

# Unifying Wildfire Models from Ecology and Statistical Physics

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**ABSTRACT:** Understanding the dynamics of wildfire regimes is crucial for both regional forest management and predicting global interactions between fire regimes and climate. Accordingly, spatially explicit modeling of forest fire ecosystems is a very active field of research, including both generic and highly specific models. There is, however, a second field in which wildfire has served as a metaphor for more than 20 years: statistical physics. So far, there has been only limited interaction between these two fields of wildfire modeling. Here we show that two typical generic wildfire models from ecology are structurally equivalent to the most commonly used model from statistical physics. All three models can be unified to a single model in which they appear as special cases of regrowth-dependent flammability. This local “ecological memory” of former fire events is key to self-organization in wildfire ecosystems. The unified model is able to reproduce three different patterns observed in real boreal forests: fire size distributions, fire shapes, and a hump-shaped relationship between disturbance intensity (average annual area burned) and diversity of succession stages. The unification enables us to bring together insights from both disciplines in a novel way and to identify limitations that provide starting points for further research.

**Keywords:** wildfire models, landscape ecology, statistical physics, self-organization, ecological memory, pattern-oriented modeling.

## Introduction

Wildfire regimes shape many ecosystems. The interaction between fire, vegetation composition and structure, other landscape features, and climatic conditions is a major determinant of these ecosystems’ spatiotemporal dynamics and function (Clark 1993; Swetnam 2003). Understanding and predicting these interactions is crucial both on the scale of local and regional management—for example, of boreal forests—and on a global scale because climate is believed to affect and be affected by wildfire regimes (Pueyo 2007).

Empirical studies of wildfire regimes are invaluable, but fieldwork is usually limited to spatial and temporal scales much smaller than those on which fire ecosystems develop. Remote sensing is used to capture patterns at larger scales,

but the mechanisms causing these patterns are not immediately evident. Simulation models are therefore widely used for exploring wildfire regimes (Baker 1999; Mladenoff and Baker 1999; Keane and Finney 2003). For example, there are more than 40 landscape fire succession models that “simulate the dynamic interaction of fire, vegetation, and often climate” (Keane et al. 2004, p. 4). They cover a wide range of ecosystems, geographic areas, and scales. Most of them are quite specific, addressing small scales, but some are more generic, addressing larger scales.

There is, however, a second field of research in which the development and discussion of simulation models of fire regimes have been ongoing during the past 20 years or so: statistical physics. Here, the models reflect a systems theory perspective and have become an important metaphor in complex-systems science. In these models, vegetation recovery and fire spread are represented in a highly abstract manner. They are used to reproduce and explain observed fire size distributions that resemble power laws (Clar et al. 1996). These models are usually discussed in the context of general theories such as self-organized criticality (SOC; Bak et al. 1988) or, more recently, highly optimized tolerance (Carlson and Doyle 1999). Most of these models are related to forests but not to any specific type of forest or geographic region.

So far, there has been only limited direct interaction between these two fields of wildfire modeling. One notable exception is Malamud et al. (1998), who use a model from statistical physics (Henley 1989; Drossel and Schwabl 1992) not only to explain the frequency-area distributions of actual wildfires but also to make inferences about recent fire history in the Yellowstone National Park. However, it seems that this work has not yet been acknowledged by many landscape and fire ecologists.

This limited interaction of wildfire modeling in ecology and statistical physics reflects limitations on both sides: ecological wildfire models are often so detailed or tailored to specific regions that they hardly allow the identification of general principles and predictions, for example, how climate change would affect the frequency of large fires (Meyn et al. 2007). On the other hand, wildfire models from statistical physics are so abstract that they hardly

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relate to any of those factors known to be important in fire ecology. In particular, a process considered essential in landscape fire models seems to be completely ignored: vegetation succession. Thus, it remains unclear whether theories such as SOC have any bearings in real fire ecosystems. Linking these two types of models would further understanding by the integration of insights gained in both fields and would overcome their mutual limitations.

Here, we systematically compare two generic and parsimonious ecological wildfire models (Ratz 1995; Peterson 2002) with the most widely used model from statistical physics, the Drossel-Schwabl (1992) model (hereafter DSM). Further, we include a dynamic percolation model (Stauffer and Aharony 1992) as a theoretical reference.

The Ratz (1995) model (hereafter RM) is a grid-based landscape fire model that considers succession, represented by a sequence of characteristic species communities appearing after disturbance, that is, after a grid cell burned. The Peterson (2002) model (hereafter PM) is also grid based and focuses on spatiotemporal correlations in vegetation caused by fire. It was designed to study local ecological memory, that is, the effect of former fires on current fires due to the time needed for regrowth of fuel. However, it does not consider landscape succession.

The RM and PM, representative of several generic ecological fire models, and the DSM appear to be very different and even contradictory. In the RM and PM, fire spread is modeled as a stochastic process in which the flammability of a grid cell depends on the time since its last fire. In the DSM, fire spread itself is deterministic. It immediately consumes the entire cluster of connected fuel cells. Further, although the DSM includes a stochastic recovery process of fuel, it does not explicitly address succession. The RM, on the other hand, considers succession yet does not include an explicit spatial regrowth process from which it emerges. Is there, therefore, any chance of unifying these seemingly very different models from ecology and statistical physics?

The surprising answer is yes. To demonstrate this, we first present the four models (including the percolation model) in a more formal way. Analyzing the structure of the models leads to the surprising insight that the DSM, RM, and PM actually are based on the same conceptualization of wildfire systems. We show that the differences between the models lie solely in the assumptions made about ecological memory, that is, susceptibility to disturbance as a function of recovery since the last fire. In the RM and PM, this relationship is imposed (Railsback 2001; Grimm and Railsback 2005); that is, a certain functional relationship is assumed. In contrast, in the DSM ecological memory emerges from a spatial stochastic process.

Finally, we investigate the effect of the different memory functions on the large-scale patterns presented in figure

1: fire shape properties, the relation between the diversity of succession stages and the average annual area burned, and fire size distributions. Surprisingly, the capability of the three models to reproduce the observed patterns is largely independent of the details of their functions representing ecological memory. This robustness in producing observed patterns suggests that the unified model indeed captures some essential aspects of wildfire ecosystems.

## The Models

### *Ecology: The RM and PM*

The fire spread processes represented in the RM and PM are very similar: every grid cell keeps track of the time since its last fire. Fires are initiated by sparks that hit the landscape at random locations. Whether a spark starts a fire depends on the local conditions, which, in turn, are tied to the time since the last fire at this location. The success of fire spread, going from a cell to its direct four neighbors, is determined by the same property. Only one fire burns at a time. (A very similar model is used by Pueyo [2007].)

The main simulation loop used by Ratz (1995) and Peterson (2002) consists of the following rules (see fig. 2).

*Rule 1.* Execute rule 2  $n$  times. Then increase the age  $a$  of all cells by a fixed time interval (10 years for RM, 1 year for PM).

*Rule 2.* Choose a site at random. Ignite if susceptible; that is, with probability  $\text{Pr}_i(a)$ , set its age to 0 and execute rule 3.

*Rule 3.* For all four direct neighbors of the burned site: burn with the local  $\text{Pr}_i(a)$ . Set the age of all ignited neighbors to 0 and execute rule 3 for each of them.

To calculate  $\text{Pr}_i(a)$  of a certain cell, Ratz (1995) assumed a function that increases quadratically with time since fire, or age  $a$ :

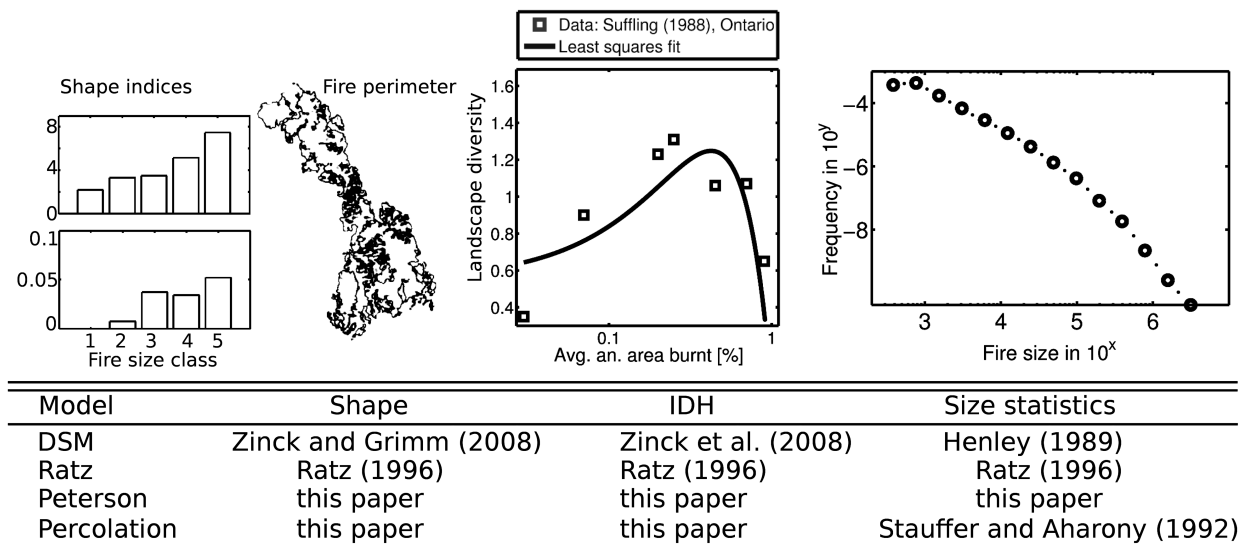
$$f(a)_{a \leq 500} = i + c \left( \frac{a}{10} \right)^2,$$

with parameters  $i$  and  $c$  and assuming a cutoff age at  $a = 500$  such that  $f(a)_{a > 500} = f(500)$ . The probability of ignition is thus

$$\text{Pr}_i(a) = \begin{cases} f(a) & 0 \leq f(a) \leq 1 \\ 1 & f(a) > 1 \\ 0 & f(a) < 0 \end{cases}. \quad (1)$$

Peterson (2002) uses the following ignition function:

$$\text{Pr}_i(a) = \begin{cases} (1 + \text{Pr}_{\max})^{(a/t_{\max})^\alpha} - 1 & a < t_{\max} \\ \text{Pr}_{\max} & \text{otherwise} \end{cases}, \quad (2)$$



**Figure 1:** Patterns that emerge in fire-disturbed boreal forests on landscape scales. The average geometrical properties of area burned in a fire (burn scars) were studied by Eberhart and Woodard (1987) in Alberta (*left*). After an empirical study in Ontario, Suffling et al. (1988) concluded that the intermediate disturbance hypothesis (IDH) is valid for fire-disturbed boreal forests (*middle*). The frequency-area distribution of wildfires is heavy tailed. A histogram of all fires in the Canadian large-fire database is plotted on a log-log scale (*right*). The table provides information as to where these patterns were tested for the four models. All patterns were tested again in this article. DSM = Drossel-Schwabl model.

where  $a$  again is the time since last fire at the cell considered,  $t_{\max}$  is the maximum time considered to still have an increasing effect on  $\text{Pr}_i(a)$ , and  $\text{Pr}_{\max}$  is the maximum probability of fire spread. The advantage of this function is that it can be tailored to just about any shape that smoothly approaches the maximum probability, mainly by adjusting the parameter  $\alpha$ .

The RM is linked to succession by assuming a typical sequence of species communities, or succession stages, after a fire. Each type of community, or stage, is characterized by a certain transition time, after which it is followed by the next type of community. Succession is assumed to reach a climax stage that does not change any more after 500 years.

#### *Statistical Physics: The DSM and Dynamic Percolation*

In the original DSM, each grid cell can take one of the three states: burning, empty, or tree. Contrary to the RM, an occupied cell is not considered to contain a forest stand. Rather, it refers to a single “tree,” where the only aspect of real trees represented is their flammability. Fire destroys a tree completely, leaving the cell empty. Trees pop up spontaneously on empty sites. Fire spreads among direct

neighbors; only cells occupied by a tree can burn. The simulation loop is outlined below (fig. 2).

**Rule 1.** Effect of burning: a site with a burning tree turns into an empty site.

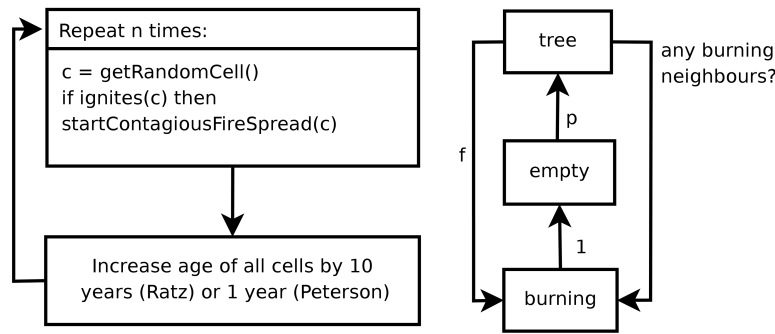
**Rule 2.** Tree growth: a new tree is established with probability  $p$  in an empty cell.

**Rule 3.** Fire initiation: a site with a tree burns spontaneously with probability  $f$ .

**Rule 4.** Fire spread: a site with a tree will burn if at least one of its four direct neighbors is burning.

The behavior of the DSM depends on the spontaneous ignition probability  $f$  and the recovery parameter  $p$ . Only if the timescales of recovery and fire initiation by sparks are separated such that  $p/f \rightarrow \infty$  as  $p \rightarrow 0$  will the model produce clusters of a wide range of sizes (Clar et al. 1996). The first condition,  $p/f \rightarrow \infty$ , guarantees that the rate of regrowth is a lot faster than that of sparking. The second condition,  $p \rightarrow 0$ , leads to a recovery rate that, although faster than the sparking rate, is still lower than the speed at which fires burn. Together, both conditions assure that there are fuel clusters of all sizes. We provide a short description as to how the parameter  $p$  can be estimated from data on average annual area burned in appendix D.

The fourth model in our analysis is based on the concept of dynamic percolation (Stauffer and Aharony 1992). Fires



**Figure 2:** *Left*, simulation loop used by Ratz (1996) and Peterson (2002) for their landscape fire models. After trying to ignite fires for a fixed number of trials, the age of all cells is increased by a predefined amount. Once a fire ignites, it spreads via nearest-neighbor interaction in a probabilistic manner. *Right*, illustration of the basic rules of the Drossel and Schwabl (1992) cellular automaton model. These rules are applied to all cells as  $t \rightarrow t + 1$ .

start at a random location on the grid and spread to neighboring sites according to a flammability  $\text{Pr}_i = p_b$ , assumed to be constant and the same for all cells. Hence, it neglects temporal correlations completely. This assumption is so unrealistic that the percolation model serves as a kind of null model in our analysis. Scaling laws emerge near the critical value of  $p_b \approx 0.59$ . This value does not, however, emerge via self-organization, as in SOC models (Drossel and Schwabl 1992; Ratz 1996), but has to be set explicitly by the modeler.

### Devising a Unified Forest Fire Model

The connection between RM and PM is easy to see: they just assume different functions for flammability  $\text{Pr}_i(a)$ . Ratz (1996) refers to this function as age-dependent flammability, whereas Peterson (2002) calls it ecological memory. The challenge is to show that the DSM falls into the same class of wildfire models as do RM and PM. The solution lies in reformulating the DSM into a version that, unlike the original version, incorporates cell age as time since it burned the last time.

Provided that timescales of fire sparks and tree regrowth are separated, the DSM's dynamic is not determined by parameters  $f$  and  $p$  separately but by their ratio  $\theta = p/f$  (Clar et al. 1996). It can be shown that the average number of trees regrowing between sparks is proportional to  $\theta$  (Clar et al. 1996). On a conceptual level,  $\theta$  corresponds to the number of grid cells that become susceptible, on average, between two fires. This would link the parameter  $p$  to the primary productivity of an ecoregion, while  $f$  is related to the number of lightnings and other ignition sources, such as arson. Because this ratio of  $p/f$  controls the models dynamics, it can be simulated using the rules below (Grassberger 1993).

**Rule 1.** Choose a site at random. If empty, proceed with rule 2. If occupied by a tree, determine the tree cluster connected to it. Set all cluster sites to empty.

**Rule 2.** Choose  $p/f$  sites at random and grow a tree at all of these sites that are empty. Employ rule 1.

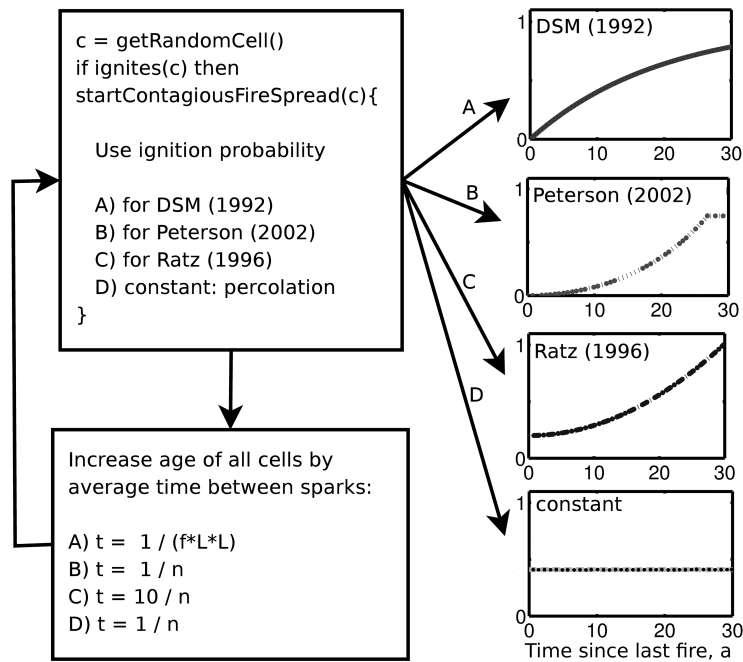
This version of the DSM uses rule 1, which is essentially a sparking event, as its main simulation timer. We now derive an age-dependent flammability,  $\text{Pr}_i(a)$ , for the DSM by noting that the proportion of cells that regain susceptibility to fire at any given time is  $p(1 - p_t)$ , where  $p_t \approx 0.41$  is the average proportion of occupied cells in the quasi-stationary state (Grassberger 1993). This proportion is constant in the time average over the entire range of parameters when timescales are separated. The probability of not turning susceptible is the opposite:  $1 - p(1 - p_t)$ . Hence, the probability of not having turned susceptible in the time since the last fire, corresponding to  $a$  time steps, is  $[1 - p(1 - p_t)]^a$  because each time step constitutes an independent trial. The probability of ignition,  $\text{Pr}_i(a)$ , which emerges for a cell  $a$  years after it burned the last time, is thus

$$\text{Pr}_i(a) = 1 - [1 - p(1 - p_t)]^a. \quad (3)$$

Equation (3) allows simulating the DSM in a novel way. The only information we need to store per cell is its time since last fire,  $a$ , which is increased in every loop of the simulation, that is, after each spark. The flammability of a cell is then calculated using equation (3) and determines fire spread. This allows us to formulate the common core behind all four models (fig. 3).

**Rule 1.** Choose a grid cell at random. If susceptible, ignite and set its age to 0 and execute rule 2.

**Rule 2.** For all four direct neighbors: burn if susceptible,



**Figure 3:** Core abstraction of wildfire systems underlying both the Drossel-Schwabl model (DSM) from statistical physics (implicitly via the recovery of trees; see eq. [3]) and the Ratz model (RM) and the Peterson model (PM) in landscape ecology (explicitly; see eqq. [1], [2]). The time step is shifted to the time between sparks for all models; the necessary mapping is shown in the lower box ( $n$  refers to the number of fires). The main iteration loop is defined by the sparks that start fires. These fires then spread among neighboring sites in a probabilistic fashion. The models differ only in how they define this probability. The DSM, RM, and PM assume a dependence of flammability on the time since the last fire. The dynamic percolation model does not make that assumption and serves as a null model.

that is, according to probability  $\text{Pr}_i(a)$ . Set the age of all ignited neighbors to 0 and execute rule 2 for each of them.

**Rule 3.** Increase the age of all cells by the average time between sparks  $\tau$ .

This algorithm constitutes the unified model, which contains the four models considered so far as special cases. The four models are obtained by plugging in the corresponding probability of ignition  $\text{Pr}_i(a)$  and time step into the general model (fig. 4). The time step of the DSM changed from that of fire spread and recovery per cell, as in the original formulation, to that of the average time between sparks  $\tau = 1/(fL^2)$  on a lattice of length  $L$ .

#### Effect of Different Memory Functions on Patterns: Methods and Data

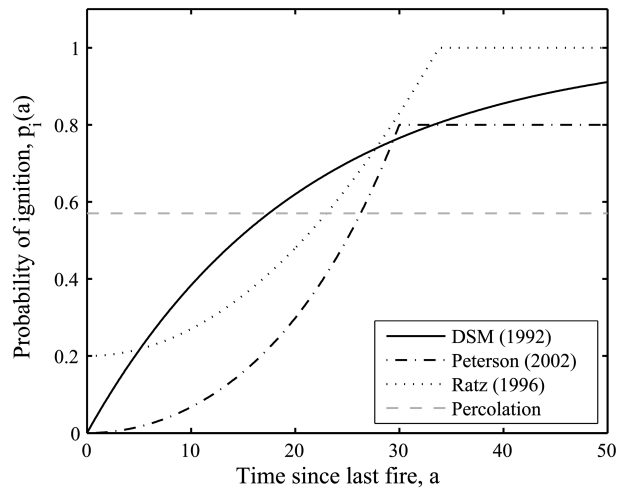
Figure 1 shows three different patterns with which the RM has been evaluated (Ratz 1995, 1996). In the following we show how the patterns emerge in the unified model in a boreal forest scenario, for which the data in figure 1 have been gathered. The parameters used for the function  $\text{Pr}_i(a)$  were chosen to reveal the effect of different assumptions about ecological memory (fig. 4; for details see app. B).

Models were run on a  $300 \times 300$  square grid. We ran the models for at least 15,000 time steps on a random initial landscape before taking samples. The details are discussed separately for each pattern. Algorithms in pseudocode notation describing the models and the corresponding source code are provided in appendix F.

#### Frequency-Area Distributions of Fires and Fire Shapes

The frequency-area distributions of real wildfires have been successfully fitted with power laws. The exponents found lie between  $-1.2$  and  $-1.9$  (Malamud et al. 1998, 2005; Millington et al. 2006). We did the same fitting for the four special cases of the unified model. We used a sample of 3,000 fires for every flammability function. For visualization, we used multiplicative binning (Pueyo and Jovani 2006); to determine the exponent  $\alpha$  of the power law, we used the maximum likelihood estimator directly from the sample data (Clauset et al. 2009; for details see app. A).

Ratz (1995) found that his model reproduced the shape of fires in boreal forests (Eberhart and Woodard 1987). Here we used the same data to evaluate all four models.



**Figure 4:** Flammability, that is, probability  $\Pr_i(a)$  of a grid cell of age  $a$  (i.e., the time since that cell last burned) to ignite if hit by lightning or being adjacent to a burning cell, also referred to as ecological memory. We derived the flammability that emerges in the Drossel-Schwabl model (DSM; see eq. [3]). In the Ratz model and the Peterson model, flammability is imposed. For the test of shape- and frequency-area distribution, parameters for the increasing functions were chosen so that the flammability is fairly high ( $>0.8$ ) after 30 years. This is the time it takes for boreal forests to develop a critical bulk density.

Data were obtained by Eberhart and Woodard (1987) by analyzing 68 fires that burned, without human intervention, in the boreal forest of Alberta. The fires were grouped in size classes of 20–40, 41–200, 201–400, 401–2,000, and 2,001–20,000 ha. Structural properties studied by Eberhart and Woodard (1987) and Ratz (1995) were fire shape and number and size of islands of unburned vegetation within fires. We evaluated the same properties in the models, using 2,000 fires for each size class (see app. C for details): unburned area, shape index, edge index, and median island size per fire. An island within a burned cluster of the forest fire model was defined as a coherent area of grid cells surrounded by cells that burned in the last fire event. The area of a grid cell corresponded to 6 ha/cell, according to a calibration done for the RM (Ratz 1995; Zinck and Grimm 2008). Associating a cell with a forest stand rather than a single tree in the DSM is useful in the interpretation of  $\Pr_i(a)$  (see “Discussion”). We chose a grid cell in the PM and percolation model to represent the same area as in the RM and DSM, 6 ha.

#### Disturbance-Diversity Relationship

For boreal forests of northwestern Ontario, Suffling et al. (1988) used eight sample sites of 250 km<sup>2</sup> each, selected along a gradient transect of disturbance level, quantified

by the average annual area burned. Each site was divided into 16 squares, the landscape diversity of which was assessed regarding different succession stages and averaged for the sample using the Shannon index. The obtained data were best fitted by a hump-shaped curve, as suggested by the intermediate disturbance hypothesis (IDH; Connell 1978).

In order to test the models for this pattern, we add a succession aspect to the models. A succession stage is characterized by a specific composition of species. Succession is modeled as a chain of characteristic communities appearing after fire (table 1). It ends in a climax stage. Transitions take place in a characteristic sequence after the average lifetime of a certain community has passed. In the models studied here, the succession stage of a cell is identified via the time since its last disturbance. Every state can be thrown back to the initial state by another fire.

We use four different succession scenarios from the literature to explore the effect of varying transition times and the number of succession stages on the resulting landscape pattern (table 1). The succession scenarios were all developed for actual forests. We use the same diversity measure as did Suffling et al. (1988), the Shannon index:

$$\text{SHX}(S) = - \sum_{i \in N} s_i \ln(s_i),$$

where  $S$  is the frequency distribution of  $N$  succession stages and  $s_i$  the  $i$ th succession stage.

Samples for landscape patterns are taken at intervals of 10 time steps each, as by Ratz (1996). The landscape diversity was determined for every disturbance intensity (i.e., average annual area burned) by averaging 1,000 samples. To systematically vary the disturbance level, we varied parameter  $p$  in the DSM from 0.0001 to 0.1 while keeping the ratio  $p/f$  constant at 200. To obtain the same effect in the RM, we varied parameter  $c$  from 0.0001 to 0.2 while setting  $i = 0.2$ . In the percolation model we varied the flammability from 0.31 to 0.57. The disturbance intensity in the PM was increased by varying  $t_{\max}$  from 1,700 to 10 years while setting the maximum flammability,  $\Pr_{\max}$ , to 0.8 and using  $\alpha = 2$ . Note that for the DSM, it is also possible to calculate the diversity-disturbance relationship analytically (Zinck et al. 2009; see app. E).

#### Results

The frequency-area statistics of all four models can be fitted successfully with a power law (fig. 5). This has been known to be so for the DSM and the RM for a large range of parameters for both models. The constant scenario near the percolation threshold produces power-law-like statis-

**Table 1:** Transition times of the four succession models used in creating the disturbance-diversity curves

Forest type	Transition times	Reference
Mountain forest, Wyoming	10, 40, 150, 300, 600	Romme and Despain 1991
Coniferous forest, Montana	20, 50, 150, 300, 320, 350	Kessel 1980
Oak forest, Michigan	100, 150, 200, 350, 500	Shugart et al. 1973
Douglas-hemlock forest, British Columbia	6, 60, 150, 200, 300, 400	Otto 1994

tics, as expected (Stauffer and Aharony 1992), but with a considerably smaller exponent. The exponent of the other three models, including the PM, is very similar.

The statistical properties of fire shapes generally follow the trend seen in the data by Eberhart and Woodard (1987) for all models (fig. 6). The overall performance of all models is similar and depends more on the fire size than on the model. The best fit is attained in size class 4, that is, for fires of the size of 401–2,000 ha. The largest deviation is seen for very large fires in size class 5 (2,001–20,000 ha) in the edge index, shape index, and median island size.

The diversity-disturbance curves of all four models are hump shaped, as predicted by the IDH (fig. 7; see also fig. 1). The exact shape varies with the succession scenario (table 1). For all three models that include ecological memory, the maximum diversity is attained at lower values of average annual area burned, as compared with the dynamic percolation model, which assumes constant flammability. Further, the slight bimodality seen for the Wyoming scenario (Yellowstone National Park) does not appear in the percolation model. Ecological memory increases the diversity at lower disturbance levels. Simulation results are supported by analytical calculations of the diversity-disturbance curves for the DSM (app. E; Zinck et al. 2009).

### Discussion

The aim of this work was to unify simple models of forest fire ecosystems from ecology and statistical physics that have so far existed almost independently of each other. The idea that a unification might be possible came about gradually, on the basis of the following series of insights. First, the DSM reproduced fire shapes just as well as the RM (Zinck and Grimm 2008). To compare the DSM and the RM, the size of a grid cell in the DSM had to be rescaled so that it no longer corresponded to a single tree but to a forest stand of about 6 ha.

Next, it turned out that the DSM, like the RM, was capable of reproducing data to realistically describe the relationship between disturbance level (average annual area burned) and diversity of succession stages (Suffling et al. 1988; Zinck et al. 2009). To test this, succession had to be introduced in the DSM. This was achieved by de-

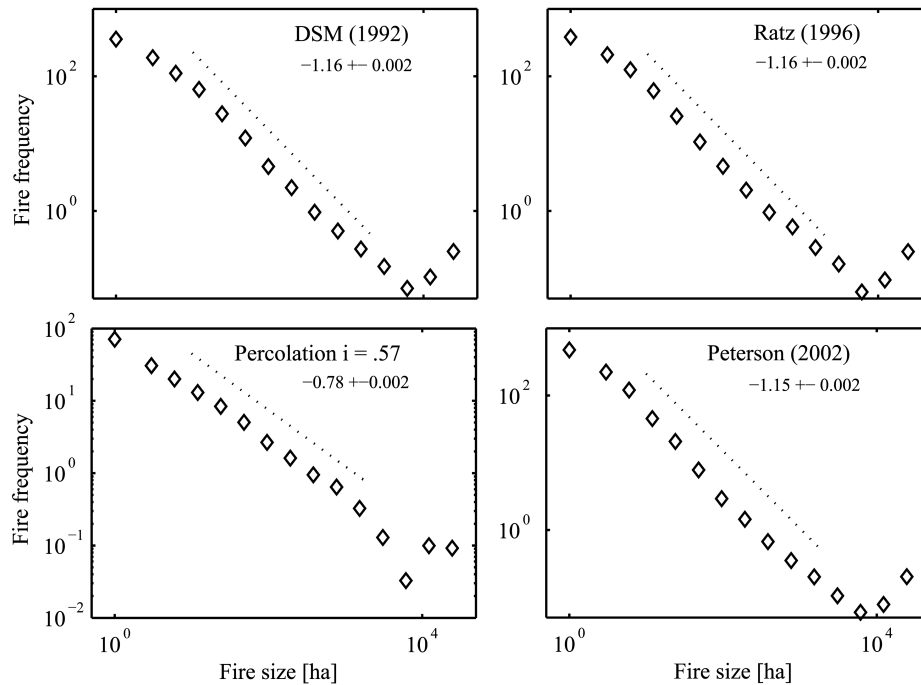
riving the flammability  $Pr_i(a)$  of a grid cell, which emerges from its fuel-replenishing process.

The final step was to take into account the notion and specific formulation of ecological memory introduced by Peterson (2002). This led to further insights: to capture key features of a real forest fire system, it appears that such a memory function describing flammability as function of the time since the last disturbance is needed. Further, by setting  $t_{\max}$ , the time after which flammability reaches its maximum, to 30 years instead of 500 years, as in the RM, a possible criticism in models including age-dependent flammability is met: time since fire might not matter for flammability for a wide range of stand ages, but it certainly matters for the first few decades, and this is necessary and sufficient to reproduce observed patterns without external parameter fine-tuning, as in the percolation model.

Our unified model shows that three so far unrelated models are indeed basically the same and that ecological memory, that is, a susceptibility to fire dependent on the time since the last fire, is the core feature of each of these models. Ecological memory is the reason why the difference between imposing  $Pr_i(a)$  in the RM and PM and letting it emerge in the DSM is superficial. In the DSM with separation of timescales, fire spread is deterministic, yet fuel patches emerge in a stochastic manner. In the RM and PM, fuel patches emerge during fire spread. It is just a question of when the fuel patch is determined: at the time of fire spread or in between fires, as in the DSM. In the end, the effect is the same because in both approaches it is the time lag in reestablishing flammability that determines  $Pr_i(a)$ . This is an astonishing and nontrivial insight. In the following we will discuss the benefits of our synthesis.

### *Simple Theoretical Forest Fire Models Are More Realistic than Anticipated*

Most fire models developed for landscape and fire ecology are more detailed than the four simple and generic models that we explored here. The RM (Ratz 1995, 1996) was designed for understanding generic patterns in forest fire ecosystems but was corroborated by comparing its output to data on fire shapes and landscape-level disturbance-diversity relationships (the latter was published only in a thesis and therefore was not generally known so far; Ratz



**Figure 5:** Frequency-size distribution of fires fitted by a power law. The exponents of the Drossel-Schwabl (1992) model (DSM), the Ratz (1996) model, and the Peterson (2002) model differ only marginally, although these models vary in their assumptions about flammability as a function of time since last fire (standard error of estimation depends on sample size; see app. A). The parameters (see fig. 4) were not fine-tuned to this exponent. The exponent obtained for the constant percolation scenario for the subcritical value of  $i = 0.57$  is, however, considerably lower.

1996). The RM therefore is acknowledged by landscape and fire ecologists as being ecologically significant to some degree, although it ignores virtually all details that are discussed in more detailed forest fire models, for example, topography, fuel and soil moisture, wind directions, weather, species composition, and individual trees (Keane et al. 2004).

In contrast, the PM (Peterson 2002) was designed to explore the effect of ecological memory on the persistence of patterns in the landscape. In this context, ecological memory refers to the marks in the landscape left by previous fires. If the parameter  $t_{\max}$  in the PM is, for example, assumed to be 30 years, the memory of the landscape in terms of flammability fades away after 30 years. Realism was not a primary issue for the PM. The DSM has so far been used rarely by landscape and fire ecologists (but see Malamud et al. 1998) because the assumptions of the DSM seem unrealistic, for example, trees “popping up” on empty grid cells, and because succession was completely ignored.

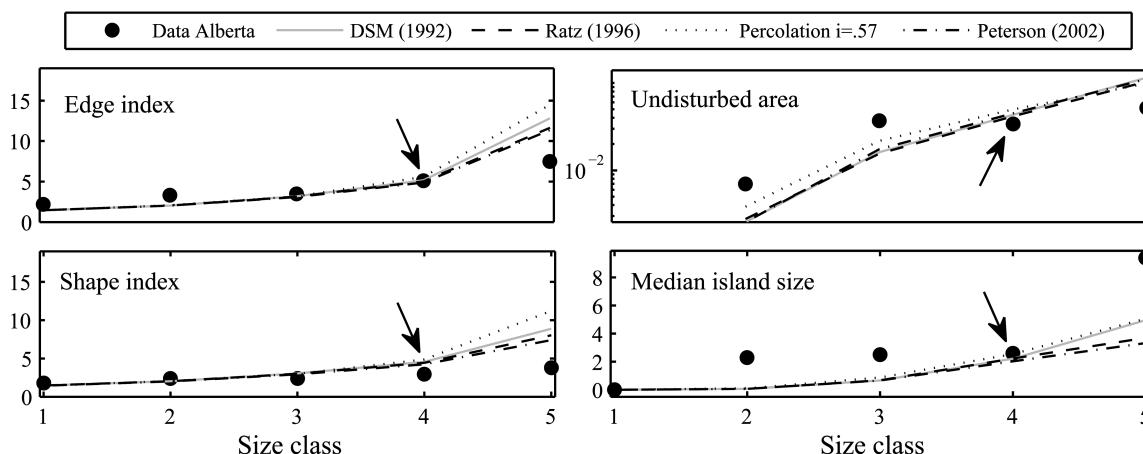
There was thus a wide gap between realistic forest fire models and the three generic models explored here. Our synthesis now shows that in fact these three models are equivalent. Further, we were led to this synthesis by rec-

ognizing that all three models are capable of reproducing three patterns observed in real forest fire ecosystems: fire shapes, disturbance-diversity relationships, and power-law-like fire size distributions. Thus, simple forest fire models, which are stochastic cellular automata that ignore virtually every detail in structure and process of real forests, are much more realistic than anticipated (even by their creators, as in the case of PM and DSM). “Realistic” here means that these models capture key processes that already explain a great deal of what is observed in real systems on larger spatial and temporal scales. The key processes are related to limitations to fire spread by the fuel mosaic, which, in turn, has been shaped by both former fires and the fuel regeneration process. The memory of previous fire events and the subsequent fuel mosaic are thus more important than previously believed.

#### *The Role of Fuel Mosaics and Ecological Memory*

The spread of wildfire is determined by a variety of factors such as fuel moisture, topography, exposition, fuel connectivity, and wind. Our unified model considers fuel connectivity only. The role of fuel connectivity in fire spread is controlled mainly by weather (Turner and Romme 1994;





**Figure 6:** Characteristics of fire shapes as produced by the models and as observed by Eberhart and Woodard (1987) in Alberta, Canada. The size classes are defined as (1) 20–40 ha, (2) 41–200 ha, (3) 201–400 ha, (4) 401–2,000 ha, and (5) 2,001–20,000 ha. All four models perform similarly; the largest deviation is seen for the median island size. The best fit is obtained for fires of size class 4 for all indexes. The robustness of this pattern is studied in detail for the Drossel-Schwabl model (DSM) by Zinck and Grimm (2008) and for the Ratz model by Ratz (1995). The goodness of fit was assessed by comparing the standard deviation of the model and data for each size class. The data from Alberta are available only as summary statistics. Error bars are not shown here to avoid clutter.

Turner 2005). Under poor burning conditions, that is, when fuel is wet, fires tend to die out rapidly. Under good burning conditions, fires spread, consuming the entire cluster of connected fuel on which they were initiated (Despain and Sellers 1977; Minnich 1983; Romme and Despain 1989; Turner 2005). These conditions may exist only for a few weeks in the year, yet they account for most large fires during the burning season. Under extreme conditions, fire spread is not restricted to the fuel mosaic, and large areas can burn, irrespective of stand structure and accumulated biomass (Moritz 1997). The role of weather as an enabler is also stressed by Johnson et al. (2001), who differentiate between poor and extreme burning conditions in boreal forests. Our unified model thus implicitly assumes that fires burn under good conditions.

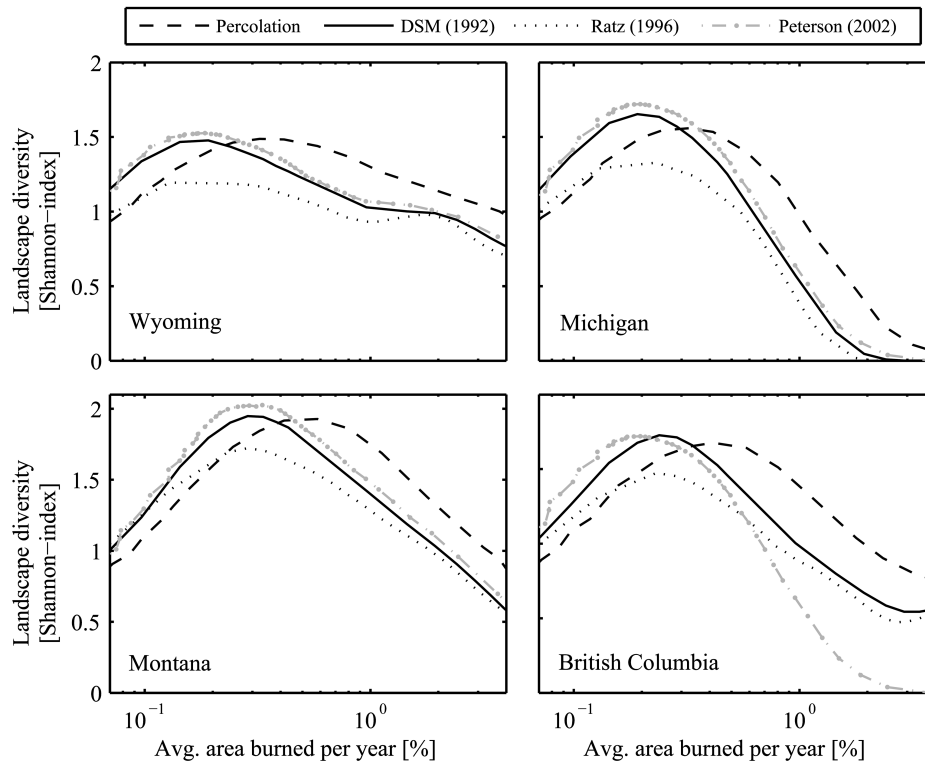
Stand-replacing fires start and spread in forests where a critical surface-fire intensity (kW/m; Van Wagner 1977) is reached and that exhibit a structure that enables the fire to reach and consume the crowns. A forest stand develops this susceptibility, characterized by a critical crown height and bulk density in closed canopy forests, at some time during its maturation (Johnson et al. 2001). The time needed for transition from nonsusceptible to susceptible depends on conditions that differ locally because some sites might have been affected more severely by a fire than others or are more rapidly reclaimed. Dispersal or sprouting, herbivores, and local climatic and soil conditions all play a role (Whelan 1995). Our unified model aggregates all these mechanisms into a function describing flammability depending on stand age, or time since last fire.

It is surprising that the shape of this function (fig. 3) has such a small effect on large-scale patterns. The largest effect can be seen in the disturbance-diversity curve, which rises to its maximum more rapidly with increasing ecological memory (the sequence is percolation model, DSM, RM, and PM) and appears to promote bimodality. The processes of fire spread and subsequent vegetation recovery shape the landscape and at the same time determine its spatiotemporal susceptibility for future disturbances.

In fire ecology, there has been some controversy over the role of age-dependent flammability. “Researchers working in different locations have variously proposed that the probability of fire spread is either independent of time since fire (Bessie and Johnson 1995), or that it increases with time since fire (Minnich 1983). These differences suggest that some forests have little ecological memory, whereas others have a significant amount” (Peterson 2002, p. 330). Our unified model shows that this controversy can be resolved to a large extent: “age dependent” does not necessarily mean that flammability changes with stand age for 100 years or more. A few decades, needed to build up critical bulk density, are sufficient to reproduce observed patterns, at least in boreal forests.

#### *Theoretical Insights Have Bearings for Real Forest Fire Ecosystems*

So far, it has remained unclear whether the body of theoretical insights on wildfire systems gained in statistical physics had any bearings for real forest fire ecosystems.



**Figure 7:** Diversity-disturbance relationship for succession scenarios representing four different real forests (table 1). The hump-shaped curve predicted by the intermediate disturbance hypothesis emerges for all scenarios (see also field study by Suffling et al. [1988]; fig. 1). The exact shape depends on the assumptions of the models about flammability as a function of time since last disturbance. For all three models that include ecological memory, the maximum diversity is attained at lower values of average annual area burned as compared with the dynamic percolation model, which assumes constant flammability. All curves were obtained by simulation (no interpolation used). DSM = Drossel-Schwabl model.

Our synthesis shows that it does. Real fire ecosystems can in fact be related to the ideas underlying the concept of SOC (Bak et al. 1988). SOC of wildfire systems has been discussed intensively in statistical physics (Grassberger 2002), and it seems clear by now that it is more appropriate to speak of power-law-like fire-frequency distributions rather than power laws (Reed and McKelvey 2002; Millington et al. 2006) and to assume that fire ecosystems are not necessarily fulfilling all theoretical requirements of SOC. Nevertheless, key elements of SOC seem to play a role: self-organization driven by ecological memory, that is, the feedback between a process (fire) and its effects (landscape structure), is indeed a key driver of these systems. The point is not whether we have critical states in the strict sense of phase transitions, as described in physics (Pascual and Guichard 2005), but whether we have a self-organized process that drives certain classes of systems, for example, wildfire systems, into a preferred range of states. This could explain the resilience of these states (Gunderson 2000).

Our unification allows, with caution, the direct transfer

of insights from statistical physics to real systems, for example, that an increase in the sparking rate leads to smaller average fire size and that the average annual area burned remains largely constant (see derivation in app. D). This finding is important in the context of discussion on the effects of increased natural and anthropogenic sparking rates and of fire management in general. Furthermore, many of the insights based on the DSM can be devised analytically, allowing more rigorous and comprehensive analyses than simulations. For example, the diversity-disturbance curves, which we determined here via simulation, can also be derived analytically (app. E; Zinck et al. 2009).

#### *Focusing on the Real Limitations of Simple Forest Fire Models*

The most important consequence of our synthesis might be that it allows us to identify the most pressing gap in current generic research on fire ecosystems. Our pattern analysis showed that the three models containing ecolog-

ical memory were well capable of reproducing fire shapes and the diversity-disturbance relationship of succession stage. However, the exponent of the power law part of the frequency–fire size relation is consistently too large in all three models; that is, these models produce too many large fires. The DSM, for example, typically produces an exponent of around  $-1.16$  (Grassberger 1993), whereas real wildfires have an exponent of between  $-1.2$  and  $-1.9$  (Malamud et al. 2005; Millington et al. 2006).

The exponent of the unified model is extremely robust. We tried many different features that would make the model more realistic, for example, introducing spatial heterogeneity, spatially inhomogeneous spark distribution, and wind, but all this had virtually no effect on the exponent (data not shown; listing in app. A). As yet, there is only one known way to get the exponent right in cellular automata fire models: the inclusion of stochastic effects of weather (Pueyo 2007) by adding a value, drawn from a weather distribution, to the flammability of all cells during a fire (see also Ratz 1996). Such a value is drawn for each fire and leads to frequency-area distributions of varying steepness. Nevertheless, there is evidence of the existence of nonlinear thresholds in environmental conditions, after which weather effects can no longer be correlated to fire size (Schoenberg et al. 2003). Hence, there is still considerable work to be done in explaining the frequency-area distributions of wildfires.

### Conclusions

We showed here that linking theoretical models, which focus on only one pattern observed in reality (power laws),

to further patterns (fire shapes, diversity-disturbance relationships) broadens the view, allows the identification of key mechanisms, and makes the models testable at different scales and hierarchical levels. This way of linking the process of formulating and testing models to entire sets of patterns, instead of to individual ones, has been referred to as pattern-oriented modeling (Grimm et al. 1996, 2005; Wiegand et al. 2003; Grimm and Railsback 2005). The search for mechanisms generating the right exponents suggests that we might need to include further patterns, for example, in fire spread or in the relationship between weather and fire sizes. It might even be possible to check the many existing, more detailed, and well-tested fire-succession models that focus on smaller scales for these additional patterns. This could be an example of the complimentary, unifying use of exploratory and predictive modeling that recently has been recommended by Perry and Millington (2008). In the future, therefore, perhaps even generic and specific fire models might be unified to provide general insights into what determines fire size distribution and to formulate specific management recommendations.

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## APPENDIX A

### Fitting with Power Laws: Listing of What We Tried to Change the Exponent

We used multiplicative binning, that is, bin sizes of 2, 4, 8, 16, ..., to plot the frequency-area data on log-log scales (Pueyo and Jovani 2006). The value for each bin was normalized by its size and plotted in the center of the bin. To determine the exponent  $\alpha$  of the power law, we used the maximum likelihood estimator directly from the  $n$  data (without the need for binning):

$$\alpha = 1 + n \left[ \sum_{i=1}^n \ln \left( \frac{x_i}{x_{\min} - 0.5} \right) \right]^{-1}, \quad (\text{A1})$$

where the estimate of the associated statistical error is given by

$$\sigma = \frac{\alpha - 1}{\sqrt{n}}. \quad (\text{A2})$$

This approach appeared to be the most suitable because it allowed a good fitting of the distribution even at its tail (Newman 2005; Clauset et al. 2009). The value  $x_{\min}$  was set to the smallest value for which the power law behavior

held. We cut off the first and last 10% of the sorted fire size data and plotted a power law with the determined exponent into the figure as test of accuracy.

The exponents we obtained for the Ratz model and Peterson model were around the same as that of the Drossel-Schwabl model (DSM). Fitting heavy-tailed distributions with power laws is prone to error, so we plotted the obtained distributions against each other to confirm that the distributions are almost identical over a wide range of parameters (see also the exponents documented by Ratz [1996] and the report of Pueyo [2007]). The challenge is to obtain distributions that are steeper to better fit the data. With this aim, we tried to modify the DSM by (1) using two different types of vegetation characterized by a different flammability (value of parameter  $p$ ); (2) randomly adding nonflammable cells to the landscape (up to 60%; we got the heavy tail of the distribution to break down in this way, yet the steepness remained the same almost to the point of breakdown); (3) including a dominant direction of wind such that the fire preferentially burns into one direction, leaving part of the fuel unburned; (4) introducing a time lag in which burned cells do not regenerate after a fire (after this time lag, the cell can regenerate with probability  $p$  per time step, as in the original model); and (5) using a spatially inhomogeneous spark distribution (highest probability of lightning spark in the middle, exponential drop off from there).

All of these modifications had, at best, limited effects on the exponent. This is highly counterintuitive and nontrivial. Pueyo (2007) attributes this effect to universality. To our knowledge, there is no more rigorous argument as yet.

## APPENDIX B

### Frequency-Area Distribution and Fire Shapes: Parameters Used

In the Ratz model and the Peterson (2002) model (PM), the probability of ignition  $\text{Pr}_i(a)$  can be freely imposed, whereas  $\text{Pr}_i(a)$  emerges in the Drossel-Schwabl model (DSM), depending on the regrowth parameter  $p$  (DSM rule 2; eq. [3]). Ratz (1995) assumed a constant base flammability  $i$ , which we set to  $i = 0.2$ , and an age-dependent part, for which we used  $c = 0.07$  (eq. [1]). In the PM (eq. [2]), we set  $\alpha = 2$ ,  $t_{\max} = 30$ , and the maximal flammability to  $\text{Pr}_{\max} = 0.8$ , considering the good burning conditions assumed for our scenario. We set the time to reach the maximal flammability to 30 years because critical bulk density—which is associated with the development of crown fires—is fairly stable in boreal forests after the first 2 decades (Rothermel 1972; Brown and Bevins 1986; Bessie and Johnson 1995). For the percolation model, we used  $\text{Pr}_i = 0.57$ , which is close to the percolation threshold of about 0.59 (Stauffer and Aharony 1992). We chose  $p = 0.08$  for the DSM in order to get a steep increase in the emerging flammability.

## APPENDIX C

### Indexes for Fire Shape

The following indexes were used in a field study by Eberhart and Woodard (1987). We provide the formal definition here. The unburned area, UA, consists of unburned islands relative to the area  $a_o$  enclosed by the outer perimeter of the fire. Let  $a_b$  be the area that was actually burned; then, the total unburned area is

$$\text{UA} = 1 - \frac{a_b}{a_o}. \quad (\text{C1})$$

The shape index, SI, is the ratio of the outer perimeter  $p_o$  of the fire to the perimeter of a circle that encloses the same area,  $a_o$ :

$$\text{SI} = \frac{p_o}{2\sqrt{\pi a_o}}. \quad (\text{C2})$$

The edge index, EI, is the sum of the outer perimeter plus the perimeter of all enclosed islands,  $p_w$ , compared with the perimeter of a circle that encloses an area equal to the burned area,  $a_b$ :

$$EI = \frac{p_w}{2\sqrt{\pi a_b}}. \quad (C3)$$

#### APPENDIX D

##### Average Area Burned and Sparking Frequency in the Drossel-Schwabl Model

The average area burned in the Drossel-Schwabl model (DSM) is equal to that which regrows on average  $c = p(1 - p_t)$ , where  $p_t = 0.41$  is the quasi-stationary state (Grassberger 1993) and is independent of  $p$  and  $f$ , as long as  $f \ll p$  and  $p \ll 1$ . This mean field equation enables us to calculate  $p$  directly from the average annual area burned, a quantity commonly monitored by conservation agencies. An average area burned of 1%, for example, would indicate a  $p$  value of  $0.01/0.59 = 0.016$ . The sparking rate does not enter this equation and does not influence the average area burned, controlled by the regrowth parameter  $p$ . The average fire size is a function of both the regrowth parameter  $p$  and the sparking rate  $f$ , such that (Clar et al. 1996)

$$s = \frac{p}{f} \frac{1 - p_t}{p_t} = \frac{c}{fp_t}. \quad (D1)$$

Increasing the sparking rate leads to smaller average fire size. In the DSM, the average annual area burned is not affected because the average fuel load on the landscape,  $p$ , remains at the same value.

#### APPENDIX E

##### Analytical Prediction of the Disturbance-Diversity Relationship

The age-class distribution is the basis for determining landscape diversity. The behavior of the age-class distribution can be modeled using a one-dimensional drift equation (Sinko and Streifer 1967). The state of the model landscape is characterized by the distribution of the age classes,  $s(a, t)$ , which is the area with time since last fire  $a$  at time  $t$ . The probability of perishing in a fire depends on the age class and the disturbance intensity of the system, which can be quantified by the average annual area burned,  $c$ . The drift equation is

$$\frac{\partial}{\partial t} s(a, t) + \frac{\partial}{\partial a} s(a, t) = -m(a, c)s(a, t), \quad (E1)$$

where  $m$  is mortality. The average number of sites with time since last fire  $a$  that are consumed by fire is (Zinck et al. 2009)

$$m(a, C)s(a) = \frac{c}{p_t} \text{Pr}_i(a)s(a). \quad (E2)$$

#### APPENDIX F

##### Pseudocode for Wildfire Models

In this appendix we provide a pseudocode that illustrates how the wildfire models and the unified model can be implemented independently of the programming language. The actual source code used in the simulations is available as a zip file. A sketch of the general fire model is provided in algorithm 1. The fire spread algorithm is provided in recursive form in algorithm 2 and using a queue in algorithm 3. Algorithms 4, 5, and 6 provide the framework for the model of Ratz (1996) and Peterson (2002), as well as that of Drossel and Schwabl (1992), respectively.

**Algorithm 1:** The general wildfire model

```

1: s <= rectangular landscape grid
2: repeat
3:   c <= s.getRandomCell()
4:   if c.isSusceptible() then
5:     FireSpread(c)
6:   end if
7:   for all cells c in the landscape do
8:     c.increaseAgeBy(timeBetweenSparks)
9:   end for
10: until Simulation is done

```

**Algorithm 2:** Recursive implementation of fire spread

```

1: procedure RecursiveFireSpread(cell c)
2:   if ignites(c) then
3:     c.setAge(0)
4:     for all von Neumann neighbors n of c do
5:       RecursiveFireSpread(n)
6:     end for
7:   end if
8: end procedure

```

**Algorithm 3:** Nonrecursive implementation of fire spread

**Require:** Cell c has been struck by lightning and ignited

```

1: procedure FireSpread(cell c)
2:   c.setAge(0)
3:   queue.add(c)
4:   while queue is not empty do
5:     c = queue.poll()
6:     for all von Neumann neighbors n of c with n not in queue do
7:       if ignites(c) then
8:         n.setAge(0)
9:         queue.add(n)
10:      end if
11:    end for
12:   end while
13: end procedure

```

**Algorithm 4:** The basic simulation loop of Ratz (1995) and Peterson (2002)

```

1: s <= rectangular landscape grid
2: repeat
3:   for a fixed number n do
4:     c <= s.getRandomCell()
5:     if ignites(c) then FireSpread(c)
6:   end if
7:   end for
8:   for all cells c in s do
9:     c.increaseAgeByYears(timeStep)
10:  end for
11: until Simulation is done

```

**Algorithm 5:** The DSM cellular automaton

```

1: s <= rectangular landscape grid
2: repeat
3:   for all cells c in s do
4:     if c.isBurning() then c.nextState('empty')
5:     end if
6:     if c.isTree() AND at least one direct neighbor is burning then
7:       c.nextState('burning')
8:     end if
9:     if c.isTree() AND random() < f then c.nextState('burning')
10:    end if
11:    if c.isEmpty() AND random() < p then c.nextState('tree')
12:    end if
13:  end for
14:  for all cells c in s do
15:    c.assumeNewState()
16:  end for
17: until Simulation is done

```

**Algorithm 6:** DS-FFM after Grassberger (1993); Henley (1993)

```

1: s <= rectangular landscape grid
2: repeat
3:   c <= s.getRandomCell()
4:   if c.isTree() then FireSpread(c)
5:   end if
6:   for a number of  $\theta = p/f$  do
7:     d <= s.getRandomCell()
8:     if d.isEmpty() then d.setState('Tree');
9:     end if
10:  end for
11: until Simulation is done

```

## Literature Cited

- Bak, P., C. Tang, and K. Wiesenfeld. 1988. Self-organized criticality. *Physical Review A* 38:364–374.
- Baker, W. L. 1999. Spatial simulation of the effects of human and natural disturbance regimes on landscape structure. Pages 277–308 in D. J. Mladenoff and W. L. Baker, eds. *Spatial modeling of forest landscape change: approaches and applications*. Cambridge University Press, Cambridge.
- Bessie, W., and E. Johnson. 1995. The relative importance of fuels and weather on fire behavior in subalpine forests. *Ecology* 76:747–762.
- Brown, J., and C. Bevins. 1986. Surface fuel loadings and predicted fire behavior for vegetation types in the northern Rocky Mountains. Research note INT-358. USDA Forest Service, Missoula, MT.
- Carlson, J. M., and J. Doyle. 1999. Highly optimized tolerance: a mechanism for power laws in designed systems. *Physical Review E* 60:1412–1427.
- Clar, S., B. Drossel, and F. Schwabl. 1996. Forest fires and other examples of self-organized criticality. *Journal of Physics: Condensed Matter* 8:6803–6824.
- Clark, J. S. 1993. Fire, climate change, and forest processes during the last 2000 years. *Geological Society of America Special Paper* 276:295–308.
- Clauset, A., C. R. Shalizi, and M. E. J. Newman. 2009. Power-law distributions in empirical data. *SIAM Review* (forthcoming), arXiv:0706.1062.
- Connell, J. H. 1978. Diversity in tropical rain forests and coral reefs. *Science* 199:1302–1310.
- Despain, D., and R. Sellers. 1977. Natural fire in Yellowstone National Park. *Western Wildlands* 4:20–24.
- Drossel, B., and F. Schwabl. 1992. Self-organized critical forest fire model. *Physical Review Letters* 69:1629–1632.
- Eberhart, K., and P. Woodard. 1987. Distribution of residual vegetation associated with large fires in Alberta. *Canadian Journal of Forest Research* 17:1207–1212.
- Grassberger, P. 1993. On a self-organized critical forest fire model. *Journal of Physics A: Mathematical and General* 26:2081–2089.
- . 2002. Critical behaviour of the Drossel-Schwabl forest fire model. *New Journal of Physics* 4:17.1–17.15.
- Grimm, V., and S. F. Railsback. 2005. *Individual-based modeling and ecology*. Princeton University Press, Princeton, NJ.
- Grimm, V., K. Frank, F. Jeltsch, R. Brandl, J. Uchmanski, and C. Wissel. 1996. Pattern-oriented modelling in population ecology. *Science of the Total Environment* 183:151–166.
- Grimm, V., E. Revilla, U. Berger, F. Jeltsch, W. M. Mooij, S. F. Railsback, H.-H. Thulke, J. Weiner, T. Wiegand, and D. L. DeAngelis. 2005. Pattern-oriented modeling of agent-based complex systems: lessons from ecology. *Science* 310:987–991.

- Gunderson, L. H. 2000. Ecological resilience: in theory and application. *Annual Review of Ecology and Systematics* 31:425–439.
- Henley, C. L. 1989. Self-organized percolation: a simpler model. *Bulletin of the American Physical Society* 34:838.
- . 1993. Statics of a “self-organized” percolation model. *Physical Review Letters* 71:2741–2744.
- Johnson, E., K. Miyanishi, and S. Bridge. 2001. Wildfire regime in the boreal forest and the idea of suppression and fuel buildup. *Conservation Biology* 15:1554–1557.
- Keane, R. E., and M. A. Finney. 2003. The simulation design for modeling landscape fire, climate, ecosystem dynamics. Pages 32–68 in T. W. Swetnam, ed. *Fire and climatic change in temperate ecosystems of the western Americas*. Springer, New York.
- Keane, R. E., G. J. Cary, I. D. Davies, M. D. Flannigan, R. H. Gardner, S. Lavorel, J. M. Lenihan, C. Li, and T. S. Rupp. 2004. A classification of landscape fire succession models: spatial simulations of fire and vegetation dynamics. *Ecological Modelling* 179:3–27.
- Kessel, S. R. 1980. A quantitative succession model for nine Montana forest communities. *Environmental Management* 4:227–240.
- Malamud, B., G. Morein, and D. L. Turcotte. 1998. Forest fires: an example of self-organized critical behavior. *Science* 281:1840–1842.
- Malamud, B. D., G. Morein, and D. L. Turcotte. 2005. Log-periodic behavior in a forest fire model. *Nonlinear Processes in Geophysics* 12:575–585.
- Meyn, A., C. Buhk, P. S. White, and A. Jentsch. 2007. Environmental drivers of large, infrequent wildfires: the emerging conceptual model. *Progress in Physical Geography* 31:287–312.
- Millington, J. D. A., G. L. W. Perry, and B. D. Malamud. 2006. Models, data and mechanisms: power laws in wildfire regime frequency-area statistics. Pages 155–167 in G. Cello and B. D. Malamud, eds. *Fractal analysis of natural hazards*. Geological Society of London Special Publication 261:155–167.
- Minnich, R. 1983. Fire mosaics in southern California and northern Baja California. *Science* 219:1287–1294.
- Mladenoff, D. J., and W. L. Baker, eds. 1999. *Spatial modeling of forest landscape change: approaches and applications*. Cambridge University Press, Cambridge.
- Moritz, M. 1997. Analyzing extreme disturbance events: fire in Los Padres National Forest. *Ecological Applications* 7:1252–1262.
- Newman, M. E. J. 2005. Power laws, Pareto distributions and Zipf's law. *Contemporary Physics* 46:323–351.
- Otto, H. 1994. *Waldökologie*. Ullmer, Stuttgart.
- Pascual, M., and F. Guichard. 2005. Criticality and disturbance in spatial ecological systems. *Trends in Ecology & Evolution* 20:88–95.
- Perry, G. L. W., and J. D. A. Millington. 2008. Spatial modelling of succession-disturbance dynamics in forest ecosystems: concepts and examples. *Perspectives in Plant Ecology, Evolution and Systematics* 9:191–210.
- Peterson, G. D. 2002. Contagious disturbance, ecological memory, and the emergence of landscape pattern. *Ecosystems* 5:329–338.
- Pueyo, S. 2007. Self-organised criticality and the response of wildland fires to climate change. *Climatic Change* 82:131–161.
- Pueyo, S., and R. Jovani. 2006. Comment on “A keystone mutualism drives pattern in a power function.” *Science* 313:1739c.
- Railsback, S. F. 2001. Concepts from complex adaptive systems as a framework for individual-based modelling. *Ecological Modelling* 139:47–62.
- Ratz, A. 1995. Long-term spatial patterns created by fire: a model oriented towards boreal forests. *Journal of Wildland Fire* 5:25–34.
- . 1996. A generic forest fire model: spatial patterns in forest fire ecosystems. PhD thesis. Philipps-Universität Marburg.
- Reed, W. J., and K. S. McKelvey. 2002. Power law behaviour and parametric models for the size distribution of forest fires. *Ecological Modelling* 150:239–254.
- Romme, W. H., and D. G. Despain. 1989. Historical perspective on the Yellowstone fires of 1988. *BioScience* 39:695–699.
- . 1991. The long history of fire in the greater Yellowstone National Park. *Western Wildlands* 15:10–17.
- Rothermel, R. 1972. A mathematical model for predicting fire spread in wildland fuels. Research paper INT-115. USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, UT.
- Schoenberg, F. P., R. Peng, Z. Huang, and P. Rundel. 2003. Detection of nonlinearities in the dependence of burn area on fuel age and climatic variables. *International Journal of Wildland Fire* 12:1–6.
- Shugart, H. H., T. Crow, and J. Hett. 1973. Forest succession models: a rationale and methodology for modelling forest succession over large regions. *Forest Science* 19:203–212.
- Sinko, J., and W. Streifer. 1967. A new model for age size structure of a population. *Ecology* 48:910–918.
- Stauffer, D., and A. Aharony. 1992. *Introduction to percolation theory*. Taylor & Francis, London.
- Suffling, R., C. Lihou, and Y. Morand. 1988. Control of landscape diversity by catastrophic disturbance: a theory and case study of fire in Canadian boreal forest. *Environmental Management* 12:73–78.
- Swetnam, T. W., ed. 2003. *Fire and climatic change in temperate ecosystems of the western Americas*. Springer, New York.
- Turner, M. G. 2005. Landscape ecology: what is the state of the science? *Annual Review of Ecology and Systematics* 36:319–344.
- Turner, M. G., and W. H. Romme. 1994. Landscape dynamics in crown fire ecosystems. *Landscape Ecology* 9:59–77.
- Van Wagner, C. 1977. Conditions for the start and spread of crownfire. *Canadian Journal of Forest Research* 7:23–34.
- Whelan, R. J. 1995. *The ecology of fire*. Cambridge University Press, Cambridge.
- Wiegand, T., F. Jeltsch, I. Hanski, and V. Grimm. 2003. Using pattern-oriented modeling for revealing hidden information: a key for reconciling ecological theory and application. *Oikos* 100:209–222.
- Zinck, R., and V. Grimm. 2008. More realistic than anticipated: a classical forest fire model from statistical physics captures real fire shapes. *Open Ecology Journal* 1:8–13.
- Zinck, R., K. Johst, and V. Grimm. 2009. Wildfire, landscape diversity and the Drossel-Schwabl model. *Ecological Modelling*, doi: 10.1016/j.ecolmodel.2008.12.026.

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