Poisson: $f(y|\mu) = \frac{\mu^y e^{-\mu}}{y!} = \exp\left[y \cdot \underbrace{\log(\mu)}_{\theta} - \mu + \log\left(\frac{1}{y!}\right) \right]$

$b(\theta) = e^\theta$

$\eta = \log(\mu)$ ⟷ Canonical link ⟷ $b'(\theta) = e^\theta = \mu$

$\theta = b'^{(-1)}(\mu) = \log(\mu)$

$\frac{d\eta}{d\mu} = \frac{1}{\mu}$

$V(\mu) = b''(\theta) = e^\theta = \mu$

$\text{Weights} = \frac{1}{\left(\frac{d\eta}{d\mu}\right)^2 V(\mu)} = \frac{1}{\left(\frac{1}{\mu}\right)^2 \cdot \mu} = \frac{1}{\left(\frac{1}{\mu}\right)} = \mu$

Adjusted Dependent variable

$\eta + (y - \mu)$

$\log(\mu) + (y - \mu)\left(\frac{1}{\mu}\right)$

$= \log(\mu) + \frac{(y-\mu)}{\mu}$