ECE 634 Hunter W. Nick S. Andrew P.

Homework 7

5.1-1 $x(n) = (-1)^n \left(u(n) - u(n-8)\right)$

$\Rightarrow \mathcal{Z}\{x(n)\} = \sum_{-\infty}^{\infty} (-1)^n z^{-n} \left(u(n) - u(n-8)\right) = \sum_{n=0}^{7} (-1)^n z^{-n} = \sum_{0}^{7} (-1/z)^n$

$\Rightarrow \mathcal{Z}\{x(n)\} = \sum_{0}^{7} (-1/z)^n = \dfrac{1 - (-1/z)^{7+1}}{1 - (-1/z)} = \dfrac{1 - z^{-8}}{1 + z^{-1}}$ ROC $|z| > 0$

$\Rightarrow$ pole $z = -1$ zeros at intervals of $\pi/8$ cause $z = e^{[illegible]}$ from $k = 0, 1, 2, 3, 4, 5, 6, 7$

$\Rightarrow$ zeros $= \pi/8, \pi/4, 3\pi/8, \pi/2, 5\pi/8, 3\pi/4, 7\pi/8, 0$

5.1-6

$\mathcal{Z}\{u[n] - u[n-2]\} = \dfrac{z^2 - 1}{z(z-1)} = \dfrac{(z-1)(z+1)}{z(z-1)} = \dfrac{z+1}{z}$

b) $\mathcal{Z}\{\gamma^{n-2} u[n-2]\} = \dfrac{1}{z^2}\left(\dfrac{z}{z-\gamma}\right) = \dfrac{1}{z(z-\gamma)}$

c) $x(n) = 2^{n+1} u(n-1) + e^{n-1} u(n) = 2^{(n+2-1)} u(n-1) + e^{n} e^{-1} u(n)$

$\Rightarrow x(n) = 2^2 2^{(n-1)} u(n-1)$

$\mathcal{Z}\{x(n)\} = \dfrac{4}{z(z-2)} + \dfrac{1}{e} \dfrac{z}{z-e}$

d) $x(n) = \left[2^{-n} \cos(\pi n/3)\right] u[n-1] = (1/2)^n \cos(\pi n/3) u[n] - \delta[n]$

$\Rightarrow \mathcal{Z}\{x(n)\} = \dfrac{z(z-0.5)}{(z^2 - 1.0z + 0.25)}$

5-1.6

e) $x[n] = n\gamma^n u[n-1] = (n+1-1)\gamma^{n+1-1} u[n-1] = \gamma u[n-1] + (n-1)\gamma^{n-1} u[n-1]$

$$\mathcal{Z}\{x[n]\} = \frac{\gamma}{z-1} + \frac{\gamma}{(z-\gamma)^2}$$

f) $x[n] = n(n-1)(n-2)2^{n-3}u[n-m]$, for $m=3$

$\triangleright\ x[n] = 2^{-3}\left[n(n-1)(n-2)2^n u[n]\right]$

$\triangleright\ x[n] = 2^{-3}\left[\frac{3!2^3\,(n(n-1)(n-2)2^n u[n]}{3!2^3}\right]$

$\triangleright\ \mathcal{Z}\{x[n]\} = 2^{-3}\frac{3!2^3 z}{(z-2)^4} = \frac{3!z}{(z-2)^4} = \frac{6z}{(z-2)^4}$

g) $x[n] = (-1)^n n u[n]$

$$\mathcal{Z}\{x[n]\} = \frac{-z}{(z+1)^2}$$

h) $x[n] = \sum_{n=0}^{\infty} k\delta[n-2k+1] = k\sum_{n=0}^{\infty}\delta[n-2k+1]$

$\triangleright\ \sum_{n=0}^{\infty} k\left(\sum_{n=0}^{\infty}\delta[n-2k+1]z^{-n}\right) = \sum_{n=0}^{\infty} kz^{-(2k-1)} = z\sum kz^{-2k}$

$\triangleright\ z\,\frac{1/z^2}{[(1/z)^2-1]^2}$

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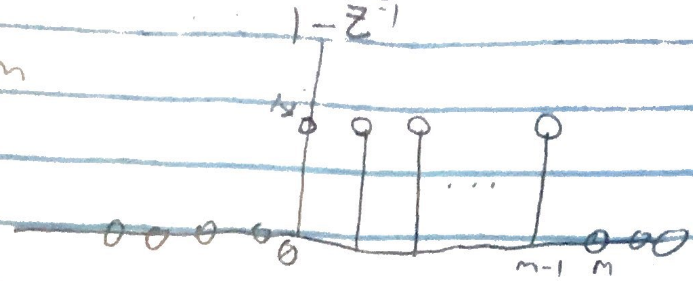
5.2-1 Show $X[z] = \frac{1-z^{-m}}{1-z^{-1}}$

from



from the Graph I can tell this is a combination of two $U(n)$ functions specifically $U(n) - U(n-m)$: $n=m$ Start of Second function

Now $\mathcal{Z}\{U(n) - U(n-m)\}$

$$\mathcal{Z}\{U(n)\} - \mathcal{Z}\{U(n-m)\}$$

$$= \frac{z}{z-1} - z^{-m}\frac{z}{z-1} \rightarrow \frac{z - z^{-m+1}z}{z-1}$$ Divide each by z

$$\frac{\frac{z}{\cancel{z}} - \frac{z^{-m}z}{z}}{\frac{z}{z} - \frac{1}{z}} = \frac{1 - z^{-m}}{1 - \frac{1}{z}}$$

Which is:

$$X(z) = \frac{1-z^{-m}}{1-z^{-1}}$$

Reference: Table 5.1 textbook

5.2-13 a.) $x(n) \to X(z)$ Show

$$\sum_{k=0}^{n} x[k] \to \frac{zX(z)}{z-1}$$

$$x(n) \to \sum_{k=0}^{n} x(k)$$ same as $$\sum_{k=0}^{\infty} x(k)u(n-k)$$

using convolution property and table 5.1

we get $u(n) * x[n]$

$$\mathcal{Z}\{u(n) * x(n)\} = \frac{z}{z-1} X(z)$$

b.) $\delta(n-k) \to u(n)$

also know $\mathcal{Z}\{\delta(n)\} = 1$

using a.) $u[n] * x(n) = \frac{z}{z-1} X(z)$

$$x(n) = \delta(n) \to 1 = X(z)$$

$$\frac{z}{z-1} X(z) = \frac{z}{z-1}(1)$$

5.2-14 A number of causal time-domain functions are shown in Fig. P5.2-14. List the function of time that corresponds to each of the following functions of z. Few or no calculations are necessary! Be careful, the graphs may be scaled differently.

a) $\frac{z^2}{(z-0.75)^2}$ ~~X~~ 13

b) $\frac{z^2-0.9z/\sqrt{2}}{z^2-0.9\sqrt{2}z+0.81}$ 9

c) $\sum_{k=0}^{4} z^{-2k}$ 18

d) $\frac{z^{-5}}{1-z^{-1}}$ 10

e) $\frac{z^2}{z^4-1}$ 15

f) $\frac{0.75z}{(z-0.75)^2}$ 7

g) $\frac{z^2-z/\sqrt{2}}{z^2-\sqrt{2}z+1}$ 4

h) $\frac{z^{-1}-5z^{-5}+4z^{-6}}{5(1-z^{-1})^2}$ 6

i) $\frac{z}{z-1.1}$ 16