Homework # 6

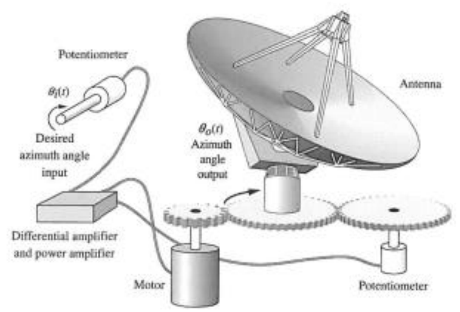
<u>**Instructions:**</u> Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and <u>box all your solutions</u>. All Matlab code needs to be published, with your name and date at the top of the script, and <u>all figures needs to have proper axis labeling and legends</u>. Homework assignments will be collected during class time on the due date. *Late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- <u>**Homework problems not to be graded**</u>
  - From textbook (Lathi):
    - Ch 4: 2-2, 3-1, 3-5, 3-13, 3-17, 5-4, 8-2, 8-4

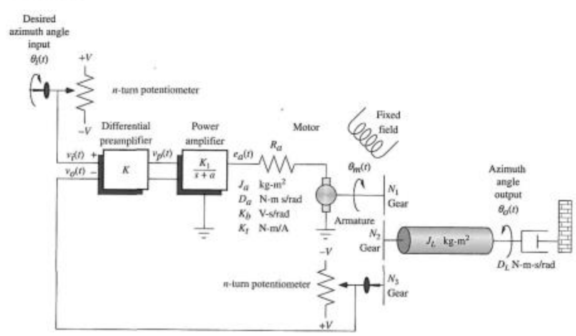
- <u>**Homework problems to be graded**</u>

Consider the azimuth antenna position control problem, where a potentiometer angle is the reference input to control the orientation of the antenna. The layout and schematic drawings of the position control system is shown below.

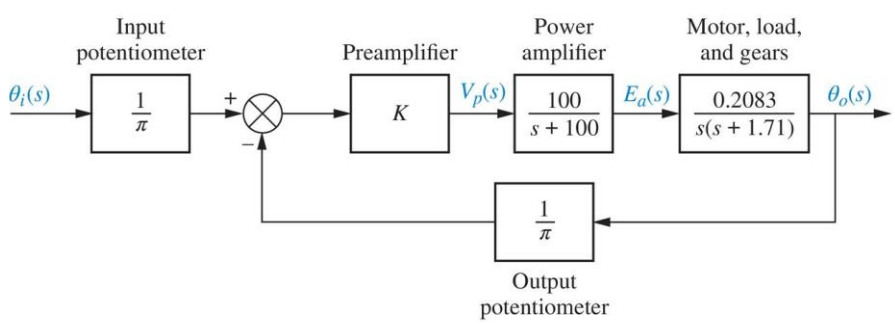
Layout



Schematic



The simplified block diagram of the antenna position control system is provided below



a) Find the transfer function $T(s)$ corresponding to the closed-loop system from reference input angle $\theta_i(t)$ to output angle $\theta_o(t)$. <u>Show all work</u> $\left(\text{Hint: } T(s) = \frac{20.83K}{\pi s(s+100)(s+1.71)+20.83K}\right)$

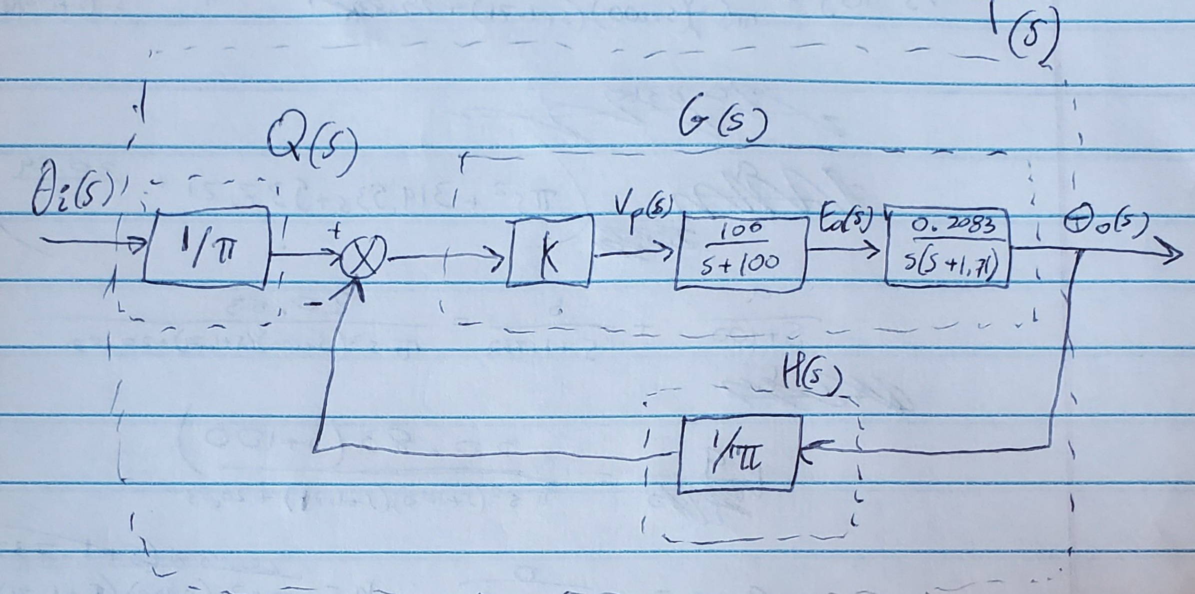
b) Find the unit step response of $\theta_o(t)$ for $K = 1$. Using the Final Value property of the Laplace transform, find the steady state value of the error signal $e(t) = \theta_o(t) - \theta_i(t)$?

c) Compute the unit step response of the closed-loop system (a) using the *step* command in Matlab. Transfer functions can be defined using the *tf* command. Is this the same response you found in Part (b)? Plot and compare to your solution to Part (a).

d) Increase the preamplifier gain to $K = 100$ and use the *step* command to obtain the step response in Matlab. How did the response change from Part (c)? Plot and compare.

a)

$$\left(\theta_i(s) * \frac{1}{\pi}\right) - \left(\theta_o(s) * \frac{1}{\pi}\right)$$



$$T(s) = \theta_o(s)/\theta_i(s) = \frac{G(s)\,Q(s)}{1 + G(s)\,H(s)} \qquad G(s) = \frac{20.83K}{(s+100)(s)(s+1.71)}$$

$$H(s) = 1/\pi \qquad Q(s) = 1/\pi$$

$$G(s) * Q(s) = \frac{20.83k}{\pi(s+100)(s)(s+1.71)} = G(s) * H(s)$$

$$\frac{\pi(s+100)(s)(s+1.71)}{\pi(s+100)(s)(s+1.71)} + \frac{20.83k}{\pi(s+100)(s)(s+1.71)}$$

$$= \frac{\pi(s+100)(s)(s+1.71) + 20.83K}{\pi(s+100)(s)(s+1.71)}$$

$$T(s) = \frac{20.83k}{\pi\cancel{(s+100)(s)(s+1.71)}} \cdot \frac{\cancel{\pi(s+100)(s)(s+1.71)}}{\pi(s+100)(s)(s+1.71) + 20.83k} \qquad V(s) =$$

$$\boxed{= \frac{20.83k}{\pi(s+100)(s)(s+1.71) + 20.83K}}$$

b) $\theta_o(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}T(s)\right\}$

$$\frac{1}{s}T(s) = \frac{20.83K}{\pi(s^2)(s+100)(s+1.71)+20.83K} = \frac{20.83}{\pi}\cdot\frac{1}{s^2(s+100)(s+1.71)+\frac{20.83}{\pi}}$$

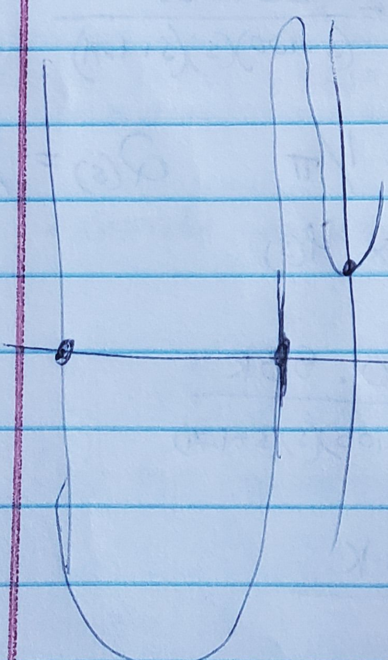
$$\left(\pi s^2 + 319.53s + 537.21 + \frac{20.84}{s^2}\right)s^2$$

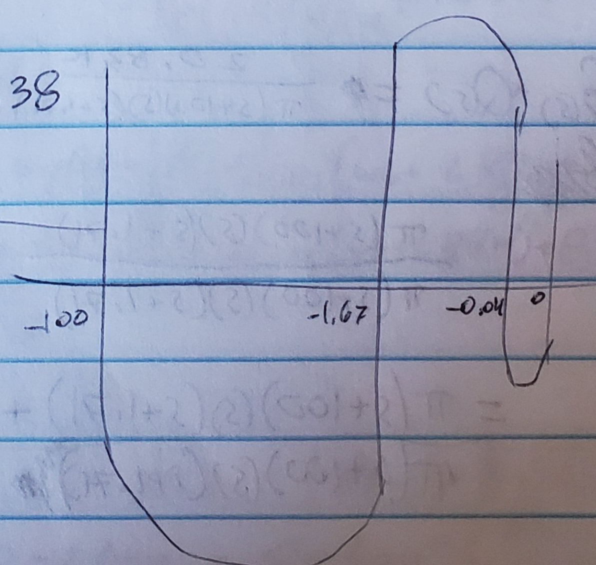
$$\frac{a}{s+100} + \frac{b}{s+1.732} = \frac{20.83}{\pi s^2(s+100)(s+1.71)+20.83}$$

$$a = \frac{20.83(s+100)}{\pi s^2(s+100)(s+1.71)+20.83} \quad 0$$

$$a = 0 = \frac{0}{20.83} \qquad b = \frac{20.83(s+1.732)}{\pi s^2(s+100)(s+1.71)+20.83}$$

$$\frac{20.83}{1}\cdot\frac{1}{\pi(s^2)(s+100)(s+1.71)+20.83s}$$





$$\frac{20.83}{\pi s^4 + 101.71\pi s^3 + 171\pi s^2 + 20.83s}$$

$$\frac{1}{(s)(s+0.04)(s+1.67)(s+100)}\cdot\frac{20.83}{\pi}$$

$$\frac{6.63}{s(s+0.04)(s+1.67)(s+100)} = \frac{a}{s} + \frac{b}{s+0.04} + \frac{c}{s+1.67} + \frac{d}{s+100}$$

$$= \frac{0.96}{s} - \frac{1.02}{s+0.04} + \frac{0.02}{s+1.67} - \frac{0}{s+100}$$

$$\boxed{y(t) = 0.96 - 1.02e^{-0.04t} + 0.02e^{-1.67t} - 0e^{-100t}}$$

$$E(s) = \theta_i - \theta_o = \theta_i - G(s)K\,E(s) \Rightarrow (1 + G(s)K)E(s) = \theta_i(t)$$

$$E(s)/\theta_i(t) = \frac{1}{1+G(s)K} = \frac{1}{1 + \frac{20.83K^2}{s(s+100)(s+1.71)}} \qquad G(s) = \frac{20.83K}{(s+100)s(s+1.71)}$$

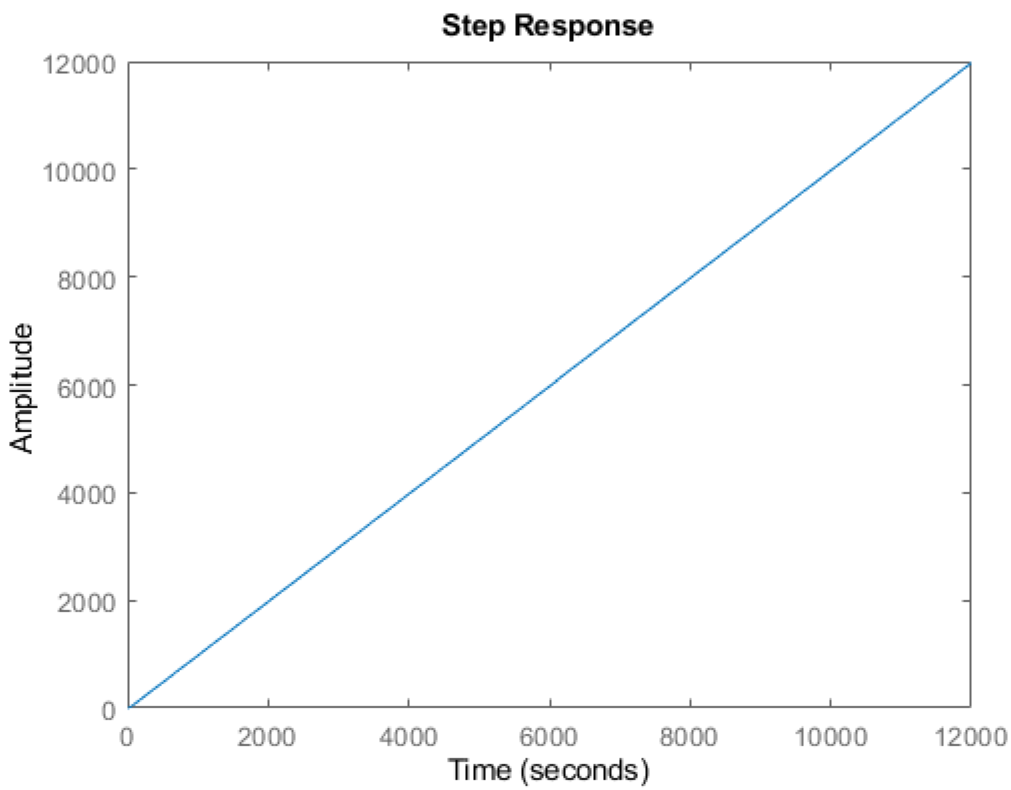
$$\frac{1}{\frac{s(s+100)(s+1.71)+20.83K^2}{s(s+100)(s+1.71)}} = \frac{s(s+100)(s+1.71)}{s(s+100)(s+1.71)+20.83K^2}$$

$$E(s) = \frac{s(s+100)(s+1.71)}{s(s+100)(s+1.71)+20.83K^2} \cdot \frac{1}{s^2} = \frac{(s+100)(s+1.71)}{s^2(s+100)(s+1.71)+20.83K^2 s}$$

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0}\left(\frac{(s+100)(s+1.71)}{s(s+100)(s+1.71)+20.83K}\right)$$

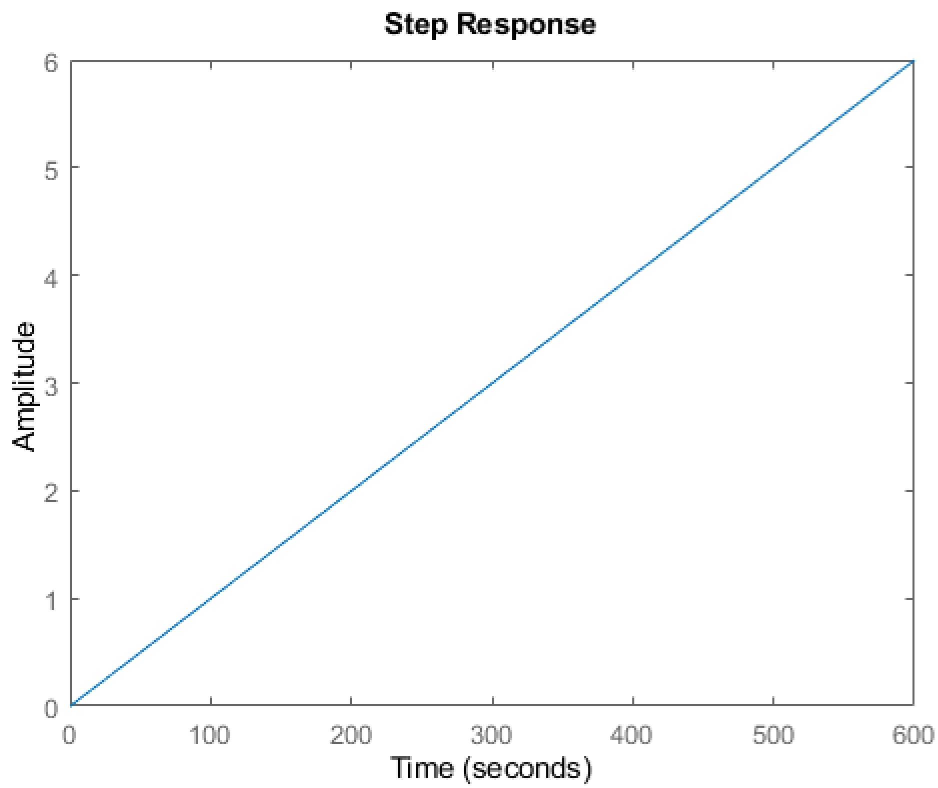
$$= 171/20.83K,\ K=1 \rightarrow \boxed{8.21}$$

c)




```
>> one=1;
>> two=101.71;
>> three=171;
>> four=20.83/pi;
>> five=0;
>> num=20.83/pi;
>> den=[one two three four five];
>> sys=tf(num,den)
sys =
                6.63
  ----------------------------------
  s^4 + 101.7 s^3 + 171 s^2 + 6.63 s
Continuous-time transfer function.
>> step(sys);
```

d)




```
>> four=2083/pi;
>> den=[one two three four five];
>> sys=tf(num,den)
sys =
                 6.63
  ----------------------------------
  s^4 + 101.7 s^3 + 171 s^2 + 663 s
Continuous-time transfer function.
>> step(sys);
```

It doesn't look like it changed at all but that may be because of a mistake on my part