

MATH 527

① $y' + y \cos t = 0$ $\Rightarrow y = e^{\sin t}$

$\frac{d}{dt}[y e^{\sin t}] = 0 \Rightarrow y e^{\sin t} = C$

$a'b + b'a = (ab)'$ $\Rightarrow y e^{\sin t} = C$

$y = \frac{C}{e^{\sin t}}$

② $y' + \frac{2x+y}{1+x^2} = \frac{1}{1+x^2}$ $\Rightarrow y = e^{\int \frac{2x+y}{1+x^2} dx} \ln(x^2+1)$

$\frac{d}{dx}[y e^{\int \frac{2x+y}{1+x^2} dx}] = \frac{1}{1+x^2}$ $\Rightarrow y = \frac{1}{x^2+1} + C$

③ $y' - 2xy = x$ $\Rightarrow y = e^{\int -2x dx} = e^{-x^2}$ $\Rightarrow y = \frac{1+C}{x^2+1}$

$\frac{d}{dx}[y e^{-x^2}] = x e^{-x^2}$ $\Rightarrow y = \frac{1+C}{x^2+1}$

$\int u dv = v u - \int v du$ $\Rightarrow y = \frac{1+C}{x^2+1}$

$C = 1/2$ $\Rightarrow y = \frac{1+C}{x^2+1}$

④ $y' = x + 5y$ $y(0) = 3$ $\Rightarrow y = \frac{1}{5} e^{5x} - \frac{1}{5}$

$y' - 5y = x$ $\Rightarrow y = \frac{1}{5} e^{5x} - \frac{1}{5}$

$C = 3/25$ $\Rightarrow y = \frac{1}{5} e^{5x} - \frac{1}{5}$

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⑤ $\frac{2x-1}{x} + (3y+7)y' = 0$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int (2x-1) dx = x^2 - x + C_1$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial y}[x^2 - x + C_1] = 0$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int (3y+7) dy = \frac{3}{2}y^2 + 7y + C_2$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

⑥ $\tan(x) - \sin(x) \sin(y) + (\cos(x) \cos(y))y' = 0$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial x}[\tan(x) - \sin(x) \sin(y)] = \sec(x) \cos(y)$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial y}[\cos(x) \cos(y)] = -\cos(x) \sin(y)$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial x}[\cos(x) \cos(y)] = -\sin(x) \cos(y)$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

⑦ $x^2 - y^2 + (x^2 - 2xy)y' = 0$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial y}[x^2 - y^2] = -2y$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial x}[(x^2 - 2xy)y'] = 2x - 2y$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

⑧ $xy' = 2te^t - y + 6t^2$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial y}[xy] = x$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

⑨ $(x+y)^2 + (2xy + x^2 - 1)y' = 0$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial y}[(x+y)^2] = 2(x+y)$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$