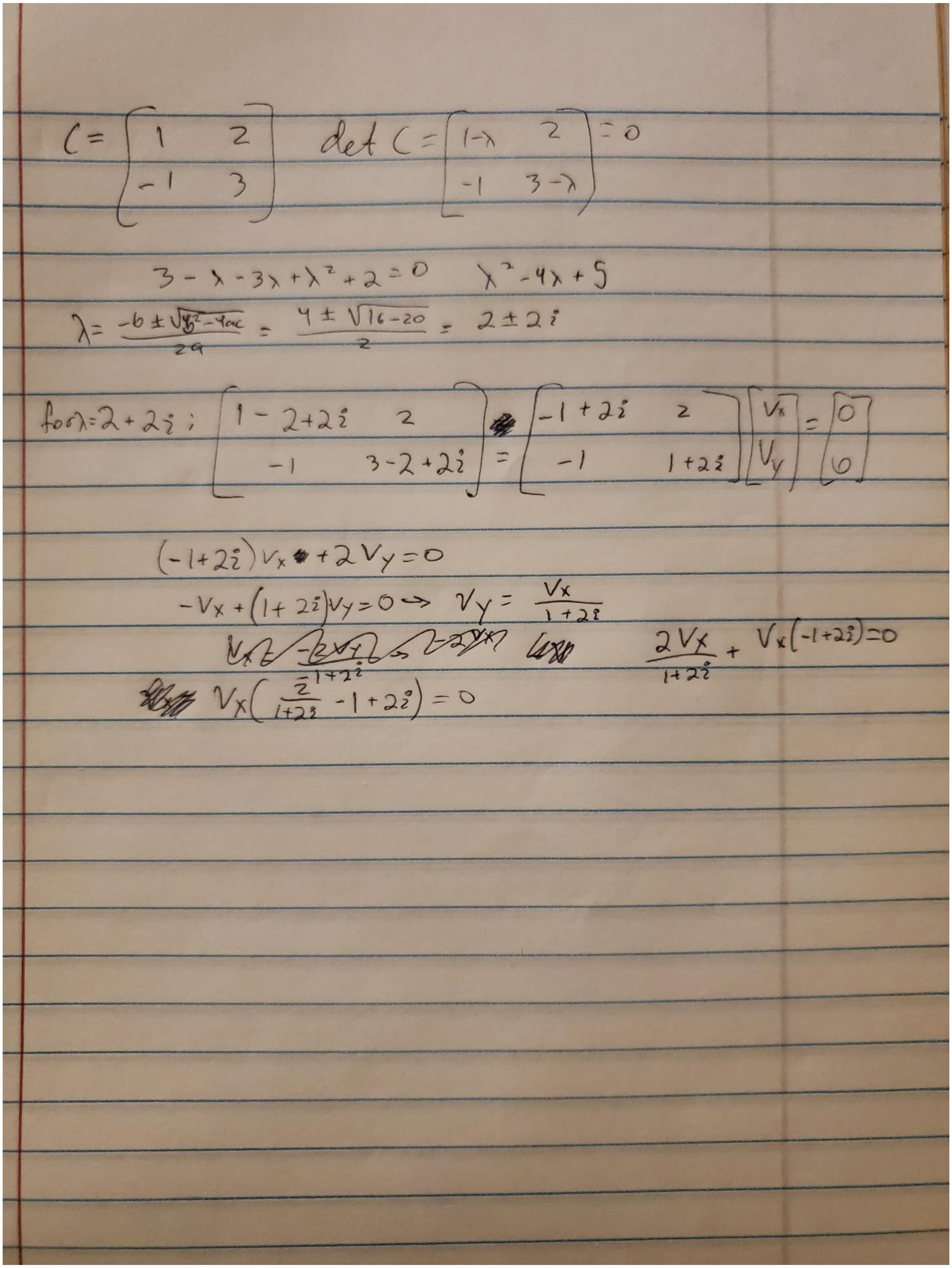
$= \frac{-1 \pm \sqrt{-7}}{-2}$

$= \frac{1}{2} \pm \frac{\sqrt{7}}{2} i$

$\lambda = 2,\ \frac{1}{2} \pm \frac{\sqrt{7}}{2} i$

for $\lambda = 2$: $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 0 & 0 \\ -2 & 2 & -1 \end{bmatrix} \to \begin{bmatrix} -2 & 2 & -1 \\ 0 & 0 & 0 \\ 6 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\vec{V_1} = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$



$C = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ $\det C = \begin{bmatrix} 1-\lambda & 2 \\ -1 & 3-\lambda \end{bmatrix} = 0$

$3 - \lambda - 3\lambda + \lambda^2 + 2 = 0$ $\lambda^2 - 4\lambda + 5$

$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16-20}}{2} = 2 \pm 2i$

for $\lambda = 2 + 2i$; $\begin{bmatrix} 1 - 2+2i & 2 \\ -1 & 3-2+2i \end{bmatrix} = \begin{bmatrix} -1+2i & 2 \\ -1 & 1+2i \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(-1+2i)V_x + 2V_y = 0$

$-V_x + (1+2i)V_y = 0 \rightarrow V_y = \frac{V_x}{1+2i}$

~~$V_x = \frac{-2V_y}{-1+2i}$~~ [illegible] $\frac{2V_x}{1+2i} + V_x(-1+2i) = 0$

$V_x\left(\frac{2}{1+2i} - 1 + 2i\right) = 0$

```
>> A=[1 1 1; 1 -1 0; 0 0 1];
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```
>> [V,D]=eig(A)
V =
    0.9239   -0.3827   -0.8165
    0.3827    0.9239   -0.4082
         0         0    0.4082
D =
    1.4142         0         0
         0   -1.4142         0
         0         0    1.0000
>> B=[0 2 -1; 0 2 0; -2 2 1];
>> [V,D]=eig(B)
V =
   -0.7071    0.4472    0.4851
         0         0    0.7276
   -0.7071   -0.8944    0.4851
D =
    -1     0     0
     0     2     0
     0     0     2
>> C=[1 2; -1 3];
>> [V,D]=eig(C)
V =
   0.8165 + 0.0000i   0.8165 + 0.0000i
   0.4082 + 0.4082i   0.4082 - 0.4082i
D =
   2.0000 + 1.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   2.0000 - 1.0000i
```