

HW #1

1. compute $2e^{-j\frac{\pi}{3}} + 2\sqrt{3}e^{j\frac{\pi}{6}}$

$$M_1 e^{j\theta_1} + M_2 e^{j\theta_2}$$

$$2\left(\cos\left(\frac{5\pi}{3}\right) + j\sin\left(\frac{5\pi}{3}\right)\right) + 2\left(\cos\left(\frac{\pi}{6}\right) + j\sin\left(\frac{\pi}{6}\right)\right)$$

$$2\cos\left(\frac{5\pi}{3}\right) + 2j\sin\left(\frac{5\pi}{3}\right) + 2\cos\left(\frac{\pi}{6}\right) + 2j\sin\left(\frac{\pi}{6}\right)$$

$$2\cos\left(\frac{5\pi}{3}\right) + 2\cos\left(\frac{\pi}{6}\right) + j\left(2\sin\left(\frac{5\pi}{3}\right) + 2\sin\left(\frac{\pi}{6}\right)\right)$$

$$1 + 3 + j(-\sqrt{3} + \sqrt{3})$$

$$\boxed{4}$$

1. Derive $\cos(2x) = \cos^2(x) - \sin^2(x)$ and $\sin(2x) = 2\cos(x)\sin(x)$

$$e^{ix} = \sum_{n=1}^{\infty} \frac{(ix)^n}{n!} = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) + j\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right)$$

$$= \cos(x) + j\sin(x)$$

$$e^{ix} e^{iy} = (\cos(x) + j\sin(x))(\cos(y) + j\sin(y))$$

$$= \cos(x)\cos(y) + j\sin(x)\cos(y) + j\sin(y)\cos(x) + j^2\sin(x)\sin(y)$$

$$= (\cos(x)\cos(y) - \sin(x)\sin(y)) + (j\sin(x)\cos(y) + j\sin(y)\cos(x))$$

$$e^{ix} e^{iy} = e^{i(x+y)} = \cos(x+y) + j\sin(x+y)$$

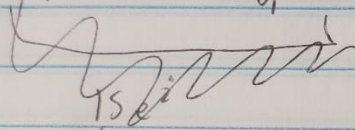
$$= (\cos(x) + j\sin(x))(\cos(y) + j\sin(y))$$

$$= \cos(x)\cos(y) + j\sin(x)\cos(y) + j\sin(y)\cos(x) + j^2\sin(x)\sin(y)$$

$$\boxed{\begin{aligned}\cos(2x) &= \cos^2(x) - \sin^2(x) \\ \sin(2x) &= 2\sin(x)\cos(x)\end{aligned}}$$

1. Compute $15 \cos(377t + \frac{\pi}{4}) + 15 \cos(377t + \frac{\pi}{12})$

$\begin{matrix} A_1 & \omega & \theta_1 & A_2 & \omega & \theta_2 \end{matrix}$



$$\operatorname{Re}\{15e^{i(377t + \frac{\pi}{4})}\} + \operatorname{Re}\{15e^{i(377t + \frac{\pi}{12})}\}$$

$$\operatorname{Re}\{e^{i377t} (15e^{i\frac{\pi}{4}} + 15e^{i\frac{\pi}{12}})\} \quad \frac{\pi}{12} + \frac{\pi}{4} \cdot \frac{2}{2} = \frac{3\pi}{12} + \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$$

$$\operatorname{Re}\{e^{i377t} 30e^{i\frac{\pi}{3}}\} = 30 \cos(377t + \frac{\pi}{3})$$

2. MATLAB

```
>> t=-pi/96:0.001:pi/96;
>> X = 15*cos(377*t+pi/4);
>> Y=15*cos(377*t+pi/12);
>> A=X+Y;
>> plot(t,X,t,Y,t,A)
>> legend('X=15*cos(377*t+pi/4)', 'Y=15*cos(377*t+pi/12)', 'A=X+Y')
>> xlabel('t')
>> ylabel('x(t)')
>> title('t vs. x(t)')
```

