Chapters 1: Probability Basics

Set Theory (1.1)

- **Definition**: a **set** is a collection of things/elements

  Example: discrete sets

  Example: continuous sets

  - Set inclusion is denoted by $\in$

  - *Subset*: a set with all its elements also being elements of another set

  - *Null set:* empty set

  - *Universal set:* set of all things that are considered in a given context

    Note: 1) every set is a subset of the universal set, and 2) the null set is a subset of every set

- *Venn Diagrams* are used to graphically display relationships among sets

- ***Set algebra*:**
  - Union: the union of two sets *A* and *B* is the set of all elements in *A or B*
  - Intersection: the intersection of two sets *A* and *B* is the set of all elements in both *A and B*
  - Complement: the complement of a set *A* is the set of all elements in the universal set *S* that are not in *A*
- Set algebra properties:
  - Commutative:
  - Associative:
  - Distributive:
  - De Morgan's Law:
- **Definition**: sets *A* and *B* are **mutually exclusive (disjoint)** if

- **Definition**: a collection of sets $A_1, A_2, \dots, A_n$ is a **partition** if

Set Theory in Probability (1.2, 1.3)

A probability is based upon a repeatable experiment that consists of a procedure and observations (outcomes). *Example: roll a die*

- Outcome: any possible observation of the experiment (set elements)

- Sample space: set of all possible outcomes (universal set)

- Event: a set of outcomes of an experiment (a subset of the sample space)

Probability axiom: a probability measure $\mathrm{P}[\cdot]$ is a function that maps events to a real number such that

  - Axiom 1 (non-negative):
  - Axiom 2 (normalized):
  - Axiom 3 (additive):

- Properties of probability measure (*Thm 1.4*):
  - $\mathrm{P}[A^C] =$
  - $\mathrm{P}[\emptyset] =$
  - $\mathrm{P}[A \cup B] =$

- Discrete Probability Law (*Thm 1.5*): the probability of an event $B = \{s_1, s_2, \dots, s_n\}$ equals

- <u>Discrete Uniform Probability Law (*Thm 1.6*):</u> for an experiment with sample space $S = \{s_1, s_2, \dots, s_n\}$ with equally likely outcomes s_i

- <u>Example:</u> experiment where a fair coin is tossed four times
  - Sample space
  - P[*A*] where *A*={"exactly 1 heads"}
  - P["exactly *n* heads"]=

- <u>Example</u>: experiment where a die is tossed, and the face number is noted
  - Sample space
  - $P[A \cup B]$ where A={"face # $\leq$ 3"} and $B = \{\text{"face \# = 3"}\}$

- <u>Example</u>: reliability analysis of a circuit with two switches

  At least one switch needs to be closed for the circuit to operate

  - Sample space

<u>Conditional Probability (1.4)</u>

- **<u>Definition</u>**: the **conditional probability** of an event *A* given the occurrence of an event *B* is given as

  <u>Note</u>: P[*A*] and P[*B*] must be nonzero

- **<u>Properties of conditional probability</u>**
  - Axiom 1:

- Axiom 2:
- Axiom 3:

- Example: roll two fair four-sided dice
  - Sample space
  - Let events $A = \{X_1 \geq 2\}$ and $B = \{X_2 \geq X_1\}$

Partitions and the Law of Total Probability

A partition divides the sample space of an experiment into mutually exclusive sets

- Facilitates the calculation of probability functions.
- Enabled by the Law of Total Probability

- Consider the sample space S with partition $\{A_1, A_2, A_3, A_4\}$, and event B
  - Suppose we want to calculate $\mathrm{P}[B]$

- Law of Total Probability (*Thm 1.10*): For a partition $\{A_1, A_2, A_3, A_4\}$ with $\mathrm{P}[A_i] > 0$ for all i

    - Note: for a set A, $\{A, A^c\}$ is a partition and

- Example: a manufacturer makes resistors from 3 machine $\{B_1, B_2, B_3\}$. Resistors within 50Ω of nominal are considered acceptable. The following data is known:
    - 80% of resistors from B_1 are acceptable
    - 90% of resistors from B_2 are acceptable
    - 60% of resistors from B_3 are acceptable

  Each hour, B_1 produces 3000 resistors, B_2 produces 4000 resistors, and B_3 produces 3000 resistors. All resistors are mixed and shipped together. What is the probability that the manufacturer's resistor in a shipment is acceptable.

- Example: internet packets are sent from Durham to Los Angeles, with a probability of 75% that they are routed through New York. Of routed through New York, 33% of the packets are dropped, otherwise the probability of dropped packets is 25%. What is the total probability of a dropped packet?

- Bayes' Theorem (*Thm 1.11*):

- Example: a patient is tested for a disease that affects 1% of the population, and the is 95% accurate. What is the probability that a patient has the disease after a positive test?

<u>Statistical Independence (1.6)</u>

Definition: events A and B are **independent** if and only if

- <u>Example</u>: experiment is to draw a random card from a deck
  - Event *A*={"red card”} and *B*={“spade card”}
  - Event *A*={“red card”} and *B*={“ace card”}