HawkesModel

Fitting Hawkes model to marathon record data

Produce estimates for model, exponential Hawkes process, using MLE: - baseline intensity, - reproduction mean
- exponential fertility function rate

Note: originally data was in days from the first world record which is set as time 0. The model was fit through day 40,300 (the last record at day 40,231). Rescaling helps with visualization - several options, one put in years by dividing by 365.

Three parameter estimates are first output.

NOTE: the results change if one doesn't set a seed…rescaling seems to help variability in that.

```
record_table_mod<-read_rds("record_table_mod.rds")

days_between = as.numeric(diff(record_table_mod$Date_ymd))
daysfromstart <- cumsum(days_between)
daysfromstart <- c(0,daysfromstart)  ### get data in terms of days from first record (time 0)

### Rescale

  # max time is 400 (about 402 days per unit)
#daysfromstart_mod <- daysfromstart/max(daysfromstart)*100
#days_between_mod = diff(daysfromstart_mod)
  # units of year
daysfromstart_mod2 <- daysfromstart/365
days_between_mod2 = diff(daysfromstart_mod2)

set.seed(1234)
optMarathon<-mle(daysfromstart_mod2,"Exponential",114)  # end date picked number greater than longest t
optMarathon$par
```

```
## [1] 0.3819822 0.1290806 3.5829214
```

```
summary(optMarathon)
```

```
##        Length Class            Mode
## par     3     -none-           numeric
## model   1     Rcpp_Exponential S4
## events 50     -none-           numeric
## end     1     -none-           numeric
## opt    20     nloptr           list
```

```r
optMarathon$events
```

```
##  [1]   0.0000000   0.4410959   0.5561644   0.7890411   0.7945205   1.1041096
##  [7]   4.8027397   4.8547945   6.3534247  12.0876712  17.2301370  20.9616438
## [13]  26.6739726  26.7013699  26.7095890  27.2958904  38.7616438  43.9205479
## [19]  44.9178082  45.2273973  45.9534247  48.0849315  50.1178082  52.1671233
## [25]  54.6054795  54.9287671  54.9863014  55.9260274  56.2821918  56.9232877
## [31]  59.4000000  60.8904110  62.0383562  65.5671233  69.5835616  71.8054795
## [37]  73.4191781  76.2958904  76.7917808  79.7863014  90.2191781  91.3123288
## [43]  93.7863014  95.2438356  99.2520548 100.2493151 103.2410959 105.2547945
## [49] 106.2520548 110.2219178
```

```r
optMarathon$end
```

```
## [1] 114
```

```r
optMarathon$model$param
```

```
##           [,1]
## [1,] 0.3819822
## [2,] 0.1290806
## [3,] 3.5829214
```

```r
optMarathon$model$mean()  # expected value
```

```
## [1] 0.4385965
```

```r
optMarathon$model$dmean()  # Jacobian matrix of expected value
```

```
##           [,1]
## [1,] 1.1482119
## [2,] 0.5036017
## [3,] 0.0000000
```

```r
optMarathon$model$ddmean()  # Hessian matrix of expected value
```

```
##          [,1]     [,2] [,3]
## [1,] 0.000000 1.318391    0
## [2,] 1.318391 1.156483    0
## [3,] 0.000000 0.000000    0
```

```r
optMarathon$model$loglik(daysfromstart_mod2, optMarathon$end)  #  log-likelihood
```

```
## [1] -89.20395
```

```r
optMarathon$model$dloglik(daysfromstart_mod2, optMarathon$end)  # Jacobian matrix of log-lik
```

```
##                 [,1]
## [1,] -2.130966e-07
## [2,] -4.860773e-07
## [3,]  9.962603e-10
```

```
optMarathon$model$ddloglik(daysfromstart_mod2, optMarathon$end) # hessian matrix of log-lik
```

```
##              [,1]        [,2]        [,3]
## [1,] -270.2800222  -70.807871  0.66447926
## [2,]  -70.8078705 -177.816385 -1.96636666
## [3,]    0.6644793   -1.966367 -0.07411144
```

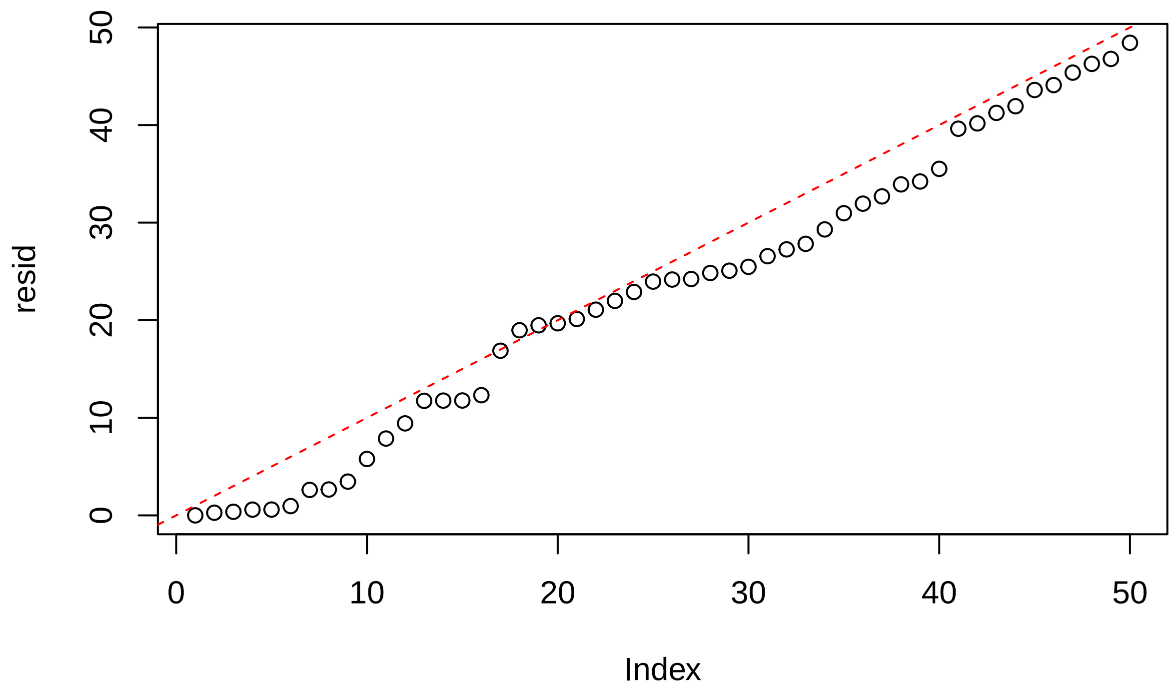
Residuals

From the help: "Outputs the residuals (values of the compensator at the times of arrival) of a Hawkes process. Useful function for diagnosis through the random time change theorem: the residuals should follow a unit rate Poisson process" Based on the example in the help I assume should follow the y=x line... we see divergence here suggesting an issue (in what direction?)

```
resid<-residuals(daysfromstart_mod2, fun = optMarathon$par[1], repr = optMarathon$par[2],
                 family = "exp", rate = optMarathon$par[3])
resid
```

```
##  [1]  0.0000000  0.2709950  0.3675397  0.5878563  0.5944182  0.9502425
##  [7]  2.6090444  2.6508913  3.4584403  5.7790008  7.8724117  9.4268612
## [13] 11.7379498 11.7604839 11.7707649 12.3177157 16.8715474 18.9712374
## [19] 19.4776306 19.6848246 20.1221807 21.0782119 21.9837949 22.8956825
## [25] 23.9562346 24.1682863 24.2218605 24.8387095 25.0744513 25.4701579
## [31] 26.5621402 27.2599325 27.8260052 29.3051333 30.9684221 31.9461908
## [37] 32.6913225 33.9196498 34.2163162 35.5110869 39.6253438 40.1694188
## [43] 41.2460640 41.9312188 43.5920642 44.0984574 45.3739652 46.2721508
## [49] 46.7786363 48.4277595
```

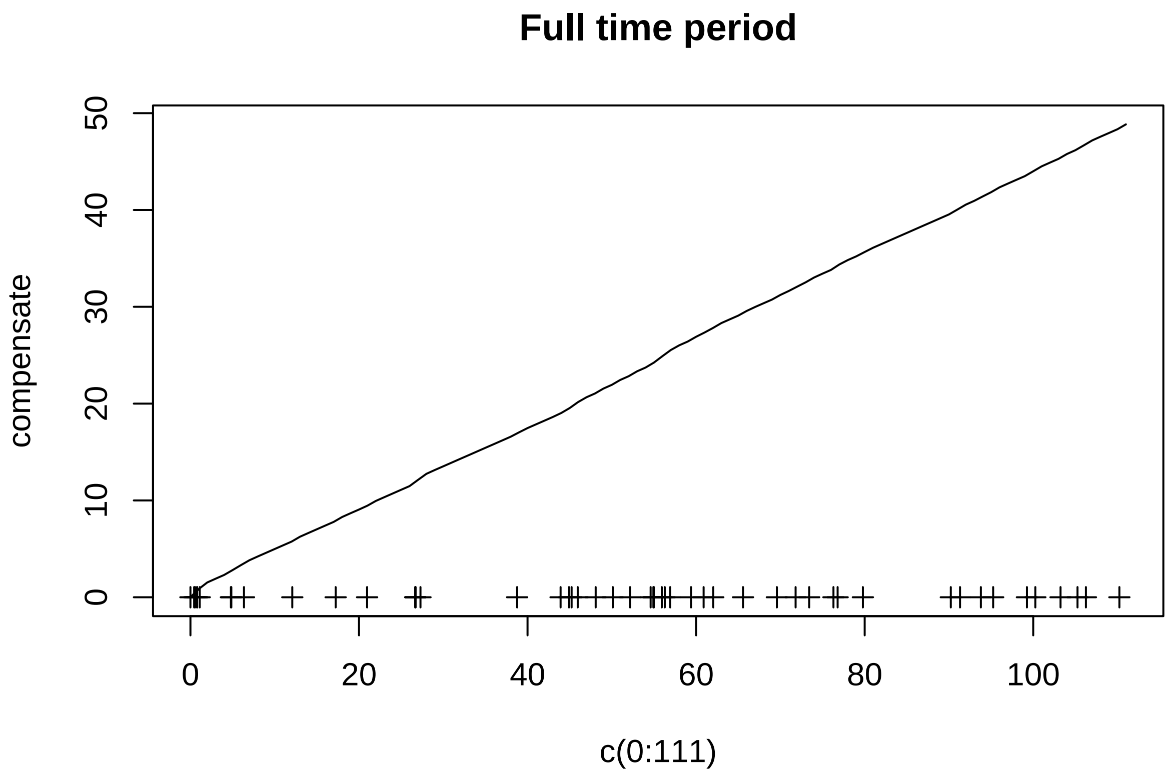
```
plot(resid)
abline(0, 1, col="red", lty="dashed")
```



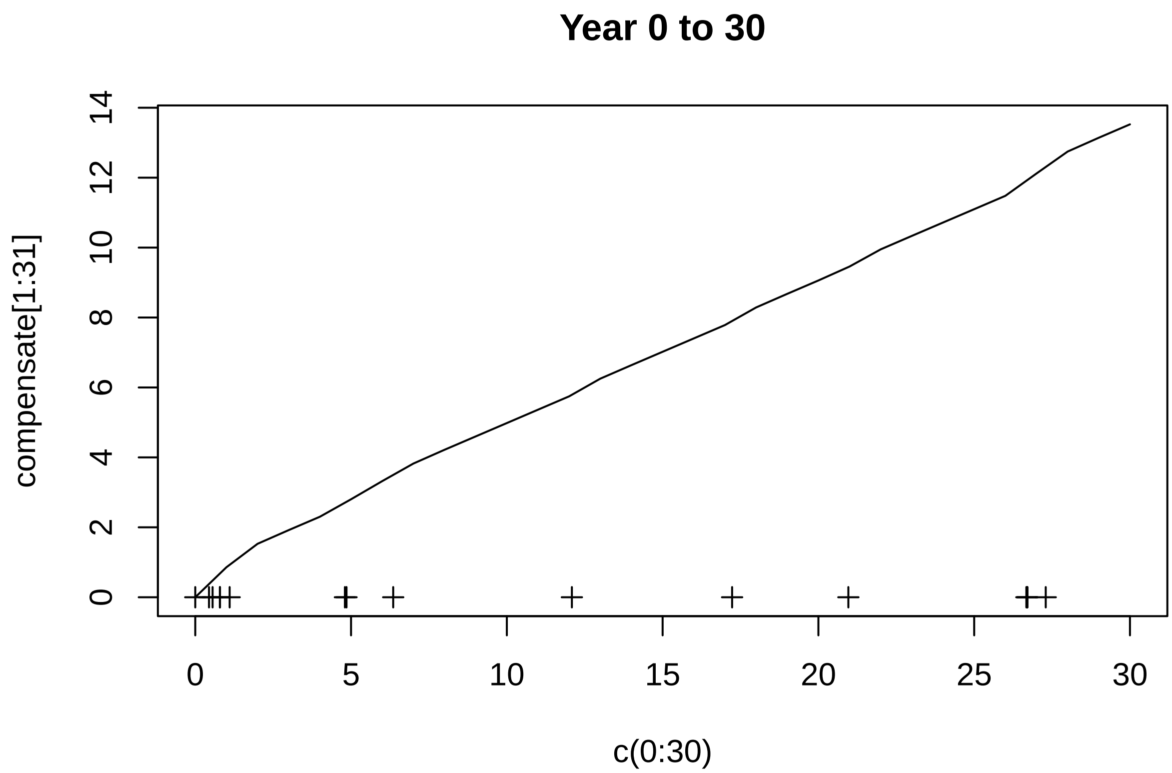
Compensator

From the help: the compensator (integrated intensity) of a Hawkes process. . . kind of cool, can see how the events (world records) up the intensity. Event times are the plus symbols plotted below the intensity function line.

```
compensate<-compensator(daysfromstart_mod2, t=0:111, fun = optMarathon$par[1],
                        repr = optMarathon$par[2], family = "exp", rate = optMarathon$par[3])
plot(c(0:111),compensate,main="Full time period", type = "l")
points(daysfromstart_mod2,rep(0,length(daysfromstart_mod2)), pch=3)
```




```
### zoom in...
plot(c(0:30),compensate[1:31],main="Year 0 to 30", type = "l" )
points(daysfromstart_mod2[0:16],rep(0,length(daysfromstart_mod2[0:16])), pch=3)
```

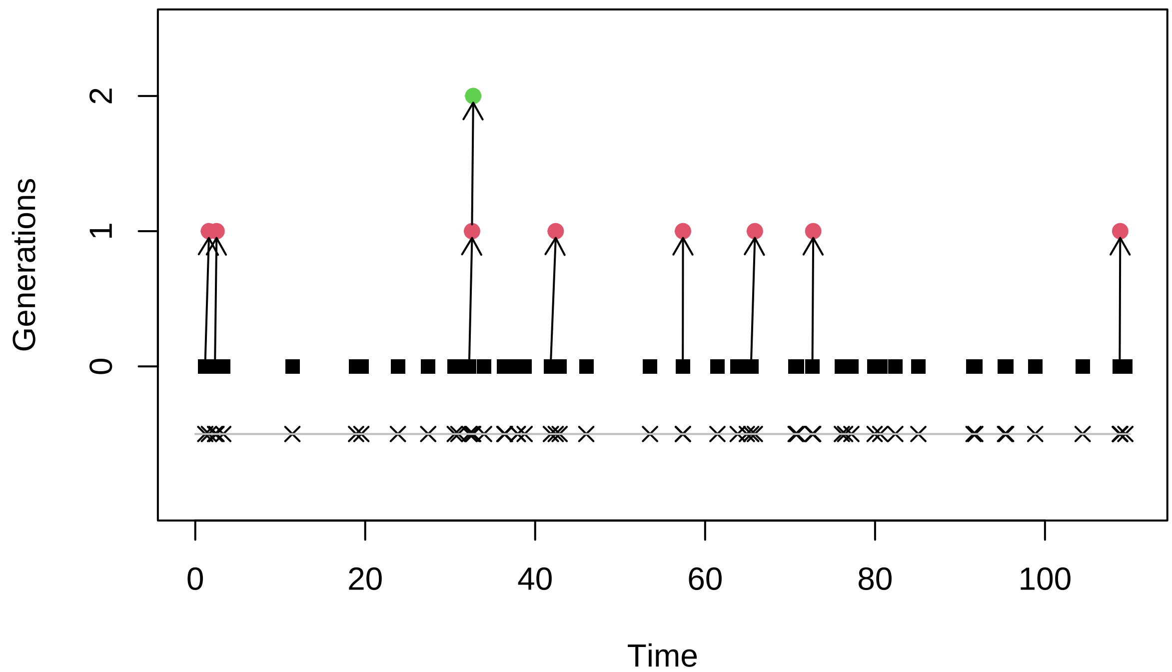


Simulation using Hawkes model with MLE parameter estimates from data

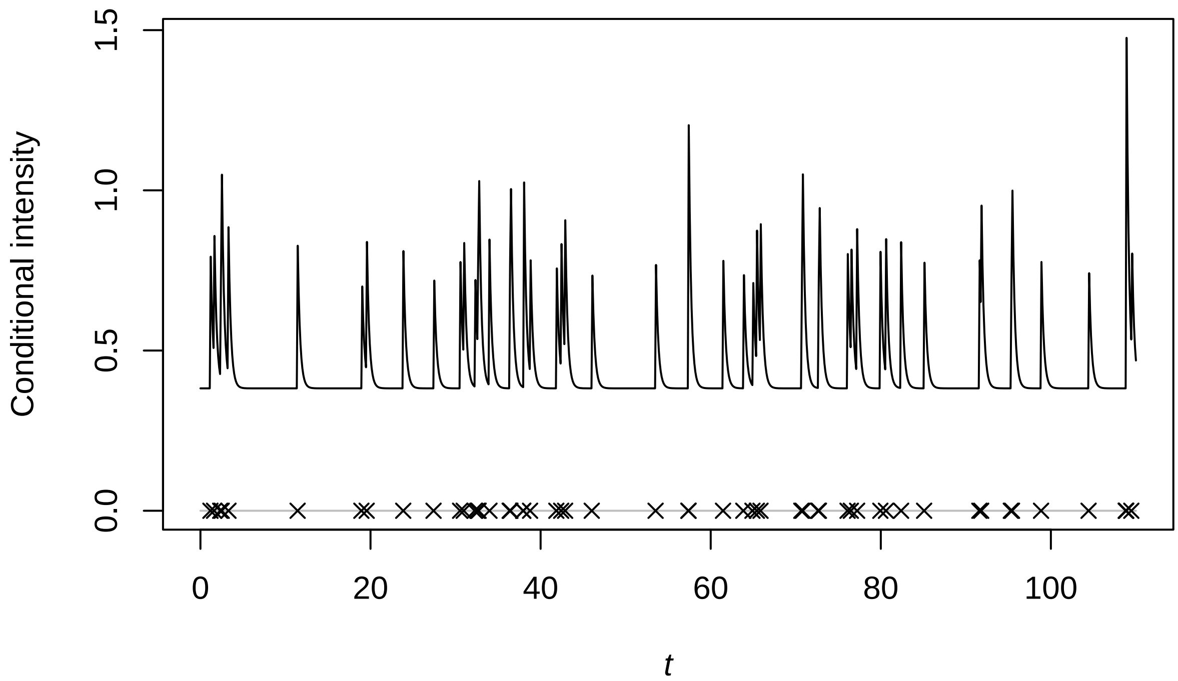
Simulation for the same number of years used in the estimation. Last plot is the residuals - we see since from the model we simulated generally seem to follow the y=x line.

Intensity plots with the rescaling are much easier to see. Different seeds different results, some have second "child".

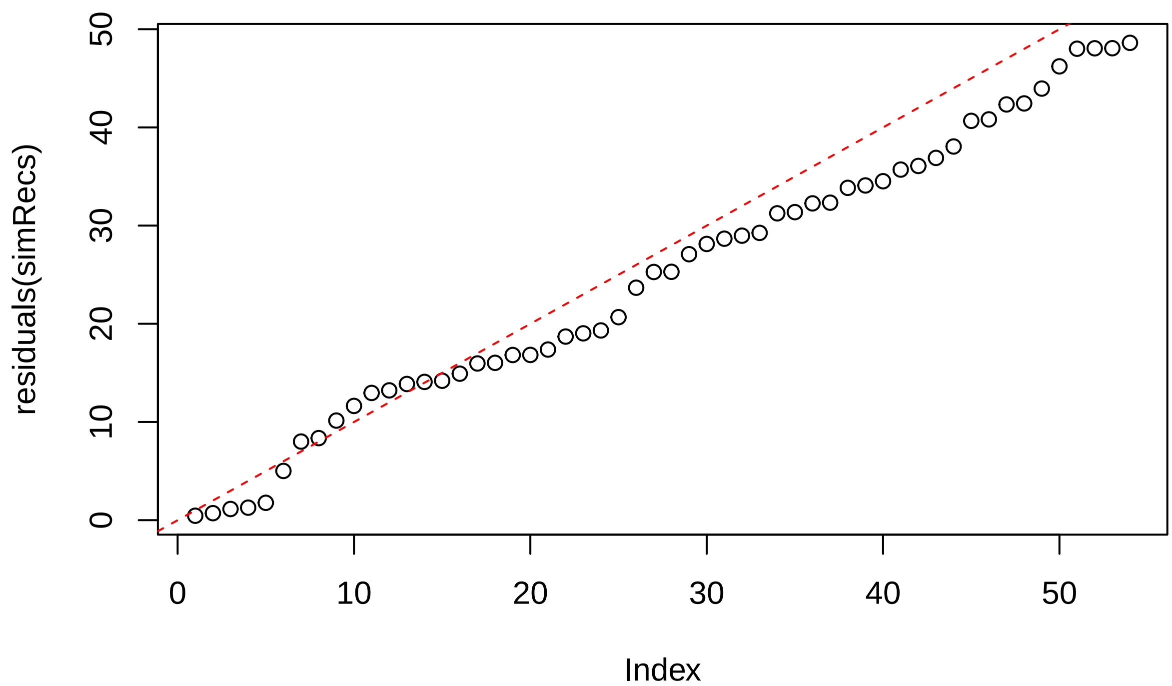
```
set.seed(777)
simRecs <- hawkes(110, fun = optMarathon$par[1], repr = optMarathon$par[2],
                  family = "exp", rate = optMarathon$par[3])
plot(simRecs, intensity = FALSE)
```




```
plot(simRecs, intensity = TRUE)
```




```r
plot(residuals(simRecs))
abline(0, 1, col="red", lty="dashed")
```



Comparing Hawkes model to constant rate model

The estimated constant rate is 0.44 using the year scaled data. The baseline parameter rate from the Hawkes model is lower; depends slightly on the seed but around 0.396. This makes sense - the conditional "excite" portion of the model will increase the rate at times so the baseline is set lower. The other two parameters determine how much the rate jumps…max will be at T1-t = 0 or at (in example for seed I used) around 0.47…so an immediate doubling of the rate. Decay is fairly rapid though (see intensity from simulation) as the parameter for the exponential decay is large (3.9).

```
expfit=fitdistr(days_between_mod2,"exponential")
exprate<-expfit$estimate
exprate
```

```
##      rate
## 0.4445577
```

```
optMarathon$par
```

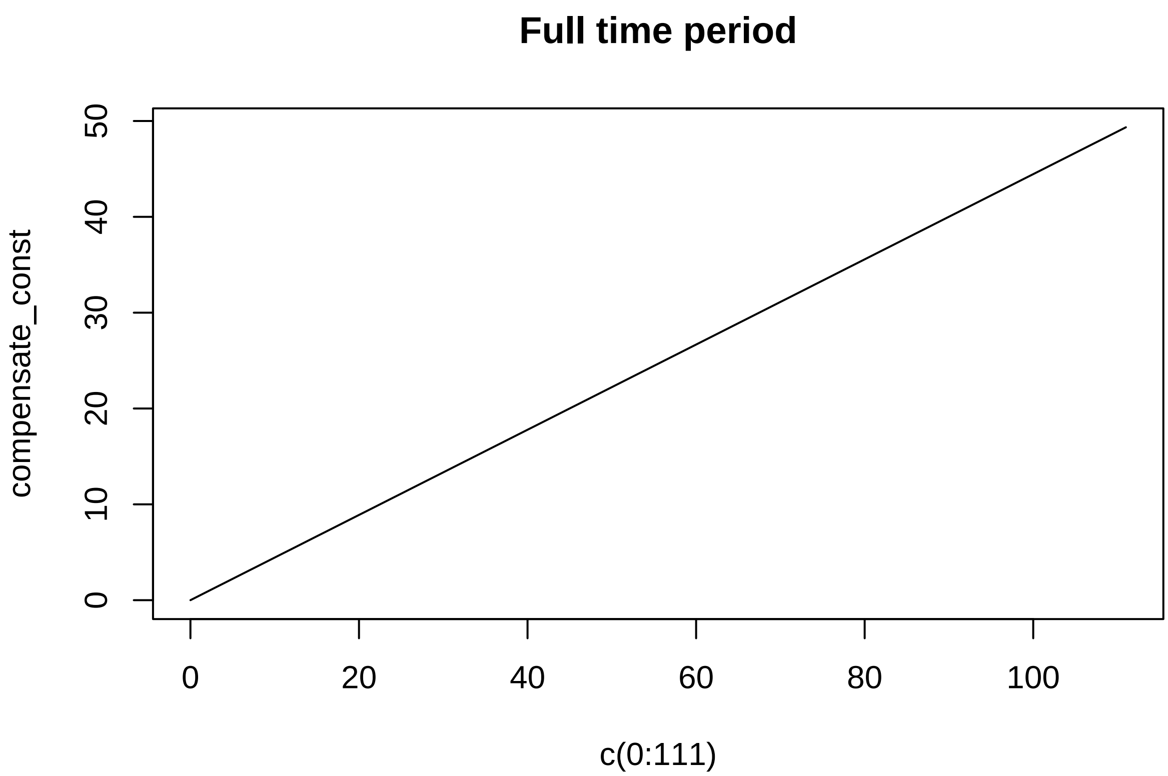
```
## [1] 0.3819822 0.1290806 3.5829214
```

```
optMarathon$par[2]*optMarathon$par[3]
```

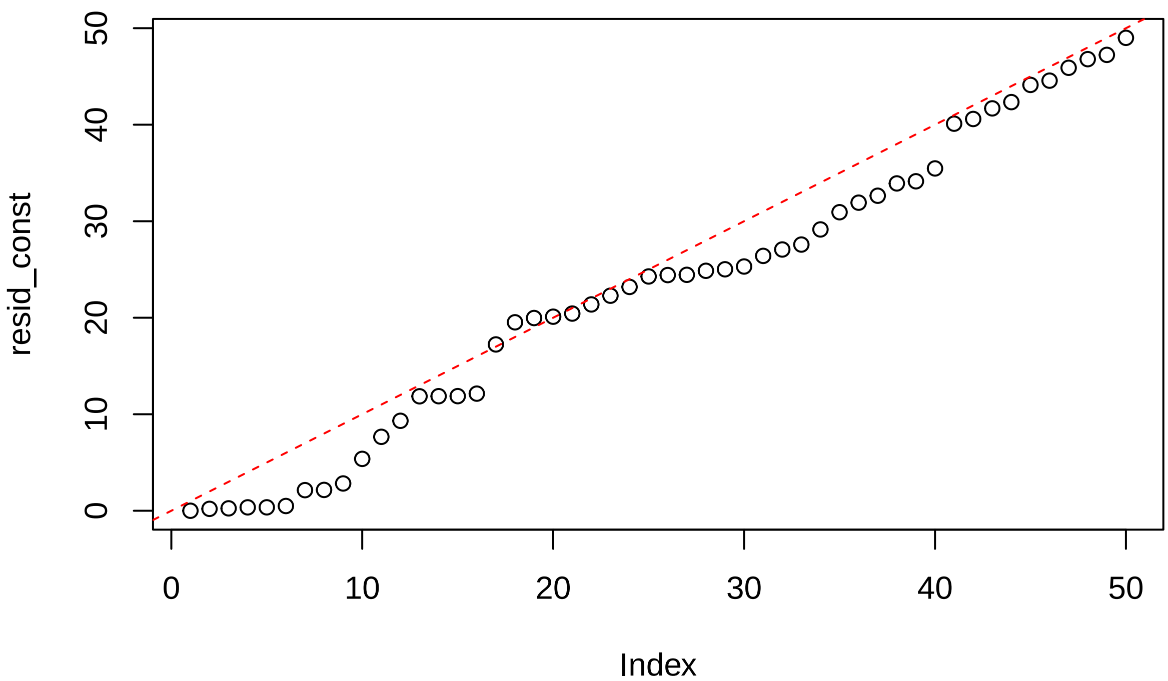
```
## [1] 0.4624858
```

CONSTANT MODEL compensator and residual plot are shown below. Note the features of Hawkesbow package make it easy to do this…simply set the baseline rate to the rate estimated for the Poisson process model and set the other two rates to 0.

```
compensate_const<-compensator(daysfromstart_mod2, t=0:111, fun = exprate, repr = 0,
                              family = "exp", rate = 0)
plot(c(0:111),compensate_const,main="Full time period", type = "l")
```

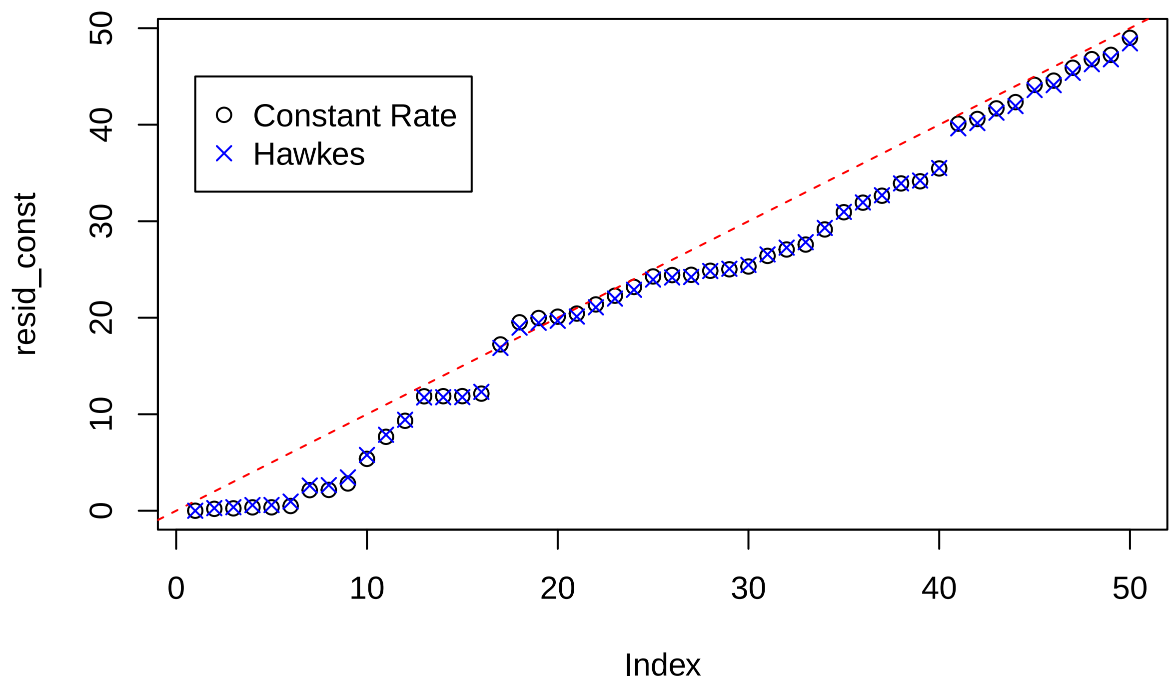



```
resid_const<-residuals(daysfromstart_mod2, fun = exprate, repr = 0,
                     family = "exp", rate = 0)
plot(resid_const)
abline(0, 1, col="red", lty="dashed")
```



Residual comparison is below. We see Hawkes model generally better but maybe not by a "significant" amount?

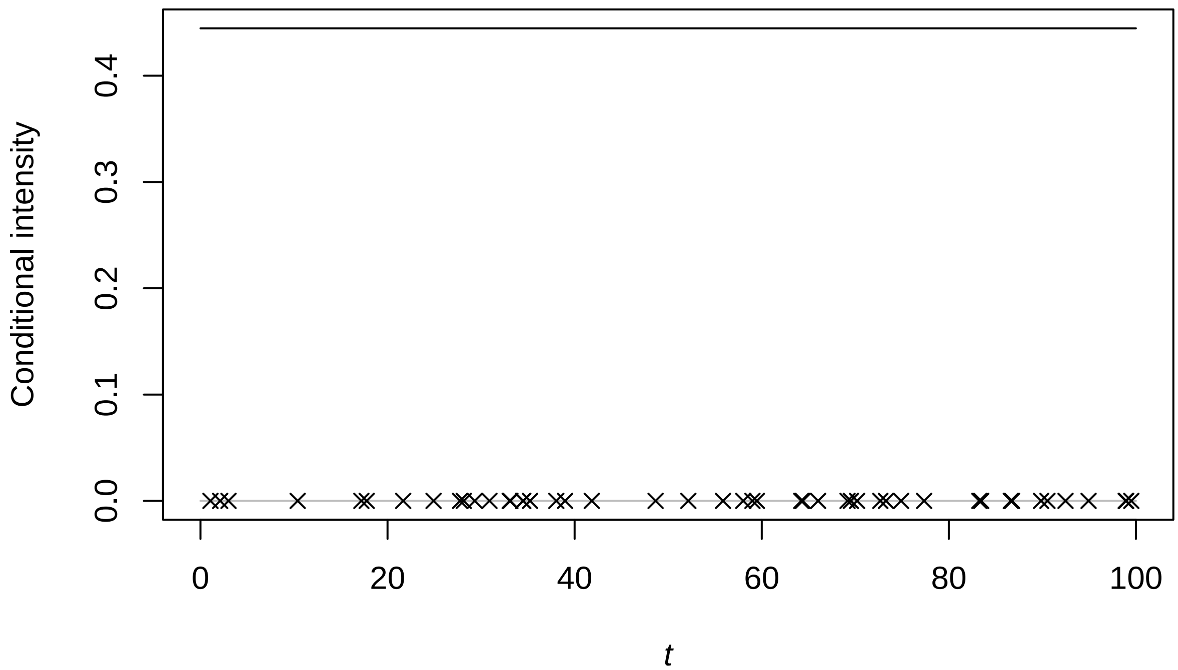
```
plot(resid_const)
abline(0, 1, col="red", lty="dashed")
points(resid, col ="blue", pch = 4)
legend(1, 45, legend=c("Constant Rate", "Hawkes"),
       col=c("black", "blue"), pch=c(1,4))
```



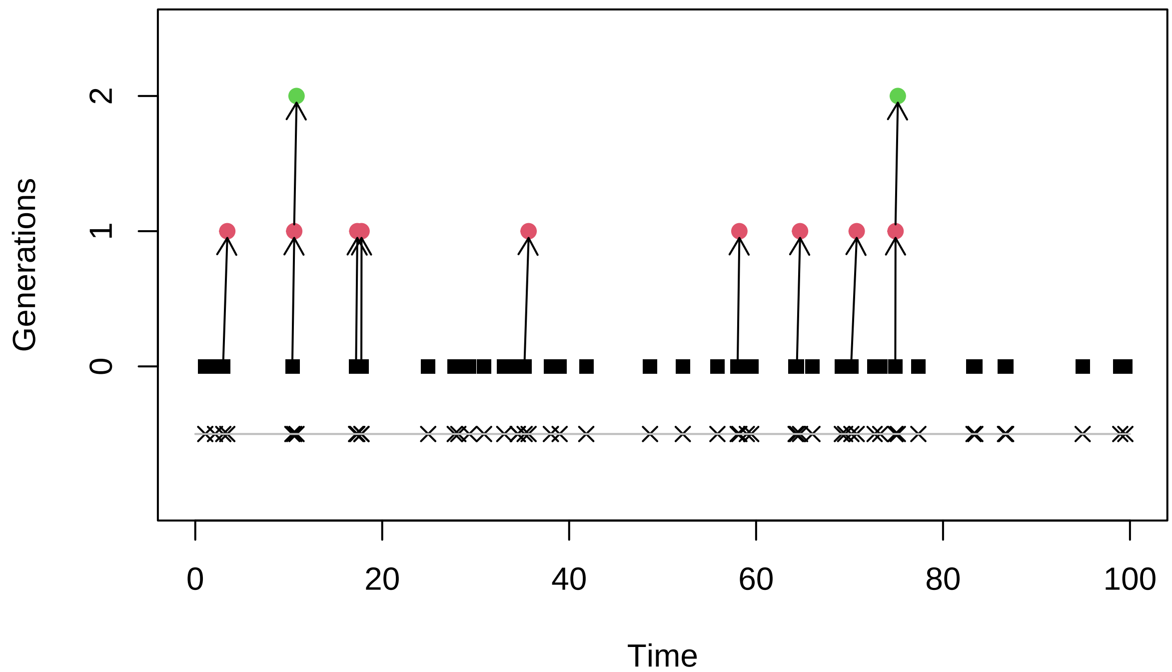
Simulations from both models

An example of simulations based on the two models - 100 years here. We can see the intensity roughly doubling in the Hawkes process.

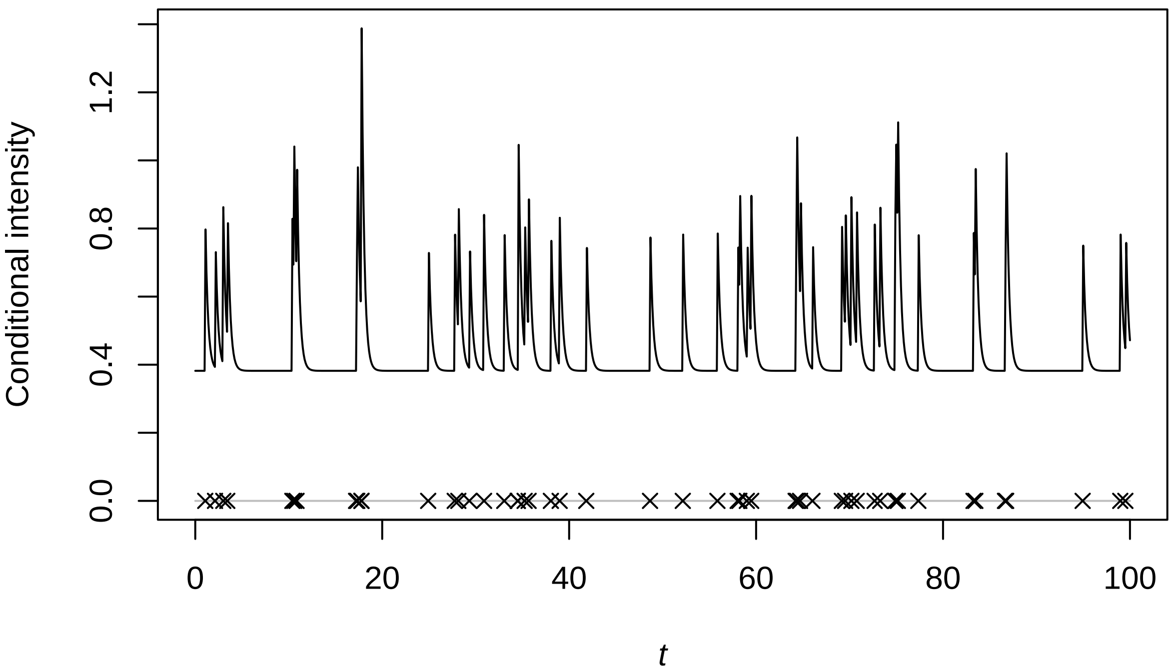
```
set.seed(777)
simRecs_const <- hawkes(100, fun = exprate, repr = 0, family = "exp", rate = 0)
#plot(simRecs_const, intensity = FALSE)
plot(simRecs_const, intensity = TRUE)
```




```
set.seed(777)
simRecs_Hawkes <- hawkes(100, fun = optMarathon$par[1], repr = optMarathon$par[2], family = "exp", rate
plot(simRecs_Hawkes, intensity = FALSE)
```




```
plot(simRecs_Hawkes, intensity = TRUE)
```


Conditional intensity
0.0
0.4
0.8
1.2
0
20
40
60
80
100
t