

Poisson:  $f(y|m) = \frac{m^y e^{-m}}{y!} = \exp \left[ \underbrace{y \cdot \log(m)}_{\theta} - m + \log\left(\frac{1}{y!}\right) \right]$

$b(\theta) = e^{\theta}$   
 $b'(\theta) = e^{\theta} = m$

$$\eta = \log(m)$$

Canonical link

$$\frac{d\eta}{dm} = \frac{1}{m}$$

$$\theta = b'^{-1}(m) = \log(m)$$

$$v(m) = b''(\theta) = e^{\theta} = m$$

$$\text{Weights} = \frac{1}{\left(\frac{d\eta}{dm}\right)^2 v(m)} = \frac{1}{\left(\frac{1}{m}\right)^2 \cdot m} = \frac{1}{\left(\frac{1}{m}\right)} = m$$

Adjusted Dependent variable

$$\frac{\eta}{\log(m)} + \frac{(y-m)}{(y-m) \left(\frac{1}{m}\right)}$$

$$= \log(m) + \frac{(y-m)}{m}$$