The Large Scale Universe

Nick van Eijndhoven

nick@icecube.wisc.edu http://www.iihe.ac.be

Vrije Universiteit Brussel - IIHE(ULB-VUB)
Pleinlaan 2, B-1050 Brussel, Belgium

Contents

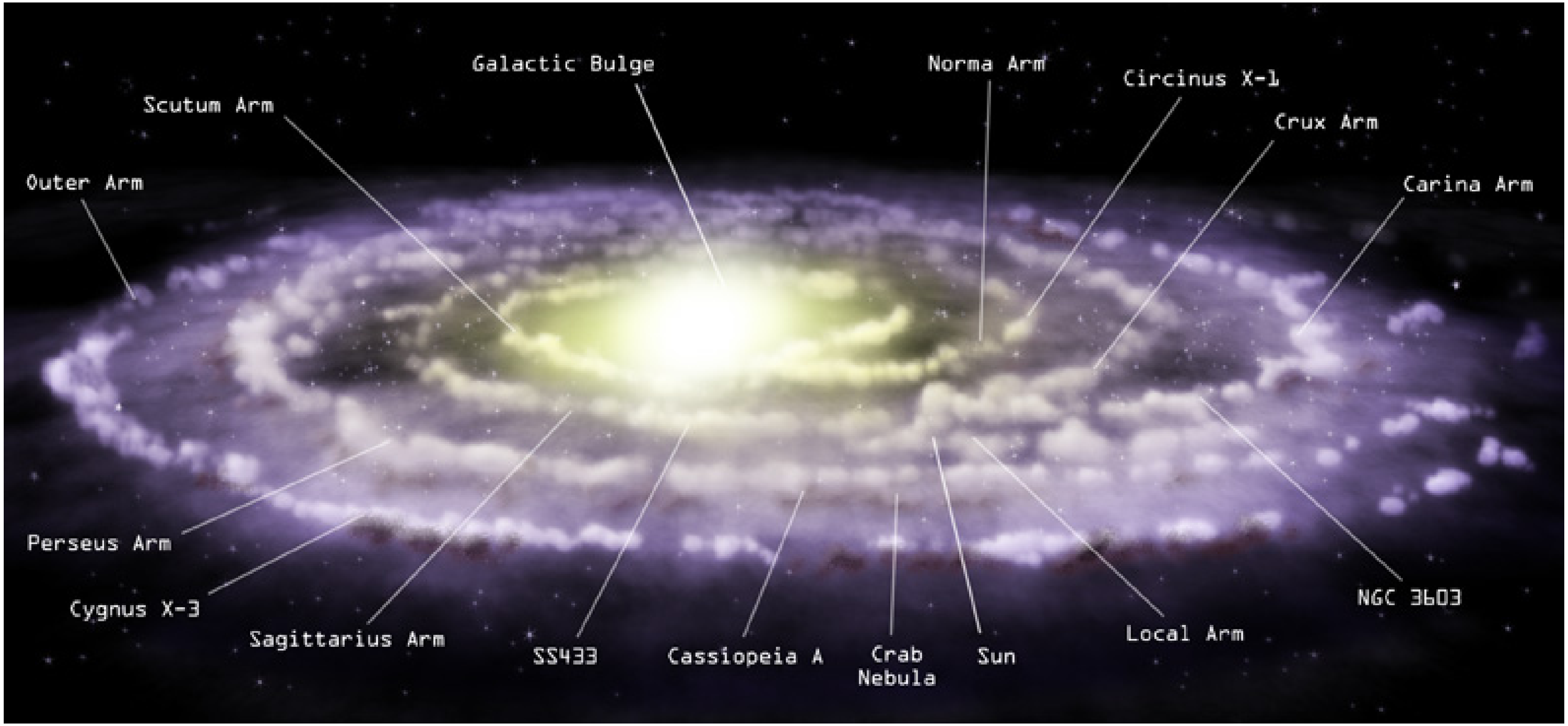
Introduction	1
Spatial curvature	4
The Robertson-Walker metric	7
The Friedmann-LeMaître equation	10
Critical density, Dark energy and Cosmological constant	12
Cosmological redshift	14
Measurement of Cosmological distances in a Flat Universe	19

Introduction

- Observations in the neighbourhood of our solar system and galaxy

  Concentrations of matter (Sun, planets, stars,...) and rather empty regions

  Things look different in different directions (e.g. galactic plane and celestial poles)



Introduction

- Somewhat larger scale : Clusters of galaxies $\longrightarrow$ structures of stars etc... disappear

  Still a bit "clumpy" but things start to look the same in whatever direction one looks

Introduction

- Quantitative investigation of the structure of the universe

  Consider spheres with radius R and place them at random locations in the universe

  Determine the energy (incl. mass) density ρ_i within each sphere

  Calculate the average density $\bar{\rho}$ from all these spheres

  Observation : fractional density fluctuations $(\rho_i - \bar{\rho})/\bar{\rho} \propto R^{-\alpha} \qquad (\alpha > 0)$

* At very large scales : Uniform density $\longrightarrow$ homogeneous universe

  Random placement of our test sphere $\longrightarrow$ Same observations from any location

* At very large scales the universe becomes isotropic

- Large Scale Universe can be described as a cosmic fluid and we are part of it

  How to describe the space-time structure (metric) of this cosmic fluid ?

Spatial curvature

- At large scales the universe is homogeneous and isotropic

  If space is curved : curvature should be the same everywhere $\longrightarrow$ constant curvature K

$*$ Metric can be determined in the same way as for the Schwarzschild case

  Isotropic $\longrightarrow$ use spherical coordinates and write spatial metric as

$$\mathrm{d}s^2 = f(r)(\mathrm{d}r)^2 + r^2(\mathrm{d}\theta)^2 + r^2\sin^2(\theta)(\mathrm{d}\varphi)^2$$

- Reduce to 2-dim. case using $\theta = \frac{1}{2}\pi$ and $\mathrm{d}\theta = 0 \longrightarrow \mathrm{d}s^2 = f(r)(\mathrm{d}r)^2 + r^2(\mathrm{d}\varphi)^2$

  For this 2D-surface : $x_1 = r \quad x_2 = \varphi \longrightarrow g_{\mu\nu} = \mathrm{diag}(f(r), r^2)$

- Gauss : $K = \dfrac{1}{2rf^2(r)} \cdot \dfrac{\mathrm{d}f(r)}{\mathrm{d}r} \longrightarrow f(r) = \dfrac{1}{C - Kr^2} \quad (C = \text{constant})$

  Flat space : $K \equiv 0$ and $f(r) \equiv 1 \longrightarrow C = 1$

$*$ Spatial metric for constant curvature K

$$\boxed{\mathrm{d}s^2 = \frac{(\mathrm{d}r)^2}{1 - Kr^2} + r^2(\mathrm{d}\theta)^2 + r^2\sin^2(\theta)(\mathrm{d}\varphi)^2}$$

Spatial curvature

Some aspects of space with constant curvature K

- Relation between radial coordinate r and physical distance $D(r)$

$$D(r) = \int \mathrm{d}s = \int_0^r \frac{1}{\sqrt{1-Kr^2}}\,\mathrm{d}r = \frac{1}{\sqrt{K}} \arcsin(r\sqrt{K}) \quad (K>0)$$

$$\boxed{D(r) = \frac{1}{\sqrt{K}} \arcsin(r\sqrt{K})} \qquad \boxed{r(D) = \frac{1}{\sqrt{K}} \sin(D\sqrt{K})}$$

- Area A of an r-sphere : $A = 4\pi r^2 = \dfrac{4\pi}{K} \sin^2(D\sqrt{K})$

Small D $\quad (D\sqrt{K} \ll 1) \rightarrow A \approx 4\pi D^2 \quad$ (Euclidean value)

Large D $\quad (D\sqrt{K} \geqq 1) \rightarrow A$ increases more slowly than $4\pi D^2$

$*$ In case $K>0$ the area A of an r-sphere reaches two extreme values

$$D\sqrt{K} = (n+\tfrac{1}{2})\pi \rightarrow A_{max} = \tfrac{4\pi}{K} \quad (n \in \mathbb{N})$$

$$D\sqrt{K} = n\pi \rightarrow A_{min} = 0$$

Positively-curved $(K>0)$ space is closed

Spatial curvature

- For $K < 0$ we get

$$D(r) = \int \mathrm{d}s = \int_0^r \frac{1}{\sqrt{1+|K|r^2}}\,\mathrm{d}r = \frac{1}{\sqrt{|K|}}\operatorname{arcsinh}(r\sqrt{|K|})$$

$$\boxed{D(r) = \frac{1}{\sqrt{|K|}}\operatorname{arcsinh}(r\sqrt{|K|})} \qquad \boxed{r(D) = \frac{1}{\sqrt{|K|}}\sinh(D\sqrt{|K|})}$$

- So the area of an r-sphere becomes : $A = \dfrac{4\pi}{|K|}\sinh^2(D\sqrt{|K|})$

  A increases faster than in flat space

  A increases to ∞ when $D \to \infty$

∗ Negatively-curved $(K < 0)$ space is open

- Can we now also derive the deformation of the time coordinate ?

  Depends on the space-time evolution of the universe

  $r \to r(t)$ and consequently $K \to K(t)$

∗ Need for observational data w.r.t. the space-time evolution of the universe

The Robertson-Walker metric

- Observation : All galaxies at large distances are moving away from us

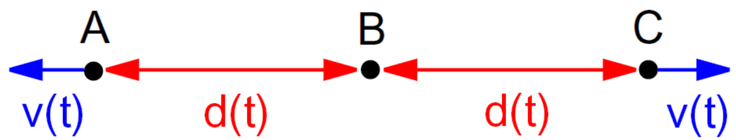
  Isotropic universe $\longrightarrow$ All distant galaxies are moving away from each other

$\ast$ Consequence : The universe is expanding !

  Comparison : Cooking of a pudding with raisins in it

- Expansion in an isotropic homogeneous universe

  Observer B observes objects A and C at physical distances $d(t)$ as shown below



  Expansion as seen from B : A and C move with equal speed $v(t)$ in opposite directions

  Expansion as seen from A : B moves away with speed $v(t)$ and C with speed $2v(t)$

  Expansion as seen from C : B moves away with speed $v(t)$ and A with speed $2v(t)$

$\ast$ Consequently : $v(t) = H(t)d(t)$ where $H(t) \equiv$ Hubble parameter

  Using $v(t) = \dot{d}(t)$ we can write $\dot{d}(t) = H(t)d(t)$

  Can we obtain an expression for the spatial evolution d(t) of the universe from this ?

The Robertson-Walker metric

- Starting expression for the space-time description of the universe : $\dot{d}(t) = H(t)d(t)$

  Solution to this differential equation : $d(t) = a(t)\chi$ with $\chi \equiv$ constant

∗ Interpretation of this solution

  At some time origin every object is given a fixed radial distance χ

  Expansion is then described by the cosmic expansion factor $a(t)$ via $d(t) = a(t)\chi$

  $$\rightarrow \dot{d}(t) = \dot{a}(t)\chi = \frac{\dot{a}(t)}{a(t)}a(t)\chi = \frac{\dot{a}(t)}{a(t)}d(t) \Rightarrow H(t) = \frac{\dot{a}(t)}{a(t)}$$

∗ All distances in the universe increase by the cosmic expansion factor $a(t)$

- The same procedure can be performed for coordinates

  At some time origin every object is given a fixed comoving radial coordinate σ

  Expansion is then described via $r(t) = a(t)\sigma$

∗ Description in comoving spherical coordinates $(\sigma, \theta, \varphi)$

  Always proper time $\mathrm{d}\tau$ on comoving spherical shell $\rightarrow$ No time distortion in the metric

  This reference frame is at rest w.r.t. the average matter of the universe (i.e. CMBR)

The Robertson-Walker metric

- Expanding sphere : $K(t) = \dfrac{1}{R^2(t)} = \dfrac{1}{a^2(t)R_0^2}$ and arbitrary scale of σ yields

$$K(t)r^2(t) = \frac{a^2(t)\sigma^2}{a^2(t)R_0^2} \equiv k\sigma^2 \text{ with } k = \begin{cases} 1 & \text{positive curvature (i.e. closed space)} \\ 0 & \text{no curvature (i.e. flat space)} \\ -1 & \text{negative curvature (i.e. open space)} \end{cases}$$

With $\mathrm{d}r = a(t)\mathrm{d}\sigma$ we get the space-time metric for const. curvature in comoving coord.

$$\boxed{\mathrm{d}s^2 = (c\,\mathrm{d}t)^2 - a^2(t)\left[\frac{(\mathrm{d}\sigma)^2}{1-k\sigma^2} + \sigma^2(\mathrm{d}\theta)^2 + \sigma^2\sin^2(\theta)(\mathrm{d}\varphi)^2\right]}$$

which is called the Robertson-Walker metric

- Describes the universe in comoving coordinates

$\longrightarrow$ The most natural reference frame : the one at rest w.r.t. the CMBR

$*$ It is the only solution to Einstein's equations for a homogeneous and isotropic universe

Can we also obtain an expression for a(t) to make our description complete ?

The Friedmann-LeMaître equation

- Determination of $a(t)$ and k : need energy distr. and full treatment of Einstein's equations

  Energy distribution not very well known $\rightarrow$ Need some assumptions

  Assume various expressions for $a(t)$ $\rightarrow$ Various models for the universe

  Make predictions for certain observables within these models

  Measure the observables in real life $\rightarrow$ Accept or reject corresponding model(s)

* We will use classical arguments to derive an expression for $a(t)$

- Consider a test particle with mass m at a location $\vec{r}$ w.r.t. some origin O

  m feels gravitational potential of the mass M contained in the r-sphere as located in O

  Spherical symmetry : $\vec{g}(\vec{r}) = \vec{g}(r)$ and $\vec{\nabla} \times \vec{g}(r) = 0$

  Conservative gravitational potential $\rightarrow$ E_{tot} of m is constant

  $$\rightarrow \frac{1}{2}mv^2 - \frac{GMm}{r} \equiv -mC \qquad (C = \text{constant})$$

- Uniform density : $M = \frac{4}{3}\pi r^3 \rho \rightarrow \frac{1}{2}v^2 - \frac{4}{3}\pi G \rho r^2 = -C$

The Friedmann-LeMaître equation

- Using $\vec{v}(t) = \dot{\vec{r}}(t)$ and $\vec{r}(t) = a(t)\sigma$ we can write the previous formula as

$$\tfrac{1}{2}\dot{a}^2(t)\sigma^2 - \tfrac{4}{3}\pi G\rho(t)a^2(t)\sigma^2 = -C$$

- This yields : $\dot{a}^2(t) - \frac{8}{3}\pi G\rho(t)a^2(t) = -k \qquad (k = \text{constant})$

  which is called the Friedmann-LeMaître equation

$$\boxed{\left[\frac{\dot{a}(t)}{a(t)}\right]^2 = \frac{8\pi G\rho(t)}{3} - \frac{k}{a^2(t)}}$$

∗ This is the exact form obtained from Einstein's equations for a homogeneous and isotropic universe with total energy density $\rho(t)$ and curvature k

- The Friedmann-LeMaître equation describes the dynamical evolution of the universe

  Evolution of the universe completely determined by $\rho(t)$ and k

- Note : $H(t) = \dfrac{\dot{a}(t)}{a(t)} \rightarrow H^2(t) = \dfrac{8\pi G\rho(t)}{3} - \dfrac{k}{a^2(t)}$

Critical density, Dark energy and Cosmological constant

- Universe can be closed $(k > 0)$, flat $(k = 0)$ or open $(k < 0)$

  $\rightarrow$ Flat universe can be considered as a special (critical) case

- Flat universe : $H^2(t) = \dfrac{8\pi G \rho(t)}{3} \rightarrow$ critical density : $\rho_c(t) \equiv \dfrac{3H^2(t)}{8\pi G}$

$*$ Indicate present day values as ρ_0 and $H_0 \rightarrow \rho_{c,0} = \dfrac{3H_0^2}{8\pi G}$

- What are the components which make up the total energy density $\rho(t)$?

  Universe contains matter $\rightarrow$ matter energy density $\rho_m(t)$

  Universe contains radiation $\rightarrow$ radiation energy density $\rho_r(t)$

  Unknown influence of the vacuum $\rightarrow$ vacuum energy density $\rho_v(t)$

- Fractional density parameters : $\Omega_m(t) \equiv \dfrac{\rho_m(t)}{\rho_c(t)} \quad \Omega_r(t) \equiv \dfrac{\rho_r(t)}{\rho_c(t)} \quad \Omega_v(t) \equiv \dfrac{\rho_v(t)}{\rho_c(t)}$

$$\Rightarrow \rho(t) = \rho_m(t) + \rho_r(t) + \rho_v(t) \qquad \Omega(t) = \frac{\rho(t)}{\rho_c(t)} = \Omega_m(t) + \Omega_r(t) + \Omega_v(t)$$

- Note : Flat universe $\leftrightarrow \Omega(t) = 1$

Critical density, Dark energy and Cosmological constant

- The vacuum energy density is also called dark energy density

  Note : Don't confuse this dark energy density with dark matter !

∗ Einstein's equations allow only a time-independent vacuum effect : $\rho_v(t) \to \rho_v$

$$\to \left[\frac{\dot{a}(t)}{a(t)}\right]^2 = \frac{8\pi G\rho_m(t)}{3} + \frac{8\pi G\rho_r(t)}{3} + \frac{8\pi G\rho_v}{3} - \frac{k}{a^2(t)}$$

- It is custom to define : $\rho(t) \equiv \rho_m(t) + \rho_r(t)$ and $\Lambda \equiv 8\pi G\rho_v$

  so that the Friedmann-LeMaître equation can be written as

$$\boxed{\left[\frac{\dot{a}(t)}{a(t)}\right]^2 = \frac{8\pi G\rho(t)}{3} + \frac{\Lambda}{3} - \frac{k}{a^2(t)}}$$

  where Λ is called the Cosmological constant

- Currently $\rho_{r,0} \ll \rho_{m,0} \to$ Only 2 relevant parameters : $\Omega_M \equiv \frac{\rho_{m,0}}{\rho_{c,0}}$ $\quad \Omega_\Lambda \equiv \frac{\rho_v}{\rho_{c,0}}$

∗ Planck results (2018) : $\Omega_M = 0.315 \pm 0.007$ $\quad \Omega_\Lambda = 0.685 \pm 0.007$

  It seems that we live in a flat universe !

Cosmological redshift

- Expansion of the universe $\longrightarrow$ Distant objects should exhibit a redshift
- Consider the following events of information transfer via EM radiation

  A galaxy at comoving coordinate $\sigma = \sigma_e$ emits 2 wave crests at t_e and $t_e + \Delta t_e$

  An observer at comoving coordinate $\sigma = 0$ observes the wave crests at t_o and $t_o + \Delta t_o$

$*$ Light travels radially from the galaxy to the observer

Robertson-Walker metric $\longrightarrow$ $\mathrm{d}s^2 = (c\,\mathrm{d}t)^2 - a^2(t)\left[\frac{(\mathrm{d}\sigma)^2}{1-k\sigma^2}\right] \equiv 0$

which yields for the emission and observation of the 2 wave crests

$$\int_{t_e}^{t_o} \frac{c}{a(t)}\,\mathrm{d}t = \int_{\sigma_e}^{0} \frac{1}{\sqrt{1-k\sigma^2}}\,\mathrm{d}\sigma \qquad \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{c}{a(t)}\,\mathrm{d}t = \int_{\sigma_e}^{0} \frac{1}{\sqrt{1-k\sigma^2}}\,\mathrm{d}\sigma$$

$$\rightarrow \int_{t_e}^{t_o} \frac{c}{a(t)}\,\mathrm{d}t = \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{c}{a(t)}\,\mathrm{d}t = \int_{t_e}^{t_o}\ldots - \int_{t_e}^{t_e+\Delta t_e}\ldots + \int_{t_o}^{t_o+\Delta t_o}\ldots$$

So we obtain : $\int_{t_e}^{t_e+\Delta t_e} \frac{c}{a(t)}\,\mathrm{d}t = \int_{t_o}^{t_o+\Delta t_o} \frac{c}{a(t)}\,\mathrm{d}t$

Cosmological redshift

- Two consecutive wave crests of EM radiation : $\Delta t = \nu^{-1} \rightarrow \Delta t \ll 1$ sec.

  which implies $a(t_e + \Delta t_e) \approx a(t_e)$ and $a(t_o + \Delta t_o) \approx a(t_o)$

  So we obtain : $\displaystyle\int_{t_e}^{t_e+\Delta t_e} \frac{c}{a(t_e)}\,\mathrm{d}t = \int_{t_o}^{t_o+\Delta t_o} \frac{c}{a(t_o)}\,\mathrm{d}t \rightarrow \frac{\Delta t_e}{a(t_e)} = \frac{\Delta t_o}{a(t_o)}$

  Using $\Delta t = \dfrac{1}{\nu} = \dfrac{\lambda}{c}$ we directly obtain : $\dfrac{\lambda_o}{\lambda_e} = \dfrac{a(t_o)}{a(t_e)}$

$\ast$ Also the wavelength of radiation is stretched by the cosmic scale factor $a(t)$

- The redshift z was defined as $z \equiv \dfrac{\lambda_o - \lambda_e}{\lambda_e} = \dfrac{\lambda_o}{\lambda_e} - 1$

  which yields for the Cosmological redshift

$$\boxed{z = \frac{a(t_o)}{a(t_e)} - 1}$$

- Since $t_o > t_e \rightarrow a(t_o) > a(t_e)$ so indeed distant galaxies appear redshifted

Cosmological redshift

- Most of the observed redshifts are rather small (e.g. $z < 0.1$)

  $\rightarrow a(t_o) \approx a(t_e) \rightarrow$ on a cosmological timescale $t_o \approx t_e$

  This implies that we can use a Taylor expansion to investigate $a(t_e)$ w.r.t. $a(t_o)$

  $a(t_e) = a(t_o) + (t_e - t_o)\dot{a}(t_o) + \frac{1}{2}(t_e - t_o)^2\ddot{a}(t_o) + \ldots$

- Using again $H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$ and $H_0 \equiv \dfrac{\dot{a}(t_o)}{a(t_o)}$ we can write for small z

$$\boxed{a(t_e) = a(t_o) - a(t_o)\left[H_0(t_o - t_e) + \frac{1}{2}q_0 H_0^2(t_o - t_e)^2 + \ldots\right]}$$

where we have introduced the deceleration parameter

$$\boxed{q(t) \equiv \frac{-\ddot{a}(t)}{H^2(t)a(t)}}$$

$*$ As usual the present day value is indicated as $q_0 \equiv \dfrac{-\ddot{a}(t_o)}{H^2(t_o)a(t_o)} = \dfrac{-\ddot{a}_0}{H_0^2 a_0}$

Cosmological redshift

- Using the previous Taylor expansion we can obtain a similar expression for the redshift

  $a(t_o) - a(t_e) = a(t_o)\left[H_0(t_o - t_e) + \frac{1}{2}q_0 H_0^2 (t_o - t_e)^2 + \ldots\right]$

  with $z = \dfrac{a(t_o)}{a(t_e)} - 1 = \dfrac{a(t_o) - a(t_e)}{a(t_e)}$ and $\dfrac{a(t_o)}{a(t_e)} \approx 1$ we get

$$\boxed{z = H_0(t_o - t_e) + \tfrac{1}{2}q_0 H_0^2 (t_o - t_e)^2 + \ldots}$$

- Using $z \approx H_0(t_o - t_e)$ we can obtain an expression for the light travel time

$$\boxed{(t_o - t_e) = \frac{1}{H_0}\left[z - \tfrac{1}{2}q_0 z^2 - \ldots\right]}$$

$\ast$ z vs. distance plot : Departure from a straight line for large distances

Accurate measurements may enable determination of q_0

Hubble was lucky to have only data from rather nearby $(z \ll 1)$ objects !

- As mentioned before : these Taylor expansions are only valid for small z

Cosmological redshift

∗ Exercise : Consider the Cosmic Microwave Background Radiation (CMBR) which has a blackbody spectrum corresponding currently to a temperature of about 2.73 K.

- Show that due to the expansion of the universe the CMBR maintains the blackbody spectrum but that the corresponding temperature decreases with time.

Measurement of Cosmological distances in a Flat Universe

- Quite often it is necessary to know the physical distance of a certain object

  E.g. to determine the actual energy output of a Gamma Ray Burst from observed fluence

- WMAP & Planck measurements : Consistent with a flat universe $\rightarrow k = 0$

  This yields for the Robertson-Walker metric describing a flat universe :

$$\mathrm{d}s^2 = (c\,\mathrm{d}t)^2 - a^2(t)\left[(\mathrm{d}\sigma)^2 + \sigma^2(\mathrm{d}\theta)^2 + \sigma^2\sin^2(\theta)(\mathrm{d}\varphi)^2\right]$$

- Actual 3D spatial distances can only be measured correctly if $\mathrm{d}t = 0$ (i.e. at the same time)

  Due to the finite speed of light this is impossible for cosmological objects

- Can we find an observable which can be related to the physical distance ?

  Use EM radiation for observations $\rightarrow \mathrm{d}s^2 \equiv 0$

  Distant objects may show emission/absorbtion lines $\rightarrow$ Use redshift z as observable

* Can we relate the observed redshift z to the physical distance of the object ?

  Consider the case that we observe at $t = t_o$ a light signal from a distant source that was emitted at some time $t < t_o$

Measurement of Cosmological distances in a Flat Universe

- Robertson-Walker for a flat universe : $\mathrm{d}s^2 = 0 \rightarrow \mathrm{d}\sigma = \dfrac{c}{a(t)}\mathrm{d}t \qquad (1)$

- For the cosmological redshift we have : $z(t) = \dfrac{a(t_o)}{a(t)} - 1 \rightarrow \dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{-a(t_0)\dot{a}(t)}{a^2(t)}$

  Substitution of $\mathrm{d}t = \dfrac{-a^2(t)}{a(t_o)\dot{a}(t)}\mathrm{d}z$ in (1) yields : $\mathrm{d}\sigma = \dfrac{c}{a(t_o)} \cdot \dfrac{a(t)}{\dot{a}(t)}\mathrm{d}z \qquad (2)$

- Friedmann-LeMaître for a flat universe : $\left[\dfrac{\dot{a}(t)}{a(t)}\right]^2 = \dfrac{8\pi G\rho(t)}{3} + \dfrac{\Lambda}{3}$

$*$ Total energy conservation within a comoving volume : $\rho(t)a^3(t) = \rho(t_o)a^3(t_o)$

  Flat matter-dominated universe : $\rho(t_o) = \rho_{c,0}\,\Omega_M \rightarrow \rho(t) = \left(\dfrac{a(t_o)}{a(t)}\right)^3 \dfrac{3H_0^2}{8\pi G}\Omega_M$

  Using $\Lambda = 3H_0^2\,\Omega_\Lambda$ we obtain : $\left[\dfrac{\dot{a}(t)}{a(t)}\right]^2 = H_0^2\left[\Omega_M\left(\dfrac{a(t_o)}{a(t)}\right)^3 + \Omega_\Lambda\right] \qquad (3)$

Measurement of Cosmological distances in a Flat Universe

- Using $\dfrac{a(t_o)}{a(t)} = 1 + z$ in (3) yields for the Friedmann-LeMaître equation :

$$\left[\frac{\dot{a}(t)}{a(t)}\right]^2 = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_\Lambda\right] \qquad (4)$$

- Combination of (2) and (4) yields :

$$\mathrm{d}\sigma = \frac{c}{a(t_o)H_0} \cdot \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}}\,\mathrm{d}z \qquad (5)$$

- Combination of (1) and (5) yields : $c\,\mathrm{d}t = \dfrac{c}{H_o} \cdot \dfrac{a(t)}{a(t_o)} \cdot \dfrac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}}\,\mathrm{d}z$

$$c\,\mathrm{d}t = \frac{c}{H_o} \cdot \frac{1}{(1+z)} \cdot \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}}\,\mathrm{d}z \qquad (6)$$

The equations (5) and (6) provide the basis to define various cosmological distances

Measurement of Cosmological distances in a Flat Universe

Definitions of cosmological distances between source and observer

∗ Light emitted at (σ_e, t_e) is observed at $(0, t_o)$ with a redshift z_o

Various cosmological distances could be of interest in studying the source

- What was the physical distance when the light was emitted ?
- What is the physical distance when the light is observed ?
- How long has the light traveled before it reached us ?
- What is the luminosity of the source based on the observed flux ?

• The Comoving Distance D_C : This is the (constant) distance in comoving coordinates

$D_C = \sigma_e = \int_0^{\sigma_e} \mathrm{d}\sigma$ and using (5) yields :

$$\boxed{D_C(z_o) = \frac{c}{a(t_o)H_0} \int_0^{z_o} \frac{1}{\sqrt{\Omega_M\,(1+z)^3 + \Omega_\Lambda}}\,\mathrm{d}z}$$

Note : It is customary to "calibrate" the "σ-scale" by defining $a(t_o) \equiv 1$

Measurement of Cosmological distances in a Flat Universe

- The Proper Distance D_P : (also called Physical Distance)

  This is the (time-dependent) distance that would be measured by a ruler at a specific time $t \leq t_o$. In other words : $D_P(t) = a(t) D_C$

  $$\rightarrow D_P(t, z_o) = \frac{a(t)}{a(t_o)} \cdot \frac{c}{H_0} \int_0^{z_o} \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}} \mathrm{d}z$$

  So, $D_P(t)$ represents the physical distance of D_C at a specific time t in the past

∗ We get for the Proper Distance at the time of observation $D_P(t_o, z_o)$:

$$\boxed{D_P(t_o, z_o) = \frac{c}{H_0} \int_0^{z_o} \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}} \mathrm{d}z}$$

and for the Proper Distance at the time of emission $D_P(t_e, z_o)$:

$$\boxed{D_P(t_e, z_o) = \frac{1}{(1+z_o)} \cdot \frac{c}{H_0} \int_0^{z_o} \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}} \mathrm{d}z}$$

∗ With $a(t_o) \equiv 1 \rightarrow D_P(t_o, z_o) = D_C(z_o)$ and $D_P(t_e, z_o) = \frac{D_C(z_o)}{(1+z_o)}$

Measurement of Cosmological distances in a Flat Universe

- The Light Travel Distance D_{LT} :

  This is the distance that light has traveled to reach us from an object with redshift z_o

$$D_{LT} = \int_{t_e}^{t_o} c\, dt$$ which yields with (6) :

$$\boxed{D_{LT}(z_o) = \frac{c}{H_0} \int_0^{z_o} \frac{1}{(1+z)} \cdot \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}} \mathrm{d}z}$$

∗ The Size of the Universe R_U can be expressed by $R_U = D_{LT}(z = \infty)$

- The Light Travel Time T_L is defined as : $T_L(z_o) = D_{LT}(z_o)/c$

  So we have :

$$\boxed{T_L(z_o) = \frac{1}{H_0} \int_0^{z_o} \frac{1}{(1+z)} \cdot \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}} \mathrm{d}z}$$

∗ Note : T_L is also called the Look Back Time

∗ The Age of the Universe T_U can be expressed by $T_U = T_L(z = \infty)$

Measurement of Cosmological distances in a Flat Universe

- The Luminosity Distance D_L :

  This is the distance relating the intrinsic source luminosity (L) to the observed flux (F)

* Consider a source with an intrinsic luminosity of L (erg sec^{-1}) of which we observe a flux F (erg sec^{-1} cm^{-2}) Note : 1 erg$=10^{-7}$ J

* Definition of luminosity distance : $F = \dfrac{L}{4\pi D_L^2}$

  Concerning the observed flux F we have the following cosmological effects

  – Wavelength stretching $\rightarrow$ The observed energy is reduced by a factor $(1+z_o)$

  – Time dilation $\rightarrow$ The observational time intervals are stretched by a factor $(1+z_o)$

  – We should of course use the physical distance at the time of observation

* So we have $F = \dfrac{L}{4\pi D_L^2} = \dfrac{L}{4\pi(1+z_o)^2 D_P^2(t_o, z_o)}$ yielding :

$$\boxed{D_L = (1+z_o) D_P(t_o, z_o) = (1+z_o) \cdot \frac{c}{H_0} \int_0^{z_o} \frac{1}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}} \mathrm{d}z}$$

Measurement of Cosmological distances in a Flat Universe

Some notes about the Luminosity Distance D_L

- In case the observed Fluence S (=time-integrated flux in erg/cm^2) is used in an analysis one has to use $\sqrt{(1+z_o)} \cdot D_P(t_o, z_o)$ as the corresponding distance
- D_L also relates the absolute magnitude M of an astronomical object to its observed apparent magnitude m via :

$$\boxed{M = m - 5[{}^{10}\log(D_L) - 1] \quad \rightarrow \quad D_L = 10^{[1+(m-M)/5]}}$$

where $[D_L]$=parsec (pc) and $1\,\mathrm{pc} \approx 3.09 \cdot 10^{16}\,\mathrm{m} \approx$ 3.26 light year (ly)

Measurement of Cosmological distances in a Flat Universe

Exercise

- Consider very nearby objects such that the observed redshift z can be interpreted as a Doppler shift due to the recession velocity v of the object.

∗ Derive that the distance r of the object can be determined as $r = \frac{c}{H_0} \cdot \frac{(1+z)^2-1}{(1+z)^2+1}$

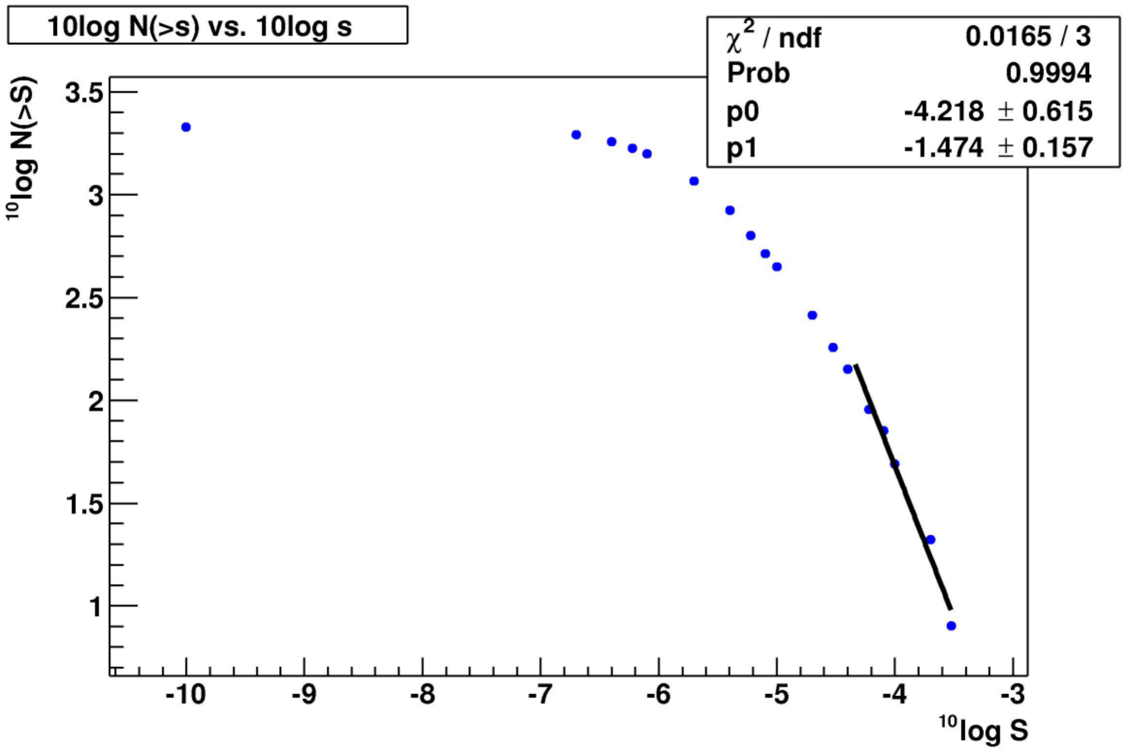
- For some relatively nearby Gamma Ray Bursts (GRBs) the fluence S in gamma rays and z have been measured by the Batse and BeppoSax satellites. Assuming isotropic emission over the full solid angle, all these bursts seem to have more or less the same energy output E_0 of about 10^{52} erg.

∗ Assume a characteristic isotropic energy output of E_0 for all GRBs and a homogeneous GRB number density n. Neglect redshift effects.
Show how n can be determined by only a measurement of the fluence of the various bursts.
Hint : Investigate the cumulative GRB count above a certain reference fluence

- In the plot below the cumulative GRB count $N(> S)$ of GRBs with a fluence exceeding a certain reference fluence S is presented as a function of this reference fluence S.
It is seen that at high S the plot approaches a straight line with a slope of about -1.5.

Measurement of Cosmological distances in a Flat Universe

* Explain the linear behaviour and slope value at high S.
* Give a possible explanation for the flattening at lower S values.
* What could be a reason for the saturation effect at the lowest S values ?



Measurement of Cosmological distances in a Flat Universe

Using the NcAstrolab facility of NCFS-Pack

- At the IIHE everything has been centrally pre-installed in NCFS-Pack

  Login to the central IIHE computer portal as indicated at the lectures

  For once issue the command `cp /ice3/software/iihe/.rootrc $HOME`

  At the command prompt enter `source /ice3/software/iihe/ncfs.sh`

  This initialises the package and sets the prompt to `ncfs>`

* Now you are able to use the ROOT framework

  ROOT session : at the command prompt `ncfs>` just type `root`

  Running a ROOT macro : `ncfs> root -b -q test.cc >test.log`

- Loading NCFS-Pack into a ROOT session or macro

  `gSystem->Load("ncfspack");`

- Online ROOT docs are available via http://root.cern.ch
- Online NCFS-Pack docs are available via http://www.iihe.ac.be/ice3/ncfsdoc

Measurement of Cosmological distances in a Flat Universe

The NcAstrolab facility

- NcCollider provides a tool to simulate various high-energy collision processes
- NcAstrolab provides a virtual laboratory to analyse (astro)physical phemomena

  It contains various analysis tools like :

  - Conversion between various (astrophysical) coordinate systems
  - Various date and time systems (e.g. Julian dates and siderial times)
  - Determination of nuclear masses and binding energies
  - Determination of distances on cosmological scales
  - Matching of lab. observations with astrophysical objects and skymap displays

Exercise

- Consider a flat Friedmann-LeMaître universe.
- Use the NcAstrolab facility to produce a plot with on the X-axis the redshift z and on the Y-axis the physical distance $D(z)$ for objects with $0 < z < 15$.

Measurement of Cosmological distances in a Flat Universe

Analysis of the NASA Swift satellite GRB data

- GRB data of the NASA Swift satellite are available at
  http://swift.gsfc.nasa.gov/docs/swift/archive/grb_table
- Assume that GRBs emit their energy isotropically over the full 4π solid angle

∗ Investigate whether every GRB emits about the same amount of energy

- Assume that GRBs emit their energy in 2 back to back jets with a jet cone of 3°

∗ What will be the difference w.r.t. the previous analysis ?

∗ What would this imply for the GRB rate in the Universe ?