Is a sub 2 hour marathon in the near future? Modeling rare events in sports.

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Outline

- Baseball Rare Events (if needed only)
- Background
- Marathon Data
- Simple Model
- Self-Exciting Model
- Further Research
- SCORE

Background

The New York Times

Records Fell at the Track Worlds. A Trend? Not So Fast.

Some have referred to this era as a golden age of better and better times. But a deeper look at the data shows the simple shorthand conclusion is incomplete.

Golden Age?

Are we living in a time of records?

- Idea: seems like an increase in records falling, but is it just the nature of randomness?

How can we address this question?

What would randomness look like?

Rod Aloha 10K Run (San Diego, 2018), 2nd Age Group

Rod Last Marathon (LA, 2018), 1st, Glendora CA Runners

Marathon World Record Data

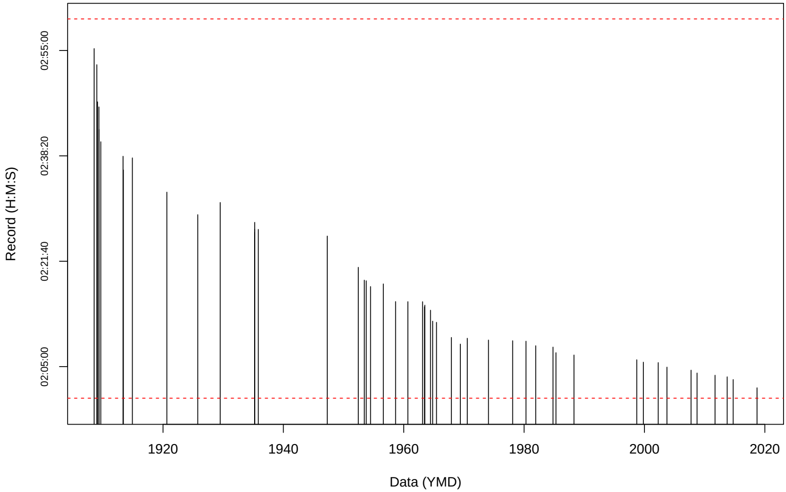
Men's Marathon world records since 1908

NEED TO CLEAN UP - NICER TABLE WITH JUST TIME NAME NATIONALITY DATE MAYBE INCLUDE A COUPLE OF PICTURES OF PEOPLE

Time	Name	Nationality	Date	Event/Place	Source	Notes	Time	Time_in_sec	Date_ymd	Date_end
2:55:18.4	Johnny Hayes	United States	July 24, 1908	London, United Kingdom	IAAF[53]	Time was officially recorded as 2:55:18 2/5.[54]Italian Dorando Pietri finished in 2:54:46.4, but was disqualified for receiving assistance from race officials near the finish.[55] Note.[56]	2H 55M 18.4S	10518.4	1908-07-24	1909-01-01
2:52:45.4	Robert Fowler	United States	January 1, 1909	Yonkers,[Nb 5]United States	IAAF[53]	Note.[56]	2H 52M 45.4S	10365.4	1909-01-01	1909-02-12

Visualizing the data

- Two and three hour times shown as horizontal lines



SIMPLE MODEL

POISSON PROCESS

A model for a series of discrete events where the average time between events is known, but the exact timing of events is "random" meeting the following criteria:

- Events are independent of each other. The occurrence of one event does not affect the probability another event will occur.
- The average rate (events per time period) is constant.
- Two events cannot occur at the same time.

Poisson Process Interarrival Times

The time between events (known as the interarrival times) follow an exponential distribution defined as:

$$P(T > t) = e^{-\lambda t}$$

- T is the random variable of the time until the next event
- t is a specific time for the next event
- λ is the rate: the average number of events per unit of time.

Note the possible values of T are greater than 0 (positive only).

Reasonableness of Exponential Interarrivals

The exponential distribution has certain attributes, for example:

$E(T) = 1/\lambda$ $SD(T) = 1/\lambda$

The mean and standard deviation of the years between records:

```
## [1] 2.249427
```

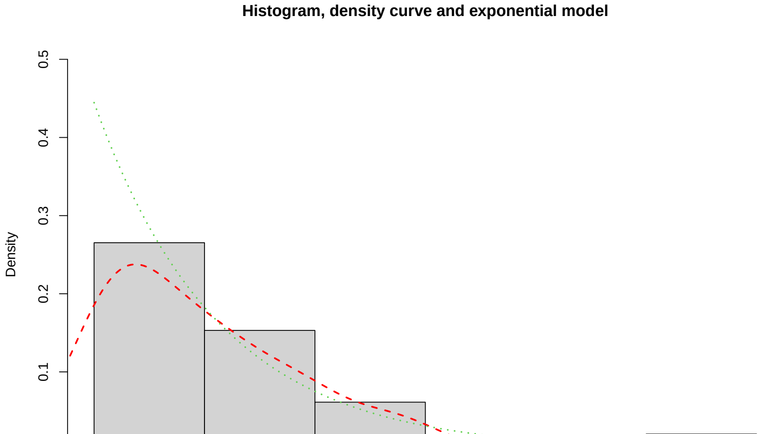
```
## [1] 2.428191
```

MORE ON THE SIMPLE MODEL

We estimate (MLE) $\lambda = 1/E(T)$

```
##      rate
## 0.4445577
```

Model fit



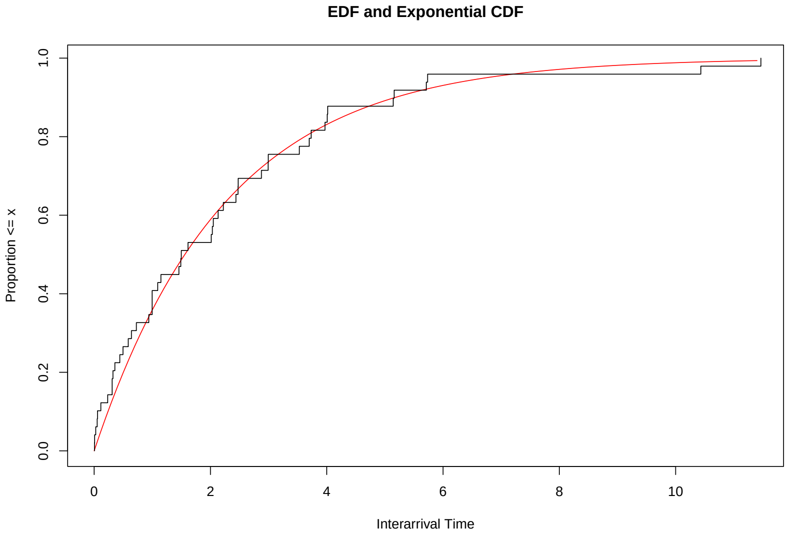
Fit of simple model

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  days_between_mod2
## D = 0.078053, p-value = 0.9264
## alternative hypothesis: two-sided
```

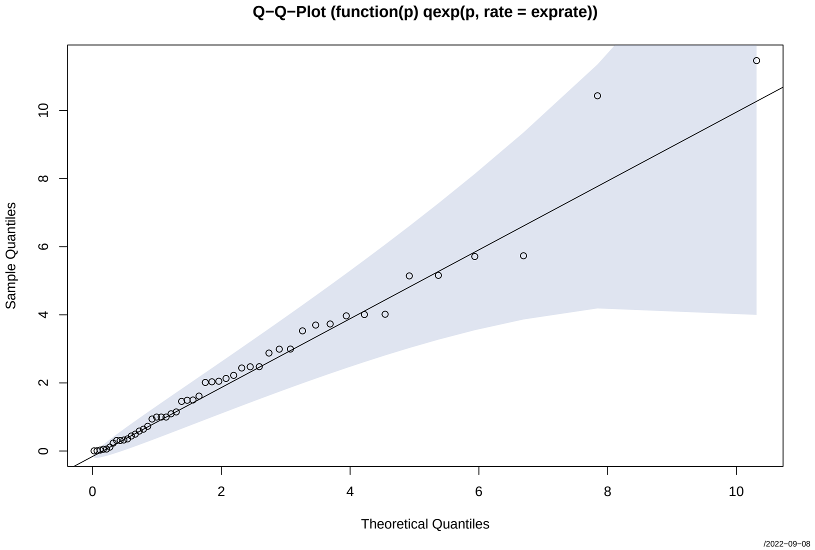
```
##
##  Cramer-von Mises test of goodness-of-fit
##  Braun's adjustment using 7 groups
##  Null hypothesis: exponential distribution
##  with parameter rate = 0.444557679401457
##  Parameters assumed to have been estimated from data
##
## data:  days_between_mod2
## omega2max = 0.43594, p-value = 0.3284
```

```
##
##  Anderson-Darling test of goodness-of-fit
```

Fit of simple model B



Are records then random?



What are the poorly fit points?

LOOK BACK AT THE ORIGINAL DATA HERE. . . LONGEST TIMES BETWEEN EVENTS (I THINK - NEED TO LOOK MORE CLOSELY). . . ONE IS WW2 PRETTY SURE. . . THE OTHER NEED TO LOOK AGAIN - MAYBE AN UNUSUALLY LARGE LOWERING OF THE RECORD OR SOMETHING?

A "Self-Exciting" Model

Hawkes Processes

- Let H_t be the history of events up to time t. The Hawkes (1971) model of the conditional intensity is:

$$\lambda(t|H_t) = \nu + \sum_{i:t_i<t} g(t - t_i)$$

where ν is the background rate of events and g is the "triggering function".

- The "triggering" function can be further decomposed:

$$g = \mu g^*$$

where g^* is a density function known as the "reproduction kernel" and μ is known as the "reproduction" mean.

- A common choice for the "reproduction kernel" is the exponential density given by:

$$g^*(t) = \beta e^{-\beta t}$$

Fitting the model

Parameter estimates for marathon data (exponential) Hawkes process, using MLE:

- baseline intensity 0.382
- reproduction mean 0.129
- exponential reproduction function rate 3.583

Note the baseline intensity is slightly lower than the constant model rate estimate of 0.445

The estimated reproduction function is then:

$$
\begin{aligned}
g(t) = \mu g^*(t) &= \mu\beta e^{-\beta t} \\
&= 0.13 * 3.58 e^{-3.58 t}
\end{aligned}
$$

Model implications

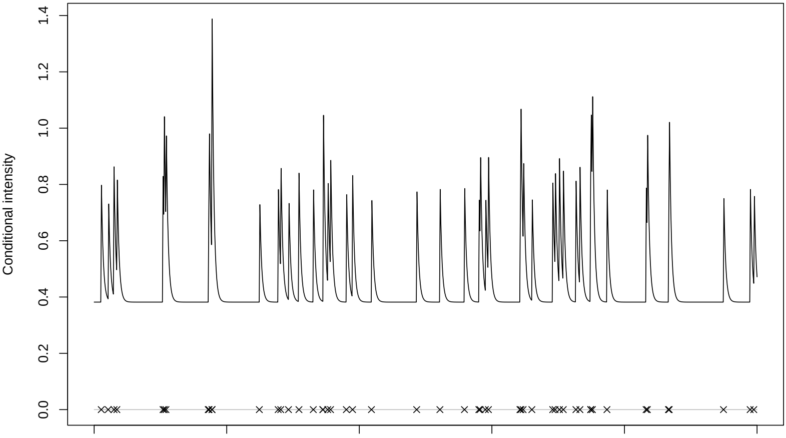
At the instant of the first event (world record), $t = t_1$ so $g(t - t_1 = 0)$ and the reproduction rate is:

$$g(0) = 0.13 * 3.58e^{-3.580} = 0.13 * 3.58 = 0.462$$

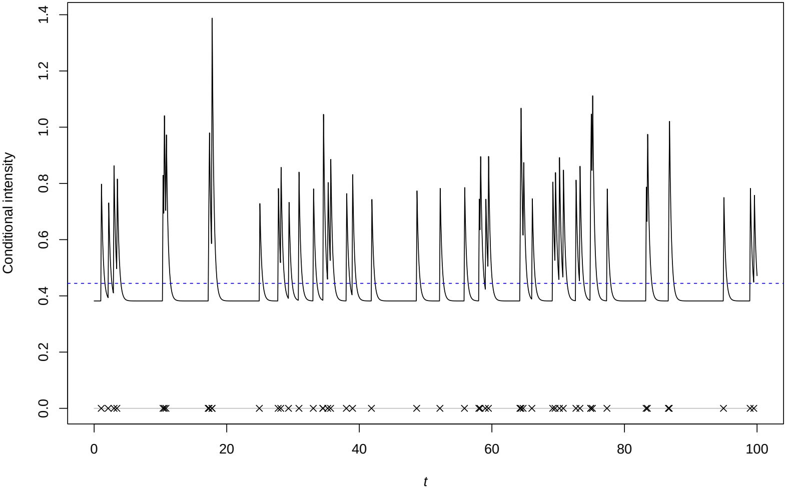
- The rate increases from the baseline rate of 0.382 by this amount at the moment of this occurrence
- The rate then decays back to baseline over time (unless a new event occurs).
- Each new event “excites” the rate to increase and then decay

The Intensity Function over Time

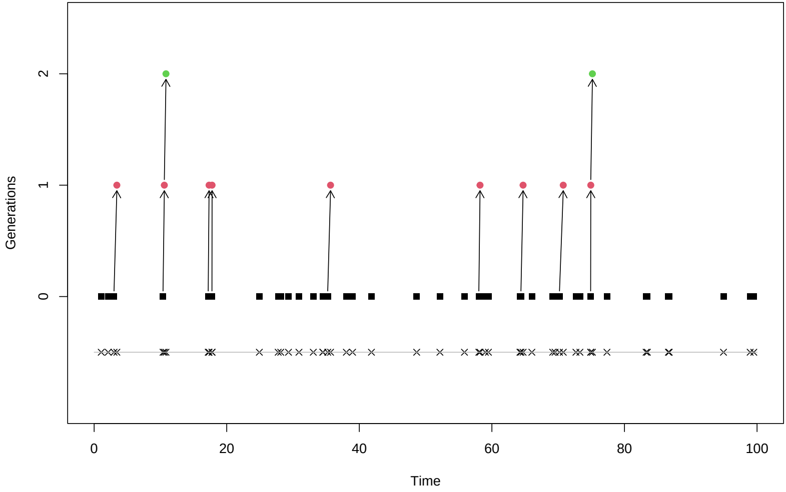
Below is based on a simulation of the intensity function over a 100 year period. NOTE: HERE WOULD BE NICE TO SHOW FOR OUR DATA ALTHOUGH MIGHT NOT GIVE THE FULL PICTURE ANYWAY



Intensity compared to the constant rate model

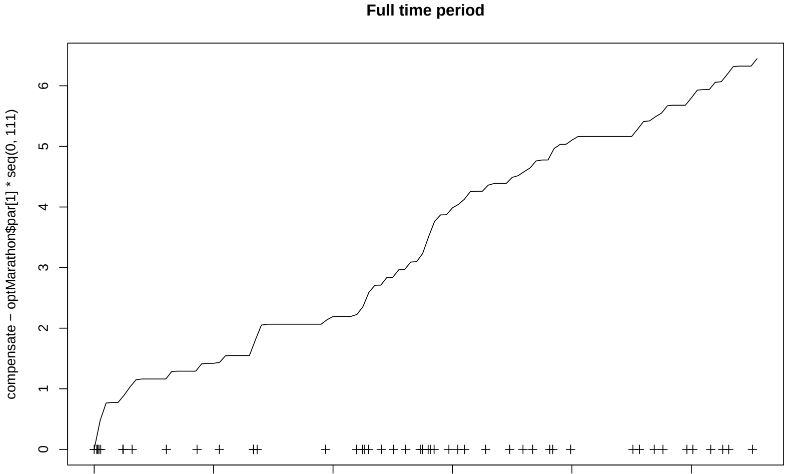


Process as “Generations”



The Compensator Function

NEED TO WORK ON EXPLAINING - BELOW IS THE VERSION TAKING OUT BASELINE. . . MAYBE START WITH CONSTANT (CAN DO Poisson MODEL AND THEN THE BASELINE RATE HERE)



Residuals

References

Data source: Wikipedia (https://en.wikipedia.org/wiki/Marathon_world_record_progression) scraped August 12, 2022

Poisson process: https://towardsdatascience.com/the-poisson-distribution-and-poisson-process-explained-4e2cb17d459

Hawkes, Alan G. 1971. “Spectra of Some Self-Exciting and Mutually Exciting Point Processes.” Biometrika 58 (1): 83–90. https://doi.org/10.2307/2334319.

“Hawkesbow” package…