Is a sub 2 hour marathon in the near future? Modeling rare events in sports.

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Outline

- Background
- Marathon Data
- Simple Model
- Self-Exciting Model
- Model Comparisons (preliminary results)
- Further Research
- SCORE

Background


The New York Times

Records Fell at the Track Worlds. A Trend? Not So Fast.

Some have referred to this era as a golden age of better and better times. But a deeper look at the data shows the simple shorthand conclusion is incomplete.

Figure 1: Golden Age?

- Are we living in a time of records?
  - Idea: seems like an increase in records falling, but is it just the nature of randomness?
- How can we address this question?
- What would randomness look like?

Figure 2: Rod Aloha 10K Run (San Diego, 2018), 2nd Age Group

Figure 3: Rod Last Marathon (LA, 2018), 1st, all city of Glendora CA runners

Marathon World Record Data

Men's Marathon world records since 1908

- 50 total
- First 5

Time	Name	Nationality	Date	Event/Place
2:55:18.4	Johnny Hayes	United States	July 24, 1908	London, United Kingdom
2:52:45.4	Robert Fowler	United States	January 1, 1909	Yonkers,[nb 5]United States
2:46:52.8	James Clark	United States	February 12, 1909	New York City, United States
2:46:04.6	Albert Raines	United States	May 8, 1909	New York City, United States
2:42:31.0	Henry Barrett	United Kingdom	May 8, 1909[nb 6]	Polytechnic Marathon, London, United

- Last 5

Time	Name	Nationality	Date	Event/Place
2:03:59	Haile Gebrselassie	Ethiopia	September 28, 2008	Berlin Marathon
2:03:38	Patrick Makau	Kenya	September 25, 2011	Berlin Marathon
2:03:23	Wilson Kipsang	Kenya	September 29, 2013	Berlin Marathon
2:02:57	Dennis Kimetto	Kenya	September 28, 2014	Berlin Marathon
2:01:39	Eliud Kipchoge	Kenya	September 16, 2018	Berlin Marathon

Breaking News. . .

SPORTS

He was so fast, he had time to celebrate long before the second-place runner arrived

Updated September 25, 2022 · 2:36 PM ET

Kenya's Eliud Kipchoge crosses the line to win the Berlin Marathon in Berlin on Sunday.

Christoph Soeder/AP

Figure 4: Eliud Kipchoge, Berlin, 2:01:09, September 25, 2022

Fun Facts

- Countries with most records
  - U.K. 12
  - U.S. 7
  - Kenya 6
  - Japan/Ethiopia 5
- Largest decrease:
  - James Clark, US, 1909
  - 2:46:52.8
  - Nearly 6 minutes faster than previous

Figure 5: Khalid Khannouchi, last US record holder (2:05:38, 2002)

Fun Facts

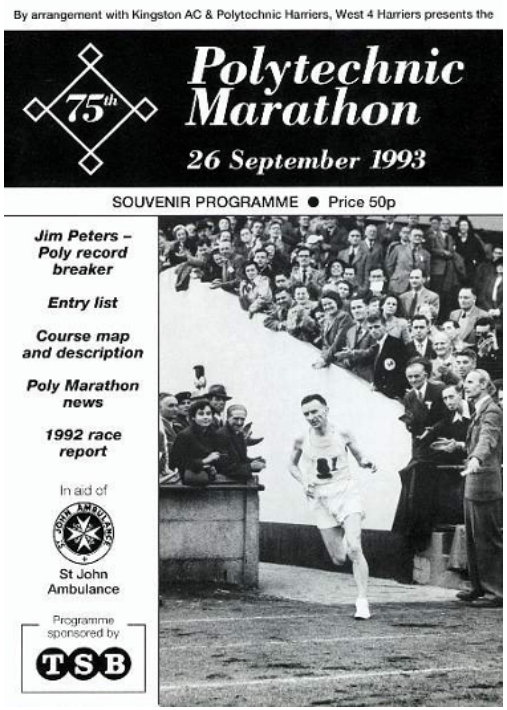
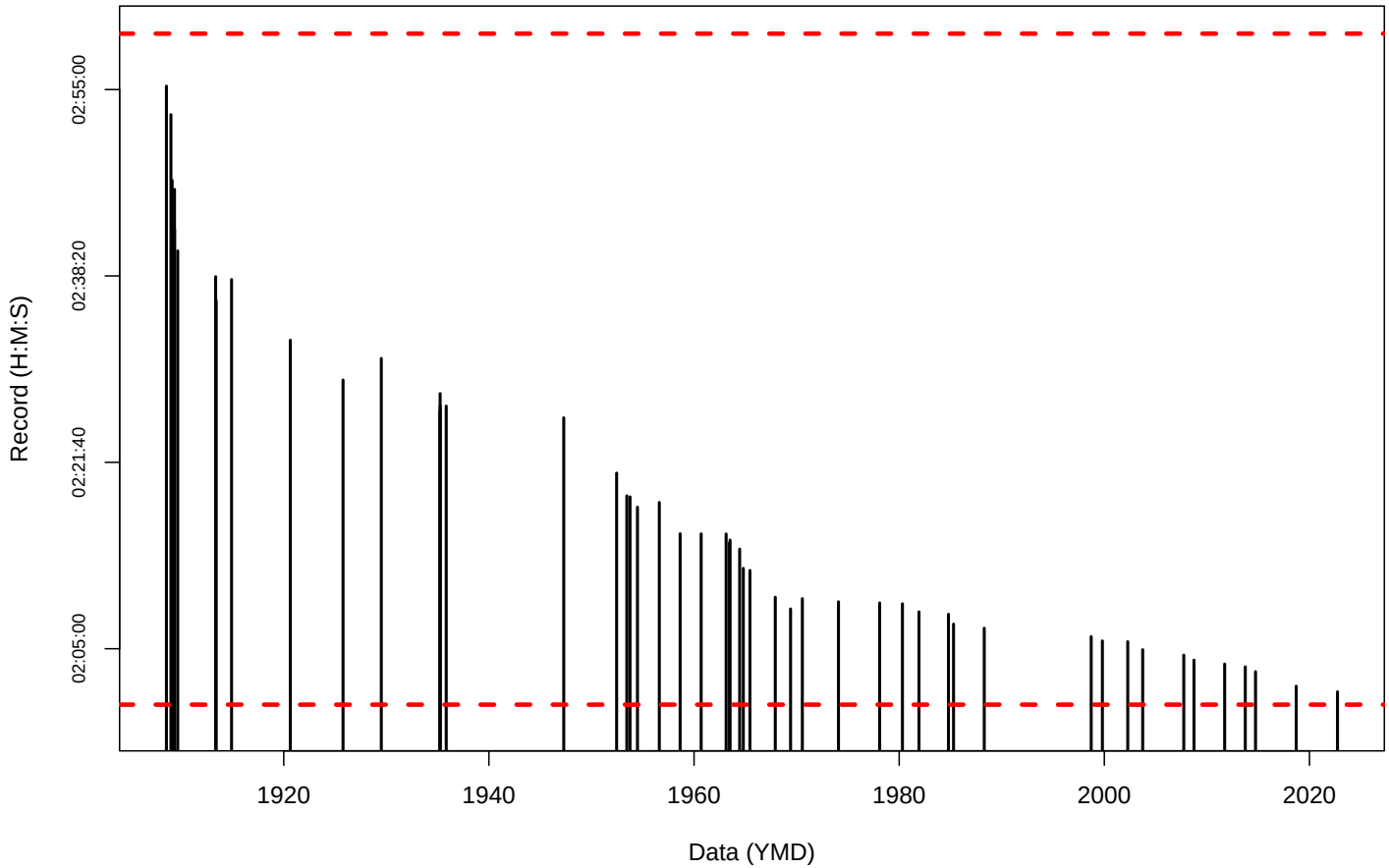


Figure 6: Polytechnic Marathon Program 1993 (Jim Peters)

- Most records
  - Jim Peters, UK,
  - Set records 4 times from 1952-1954
- Locations with most records
  - Berlin Marathon - 9
    - Last 8 records
  - Polytechnic Marathon (London) - 8
  - Tokyo Marathon - 5

Visualizing the data

- Two and three hour times shown as horizontal lines



- 2 hour marathon pace: 4:35 per mile
- 3 hour pace: 6:52 per mile

SIMPLE MODEL

POISSON PROCESS

A model for a series of discrete events where the average time between events is known, but the exact timing of events is "random" meeting the following criteria:

- Events are independent of each other. The occurrence of one event does not affect the probability another event will occur.
- The average rate (events per time period) is constant.
- Two events cannot occur at the same time.

Poisson Process Interarrival Times

The time between events (known as the interarrival times) follow an exponential distribution defined as:

$$P(T > t) = e^{-\lambda t}$$

- T is the random variable of the time until the next event
- t is a specific time for the next event
- λ is the rate: the average number of events per unit of time.

Note the possible values of T are greater than 0 (positive only).

Reasonableness of Exponential Interarrivals

The exponential distribution has certain attributes, for example:

$$E(T) = SD(T) = 1/\lambda$$

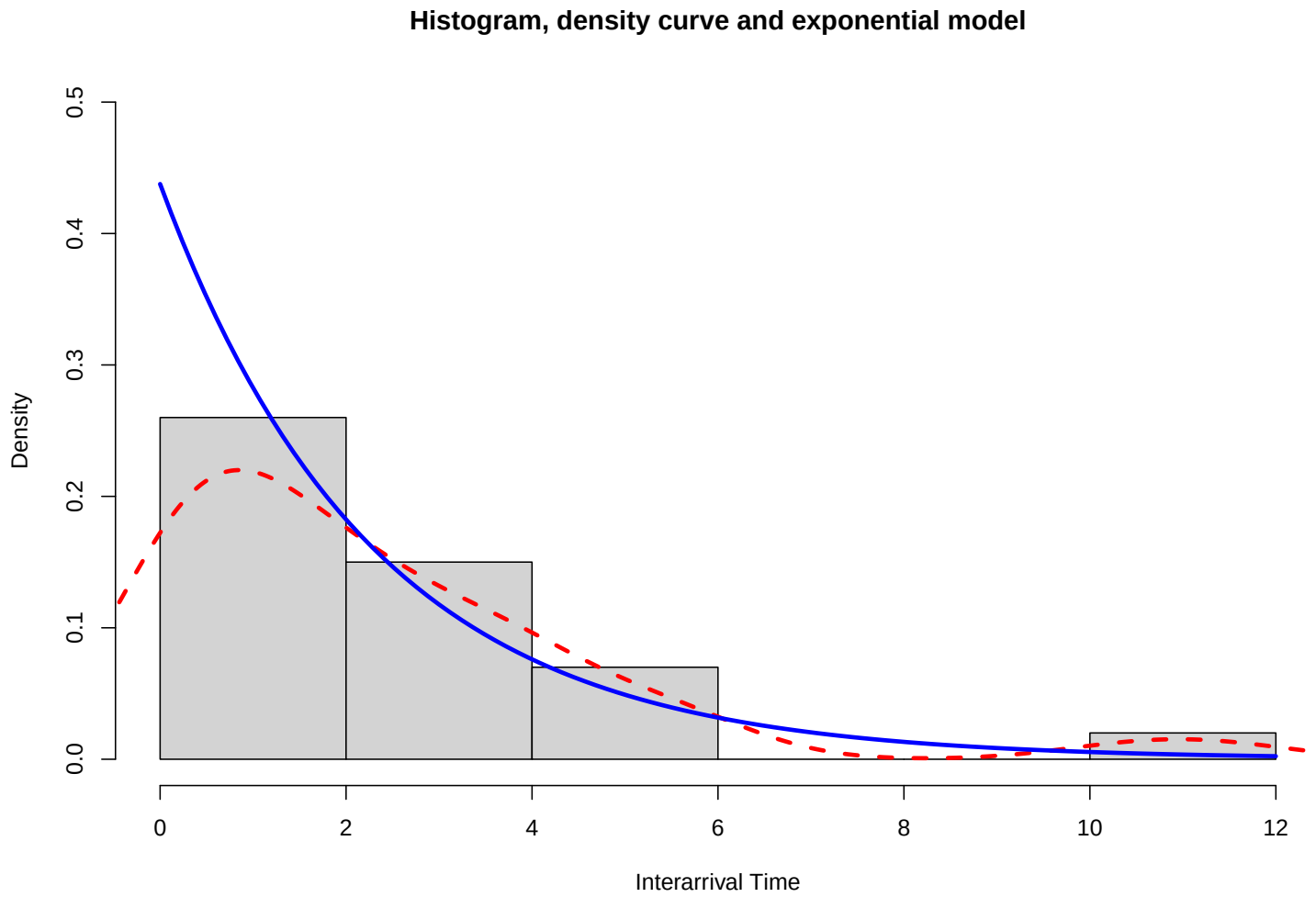
For the time between record data:

- Mean: 2.28 years
- SD: 2.42 years

Reasonable. . . slightly "overdispersed"

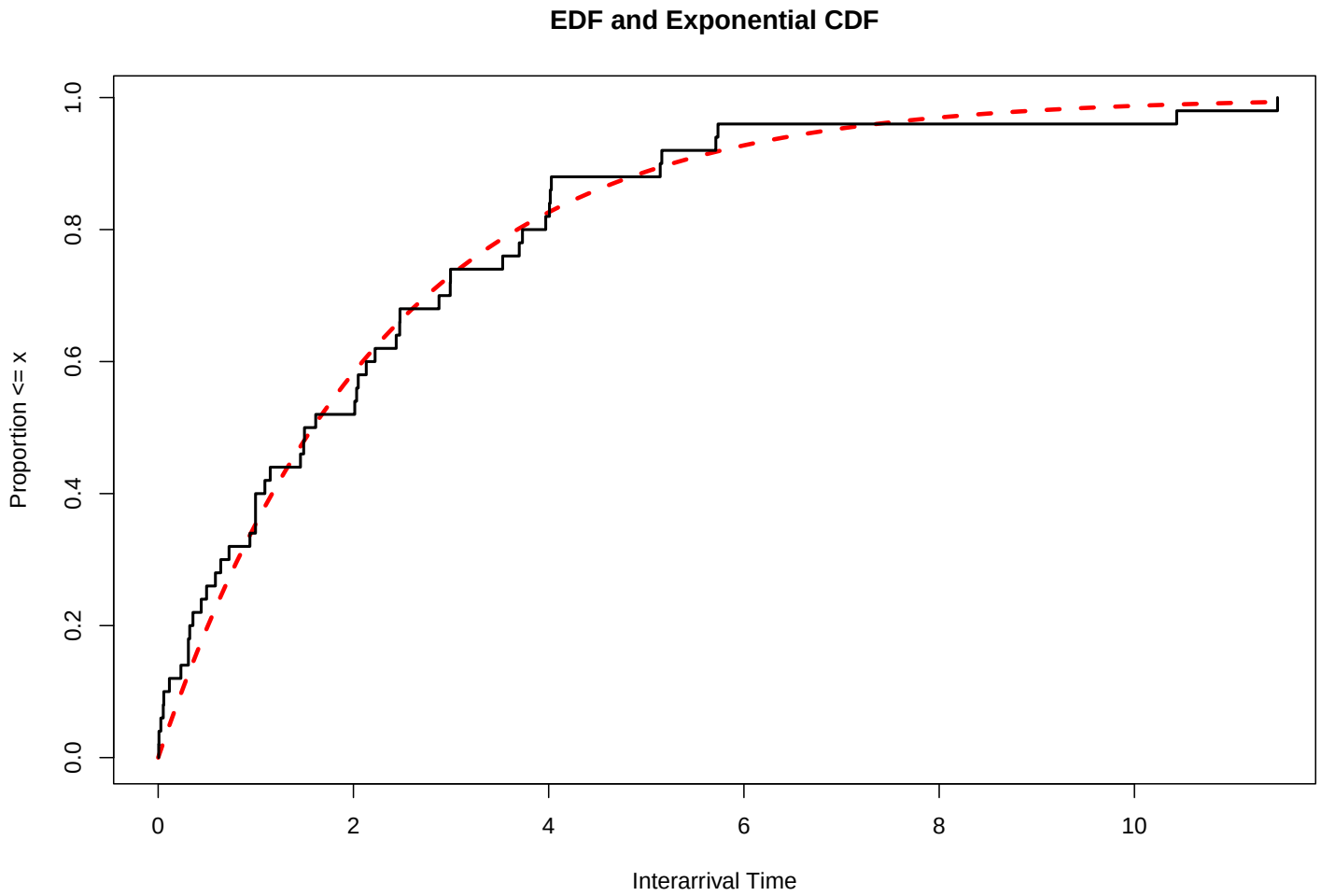
Estimating the Model

We estimate (MLE) $\lambda = 1/E(T) = 0.438$



Graphical Assessment of Fit

Empirical Distribution Function (EDF) and Cumulative Distribution Function (CDF)



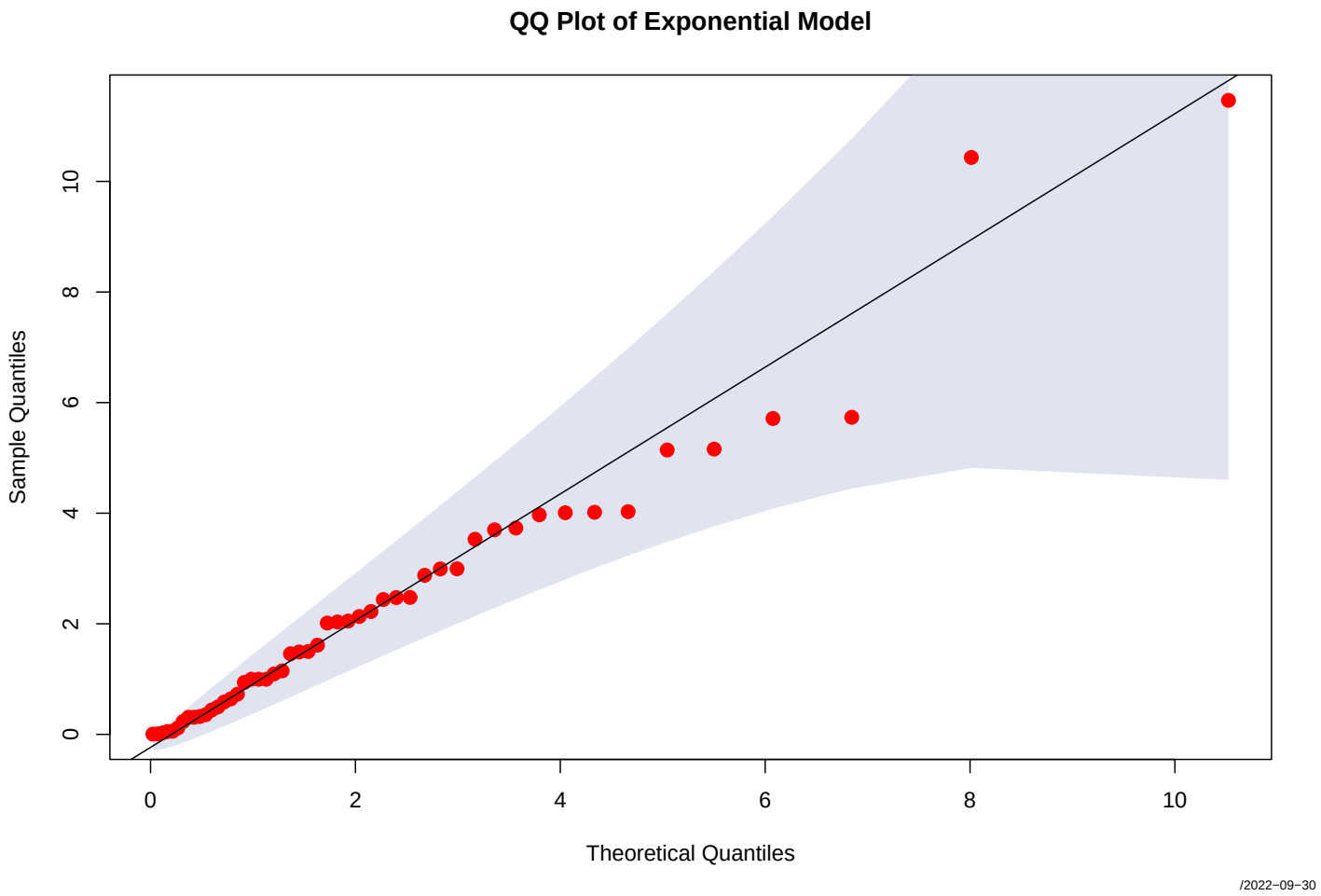
Testing Model Fit

Goodness of fit tests

- Kolmogorov-Smirnov: $p = 0.937$
- Cramer-Von Mises: $p = 0.846$
- Anderson-Darling: $p = 0.563$

All fail to reject the null hypothesis of model fit

Are records then random?



What are the poorly fit points?

- 11.5 year gap

Time	Name	Nationality	Date	Event/Place
2:26:42	Sohn Kee-chung	Japanese Korea	November 3, 1935	Tokyo, Japan
2:25:39	Suh Yun-bok	Korea	April 19, 1947	Boston Marathon

- 10.4 year gap

Time	Name	Nationality	Date	Event/Place
2:06:50	Belayneh Dinsamo	Ethiopia	April 17, 1988	Rotterdam Marathon
2:06:05	Ronaldo da Costa	Brazil	September 20, 1998	Berlin Marathon

Have we answered the question?

- Model fit seems to suggest random events (no "golden age" of records)
- Assumptions such as independence very hard to assess
  - Could model miss periods of increased records and still look reasonable?
  - There may be factors that increase the probability of records...

A "Self-Exciting" Model

Self-Exciting Point Processes

- Events "trigger" more events
- Examples of use include earthquakes, crime waves

Hawkes Processes

- Let H_t be the history of events up to time t. The Hawkes (1971) model of the conditional intensity is:

$$\lambda(t|H_t) = \nu + \sum_{i:t_i<t} g(t - t_i)$$

where ν is the background rate of events and g is the "triggering function".

Exponential Triggering Function

- The "triggering" function can be further decomposed:

$$g = \mu g^*$$

where g^* is a density function known as the "reproduction kernel" and μ is known as the "reproduction" mean.

- A common choice for the "reproduction kernel" is the exponential density given by:

$$g^*(t) = \beta e^{-\beta t}$$

Fitting the model

Parameter estimates for marathon data (exponential) Hawkes process, using MLE (Cheyson and Lang, 2020):

- baseline intensity 0.393
- reproduction mean 0.121
- exponential reproduction function rate 3.844

Note the baseline intensity is slightly lower than the constant model rate estimate of 0.438

The estimated reproduction function is then:

$$g(t) = \mu g^*(t) = \mu\beta e^{-\beta t}$$
$$= 0.12 * 3.84 e^{-3.84t}$$

Model implications

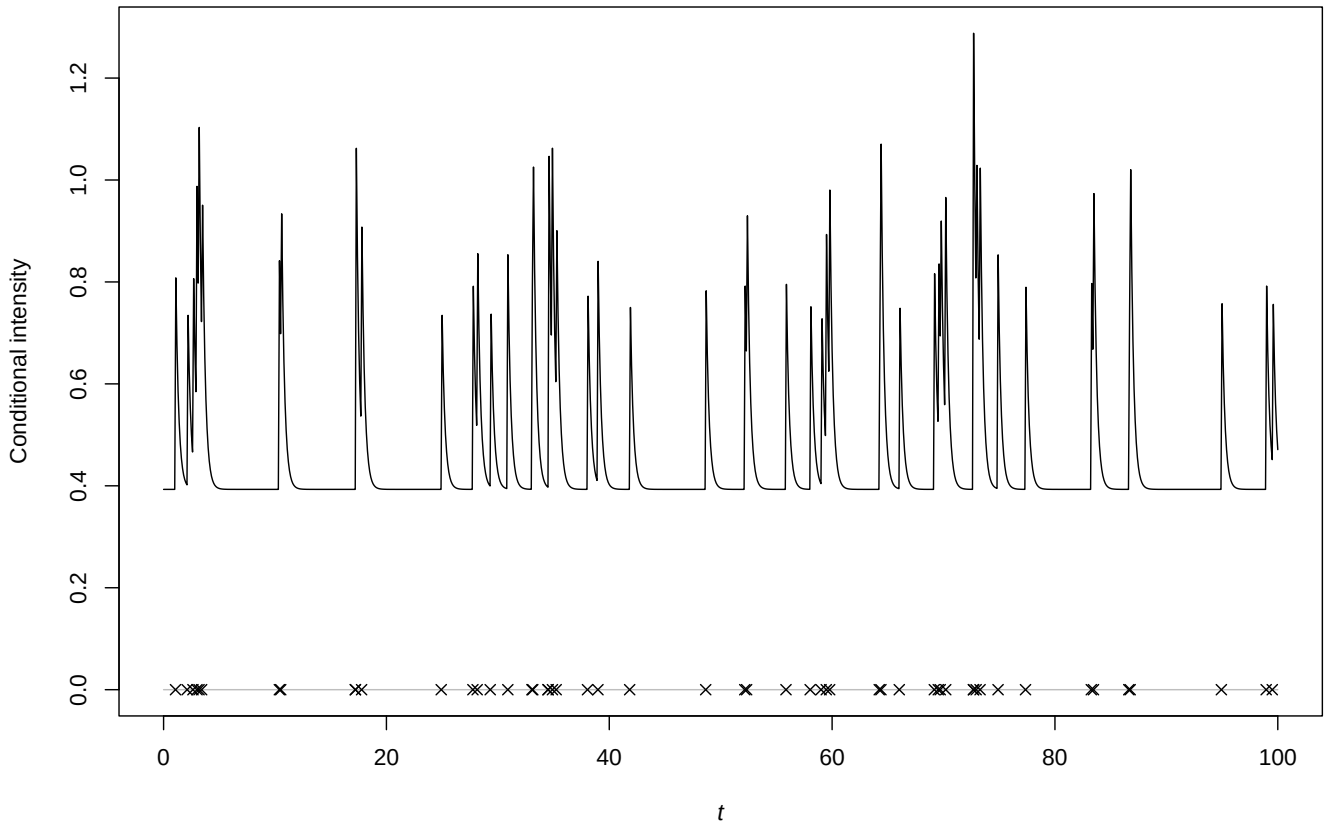
At the instant of the first event (world record), $t = t_1$ so $g(t - t_1 = 0)$ and the reproduction rate is:

$$g(0) = 0.12 * 3.84e^{-3.84*0} = 0.12 * 3.84 = 0.466$$

- The rate increases from the baseline rate of 0.393 by this amount at the moment of this occurrence
- The rate then decays back to baseline over time (unless a new event occurs)
- Each new event "excites" the rate to increase and then decay

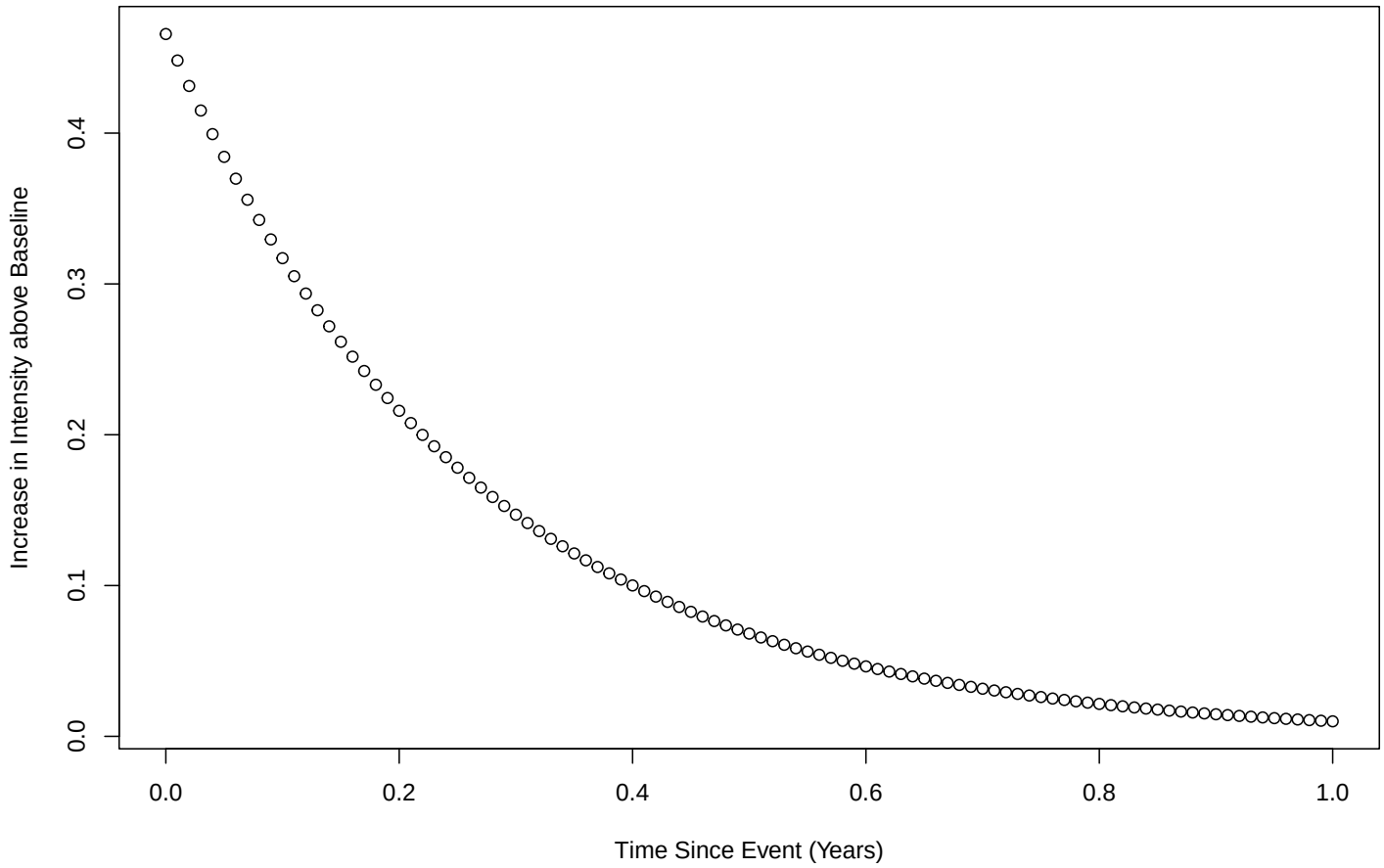
The Intensity Function over Time

- The intensity function gives the value of the rate at any time
- Here we simulate 100 years based on the fitted model.
  - The rate is at the baseline of 0.393 until a new event occurs
  - We see the jump in rate with each new event
  - The rate decays to baseline unless another event happens



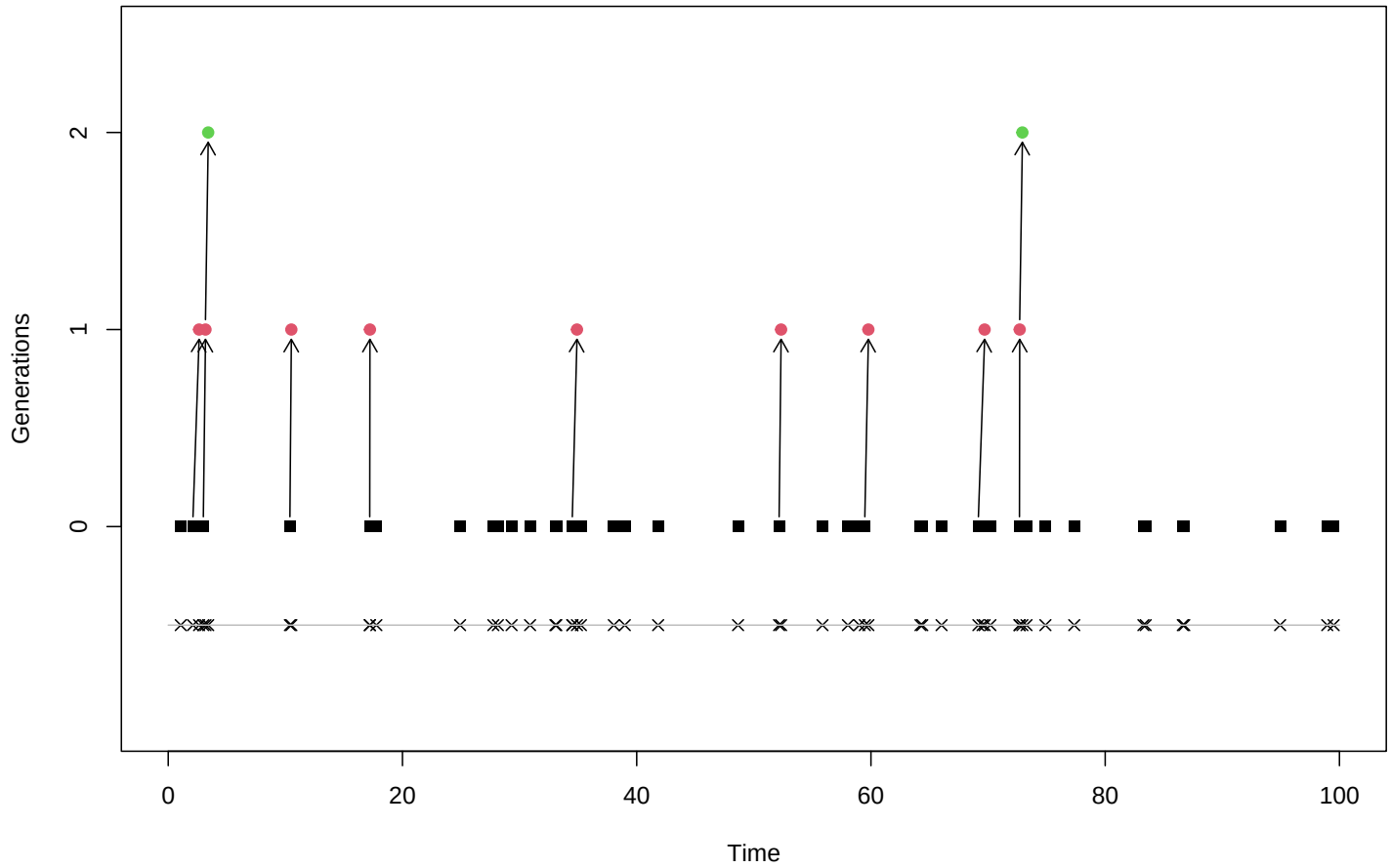
How Fast is the rate "decay"

- Within 1 year essentially return to baseline



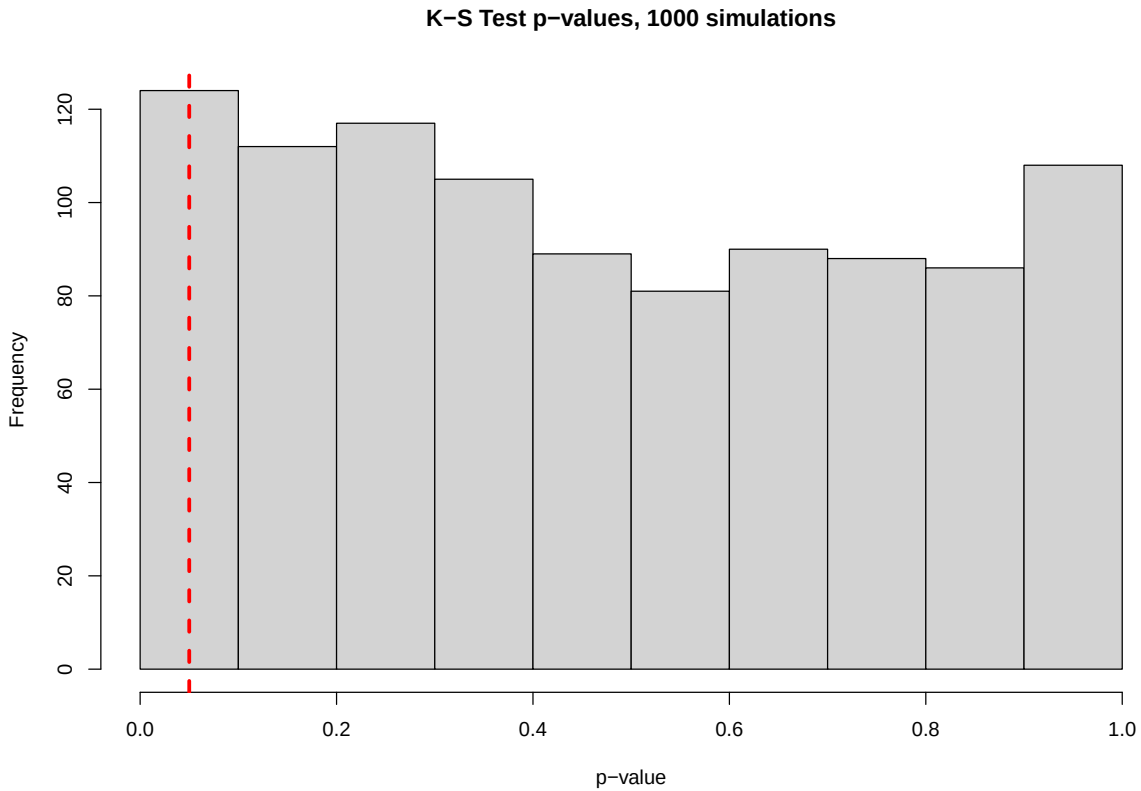
Process as "Generations"

- An event (1st generation) may spur immediate new event (2nd generation). . . etc
- Cannot truly identify in our data; below is simulated



Would a Poisson Process model detect "excited" process?

- Simulated Hawkes process using estimates from our data
  - 1000 runs of 100 years each
- Fit exponential (constant) rate interrarival model
  - Tested fit using K-S Test



- Rejected fit in only 7 percent of simulations.

Inference for parameter estimates

- Ogata (1978) asymptotic result for the parameters (θ):

$$M^{1/2}(\hat{\theta}_M - \theta_o) \xrightarrow{d} N(0, I(\theta_o)^{-1})$$

where M is the total time for the process and:

$$-M^{-1}H_M(\theta_o) \xrightarrow{p} I(\theta_o)$$

- We can estimate the Hessian, H_M, from ML estimation
  - Use the result to produce confidence intervals for the parameters.
  - Asymptotic result may not hold (Cavaliere et al, 2021)

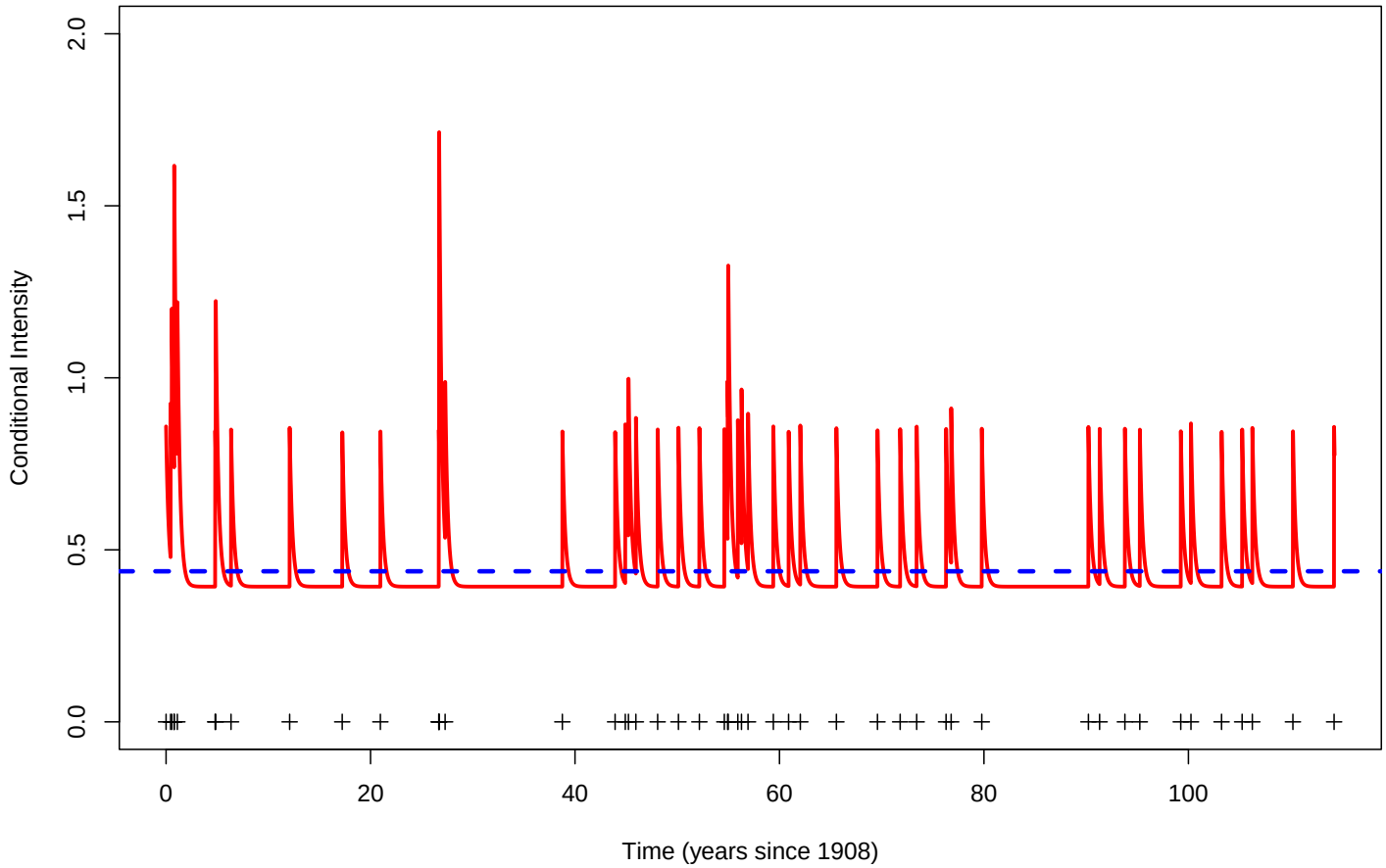
Estimated Ogata confidence intervals

95% confidence intervals based on Ogata for the three model parameters

- Baseline intensity estimate: 0.393
  - 95% CI: 0.253, 0.533
- Reproduction mean estimate: 0.121
  - 95% CI: -0.082, 0.324
- Exponential reproduction function rate estimate: 3.844
  - 95% CI: -7.555, 15.243

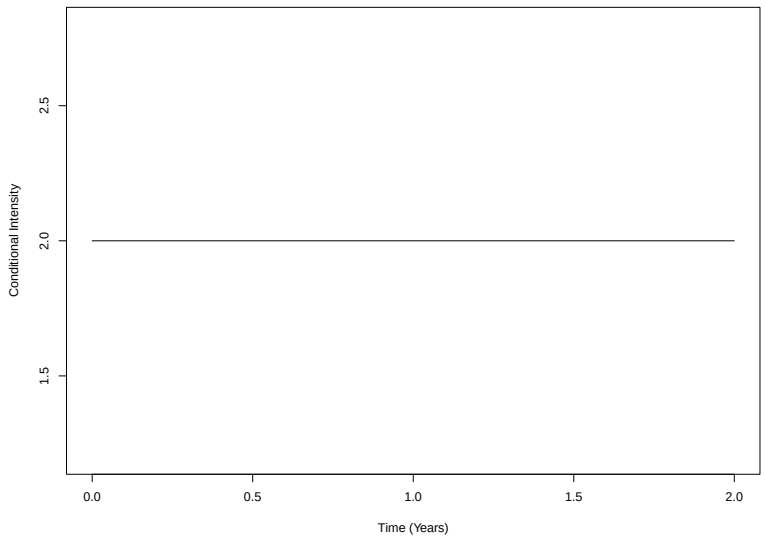
Intensity Functions for Both Models

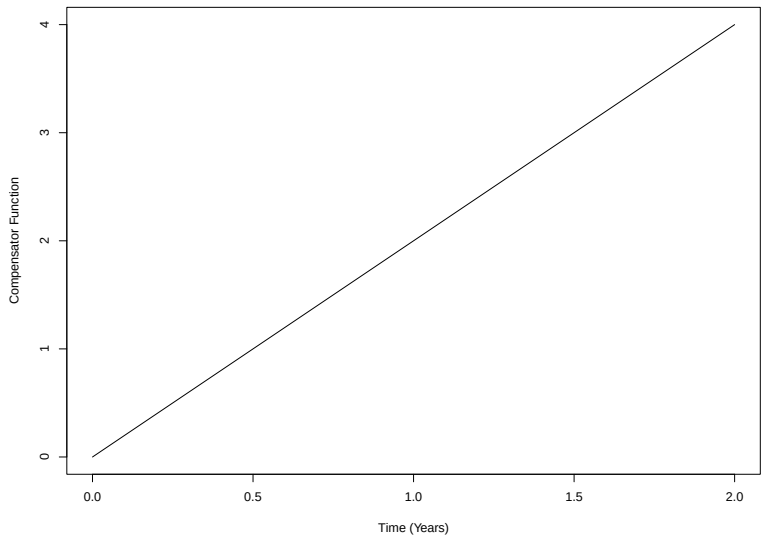
- The Hawkes intensity function is the model applied to the actual data
- The function for the Poisson model is a constant rate of 0.438



The Compensator Function

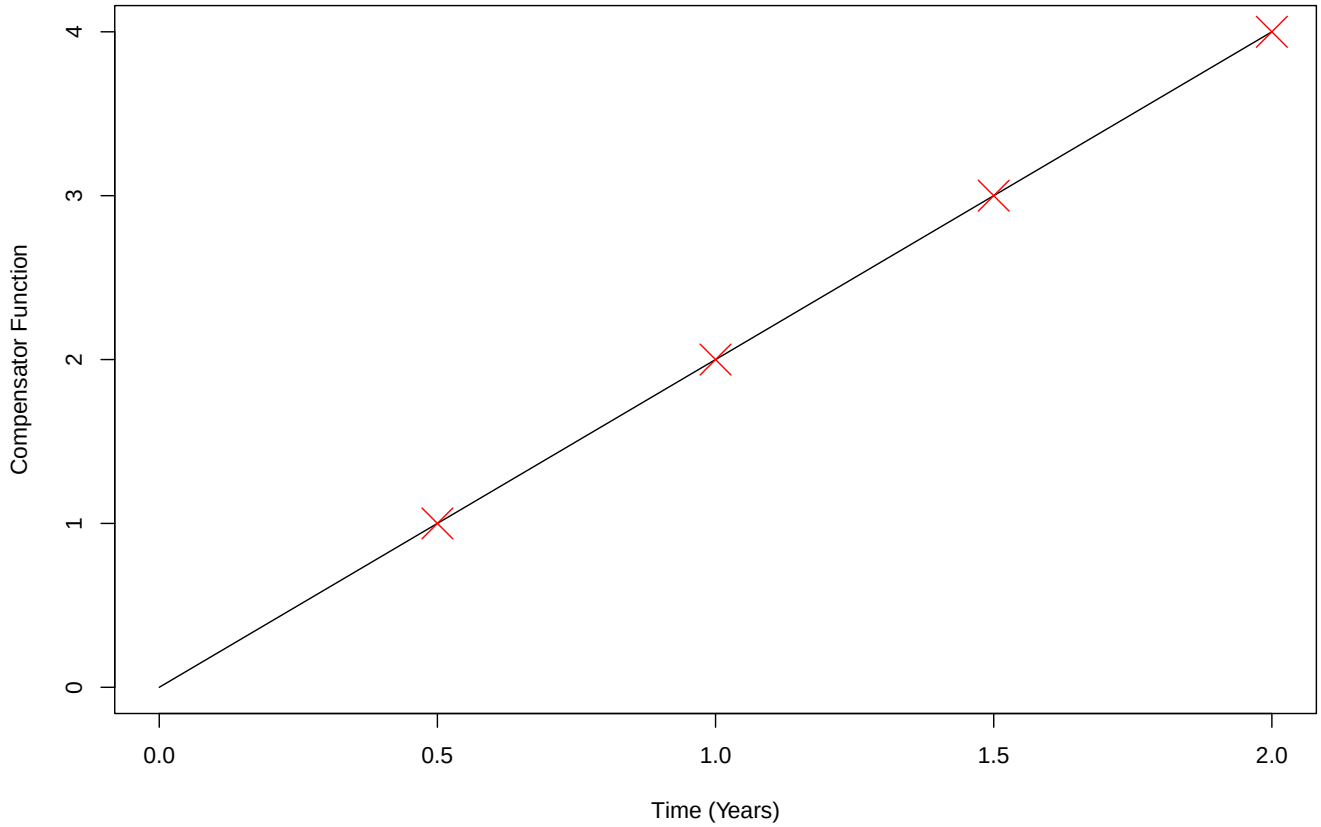
- Integrated intensity function. . . cumulative rate
- Simple example: constant intensity with rate 2 per year
- Compensator grows at constant rate (linear)





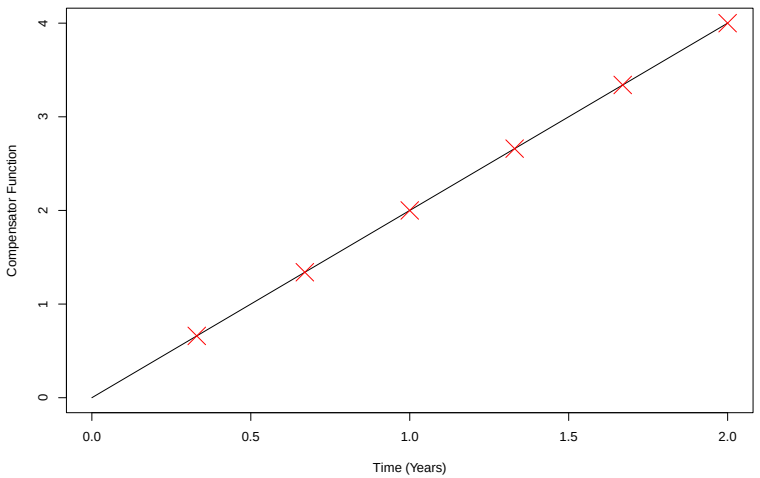
Arrivals Exactly on Schedule

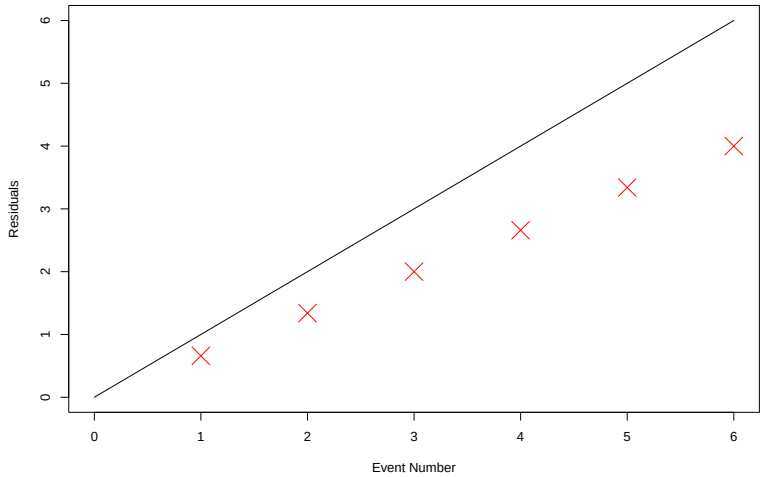
- Two arrivals per year spaced perfectly at 6 months
- Values of compensator for points 1, 2, 3, 4 are 1, 2, 3, 4
- Compensator values at arrivals are "residuals" (Paparoditis, 2000)
  - if "on schedule" Poisson process with rate 1



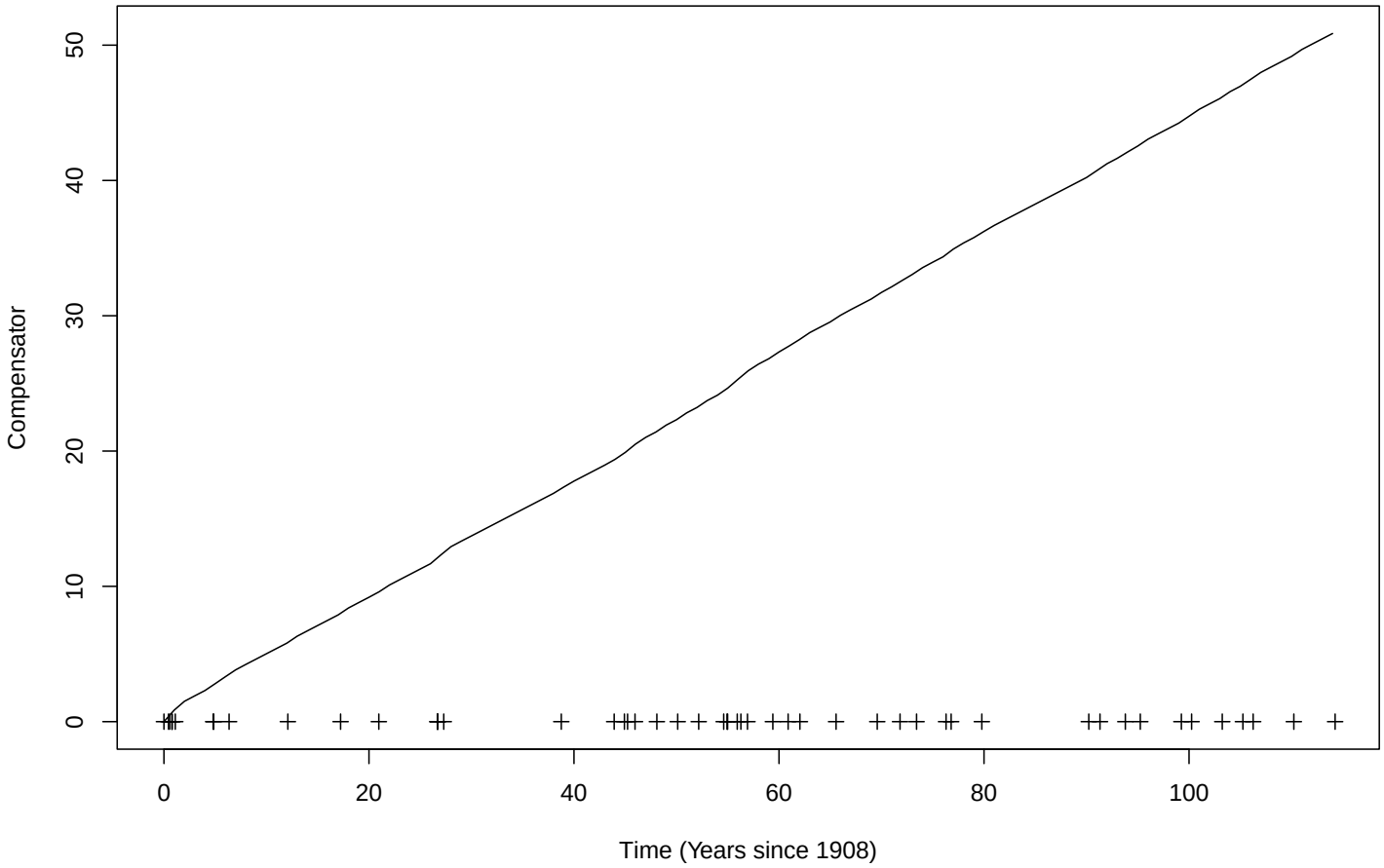
“Residuals” based on Compensator

- Suppose actual arrivals still constant but 3 per year (model is 2 per year)
- Residuals below the y = x (Poisson rate 1) line



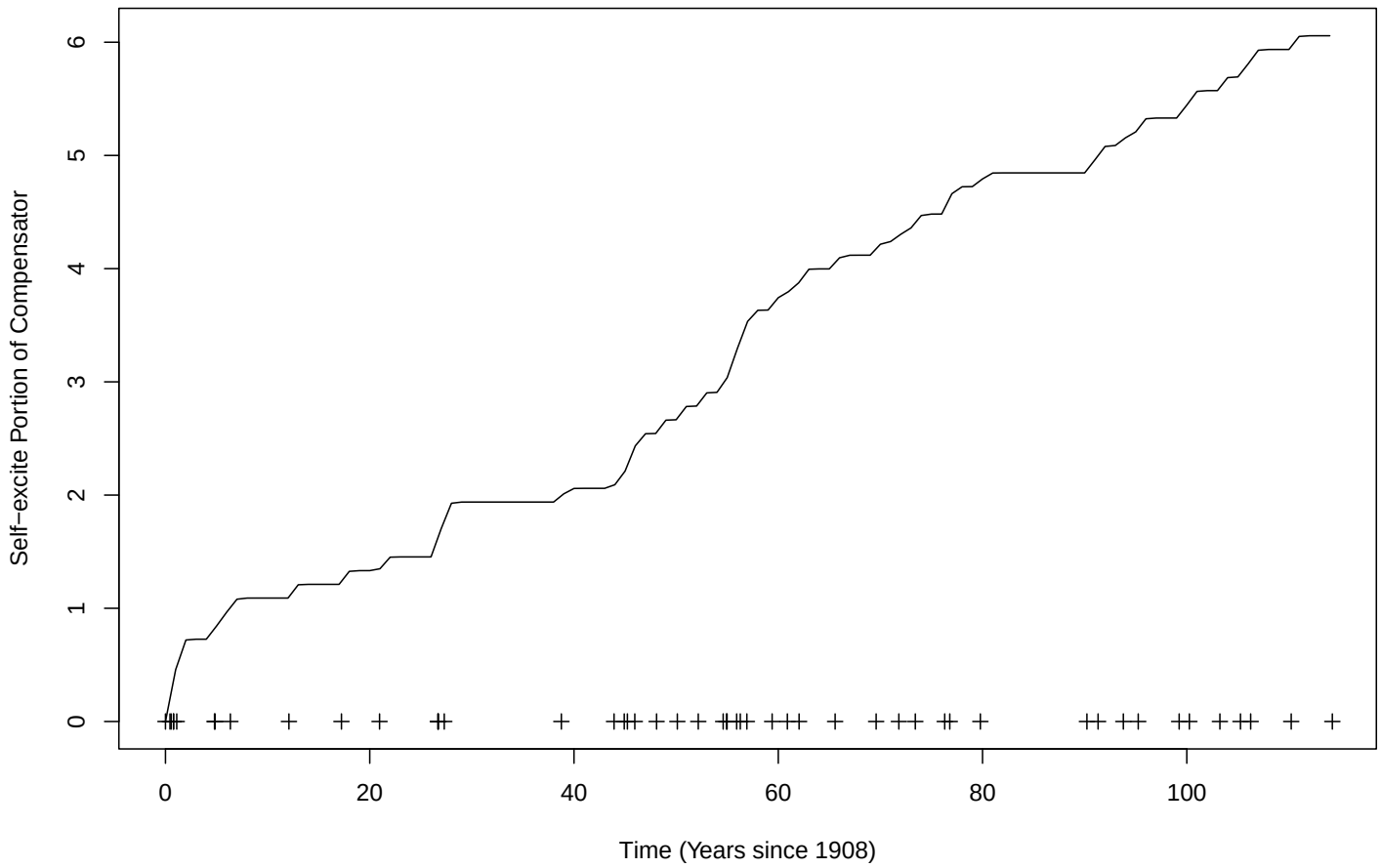


Compensator Function for Fitted Hawkes Model



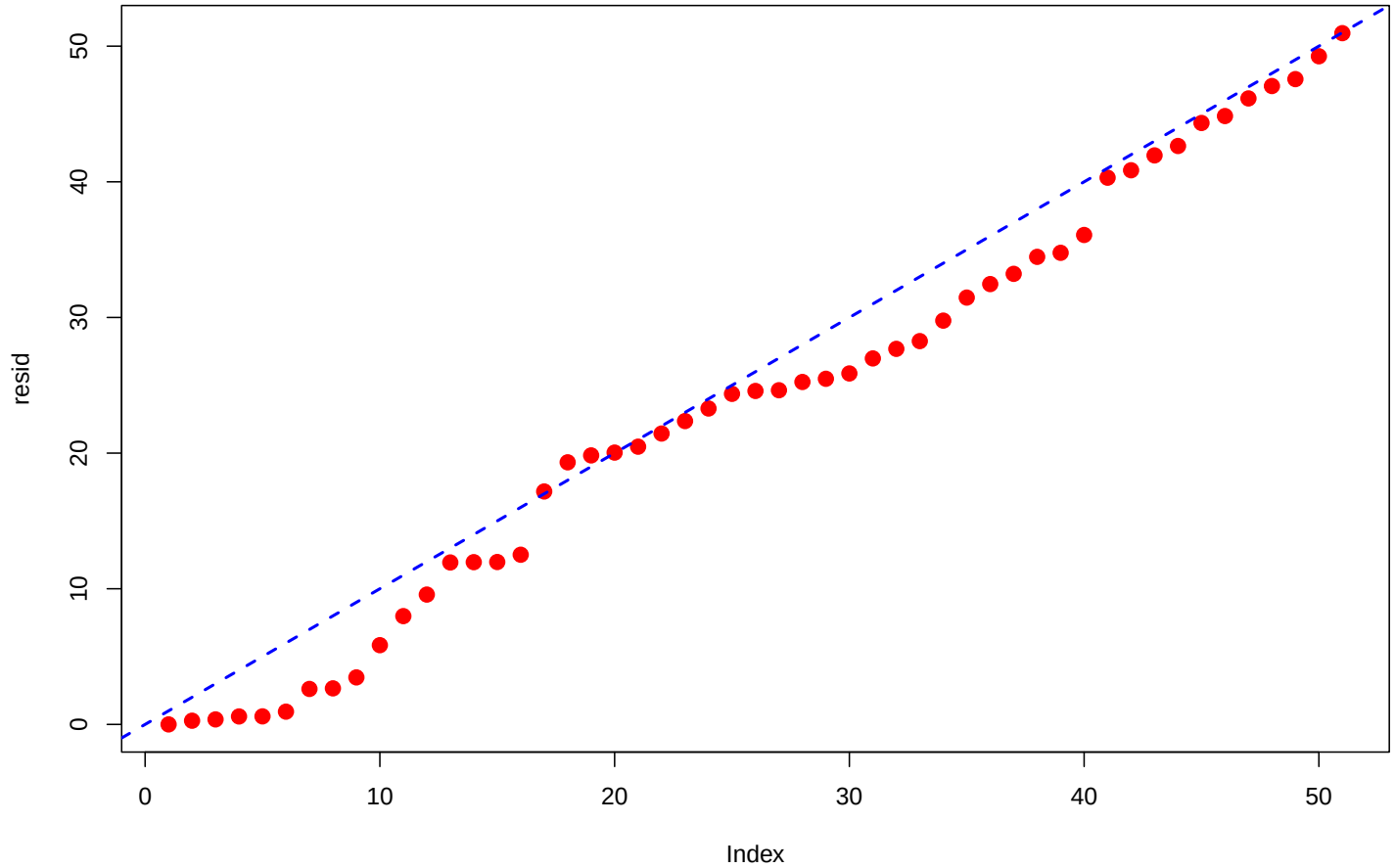
"Self Exciting" Portion of Compensator Function

- We remove the baseline (cumulative) rate to better see the Hawkes process

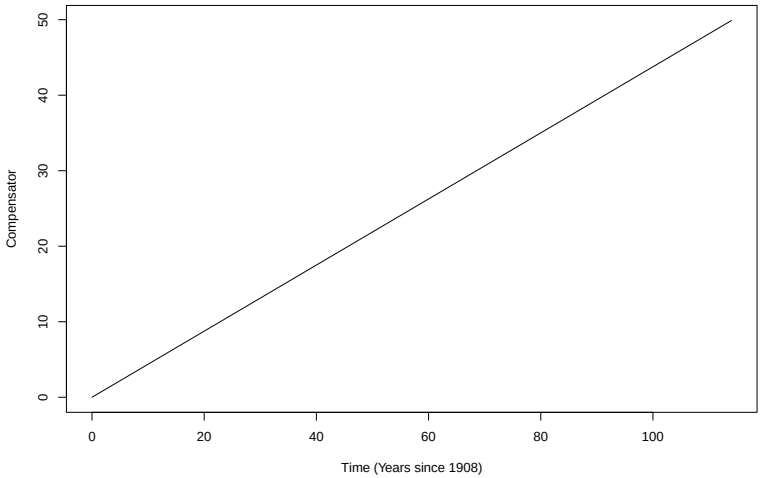


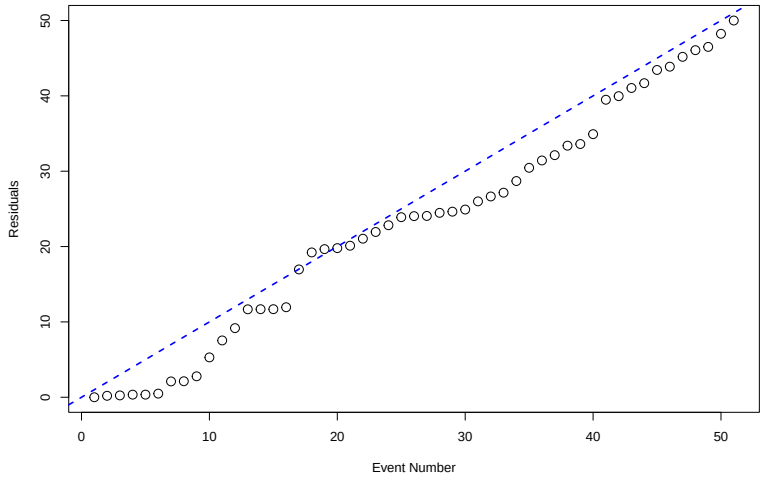
Residuals for Fitted Hawkes Model

- Arrivals at times "faster" than model
- Jump up event 17: 1947 record (after 12 year gap)
- Jump up event 41: 1998 record (after 10 year gap)



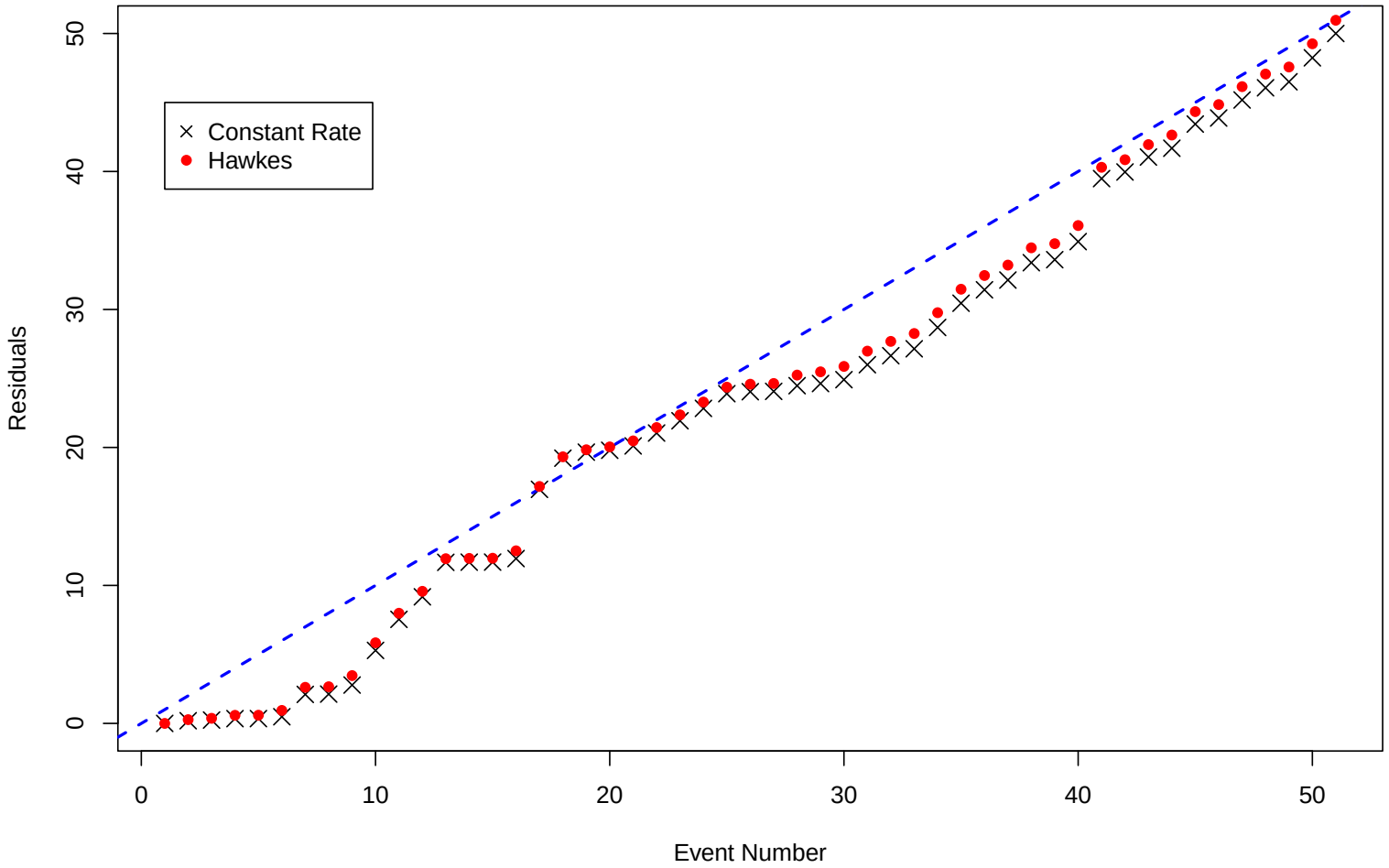
Constant Rate Model Compensator and Residuals





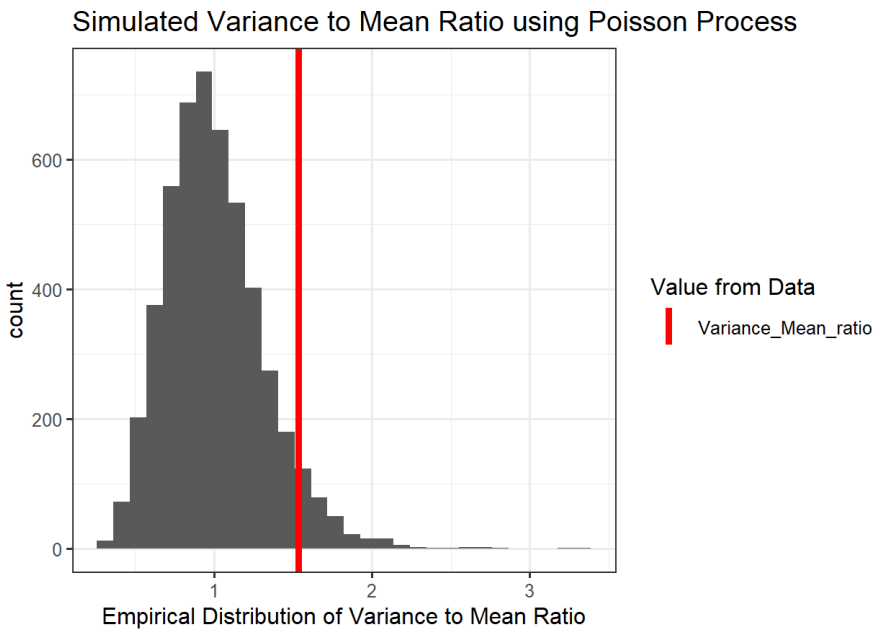
Comparing Model Residuals

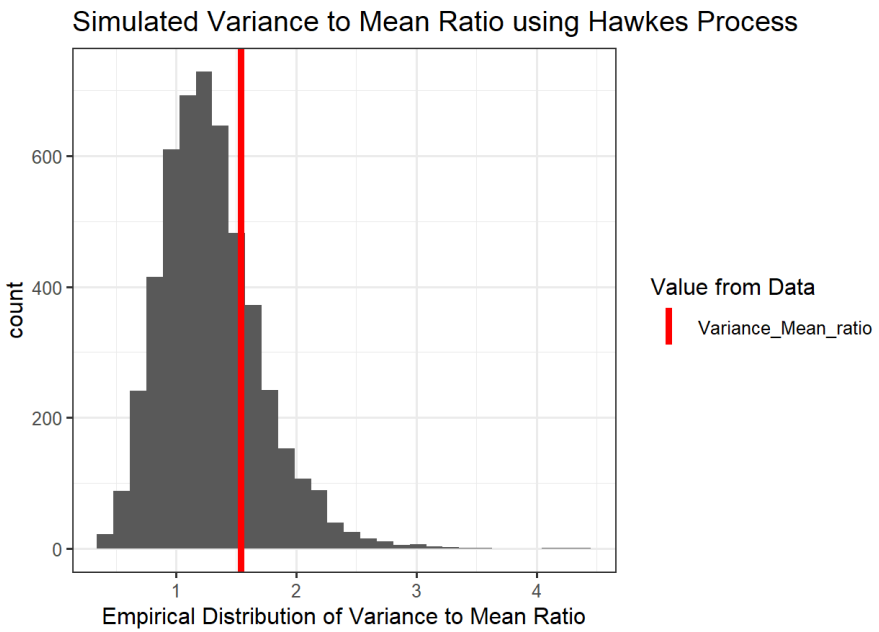
- Hawkes generally "better"
- Two large gaps clear impact on both models
- Hard to tell degree of improvement



Comparing Models - Overdispersion

- "Posterior predictive P values" (Gelman et al, 1996)
- Simulate from based on the fitted model parameters
- Here we use the "overdispersion" (measured by variance/mean ratio)





Conclusions

- Men's marathon records reasonably modeled using Poisson process
- Some indication a "self-exciting" process could explain arrivals better
- Other factors not in model
  - As times get faster harder to break the record?
  - Two "outlier" periods impact the models
  - New technology ("super shoes"), training methods etc.
  - Magnitude of event: how much faster than previous (inverse?)

Further Work

- Explore models for other events: women's marathon, other distances, swimming
- Develop better metrics to compare and assess models

SCORE

- NSF Grant, 2022 - 2025
- SCORE with Data: Building a sustainable national network for developing and disseminating **S**ports **C**ontent for **O**utreach, **R**esearch, and **E**ducation in data science
- Carnegie Mellon, Baylor, West Point, University of Pittsburgh, St. Lawrence University, Yale
- Partners across sports industry
  - NBA, NFL, MLB, NHL teams
  - ESPN
  - Other sports related organizations (analytics, etc)

SCORE Goals

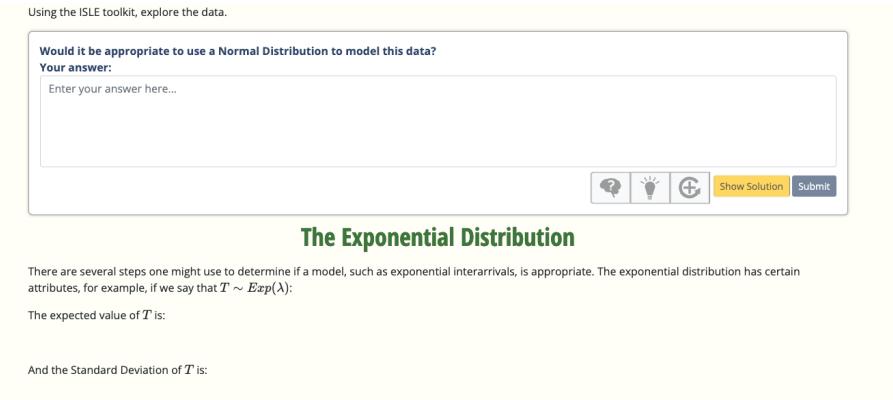
- Create a national network
  - academic, industry, media and government partners
  - elevating data science education
  - particularly in underrepresented populations and minorities
- Modules based on sports
  - Develop, implement, evaluate and disseminate an educational framework
  - Case-based Learning involving real-world problems and applications
- Educational research on data science education delivery modalities

SCORE Module

https://isle.stat.cmu.edu/SCORE/marathons/

Figure 7: Kate Sanborn, youngest qualifier US Olympic marathon trials, introduces module

Figure 8: Students explore the data interactively in ISLE



SCORE Participation

- Participate as part of network
- Use and test modules
- Create new modules
  - Example modules soon available
  - "HandbooK" with criteria in development
  - Peer-review similar to R package contribution

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Notes

- Data sources:

Wikipedia scraped August 12, 2022
(https://en.wikipedia.org/wiki/Marathon_world_record_progression)

- Poisson process:

https://towardsdatascience.com/the-poisson-distribution-and-poisson-process-explained-4e2cb17d459

- Hawkesbow, R Package for Hawkes process:

Felix Cheysson (2021). hawkesbow: Estimation of Hawkes Processes from Binned Observations. R package version 1.0.2.
https://CRAN.R-project.org/package=hawkesbow

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Gelman, A., Meng, X.L. and Stern, H.S. (1996). Posterior Predictive Assessment of Model Fitness via Realized Discrepancies. Statistica Sinica, 6, 733-807.

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