Homework # 8

<u>**Instructions:**</u> Prepare your deliverables in clean letter size printer-quality papers with a high-contrast pencil (engineering pads are also accepted). Attach this assignment sheet as cover page, show all your work, and <u>box all your solutions</u>. All Matlab code needs to be published, with your name and date at the top of the script, and <u>all figures needs to have proper axis labeling and legends</u>. Homework assignments will be collected during class time on the due date. *Late homework or submission that do not strictly follow the provided instructions will not be accepted.*

- **<u>Homework problems not to be graded</u>**
  - o From textbook (Lathi):
    - Ch 4: 9-4, 9-5, 9-6

- **<u>Homework problems to be graded</u>**
  - o Consider a system with transfer function

$$H(s) = \frac{s(s + 10)}{(s^2 + 50s + 2500)}$$

  - a) Find the poles, zeros and gain of the transfer function. Re-write the transfer function in the normalized form needed to sketch the Bode plots.
  - b) Sketch the amplitude and phase asymptotes corresponding to the contributions of the poles to the Bode plot. Clearly mark each asymptote the corresponding pole, slope and all relevant frequencies
  - c) Sketch the amplitude and phase asymptotes corresponding to the contributions of the zeros to the Bode plot. Clearly mark each asymptote the corresponding zero, slope and all relevant frequencies
  - d) Combine solutions to (b) and (c) to sketch the asymptotes corresponding to the final Bode plots. Label all axes

a)

$$H(s) = \frac{s(s+10)}{(s^2 + 50s + 2500)} = \frac{\phi(10)(\text{zeros: } 0, -10)}{-25 - \frac{i}{2}25\sqrt{3}(25 + i\,225\sqrt{3})}$$

$$s = \frac{-50 \pm \sqrt{2900 - 10000}}{2} = \frac{-50 \pm i\,50\sqrt{3}}{2} = -25 \pm i\,25\sqrt{3}$$

Poles: $-25 \pm i\,25\sqrt{3}$ $\omega_n = 100$

$$\left(-25 - i\,25\sqrt{3}\right)\left(-25 + i\,25\sqrt{3}\right) = 629 - i\,625\sqrt{3} + i\,625\sqrt{3} + 1875$$

$$= 2500$$

$$\frac{\phi(10)}{2500} = \frac{\phi}{250} = K = \frac{1}{250}$$

$$H(s) = \frac{1}{250} \cdot \frac{s\left(\frac{s}{10} + 1\right)}{\left(0.1 - i\frac{\sqrt{3}}{10}\right)}$$

$$H(s) = \frac{1}{250} \cdot \frac{s\left(\frac{s}{10} + 1\right)}{\left(\frac{s}{250} + \frac{1}{10} - \frac{i\sqrt{3}}{10}\right)\left(\frac{s}{250} + \frac{1}{10} + \frac{i\sqrt{3}}{10}\right)}$$

$$= \frac{1}{250}\left(\frac{s\left(\frac{s}{10} + 1\right)}{\frac{s^2}{250} + \frac{2s}{10} + 10}\right) = \frac{10}{2500} \cdot \frac{s\left(\frac{s}{10} + 1\right)}{\left(\frac{s^2}{2500} + \frac{2s}{100} + 1\right)}$$

$$\frac{s^2}{50^2} + 2\frac{0.5}{50}s + 1$$

$$K = \frac{1}{250} = 20\log_{10}\left(\frac{1}{250}\right)\,dB = -48\,dB$$

b) Amplitude (dB) / Phase (deg)

Amplitude axis: 0, -20, -40, -60, -80
Frequency axis: 0.1, 0.51, 5 10, ωn 100, 1000 ω, 50, 500

P@ωn=50
-40dB/dec

Phase axis: 0, -180
Frequency: 0.1, 0.51, 5 10, 50 100, 501 000 ω
P@ωn=50

c) Amplitude (dB)

Amplitude axis: 30, 20, 10, 0, -10, -20
Frequency axis: 0.1, 1, 10, 100, 1000 ω

z@0, 20dB/dec
z@-10, 20dB/dec

Phase (deg)

Phase axis: 135, 90, 45, 0, -45
Frequency: 0.1, 1, 10, 100, 1000 ω

z@0
z@-10

d) Amplitude (dB)

Amplitude axis: -48 +40, -48 +20, -48 +0, -48 -20, -48 -40, -48 -60
Frequency axis: 0.1, 0.51, 5 10, 50 100, 500 000 ω

z@0, 20dB/dec
z@-10, 20dB/dec
P@50=ωn, -40dB/dec

Phase (deg)

Phase axis: 90, 0, -90, -180
Frequency: 0.1, 0.51, 5 10, 50 100, 501 000 ω

z@0
z@-10,
P@ωn=50