1\. a) $f(x,y,z) = x^2 + 2yz$

$$\vec{\nabla} = \delta/\delta x \, \hat{a}_x + \delta/\delta y \, \hat{a}_y + \delta/\delta z \, \hat{a}_z$$

$$= 2x\,\hat{a}_x + 2z\,\hat{a}_y + 2y\,\hat{a}_z$$

$$\boxed{\vec{\nabla} f = \begin{bmatrix} 2x \\ 2z \\ 2y \end{bmatrix}}$$

b) $f(x,y,z) = x\sin(yz)$

$$\vec{\nabla} = \sin(yz)\,\hat{a}_x + xz\cos(yz)\,\hat{a}_y + xy\sin(yz)\,\hat{a}_z$$

$$\boxed{\vec{\nabla} f = \begin{bmatrix} \sin(yz) \\ xz\sin(yz) \\ xy\sin(yz) \end{bmatrix}}$$

c) $f(x,y) = x^3 - 3x - 2y^2$

$$\boxed{\vec{\nabla} f = \begin{bmatrix} 3x^2 - 3 \\ -4y \end{bmatrix}}$$

$$\vec{\nabla} = (3x^2 - 3)\,\hat{a}_x + (-4y)\,\hat{a}_y$$

2\. $f(x,y,z) = 2xy + z^2 \; (1,-1,3)$ $\quad \vec{v} = \hat{a}_x + 2\hat{a}_y + 2\hat{a}_z$

$$\hat{a}_l = \frac{\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z}{3}$$

$$\vec{\nabla} f = 2y\,\hat{a}_x + 2x\,\hat{a}_y + 2z\,\hat{a}_z$$

$$\vec{\nabla} f \cdot \hat{a}_l = \left(\tfrac{1}{3} \cdot 2y\right) + \left(\tfrac{2}{3} \cdot 2x\right) + \left(\tfrac{2}{3} \cdot 2z\right)$$

$$-1 \qquad\qquad 1 \qquad\qquad 3$$

$$-2/3 + 4/3 + 4 = \boxed{4\tfrac{2}{3}}$$

3. $z = x^2 + y^2 \quad (1, -2, 5) \quad f(x, y, z) = x^2 + y^2 - z$

$$\vec{A}_{norm} = \vec{A} - \vec{A}_{tangent} \qquad \vec{A}_{tangent} = \left(\vec{A} \cdot \vec{v}\right) / \left(|\vec{v}|^2\right) \cdot \vec{v}$$

$$[redacted] = x^2 \hat{a}_x + y^2 \hat{a}_y - \hat{a}_z$$

$$\vec{\nabla} f = 2x \hat{a}_x + 2y \hat{a}_y - \hat{a}_z$$

$$\hat{a}_{l\,norm} = \hat{a}_l - \hat{a}_{l\,tan}$$

unit vector: $\dfrac{\vec{\nabla} f}{|\vec{\nabla} f|} = \dfrac{2x \hat{a}_x + 2y \hat{a}_y - 1 \hat{a}_z}{\sqrt{(2x)^2 + (2y)^2 + (-1)^2}}$

$$= \left. \frac{2x \hat{a}_x + 2y \hat{a}_y - \hat{a}_z}{\sqrt{4x^2 + 4y^2 + 1}} \right|_{1, -2, 5}$$

$$\frac{2 \hat{a}_x + (-4) \hat{a}_y - \hat{a}_z}{\sqrt{4 + 8 + 1}}$$

$$\boxed{\frac{2}{\sqrt{13}} \hat{a}_x - \frac{4}{\sqrt{13}} \hat{a}_y - \frac{1}{\sqrt{13}} \hat{a}_z}$$

4. $z = x^2 + y^2 \quad (1, 2, 5)$ and $(2, 4, 2)$

$f(x, y, z) = x^2 + y^2 - z$

$$\left. \frac{2x \hat{a}_x + 2y \hat{a}_y - \hat{a}_z}{\sqrt{4x^2 + 4y^2 + 1}} \right|_{2, 4, 2}$$

$$\frac{4 \hat{a}_x + 8 \hat{a}_y - 1}{\sqrt{16 + 64 + 1} = \sqrt{81} = 9} = \frac{4}{9} \hat{a}_x + \frac{8}{9} \hat{a}_y - \frac{1}{9} \hat{a}_z$$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right) = \cos^{-1}\left(\frac{\left(\frac{4}{9} \cdot \frac{2}{\sqrt{13}}\right) + \left(\frac{8}{9} \cdot \frac{-4}{\sqrt{13}}\right) + \left(\frac{-1}{9} \cdot \frac{-1}{\sqrt{13}}\right)}{\underbrace{\sqrt{\left(\frac{4}{9}\right)^2 + \left(\frac{8}{9}\right)^2 + \left(\frac{-1}{9}\right)^2}}_{1} \underbrace{\sqrt{\left(\frac{2}{\sqrt{13}}\right)^2 + \left(\frac{-4}{\sqrt{13}}\right)^2 + \left(\frac{-1}{\sqrt{13}}\right)^2}}_{4/13 + 8/13 + 1/13 \;\; 1}}\right)$$

$$= \cos^{-1}(-0.709) = \boxed{2.359 \text{ radians}}$$

~~messed up~~ switched $(-1, -1, 2)$ with $(2, 4, 2)$ from #5

4. $$\left.\frac{2x\hat{a}_x + 2y\hat{a}_y - \hat{a}_z}{\sqrt{4x^2 + 4y^2 + 1}}\right|_{-1,-1,2} \longrightarrow \frac{-2\hat{a}_x - 2\hat{a}_y - \hat{a}_z}{\sqrt{4 + 4 + 1} = 3}$$

$$-\frac{2}{3}\hat{a}_x - \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_z$$

$$\theta = \cos^{-1}\left(\frac{\left(\frac{-2}{3}\cdot\frac{2}{\sqrt{13}}\right) + \left(\frac{-2}{3}\cdot\frac{-4}{\sqrt{13}}\right) + \left(\frac{-1}{3}\cdot\frac{-1}{\sqrt{13}}\right)}{\underbrace{\sqrt{\left(\frac{-2}{3}\right)^2 + \left(\frac{-2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2}}_{1}\ \underbrace{\sqrt{\left(\frac{2}{\sqrt{13}}\right)^2 + \left(\frac{-4}{\sqrt{13}}\right)^2 + \left(\frac{-1}{\sqrt{13}}\right)^2}}_{1}}\right) = \cos^{-1}(0.462) = \boxed{1.09 \text{ radians}}$$

5. $3z^3 - 2x^2 - y^2 = 0$ $\quad f(x,y,z) = -2x^2 - y^2 + 3z^3$ $\quad (2,4,2)$

$\vec{\nabla} f = -4x\hat{a}_x - 2y\hat{a}_y + 9z^2\hat{a}_z$

$-4(x-2) - 2(y-4) + 9(z-2)^2 =$ tangent plane

$-4x + \underbrace{8} - 2y + \underbrace{8} + 9z^2 - 36z + \underbrace{36}, \quad 9(z^2 - 4z + 4)$

tangent plane $= \boxed{-4x - 2y + 9z^2 - 36z = 52}$